A new approach for designing dynamic balanced serial mechanisms

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Abstract

Balancing is an important issue related to the design of mechanical systems in general, and also parallel mechanisms, in particular. In fact, the performance of parallel mechanisms associated to specific applications depends on the choice of the balancing method, namely, either static or dynamic, either passive or active, whether it is valid for a given trajectory or even for any motion. The main contribution of this work is to highlight the importance of the dynamic modelling process in order to achieve the compensation conditions associated to the chosen balancing technique. Due to the fact that parallel mechanisms have highly complex structures, the use of dynamic formalisms that employ redundant generalized coordinates, in association with the successive coupling of additional balancing elements to the original system model, can bring remarkable benefits. Additionally, this book chapter also discusses the impact of the dynamic model, developed in accordance with the methodology shown here, for the control strategy of parallel mechanisms. Finally, the simulation results demonstrates how effective is the presented methodology for the planar 5-bar with revolute joints (5R).

KEYWORDS: Dynamic balancing, serial mechanisms

1 Introduction and literature review

2 Mothodology

2.1 Static Balancing

2.2 Dynamic Balancing

3 Applying the technique

$$\mathbb{M}(\mathbb{q})\ddot{\mathbb{q}} + \mathbb{v}(\mathbb{q},\dot{\mathbb{q}}) + \mathbb{g}(\mathbb{q}) = \mathbb{u} \tag{1}$$

For a 3-DOF serial mechanism:

$$\mathbb{M} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} \tag{2}$$

$$\mathbb{v} = \begin{bmatrix} D_{111} & D_{122} & D_{133} \\ D_{211} & D_{222} & D_{233} \\ D_{311} & D_{322} & D_{333} \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \\ \dot{q}_3^2 \end{bmatrix} + 2 \begin{bmatrix} D_{112} & D_{113} & D_{123} \\ D_{212} & D_{213} & D_{223} \\ D_{312} & D_{313} & D_{323} \end{bmatrix} \begin{bmatrix} \dot{q}_1\dot{q}_2 \\ \dot{q}_1\dot{q}_3 \\ \dot{q}_2\dot{q}_3 \end{bmatrix} \tag{3}$$

$$\mathbb{g} = \begin{bmatrix} D_1 & D_2 & D_3 \end{bmatrix}^\top \tag{4}$$

$$\mathbb{q} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^\top \tag{5}$$

$$\mathbb{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^\top \tag{6}$$

For a rotational joint we call $q_i = \theta_i$ and $u_i = \tau_i$, and for a prismatic joint we call $q_i = d_i$ and $u_i = f_i$.

3.1 3-DOF RRR planar serial mechanism

$$\begin{cases} D_1 = g[(m_1 l_{g1} + m_2 l_1 + m_3 l_1)\mathsf{c}(\theta_1) + (m_2 l_{g2} + m_3 l_2)\mathsf{c}(\theta_1 + \theta_2) + m_3 l_{g3}\mathsf{c}(\theta_1 + \theta_2 + \theta_3)] \\ D_2 = g[(m_2 l_{g2} + m_3 l_2)\mathsf{c}(\theta_1 + \theta_2) + m_3 l_{g3}\mathsf{c}(\theta_1 + \theta_2 + \theta_3)] \\ D_3 = g[m_3 l_{g3}\mathsf{c}(\theta_1 + \theta_2 + \theta_3)] \end{cases} \tag{7}$$

Static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g2} = -\frac{l_2 m_3}{m_2} \\ l_{g3} = 0 \end{cases} \tag{8}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{x_1} + J_{x_2} + J_{x_3} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ D_{22} = J_{x_2} + J_{x_3} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{x_3} \\ D_{12} = D_{22} \\ D_{13} = D_{23} = D_{33} \\ \mathbb{v} = \mathbb{0} \\ \mathbb{g} = \mathbb{0} \end{cases} \tag{9}$$

Coupling 4 discs:

$$\begin{cases} D'_{11} = D_{11} + J_{x_4} + J_{x_5} + J_{x_6} + J_{x_7} \\ D'_{22} = D_{22} + J_{x_4} + J_{x_5}\beta^2 + J_{x_6} + J_{x_7} \\ D'_{33} = D_{33} + J_{x_6} + J_{x_7}\gamma^2 \\ D'_{12} = D_{12} + J_{x_4} + J_{x_5}\beta + J_{x_6} + J_{x_7} \\ D'_{13} = D_{13} + J_{x_6} + J_{x_7}\gamma \\ D'_{23} = D'_{13} \\ \mathbb{v}' = \mathbb{0} \end{cases} \tag{10}$$

