

Introduction to **Information Retrieval**

<http://nlp.stanford.edu/IR-book/>

Unstructured data in 1680

- Which plays of Shakespeare contain the words ***Brutus*** ***AND Caesar*** but ***NOT Calpurnia***?
- One could `grep` all of Shakespeare's plays for ***Brutus*** and ***Caesar***, then strip out lines containing ***Calpurnia***?
- Why is that not the answer?
 - Slow (for large corpora)
 - ***NOT Calpurnia*** is non-trivial
 - Other operations (e.g., find the word ***Romans*** near ***countrymen***) not feasible
 - Ranked retrieval (best documents to return)
 - Later lectures

Term-document incidence

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

*Brutus AND Caesar BUT NOT
Calpurnia*

1 if **play** contains
word, 0 otherwise

Incidence vectors

- So we have a 0/1 vector for each term.
- To answer query: take the vectors for ***Brutus***, ***Caesar*** and ***Calpurnia*** (complemented) → bitwise *AND*.
- $110100 \text{ AND } 110111 \text{ AND } 101111 = 100100$.

Answers to query

- Antony and Cleopatra, Act III, Scene ii

Agrippa [Aside to DOMITIUS ENOBARBUS]: Why, Enobarbus,
When Antony found Julius **Caesar** dead,
He cried almost to roaring; and he wept
When at Philippi he found **Brutus** slain.

- Hamlet, Act III, Scene ii

Lord Polonius: I did enact Julius **Caesar** I was killed i' the
Capitol; **Brutus** killed me.



Basic assumptions of Information Retrieval

- **Collection**: Fixed set of documents
- **Goal**: Retrieve documents with information that is relevant to the user's **information need** and helps the user complete a **task**

How good are the retrieved docs?

- *Precision* : Fraction of retrieved docs that are relevant to user's information need
- *Recall* : Fraction of relevant docs in collection that are retrieved

Bigger collections

- Consider $N = 1$ million documents, each with about 1000 words.
- Avg 6 bytes/word including spaces/punctuation
 - 6GB of data in the documents.
- Say there are $M = 500K$ *distinct* terms among these.

Can't build the matrix

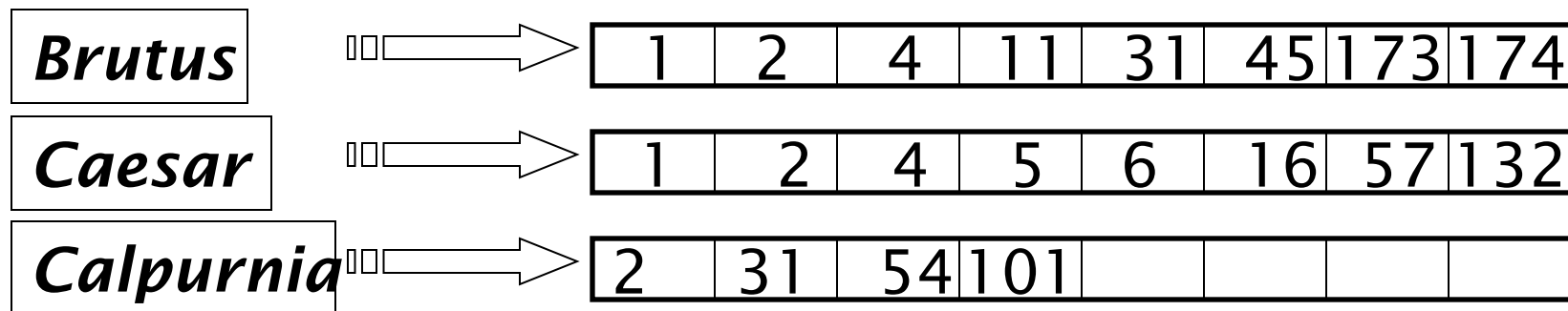
- 500K x 1M matrix has half-a-trillion 0's and 1's.
- But it has no more than one billion 1's.
 - matrix is extremely sparse.
- What's a better representation?
 - We only record the 1 positions.



Why?

Inverted index

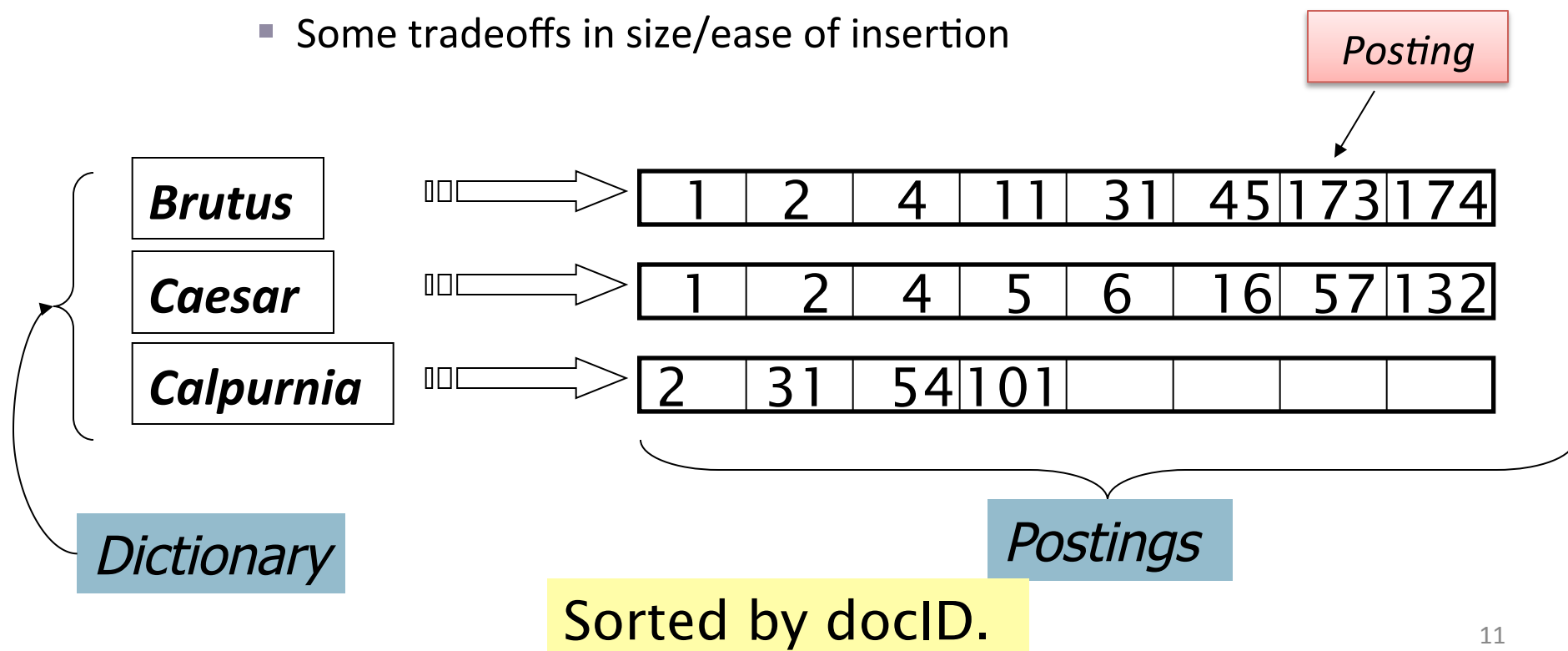
- For each term t , we must store a list of all documents that contain t .
 - Identify each by a **docID**, a document serial number
- Can we use fixed-size arrays for this?



