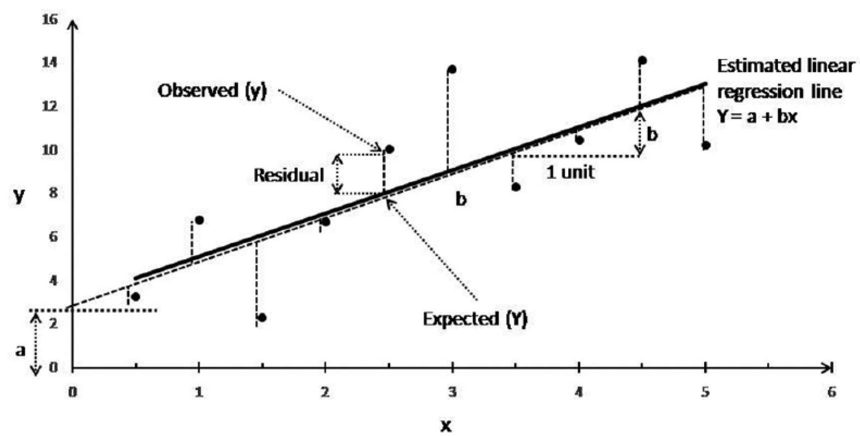
Linear Regression

- https://scikit-learn.org/stable/modules/linear_model.html (https://scikit-learn.org/stable/modules/linear_model.html)
  - the target value is expected to be a linear combination of the input variables
  - if $\hat{y}$ is the predicted value, $\hat{y}(w, x) = w_0 + w_1 x_1 + w_2 x_2 + ... + w_p x_p$
  - the vector $w = (w_1, w_2, \ldots w_p)$ is denoted by coef_ and w_0 as intercept_
  - fits a linear model with coefficients to minimize the residual sum of squares between the observed responses in the dataset, and the responses predicted by the linear approximation



In [1]:
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

np.set_printoptions(precision=4, suppress=True)
```

In [2]:
```python
from sklearn import datasets

from sklearn.linear_model import LinearRegression

from sklearn.metrics import mean_squared_error, r2_score, scorer

from sklearn.model_selection import train_test_split, cross_val_score
```

Example: $y = w_0 + w_1 x_1 + w_2 x_2$

In [3]:
```
reg = LinearRegression()
```

In [4]:
```
X = [[0, 0], [1, 1], [2, 2], [3, 3]]
y = [0, 1, 2, 3]
```

In [5]:
```
reg.fit(X, y)
```

```
/Library/Frameworks/Python.framework/Versions/3.7/lib/python3.7/site-packages/sklearn/linear_model/base.py:485:
RuntimeWarning: internal gelsd driver lwork query error, required iwork dimension not returned. This is likely
the result of LAPACK bug 0038, fixed in LAPACK 3.2.2 (released July 21, 2010). Falling back to 'gelss' driver.
  linalg.lstsq(X, y)
```

Out[5]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None,
         normalize=False)
```

In [6]:
```
reg.coef_
```

Out[6]:
```
array([0.5, 0.5])
```

In [7]:
```
reg.intercept_
```

Out[7]:
```
4.440892098500626e-16
```

In [8]:
```
np.allclose(reg.intercept_, 0)
```

Out[8]:
```
True
```

In [9]:
```
reg.predict([[1, 2], [2, 3]])
```

Out[9]:
```
array([1.5, 2.5])
```

In [10]:
```
reg.predict(X)
```

Out[10]:
```
array([0., 1., 2., 3.])
```

In [11]:
```
reg.score(X, y)
```

Out[11]:
```
1.0
```

Example - Pizza Price Predictor

Training instance	Diameter (in inches)	Price (in dollars)
1	6	7
2	8	9
3	10	13
4	14	17.5
5	18	18

In [12]:
```
# X represents the features of our training data, the diameters of the pizzas.

x = np.array([6, 8, 10, 14, 18])
x
```

Out[12]:
```
array([ 6,  8, 10, 14, 18])
```

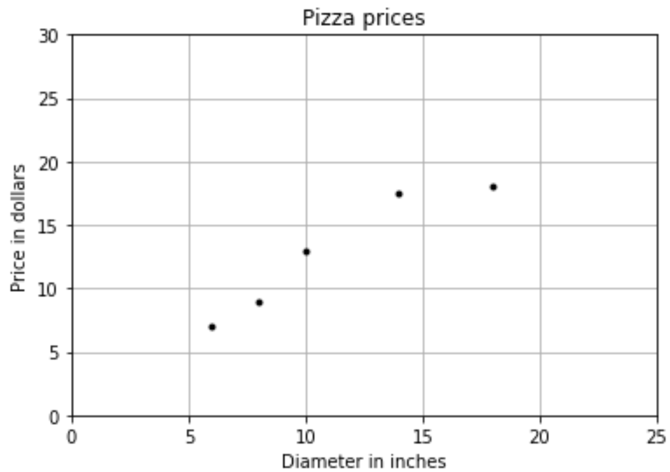
In [13]:
```
# reshape as a matrix

X = x[:, np.newaxis]
X
```

Out[13]:
```
array([[ 6],
       [ 8],
       [10],
       [14],
       [18]])
```

In [14]:
```
# y is a vector representing the prices of the pizzas.

y = [7, 9, 13, 17.5, 18]
y
```

Out[14]:
```
[7, 9, 13, 17.5, 18]
```

In [15]:
```
plt.figure()
plt.title('Pizza prices')
plt.xlabel('Diameter in inches')
plt.ylabel('Price in dollars')
plt.plot(X, y, 'k.')
plt.axis([0, 25, 0, 30])
plt.grid(True)
plt.show()
```



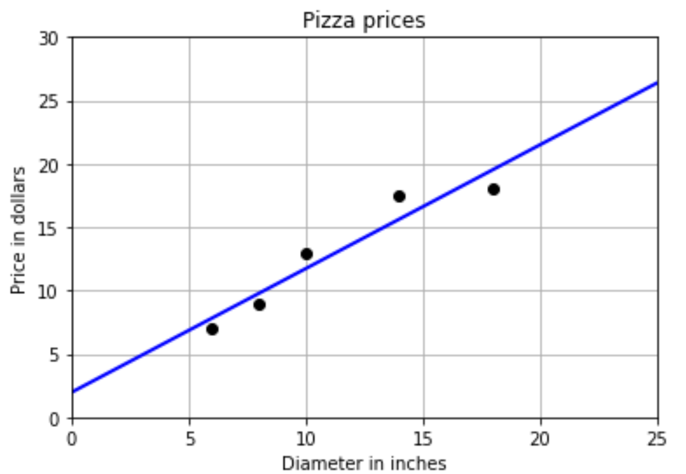
In [16]:
```
# Create an instance of the estimator, LinearRegression
model = LinearRegression()

# Fit the model on the training data
model.fit(X, y)

# Predict the price of a pizza with a diameter that has never been seen before
test_pizza = np.array([[12]])
predicted_price = model.predict(test_pizza)[0]

print('A 12" pizza should cost: ${:.2f}'.format(predicted_price))

test_pizza = np.array([[25]])
predicted_price = model.predict(test_pizza)[0]
print('A 25" pizza should cost: ${:.2f}'.format(predicted_price))
```

```
A 12" pizza should cost: $13.68
A 25" pizza should cost: $26.37
```

In [17]:
```
model.coef_ , model.intercept_
```
Out[17]:
```
(array([0.9763]), 1.9655172413793114)
```

