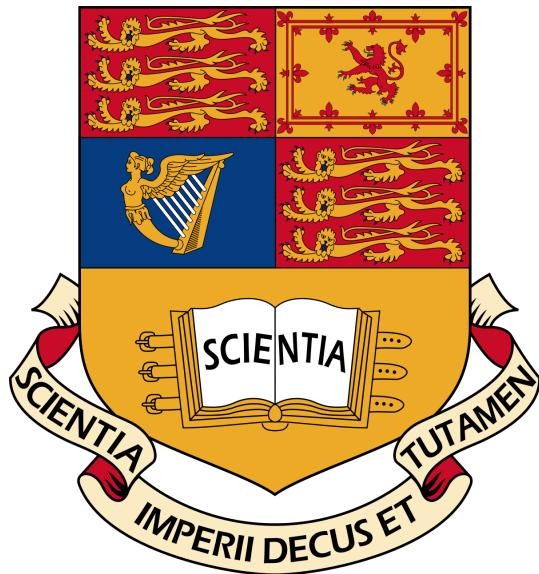

Analysis of modelling techniques used in the HIV epidemic

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1 Introduction

UNAIDS currently uses the Estimation and projection package (EPP) to evaluate and predict trends in HIV incidence and prevalence within countries. This model has evolved markedly over the years, incorporating Bayesian melding for parameter estimation and using various techniques to estimate the transmission parameter kappa.

In this report we aim to compare two of the most commonly used methods for modelling the kappa parameter and incidence: penalized B splines and the gaussian random walk (RW). We systematically evaluate how each technique performs under different data configurations, to better inform future modelling directions for the EPP package.

2 Methods

We will simulate data for a HIV epidemic from our deterministic simple EPP model. This models the transmission parameter as a logistic curve through time and we can incorporate ART treatment into this framework.

We will initially test the goodness of fit to simulated data, from our deterministic model, of both first order and second penalized splines, and first and second order penalized RW, with complete data for prevalence from the beginning of the epidemic. This will be performed over a set of different sample sizes from the population: 100, 500, 1,000 and 5,000 people. These random samples from the population will be repeated 100 times in each case and each of the four techniques will fit to the same set of four samples.

We will test how well our modelled fitted values match the true epidemic curve via a comparison of the root mean squared error (RMSE) of the true epidemic to the fitted values. This will be compared for the three output curves from our model representing: prevalence, incidence and kappa. We will compare this over the whole time period of the epidemic and specifically in the last 5 years of the epidemic when we have no sample data, to assess which modelling technique is best for predicting future trends, and we will look at the fit during the peak epidemic years, to gauge which technique can assess this peak and decline best.

Furthermore we will evaluate how often the true values for the epidemic parameters fall within our 95% confidence interval produced from the model fitting, again this will be performed for each of the three parameters: Prevalence, incidence and the kappa parameter.

Finally we will also assess how the four different models maybe overfitting to the data. We attempt to understand this via a comparison of the RMSE between the model fitted values and the true epidemic values, and the RMSE between the model fitted values and the values of the prevalence used in the sample to fit the data with.

3 Results

3.1 Single peak epidemic

3.1.1 Plots of mean results

First we will plot the mean output over the 100 runs of the sampled data for each of the four techniques for the model fitting, below in figure (1) is the mean fit with respect to prevalence, the red line represents the true epidemic and the blue line represents the fitted values with their 95% confidence intervals in shaded blue.

