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ABSTRACT

In this thesis, some algebraic operators are studied and some examples of their application in semigroup theory are presented. This study contains properties of the following algebraic operators: direct product, semidirect product, wreath product and λ-semidirect product. Characterisations of certain semigroups are provided using the operators studied.

RESUMO

Nesta tese estudamos alguns operadores algébricos e apresentamos exemplos de suas aplicações. No estudo efetuado estabelecemos propriedades dos seguintes operadores algébricos: produto direto, produto semidireto, produto de wreath e produto λ-semidireto. São também estabelecidas caracterizações de certos semigrupos usando os operadores estudados.

Contents

Introduction

The main objectives of this dissertation are the study of some algebraic operators and of their importance for the development of semigroup theory, and the presentation of some examples of their application in this theory. Some of this operators are universal in the sense they are used in classes of any kind of algebras. An example of this is the direct product. Other operators were introduced only for classes of semigroups. That is the case, for example, of the λ-semidirect product. The studies about this last kind of operators can be found in several articles and in certain cases with very different terminology and notation. Thus, in the present study, we present a brief review of this knowledge.

In the preliminary phase, we study basic concepts and results concerning arbitrary semigroups as well as regular semigroups, orthodox and inverse semigroups, which are necessary to understand the subsequent chapters. For all the notations, terminologies and notions not defined in this thesis, and for the proofs of the results presented in Chapter **??**, the reader is referred to [**?**], [**?**], [**?**] and [**?**]. The following chapters contain a study of the direct product, semidirect product, wreath product and λ-semidirect product: some properties and some applications.

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