In [18]:
```
25 * model.coef_ + model.intercept_
```
Out[18]:
```
array([26.3728])
```

In [19]:
```
plt.figure()
plt.title('Pizza prices')
plt.xlabel('Diameter in inches')
plt.ylabel('Price in dollars')
plt.plot(X, y, 'ko')
plt.axis([0, 25, 0, 30])
plt.grid(True)
plt.plot([0, 25], [model.intercept_, predicted_price],
         color='b', linestyle='-', linewidth=2)
plt.show()
```



In [20]:
```
print('Residual sum of squares: {:.2f}'.format(
    np.mean((model.predict(X) - y) ** 2)))
```
```
Residual sum of squares: 1.75
```

Evaluating the model using test data

Test Instance	Diameter (in inches)	Observed price (in dollars)	Predicted price (in dollars)
1	8	11	9.7759
2	9	8.5	10.7522
3	11	15	12.7048
4	16	18	17.5863
5	12	11	13.6811

In [21]:
```
# Training and test data

X = [[6], [8], [10], [14],   [18]]
y = [7, 9, 13, 17.5, 18]

X_test = [[8],  [9],   [11], [16], [12]]
y_test = [11, 8.5, 15, 18, 11]

model = LinearRegression()
model.fit(X, y)

y_pred = model.predict(X_test)


print("Mean squared error: {:.4f}".format(
        mean_squared_error(y_test, y_pred)))

print("Variance score: {:.4f}".format(
    r2_score(y_test, y_pred)))

print("R-squared score: {:.4f}".format(
    model.score(X_test, y_test)))
```

```
Mean squared error: 3.8396
Variance score: 0.6620
R-squared score: 0.6620
```

In [22]:
```
list(zip(y_test, y_pred))
```

Out[22]:
```
[(11, 9.77586206896552),
 (8.5, 10.752155172413795),
 (15, 12.704741379310347),
 (18, 17.586206896551726),
 (11, 13.681034482758623)]
```

In [23]:
```
sum_of_sqaures_residuals = np.sum((y_test - y_pred)**2)
sum_of_sqaures_residuals
```

Out[23]:
```
19.198099360879908
```

In [24]:
```
sum_of_squares_total = np.sum((y_test - np.mean(y_test))**2)
sum_of_squares_total
```

Out[24]:
```
56.8
```

In [25]:
```
r2 = 1 - (sum_of_sqaures_residuals/sum_of_squares_total)
r2
```

Out[25]:
```
0.6620052929422551
```

Diabetes dataset

In [26]:
```
diabetes = datasets.load_diabetes()
```

In [27]:
```
print(diabetes.DESCR)
```

```
.. _diabetes_dataset:

Diabetes dataset
----------------

Ten baseline variables, age, sex, body mass index, average blood
pressure, and six blood serum measurements were obtained for each of n =
442 diabetes patients, as well as the response of interest, a
quantitative measure of disease progression one year after baseline.

**Data Set Characteristics:**

  :Number of Instances: 442

  :Number of Attributes: First 10 columns are numeric predictive values

  :Target: Column 11 is a quantitative measure of disease progression one year after baseline

  :Attribute Information:
      - Age
      - Sex
      - Body mass index
      - Average blood pressure
      - S1
      - S2
      - S3
      - S4
      - S5
      - S6

Note: Each of these 10 feature variables have been mean centered and scaled by the standard deviation times `n_
samples` (i.e. the sum of squares of each column totals 1).

Source URL:
http://www4.stat.ncsu.edu/~boos/var.select/diabetes.html (http://www4.stat.ncsu.edu/~boos/var.select/diabetes.h
tml)

For more information see:
Bradley Efron, Trevor Hastie, Iain Johnstone and Robert Tibshirani (2004) "Least Angle Regression," Annals of S
tatistics (with discussion), 407-499.
(http://web.stanford.edu/~hastie/Papers/LARS/LeastAngle_2002.pdf)
```

In [28]:
```
diabetes.feature_names
```

Out[28]:
```
['age', 'sex', 'bmi', 'bp', 's1', 's2', 's3', 's4', 's5', 's6']
```

In [29]:
```
diabetes.target[:5]
```

Out[29]:
```
array([151.,  75., 141., 206., 135.])
```

In [30]:
```
diabetes.data[:5]
```

Out[30]:
```
array([[ 0.0381,  0.0507,  0.0617,  0.0219, -0.0442, -0.0348, -0.0434,
        -0.0026,  0.0199, -0.0176],
       [-0.0019, -0.0446, -0.0515, -0.0263, -0.0084, -0.0192,  0.0744,
        -0.0395, -0.0683, -0.0922],
       [ 0.0853,  0.0507,  0.0445, -0.0057, -0.0456, -0.0342, -0.0324,
        -0.0026,  0.0029, -0.0259],
       [-0.0891, -0.0446, -0.0116, -0.0367,  0.0122,  0.025 , -0.036 ,
         0.0343,  0.0227, -0.0094],
       [ 0.0054, -0.0446, -0.0364,  0.0219,  0.0039,  0.0156,  0.0081,
        -0.0026, -0.032 , -0.0466]])
```

In [ ]:

In [31]:
```
# Use only the bmi feature

# diabetes_X = diabetes.data[:, np.newaxis, 2]

diabetes_X = diabetes.data[:,  [2]]

diabetes_X[:5]
```

Out[31]:
```
array([[ 0.0617],
       [-0.0515],
       [ 0.0445],
       [-0.0116],
       [-0.0364]])
```

In [32]:
```
# Split the data into training/testing sets

diabetes_X_train = diabetes_X[:-20]
diabetes_X_test  = diabetes_X[-20:]
```

In [33]:
```
# Split the targets into training/testing sets

diabetes_y_train = diabetes.target[:-20]
diabetes_y_test = diabetes.target[-20:]
```

In [34]:
```
# Create linear regression object

lr = LinearRegression()
```

In [35]:
```
# Train the model using the training sets

lr.fit(diabetes_X_train, diabetes_y_train)
```

Out[35]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None,
         normalize=False)
```

In [36]:
```
# Make predictions using the testing set

diabetes_y_pred = lr.predict(diabetes_X_test)
```

In [37]:
```
# The coefficients

print('Coefficients: \n', lr.coef_)
```

```
Coefficients:
 [938.2379]
```

In [38]:
```
# The intercept

print('Intercept: \n', lr.intercept_)
```

```
Intercept:
 152.91886182616167
```

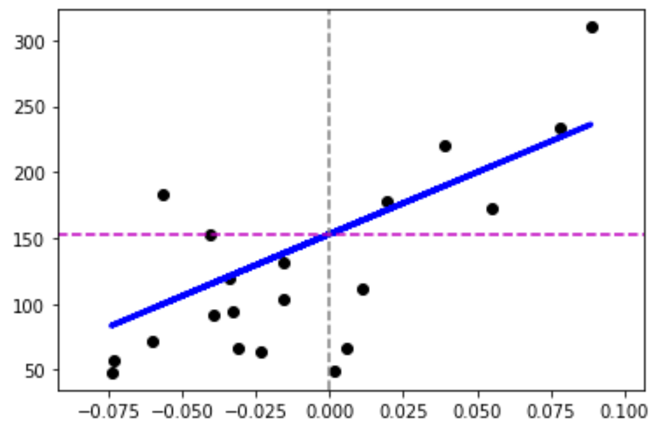
In [39]:
```
# The mean squared error

print("Mean squared error: {:.2f}".format(
        mean_squared_error(diabetes_y_test, diabetes_y_pred)))
```