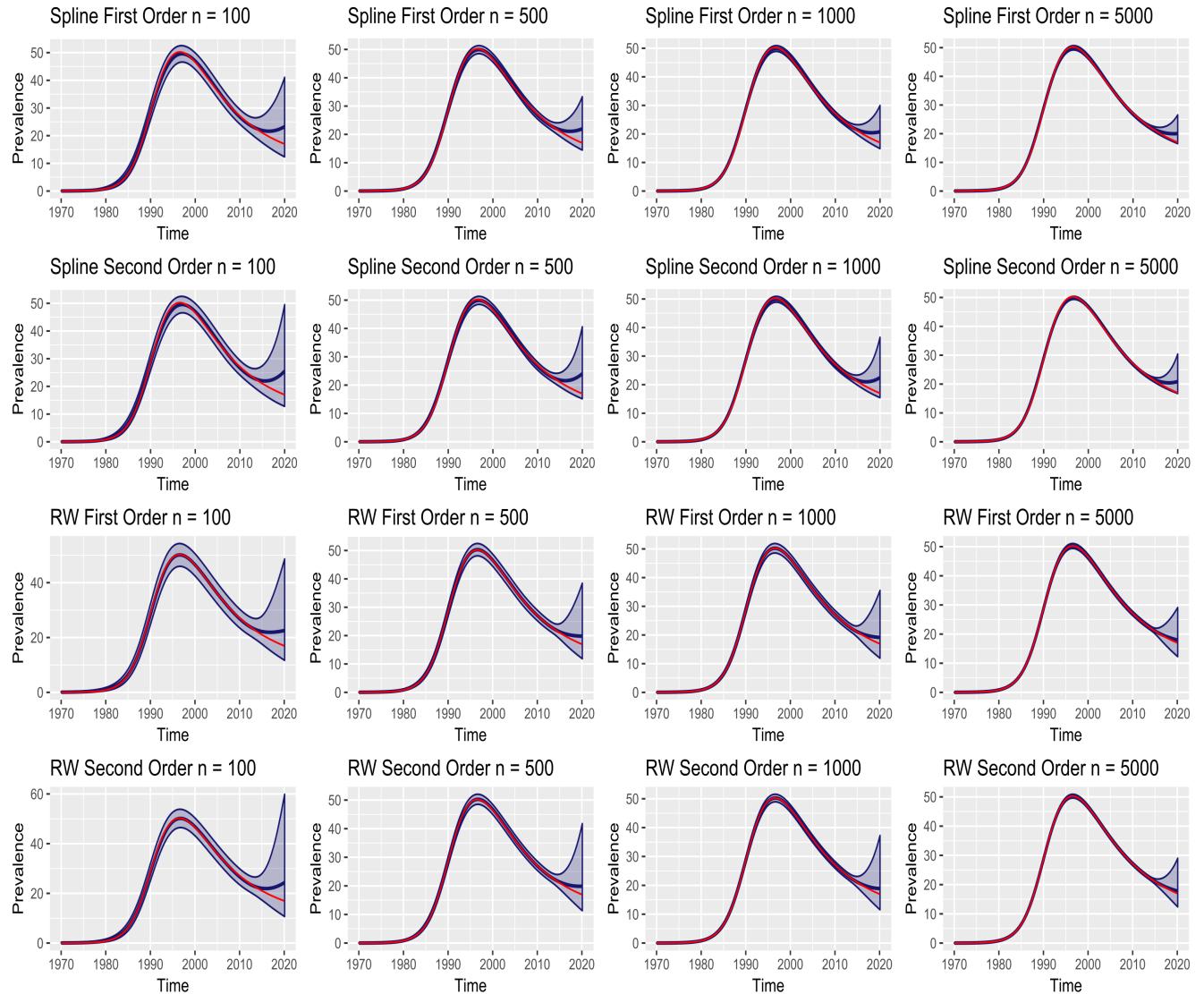


Figure 1: Prevalence mean for the 4 modelling techniques for 4 different sample sizes

From figure(1) we can initially see that all the different fitting methods capture the prevalence of the true epidemic very well during the phase from 1970 to 2015 when we have data to fit to. All models also have a narrowing of their credibility bounds as the sample size goes up from left to right. During the prediction period however the models differ in how well they seem to match the true epidemic. Both spline models seem to predict a levelling off of the prevalence or a slight uptick

depending on their sample sizes, whereas in reality the prevalence is seen to decrease during this period. The RW models begin to more closely match the epidemic qualitatively with larger sample sizes, with the RW second order model at sample size 5000 very similar indeed.

The mean values over the 100 runs on the different data sets are depicted below in figure(2), again the blue line is the fitted data while the red line indicates the true value from the simulated epidemic.

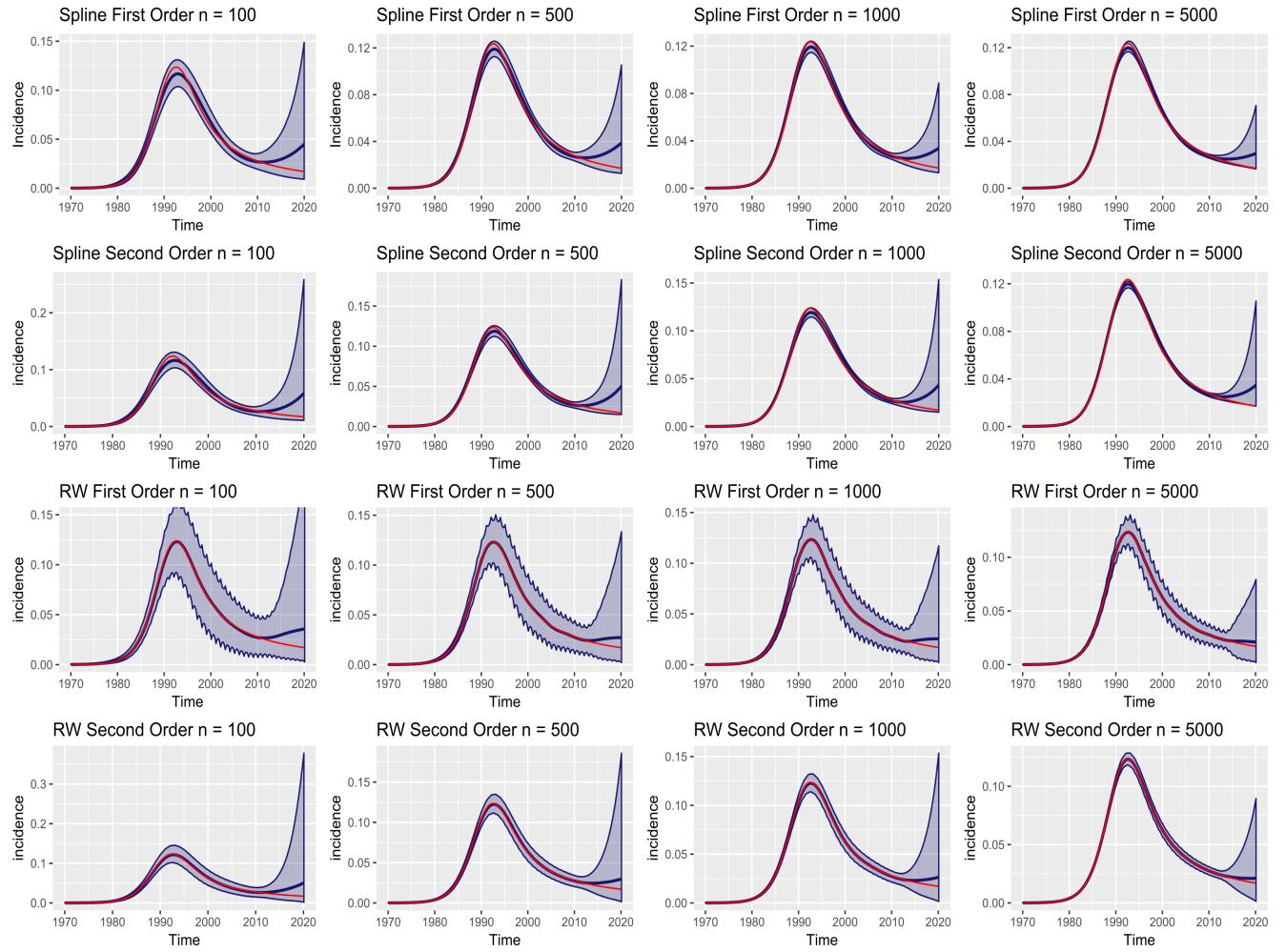


Figure 2: Incidence plots for the four different fitting methods over four different sample sizes from the population

From this we see that all four methods generally fit well to the true epidemic. Both spline methods however seem to underestimate the peak incidence reached during the early to mid 1990s. The credibility intervals for the RW methods also remain fairly constant, especially for the first order RW, despite an increase in sample size from the population. Once again the RW methods also seem to more closely match the true epidemic incidence during our prediction period from 2015 to 2020. The Second order RW again matches most closely the slight decline seen in the true simulated epidemic.