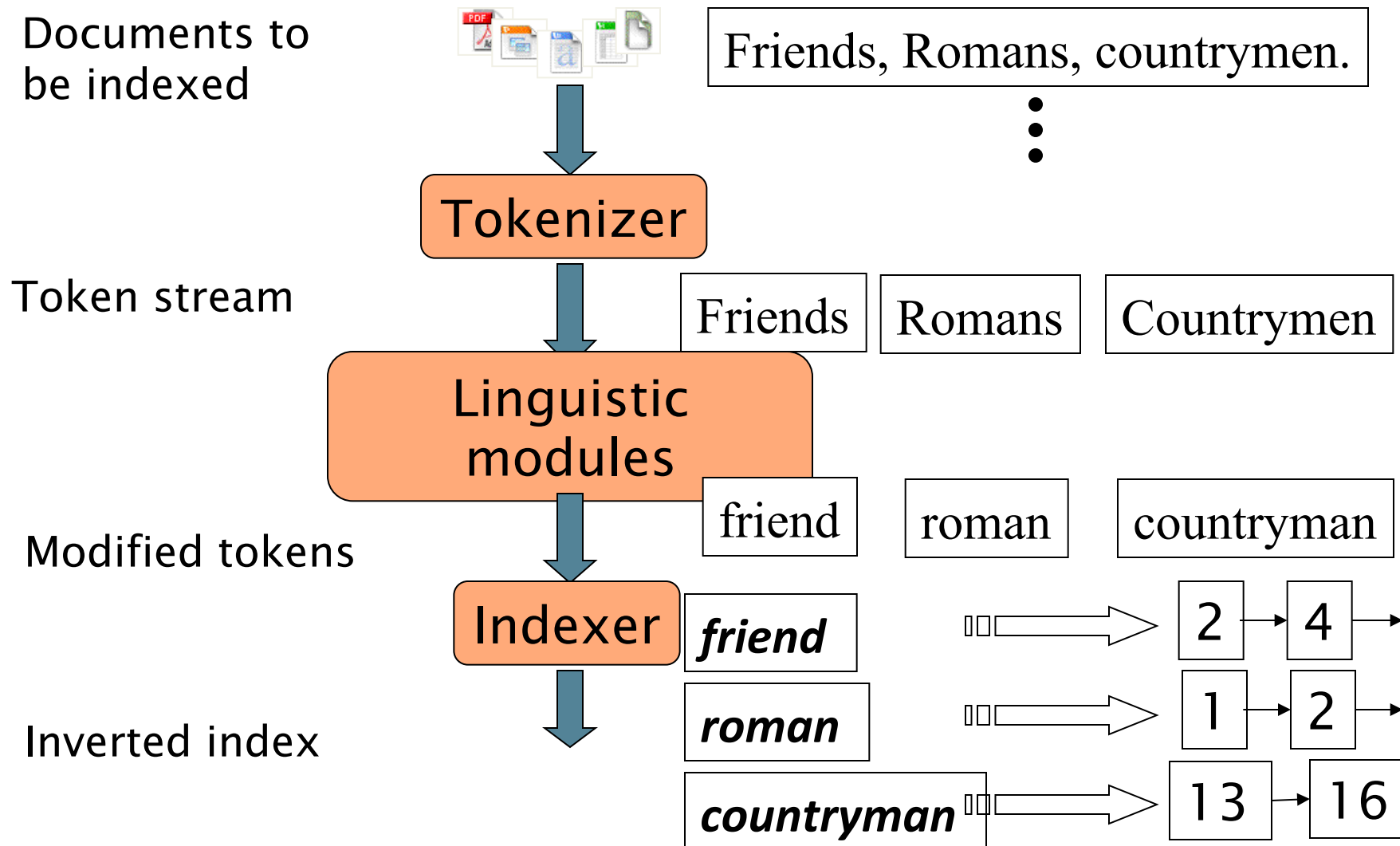
What happens if the word *Caesar* is added to document 14?

Inverted index

- We need variable-size postings lists
 - On disk, a continuous run of postings is normal and best
 - In memory, can use linked lists or variable length arrays
 - Some tradeoffs in size/ease of insertion



Inverted index construction



Indexer steps: Token sequence

- Sequence of (Modified token, Document ID) pairs.

Doc 1

I did enact Julius
Caesar I was killed
i' the Capitol;
Brutus killed me.

Doc 2

So let it be with
Caesar. The noble
Brutus hath told you
Caesar was ambitious



Term	docID
I	1
did	1
enact	1
julius	1
caesar	1
I	1
was	1
killed	1
i'	1
the	1
capitol	1
brutus	1
killed	1
me	1
so	2
let	2
it	2
be	2
with	2
caesar	2
the	2
noble	2
brutus	2
hath	2
told	2
you	2
caesar	2
was	2
ambitious	2

Indexer steps: Sort

- Sort by terms
 - And then docID



Term	docID
I	1
did	1
enact	1
julius	1
caesar	1
I	1
was	1
killed	1
i'	1
the	1
capitol	1
brutus	1
killed	1
me	1
so	2
let	2
it	2
be	2
with	2
caesar	2
the	2
noble	2
brutus	2
hath	2
told	2
you	2
caesar	2
was	2
ambitious	2



Term	docID
ambitious	2
be	2
brutus	1
brutus	2
capitol	1
caesar	1
caesar	2
caesar	2
did	1
enact	1
hath	1
I	1
I	1
i'	1
it	2
julius	1
killed	1
killed	1
let	2
me	1
noble	2
so	2
the	1
the	2
told	2
you	2
was	1
was	2
with	2

Indexer steps: Dictionary & Postings

- Multiple term entries in a single document are merged.
- Split into Dictionary and Postings
- Doc. frequency information is added.