```
Mean squared error: 2548.07
```

In [40]:
```
# Explained variance score: 1 is perfect prediction

print("Variance score: {:.2f}".format(
    r2_score(diabetes_y_test, diabetes_y_pred)))
```

```
Variance score: 0.47
```

In [41]:
```
plt.scatter(diabetes_X_test, diabetes_y_test,  color='black')

plt.plot(diabetes_X_test, diabetes_y_pred, color='blue', linewidth=3)

plt.axvline(0, c='gray', ls='--')
plt.axhline(lr.intercept_, c='m', ls='--');
```



In [ ]:

In [ ]:

Boston Housing Dataset

In [42]:
```
boston = datasets.load_boston()
```

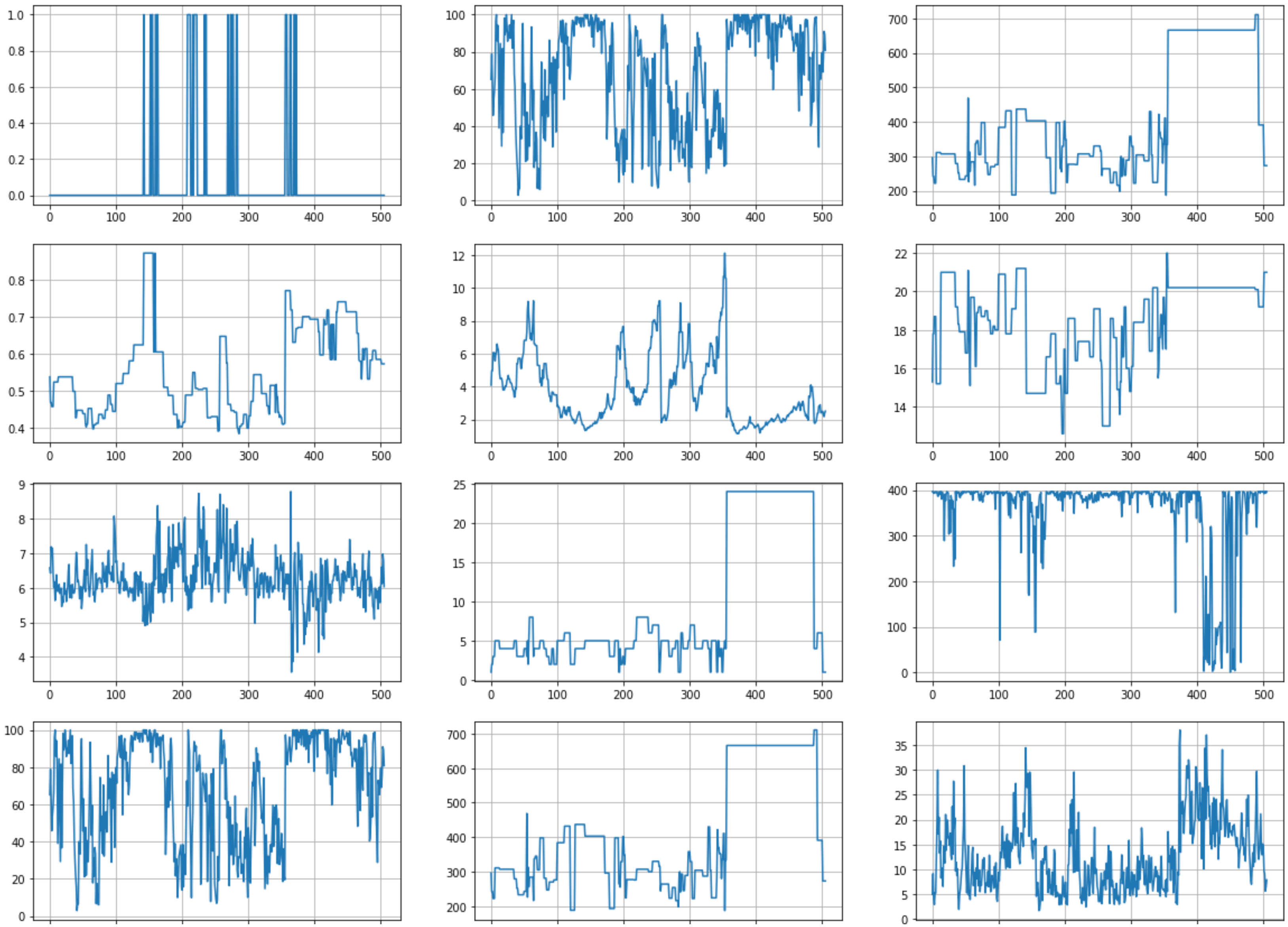
In [43]:
```
boston.data.shape
```

Out[43]:
```
(506, 13)
```

In [44]:
```
print(boston.DESCR)
```

```
.. _boston_dataset:

Boston house prices dataset
---------------------------

**Data Set Characteristics:**

    :Number of Instances: 506

    :Number of Attributes: 13 numeric/categorical predictive. Median Value (attribute 14) is usually the targe
t.

    :Attribute Information (in order):
        - CRIM     per capita crime rate by town
        - ZN       proportion of residential land zoned for lots over 25,000 sq.ft.
        - INDUS    proportion of non-retail business acres per town
        - CHAS     Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
        - NOX      nitric oxides concentration (parts per 10 million)
        - RM       average number of rooms per dwelling
        - AGE      proportion of owner-occupied units built prior to 1940
        - DIS      weighted distances to five Boston employment centres
        - RAD      index of accessibility to radial highways
        - TAX      full-value property-tax rate per $10,000
        - PTRATIO  pupil-teacher ratio by town
        - B        1000(Bk - 0.63)^2 where Bk is the proportion of blacks by town
        - LSTAT    % lower status of the population
        - MEDV     Median value of owner-occupied homes in $1000's

    :Missing Attribute Values: None

    :Creator: Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset.
https://archive.ics.uci.edu/ml/machine-learning-databases/housing/ (https://archive.ics.uci.edu/ml/machine-lear
ning-databases/housing/)


This dataset was taken from the StatLib library which is maintained at Carnegie Mellon University.

The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedonic
prices and the demand for clean air', J. Environ. Economics & Management,
vol.5, 81-102, 1978.   Used in Belsley, Kuh & Welsch, 'Regression diagnostics
...', Wiley, 1980.   N.B. Various transformations are used in the table on
pages 244-261 of the latter.
```

In [45]:
```
fig, ax = plt.subplots(4, 3, figsize=(20, 15))

for i in range(4):
    for j in range(3):
        ax[i, j].plot(boston.data[:, i + (j + 1) * 3])
        ax[i, j].grid()
```



In [46]:
```
boston.data[:5]
```

Out[46]:
```
array([[  0.0063,  18.    ,   2.31  ,   0.    ,   0.538 ,   6.575 ,
         65.2   ,   4.09  ,   1.    , 296.    ,  15.3   , 396.9   ,
          4.98  ],
       [  0.0273,   0.    ,   7.07  ,   0.    ,   0.469 ,   6.421 ,
         78.9   ,   4.9671,   2.    , 242.    ,  17.8   , 396.9   ,
          9.14  ],
       [  0.0273,   0.    ,   7.07  ,   0.    ,   0.469 ,   7.185 ,
         61.1   ,   4.9671,   2.    , 242.    ,  17.8   , 392.83  ,
          4.03  ],
       [  0.0324,   0.    ,   2.18  ,   0.    ,   0.458 ,   6.998 ,
         45.8   ,   6.0622,   3.    , 222.    ,  18.7   , 394.63  ,
          2.94  ],
       [  0.0691,   0.    ,   2.18  ,   0.    ,   0.458 ,   7.147 ,
         54.2   ,   6.0622,   3.    , 222.    ,  18.7   , 396.9   ,
          5.33  ]])
```

In [47]:
```
lr = LinearRegression(normalize=True)
```

In [48]:
```
# Split dataset

