Università Commerciale "Luigi Bocconi"

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Group 5 Assignment

CERTIFICATE EVALUATION

10052 Derivatives
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1 Abstract

The following paper consists in an advanced analysis of the Certificate issued by Commerzbank and based on the *Efficiency Growth Index* (ISIN DE000CZ37TL7). This structured derivative product can be viewed as a sort of call option for the nature of its payoff. Commerzbank played the role of the seller (writer) of the instrument, while the target customers (buyers) are assumed to be institutional investors. The report is divided into two main sections:

- Quantitative
- Qualitative

In the *Quantitative Chapter* (Section 2) we first present the formulas related to the contract as reported in the Certificate factsheet, together with the method we pursued to carry out our analysis. Next, we pass through the valuation procedure, by adopting a retrospective perspective; this has been done by means of the famous Black&Scholes model (1973), which returned us an estimated price at issuance of almost 33€. We have improved our analysis by taking into consideration the main *Greeks*. Finally, we conclude this section by the assessment of the required assumptions of such a model in order to verify the robustness of our computations. We find out that only one out of the six fundamental hypotheses are satisfied and so this could undermine our methodology.

In the *Qualitative Chapter* (Section 3) we make our considerations deeming the assumptions of the B&S model to be verified. We further distinguish three main parts: first, we put ourselves in the shoes of a potential buyer trying to find out potential reasons to invest in the Certificate at the issuance date. An investor with strong market expectations consisting of low volatility and decreasing term structures of interest rates would have greatly benefited from the contract purchase, both with speculation or hedging purposes. Secondly, we investigate the convenience to issue such a derivatives instrument from Commerzbank's perspective. The bank devised the contract in such a way to be protected from extreme scenarios; in this respect, it issued the products at a great premium (57€), thus representing a sort of "protective-buffer" against adverse circumstances (in the specific case, it feared potential drops in interest rates). Finally, we conclude our report by making a retrospective assessment in light of what occurred in those years. For sure, the investors in the Certificate took a great profit (40€ per certificate), while this issuance was not a big deal for Commerzbank, which recorded a loss of €1.2 mln.

2 Quantitative Analysis

2.1 Contract Overview

The analyzed contract is a structured derivatives product that relates to the *Commerzbank Efficiency Growth Index* (ISIN DE000CZ37TL7); given the intrinsic nature of its payoff, we can consider the Certificate as a call option in which the index itself assumes the role of the underlying. The contract was issued on 25 April 2014 with maturity of 4 years (25 April 2018); the index's initial calculation date was the 14th April 2014. Commerzbank issued 30,000 certificates at an initial issue price of 90€ per certificate; in addition, each one was sold and traded by the Issuer only through the regulated market of Borsa Italiana S.p.A. (i.e. SeDeX). On the Maturity Date, each contract has been redeemed through the payment of a "Settlement Amount" in EUR computed in the following way:

$$SA = CA \times Max\left(0; \frac{Underlying_{Final}}{Underlying_{Strike}} \times (1-0.011)^4 - 1\right) \tag{1}$$

where:

- SA is the settlement amount per certificate (rounded, if necessary, to the next €0.01).
- CA is the calculation amount per certificate set equal to 1000€.
- $Underlying_{Final}$ is the value of the Index on the maturity date (25 April 2018).
- $Underlying_{Strike}$ is the strike price computed as the average of the index value on the dates 22nd, 23rd and 25th April 2014.

2.2 Method

As already mentioned, each contract can be viewed as a call option on the underlying index; therefore, for our investigation and analysis purposes, we rely on the Black & Scholes model (1973), which allows us to calculate the market value of the option with the well-known PDE, which assumes the following form:

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} = rC \tag{2}$$

where C is the price of the call option as a function of the underlying price S (in this case the Index) and time t. r is the risk-free interest rate, and σ is the volatility of the underlying. It is important to remark that such a model is efficient provided that some fundamental hypotheses are verified:

1. The underlying price follows a *Geometric Brownian Motion* stochastic process with constant drift μ and volatility σ. [1]

2. No transaction costs or taxes are charged.

3. Securities are perfectly fractionable.

4. Short selling of the securities is allowed without any restriction.

5. Risk-free rate is known and constant for all maturities.

6. No dividends are paid during the life of the contract.

In the following sections, together with the computational methods used for pricing the derivatives instruments at the different dates, we provide an in-depth analysis of each of the above hypotheses in order to test the effectiveness of the model and the related results.

2.3 Underlying Index

The Index is computed as a weighted average of two components: one share of the Efficiency Growth Fund and a reference interest rate (1M EURIBOR). Its computations are carried out using the following deterministic formula:

$$I_t = I_{t-1} \cdot \left\{ 1 + \left[w_{t-1} \cdot \left(\frac{fund_t}{fund_{t-1}} \right) \right] + \left[(1 - w_{t-1}) \cdot EURIBOR_{t-1} \cdot \frac{Act}{conv} \right] \right\} \tag{3}$$

where:

- I_{t-1} is the Index value at previous time $t-1$;
- w_{t-1} is the weight w at previous time $t-1$;
- $EURIBOR_{t-1}$ is the reference interest rate at previous time $t-1$;
- Act is the difference, expressed in days, between the current and the previous date;
- $conv$ is a conversion factor for the number of day in a year (360);