Finally the mean kappa values are seen in figure (3) below. Again the red line indicates the true value and the blue line the fitted value with associated shaded region the 95% credible interval.

The fitted values from the model produced for kappa are much less accurate than our previous two parameters. All

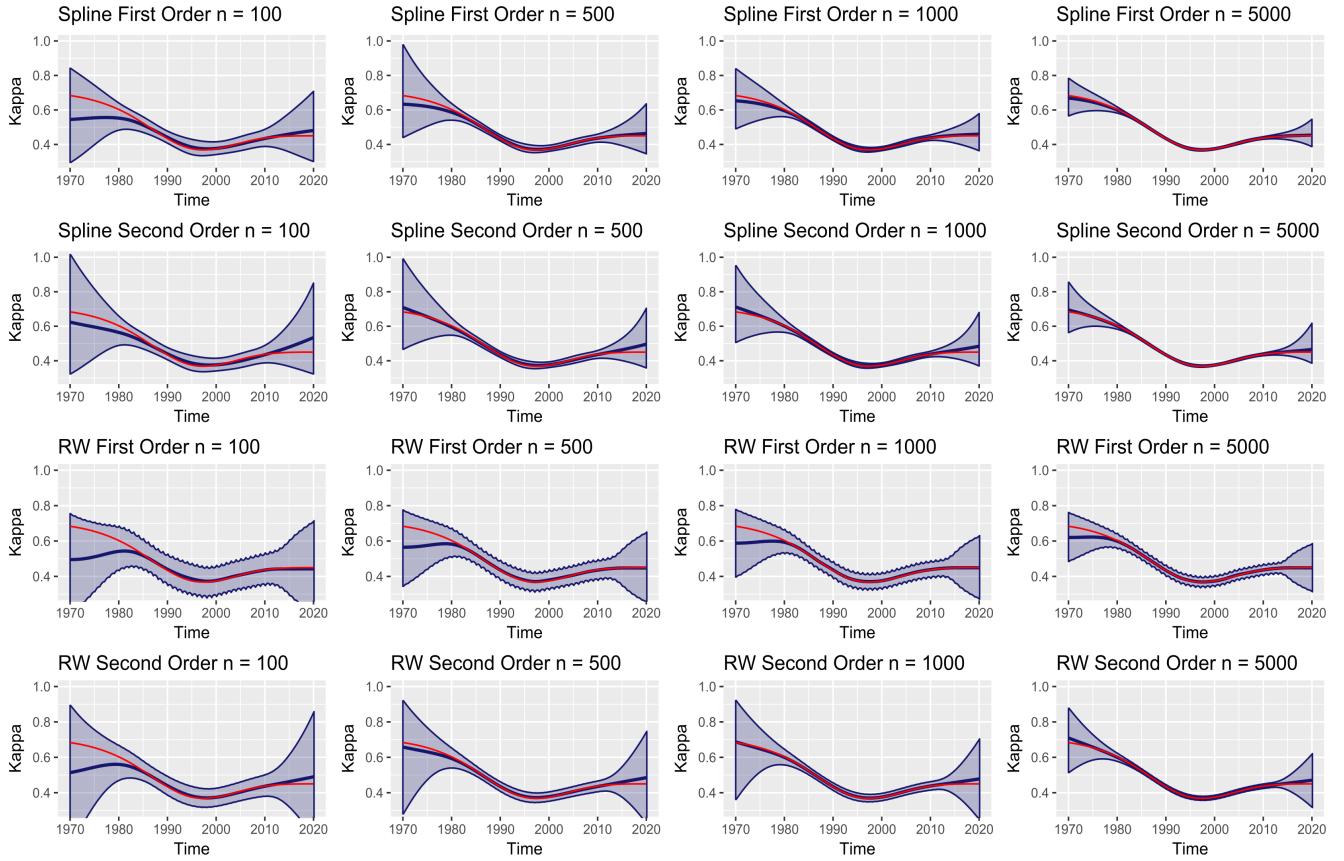


Figure 3: Kappa plots produced for the four different fitting methods, over four different sample sizes from the population

methods are seen to underestimate the initial kappa value, with first order splines and in particular RW second order fitting to a sample size of 100 very low estimates. All techniques though fit well during the decline phase of kappa from the late 1980s to the early 2000s. During the prediction period, both spline methods predict an uptick in kappa, with this being most pronounced in smaller sample sizes. The RW methods are seen to predict a levelling off of the kappa during this period, with the RW second order fitting particularly well at sample size 5000.

3.1.2 RMSE to true values

Below in table (1) is the RMSE values for prevalence of the true epidemic against the fitted median values for the whole data series, averaged over 100 iterations. One broad trend is that as the sample size increases, we see a better fit to the true epidemic as witnessed by a decrease in RMSE. In general at lower sample sizes we see the first order penalized splines and RW outperform their second order versions, while at higher sample sizes both RW techniques fit better to the true epidemic than the spline methods, with second order RW fitted with a sample size of 5,000 having the lowest RMSE value.

In table (2) we see the RMSE values over the whole course of the epidemic for the true incidence value against the fitted incidence values averaged over 100 iterations. Similar to what we see for prevalence, the first order methods are slightly better at fitting with lower sample sizes, while at the highest sample sizes, RW methods fit to true incidence better, with the best fit to incidence produced by the second order penalized RW at a sample size of 5,000.