X_train, X_test, y_train, y_test = train_test_split(
    boston.data, boston.target, test_size=0.1)
```

In [49]:
```
# Train the model

lr.fit(X_train, y_train)
```

Out[49]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=True)
```

In [50]:
```
# Make predictions using the testing set

y_pred = lr.predict(X_test)
```

In [51]:
```
lr.coef_
```

Out[51]:
```
array([ -0.1117,   0.0533,   0.0015,   3.1766, -19.5664,   3.5038,
        -0.0003,  -1.6514,   0.318 ,  -0.0119,  -0.9905,   0.0096,
        -0.539 ])
```

In [52]:
```
lr.intercept_
```
Out[52]:
```
40.799144686190445
```

In [53]:
```
print("Mean squared error: {:.2f}".format(
        mean_squared_error(y_test, y_pred)))
```
```
Mean squared error: 15.98
```

In [54]:
```
# Explained variance score: 1 is perfect prediction

print("Variance score: {:.2f}".format(
    r2_score(y_test, y_pred)))
```
```
Variance score: 0.75
```

In [55]:
```
lr.score(X_test, y_test)
```
Out[55]:
```
0.7467758847858519
```

In [56]:
```
# CV score (10-fold validation)

scores = cross_val_score(lr, boston.data, boston.target, cv=10, scoring='r2')
```

In [57]:
```
scores
```
Out[57]:
```
array([ 0.7338,  0.4731, -1.0063,  0.6411,  0.5477,  0.7364,  0.3783,
       -0.1292, -0.7684,  0.4189])
```

In [58]:
```
scores.mean(), scores.std()
```
Out[58]:
```
(0.20252899006056452, 0.5952960169512267)
```

In [ ]:

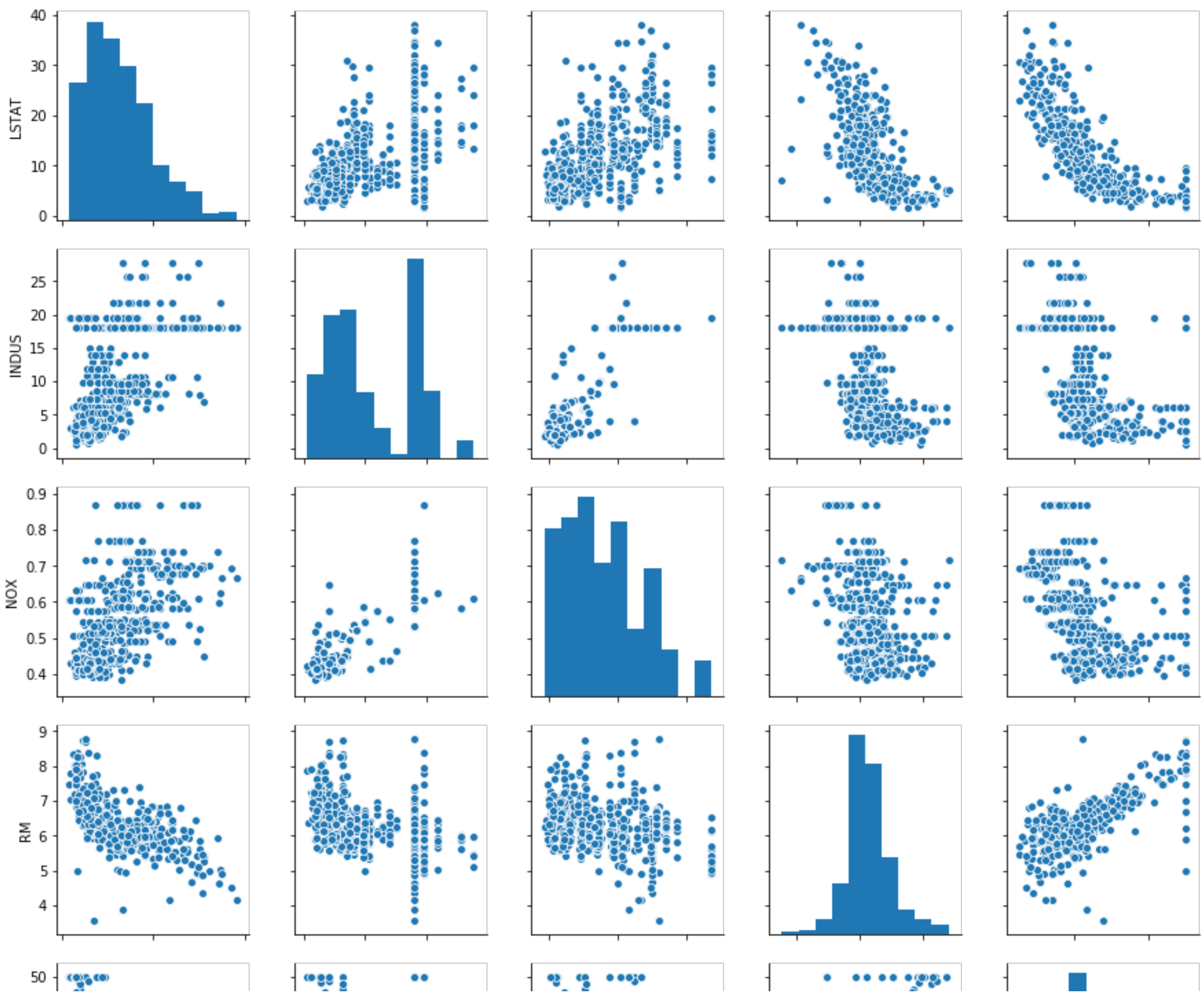
Using Dataframes

In [59]:
```
boston_df = pd.DataFrame(boston.data, columns=boston.feature_names)
boston_df['PRICE'] = boston.target

boston_df.head()
```

Out[59]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	PRICE
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98	24.0
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14	21.6
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03	34.7
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94	33.4
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33	36.2

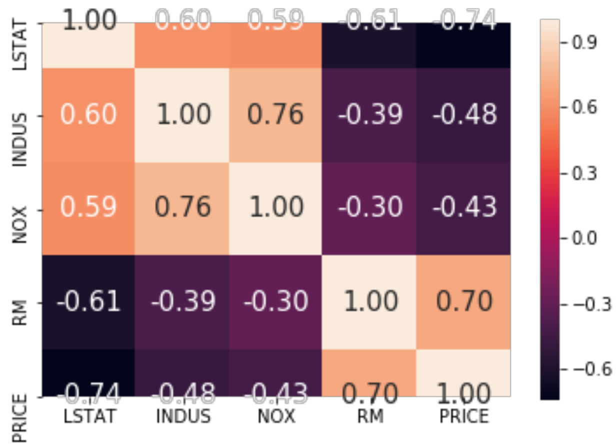
In [60]:
```
cols = ['LSTAT', 'INDUS', 'NOX', 'RM', 'PRICE']

sns.pairplot(boston_df[cols]);
```