The weighting mechanism is based on the parameter w and its complement, $1-w$: they indicate, respectively, the amounts invested in the fixed-income Basket and in 1M EURIBOR for the purposes of the index calculation; at each period t, the corresponding weight w is computed as follows:

$$w_t = min\left\{ W_{max}; \frac{Target\,Vol}{Real\,Vol_{t-lag}} \right\} \tag{4}$$

where $W_{max} = 150\%$ and $TargetVol = 6\%$, as specified in the contract. The time-dependent $Real\,Vol_{t-lag}$ parameter, instead, represents the fund log returns' volatility computed over a 20-days rolling window before each calculation date. [2]

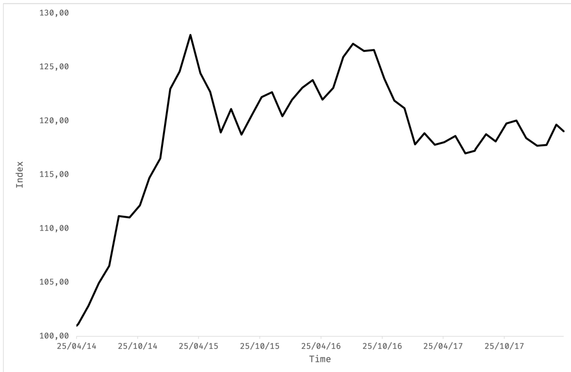

Figure 1: Index evolution over time, April 2014 - April 2018

Finally, we report in Figure 1 the realized time series of the index. It is immediate to note that the index (initial value of 100) experienced strong growth in the first 2 years of the contract, peaking at 128.74, a value reached between March and April 2015. This was mainly due to the announcement of non-conventional monetary policies by the ECB, addressing the strong period of uncertainty faced by Eurozone countries in mid-2014. After this important rise, the index retraces back a bit and more or less stabilized itself around a lower constant value (≈ 120).

[1] The GBM is a stochastic process of the form $\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$, where S_t is the underlying price at time t, μ the mean or drift rate, σ the diffusion coefficient or volatility, and W_t the Wiener process at time t.

[2] The contract fact-sheet specifies the calculation method used for computing the realized volatility. As we can see from the formula, it is basically the standard deviation of the log-returns adjusted for an initial factor, which serves to express the result in annualized term:

$$Real\,Vol_{t-lag} = \sqrt{\frac{d}{m}} \cdot \sqrt{\sum_{k=1}^{n=20} \left(\ln \frac{fund_{t+k-n-lag}}{fund_{t+k-n-lag-1}} \right)^2 - \frac{1}{n} \cdot \left(\sum_{k=1}^{n=20} \ln \frac{fund_{t+k-n-lag}}{fund_{t+k-n-lag-1}} \right)^2}.$$

2.4 Time to maturity

The computation of the time to maturity has been carried out by adopting the trading days convention, which considers 252 working days in a year.

$$T = \frac{N.\,of\,residual\,trading\,days}{252} \tag{5}$$

For completeness, we would like to stress that, during Luxembourg bank holidays, when the Index data were not available, they have been substituted with the most recent observation.

2.5 Risk-free rate

For what concerns the risk-free rate, we adopted for each time-interval of interest (so for each month of valuation) the values obtained by the linear interpolation of the available interest rates for such reference period. More specifically, for each timeframe t, the corresponding risk-free rate has been computed by means of the following formula:

$$r(t) = \frac{[TS(i) + \frac{TS(j)-TS(i)}{j-i} \cdot (t-i)]}{100} \tag{6}$$

where i and j (with $i < j$) refer respectively to the two reference time intervals that include the residual date to maturity t in which we are interested; they are both expressed in annual term (i.e. 1 month is equal to 1/12). By means of this notation it follows that $TS(i)$ and $TS(j)$ represent the two available term structures for those periods. The available term structures are the EUR Swap rates (against 6 months EURIBOR) referred to 5Y, 3Y, 2Y and the EURIBOR 12M, 6M, 3M and 1M; in the following table we have reported the actual term structures used for the linear interpolation procedure, according to the different values of the reference date t.

$Interval\,dates$	$TS(j)$	$TS(i)$
25/04/14 – 30/04/15	$Swap(vs6M)_{5Y}$	$Swap(vs6M)_{3Y}$
29/05/15 – 29/04/16	$Swap(vs6M)_{3Y}$	$Swap(vs6M)_{2Y}$
30/05/16 – 28/04/17	$Swap(vs6M)_{2Y}$	$EUR\,12M$
31/05/17 – 29/09/17	$EUR\,12M$	$EUR\,6M$
31/10/17 – 29/12/17	$EUR\,6M$	$EUR\,3M$
31/01/18 – 29/03/18	$EUR\,3M$	$EUR\,1M$

Table 1: Reference term-structures

By the above considerations, it is trivial to note that the 5^{th} B&S hypothesis is not completely satisfied, since the computed risk-free rate is not constant during the whole life of the underlying, but it can be assumed to be such (i.e. constant) only in each valuation reference timeframe (the month). For the sake of completeness, in Figure 2 is reported a visual representation of the interpolated risk-free rate term structure. We can clearly see how short-term interest rates decreases over the whole time horizon 2014-2018 and became even negative starting from 2015.