Table 1: The complete RMSE for prevalence

Sample size	Spline First Order	Spline Second Order	RW first order	RW second order
100	0.0185343	0.0214740	0.0182192	0.0210485
500	0.0120273	0.0146318	0.0097615	0.0100131
1000	0.0093654	0.0113620	0.0078701	0.0077898
5000	0.0062764	0.0073870	0.0040916	0.0038942

Table 2: The complete RMSE for incidence

Sample size	Spline First Order	Spline Second Order	RW first order	RW second order
100	0.0081221	0.0102019	0.0078942	0.0093847
500	0.0056484	0.0074060	0.0049852	0.0044005
1000	0.0044308	0.0058095	0.0044844	0.0035941
5000	0.0031688	0.0038922	0.0029155	0.0019608

For kappa, the RMSE values for the whole time series of the epidemic averaged over 100 iterations are seen in table (3). This is an interesting set of results, for both splines we see no real decrease in RMSE with an increase in sample size, unlike the trend seen previously for both incidence and prevalence. Indeed if anything first order splines decrease in their predictive power with increasing sample size. With RW there is a clear improvement with increasing sample size, with the best fit to the true value being the First order RW with a sample size of 5000. These values though are still the same order of magnitude as the values Splines produce for RMSE.

Table 3: The complete RMSE for Kappa

Sample size	Spline First Order	Spline Second Order	RW first order	RW second order
100	0.0432773	0.0450542	0.0606701	0.0633587
500	0.0427531	0.0451453	0.0332154	0.0362085
1000	0.0457103	0.0457409	0.0284724	0.0337876
5000	0.0481981	0.0434053	0.0173404	0.0185758

3.1.3 RMSE to true for prediction period

In table (4) we have the RMSE values of the true prevalence against the fitted values for prevalence during the last five year period in which no data were sampled, these are averaged values across 100 iterations of sampled data. In general from this we can see that both RW methods produce closer fits to the true data, with RW first order at a sample size of 5,000 having the best average fit. At lower sample sizes the different methods are quite similar, with RW first order having a slightly better fit each sample size.

Incidence predictions during 2015:2020 are seen in table (5). Here once again we see that the RW fitting technique produces a closer fit across the different sample sizes than the equivalent penalized spline. Additionally the first order penalized RW produces the best fit to the true data among the different fitting techniques across the range of sample sizes during this five year prediction period. It is interesting to note that the first order penalized methods in this scenario always outperform

Table 4: RMSE of Prevalence for fitted against true values during 2015:2020 prediction period

Sample size	Spline First Order	Spline Second Order	RW first order	RW second order
100	0.0413941	0.0512600	0.0386404	0.0497123
500	0.0292720	0.0383811	0.0199103	0.0220041
1000	0.0233138	0.0301396	0.0166608	0.0180795
5000	0.0167389	0.0205310	0.0086108	0.0091037

their second order counterparts. This could be due to the fact that in the absence of data a first order penalty will mean the value plateaus, something which we see the incidence parameter of the true epidemic begins to do during this prediction period. Further testing with different epidemic trajectories will allow us to test whether this better fitting of first order methods is merely an artefact from this true epidemic curve.

Table 5: RMSE of Incidence for fitted against true values during 2015:2020 prediction period

Sample size	Spline First Order	Spline Second Order	RW first order	RW second order
100	0.0184568	0.0253233	0.0144984	0.0218784
500	0.0139638	0.0199397	0.0079612	0.0093225
1000	0.0108674	0.0156428	0.0067635	0.0076223
5000	0.0081842	0.0106920	0.0036823	0.0040593

For the kappa parameter, the average RMSE values during this prediction period are displayed in table (6). These results are similar to the results for prevalence and incidence during this prediction period, in that at each sample size the best fitting technique is the first order penalized random walk, while the first order techniques also outperform their respective second order penalized versions. The improvement with increasing sample size isn't as clear cut as the previous two parameters, with all RMSE values the same order of magnitude for each of the sample sizes in this case.

Table 6: RMSE of Kappa for fitted against true values during the 2015:2020 prediction period

Sample size	Spline First Order	Spline Second Order	RW first order	RW second order
100	0.0608326	0.0807562	0.0495436	0.0692751
500	0.0496149	0.0689270	0.0307835	0.0343041
1000	0.0400896	0.0569184	0.0269582	0.0287582
5000	0.0322347	0.0420635	0.0153424	0.0166802

3.1.4 RMSE during peak epidemic years

The peak years of this simulated epidemic, both in terms of prevalence and incidence, were during the 1990s. To gauge how sensitive each of our modelling techniques were to detecting this peak and decline we evaluate the goodness of fit for the prevalence, incidence and kappa values below in tables (7 : 9) for this period.

From these we can see that there is a clear trend of increasing fit to the true epidemic with increasing sample size, this is present as well within both spline's estimation of kappa, where before for the whole time period of the epidemic we saw no clear relationship between sample size and RMSE. For all parameters across all sample sizes during this period the second

Table 7: RMSE of true to fitted values for prevalence during the peak epidemic period 1990:2000

Sample size	Spline First Order	Spline Second Order	RW first order	RW second order
100	0.0075059	0.0078810	0.0092716	0.0077592
500	0.0047457	0.0049406	0.0063034	0.0042745
1000	0.0037979	0.0039250	0.0059571	0.0035344
5000	0.0028269	0.0027916	0.0041508	0.0019855

Table 8: RMSE of true to fitted values of incidence during the peak epidemic period of 1990:2000

Sample size	Spline First Order	Spline Second Order	RW first order	RW second order
100	0.0075059	0.0078810	0.0092716	0.0077592
500	0.0047457	0.0049406	0.0063034	0.0042745
1000	0.0037979	0.0039250	0.0059571	0.0035344
5000	0.0028269	0.0027916	0.0041508	0.0019855

Table 9: RMSE of true to fitted values of kappa during the peak epidemic period of 1990:2000

Sample size	Spline First Order	Spline Second Order	RW first order	RW second order
100	0.0140984	0.0150774	0.0177704	0.0139535
500	0.0096315	0.0100801	0.0129679	0.0081949
1000	0.0082273	0.0084961	0.0125785	0.0070732
5000	0.0066278	0.0065542	0.0089335	0.0039714

order penalized RW produces the closest fit to the true epidemic's values. There is a clear difference between the RMSE values produced at sample size 5,000 for the kappa and prevalence values of the second order penalized RW, and the other techniques' RMSE values. This perhaps indicates that for a growing epidemic, a second order penalized RW is the best fitting method.

