Linear Regression - Part2

In [1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

np.set_printoptions(precision=4, suppress=True)
```

In [2]:
```
from sklearn import datasets

from sklearn.linear_model import LinearRegression

from sklearn.metrics import mean_squared_error, r2_score, scorer

from sklearn.model_selection import train_test_split, cross_val_score

from sklearn.preprocessing import PolynomialFeatures
```

Multiple Linear Regression

Training Data

Training Example	Diameter (in inches)	Number of toppings	Price (in dollars)
1	6	2	7
2	8	1	9
3	10	0	13
4	14	2	17.5
5	18	0	18

In [3]:
```python
X = [[6, 2], [8, 1], [10, 0], [14, 2], [18, 0]]
y = [7, 9, 13, 17.5, 18]

model = LinearRegression()
model.fit(X, y)
```

```
/Library/Frameworks/Python.framework/Versions/3.7/lib/python3.7/site-packages/sklearn/linear_model/base.py:485:
RuntimeWarning: internal gelsd driver lwork query error, required iwork dimension not returned. This is likely
the result of LAPACK bug 0038, fixed in LAPACK 3.2.2 (released July 21, 2010). Falling back to 'gelss' driver.
  linalg.lstsq(X, y)
```

Out[3]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None,
         normalize=False)
```

In [4]:
```python
model.coef
```

Out[4]:
```
array([1.0104, 0.3958])
```

In [5]:
```python
model.intercept
```

Out[5]:
```
1.1875000000000018
```

Test Data

Training Example	Diameter (in inches)	Number of toppings	Price (in dollars)
1	6	2	7
2	8	1	9
3	10	0	13
4	14	2	17.5
5	18	0	18

In [6]:
```
X_test = [[6, 2], [8, 1], [10, 0], [14, 2], [18, 0]]
y_test = [7, 9, 13, 17.5, 18]

predictions = model.predict(X_test)

for i, prediction in enumerate(predictions):
    print('Predicted: ${0:.2f}, With Equation: ${1:.2f}, Target: ${2:.2f}'.format(
        prediction,
        np.dot(model.coef_, X_test[i]) + model.intercept_,
        y_test[i]))
```

```
Predicted: $8.04, With Equation: $8.04, Target: $7.00
Predicted: $9.67, With Equation: $9.67, Target: $9.00
Predicted: $11.29, With Equation: $11.29, Target: $13.00
Predicted: $16.12, With Equation: $16.12, Target: $17.50
Predicted: $19.38, With Equation: $19.38, Target: $18.00
```

In [7]:
```
print("R-squared score: {:.4f}".format(
    model.score(X_test, y_test)))
```

```
R-squared score: 0.9153
```

Polynomial Fit

- Quadratic: $y = \alpha + \beta_1 x + \beta_2 x^2$

Training Data

Training instance	Diameter (in inches)	Price (in dollars)
1	6	7
2	8	9
3	10	13
4	14	17.5
5	18	18

In [8]:
```
X_train = [[6], [8], [10], [14], [18]]
y_train = [7, 9, 13, 17.5, 18]
```

In [9]:
```
X_test = [[6], [8], [11], [16]]
y_test = [8, 12, 15, 18]
```

In [10]:
```
lr = LinearRegression()
```

In [11]:
```
# For plotting

X_fit = np.linspace(0, 26, 100)[:, np.newaxis]
X_fit[:5]
```

Out[11]:
```
array([[0.    ],
       [0.2626],
       [0.5253],
       [0.7879],
       [1.0505]])
```

In [12]:
```
lr.fit(X_train, y_train)

# For plotting
y_lin_fit = lr.predict(X_fit)
```

In [13]:
```
linear_r2 = lr.score(X_test, y_test)
print("Linear Fit R-squared score: {:.4f}".format(
    linear_r2))
```

```
Linear Fit R-squared score: 0.8097
```

In [ ]:
```
```

In [14]:
```
quadratic = PolynomialFeatures(degree=2)
X_train_quadratic = quadratic.fit_transform(X_train)
X_train_quadratic
```

Out[14]:
```
array([[  1.,   6.,  36.],
       [  1.,   8.,  64.],
       [  1.,  10., 100.],
       [  1.,  14., 196.],
       [  1.,  18., 324.]])
```

In [15]:
```
X_test_quadratic = quadratic.transform(X_test)
X_test_quadratic
```

Out[15]:
```
array([[  1.,   6.,  36.],
       [  1.,   8.,  64.],
       [  1.,  11., 121.],
       [  1.,  16., 256.]])
```

In [16]:
```
lr.fit(X_train_quadratic, y_train)

# For plotting
y_quad_fit = lr.predict(quadratic.fit_transform(X_fit))
```

In [17]:
```
lr.predict(X_test_quadratic)
```

Out[17]:
```
array([ 6.3865, 10.0021, 14.1953, 17.903 ])
```

In [18]:
```
y_test
```

Out[18]:
```
[8, 12, 15, 18]
```

In [19]:
```
quadratic_r2 = lr.score(X_test_quadratic, y_test)
print("Quadratic R-squared score: {:.4f}".format(
    quadratic_r2))
```

```
Quadratic R-squared score: 0.8675
```

In [ ]:

In [20]:
```
cubic = PolynomialFeatures(degree=3)
X_train_cubic = cubic.fit_transform(X_train)
X_train_cubic
```

Out[20]:
```
array([[   1.,    6.,   36.,  216.],
       [   1.,    8.,   64.,  512.],
       [   1.,   10.,  100., 1000.],
       [   1.,   14.,  196., 2744.],
       [   1.,   18.,  324., 5832.]])
```

In [21]:
```
X_test_cubic = cubic.transform(X_test)
X_test_cubic
```

Out[21]:
```
array([[   1.,    6.,   36.,  216.],
       [   1.,    8.,   64.,  512.],
       [   1.,   11.,  121., 1331.],
       [   1.,   16.,  256., 4096.]])
```

In [22]:
```
lr.fit(X_train_cubic, y_train)

# For plotting
y_cubic_fit = lr.predict(cubic.fit_transform(X_fit))
```

In [23]:
```
lr.predict(X_test_cubic)
```

Out[23]:
```
array([ 6.8449,  9.4962, 14.0337, 18.6776])
```

In [24]:
```
y_test
```

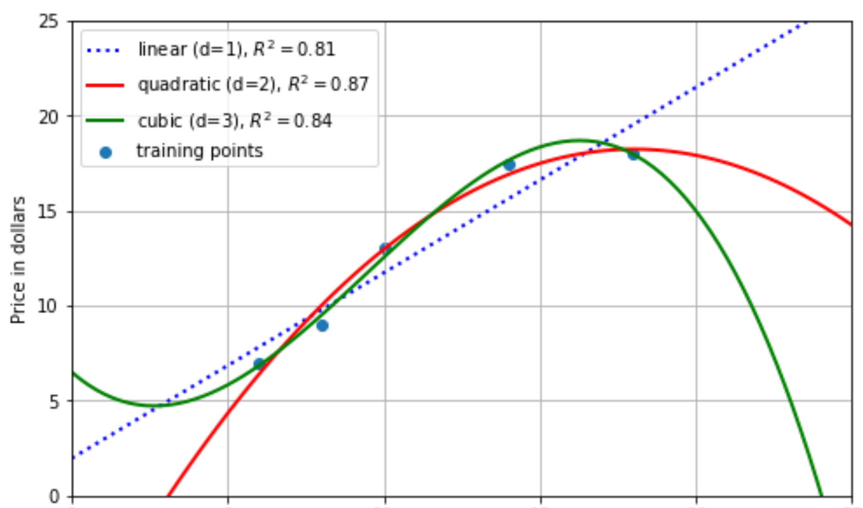
Out[24]:
```
[8, 12, 15, 18]
```

In [25]:
```
cubic_r2 = lr.score(X_test_cubic, y_test)
print("Cubic R-squared score: {:.4f}".format(
    cubic_r2))
```

```
Cubic R-squared score: 0.8357
```

In [ ]:
```

```

In [26]:
```
plt.figure(figsize=(8, 5))

plt.scatter(X_train, y_train, label='training points')

plt.plot(X_fit, y_lin_fit,
         label='linear (d=1), $R^2={:.2f}$'.format(linear_r2),
         color='blue',
         lw=2,
         linestyle=':')

plt.plot(X_fit, y_quad_fit,
         label='quadratic (d=2), $R^2={:.2f}$'.format(quadratic_r2),
         color='red',
         lw=2,
         linestyle='-')

plt.plot(X_fit, y_cubic_fit,
         label='cubic (d=3), $R^2={:.2f}$'.format(cubic_r2),
         color='green',
         lw=2,
         linestyle='-')


plt.xlabel('Diameter in inches')
plt.ylabel('Price in dollars')
plt.axis([0, 25, 0, 25])
plt.grid(True)
plt.legend(loc='upper left');
```



In [ ]:

In [27]:
```
# Degree 9 polynomial
```

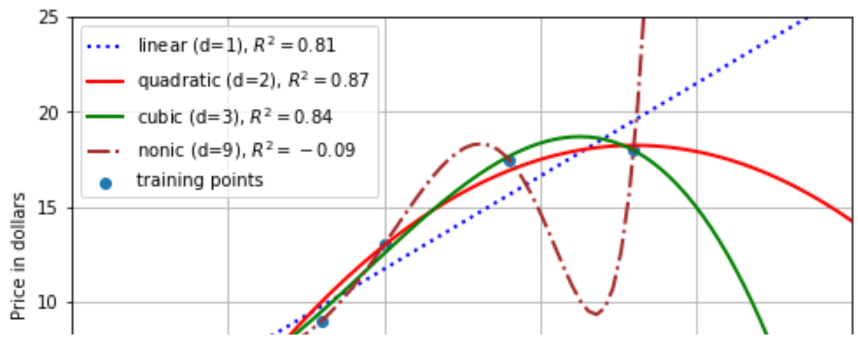
In [28]:
```
nonic = PolynomialFeatures(degree=9)
X_train_nonic = nonic.fit_transform(X_train)
X_test_nonic  = nonic.transform(X_test)
```

In [29]:
```
lr.fit(X_train_nonic, y_train)

# For plotting
y_nonic_fit = lr.predict(nonic.fit_transform(X_fit))
```

In [30]:
```
nonic_r2 = lr.score(X_test_nonic, y_test)
print("Nonic R-squared score: {:.4f}".format(
    nonic_r2))
```

```
Nonic R-squared score: -0.0944
```

In [31]:
```
plt.figure(figsize=(8, 5))

plt.scatter(X_train, y_train, label='training points')

plt.plot(X_fit, y_lin_fit,
         label='linear (d=1), $R^2={:.2f}$'.format(linear_r2),
         color='blue',
         lw=2,
         linestyle=':')

plt.plot(X_fit, y_quad_fit,
         label='quadratic (d=2), $R^2={:.2f}$'.format(quadratic_r2),
         color='red',
         lw=2,
         linestyle='-')

plt.plot(X_fit, y_cubic_fit,
         label='cubic (d=3), $R^2={:.2f}$'.format(cubic_r2),
         color='green',
         lw=2,
         linestyle='-')

plt.plot(X_fit, y_nonic_fit,
         label='nonic (d=9), $R^2={:.2f}$'.format(nonic_r2),
         color='brown',
         lw=2,
         linestyle='-.')


plt.xlabel('Diameter in inches')
plt.ylabel('Price in dollars')
plt.axis([0, 25, 0, 25])
plt.grid(True)
plt.legend(loc='upper left');
```



- The ninth-degree polynomial regression model fits the training data almost exactly!
- The model's r-squared score, however, is -0.09.
- An extremely complex model that fits the training data exactly, but fails to approximate the real relationship.
- This problem is called over-fitting.

In [32]:
```
X_pred = [[16], [20]]
X_pred_nonic = nonic.transform(X_pred)
lr.predict(X_pred_nonic)
```

Out[32]:
```
array([ 10.9449, 131.7757])
```

- Over-fitted model predicts that the 16 inch pizza should be prized $11 and a 20 inch pizza should be prized at $132

In [ ]:
```
```