7) One form of stochastic system is how an event percolates from one side to the other of a medium that contains inherent random properties. The route to cross is governed by a level of randomness p at each spatial point and so the ability to cross from one side to the other occurs at a critical probability level p_c. These systems can correspond to the spread of a fire in a forest, an electrical discharge through an insulator or the way water seeps down through mixture of sand and rock. They all exhibit an equivalent stochastic percolation property that occurs at p_c.Your task here is to evaluate the algorithm for the seeping of water through rocks with the python code RockPercolation.py (repeating this analysis to create an ensemble). The algorithm to evaluate the percolation property is: a drop of liquid starts in the middle of the top layer (row 1). It then moves according to the following four options, where options with lower numbers have higher precedence.

1. If the space directly below is sand, move there.
2. If the space below and to the left is sand, move there.
3. If the space below and to the right is sand, move there.
4. If the space directly to the right is sand, move there.

If none of these moves can be made, the drop of liquid is stuck.

In this code you can vary a) the size of the sand and rock system ($N \times N$), b) the probability level of randomness p that represents the proportion of impenetrable rock and porous sand – that is arranged randomly and c) the number of realisations *nrep* from which to evaluate the statistical ensemble properties.

(i) For $N = 100$ establish where the critical percolation level occurs p_c by varying p and use python graphics to show the 2d system. Use differing numbers of realisations. **(10 marks)**

(ii) Assess how p_c varies for $N = \{10, 50, 100, 200, 400\}$ and *nrep* $= \{100, 500, 1000, 2000, 4000\}$. In the large N limit it is expected that p_c will give a constant 'universal' value, discuss the performance of this algorithm in this context. **(10 marks)**

(iii) More general percolation algorithms are needed to evaluate the percolation threshold p_c for forest fires, electrical discharge and more widely. Suggest an algorithm you could use to find p_c in these cases, and have a go at implementing it. Read about other percolation theory applications. Explain in 400 to 500 what their main features are (hint: what types of system do they represent) and the type of applications. Include 1 figure or table from parts 7(i) or (ii) to illustrate this.

(20 marks – including 10 marks for originality of ideas)