In [61]:
```
cm = np.corrcoef(boston_df[cols].values.T)
cm
```

Out[61]:
```
array([[ 1.    ,  0.6038,  0.5909, -0.6138, -0.7377],
       [ 0.6038,  1.    ,  0.7637, -0.3917, -0.4837],
       [ 0.5909,  0.7637,  1.    , -0.3022, -0.4273],
       [-0.6138, -0.3917, -0.3022,  1.    ,  0.6954],
       [-0.7377, -0.4837, -0.4273,  0.6954,  1.    ]])
```

In [62]:
```
sns.heatmap(cm,cbar=True, annot=True,square=True,
                 fmt='.2f',annot_kws={'size': 15},
                 yticklabels=cols,xticklabels=cols);
```



In [ ]:

In [63]:
```
X = boston_df.drop('PRICE', axis = 1)
X.head()
```

Out[63]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33

In [64]:
```
lr = LinearRegression(normalize=True);
```

In [65]:
```
lr.fit(X, boston_df.PRICE)
```

Out[65]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=True)
```

In [66]:
```
lr.coef_
```

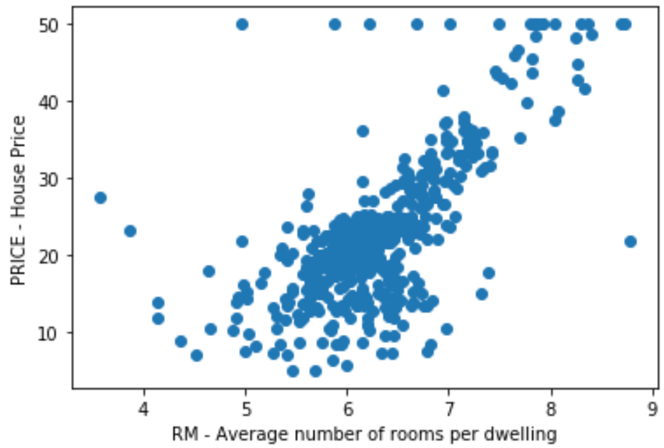
Out[66]:
```
array([ -0.108 ,   0.0464,   0.0206,   2.6867, -17.7666,   3.8099,
          0.0007,  -1.4756,   0.306 ,  -0.0123,  -0.9527,   0.0093,
        -0.5248])
```

In [67]:
```
pd.DataFrame({'feature': boston.feature_names, 'estimatedCoeff': lr.coef_})
```

Out[67]:

	feature	estimatedCoeff
0	CRIM	-0.108011
1	ZN	0.046420
2	INDUS	0.020559
3	CHAS	2.686734
4	NOX	-17.766611
5	RM	3.809865
6	AGE	0.000692
7	DIS	-1.475567
8	RAD	0.306049
9	TAX	-0.012335
10	PTRATIO	-0.952747
11	B	0.009312
12	LSTAT	-0.524758

In [68]:
```
plt.scatter(boston_df.RM, boston_df.PRICE)
plt.xlabel('RM - Average number of rooms per dwelling')
plt.ylabel('PRICE - House Price');
```



In [69]:
```
y = boston_df.PRICE

y_pred = lr.predict(X)
```

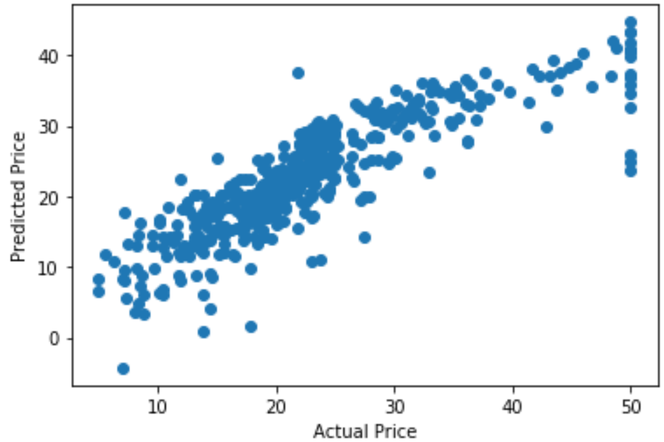
In [70]:
```
print("Mean squared error: {:.2f}".format(
        mean_squared_error(y, y_pred)))
```

```
Mean squared error: 21.89
```

In [71]:
```
np.mean((y - y_pred)**2)
```

Out[71]:
```
21.894831181729206
```

In [72]:
```
plt.scatter(y, y_pred)
plt.xlabel('Actual Price')
plt.ylabel('Predicted Price');
```



In [ ]:
```

```

Polynomial regression

- One common pattern within machine learning is to use linear models trained on nonlinear functions of the data.
- Linear model for 2-dimensional data: $\hat{y}(w, x) = w_0 + w_1 x_1 + w_2 x_2$
- To fit a paraboloid to the data instead of a plane, we can combine the features in second-order polynomials: $\hat{y}(w, x) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_1 x_2 + w_4 x_1^2 + w_5 x_2^2$

In [73]:
```
from sklearn.preprocessing import PolynomialFeatures
```

In [74]:
```
X = np.arange(6).reshape(3, 2)
X
```

Out[74]:
```
array([[0, 1],
       [2, 3],
       [4, 5]])
```

In [75]:
```
poly = PolynomialFeatures(degree=2)
poly.fit_transform(X)
```

Out[75]:
```
array([[  1.,   0.,   1.,   0.,   0.,   1.],
       [  1.,   2.,   3.,   4.,   6.,   9.],
       [  1.,   4.,   5.,  16.,  20.,  25.]])
```

The features of X have been transformed from $[x_1, x_2]$ to $[1, x_1, x_2, x_1^2, x_1 x_2, x_2^2]$, and can now be used within any linear model.

In [76]:
```
from sklearn.pipeline import Pipeline
```

In [77]:
```
# fit to an order-3 polynomial data

model = Pipeline([('poly', PolynomialFeatures(degree=3)),
                  ('linear', LinearRegression(fit_intercept=False))])
```

In [78]:
```
x = np.arange(5)
y = 3 - 2 * x + x ** 2 - x ** 3
print(list(zip(x,y)))
```

```
[(0, 3), (1, 1), (2, -5), (3, -21), (4, -53)]
```

In [79]:
```
x[:, np.newaxis]
```

Out[79]:
```
array([[0],
       [1],
       [2],
       [3],
       [4]])
```

In [80]:
```
model = model.fit(x[:, np.newaxis], y)
```

In [81]:
```
model.named_steps['linear'].coef_
```

Out[81]:
```
array([ 3., -2.,  1., -1.])
```

In [ ]:

Underfitting vs Overfitting

In [82]:
```
import numpy as np
import matplotlib.pyplot as plt