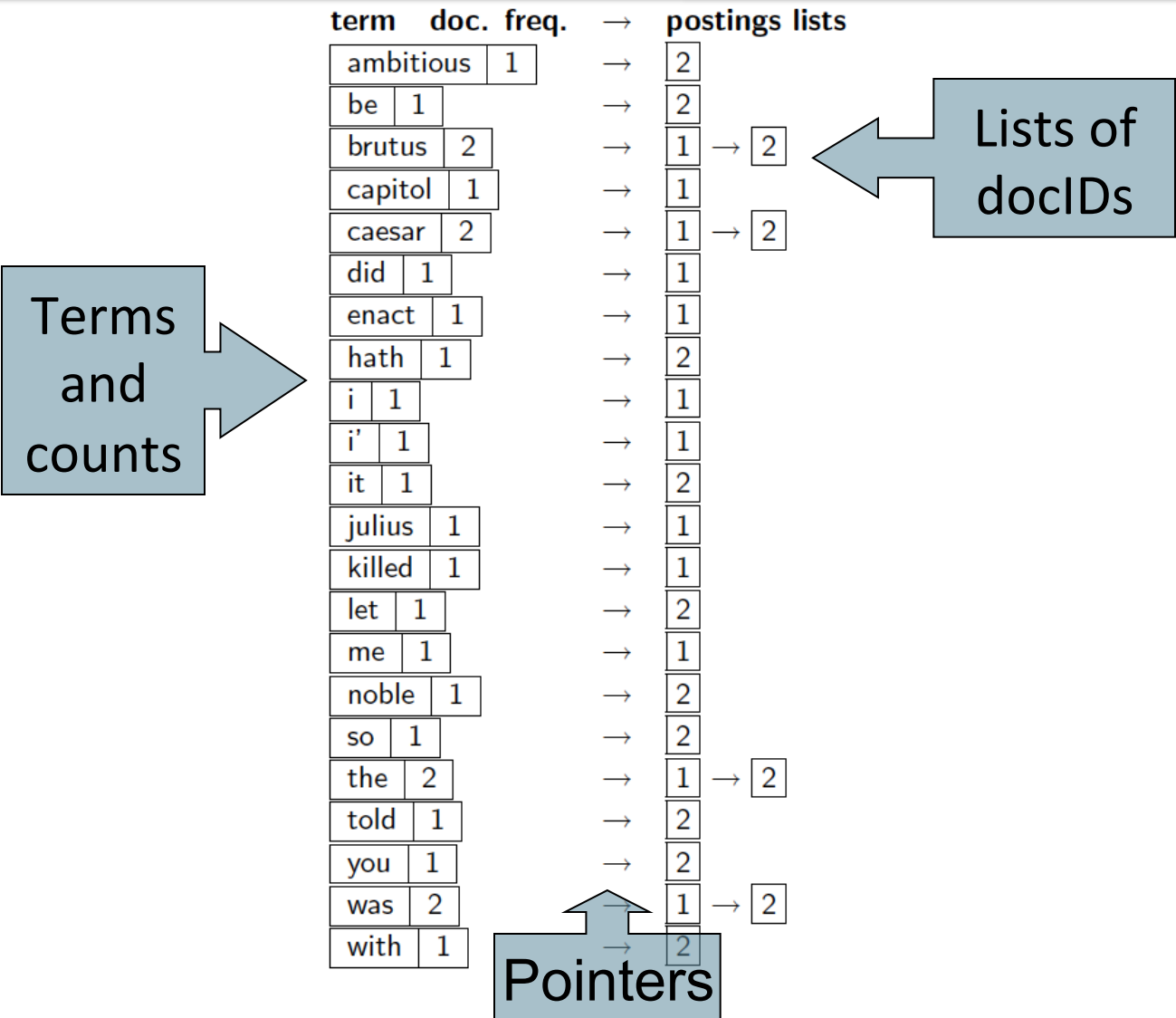
Why frequency?
Will discuss later.

Term	docID
ambitious	2
be	2
brutus	1
brutus	2
capitol	1
caesar	1
caesar	2
caesar	2
did	1
enact	1
hath	1
I	1
I	1
i'	1
it	2
julius	1
killed	1
killed	1
let	2
me	1
noble	2
so	2
the	1
the	2
told	2
you	2
was	1
was	2
with	2



term	doc. freq.	→	postings lists
ambitious	1	→	[2]
be	1	→	[2]
brutus	2	→	[1] → [2]
capitol	1	→	[1]
caesar	2	→	[1] → [2]
did	1	→	[1]
enact	1	→	[1]
hath	1	→	[2]
i	1	→	[1]
i'	1	→	[1]
it	1	→	[2]
julius	1	→	[1]
killed	1	→	[1]
let	1	→	[2]
me	1	→	[1]
noble	1	→	[2]
so	1	→	[2]
the	2	→	[1] → [2]
told	1	→	[2]
you	1	→	[2]
was	2	→	[1] → [2]
with	1	→	[2]

Where do we pay in storage?



The index we just built

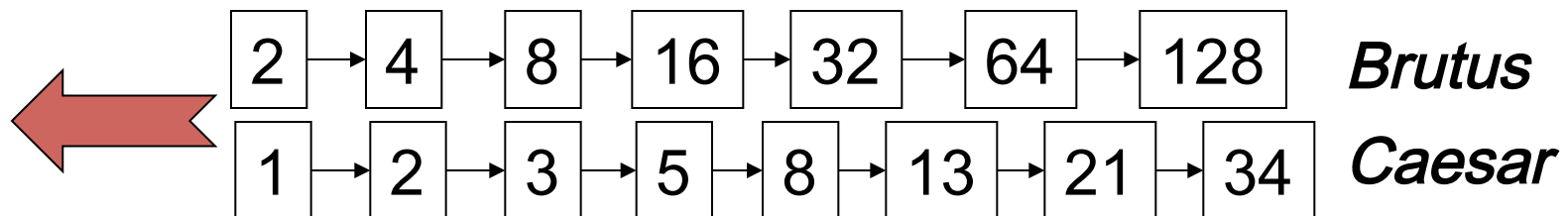
- How do we process a query?
 - Later - what kinds of queries can we process?

Query processing: AND

- Consider processing the query:

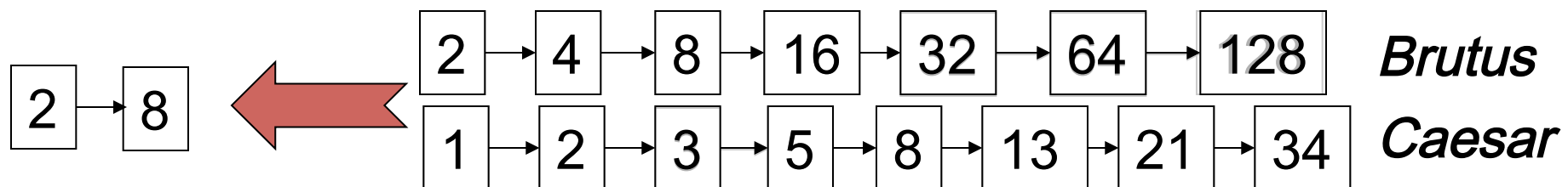
Brutus AND Caesar

- Locate ***Brutus*** in the Dictionary;
 - Retrieve its postings.
- Locate ***Caesar*** in the Dictionary;
 - Retrieve its postings.
- “Merge” the two postings:



The merge

- Walk through the two postings simultaneously, in time linear in the total number of postings entries



If list lengths are x and y , merge takes $O(x+y)$ operations.
Crucial: postings sorted by docID.