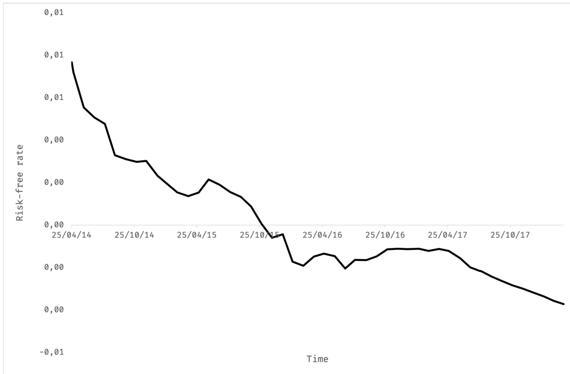

Figure 2: Risk-free rate evolution over time

2.6 Historical volatility

The volatility of a given underlying is a measure of our uncertainty about the provided returns. In real world, volatility is an unobservable component of an option's price, so options traders look instead at "implied volatility", which can be retrieved by an operation of *reverse engineering* with quoted option prices. In our work, we estimated the monthly volatility of the empirical index log-returns observed over a specific time-horizon. The formula used is the following:

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{N}(u_i - \bar{u})^2}{N-1}}\,, \tag{7}$$

where $u_i = ln(\frac{S_i}{S_{i-1}})$ is the log-return of the underlying index between date i and $i-1$, i.e. the logarithm of the ratio between each Index observation and the previous one, $\bar{u}$ is the average log-return over the sample period chosen and N is the dimension of this sample (i.e. the number of valuation days considered for the computation). Starting from the issuance date and for each month, until maturity, we took as a sample all the previous values of the Index: when possible, we took the latter 180 observations, as commonly done in practice. In order to get an annualized value for the volatility, each value has been then multiplied for the scaling factor $\sqrt{252}$. In Figure 3 is depicted the volatility evolution over the whole life of the contract.

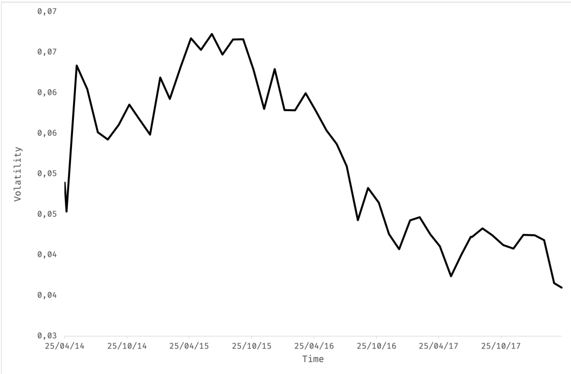

Figure 3: Volatility evolution over time

As we can appreciate from the above plot, realized volatility overtook the target value of 6% only at few dates; in the remaining ones it assumed values far below this threshold and this implied a weighting mechanism assigning high weights (or even the maximum, 150%) to the fixed-income Basket (see Equation 4).

2.7 The Certificate Value

Once we computed the key parameters (risk-free rate, time to maturity and the volatility) for each key timestamp of interest (each month), we proceeded our analysis with the pricing of the Certificate using the Black & Scholes formula for European call options, namely:

$$c = S \cdot N(d_1) - Ke^{-r_f T} \cdot N(d_2) \tag{8}$$

In this formula S is the Index underlying our derivatives product, K is the strike price[3], r_f is the risk free rate and T the time to maturity, $d_1 = \frac{ln\frac{S}{K} + (r_f + \frac{\sigma^2}{2})T}{\sigma\sqrt{t}}$, $d_2 = d_1 - \sigma\sqrt{T}$, and N the standard normal cumulative distribution function. Figure 4 plots the evolution of the Certificate value over its life period.

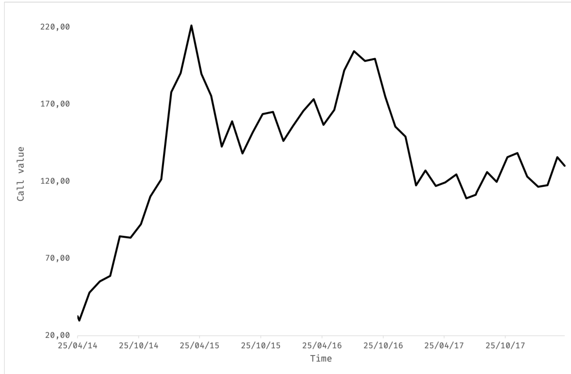

Figure 4: Call value evolution over time

On final calculation date, the call payoff is simply computed as:

$$\pi = MAX\Big(S_t - K; 0\Big) \tag{9}$$

where $S_t = 1130.21$ is the underlying value on final date, $K = 1000$ is the strike price and the final payoff π is equal to 130.21.

2.8 The Greeks

From the B&S equation (Equation (2)) it is possible to derive some key parameters known as *Greeks*, which represent the partial sensitivities of the price of the option (call, in this case) to a change in an underlying parameter and keeping fixed the other components. Each of them explains how the price function behaves when one of its variables change and provide a very useful and synthetic representation of the associated risk. The first considered Greek is the so-called *Delta*, and for a call option can be computed as

$$\Delta = \frac{\partial C}{\partial S} = N(d_1) \tag{10}$$

More specifically, it measures the sensitivity of the call value with respect to its underlying price.