from sklearn.pipeline import Pipeline
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import cross_val_score
```

In [83]:
```
def true_fun(X):
    return np.cos(1.5 * np.pi * X)
```

In [84]:
```
np.random.seed(0)
```

In [85]:
```
n_samples = 30
degrees = [1, 4, 15]
```

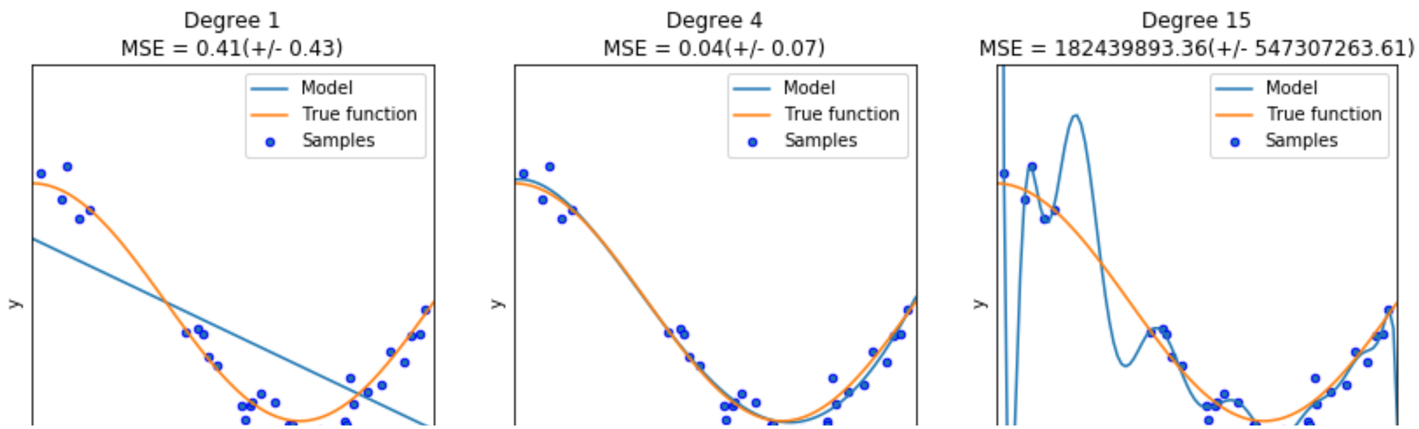
In [86]:
```
X = np.sort(np.random.rand(n_samples))
y = true_fun(X) + np.random.randn(n_samples) * 0.1
```

In [ ]:
```

```

In [87]:
```
X
```

Out[87]:
```
array([0.0202, 0.071 , 0.0871, 0.1183, 0.1434, 0.3834, 0.4147, 0.4237,
       0.4376, 0.4615, 0.5218, 0.5289, 0.5449, 0.5488, 0.568 , 0.6028,
       0.6399, 0.6459, 0.7152, 0.7782, 0.7805, 0.7917, 0.7992, 0.8326,
       0.87  , 0.8918, 0.9256, 0.9447, 0.9637, 0.9786])
```

In [88]:
```
plt.figure(figsize=(14, 5))

for i in range(len(degrees)):
    ax = plt.subplot(1, len(degrees), i + 1)
    plt.setp(ax, xticks=(), yticks=())

    polynomial_features = PolynomialFeatures(degree=degrees[i],
                                             include_bias=False)
    linear_regression = LinearRegression()
    pipeline = Pipeline([("polynomial_features", polynomial_features),
                         ("linear_regression", linear_regression)])
    pipeline.fit(X[:, np.newaxis], y)

    # Evaluate the models using crossvalidation
    scores = cross_val_score(pipeline, X[:, np.newaxis], y,
                             scoring="neg_mean_squared_error", cv=10)


    X_test = np.linspace(0, 1, 100)

    plt.plot(X_test, pipeline.predict(X_test[:, np.newaxis]), label="Model")
    plt.plot(X_test, true_fun(X_test), label="True function")
    plt.scatter(X, y, edgecolor='b', s=20, label="Samples")
    plt.xlabel("x")
    plt.ylabel("y")
    plt.xlim((0, 1))
    plt.ylim((-2, 2))
    plt.legend(loc="best")
    plt.title("Degree {}\nMSE = {:.2f}(+/- {:.2f})".format(
        degrees[i], -scores.mean(), scores.std()))
plt.show()
```



In [ ]:

Modeling Non-linear relationships in Boston Housing dataset

In [89]:
```
boston_df.head()
```

Out[89]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	PRICE
0	0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98	24.0
1	0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14	21.6
2	0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03	34.7
3	0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94	33.4
4	0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33	36.2

In [90]:
```
X = boston_df[['LSTAT']].values
X[:5]
```

Out[90]:
```
array([[4.98],
       [9.14],
       [4.03],
       [2.94],
       [5.33]])
```

In [91]:
```
y = boston_df['PRICE'].values
y[:5]
```

Out[91]:
```
array([24. , 21.6, 34.7, 33.4, 36.2])
```

In [92]:
```
# create quadratic features

quadratic = PolynomialFeatures(degree=2)

cubic = PolynomialFeatures(degree=3)

X_quad = quadratic.fit_transform(X)

X_cubic = cubic.fit_transform(X)
```

In [93]:
```
lr = LinearRegression()
```

In [94]:
```
X_fit = np.arange(X.min(), X.max())[:, np.newaxis]
X_fit[:5]
```

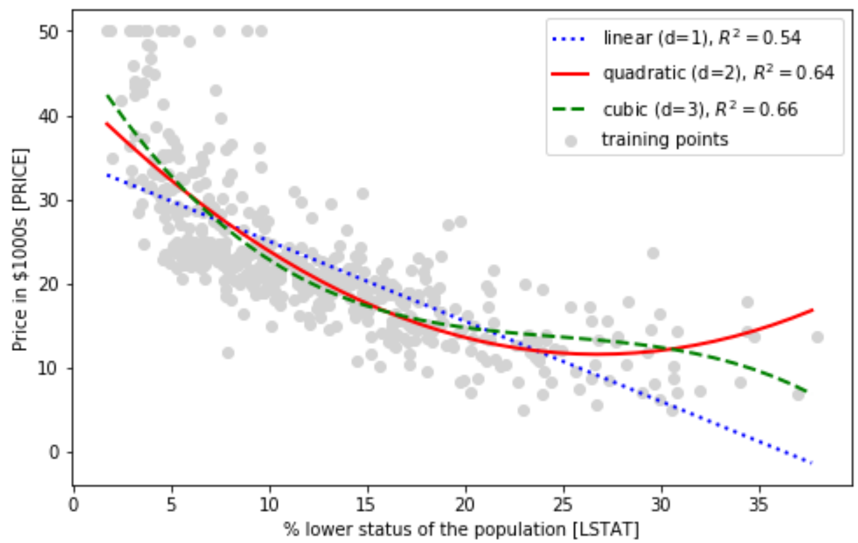
Out[94]:
```
array([[1.73],
       [2.73],
       [3.73],
       [4.73],
       [5.73]])
```

In [95]:
```
lr.fit(X, y)

y_lin_fit = lr.predict(X_fit)

linear_r2 = r2_score(y, lr.predict(X))
```

In [96]:
```
lr.fit(X_quad, y)

y_quad_fit = lr.predict(quadratic.fit_transform(X_fit))

quadratic_r2 = r2_score(y, lr.predict(X_quad))
```

In [97]:
```
lr.fit(X_cubic, y)

y_cubic_fit = lr.predict(cubic.fit_transform(X_fit))

cubic_r2 = r2_score(y, lr.predict(X_cubic))
```

In [98]:
```
plt.figure(figsize=(8, 5))

plt.scatter(X, y, label='training points', color='lightgray')

plt.plot(X_fit, y_lin_fit,
         label='linear (d=1), $R^2={:.2f}$'.format(linear_r2),
         color='blue',
         lw=2,
         linestyle=':')