3.1.5 Credibility interval analyses

To test how robust the 95% credible intervals produced from our model fitting are, we detected how often the true epidemic parameter values fell within the 95% credible interval of our model fits, across 100 different sample set iterations. These are depicted in tables (10 : 12).

Table 10: Percentage of time the true prevalence values fall within the 95 percent credible interval

Sample size	Spline first order	Spline second order	RW first order	RW second order
100	95.72655	95.27545	94.96008	96.27545
500	92.43713	91.84232	97.47505	96.64671
1000	90.13174	89.25349	96.95409	96.18563
5000	74.45309	75.58283	97.99401	97.37325

These are interesting results, seemingly depicting that RW methods of fitting are more reliable in estimating the true epidemic values. With the second order peanlized RW, all the produced credible intervals contain the true value at least 96% of the time, while first order penalized RWs perform best for including the true values of incidence and the kappa parameter.

Table 11: Percentage of time the true epidemic incidence values fall within the 95 percent credible interval

Sample size	Spline first order	Spline second order	RW first order	RW second order
100	94.35928	93.21756	98.25349	97.89421
500	89.10379	87.17964	99.68263	98.70459
1000	82.77246	80.94611	99.45509	97.76247
5000	60.01996	60.59481	99.74651	98.96008

Table 12: Percentage of time the true epidemic Kappa values fall within the 95 percent credible interval

Sample size	Spline first order	Spline second order	RW first order	RW second order
100	93.04790	91.56687	99.18762	97.95609
500	84.46108	83.39321	99.82834	98.87824
1000	75.16966	74.67066	99.89621	98.39521
5000	48.64671	52.49102	99.87026	98.96607

The Splines perform adequately at the lowest sample size of 100, but as the sample size increases, and subsequently their produced credible intervals contract, they end up holding the true value within their bounds far less frequently than they should. This is most pronounced when estimating the Kappa parameter, with the frequency at which the true kappa value lies within the credible bounds of the first order penalized Spline when the population sample size is 5,000 being only 48.6%, almost half the frequency it should be.

These results, at least for this epidemic, suggest that the RW methods for modelling the kappa parameter are far more reliable in assessing the true epidemic parameters than the Spline methods.

3.1.6 Overfitting and bias

To try and understand how systematically biased data may affect each model's ability to infer the true epidemic, we came up with a simple test statistic to determine if fitted prevalence estimates were overfitting to the sampled prevalence points each year:

$$\frac{RMSE_{fitted-true}}{RMSE_{fitted-sample}}$$

When the fitted prevalence more closely matches the true epidemic prevalence, this statistic tends to 0, while when the fitted prevalence matches the sampled prevalence more closely than the true prevalence this will tend to positive ∞ .

The results of our analysis of the potential overfitting of the model are seen in table (13). These are somewhat in line with our expectations, as we see that generally the RW models, which estimate more parameters when recreating the curve of the kappa parameter, are tending to overfit more to the prevalence data than the spline models. These RW values are higher, indicating they match closer to the sampled values than the spline models do. However these are in many cases only small differences and the second order penalized RW at sample size 5,000 actually fits closer to the true prevalence than the three other modelling techniques do.

Given these results, with a biased dataset, splines may be able to better predict the true epidemic values, although further work testing with biased datasets will be needed to test this theory more rigorously

Table 13: Analysis of overfitting to prevalence sampled data

Sample size	Spline first order	Spline second order	RW first order	RW second order
100	0.3405751	0.3506141	0.3980836	0.3793522
500	0.4334250	0.4334250	0.5206535	0.4465201
1000	0.4513946	0.4612842	0.5804345	0.4679472
5000	0.5501180	0.5520179	0.7016347	0.4958705

3.2 Double peak epidemic

Given that there is evidence for more complex epidemic trajectories existing for HIV, especially within Western Europe where a second peak of incidence has been observed during the 2000s, we sought to test how the different modelling approaches capture an epidemic with a double peak in incidence. We simulated this by modeling kappa as a spline with 7 knot points. We simulated Kappa declining initially up to the late 1990s, this decline was then followed by an increase and subsequent plateau of kappa by the early 2010s. This decline and rise matches the narrative that increasing awareness of AIDS and HIV as a deadly disease up to the mid 1990s caused transmission rates to decline, however following the advent of effective drugs for managing HIV infection, attitudes laxed concerning its spread, allowing for a modest uptick in incidence.

Figure (4) depicts the average predicted prevalence from our 100 different samples taken from the true epidemic at four different population sizes. We see a good, almost perfect, fit to the true epidemic during the period for which we have data, 1970-2015. During the prediction period there is in general a good fit, with first order penalized methods fitting closer to the true epidemic than their respective second order penalized methods, although this difference decreases with increasing sample size from the population.

In figure (5) we have plotted the average fitted incidence values from our 100 samples taken from the true epidemic. Similar to prevalence we see a good fit during the period for which we have data in the epidemic, from 1970-2015. However towards the end of this period we see more divergence between the fitted median value and the true epidemic curve. This divergence is most pronounced in second order penalized fitting methods, especially at lower sample sizes.

Finally in figure (6) the average fitted results for kappa are displayed over 100 different samples from our true epidemic. These results further what we have seen previously, that in this epidemic first order penalized methods produce closer fits to the true epidemic, especially during the prediction period of 2015-2020. This is due to the kappa parameter rising and then levelling off during the later period. First order penalized methods, in the absence of data, will by default produce a flat line, which is what this epidemic naturally falls to. Second order penalized methods, in the absence of data, maintain the gradient from previous data points. In this case, leading up to 2015 the kappa parameter increases slightly, in this case second order penalized methods continue this trend into the prediction period.