Dynamic balancing:

$$\begin{cases} D'_{12} = 0 \\ D'_{13} = 0 \\ D'_{23} = 0 \end{cases} \Rightarrow \begin{cases} \beta = -\frac{J_{x_2}+J_{x_3}+J_{x_4}+J_{x_6}+J_{x_7}+m_3 l_2^2+\frac{m_3^2 l_2^2}{m_2}}{J_{x_5}} \\ \gamma = -\frac{J_{x_3}+J_{x_6}}{J_{x_7}} \end{cases} \tag{11}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1 = k_1\ddot{\theta}_1 \\ \tau_2 = k_2\ddot{\theta}_2 \\ \tau_3 = k_3\ddot{\theta}_3 \end{cases} \tag{12}$$

Being:

$$\begin{cases} k_1 = J_{x_1} + J_{x_2} + J_{x_3} + J_{x_4} + J_{x_5} + J_{x_6} + J_{x_7} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ k_2 = J_{x_2} + J_{x_3} + J_{x_4} + J_{x_6} + J_{x_7} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} + \frac{\left(J_{x_2}+J_{x_3}+J_{x_4}+J_{x_6}+J_{x_7}+m_3 l_2^2 + \frac{m_3^2 l_2^2}{m_2}\right)^2}{J_{x_5}} \\ k_3 = \frac{(J_{x_3}+J_{x_6})(J_{x_3}+J_{x_6}+J_{x_7})}{J_{x_7}} \end{cases} \tag{13}$$

3.2 3-DOF RRR spatial serial mechanism

$$\begin{cases} D_1 = 0 \\ D_2 = g[(m_2 l_{g2} + m_3 l_2)\mathsf{c}(\theta_2) + m_3 l_{g3}\mathsf{c}(\theta_2+\theta_3)] \\ D_3 = g[m_3 l_{g3}\mathsf{c}(\theta_2+\theta_3)] \end{cases} \tag{14}$$

Static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g2} = -\frac{l_2 m_3}{m_2} \\ l_{g3} = 0 \end{cases} \tag{15}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{y_2}\mathsf{s}^2(\theta_2) + J_{y_3}\mathsf{s}^2(\theta_2+\theta_3) + J_{z_1} + J_{z_2}\mathsf{c}^2(\theta_2) + J_{z_3}\mathsf{c}^2(\theta_2+\theta_3) + m_3(l_1 + l_2\mathsf{c}(\theta_2))^2 + \frac{(m_2 l_1 - m_3 l_2 \mathsf{c}(\theta_2))^2}{m_2} \\ D_{22} = J_{x_2} + J_{x_3} + m_2 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{x_3} \\ D_{12} = D_{13} = 0 \\ D_{23} = D_{33} \\ D_{211} = -\frac{1}{2}\Big(\big(J_{y_2} - J_{z_2}\big)\mathsf{s}(2\theta_2) + \big(J_{y_3} - J_{z_3} - m_3 l_2^2(1 + \frac{m_3}{m_2})\big)\mathsf{s}(2\theta_2 + 2\theta_3)\Big) \\ D_{311} = \frac{1}{2}\Big(\big(J_{z_3} - J_{y_3}\big)\mathsf{s}(2\theta_2 + 2\theta_3)\Big) \\ D_{111} = D_{122} = D_{133} = D_{222} = D_{233} = D_{322} = D_{333} = 0 \\ D_{112} = -D_{211} \\ D_{113} = -D_{311} \\ D_{123} = D_{212} = D_{213} = D_{223} = D_{312} = D_{313} = D_{323} = 0 \\ \mathfrak{g} = \mathbb{0} \end{cases} \tag{16}$$

Coupling 2 discs:

$$\begin{cases} D'_{11} = D_{11} + J_{y_4}\mathsf{s}^2(\theta_2+\theta_2) + J_{y_5}\mathsf{s}^2(\beta\theta_2+\theta_2) + J_{z_4}\mathsf{c}^2(\theta_2+\theta_2) + J_{z_5}\mathsf{c}^2(\beta\theta_2+\theta_2) \\ D'_{22} = D_{22} + J_{x_4} + J_{x_5} \\ D'_{33} = D_{33} + J_{x_4} + J_{x_5}\beta^2 \\ D'_{12} = D'_{13} = 0 \\ D'_{23} = D_{23} + J_{x_4} + J_{x_5}\beta \\ D'_{211} = D_{211} \\ D'_{311} = D_{311} \\ D'_{111} = D'_{122} = D'_{133} = D'_{222} = D'_{233} = D'_{322} = D'_{333} = 0 \\ D'_{112} = D_{112} + \frac{1}{4}\Big(\big(J_{y_4} - J_{z_4}\big)\mathsf{s}(2\theta_2 + 2\theta_3) + \big(J_{y_5} - J_{z_5}\big)\mathsf{s}(2\beta\theta_2 + 2\theta_3)\Big) \\ D'_{113} = D_{113} + \frac{1}{4}\Big((J_{y_4} - J_{z_4})\mathsf{s}(2\theta_2 + 2\theta_3) + (J_{y_5} - J_{z_5})\mathsf{s}(2\beta\theta_2 + 2\theta_3)\Big) \\ D'_{123} = D'_{212} = D'_{213} = D'_{223} = D'_{312} = D'_{313} = D'_{323} = 0 \end{cases} \tag{17}$$

Dynamic balancing:

$$\begin{cases} D'_{23} = 0 \\ D'_{211} = 0 \\ D'_{311} = 0 \\ D'_{112} = 0 \\ D'_{113} = 0 \end{cases} \Rightarrow \begin{cases} \beta = -\frac{J_{x_3}+J_{x_4}}{J_{x_5}} \\ J_{y_2} = J_{z_2} \\ J_{y_3} = J_{z_3} + m_3 l_2^2 (1 + \frac{m_3}{m_2}) \\ J_{y_4} = J_{z_4} \\ J_{y_5} = J_{z_5} \end{cases} \tag{18}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1 = k_1 \ddot{\theta}_1 \\ \tau_2 = k_2 \ddot{\theta}_2 \\ f_3 = k_3 \ddot{d}_3 \end{cases} \tag{19}$$

Being:

$$\begin{cases} k_1 = J_{z_1} + J_{z_2} + J_{z_3} + J_{z_4} + J_{z_5} + m_2 l_1^2 + m_3 (l_1^2 + l_2^2) + \frac{l_1^2 m_3^2}{m_2} \\ k_2 = J_{x_2} + J_{x_3} + J_{x_4} + J_{x_5} + m_3 l_2^2 + \frac{l_1^2 m_3^2}{m_2} \\ k_3 = \frac{(J_{x_3}+J_{x_4})(J_{x_3}+J_{x_4}+J_{x_5})}{J_{x_5}} \end{cases} \tag{20}$$

3.3 3-DOF RRP spatial serial mechanism (SCARA)

$$\begin{cases} D_1 = g[(m_1 l_{g1} + m_2 l_1 + m_3 l_1)\mathsf{c}_1 + m_2 l_{g2} \mathsf{c}_{1+2}] \\ D_2 = g[m_2 l_{g2} \mathsf{c}_{1+2}] \\ D_3 = 0 \end{cases} \tag{21}$$

Static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g2} = 0 \end{cases} \tag{22}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{x_1} + J_{x_2} + J_{x_3} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2 (m_2+m_3)^2}{m_1} \\ D_{22} = J_{x_2} + J_{x_3} \\ D_{33} = m_3 \\ D_{12} = D_{22} \\ D_{13} = D_{23} = 0 \\ \mathbb{v} = \mathbb{0} \\ \mathbb{g} = \mathbb{0} \end{cases} \tag{23}$$

Coupling 2 discs:

$$\begin{cases} D'_{11} = D_{11} + J_{x_4} + J_{x_5} \\ D'_{22} = D_{22} + J_{x_4} + J_{x_5} \beta^2 \\ D'_{33} = D_{33} \\ D'_{12} = D_{12} + J_{x_4} + J_{x_5} \beta \\ D'_{13} = 0 \\ D'_{23} = 0 \\ \mathbb{v}' = \mathbb{0} \end{cases} \tag{24}$$

Dynamic balancing:

$$D'_{12} = 0 \Rightarrow \beta = -\frac{J_{x_2} + J_{x_3} + J_{x_4}}{J_{x_5}} \tag{25}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1 = k_1 \ddot{\theta}_1 \\ \tau_2 = k_2 \ddot{\theta}_2 \\ f_3 = k_3 \ddot{d}_3 \end{cases} \tag{26}$$

Being:

$$\begin{cases} k_1 = J_{x_1} + J_{x_2} + J_{x_3} + J_{x_4} + J_{x_5} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2 (m_2+m_3)^2}{m_1} \\ k_2 = \frac{(J_{x_2}+J_{x_3}+J_{x_4})(J_{x_2}+J_{x_3}+J_{x_4}+J_{x_5})}{J_{x_5}} \\ k_3 = m_3 \end{cases} \tag{27}$$

4 Conclusions

References

[1] V. Van der Wijk, Shaking moment balancing of mechanisms with principal vectors and moments *Front. Mech. Eng.*, 8(1): 10–16, 2013.

[2] V. H. Arakelian V. , M. R. Smith, Design of planar 3-dof 3-RRR reactionless parallel manipulators *Mechatronics*, 18: 601–606, 2008.

[3] J.-T. Seo, J. H. Woo, H. Lim, J. Chung, W. K. Kim, and B.-J. Yi, Design of an Antagonistically Counter-Balancing Parallel Mechanism *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Tokyo, November 3-7: 2882–2887, 2013.

[4] Y. Wu, C. M. Gosselin, Design of reactionless 3-dof and 6-dof parallel manipulators using parallelepiped mechanisms *IEEE Transactions on Robotics*, 21(5): 821–833, 2005.

[5] C. M. Gosselin, F. Vollmer, G. Ct, Y. Wu, Synthesis and design of reactionless three-degree-of-freedom parallel mechanisms *IEEE Transactions on Robotics and Automation*, 20(2): 191–199, 2004.

[6] J. Wang, C. M. Gosselin, Static balancing of spatial four-degree-of-freedom parallel mechanisms *Mech. Mach. Theory*, 35: 563–592, 2000.

[7] J. Wang, C. M. Gosselin, Static balancing of spatial three-degree-of-freedom parallel mechanisms *Mech. Mach. Theory*, 34: 437–452, 1999.

[8] G. Alici, B. Shirinzadeh, Optimum Force Balancing with Mass Distribution and a Single Elastic Element for a Five-bar Parallel Manipulator *Proceedings of the IEEE International Conference on Robotics and Automation*,Taipei, September 14-19: 3666–3671, 2003.

[9] G. Alici, B. Shirinzadeh, Optimum dynamic balancing of planar parallel manipulators based on sensitivity analysis *Mech. Mach. Theory*, 41: 1520–1532, 2006.

[10] M. B. Dehkordi, A. Frisoli, E. Sotgiu, M. Bergamasco, Modelling and Experimental Evaluation of a Static Balancing Technique for a new Horizontally Mounted 3-UPU Parallel Mechanism *International Journal of Advanced Robotic Systems*, 9: 193–205, 2012.

[11] K. Wang, M. Luo, T. Mei, J. Zhao, Y. Cao, Dynamics Analysis of a Three-DOF Planar Serial-Parallel Mechanism for Active Dynamic Balancing with Respect to a Given Trajectory *International Journal of Advanced Robotic Systems*, 10: 23–33, 2013.

[12] A. Russo, R. Sinatra, F. Xi, Static balancing of parallel robots *Mech. Mach. Theory*, 40: 191–202, 2005.

[13] S. K. Agrawal, A. Fattah, Gravity-balancing of spatial robotic manipulators *Mech. Mach. Theory*, 39: 1331–1344, 2004.

[14] S. Briot, V. Arakelian, J.-P. Le Baron, Shaking force minimization of high-speed robots via centre of mass acceleration control *Mech. Mach. Theory*, 57: 1–12, 2012.

[15] T. A. H. Coelho, L. Yong, V. F. A. Alves, Decoupling of dynamic equations by means of adaptive balancing of 2-dof open-loop mechanisms *Mech. Mach. Theory*, 39: 871–881, 2004.

[16] M. Moradi, A. Nikoobin, S. Azadi, Adaptive Decoupling for Open Chain Planar Robots *Transaction B: Mechanical Engineering*, 17(5): 376–386, 2010.