plt.plot(X_fit, y_quad_fit,
         label='quadratic (d=2), $R^2={:.2f}$'.format(quadratic_r2),
         color='red',
         lw=2,
         linestyle='-')

plt.plot(X_fit, y_cubic_fit,
         label='cubic (d=3), $R^2={:.2f}$'.format(cubic_r2),
         color='green',
         lw=2,
         linestyle='--')

plt.xlabel('% lower status of the population [LSTAT]')
plt.ylabel('Price in $1000s [PRICE]')
plt.legend(loc='upper right');
```



In [ ]:

Bike Traffic Example

In [99]:
```
counts = pd.read_csv('http://people.bu.edu/kalathur/datasets/Fremont_Bridge_Hourly_Bicycle_Counts.csv',
                     index_col='Date', parse_dates=True)
counts.head()
```

Out[99]:

Date	Fremont Bridge East Sidewalk	Fremont Bridge West Sidewalk
2019-01-01 00:00:00	0.0	9.0
2019-01-01 01:00:00	2.0	22.0
2019-01-01 02:00:00	1.0	11.0
2019-01-01 03:00:00	1.0	2.0
2019-01-01 04:00:00	2.0	1.0

In [100]:
```
counts.tail()
```

Out[100]:

Date	Fremont Bridge East Sidewalk	Fremont Bridge West Sidewalk
2016-02-29 00:00:00	2.0	2.0
2013-09-13 03:00:00	1.0	1.0
2016-12-07 00:00:00	3.0	3.0
2013-03-29 04:00:00	1.0	1.0
2017-05-24 01:00:00	4.0	4.0

In [101]:
```
weather = pd.read_csv('http://people.bu.edu/kalathur/datasets/seattle_weather.csv',
                      index_col='DATE', parse_dates=True)
weather.head()
```

Out[101]:

DATE	AWND	PRCP	TMAX	TMIN
2012-01-01	10.51	0.00	55	41
2012-01-02	10.07	0.43	51	37
2012-01-03	5.14	0.03	53	45
2012-01-04	10.51	0.80	54	42
2012-01-05	13.65	0.05	48	37

In [102]:
```
#  compute the total daily bicycle traffic, and put this in its own dataframe

daily = counts.resample('d').sum()
daily['Total'] = daily.sum(axis=1)
daily = daily[['Total']] # remove other columns

daily.head()
```

Out[102]:

Date	Total
2012-10-03	3521.0
2012-10-04	3475.0
2012-10-05	3148.0
2012-10-06	2006.0
2012-10-07	2142.0

In [103]:
```
# add binary columns for day of week

days = ['Mon', 'Tue', 'Wed', 'Thu', 'Fri', 'Sat', 'Sun']
for i in range(7):
    daily[days[i]] = (daily.index.dayofweek == i).astype(int)

daily.head()
```

Out[103]:

Date	Total	Mon	Tue	Wed	Thu	Fri	Sat	Sun
2012-10-03	3521.0	0	0	1	0	0	0	0
2012-10-04	3475.0	0	0	0	1	0	0	0
2012-10-05	3148.0	0	0	0	0	1	0	0
2012-10-06	2006.0	0	0	0	0	0	1	0
2012-10-07	2142.0	0	0	0	0	0	0	1

In [104]:
```python
from pandas.tseries.holiday import USFederalHolidayCalendar

cal = USFederalHolidayCalendar()
holidays = cal.holidays('2012', '2020')

holidays
```

Out[104]:
```
DatetimeIndex(['2012-01-02', '2012-01-16', '2012-02-20', '2012-05-28',
               '2012-07-04', '2012-09-03', '2012-10-08', '2012-11-12',
               '2012-11-22', '2012-12-25', '2013-01-01', '2013-01-21',
               '2013-02-18', '2013-05-27', '2013-07-04', '2013-09-02',
               '2013-10-14', '2013-11-11', '2013-11-28', '2013-12-25',
               '2014-01-01', '2014-01-20', '2014-02-17', '2014-05-26',
               '2014-07-04', '2014-09-01', '2014-10-13', '2014-11-11',
               '2014-11-27', '2014-12-25', '2015-01-01', '2015-01-19',
               '2015-02-16', '2015-05-25', '2015-07-03', '2015-09-07',
               '2015-10-12', '2015-11-11', '2015-11-26', '2015-12-25',
               '2016-01-01', '2016-01-18', '2016-02-15', '2016-05-30',
               '2016-07-04', '2016-09-05', '2016-10-10', '2016-11-11',
               '2016-11-24', '2016-12-26', '2017-01-02', '2017-01-16',
               '2017-02-20', '2017-05-29', '2017-07-04', '2017-09-04',
               '2017-10-09', '2017-11-10', '2017-11-23', '2017-12-25',
               '2018-01-01', '2018-01-15', '2018-02-19', '2018-05-28',
               '2018-07-04', '2018-09-03', '2018-10-08', '2018-11-12',
               '2018-11-22', '2018-12-25', '2019-01-01', '2019-01-21',
               '2019-02-18', '2019-05-27', '2019-07-04', '2019-09-02',
               '2019-10-14', '2019-11-11', '2019-11-28', '2019-12-25',
               '2020-01-01'],
              dtype='datetime64[ns]', freq=None)
```

In [105]:
```python
holidays_series = pd.Series(1, index=holidays, name='holiday')

holidays_series[:5]
```

Out[105]:
```
2012-01-02    1
2012-01-16    1
2012-02-20    1
2012-05-28    1
2012-07-04    1
Name: holiday, dtype: int64
```

In [106]:
```
daily = daily.join(holidays_series)
daily.head(10)
```

Out[106]:

	Total	Mon	Tue	Wed	Thu	Fri	Sat	Sun	holiday
Date									
2012-10-03	3521.0	0	0	1	0	0	0	0	NaN
2012-10-04	3475.0	0	0	0	1	0	0	0	NaN
2012-10-05	3148.0	0	0	0	0	1	0	0	NaN
2012-10-06	2006.0	0	0	0	0	0	1	0	NaN
2012-10-07	2142.0	0	0	0	0	0	0	1	NaN
2012-10-08	3537.0	1	0	0	0	0	0	0	1.0
2012-10-09	3501.0	0	1	0	0	0	0	0	NaN
2012-10-10	3235.0	0	0	1	0	0	0	0	NaN
2012-10-11	3047.0	0	0	0	1	0	0	0	NaN
2012-10-12	2011.0	0	0	0	0	1	0	0	NaN

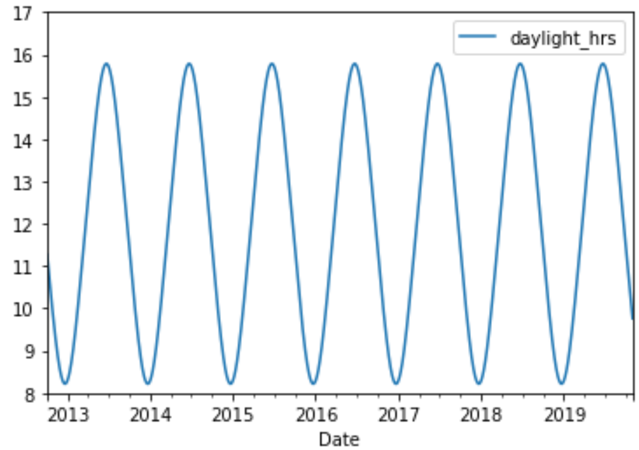
In [107]:
```
daily['holiday'].fillna(0, inplace=True)
daily.head(10)
```

Out[107]:

	Total	Mon	Tue	Wed	Thu	Fri	Sat	Sun	holiday
Date									
2012-10-03	3521.0	0	0	1	0	0	0	0	0.0
2012-10-04	3475.0	0	0	0	1	0	0	0	0.0
2012-10-05	3148.0	0	0	0	0	1	0	0	0.0
2012-10-06	2006.0	0	0	0	0	0	1	0	0.0
2012-10-07	2142.0	0	0	0	0	0	0	1	0.0
2012-10-08	3537.0	1	0	0	0	0	0	0	1.0
2012-10-09	3501.0	0	1	0	0	0	0	0	0.0
2012-10-10	3235.0	0	0	1	0	0	0	0	0.0
2012-10-11	3047.0	0	0	0	1	0	0	0	0.0
2012-10-12	2011.0	0	0	0	0	1	0	0	0.0

In [108]:
```
# Hours of daylight

