

Algorithms Cheat Sheet

Time Complexity

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Introduction

Mathematical Operations

Arithmetics

Operation	Algorithm	Input	Output	Complexity
Addition	Schoolbook	Two n -digit numbers matrices	One $n + 1$ -digit number	$O(n)$
Subtraction	Schoolbook	Two n -digit numbers matrices	One $n + 1$ -digit number	$O(n)$
Multiplication	Schoolbook	Two n -digit numbers matrices	One $2n$ -digit number	$O(n^2)$
Multiplication	3-way Toom-Cook algorithm	Two n -digit numbers matrices	One $2n$ -digit number	$O(n^{\log_3 5}) \approx O(n^{1.465})$
Multiplication	k-way Toom-Cook algorithm	Two n -digit numbers matrices	One $2n$ -digit number	$O\left(n^{\frac{\log(2k-1)}{\log k}}\right)$
Multiplication	Mixed-level Toom-Cook algorithm	Two n -digit numbers matrices	One $2n$ -digit number	$O(n2^{\sqrt{2\log n} \log n})$
Multiplication	Karatsuba algorithm	Two n -digit numbers matrices	One $2n$ -digit number	$O(n^{\log_2 3}) \approx O(n^{1.585})$
Multiplication	Schönhage–Strassen algorithm	Two n -digit numbers matrices	One $2n$ -digit number	$O(n \log n \log \log n)$
Multiplication	Harvey-Hoeven algorithm	Two n -digit numbers matrices	One $2n$ -digit number	$O(n \log n)$
Multiplication	Pointer machine ¹	Two n -digit numbers matrices	One $2n$ -digit number	$O(n)$
Multiplication	Unit Cost RAM machine ¹	Two n -digit numbers matrices	One $2n$ -digit number	$O(n)$
Division	Schoolbook	Two n -digit numbers matrices	One n -digit number	$O(n^2)$
Division	Burnikel–Ziegler Divide-and-Conquer ²	Two n -digit numbers matrices	One n -digit number	$O(M(n) \log n)$
Division	Newton–Raphson division ²	Two n -digit numbers matrices	One n -digit number	$O(M(n))$
Square root	Newton's method ²	One n -digit number	One n -digit number	$O(M(n))$
Modular exponentiation	Repeated multiplication and reduction ²	Two n -digit integers, k -bit exponent	One n -digit integer	$O(M(n)2^k)$
Modular exponentiation	Exponentiation by squaring ²	Two n -digit integers, k -bit exponent	One n -digit integer	$O(M(n)k)$
Modular exponentiation	Exponentiation with Montgomery reduction ²	Two n -digit integers, k -bit exponent	One n -digit integer	$O(M(n)k)$

¹ Theoretical model only
² $M(n)$ - The complexity of an implemented multiplication algorithm

Matrix Algebra

Operation	Algorithm	Input	Output	Complexity
Multiplication	Schoolbook	Two $n \times n$ matrices	One $n \times n$ matrix	$O(n^3)$
Multiplication	Strassen's	Two $n \times n$ matrices	One $n \times n$ matrix	$O(n^{\log_2 7}) = O(n^{2.807})$
Multiplication	Coppersmith-Winograd	Two $n \times n$ matrices	One $n \times n$ matrix	$O(n^{2.376})$
Multiplication	Alman-Williams	Two $n \times n$ matrices	One $n \times n$ matrix	$O(n^{2.3728596})$
Multiplication	Duan, Wu, Zhou	Two $n \times n$ matrices	One $n \times n$ matrix	$O(n^{2.3719})$
Multiplication	Williams, Xu, Xu, Zhou	Two $n \times n$ matrices	One $n \times n$ matrix	$O(n^{2.3716})$
Multiplication	Schoolbook	One $n \times m$ matrix, one $m \times p$ matrix	One $n \times p$ matrix	$O(nmp)$
Inversion	Gauss-Jordan elimination	One $n \times n$ matrix	One $n \times n$ matrix	$O(n^3)$
Inversion	Strassen algorithm	One $n \times n$ matrix	One $n \times n$ matrix	$O(n^{2.807})$
Inversion	Coppersmith-Winograd algorithm	One $n \times n$ matrix	One $n \times n$ matrix	$O(n^{2.376})$
Inversion	Optimised CW algorithm	One $n \times n$ matrix	One $n \times n$ matrix	$O(n^{2.373})$
SVD	Bidiagonalization, QR algorithm	One $m \times n$ matrix ($m \leq n$)	One $m \times m$	$O(m^2 n)$
			One $m \times n$ matrix	
Determinant	Laplace expansion	One $n \times n$ matrix	One number	$O(n!)$
Determinant	Division free algorithm	One $n \times n$ matrix	One number	$O(n^4)$
Determinant	LU decomposition	One $n \times n$ matrix	One number	$O(n^3)$
Determinant	Bareiss algorithm	One $n \times n$ matrix	One number	$O(n^3)$
Determinant	Fast matrix multiplication	One $n \times n$ matrix	One number	$O(n^{2.373})$
Back substitution	Back substitution algorithm	Triangular matrix	n solutions	$O(n^2)$