Intersecting two postings lists (a “merge” algorithm)

```
INTERSECT( $p_1, p_2$ )  
  1   $answer \leftarrow \langle \rangle$   
  2  while  $p_1 \neq \text{NIL}$  and  $p_2 \neq \text{NIL}$   
  3  do if  $docID(p_1) = docID(p_2)$   
  4      then  $\text{ADD}(answer, docID(p_1))$   
  5           $p_1 \leftarrow next(p_1)$   
  6           $p_2 \leftarrow next(p_2)$   
  7      else if  $docID(p_1) < docID(p_2)$   
  8          then  $p_1 \leftarrow next(p_1)$   
  9          else  $p_2 \leftarrow next(p_2)$   
 10  return  $answer$ 
```

Boolean queries: Exact match

- The **Boolean retrieval model** is being able to ask a query that is a Boolean expression:
 - Boolean Queries use *AND*, *OR* and *NOT* to join query terms
 - Views each document as a set of words
 - Is precise: document matches condition or not.
 - Perhaps the simplest model to build an IR system on
- Primary commercial retrieval tool for 3 decades.
- Many search systems you still use are Boolean:
 - Email, library catalog, Mac OS X Spotlight

Example: WestLaw <http://www.westlaw.com/>

- Largest commercial (paying subscribers) legal search service (started 1975; ranking added 1992)
- Tens of terabytes of data; 700,000 users
- Majority of users *still* use boolean queries
- Example query:
 - What is the statute of limitations in cases involving the federal tort claims act?
 - **LIMIT! /3 STATUTE ACTION /S FEDERAL /2 TORT /3 CLAIM**
 - /3 = within 3 words, /S = in same sentence

Example: WestLaw <http://www.westlaw.com/>

- Another example query:
 - Requirements for disabled people to be able to access a workplace
 - `disabl! /p access! /s work-site work-place (employment /3 place)`
- Note that SPACE is disjunction, not conjunction!
- Long, precise queries; proximity operators; incrementally developed; not like web search
- Many professional searchers still like Boolean search
 - You know exactly what you are getting
- But that doesn't mean it actually works better....

Boolean queries: More general merges

- Exercise: Adapt the merge for the queries:

Brutus AND NOT Caesar

Brutus OR NOT Caesar

Can we still run through the merge in time $O(x+y)$?

What can we achieve?

Merging

What about an arbitrary Boolean formula?

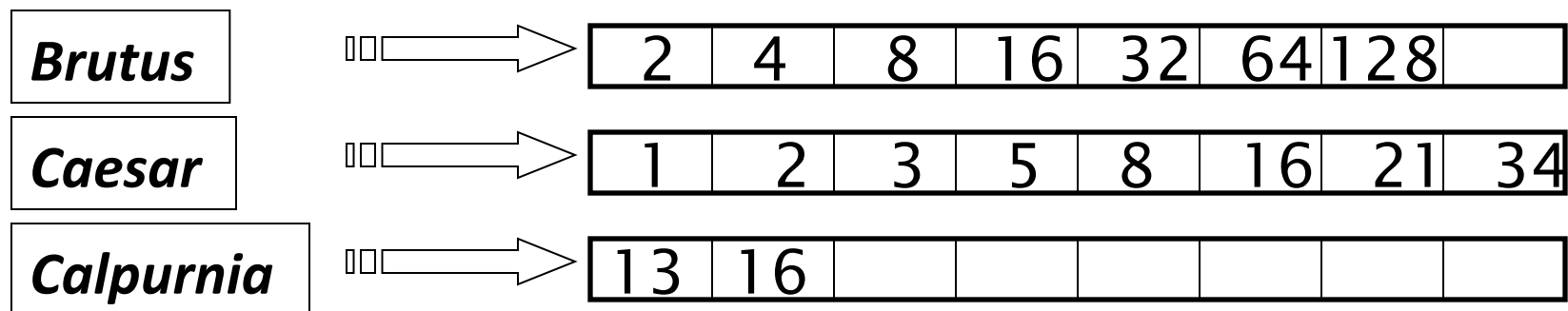
(Brutus OR Caesar) AND NOT

(Antony OR Cleopatra)

- Can we always merge in “linear” time?
 - Linear in what?
- Can we do better?

Query optimization

- What is the best order for query processing?
- Consider a query that is an *AND* of n terms.
- For each of the n terms, get its postings, then *AND* them together.



Query: **Brutus AND Calpurnia AND Caesar**

Query optimization example

- Process in order of increasing freq:
 - *start with smallest set, then keep cutting further.*

This is why we kept
document freq. in dictionary

Brutus	⇒	2	4	8	16	32	64	128	
Caesar	⇒	1	2	3	5	8	16	21	34
Calpurnia	⇒	13	16						

Execute the query as (***Calpurnia AND Brutus***) ***AND Caesar***.

More general optimization

- e.g., (*madding OR crowd*) AND (*ignoble OR strife*)
- Get doc. freq.'s for all terms.
- Estimate the size of each *OR* by the sum of its doc. freq.'s (conservative).
- Process in increasing order of *OR* sizes.

Exercise

- Recommend a query processing order for

*(tangerine OR trees) AND
(marmalade OR skies) AND
(kaleidoscope OR eyes)*

Term	Freq
eyes	213312
kaleidoscope	87009
marmalade	107913
skies	271658
tangerine	46653
trees	316812

Query processing exercises

- **Exercise:** If the query is *friends AND romans AND (NOT countrymen)*, how could we use the freq of *countrymen*?
- **Exercise:** Extend the merge to an arbitrary Boolean query. Can we always guarantee execution in time linear in the total postings size?
- **Hint:** Begin with the case of a Boolean *formula* query where each term appears only once in the query.

Exercise

- Try the search feature at <http://www.rhymezone.com/shakespeare/>
- Write down five search features you think it could do better

What's ahead in IR?

Beyond term search

- What about phrases?
 - *Stanford University*
- Proximity: Find ***Gates NEAR Microsoft.***
 - Need index to capture position information in docs.
- Zones in documents: Find documents with
(*author = **Ullman***) AND (text contains ***automata***).

Evidence accumulation

- 1 vs. 0 occurrence of a search term
 - 2 vs. 1 occurrence
 - 3 vs. 2 occurrences, etc.
 - Usually more seems better
- Need term frequency information in docs

Ranking search results

- Boolean queries give inclusion or exclusion of docs.
- Often we want to rank/group results
 - Need to measure proximity from query to each doc.
 - Need to decide whether docs presented to user are singletons, or a group of docs covering various aspects of the query.

IR vs. databases:

Structured vs unstructured data

- Structured data tends to refer to information in “tables”

Employee	Manager	Salary
Smith	Jones	50000
Chang	Smith	60000
Ivy	Smith	50000

Typically allows numerical range and exact match (for text) queries, e.g.,
Salary < 60000 AND Manager = Smith.

Unstructured data

- Typically refers to free-form text
- Allows
 - Keyword queries including operators
 - More sophisticated “concept” queries, e.g.,
 - find all web pages dealing with *drug abuse*
- Classic model for searching text documents

Semi-structured data

- In fact almost no data is “unstructured”
- E.g., this slide has distinctly identified zones such as the *Title* and *Bullets*
- Facilitates “semi-structured” search such as
 - *Title* contains data AND *Bullets* contain search

... to say nothing of linguistic structure

More sophisticated semi-structured search

- *Title* is about Object Oriented Programming AND *Author* something like stro*rup
- where * is the wild-card operator
- Issues:
 - how do you process “about”?
 - how do you rank results?
- The focus of XML search

Clustering, classification and ranking

- **Clustering:** Given a set of docs, group them into clusters based on their contents.
- **Classification:** Given a set of topics, plus a new doc D , decide which topic(s) D belongs to.
- **Ranking:** Can we learn how to best order a set of documents, e.g., a set of search results

The web and its challenges

- Unusual and diverse documents
- Unusual and diverse users, queries, information needs
- Beyond terms, exploit ideas from social networks
 - link analysis, clickstreams ...
- How do search engines work?
And how can we make them better?