This serves to highlight how the different assumptions of the modelling techniques affect predictions in the absence of data. Previously we had seen in a declining epidemic how second order penalized methods tended to produce closer fits to the true epidemic than first order penalized methods. In this scenario the reverse is true, perhaps indicating how the choice of model is an arbitrary one, and any reliance on future predictions should be held in respect to the modelling technique

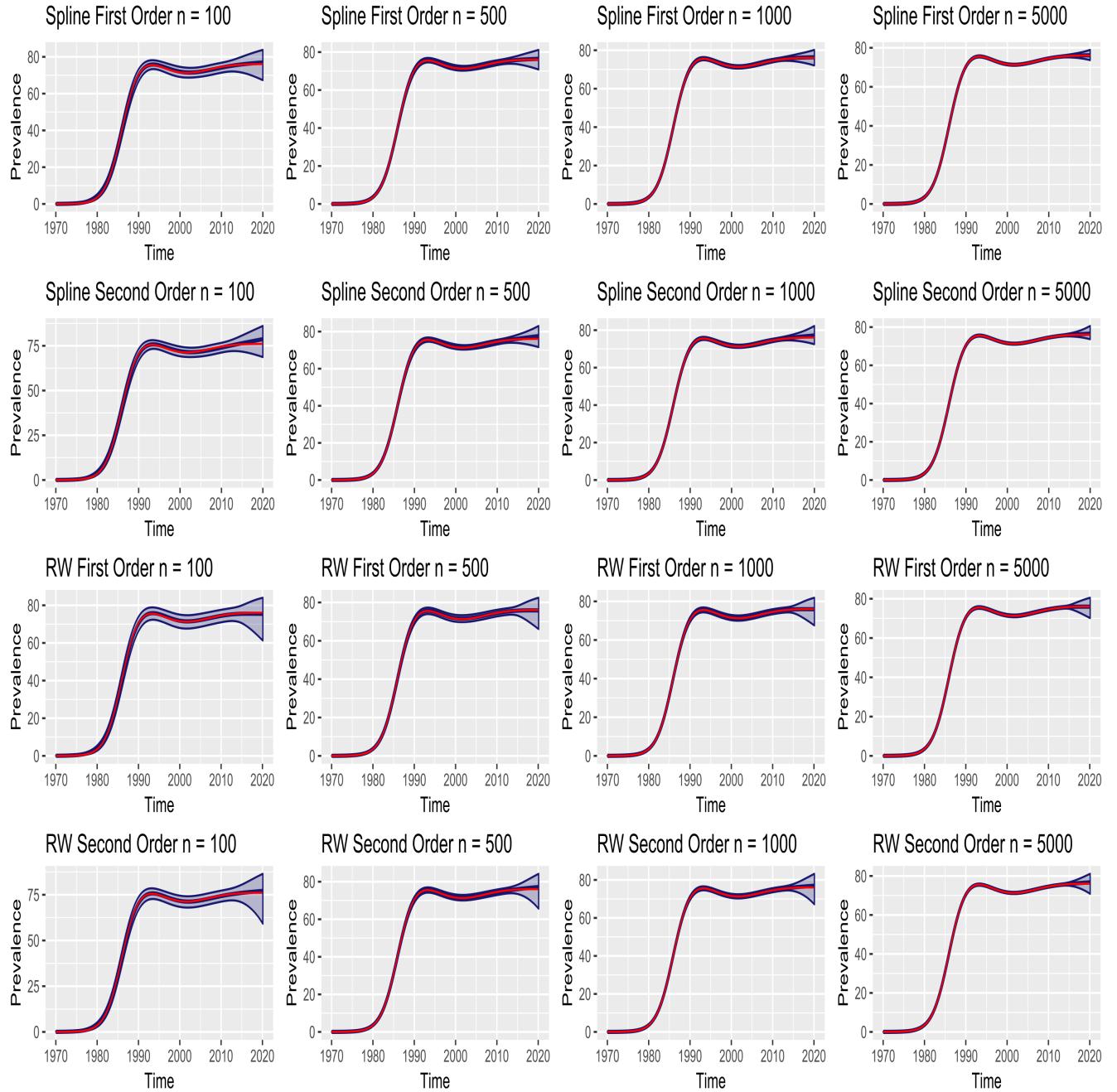


Figure 4: Prevalence results for the double peak epidemic, the true epidemic is in red with the fitted output in blue and the 95% credible interval in blue. These are the average results for 100 different samples from this true epidemic

choose.

3.2.1 RMSE to double peak epidemic

In table (14) we see the mean RMSE of the produced fits for each sample size for each of the four different modelling techniques, compared to the true epidemic values of prevalence. We see in this case the first order modelling methods outperforming their respective second order counterparts. In contrast to previous results, splines are closer to the true epidemic's