Polynomials

Operation	Algorithm	Input	Output	Complexity
Polynomial evaluation	Direct	One polynomial of degree n and integer coefficients	One number	$O(n)$
Polynomial evaluation	Horner's algorithm	One polynomial of degree n and integer coefficients	One number	$O(n)$
Polynomial gcd	Euclid's algorithm	Two polynomials of degree n and integer coefficients	One number	$O(n^2)$
Polynomial gcd	Lehmer's algorithm (Fast Euclidean) ³	Two polynomials of degree n and integer coefficients	One number	$O(M(n) \log n)$

³ $M(n)$ - The complexity of an implemented multiplication algorithm

Number theory

Operation	Algorithm	Input	Output	Complexity
Greatest common divisor	Euclidean algorithm	Two n-digit integers	One integer	$O(n^2)$
Greatest common divisor	Binary GCD	Two n-digit integers	One integer	$O(n^2)$
Greatest common divisor	Left/right k-ary binary GCD	Two n-digit integers	One integer	$O(\frac{n^2}{\log n})$
Greatest common divisor	Stehle-Zimmermann algorithm ⁴	Two n-digit integers	One integer	$O(M(n) \log n)$
Greatest common divisor	Schönhage algorithm ⁴	Two n-digit integers	One integer	$O(M(n) \log n)$
Jacobi symbol	Stehle-Zimmermann algorithm ⁴	Two n-digit integers	0, -1 or 1	$O(M(n) \log n)$
Jacobi symbol	Schönhage algorithm ⁴	Two n-digit integers	0, -1 or 1	$O(M(n) \log n)$
Factorial	Bottom-up multiplication ⁴	One positive integer less than n	One integer	$O(M(n^2) \log n)$
Factorial	Binary splitting ⁴	One positive integer less than n	One integer	$O(M(n \log n) \log n)$
Factorial	Exponentiation of the prime factors of n^4	One positive integer less than n	One integer	$O(M(n \log n) \log \log n)$
Factorial	Exponentiation of the prime factors of n^4	One positive integer less than n	One integer	$O(M(n \log n))$
Primality test	AKS primality test n	n digit integer	True or false	$O(n^{6+O(1)})$
Primality test	AKS primality test with Agrawal's conjecturen	n digit integer	True or false	$O(n^3)$
Primality test	Elliptic curve test ⁵ - heuristical approach	n digit integer	True or false	$O(n^{4+\epsilon})$
Primality test	Baillie-PSW test ⁵	n digit integer	True or false	$O(n^{2+\epsilon})$
Primality test	Miller-Rabin test ⁵ ⁶	n digit integer	True or false	$O(kn^{2+\epsilon})$
Primality test	Solovay-Strassen test ⁵ ⁶	n digit integer	True or false	$O(kn^{2+\epsilon})$
Integer factorisation	General number field sieve ⁵	b -bit input integer	A set of factors	$O((1 + \epsilon)^b)$
Integer factorisation	Shor's algorithm ⁴ ⁷	b -bit input integer	A set of factors	$O(M(b)b)$

⁴ $M(n)$ - The complexity of an implemented multiplication algorithm

⁵ ϵ - a positive constant

⁶ k - a positive constant

⁷ Theoretical model, on quantum computer

Additional Operations

Special Functions - Elementary functions

Operation	Algorithm	Input	Output	Complexity
Discrete Fourier transform	Schoolbook	Size n data sequence	Set of complex number	$O(n^2)$
Discrete Fourier transform	Fast Fourier transform	Size n data sequence	Set of complex number	$O(n \log n)$
Golden ration	Newton's method ⁸			$O(M(n))$
Square root of 2	Newton's method ⁸			$O(M(n))$
Euler's number	Taylor series binary splitting of the exp. function ⁸			$O(M(n) \log n)$
Euler's number	Newton inversion of the natural logarithm ⁸			$O(M(n) \log n)$
Pi	Arctan series binary splitting in Machin's formula ⁸			$O(M(n) \log^2 n)$
Pi	Gauss-Legendre algorithm ⁸			$O(M(n) \log n)$
Euler's constant	Sweeney's method ⁸			$O(M(n) \log^2 n)$
Gamma function	Approx. of the incomplete gamma function ⁸	n digit number		$O(M(n)n^{\frac{1}{2}} \log^2 n)$
Gamma function	Hypergeometric series ⁸	Fixed ration number		$O(M(n) \log^2 n)$
Hypergeometric function	Borwein and Borwein ⁸	n -digit number		$O(M(n)n^{\frac{1}{2}} \log^2 n)$
Hypergeometric function	Hypergeometric series ⁸	Fixed rational number		$O(M(n) \log^2 n)$
Taylor series	Repeated argument reduction ^{8 9}			$O(M(n)n^{\frac{1}{2}})$
Taylor series	FFT-based acceleration ^{8 9}			$O(M(n)n^{\frac{1}{3}} \log^2 n)$
Taylor series	Binary splitting + bit-burst ^{8 9}			$O(M(n) \log^2 n)$
Arithmetic-geometric mean iteration	Arithmetic-geometric mean iteration ^{8 9}			$O(M(n) \log n)$