More sophisticated *information* retrieval

- Cross-language information retrieval
- Question answering
- Summarization
- Text mining
- ...

TOKENS AND TERMS

Tokenization

- Issues in tokenization:
 - ***Finland's capital*** →
Finland? Finlands? Finland's?
 - ***Hewlett-Packard*** → ***Hewlett*** and ***Packard*** as two tokens?
 - ***state-of-the-art***: break up hyphenated sequence.
 - ***co-education***
 - ***lowercase, lower-case, lower case*** ?
 - It can be effective to get the user to put in possible hyphens
 - ***San Francisco***: one token or two?
 - How do you decide it is one token?

Tokenization: language issues

- French
 - *L'ensemble* → one token or two?
 - *L ? L' ? Le ?*
 - Want *l'ensemble* to match with *un ensemble*
 - Until at least 2003, it didn't on Google
 - Internationalization!
- German noun compounds are not segmented
 - *Lebensversicherungsgesellschaftsangestellter*
 - 'life insurance company employee'
 - German retrieval systems benefit greatly from a **compound splitter** module
 - Can give a 15% performance boost for German

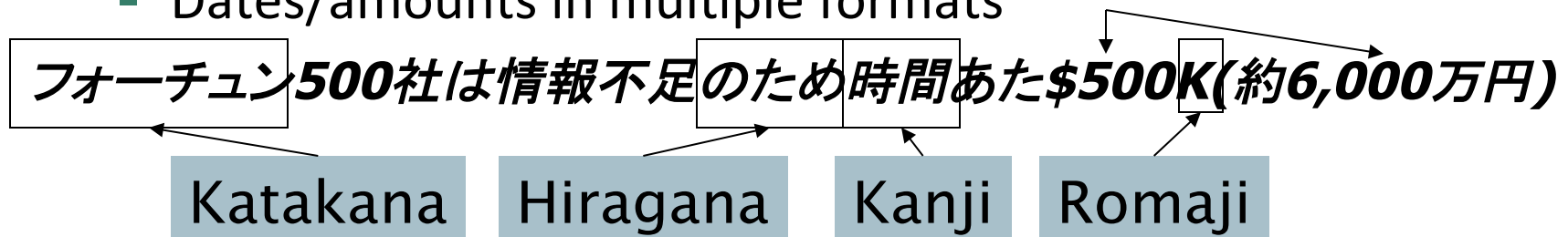
Tokenization: language issues

- Chinese and Japanese have no spaces between words:

- 莎拉波娃现在居住在美国东南部的佛罗里达。
- Not always guaranteed a unique tokenization

- Further complicated in Japanese, with multiple alphabets intermingled

- Dates/amounts in multiple formats



End-user can express query entirely in hiragana!

Tokenization: language issues

- Arabic (or Hebrew) is basically written right to left, but with certain items like numbers written left to right
- Words are separated, but letter forms within a word form complex ligatures
- استقلت الجزائر في سنة 1962 بعد 132 عام من الاحتلال الفرنسي.
- ← → ← → ← start
- ‘Algeria achieved its independence in 1962 after 132 years of French occupation.’
- With Unicode, the surface presentation is complex, but the stored form is straightforward

Stop words

- With a stop list, you exclude from the dictionary entirely the commonest words. Intuition:
 - They have little semantic content: *the, a, and, to, be*
 - There are a lot of them: ~30% of postings for top 30 words
- But the trend is away from doing this:
 - Good compression techniques means the space for including stopwords in a system is very small
 - Good query optimization techniques mean you pay little at query time for including stop words.
 - You need them for:
 - Phrase queries: “King of Denmark”
 - Various song titles, etc.: “Let it be”, “To be or not to be”
 - “Relational” queries: “flights to London”

Normalization to terms

- We need to “normalize” words in indexed text as well as query words into the same form
 - We want to match ***U.S.A.*** and ***USA***
- Result is terms: a **term** is a (normalized) word type, which is an entry in our IR system dictionary
- We most commonly implicitly define equivalence classes of terms by, e.g.,
 - deleting periods to form a term
 - ***U.S.A., USA*** → ***USA***
 - deleting hyphens to form a term
 - ***anti-discriminatory, antidiscriminatory*** → ***antidiscriminatory***

Thesauri and soundex

- Do we handle synonyms and homonyms?
 - E.g., by hand-constructed equivalence classes
 - *car* = *automobile* *color* = *colour*
 - We can rewrite to form equivalence-class terms
 - When the document contains *automobile*, index it under *car-automobile* (and vice-versa)
 - Or we can expand a query
 - When the query contains *automobile*, look under *car* as well
- What about spelling mistakes?
 - One approach is soundex, which forms equivalence classes of words based on phonetic heuristics

Lemmatization

- Reduce inflectional/variant forms to base form
- E.g.,
 - *am, are, is* → *be*
 - *car, cars, car's, cars'* → *car*
- *the boy's cars are different colors* → *the boy car be different color*
- Lemmatization implies doing “proper” reduction to dictionary headword form

Stemming

- Reduce terms to their “roots” before indexing
- “Stemming” suggest crude affix chopping
 - language dependent
 - e.g., *automate(s)*, *automatic*, *automation* all reduced to *automat*.

for example compressed and compression are both accepted as equivalent to compress.



for exampl compress and
compress ar both accept
as equival to compress

PHRASE QUERIES AND POSITIONAL INDEXES

Phrase queries

- Want to be able to answer queries such as “***stanford university***” – as a phrase
- Thus the sentence “*I went to university at Stanford*” is not a match.
 - The concept of phrase queries has proven easily understood by users; one of the few “advanced search” ideas that works
 - Many more queries are *implicit phrase queries*
- For this, it no longer suffices to store only *<term : docs>* entries

A first attempt: Biword indexes

- Index every consecutive pair of terms in the text as a phrase
- For example the text “Friends, Romans, Countrymen” would generate the biwords
 - *friends romans*
 - *romans countrymen*
- Each of these biwords is now a dictionary term
- Two-word phrase query-processing is now immediate.

Longer phrase queries

- Longer phrases are processed as we did with wild-cards:
- ***stanford university palo alto*** can be broken into the Boolean query on biwords:
stanford university AND university palo AND palo alto

Without the docs, we cannot verify that the docs matching the above Boolean query do contain the phrase.

Can have false positives!