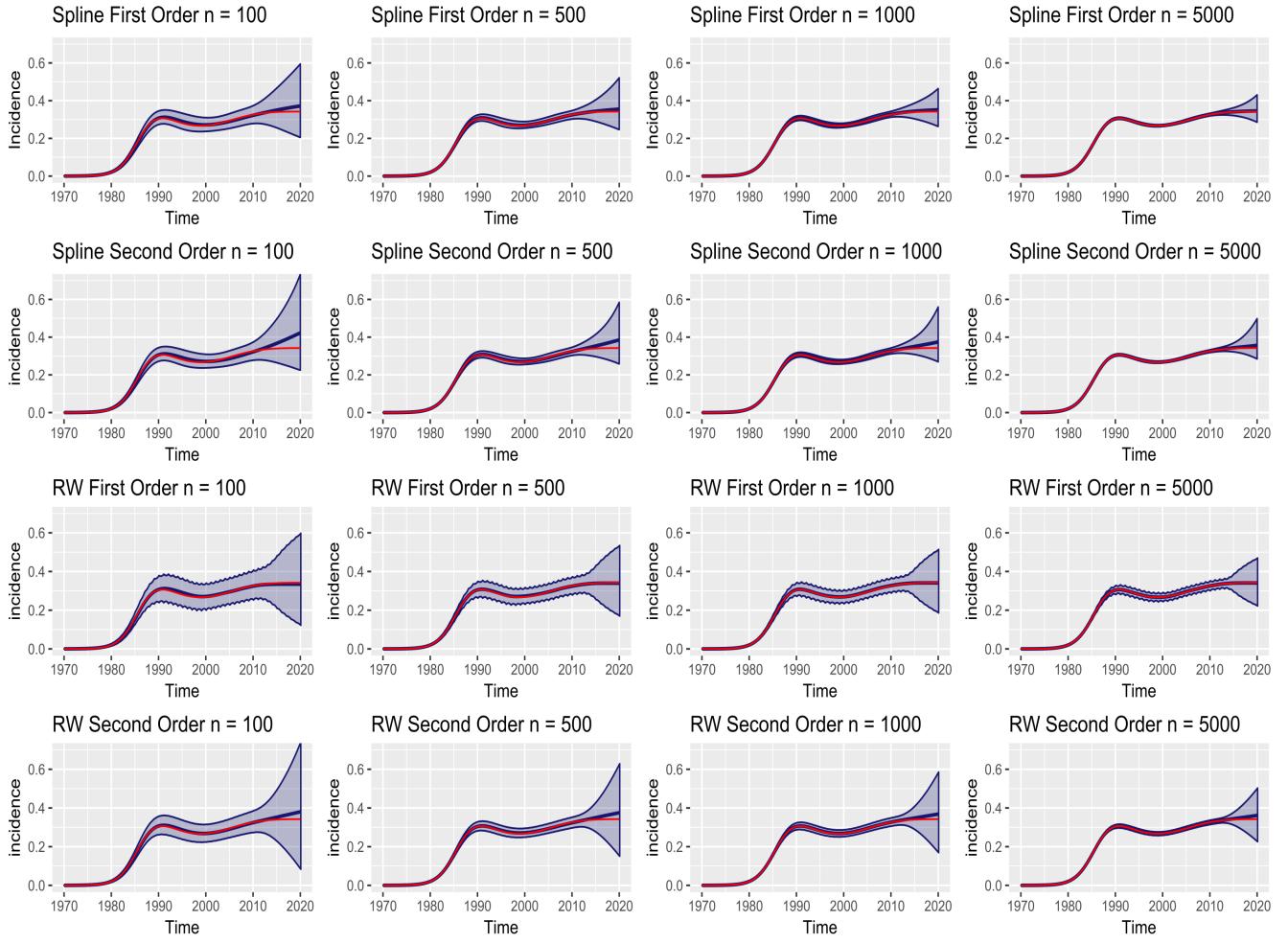


Figure 5: Incidence plots for the double peak epidemic, the true epidemic is in red, with the fitted output in blue and the 95% credible interval in shaded blue. These are average results from 100 different fits to the data

prevalence values than the RW methods are, with the 5,000 sample size of the first order splines giving the closest match of any of the techniques to the true epidemics values.

Table 14: Mean RMSE values over 100 different samples for prevalence for four different modelling techniques

Sample size	Spline first order	Spline second order	RW first order	RW second order
100	0.0139043	0.0151049	0.0143438	0.0149862
500	0.0067432	0.0075068	0.0073693	0.0079120
1000	0.0052379	0.0058424	0.0057568	0.0062697
5000	0.0024803	0.0027289	0.0032067	0.0033302

The incidence RMSE values for the whole course of the epidemic are displayed in table (15). Once again we see that first order methods fit closer to the true values of incidence than their respective second order penalized methods. While, apart from at a sample size of 100, the spline fitting methods outperform the RW methods with the first order penalized spline at a sample size of 5,000 having the closest fit to the true epidemic's incidence values. The differences in these values are in many cases very small, with all values within a sample size in general on the same order of magnitude.