⁸ $M(n)$ - The complexity of an implemented multiplication algorithm

⁹ Applicability: exp. log, sin, cos, arctan

Data Structures - Common Operations

Data Structure	Operation	Average Time Complexity	Worst Time Complexity
Array	Access	$O(1)$	$O(1)$
Array	Deletion	$O(n)$	$O(n)$
Array	Insertion	$O(n)$	$O(n)$
Array	Search	$O(n)$	$O(n)$
Binary Search Tree	Access	$O(\log n)$	$O(n)$
Binary Search Tree	Deletion	$O(\log n)$	$O(n)$
Binary Search Tree	Insertion	$O(\log n)$	$O(n)$
Binary Search Tree	Search	$O(\log n)$	$O(n)$
Doubly-Linked List	Access	$O(n)$	$O(n)$
Doubly-Linked List	Deletion	$O(1)$	$O(1)$
Doubly-Linked List	Insertion	$O(1)$	$O(1)$
Doubly-Linked List	Search	$O(n)$	$O(n)$
Hash Table	Deletion	$O(1)$	$O(n)$
Hash Table	Insertion	$O(1)$	$O(n)$
Hash Table	Search	$O(1)$	$O(n)$
Queue	Access	$O(n)$	$O(n)$
Queue	Deletion	$O(1)$	$O(1)$
Queue	Insertion	$O(1)$	$O(1)$
Queue	Search	$O(n)$	$O(n)$
Singly-Linked List	Access	$O(n)$	$O(n)$
Singly-Linked List	Deletion	$O(1)$	$O(1)$
Singly-Linked List	Insertion	$O(1)$	$O(1)$
Singly-Linked List	Search	$O(n)$	$O(n)$
Skip List	Access	$O(\log n)$	$O(n)$
Skip List	Deletion	$O(\log n)$	$O(n)$
Skip List	Insertion	$O(\log n)$	$O(n)$
Skip List	Search	$O(\log n)$	$O(n)$
Stack	Access	$O(n)$	$O(n)$
Stack	Deletion	$O(1)$	$O(1)$
Stack	Insertion	$O(1)$	$O(1)$
Stack	Search	$O(n)$	$O(n)$

Heap - Common Operations

Sorting Algorithms

Name	Average	Worst	Stable
Quicksort	$n \log n$	n^2	No
Merge sort	$n \log n$	$n \log n$	Yes
In-place merge sort	-	$n \log^2 n$	Yes
Introsort	$n \log n$	$n \log n$	No
Heapsort	$n \log n$	$n \log n$	No
Insertion sort	n^2	n^2	Yes
Block sort	$n \log n$	$n \log n$	Yes
Timsort	$n \log n$	$n \log n$	Yes
Selection sort	n^2	n^2	No
Cubesort	$n \log)$	$n \log n$	Yes
Shellsort	$n^{\frac{4}{3}}$	$n^{\frac{3}{2}}$	No
Bubble sort	n^2	n^2	Yes
Exchange sort	n^2	n^2	No
Tree sort	$n \log n$	$n \log n$	Yes
Cycle sort	n^2	n^2	No
Library sort	$n \log)$	n^2	No
Patience sort	$n \log n$	$n \log n$	No
Smoothsort	$n \log n$	$n \log n$	No
Strand sort	n^2	n^2	Yes
Tournament sort	$n \log n$	$n \log n$	No
Cocktail shaker sort	n^2	n^2	Yes
Comb sort	n^2	n^2	No
Gnome sort	n^2	n^2	Yes
Odd-even sort	n^2	n^2	Yes

Searching

Graphs

Data Structure Operations

Graph Search Algorithms

Operation	Algorithm	Input	Comment	Complexity
Explicit Graph Search	Depth First Search	V - vertices, E - edges		$O(V + E)$
Implicit Graph Search	Depth First Search	b - branching factor, d - depth		$O(b^d)$
Explicit Graph Search	Breadth First Search	V - vertices, E - edges		$O(V + E)$
Implicit Graph Search	Breadth First Search	b - branching factor, d - depth		$O(b^d)$
Shortest Path	Dijkstra's Algorithm	V - vertices, E - edges	Priority queue/heap	$O(V + E \log V)$
Shortest Path	Dijkstra's Algorithm	V - vertices, E - edges	Array	$O(V ^2)$

Other Graph Algorithms

Operations on Strings