Extended biwords

- Parse the indexed text and perform part-of-speech-tagging (POST).
- Bucket the terms into (say) Nouns (N) and articles/prepositions (X).
- Call any string of terms of the form NX^*N an extended biword.
 - Each such extended biword is now made a term in the dictionary.
- Example: ***catcher in the rye***
N X X N
- Query processing: parse it into N' s and X' s
 - Segment query into enhanced biwords
 - Look up in index: ***catcher rye***

Issues for biword indexes

- False positives, as noted before
- Index blowup due to bigger dictionary
 - Infeasible for more than biwords, big even for them
- Biword indexes are not the standard solution (for all biwords) but can be part of a compound strategy

Solution 2: Positional indexes

- In the postings, store for each ***term*** the position(s) in which tokens of it appear:

<***term***, number of docs containing ***term***;

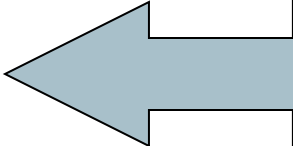
doc1: position1, position2 ... ;

doc2: position1, position2 ... ;

etc.>

Positional index example

<*be*: 993427;
1: 7, 18, 33, 72, 86, 231;
2: 3, 149;
4: 17, 191, 291, 430, 434;
5: 363, 367, ...>



Which of docs *1,2,4,5*
could contain “*to be*
or not to be”?

- For phrase queries, we use a merge algorithm recursively at the document level
- But we now need to deal with more than just equality

Processing a phrase query

- Extract inverted index entries for each distinct term:
to, be, or, not.
- Merge their *doc:position* lists to enumerate all positions with “***to be or not to be***”.
 - ***to:***
 - 2:1,17,74,222,551; 4:8,16,190,429,433; 7:13,23,191; ...
 - ***be:***
 - 1:17,19; 4:17,191,291,430,434; 5:14,19,101; ...
- Same general method for proximity searches

Positional index size

- You can compress position values/offsets
- Nevertheless, a positional index expands postings storage *substantially*
- Nevertheless, a positional index is now standardly used because of the power and usefulness of phrase and proximity queries ... whether used explicitly or implicitly in a ranking retrieval system.

Ranked retrieval

- Thus far, our queries have all been Boolean.
 - Documents either match or don't.
- Good for expert users with precise understanding of their needs and the collection.
 - Also good for applications: Applications can easily consume 1000s of results.
- Not good for the majority of users.
 - Most users incapable of writing Boolean queries (or they are, but they think it's too much work).
 - Most users don't want to wade through 1000s of results.
 - This is particularly true of web search.

Problem with Boolean search: feast or famine

- Boolean queries often result in either too few (=0) or too many (1000s) results.
- Query 1: “*standard user dlink 650*” → 200,000 hits
- Query 2: “*standard user dlink 650 no card found*”: 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
 - AND gives too few; OR gives too many

Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in **ranked retrieval**, the system returns an ordering over the (top) documents in the collection for a query
- **Free text queries**: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- In principle, there are two separate choices here, but in practice, ranked retrieval has normally been associated with free text queries and vice versa

Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set, large result sets are not an issue
 - Indeed, the size of the result set is not an issue
 - We just show the top k (≈ 10) results
 - We don't overwhelm the user

- Premise: the ranking algorithm works

Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score – say in $[0, 1]$ – to each document
- This score measures how well document and query “match”.

Query-document matching scores

- We need a way of assigning a score to a query/document pair
- **Let's start with a one-term query**
- If the query term does not occur in the document: score should be 0
- **The more frequent the query term in the document, the higher the score (should be)**
- We will look at a number of alternatives for this.

Take 1: Jaccard coefficient

- A commonly used measure of overlap of two sets A and B
- $\text{jaccard}(A, B) = |A \cap B| / |A \cup B|$
- $\text{jaccard}(A, A) = 1$
- $\text{jaccard}(A, B) = 0$ if $A \cap B = 0$
- A and B don't have to be the same size.
- Always assigns a number between 0 and 1.

Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: *ides of march*
- Document 1: *caesar died in march*
- Document 2: *the long march*

Issues with Jaccard for scoring

- It doesn't consider *term frequency* (how many times a term occurs in a document)
- Rare terms in a collection are more informative than frequent terms. Jaccard doesn't consider this information
- We need a more sophisticated way of normalizing for length
- Later in this lecture, we'll use $|A \cap B| / \sqrt{|A \cup B|}$
- ... instead of $|A \cap B| / |A \cup B|$ (Jaccard) for length normalization.

Recall: Binary term-document incidence matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector $\in \{0,1\}^{|V|}$

Term-document count matrices

- Consider the number of occurrences of a term in a document:
 - Each document is a **count vector** in \mathbb{N}^v : a column below

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

Bag of words model

- Vector representation doesn't consider the ordering of words in a document
- *John is quicker than Mary and Mary is quicker than John* have the same vectors
- This is called the bag of words model.
- In a sense, this is a step back: The positional index was able to distinguish these two documents.
- We will look at “recovering” positional information later in this course.
- For now: bag of words model

Term frequency tf

- The term frequency $tf_{t,d}$ of term t in document d is defined as the number of times that t occurs in d .
- We want to use tf when computing query-document match scores. But how?
- Raw term frequency is not what we want:
 - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term.
 - But not 10 times more relevant.
- Relevance does not increase proportionally with term frequency.

NB: frequency = count in IR

Log-frequency weighting

- The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} \text{tf}_{t,d}, & \text{if } \text{tf}_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

- $0 \rightarrow 0, 1 \rightarrow 1, 2 \rightarrow 1.3, 10 \rightarrow 2, 1000 \rightarrow 4$, etc.
- Score for a document-query pair: sum over terms t in both q and d :
- $$\text{score} = \sum_{t \in q \cap d} (1 + \log \text{tf}_{t,d})$$
- The score is 0 if none of the query terms is present in the document.

Document frequency

- Rare terms are more informative than frequent terms
 - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., *arachnocentric*)
- A document containing this term is very likely to be relevant to the query *arachnocentric*
- → We want a high weight for rare terms like *arachnocentric*.

Document frequency, continued

- Frequent terms are less informative than rare terms
- Consider a query term that is frequent in the collection (e.g., *high*, *increase*, *line*)
- A document containing such a term is more likely to be relevant than a document that doesn't
- But it's not a sure indicator of relevance.
- → For frequent terms, we want high positive weights for words like *high*, *increase*, and *line*
- But lower weights than for rare terms.
- We will use document frequency (df) to capture this.

idf weight

- df_t is the document frequency of t : the number of documents that contain t
 - df_t is an inverse measure of the informativeness of t
 - $df_t \leq N$
- We define the idf (inverse document frequency) of t by

$$idf_t = \log_{10} (N/df_t)$$

- We use $\log (N/df_t)$ instead of N/df_t to “dampen” the effect of idf.

idf example, suppose $N = 1$ million

term	df_t	idf_t
calpurnia	1	6
animal	100	4
sunday	1,000	3
fly	10,000	2
under	100,000	1
the	1,000,000	0

$$idf_t = \log_{10} (N/df_t)$$

There is one idf value for each term t in a collection.

Effect of idf on ranking

- Does idf have an effect on ranking for one-term queries, like
 - iPhone
- idf has no effect on ranking one term queries
 - idf affects the ranking of documents for queries with at least two terms
 - For the query **capricious person**, idf weighting makes occurrences of **capricious** count for much more in the final document ranking than occurrences of **person**.

Collection vs. Document frequency

- The collection frequency of t is the number of occurrences of t in the collection, counting multiple occurrences.

- Example:

Word	Collection frequency	Document frequency
<i>insurance</i>	10440	3997
<i>try</i>	10422	8760

- Which word is a better search term (and should get a higher weight)?

tf-idf weighting

- The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$w_{t,d} = \log(1 + \text{tf}_{t,d}) \times \log_{10}(N / \text{df}_t)$$

- **Best known weighting scheme in information retrieval**
 - Note: the “-” in tf-idf is a hyphen, not a minus sign!
 - **Alternative names: tf.idf, tf x idf**
- Increases with the number of occurrences within a document
- **Increases with the rarity of the term in the collection**

Score for a document given a query

$$\text{Score}(q, d) = \sum_{t \in q \cap d} \text{tf.idf}_{t,d}$$

- There are many variants
 - How “tf” is computed (with/without logs)
 - Whether the terms in the query are also weighted
 - ...

Binary \rightarrow count \rightarrow weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights $\in \mathbb{R}^{|V|}$

Documents as vectors

- So we have a $|V|$ -dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors - most entries are zero.

Queries as vectors

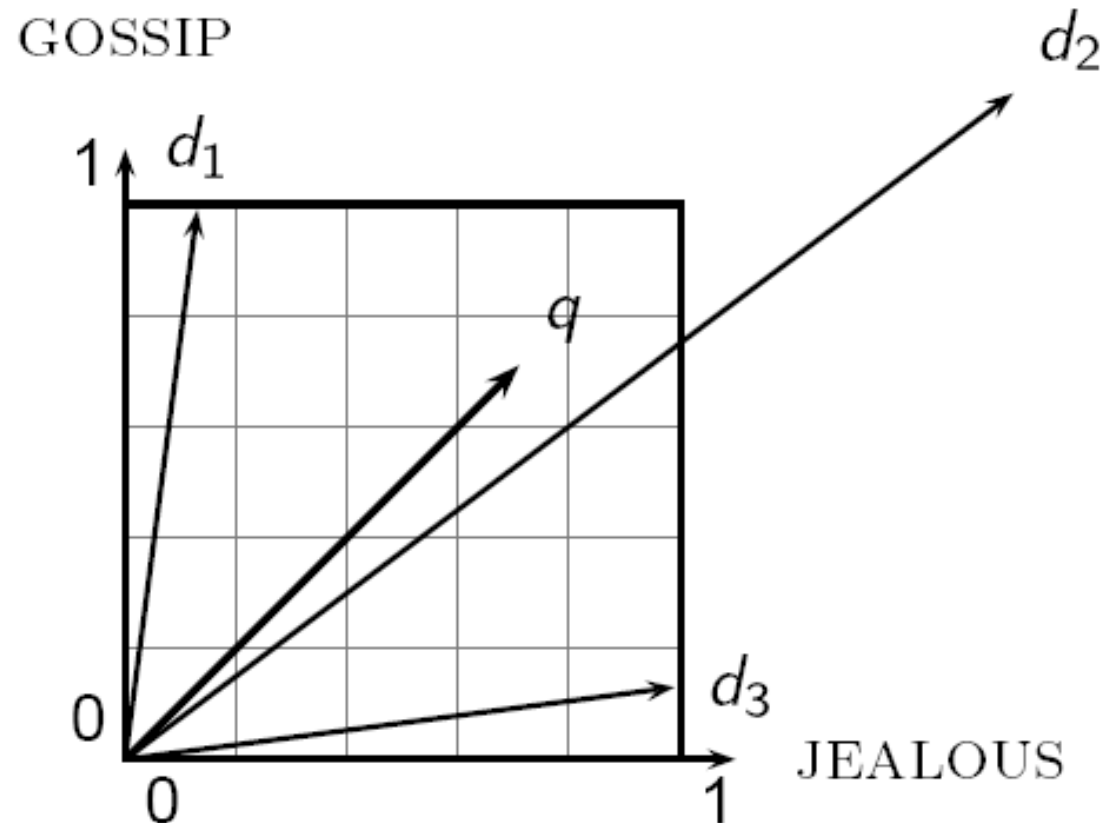
- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity \approx inverse of distance
- **Recall: We do this because we want to get away from the you' re-either-in-or-out Boolean model.**
- Instead: rank more relevant documents higher than less relevant documents

Formalizing vector space proximity

- First cut: distance between two points
 - (= distance between the end points of the two vectors)
- **Euclidean distance?**
- Euclidean distance is a bad idea . . .
- . . . because Euclidean distance is **large** for vectors of **different lengths**.

Why distance is a bad idea

The Euclidean distance between \vec{q} and \vec{d}_2 is large even though the distribution of terms in the query \vec{q} and the distribution of terms in the document \vec{d}_2 are very similar.



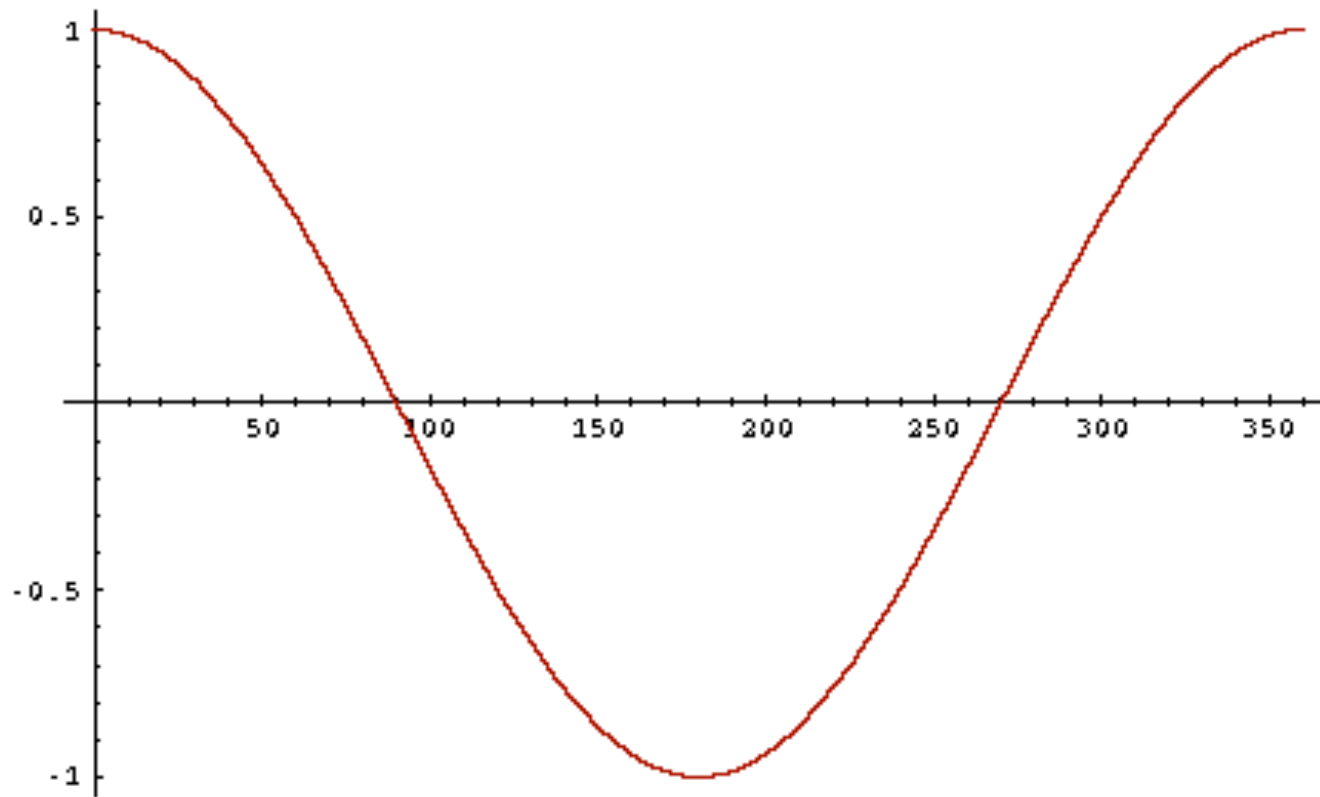
Use angle instead of distance

- Thought experiment: take a document d and append it to itself. Call this document d' .
- “Semantically” d and d' have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity.
- Key idea: Rank documents according to angle with query.

From angles to cosines

- The following two notions are equivalent.
 - Rank documents in decreasing order of the angle between query and document
 - Rank documents in increasing order of cosine (query,document)
- Cosine is a monotonically decreasing function for the interval $[0^\circ, 180^\circ]$

From angles to cosines



- But how – *and why* – should we be computing cosines?

Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length – for this we use the

L_2 norm:

$$\|\vec{x}\|_2 = \sqrt{\sum_i x_i^2}$$

- Dividing a vector by its L_2 norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d' (d appended to itself) from earlier slide: they have identical vectors after length-normalization.
 - Long and short documents now have comparable weights

cosine(query,document)

Dot product

Unit vectors

$$\cos(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{|\vec{q}| |\vec{d}|} = \frac{\vec{q}}{|\vec{q}|} \cdot \frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2} \sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

q_i is the tf-idf weight of term i in the query

d_i is the tf-idf weight of term i in the document

$\cos(\vec{q}, \vec{d})$ is the cosine similarity of \vec{q} and \vec{d} ... or,
equivalently, the cosine of the angle between \vec{q} and \vec{d} .

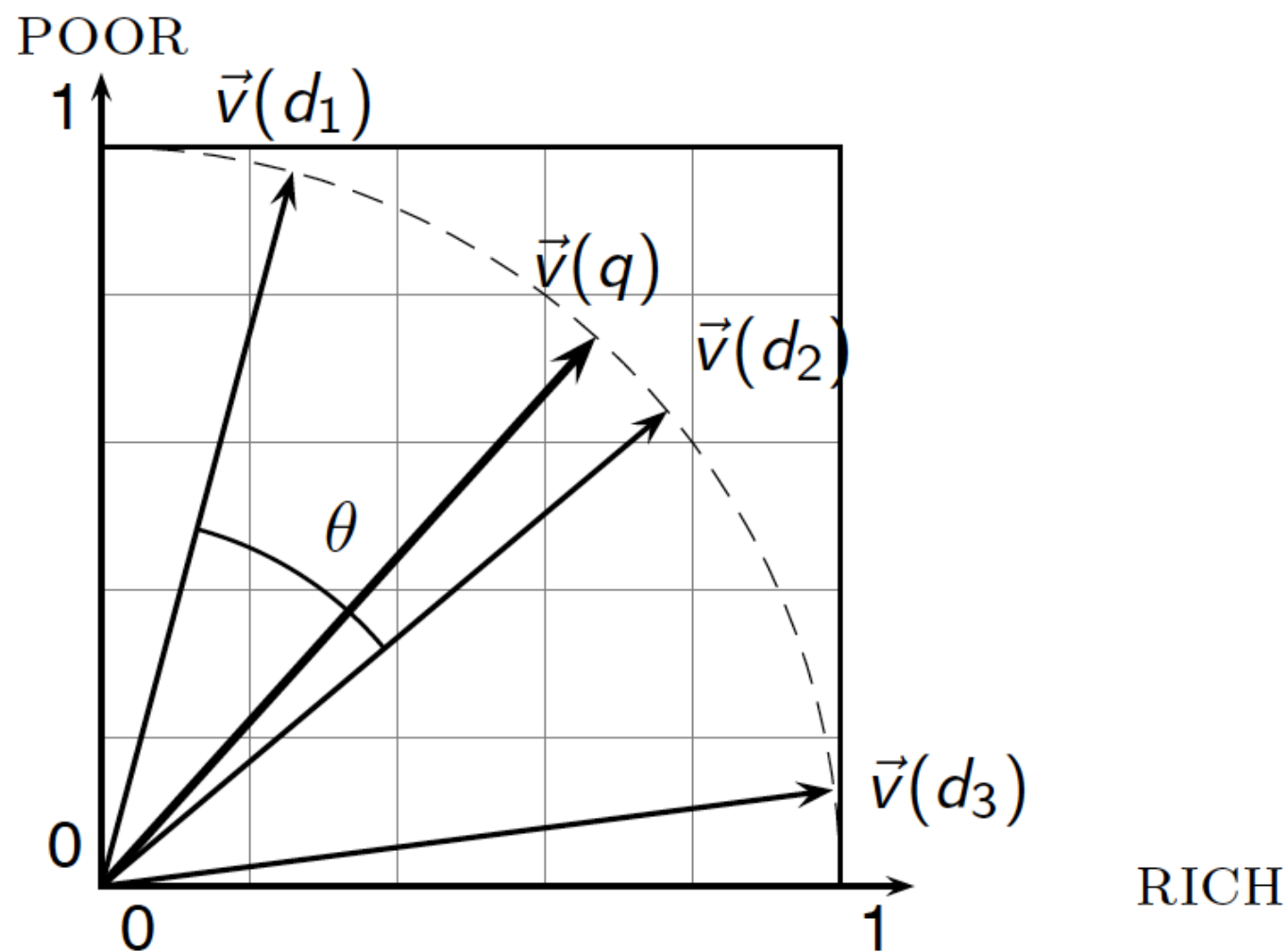
Cosine for length-normalized vectors

- For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q}, \vec{d}) = \vec{q} \cdot \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

for q, d length-normalized.

Cosine similarity illustrated



Cosine similarity amongst 3 documents

How similar are
the novels

SaS: *Sense and
Sensibility*

PaP: *Pride and
Prejudice*, and

WH: *Wuthering
Heights*?

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting.

3 documents example contd.

Log frequency weighting

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

After length normalization

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

$$\cos(\text{SaS}, \text{PaP}) \approx$$

$$0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0 \\ \approx 0.94$$

$$\cos(\text{SaS}, \text{WH}) \approx 0.79$$

$$\cos(\text{PaP}, \text{WH}) \approx 0.69$$

Why do we have $\cos(\text{SaS}, \text{PaP}) > \cos(\text{SaS}, \text{WH})$?