[3] Please note that here we are referring to a strike price of 1000; recalling Equation 1 and ignoring the maximum element for the sake of simplicity: $SA = CA \times \frac{Underlying_{Final}}{Underlying_{Strike}} \times (1 - 0.011)^4$ and $K = CA \times 1 = 1000$

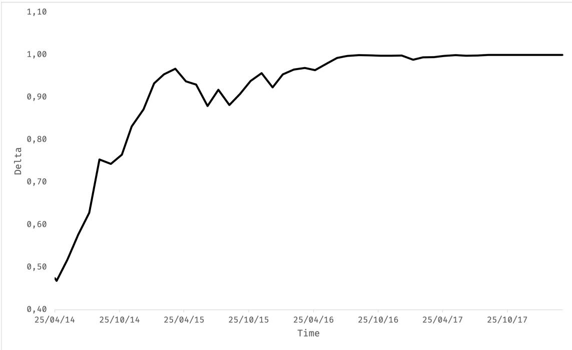

Figure 5: Δ evolution over time

Δ can be graphically interpreted as the slope of the tangent line to the call value computed at the actual underlying index value. It grows proportionally as the (forward) *moneyness* of the option grows; it assumes values in $(0, 1)$ and becomes larger and larger as the call passes from being *out-of-of-the-money* $(S < K)$, to *at-the-money* $(S = K)$ or even to *in-the-money* $(S > K)$. By observing the figure below, it can be noticed that for the great majority of the option's life, Δ has been above 0.5, meaning that the call itself became quite quickly ITM. This has been certainly a positive aspect from the investor's viewpoint because the option constantly grew in value throughout its life and faced only few light downturns.

The second greek considered is *gamma*: it is defined as a "second-order" partial derivative of the option price with respect to the underlying asset price or equivalently it represents the first partial derivative of Δ (with respect to S). Mathematically, it can be computed as follows:

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 C}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{t}} \tag{11}$$

Γ measures the sensitivity of Δ when the underlying price changes, and gives a geometrical interpretation of the convexity in the $C = C(S)$ relationship. For a long plain-vanilla call option, Γ assumes positive values, meaning that the option's delta increases as the underlying price grows; alternatively, this means that the more the underlying price grows, the more than proportional is the change in the option price. Intuitively, Γ indicates the slope of the tangent in the relationship between Δ and S: as the call gets more and more ITM and Delta tends asymptotically to 1, Gamma (almost) gradually decreases towards zero.

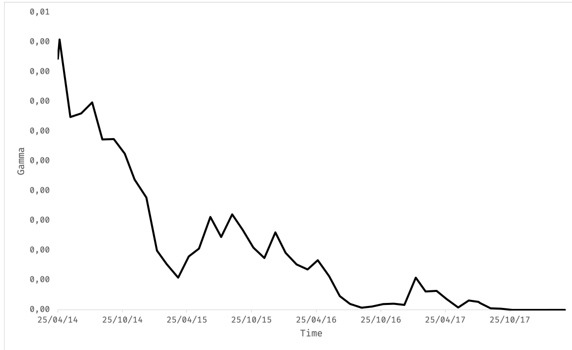

Figure 6: Γ evolution over time

When evaluating an option, it is useful to have an insight on how the time effect modifies its value. This can be achieved by considering the greek *Theta*, which is the partial derivative of the option's value with respect to its time to maturity ("time decay"). For a call option θ is computed as

$$\Theta = \frac{\partial C}{\partial t} = -\frac{S \cdot N'(d_1)\sigma}{2\sqrt{t}} - r_f K e^{-r_f t} \cdot N(d_2) \tag{12}$$

Theta assumes negative values for long call options and positive values for short call options. As can be seen in Figure 7, in our case the option's θ behaves weirdly since it starts from negative values and then constantly grows towards positive ones. This is probably due to the fact that we are dealing with a Certificate which is not at all plain-vanilla contract, but a structured product.

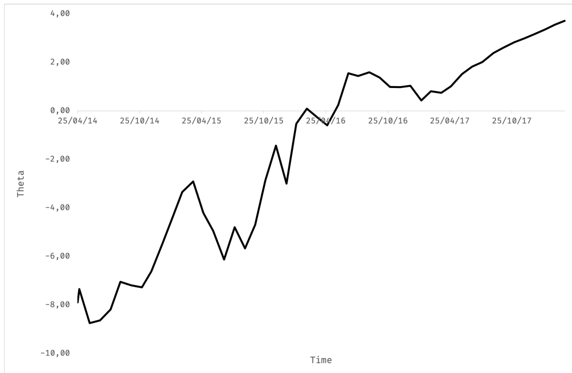

Figure 7: Θ evolution over time

The causes for this anomalous trend can be found out by observing two key facts: first, the negative and decreasing trend of the risk-free rate; this implies that the term $r_f K e^{-r_f t} \cdot N(d_2)$ in (12) becomes positive as r_f is negative). Second, the decreasing trend in the volatility σ reduces the first component in (12), allowing the second one (become positive) to prevail.

Next, the sensitivity of the call value towards the volatility is represented by the greek *Vega*, which is computed as follows:

$$\mathcal{V} = \frac{\partial C}{\partial \sigma} = S \cdot N(d_1)\sqrt{t} \tag{13}$$

For a long call option V is typically positive; this general behavior is recoverable also in our Certificate, as it can be seen in Figure 8. In our case, this greek reported a trend which is approximately coherent with the theoretical one for standard stock options: $\mathcal{V}$ peaked when the call value was ATM and then slightly decreased as maturity approached: volatility usually impacts on time value only, and its effect becomes weaker and weaker as maturity approaches.

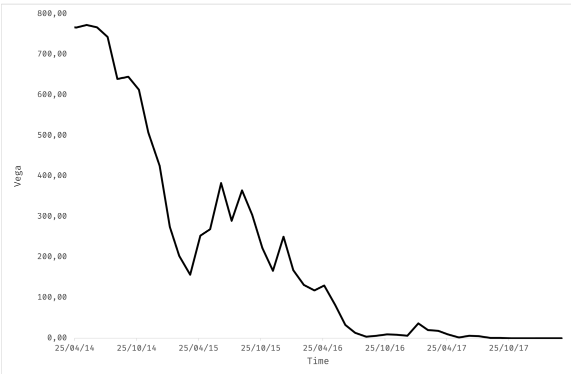

Figure 8: $\mathcal{V}$ evolution over time

The last greek considered is the so-called *Rho*, which is the partial derivative of the option's value with respect to the risk-free rate. It can be computed as:

$$\rho = \frac{\partial C}{\partial r_f} = Kte^{-r_f t} \cdot N(d_2) \tag{14}$$

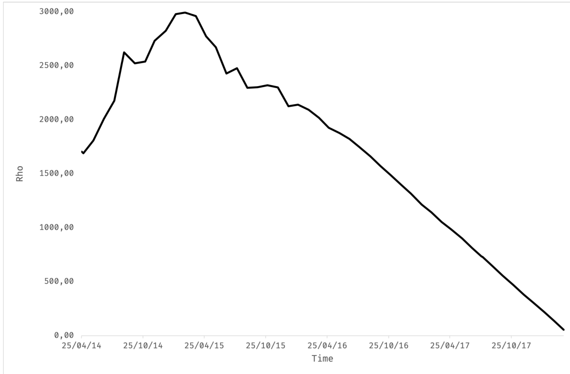

Figure 9: ρ evolution over time

Just for completeness and after having defined the five main greeks, we would like to propose the B&S equation ((2)) rewritten in terms of Δ, Γ and Θ.

$$\Theta + rS\Delta + \frac{1}{2}S^2\sigma^2\Gamma = rC \tag{15}$$

2.9 On the Black-Scholes hypotheses

The Black & Scholes model relies on some key assumptions, already mentioned in paragraph 2.2; those assumptions describes an ideal case, but it is notoriously very rare to observe all of them in the real-world. The results passed through in the previous sections can help us in discerning if and which of such assumptions hold (or not) in our framework. In particular, we summarize here our main findings:

1. As for the first assumption, we can immediately assess that the drift rate and the volatility are not constant; furthermore, by looking at Figure 10 we conclude that the underlying index is not a Geometric Brownian Motion. This because, by means of a retrospective analysis, the distribution of the log-returns on the index resulted to be not normal (or equivalently the distribution of the returns do not follow a lognormal distribution); this is clear when noting the fatter tails and the higher mass in the bulk of the distribution when compared with a normal distribution[4]. The conjecture of assuming a GBM can be made at issuance but, practically, can be back-tested only *ex-post*.

[4] We also execute a back-testing of those statements: in particular, we found out that the kurtosis of the empirical distribution of the log-returns is 7.81, value which is much higher than the target value of 3; this conclusion can be drawn also by looking at the result of the Shapiro test, which confirmed our thesis. Furthermore, the observed skewness amounts to -0.22, showing asymmetry.

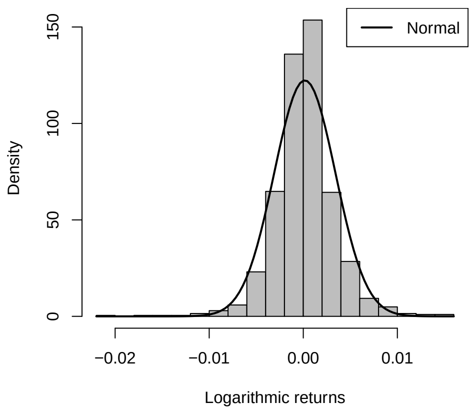

Figure 10: Histogram of log returns and fitted normal density

2. In the real world, transaction costs and taxes always exist. Requiring the absence of such elements represent a simplification of reality which can bring to misleading or even incorrect results. To this regard, relaxing the assumption induces to significantly underestimate the fair price calculated, which does not take into consideration these costs.

3. There were only 30,000 Certificates available for sale at issuance (minimum purchasable amount: one unit). Therefore, the underlying can be seen as not fractionable, keeping also in mind that it is not a standard product but the results of a weighted average between a fixed-income portfolio and the 1M EURIBOR. Recall also that the index is not managed actively by Commerzbank, but is only computed with the goal of serving as a benchmark for other derivatives products. In general, fractionable securities are required to be able to perfectly hedge the position assumed in derivatives contract; as explained above, this was not possible in our Certificate and so also this hypothesis is invalid.

4. As for the short-selling assumption we can draw the same conclusion as in the three points before, i.e. it is not verified. In the B&S model, it is assumed that investors are allowed to take short position in the underlying asset of the derivatives instruments (and also in the derivatives product). This is of course not the case for our Certificate since, as already anticipated, it is based on a non-tradable index. Furthermore, investors are only allowed to buy the derivatives and not to take a short position on it.

5. The risk-free rate is not constant: this invalidates probably the most important assumption as the B&S model is derived by imposing the return of the portfolio (composed by δ shares of the underlying assets and one derivative) equal to the risk-free rate. It turns out that this equality holds only for a short period of time, after which the portfolio must be rebalanced to ensure that the assumption is valid over the whole life of the contract. In addition, another element which contributes to further undermine the assumption is the fact that in the timespan 2014-2018 the risk-free rate experienced significant changes due to the extraordinary measures put in place by the ECB.

6. Finally, the last assumption of no-dividend payment is respected (the only one!) since we are not dealing with any kind of equity stocks which could pay any income.

Most of the B&S assumptions are violated. It is intuitive to assess that the more the market departs from Black-Scholes ideal model, the greater the potential divergence between the unknown theoretical price and the estimated with the B&S world.

After this careful analysis, from now on, we will consider as if all the assumptions of the model are satisfied; this in order to present the Qualitative Analysis in the following Section (3). Therefore, we assume as true the price estimated with our method.

3 Qualitative Analysis

In the present section, we proceed with our analysis, this time making some qualitative assessment over the Certificate. More specifically, we split the section into two parts, namely *the investor's viewpoint* and *the issuer's viewpoint*. For both cases, we will try to explain and suggest possible rationales that could have made appealing investing in or issuing/selling the derivatives instrument.

3.1 The Investor's Viewpoint

In order to understand if an investor could find the Certificate a good investment opportunity, we have to unbundle the multi-layered structure of the contract.
Recall again that the Certificate we are analyzing is in practical terms a call option. Therefore, a potential investor in this derivatives instrument would exercise

his/her right only if the spot price of the underlying asset (in this specific case the value of the Commerzbank Efficiency Growth Index) is greater than the strike price (defined as in Subsection 2.1). The payoff at maturity of the contract is reported as follow:

$$SA = CA \times Max\left(0; \frac{Underlying_{Final}}{Underlying_{Strike}} \times (1-0.011)^4 - 1\right)$$

from which we can retrieve that it would be optimal to exercise the right if:

$$\frac{Underlying_{Final} \times (1-0.011)^4}{Underlying_{Strike}} - 1 =$$
$$= \frac{Underlying_{Final} \times (1-0.011)^4 - Underlying_{Strike}}{Underlying_{Strike}} > 0$$

i.e. if the final return of the Index (adjusted for the factor $(1 - 0.011)^4$, a sort of fee) when compared to the strike price is positive. Note also that the return is magnified by the CA (Calculation Amount) equal to 1000. This can be interpreted as a sort of leveraging amount (10x) for the final return.
From these considerations, at issuance, an investor would be interested in investing in such a product only if she/he had a bullish expectation about the index value (see Equation 1) over the whole life of the contract. On a daily basis, this is equivalent to requiring that the sum of the two expressions within square brackets reported below is positive or, equivalently, that the whole sum (the "Growth Factor") is greater than 1.

$$1 + \left[w_{t-1} \cdot \left(\frac{fund_t}{fund_{t-1}}\right)\right] + \left[(1-w_{t-1}) \cdot EURIB_{t-1} \cdot \frac{Act}{conv}\right]$$

Over the whole contract life, this is instead equivalent to require that the cumulative magnitude of the Growth Factor is above the unit. Going more indepth, it is straightforward to note that the index value strictly depends on the Basket gross return (Efficiency Growth Fund) and the 1M EURIBOR, where the time-dependence is given by the weights of these two components in the index. The weighting mechanism is, in turn, based on the daily realized volatility of the Basket (as reported in Equation 4) and the exposure to the Bonds portfolio is capped to a maximum amount of 150%.
The key element in this derivatives instrument is the *realized volatility of the Basket*, since it affects both the value of the bonds in the Basket and the weighting mechanism. We can devise a general line of reasoning to approach the evolution of the *daily* index value based on the realized volatility; this would allow us to understand which scenario would be more favourable for an investor in this derivatives product.

- If the realized volatility turns out to be high (i.e. much greater than the target of 6%), the weighting mechanism assigns a weight < 100% to the Basket and starts giving some positive weight to the 1M EURIBOR. Since the component which can induce higher gains (but also losses) for an investor is the Basket, this takes under control the potential gains of investors. There are two cases to be distinguished based on the directional component of the volatility. i) If the high volatility translates in decreasing bond prices (and so, for the inverse price-interest rates relationship, in higher rates) then the overall daily return on the index would be restrained or even negative; ii) vice versa, if the high volatility translates in increasing bond prices in such a way the return of the bond portfolio is high, then the overall value of the index would be positive but always "under control" since the lower-than-100% weight given to the Basket.

- If, on the other hand, the realized volatility turns out to be low, the weighting mechanism tends to overweight the Basket, whose value however would be not so volatile given the modest movements in its price. This would reduce or at least represent a hurdle to the achievement of very high bond portfolio's returns. Moreover, a further factor that limits the increasing potential of the index value is represented by the upper constraint on the exposure to the Basket (150%). As a final remark, it is worth noting that in case of low volatility the overexposure to the index (greater than 100%) would be counterpoised by a negative exposure to the 1M EURIBOR. This fact, in the hypothesis of approximately zero (or even negative) 1M EURIBOR, represents a high advantage from the investor viewpoint, since it would guarantee an exposure only to the Basket (or even the possibility to obtain positive returns from both the Basket and the EURIBOR in negative interest rates environment[5]).

[5]Recall that in 2014 1M EURIBOR rates were very low, since the unconventional monetary policies started in the previous years by Central Banks around the world, in particular by the ECB. However, at the time it was a very strange and new situation: even negative interest rates were hardly predictable. Figure 11 plots the evolution of interest rates in Eurozone; note the low or even negative values in the right part of the plot, starting from 2014

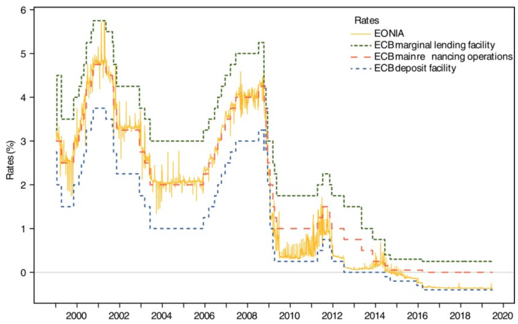

Figure 11: Evolution of interest rates in Eurozone (Period 2000-2020)

All these things said, we can understand how the structure of the derivatives is not designed to favour one party of the contract rather than the other one. It was just a matter of macroeconomic expectations about the 2014-2018 time span. Both parties could have benefited from this contractual structure, according to different conceivable scenarios. Maybe the structure was devised by Commerzbank to be safeguarded from extreme negative scenarios. This is because, very briefly, one positive factor for the investor arising from one variable is (partially) offset by a negative effect in another interrelated variable. Just to make an example in this respect, a high volatility regime could have guaranteed the investor a high return on the Basket but, at the same time, lower exposure to the same Bond portfolio.
Nevertheless, these considerations do not preclude a possible interest from an investor standpoint to buy such a derivatives instrument at issuance. An investor which in 2014 believed in lower or even negative interest rates and in prolonged accommodative monetary policies from the ECB could have been brought to buy it. These expected scenarios, together with low realized volatility of the Basket in the 4-year horizon 2014-2018, would have guaranteed a major exposure to the Basket, a positive contribution from the 1M EURIBOR (given by the combination of the negative weights and the negative benchmark interest rate) and a consequent slow but constant growth of the Index value, which is the key element triggering the decision of exercising or not the Certificate. Probably the one described would have been the best-case scenario for an investor in the derivatives. [6]

Letting aside the possible expectations of a potential investor in the Certificate, we can now try to put ourselves in the shoes of a potential investor and try to find out which practical or strategical motivations could have brought us to buy the Certificate. Before doing that, two remarks are needed: first, being the derivative a sort of European call option, the maximum loss attainable is bounded to the cost of the contract (90€ each). Second, given its complex nature, we can presume that the target clientele of this kind of product could be restricted to institutional investors, namely other banks, insurance companies or other financial institutions. These things being said, we can browse all the three main purposes generally pursued by market participants when buying non-standard derivatives instruments: hedging, speculation and arbitrage.

Arbitrages can be immediately excluded since the estimated price at issuance is 33€, which is much lower than the effective market price of 90€ at which the contracts have been traded. Furthermore, the underlying index of the Certificate is a non-traded index that is used only for benchmarking goals. This fact eliminates the possibility to exploit arbitrages, since the non-tradability of the underlying asset.

The *speculation* argument is also undermined by the great gap between the estimated theoretical price and the actual market price. Nevertheless, financial institutions with solid macroeconomic expectations favouring the growth of the index (like the one underlined above) could have devised complex speculative strategies making use of the Certificate. As an example, principal-protected strategies where the excess amount would have been invested in this Certificate could have worked well, provided that the right market outcomes were realized. These kinds of strategies would have guaranteed conspicuous gains in optimistic scenarios and re-entry in the invested capital in the most pessimistic ones.

Finally, also the *hedging* purpose could have played a role in the choice of an investor to buy the Certificate. In this regard, normally all financial institutions (especially commercial banks) prefer positive interest rates environments since these can guarantee high-interest margins: indeed, the main source of their remuneration comes from the interests paid by the borrowers on the amount requested. For an underwriter, buying the Certificate would have represented a sort of hedging to even lower (or negative) interest rates. This can be motivated by recalling again that the derivatives product allowed to assume a bullish position on a fixed-income fund, composed of a variety of European bonds. Since bond prices are inversely related to interest rates, buying the Certificate would have allowed financial institutions to benefit from an even more de-

[6] Note: this is actually the scenario that occurred, judging ex-post the historical data; the most penalizing from the issuer's perspective.

creasing term structure. In an atmosphere of financial uncertainty as to the one in 2014 (and in the years before), it would probably have been a good hedging strategy, especially if considered within a principal-protected strategy framework.[7]

To wrap up, an investor could have found attractive to invest in the Certificate. From our point of view, this interest would have arisen mainly from speculation or hedging purposes, both related to expectations or fears of a lower or even negative term structure of interest rates (in particular for short-term maturities). This would have granted a twofold positive profit on the underlying index of the derivatives contract coming from high return on the fixed-income fund and the combination of negative 1M EURIBOR and its weighting factor. The costs to be paid for entering into this exotic product are given by the huge gap (of approximately 57€ per contract) between the theoretical estimated price and the actual costs of the contract. As for the risks (in any case limited to the amount invested since it is a call option), they would have been related to adverse market developments, especially in interest rates' dynamics. In particular, an increase in interest rates could have made the value of the bond-market to plummet; this in turn, would have influenced negatively the index performance, bringing to the option being not exercised. The investor would have ended up having overpaid the position without any valuable payoff; anyhow, this scenario could have been mitigated if a proper strategy (principal-protected) were put in place.

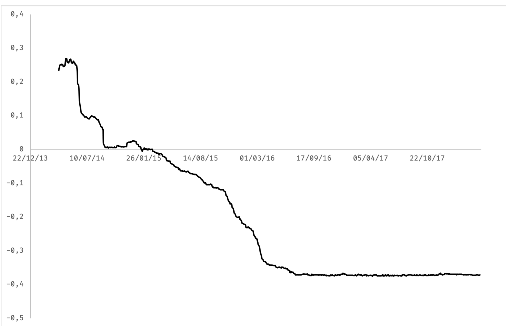

Figure 12: 1M EURIBOR after the QE announcement in mid-2014

3.2 The Issuer's Viewpoint

In this subsection, we consider the perspective of the issuer, which can be deemed as the part taking a short position in the certificates. From the deep analysis carried out in the previous subsection, we can dare to say that Commerzbank was betting on an increase in interest rates, as opposed to the investors' perspective.
Commerzbank issued 30,000 certificates at an initial unitary price of 90€: these accounted for total proceeds of €2,700,000, which is not such a relevant amount for the bank. Considering the valuation of the contract developed in Section 2 which returned us a fair price of 33€, we can clearly assess that the issuer was able to cash in a great initial profit, at least at issuance. Maybe, this great divergence could be justified by saying that the bank wanted to ensure itself a sort of "protective buffer" against adverse scenarios of even more decreasing interest rates, which in turn would have magnified the Basket's return and so the value of the index underlying the Certificate. Recall again the air of uncertainty turning around in 2014: nobody had a clear idea of what would have happened in future months. On one side, the ultra-accommodative monetary policies from the ECB would have suggested a prolonged period of very low-interest rates. On the other side, instead, this anomalous situation made quite widespread the view of rising interest rates.

It is worth also recalling how, at least theoretically, the structure of the multi-layered derivatives product has been thought to avoid extreme negative outcomes for the issuer: whatever movement in market conditions would have benefited the investor from one side but "damaged" him/her from another point of view. Furthermore, opposite considerations to the ones made for hedging and speculation purposes for the investor are valid for the issuer (in this regard, please refer to Subsection 3.1).

Finally, in addition to the purely economic benefits related to the issuance of such a product, a note must be added with reference to strategical and commercial goals that could have brought Commerzbank to issue such certificates. Financial operators are used to issuing a lot of such structured products in favour of their clients; this fact has a twofold explanation. First, big banks like Commerzbank want to give their clients the opportunity to choose among different alternatives, according to their different needs. Please note that we are not trying to suggest that big banks are charity organizations that operate in the interest of other com-

[7] Briefly, we refer to the period post-sovereign bond crises of 2011-2012 with the well-known *"Whatever it takes"* from the ECB President Mario Draghi and the first extraordinary unconventional monetary policies. Then, Quantitative Easing will be initiated in mid-2014.

panies! Of course, the products they offer are devised to return positive proceeds from their initial sale; then the final payoff is an out-of-control variable that strictly depends on macroeconomic and market outcomes. Second, these issues are aimed at strengthening the relationship among the players, which are used to subscribe financial products among each other. This was particularly true in an uncertain period as in 2014.

3.3 Concluding notes, an ex-post perspective

From an ex-post assessment of the Certificate, we can see that it turns out to be not a big deal for Commerzbank. The value of the underlying index has strongly increased during the 4 years time horizon, reaching at the expiration date a level of approximately 119. This brought investors to exercise their rights and to cash in a sound profit from the derivatives instrument. By substituting the final amounts into the Settlement Amount formula (see Equation 1), we get a per contract final payoff of:

$$\begin{aligned} SA &= 1000 \times MAX\left(0; \tfrac{119\times(1-0.011)^4}{100.78} - 1\right) \\ &= 1000 \times MAX\Big(0; 0.130\Big) = 130 \end{aligned}$$

Note how a cumulative net return on the index equal to 12.96% ($\approx$ 13.00%) is magnified by the 10x leverage represented by the Calculation Amount of 1000. Despite the great issuance gap between the theoretical price and the sale price, Commerzbank recorded an important loss, only partially covered by this "protective buffer". The loss for the whole 30,000 issued derivatives instruments can be computed as follows:

$$-(130 - 90) \times 30,000 = -1,200,000\,€$$

Looking at the attached Excel file we can investigate more in-depth the reasons that explain this final negative payoff. Over almost the whole contract life, the volatility in the Basket proved to be lower than the target volatility of 6%. In particular, the level was so low that the exposure to the Bond portfolio was almost always capped to the maximum admissible level of 150%. Furthermore, the term structure of interest rates continued to drop, especially for short-term maturities which reached also negative levels (including, of course, also the 1M EURIBOR). This triggered cumulative positive returns from the Basket and allowed the investors to exploit the mechanism consisting of both negative weights and negative 1M EURIBOR: therefore, both the component of the index contributed positively to the investors' returns. From a retrospective analysis, we can say that probably occurred the worst-case scenario for Commerzbank. However, in this respect, the loss recorded on the certificate has been compensated by the increase in the Basket (Global Euro Fund), the fixed-income fund managed by the same bank.