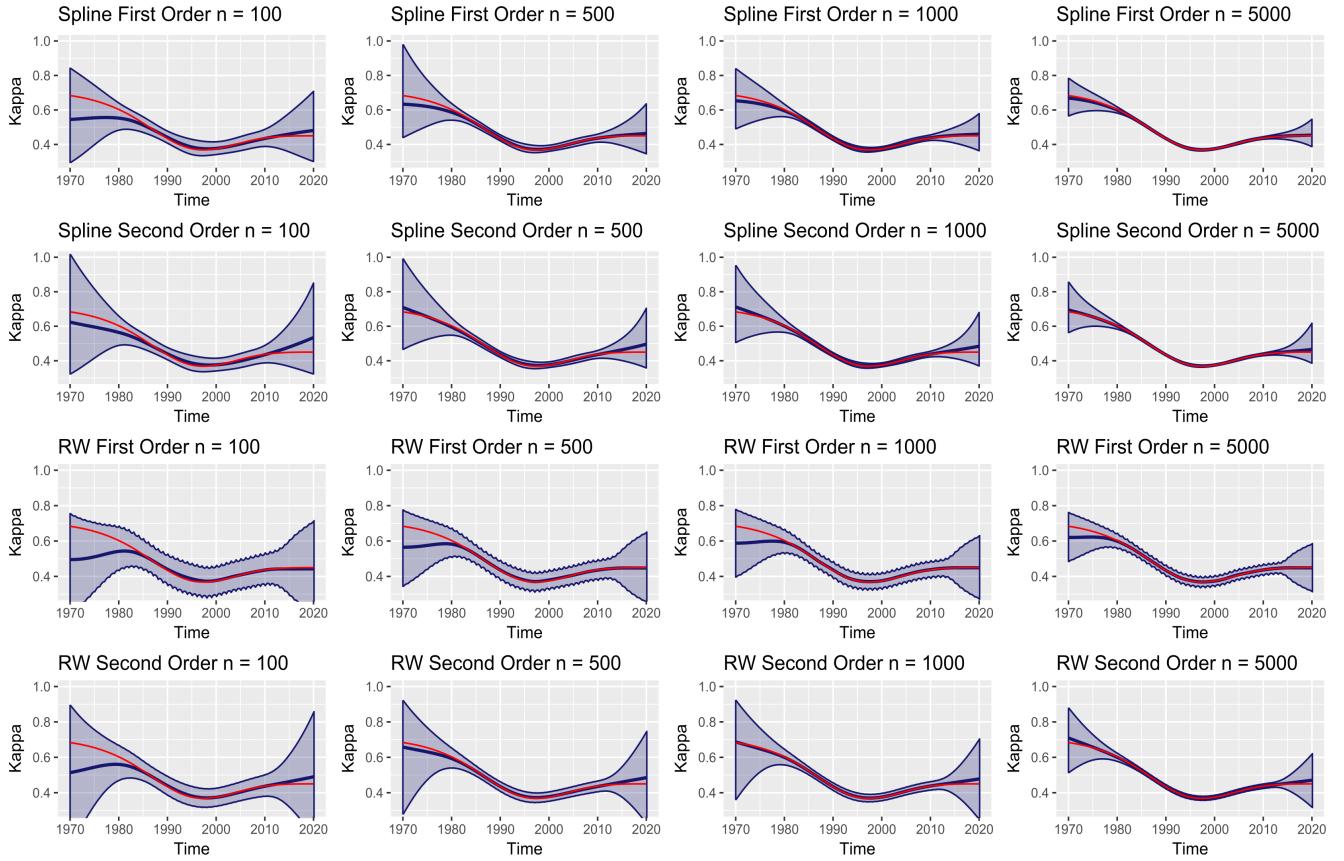


Figure 6: Kappa plots for the double peak epidemic, the true epidemic is in red, with the fitted output in blue and the 95% credible interval in shaded blue. This is the mean value over 100 samples from our true epidemic.

Table 15: Mean RMSE values over 100 different samples for incidence for four different modelling techniques

Sample size	Spline first order	Spline second order	RW first order	RW second order
100	0.0225583	0.0287197	0.0215571	0.0260806
500	0.0119939	0.0151489	0.0125286	0.0158830
1000	0.0096216	0.0119251	0.0101976	0.0128250
5000	0.0044846	0.0054875	0.0065849	0.0073253

Finally the Kappa RMSE values over the whole course of our double epidemic are displayed in table (16). The interpretation of these results is more nuanced than for the previous two parameters. In general the first order penalized spline performs the best for all sample sizes, except for the smallest of 100. In contrary to the prevalence and incidence from previously, the second order RW outperforms the first order penalized RW across all sample sizes. Given that the true kappa parameter was created using a seven knot spline function it is easy to explain why spline methods in this case fit well. However the previous epidemic's kappa parameter was generated using a simple logistic curve, a shape you would expect a seven knot spline to be able to replicate, yet in that case RW methods fit closer to the true epidemic than spline methods. Now we will move on to look in particular at the prediction period and peak epidemic period RMSE values for our different fitting methods.

Table 16: Mean RMSE values over 100 different samples for Kappa for four different modelling techniques

Sample size	Spline first order	Spline second order	RW first order	RW second order
100	0.0549530	0.0488188	0.0701690	0.0631885
500	0.0267246	0.0302877	0.0404749	0.0299723
1000	0.0202555	0.0236425	0.0318747	0.0231818
5000	0.0108782	0.0126433	0.0199110	0.0149260

3.2.2 RMSE during prediction period

In tables (17 : 19)

Table 17: Mean RMSE values over 100 different samples for Prevalence for four different modelling techniques during the prediction period 2015-2020

Sample size	Spline first order	Spline second order	RW first order	RW second order
100	0.0182540	0.0241704	0.0152911	0.0228109
500	0.0125796	0.0152657	0.0115155	0.0159198
1000	0.0100928	0.0122640	0.0083306	0.0131332
5000	0.0046704	0.0056176	0.0054258	0.0074231

Table 18: Mean RMSE values over 100 different samples for Incidence for four different modelling techniques during the prediction period 2015-2020

Sample size	Spline first order	Spline second order	RW first order	RW second order
100	0.0411505	0.0645207	0.0281549	0.0543465
500	0.0286026	0.0385944	0.0227538	0.0394510
1000	0.0232229	0.0306828	0.0165673	0.0325934
5000	0.0105306	0.0140989	0.0116581	0.0193429

Table 19: Mean RMSE values over 100 different samples for kappa for four different modelling techniques during the prediction period 2015-2020

Sample size	Spline first order	Spline second order	RW first order	RW second order
100	0.0423181	0.0675511	0.0281152	0.0568285
500	0.0301102	0.0409657	0.0233344	0.0418939
1000	0.0246058	0.0327116	0.0171234	0.0349788
5000	0.0111439	0.0152313	0.0123216	0.0210222

4 Conclusions and further work

From our results for this epidemic we can conclude that for modelling the Kappa parameter through the course of the epidemic, using a Gaussian RW is more likely to recreate the true epidemic parameters of prevalence, incidence and the kappa parameter. For predicting the future course of an epidemic a first order penalized RW produces the closest fit to this form of epidemic's trajectory. While when recapturing the peak of this epidemic, a second order penalized RW matches closest

to the true values. The 95% credible intervals produced by the RW modelling are also much more reliable than the intervals produced via modelling with a Spline.

Our work tentatively suggests though that RW may overfit to the data compared to spline methods, further testing of this for a range of epidemic trajectories and data types is needed for a better understanding of this relationship. Indeed further work is needed within this framework to understand the effects of:

- Modelling the incidence/Force of infection rather than the transmission parameter kappa.
- Different epidemic trajectories on the accuracy of the different modelling techniques.
- Different data structures, fewer sampling, biased sampling and different data types on the fits produced.
- More flexible spline modelling.
- The effect of ART and improved diagnosis on epidemic trajectory and estimation.

Completing this will allow us to provide robust feedback to the modelling community, informing them of the best methods to be used in each of the possible scenarios.