Comparative Genomic Analyses

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Bayesian inference

Bayesian inference: way to reason about probabilities.

- nothing more than a probability analysis
- a mathematical formalization of a decision process
- constitutes a different interpretation of probability

Thomas Bayes (1702-1761)



Bayesian inference

Bayesian approach to probability has some unique aspects.

- prior beliefs
- ▶ information from the data
- ▶ the idea of updated probability

Bayesian inference

The Bayes' theorem or Bayes' rule is the fundamental formula of Bayesian inference.

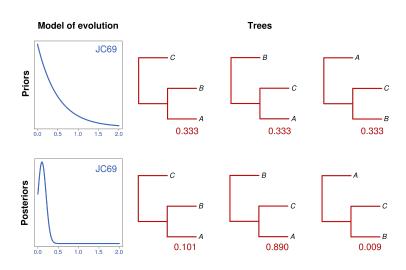
$$p(\theta|D) \propto p(\theta)p(D|\theta)$$

- \triangleright $p(\theta)$: prior distribution
- $ightharpoonup p(D|\theta)$: likelihood
- $p(\theta|D)$: **posterior distribution**, which specifies the probability after the prior has been updated with the available data

The Bayes' theorem translates quite straightforwardly to tree inference problems.

$$p(\tau, \theta|D) \propto p(\tau, \theta)p(D|\tau, \theta)$$

- ▶ *D* is the molecular sequence alignment
- ightharpoonup au and heta represent the tree and the model of evolution parameters



Estimating the posterior distribution in a phylogenetic context can be difficult.

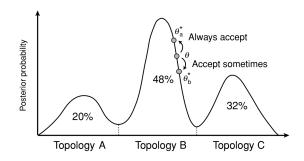
- impossible to derive the posterior probability distribution analytically
- concentrated in a small part of a vast parameter space

Estimate the posterior distribution using Markov chain Monte Carlo (MCMC) sampling

- set up a Markov chain that converges onto the posterior probability distribution
- MCMC represents random samples from the posterior

Metropolis-Hastings algorithm is an MCMC method

- make small random changes on the parameter values
- accept or reject those changes according to the appropriate probabilities



An MCMC run is a random sample of the posterior distribution.

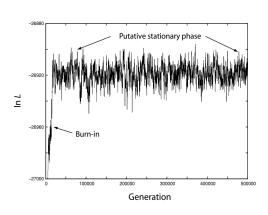
- ▶ the amount of time it spends sampling a particular region is proportional to the posterior probability of that region given that it converged to the target distribution
- convergence needs to be monitored

Burn-in:

- early phase of the run
- heavily influenced by the starting points
- likelihood increases very rapidly

Stationary phase:

- the chain approaches its stationary distribution
- the likelihood values reach a plateau



Summarizing the posterior distribution

The model parameters and the tree are summarized differently:

- model parameters are usually continuous and can be summarized as any usual statistical quantity: means, median, the credibility interval
- trees are more difficult to summarize
- posterior clade probabilities: the sum of the posterior probabilities of all trees that contain that clade

Summarizing the posterior distribution

Exercise

Bayesian phylogenetic inference in a sequence alignment with five species returned the posterior probabilities (P.p.) of 3 topologies.

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Topology P.p. (((Human,Dog),(Chicken,Lizard)),Frog) 0.76 ((((Human,Dog),Chicken),Lizard),Frog) 0.17 (((Human,Dog),Chicken),(Lizard,Frog)) 0.07
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What is the posterior probability of the following clades: (Chicken,Frog), (Chicken lizard), ((Human,Dog),Chicken) and (Human,Dog)?

Bayesian versus maximum likelihood trees

Maximum-Likelihood trees

- $ightharpoonup p(D|\tau,\theta)$
- Maximum likelihood tree
- ignores pre-existing information
- bootstrapping
- resample characters

Bayesian trees

- $ightharpoonup p(\tau,\theta|D)$
- Maximum a-posteriori tree
- considers pre-existing information
- MCMC
- resample parameters

Literature

The Phylogenetic Handbook by Lemey, Salemi and Vandamme (2009) Cambridge University Press

► Chapter 7: sections 7.1, 7.2 and 7.3, 7.4, 7.6, 7.7 and 7.9