A new approach for designing dynamic balanced serial mechanisms

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Abstract

Adaptive balancing means that the mechanical structure of the manipulator is modified in order to achieve the decoupling of dynamic equations. This work deals with a systematic methodology for the adaptive balancing. Basically, two balancing techniques are employed here: the addition of counterweight and counter-rotating disks coupled to the moving links. In addition, the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators is discussed regarding the achievement of such balancing and the complexity level of the modified mechanical structure. The balancing conditions are developed for 3-dof spatial and planar open loop-kinematic chain mechanisms, whose topologies are composed of revolute and prismatic joints.

KEYWORDS: Dynamic balancing, serial mechanisms

1 Introduction and literature review

Balancing is an important issue related to the design of mechanical systems in general, and also serial manipulators, in particular. In fact, the performance of parallel mechanisms associated to specific applications depends on the choice of the balancing method, namely, either static or dynamic, either passive or active, whether it is valid for a given trajectory or even for any motion.

Adaptive balancing means that the mechanical structure of the manipulator is modified in order to achieve the decoupling of dynamic equations. The modifications comprise the addition of either counterweights, or counter-rotating disks or even both to the original kinematic chain of the manipulator. Consequently, the terms associated to gravitational, centripetal and Coriolis efforts are completely eliminated from the dynamic equations. As a matter of fact, the effective inertias for all the actuator axes are constant and the mathematical expressions of the driving torques/forces become rather simple. One of the main advantages of this approach concerns the reduction of computing time for a closed-loop control of manipulators. Such reduction is really significant and it constitutes in a great benefit for real-time applications.

The contributions of this work are the following: to present a systematic methodology for the adaptive balancing, to discuss the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators, not only in terms of the possibility to achieve such balancing but also in terms of the increase in the complexity level of the modified mechanical structure. The analysed manipulators correspond to 3-dof spatial and planar open loop-kinematic chain, whose topologies are composed of revolute and prismatic joints.

This work is organized as follows. Section 2 describes the proposed methodology, while section 3 deals with the application of the methodology to 3 types of serial manipulators. Finally, the conclusions are drawn in section 4.

2 Mothodology

2.1 Dynamic Model

O modelo dinâmico de um mecanismo serial pode ser escrito da seguinte maneira:

$$\mathbb{M}^{\#}(\mathbb{q}^{\#})\ddot{\mathbb{q}}^{\#} + \mathbb{v}^{\#}(\mathbb{q}^{\#}, \dot{\mathbb{q}}^{\#}) + \mathbb{g}^{\#}(\mathbb{q}^{\#}) = \mathbb{u} \tag{1}$$

Sendo $\mathbb{q}^{\#}$ um conjunto de coordenadas generalizadas independentes, cujos elementos são os deslocamentos relativos das juntas e $\mathbb{u}$ os esforços generalizados nas direções das quasi-velocidades independentes $\mathbb{p}^{\#} = \dot{\mathbb{q}}^{\#}$.

Para realizar o balanceamento dinâmico de um mecanismo serial, utilizando abordagem proposta, é necessário primeiro obter o modelo do mecanismo desbalanceado. Como em um mecanismo serial é possível de expressar todas as velocidades lineares absolutas dos centros de massa das barras e todas as velocidades angulares absolutas das barras em função de $\mathbb{q}^{\#}$ e $\dot{\mathbb{q}}^{\#}$, o modelo dinâmico pode ser obtido sem grandes dificuldades utilizando métodos de mecânica analítica, como Lagrange, Kane e Orsino, aliados a programas ou bibliotecas de linguagens de programação que são capazes de utilizar manipulação simbólica, como o Mathematica e o SymPy.

2.2 Static Balancing

Depois de obtido o modelo dinâmico, realiza-se o balanceamento estático encontrando as posições dos centros de massa das barras que fazem com que $\mathbb{g}^{\#} = \mathbb{0}$. Isso é possível para mecanismos com juntas apenas rotativas e mecanismos com juntas prismáticas cuja direções são ortogonais à gravidade. O posicionamento dos centros de massa é realizado mecanicamente com o prologamento das barras do mecanismo e adição de contra-pesos.

2.3 Dynamic Balancing

O balanceamento dinâmico é obtido acoplando discos girantes ao modelo do mecanismo estáticamente balanceado. Isso é feito utilizando a técnica de acoplamento de subsistemas do Método Orsino.

Seja $\mathscr{M}_0$ um subsistema mecânico constituido por um mecanismo serial estaticamente balanceado, cuja equação de movimento é dada pela equação (1), com $\mathbb{g}^{\#} = \mathbb{0}$. Seja $\mathscr{M}_i$ um subsistema mecânico constituido de um disco girante que será acoplado ao mecanismo, cuja equação de movimento é dada por:

$$\mathbb{M}_i^{\#}\dot{\mathbb{p}}^{\#} + \mathbb{v}_i^{\#} + \mathbb{g}_i^{\#} = \mathbb{u}_i \tag{2}$$

Sendo $\mathbb{p}^{\#}$ um conjunto de quasi-velocidades independentes, cujos elementos são as componentes não nulas do vetor velocidade angular absoluta do disco, escrito em uma base solidária ao disco, e $\mathbb{v}_i^{\#} = \mathbb{g}_i^{\#} = \mathbb{u}_i = \mathbb{0}$. Nesse modelo, são considerados apenas os efeitos das inércias rotativas, sendo os efeitos da massa do disco considerados na massa e nos momentos de inércia da barra em que o disco for acoplado, efetando também no cálculo do posicionamento dos contra-pesos, de modo que o mecanismo continue estáticamente balanceado.

Supondo que irão ser acoplados n discos ao mecanismo, são feitas as seguintes definições:

$$\mathbb{M}' = \begin{bmatrix} \mathbb{M}^{\#} & \mathbb{0} & \dots & \mathbb{0} \\ \mathbb{0} & \mathbb{M}_1^{\#} & \dots & \mathbb{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{0} & \mathbb{0} & \dots & \mathbb{M}_n^{\#} \end{bmatrix} \tag{3}$$

$$\mathbb{v}' = \begin{bmatrix} \mathbb{v}^{\#\mathsf{T}} & \mathbb{v}_1^{\#\mathsf{T}} & \dots & \mathbb{v}_n^{\#\mathsf{T}} \end{bmatrix}^{\mathsf{T}} \tag{4}$$

$$\mathbb{g}' = \begin{bmatrix} \mathbb{g}^{\#\mathsf{T}} & \mathbb{g}_1^{\#\mathsf{T}} & \dots & \mathbb{g}_n^{\#\mathsf{T}} \end{bmatrix}^{\mathsf{T}} \tag{5}$$

$$\mathbb{p}^\circ = \begin{bmatrix} \mathbb{p}_1^{\#\mathsf{T}} & \dots & \mathbb{p}_n^{\#\mathsf{T}} \end{bmatrix}^\mathsf{T} \tag{6}$$

$$\mathbb{p} = \begin{bmatrix} \mathbb{p}^{\#\mathsf{T}} & \mathbb{p}^{\circ\mathsf{T}} \end{bmatrix}^\mathsf{T} \tag{7}$$

Seja $\underline{\mathbb{p}}^\circ$ o vetor $\mathbb{p}^\circ$ escrito em função de $\mathbb{q}^\#$ e $\mathbb{p}^\#$, ou seja:

$$\mathbb{p}^\circ = \underline{\mathbb{p}}^\circ(\mathbb{q}^\#, \mathbb{p}^\#) \tag{8}$$

Definimos a seguinte matriz das restrições cinemáticas:

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \dfrac{\partial \underline{\mathbb{p}}^\circ}{\partial \mathbb{p}^\#} \end{bmatrix} \tag{9}$$

O modelo dinâmico do mecanismo serial com os discos acoplados, é dado por:

$$\mathbb{M}'^\#(\mathbb{q}^\#)\ddot{\mathbb{q}}^\# + \mathbb{v}'^\#(\mathbb{q}^\#, \dot{\mathbb{q}}^\#) + \mathbb{g}'^\#(\mathbb{q}^\#) = \mathbb{u} \tag{10}$$

Sendo:

$$\mathbb{M}'^\# = \mathbb{C}^\mathsf{T}\mathbb{M}'\mathbb{C} \tag{11}$$

$$\mathbb{v}'^\# = \mathbb{C}^\mathsf{T}(\mathbb{M}'\dot{\mathbb{C}}\dot{\mathbb{q}}^\# + \mathbb{v}') \tag{12}$$

$$\mathbb{g}'^\# = \mathbb{C}^\mathsf{T}\mathbb{g}' \tag{13}$$

O balanceamento dinâmico é obtido encontrando as relações entre os parâmetros do sistema que tornam a matriz $\mathbb{M}'^\#$ diagonal e o vetor $\mathbb{v}'^\#$ nulo.

3 Applying the technique

For a 3-DOF serial mechanism:

$$\mathbb{M}^\# = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} \tag{14}$$

$$\mathbb{v}^\# = \begin{bmatrix} D_{111} & D_{122} & D_{133} \\ D_{211} & D_{222} & D_{233} \\ D_{311} & D_{322} & D_{333} \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \\ \dot{q}_3^2 \end{bmatrix} + 2 \begin{bmatrix} D_{112} & D_{113} & D_{123} \\ D_{212} & D_{213} & D_{223} \\ D_{312} & D_{313} & D_{323} \end{bmatrix} \begin{bmatrix} \dot{q}_1\dot{q}_2 \\ \dot{q}_1\dot{q}_3 \\ \dot{q}_2\dot{q}_3 \end{bmatrix} \tag{15}$$

$$\mathbb{g}^\# = \begin{bmatrix} D_1 & D_2 & D_3 \end{bmatrix}^\top \tag{16}$$

$$\mathbb{q}^\# = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^\top \tag{17}$$

$$\mathbb{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^\top \tag{18}$$

For a rotational joint we call $q_i = \theta_i$ and $u_i = \tau_i$, and for a prismatic joint we call $q_i = d_i$ and $u_i = f_i$.

3.1 3-DOF RRR planar serial mechanism

$$\begin{cases} D_1 = g[(m_1 l_{g_1} + m_2 l_1 + m_3 l_1)\mathsf{c}(\theta_1) + (m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_1+\theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_1+\theta_2+\theta_3)] \\ D_2 = g[(m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_1+\theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_1+\theta_2+\theta_3)] \\ D_3 = g[m_3 l_{g_3}\mathsf{c}(\theta_1+\theta_2+\theta_3)] \end{cases} \tag{19}$$

Static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g_2} = -\frac{l_2 m_3}{m_2} \\ l_{g_3} = 0 \end{cases} \tag{20}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{x_1} + J_{x_2} + J_{x_3} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ D_{22} = J_{x_2} + J_{x_3} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{x_3} \\ D_{12} = D_{22} \\ D_{13} = D_{23} = D_{33} \\ \mathbb{v}^{\#} = \mathbb{0} \\ \mathbb{g}^{\#} = \mathbb{0} \end{cases} \tag{21}$$

Coupling 4 discs:

Planar disc models:

$$\mathbb{M}_i^{\#} = \left[J_{x_{i+3}}\right];\ \mathbb{p}_i^{\#} = \left[\omega_{x_{i+3}}\right],\ i = 1,2,3,4 \tag{22}$$

Quasi-velocities constraints:

$$\begin{cases} \omega_{x_4} = \omega_{x_1} + \dot\theta_2 \\ \omega_{x_5} = \omega_{x_1} + \beta\dot\theta_2 \\ \omega_{x_6} = \omega_{x_2} + \dot\theta_3 \\ \omega_{x_7} = \omega_{x_2} + \gamma\dot\theta_3 \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = \dot\theta_1 + \dot\theta_2 \\ \omega_{x_5} = \dot\theta_1 + \beta\dot\theta_2 \\ \omega_{x_6} = \dot\theta_1 + \dot\theta_2 + \dot\theta_3 \\ \omega_{x_7} = \dot\theta_1 + \dot\theta_2 + \gamma\dot\theta_3 \end{cases} \Rightarrow \underline{\mathbb{p}}^{\circ} = \begin{bmatrix} \dot\theta_1 + \dot\theta_2 \\ \dot\theta_1 + \beta\dot\theta_2 \\ \dot\theta_1 + \dot\theta_2 + \dot\theta_3 \\ \dot\theta_1 + \dot\theta_2 + \gamma\dot\theta_3 \end{bmatrix} \tag{23}$$

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \dfrac{\partial \underline{\mathbb{p}}^{\circ}}{\partial \mathbb{p}^{\#}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \\ 1 & 1 & 1 \\ 1 & 1 & \gamma \end{bmatrix} \tag{24}$$

$$\begin{cases} D'_{11} = D_{11} + J_{x_4} + J_{x_5} + J_{x_6} + J_{x_7} \\ D'_{22} = D_{22} + J_{x_4} + J_{x_5}\beta^2 + J_{x_6} + J_{x_7} \\ D'_{33} = D_{33} + J_{x_6} + J_{x_7}\gamma^2 \\ D'_{12} = D_{12} + J_{x_4} + J_{x_5}\beta + J_{x_6} + J_{x_7} \\ D'_{13} = D_{13} + J_{x_6} + J_{x_7}\gamma \\ D'_{23} = D'_{13} \\ \mathbb{v}'^{\#} = \mathbb{0} \end{cases} \tag{25}$$

Dynamic balancing:

$$\begin{cases} D'_{12} = 0 \\ D'_{13} = 0 \end{cases} \Rightarrow \begin{cases} \beta = -\frac{J_{x_2}+J_{x_3}+J_{x_4}+J_{x_6}+J_{x_7}+m_3 l_2^2+\frac{m_3^2 l_2^2}{m_2}}{J_{x_5}} \\ \gamma = -\frac{J_{x_3}+J_{x_6}}{J_{x_7}} \end{cases} \tag{26}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1 = k_1 \ddot{\theta}_1 \\ \tau_2 = k_2 \ddot{\theta}_2 \\ \tau_3 = k_3 \ddot{\theta}_3 \end{cases} \tag{27}$$

Being:

$$\begin{cases} k_1 = J_{x_1} + J_{x_2} + J_{x_3} + J_{x_4} + J_{x_5} + J_{x_6} + J_{x_7} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ k_2 = J_{x_2} + J_{x_3} + J_{x_4} + J_{x_6} + J_{x_7} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} + \frac{\left(J_{x_2}+J_{x_3}+J_{x_4}+J_{x_6}+J_{x_7}+m_3 l_2^2+\frac{m_3^2 l_2^2}{m_2}\right)^2}{J_{x_5}} \\ k_3 = \frac{(J_{x_3}+J_{x_6})(J_{x_3}+J_{x_6}+J_{x_7})}{J_{x_7}} \end{cases} \tag{28}$$

3.2 3-DOF RRR spatial serial mechanism

$$\begin{cases} D_1 = 0 \\ D_2 = g[(m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_2) + m_3 l_{g_3} \mathsf{c}(\theta_2 + \theta_3)] \\ D_3 = g[m_3 l_{g_3} \mathsf{c}(\theta_2 + \theta_3)] \end{cases} \tag{29}$$

Static balancing:

$$\begin{cases} D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_2} = -\frac{l_2 m_3}{m_2} \\ l_{g_3} = 0 \end{cases} \tag{30}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{y_2}\mathsf{s}^2(\theta_2) + J_{y_3}\mathsf{s}^2(\theta_2+\theta_3) + J_{z_1} + J_{z_2}\mathsf{c}^2(\theta_2) + J_{z_3}\mathsf{c}^2(\theta_2+\theta_3) + m_3(l_1 + l_2\mathsf{c}(\theta_2))^2 + \frac{(m_2 l_1 - m_3 l_2 \mathsf{c}(\theta_2))^2}{m_2} \\ D_{22} = J_{x_2} + J_{x_3} + m_2 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{x_3} \\ D_{12} = D_{13} = 0 \\ D_{23} = D_{33} \\ D_{211} = -\frac{1}{2}\Big(\big(J_{y_2} - J_{z_2}\big)\mathsf{s}(2\theta_2) + \big(J_{y_3} - J_{z_3} - m_3 l_2^2(1 + \frac{m_3}{m_2})\big)\mathsf{s}(2\theta_2 + 2\theta_3)\Big) \\ D_{311} = \frac{1}{2}\Big(\big(J_{z_3} - J_{y_3}\big)\mathsf{s}(2\theta_2 + 2\theta_3)\Big) \\ D_{111} = D_{122} = D_{133} = D_{222} = D_{233} = D_{322} = D_{333} = 0 \\ D_{112} = -D_{211} \\ D_{113} = -D_{311} \\ D_{123} = D_{212} = D_{213} = D_{223} = D_{312} = D_{313} = D_{323} = 0 \\ \mathbb{g} = \mathbb{0} \end{cases} \tag{31}$$

Coupling 2 discs:

Spatial disc models:

$$\mathbb{M}_i^{\#} = \begin{bmatrix} J_{x_{i+3}} & 0 & 0 \\ 0 & J_{y_{i+3}} & 0 \\ 0 & 0 & J_{z_{i+3}} \end{bmatrix} ;\ \mathbb{p}_i^{\#} = \begin{bmatrix} \omega_{x_{i+3}} \\ \omega_{y_{i+3}} \\ \omega_{z_{i+3}} \end{bmatrix},\ i = 1, 2 \tag{32}$$

Quasi-velocities constraints:

$$\begin{cases} [\boldsymbol{\omega}_4]_{\mathrm{B}_4} = [\mathbb{1}]_{\mathrm{B}_4|\mathrm{B}_2}[\boldsymbol{\omega}_2]_{\mathrm{B}_2} + \begin{bmatrix} \dot\theta_3 \\ 0 \\ 0 \end{bmatrix} \\ [\boldsymbol{\omega}_5]_{\mathrm{B}_5} = [\mathbb{1}]_{\mathrm{B}_5|\mathrm{B}_2}[\boldsymbol{\omega}_2]_{\mathrm{B}_2} + \begin{bmatrix} \beta\dot\theta_3 \\ 0 \\ 0 \end{bmatrix} \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = \dot\theta_2 + \dot\theta_3 \\ \omega_{y_4} = (\dot\theta_1 \mathsf{s}(\theta_2))\mathsf{c}(\theta_3) + (\dot\theta_1 \mathsf{c}(\theta_2))\mathsf{s}(\theta_3) \\ \omega_{z_4} = -(\dot\theta_1 \mathsf{s}(\theta_2))\mathsf{s}(\theta_3) + (\dot\theta_1 \mathsf{c}(\theta_2))\mathsf{c}(\theta_3) \\ \omega_{x_5} = \dot\theta_2 + \beta\dot\theta_3 \\ \omega_{y_5} = (\dot\theta_1 \mathsf{s}(\theta_2))\mathsf{c}(\beta\theta_3) + (\dot\theta_1 \mathsf{c}(\theta_2))\mathsf{s}(\beta\theta_3) \\ \omega_{z_5} = -(\dot\theta_1 \mathsf{s}(\theta_2))\mathsf{s}(\beta\theta_3) + (\dot\theta_1 \mathsf{c}(\theta_2))\mathsf{c}(\beta\theta_3) \end{cases} \tag{33}$$

$$\Rightarrow \underline{\mathbb{p}}^{\circ} = \begin{bmatrix} \dot\theta_2 + \dot\theta_3 \\ \dot\theta_1 \mathsf{s}(\theta_2 + \theta_3) \\ \dot\theta_1 \mathsf{c}(\theta_2 + \theta_3) \\ \dot\theta_2 + \beta\dot\theta_3 \\ \dot\theta_1 \mathsf{s}(\theta_2 + \beta\theta_3) \\ \dot\theta_1 \mathsf{c}(\theta_2 + \beta\theta_3) \end{bmatrix} \tag{34}$$

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \dfrac{\partial \underline{\mathbb{p}}^{\circ}}{\partial \mathbb{p}^{\#}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ \mathsf{s}(\theta_2 + \theta_3) & 0 & 0 \\ \mathsf{c}(\theta_2 + \theta_3) & 0 & 0 \\ 0 & 1 & \beta \\ \mathsf{s}(\theta_2 + \beta\theta_3) & 0 & 0 \\ \mathsf{c}(\theta_2 + \beta\theta_3) & 0 & 0 \end{bmatrix} \tag{35}$$

$$\begin{cases} D'_{11} = D_{11} + J_{y_4}\mathsf{s}^2(\theta_2 + \theta_2) + J_{y_5}\mathsf{s}^2(\beta\theta_2 + \theta_2) + J_{z_4}\mathsf{c}^2(\theta_2 + \theta_2) + J_{z_5}\mathsf{c}^2(\beta\theta_2 + \theta_2) \\ D'_{22} = D_{22} + J_{x_4} + J_{x_5} \\ D'_{33} = D_{33} + J_{x_4} + J_{x_5}\beta^2 \\ D'_{12} = D'_{13} = 0 \\ D'_{23} = D_{23} + J_{x_4} + J_{x_5}\beta \\ D'_{211} = D_{211} \\ D'_{311} = D_{311} \\ D'_{111} = D'_{122} = D'_{133} = D'_{222} = D'_{233} = D'_{322} = D'_{333} = 0 \\ D'_{112} = D_{112} + \frac{1}{4}\Big(\big(J_{y_4} - J_{z_4}\big)\mathsf{s}(2\theta_2 + 2\theta_3) + \big(J_{y_5} - J_{z_5}\big)\mathsf{s}(2\beta\theta_2 + 2\theta_3) \Big) \\ D'_{113} = D_{113} + \frac{1}{4}\Big((J_{y_4} - J_{z_4})\mathsf{s}(2\theta_2 + 2\theta_3) + (J_{y_5} - J_{z_5})\mathsf{s}(2\beta\theta_2 + 2\theta_3) \Big) \\ D'_{123} = D'_{212} = D'_{213} = D'_{223} = D'_{312} = D'_{313} = D'_{323} = 0 \end{cases} \tag{36}$$

Dynamic balancing:

$$\begin{cases} D'_{23} = 0 \\ D'_{211} = 0 \\ D'_{311} = 0 \\ D'_{112} = 0 \\ D'_{113} = 0 \end{cases} \Rightarrow \begin{cases} \beta = -\frac{J_{x_3}+J_{x_4}}{J_{x_5}} \\ J_{y_2} = J_{z_2} \\ J_{y_3} = J_{z_3} + m_3 l_2^2 (1 + \frac{m_3}{m_2}) \\ J_{y_4} = J_{z_4} \\ J_{y_5} = J_{z_5} \end{cases} \tag{37}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1 = k_1 \ddot{\theta}_1 \\ \tau_2 = k_2 \ddot{\theta}_2 \\ f_3 = k_3 \ddot{d}_3 \end{cases} \tag{38}$$

Being:

$$\begin{cases} k_1 = J_{z_1} + J_{z_2} + J_{z_3} + J_{z_4} + J_{z_5} + m_2 l_1^2 + m_3 (l_1^2 + l_2^2) + \frac{l_1^2 m_3^2}{m_2} \\ k_2 = J_{x_2} + J_{x_3} + J_{x_4} + J_{x_5} + m_3 l_2^2 + \frac{l_1^2 m_3^2}{m_2} \\ k_3 = \frac{(J_{x_3}+J_{x_4})(J_{x_3}+J_{x_4}+J_{x_5})}{J_{x_5}} \end{cases} \tag{39}$$

3.3 3-DOF RRP spatial serial mechanism (SCARA)

$$\begin{cases} D_1 = g[(m_1 l_{g_1} + m_2 l_1 + m_3 l_1) \mathsf{c}(\theta_1) + m_2 l_{g_2} \mathsf{c}(\theta_1 + \theta_2)] \\ D_2 = g[m_2 l_{g_2} \mathsf{c}(\theta_1 + \theta_2)] \\ D_3 = 0 \end{cases} \tag{40}$$

Static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g_2} = 0 \end{cases} \tag{41}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{x_1} + J_{x_2} + J_{x_3} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2 (m_2+m_3)^2}{m_1} \\ D_{22} = J_{x_2} + J_{x_3} \\ D_{33} = m_3 \\ D_{12} = D_{22} \\ D_{13} = D_{23} = 0 \\ \mathbb{v}^\# = \mathbb{0} \\ \mathbb{g}^\# = \mathbb{0} \end{cases} \tag{42}$$

Coupling 2 discs:

Planar disc models:

$$\mathbb{M}_i^\# = \left[J_{x_{i+3}} \right]; \ \mathbb{p}_i^\# = \left[\omega_{x_{i+3}} \right], \ i = 1, 2 \tag{43}$$

Quasi-velocities constraints:

$$\begin{cases} \omega_{x_4} = \omega_{x_1} + \dot{\theta}_2 \\ \omega_{x_5} = \omega_{x_1} + \beta \dot{\theta}_2 \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = \dot{\theta}_1 + \dot{\theta}_2 \\ \omega_{x_5} = \dot{\theta}_1 + \beta \dot{\theta}_2 \end{cases} \Rightarrow \overline{\mathbb{p}} = \begin{bmatrix} \dot{\theta}_1 + \dot{\theta}_2 \\ \dot{\theta}_1 + \beta \dot{\theta}_2 \end{bmatrix} \tag{44}$$

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \frac{\partial \mathbb{p}^{\circ}}{\partial \mathbb{p}^{\#}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \end{bmatrix} \tag{45}$$

$$\begin{cases} D'_{11} = D_{11} + J_{x_4} + J_{x_5} \\ D'_{22} = D_{22} + J_{x_4} + J_{x_5}\beta^2 \\ D'_{33} = D_{33} \\ D'_{12} = D_{12} + J_{x_4} + J_{x_5}\beta \\ D'_{13} = 0 \\ D'_{23} = 0 \\ \mathbb{v}'^{\#} = \mathbb{0} \end{cases} \tag{46}$$

Dynamic balancing:

$$D'_{12} = 0 \Rightarrow \beta = -\frac{J_{x_2} + J_{x_3} + J_{x_4}}{J_{x_5}} \tag{47}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1 = k_1\ddot{\theta}_1 \\ \tau_2 = k_2\ddot{\theta}_2 \\ f_3 = k_3\ddot{d}_3 \end{cases} \tag{48}$$

Being:

$$\begin{cases} k_1 = J_{x_1} + J_{x_2} + J_{x_3} + J_{x_4} + J_{x_5} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2(m_2+m_3)^2}{m_1} \\ k_2 = \frac{(J_{x_2}+J_{x_3}+J_{x_4})(J_{x_2}+J_{x_3}+J_{x_4}+J_{x_5})}{J_{x_5}} \\ k_3 = m_3 \end{cases} \tag{49}$$

4 Conclusions

This work dealt with a systematic methodology for the adaptive balancing. Two balancing techniques were employed here: the addition of counterweight and counter-rotating disks coupled to the moving links. In addition, the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators was discussed regarding the achievement of such balancing and the complexity level of the modified mechanical structure. The balancing conditions were developed for 3-dof spatial and planar open loop-kinematic chain mechanisms, whose topologies are composed of revolute and prismatic joints. By analysing the necessary conditions, one can notice that the adaptive balancing brings great benefits for the planar RRR and the spatial RRP. However, for the spatial RRR, in spite of the achievement of the adaptive balancing, the modifications in the mechanical structure cause the increase of the link inertias and the actuator torques, accordingly. Consequently, the authors believe that the discussion provided here might help the designer to choose an adequate topology for a specific application taking advantage of the adaptive balancing whenever it brings no further consequences in terms of the added inertias.

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