def hours_of_daylight(date, axis=23.44, latitude=47.61):
    """Compute the hours of daylight for the given date"""
    days = (date - pd.datetime(2000, 12, 21)).days
    m = (1. - np.tan(np.radians(latitude))
         * np.tan(np.radians(axis) * np.cos(days * 2 * np.pi / 365.25)))
    return 24. * np.degrees(np.arccos(1 - np.clip(m, 0, 2))) / 180.

daily['daylight_hrs'] = list(map(hours_of_daylight, daily.index))
daily[['daylight_hrs']].plot()
plt.ylim(8, 17)
```

Out[108]: (8, 17)



In [109]:
```
weather['Temp'] = 0.5 * (weather['TMIN'] + weather['TMAX'])
```

In [110]:
```
# precip is in 1/10 mm; convert to inches

weather['PRCP'] /= 254
weather['dry day'] = (weather['PRCP'] == 0).astype(int)
```

In [111]:
```
weather['2012-10-03':].head()
```

Out[111]:

	AWND	PRCP	TMAX	TMIN	Temp	dry_day
DATE						
2012-10-03	16.33	0.0	66	46	56.0	1
2012-10-04	14.54	0.0	66	47	56.5	1
2012-10-05	12.75	0.0	71	48	59.5	1
2012-10-06	11.41	0.0	75	46	60.5	1
2012-10-07	2.91	0.0	75	46	60.5	1

In [112]:
```
daily = daily.join(weather[['PRCP', 'Temp', 'dry_day']])
daily.head()
```

Out[112]:

	Total	Mon	Tue	Wed	Thu	Fri	Sat	Sun	holiday	daylight_hrs	PRCP	Temp	dry_day
Date													
2012-10-03	3521.0	0	0	1	0	0	0	0	0.0	11.277359	0.0	56.0	1.0
2012-10-04	3475.0	0	0	0	1	0	0	0	0.0	11.219142	0.0	56.5	1.0
2012-10-05	3148.0	0	0	0	0	1	0	0	0.0	11.161038	0.0	59.5	1.0
2012-10-06	2006.0	0	0	0	0	0	1	0	0.0	11.103056	0.0	60.5	1.0
2012-10-07	2142.0	0	0	0	0	0	0	1	0.0	11.045208	0.0	60.5	1.0

In [113]:
```
len(daily)
```

Out[113]: 2585

In [114]:
```
# Drop any rows with null values

daily.dropna(axis=0, how='any', inplace=True)
len(daily)
```

Out[114]: 2340

In [115]:
```
column_names = ['Mon', 'Tue', 'Wed', 'Thu', 'Fri', 'Sat', 'Sun', 'holiday',
                'daylight_hrs', 'PRCP', 'dry_day', 'Temp']

X = daily[column_names]
y = daily['Total']
```

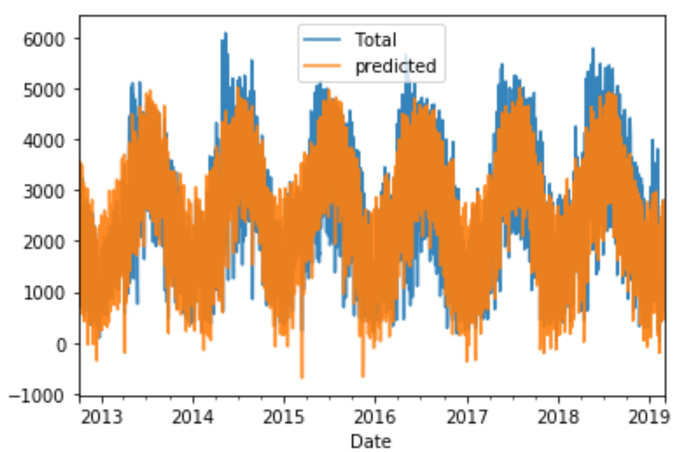
In [116]:
```
model = LinearRegression(fit_intercept=False)

model.fit(X, y)

daily['predicted'] = model.predict(X)
```

In [117]:
```
daily.head()
```

Out[117]:

Date	Total	Mon	Tue	Wed	Thu	Fri	Sat	Sun	holiday	daylight_hrs	PRCP	Temp	dry_day	predicted
2012-10-03	3521.0	0	0	1	0	0	0	0	0.0	11.277359	0.0	56.0	1.0	3634.126478
2012-10-04	3475.0	0	0	0	1	0	0	0	0.0	11.219142	0.0	56.5	1.0	3508.497435
2012-10-05	3148.0	0	0	0	0	1	0	0	0.0	11.161038	0.0	59.5	1.0	3311.313387
2012-10-06	2006.0	0	0	0	0	0	1	0	0.0	11.103056	0.0	60.5	1.0	1995.417856
2012-10-07	2142.0	0	0	0	0	0	0	1	0.0	11.045208	0.0	60.5	1.0	1909.193736

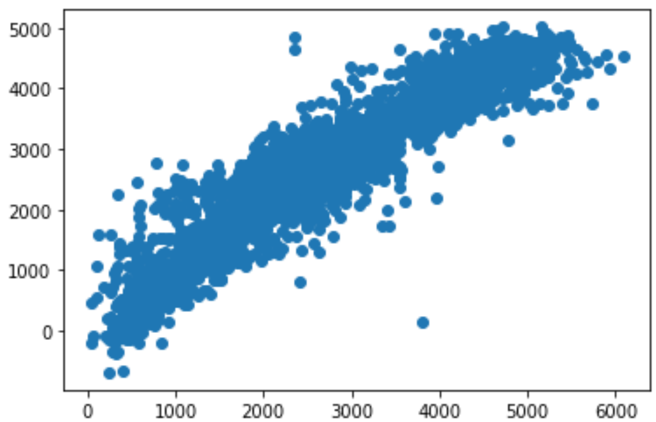
In [118]:
```
daily[['Total', 'predicted']].plot(alpha=0.9);
```



In [119]:
```
params = pd.Series(model.coef_, index=X.columns)
params
```

Out[119]:
```
Mon                -507.917183
Tue                -387.570857
Wed                -414.522840
Thu                -555.081247
Fri                -872.575854
Sat               -2224.499564
Sun               -2304.618191
holiday           -1218.439564
daylight_hrs        105.543215
PRCP            -182811.684733
dry_day             498.130283
Temp                 42.147684
dtype: float64
```

In [120]:
```
plt.scatter(daily.Total, daily.predicted);
```



In [ ]: