A new approach for designing dynamic balanced serial mechanisms

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Abstract

Adaptive balancing means that the mechanical structure of the manipulator is modified in order to achieve the decoupling of dynamic equations. This work deals with a systematic formulation for the adaptive balancing. Basically, two traditional balancing techniques are employed here: the addition of counterweight and counter-rotating disks coupled to the moving links. In addition, the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators is discussed regarding the achievement of such balancing and the complexity level of the modified mechanical structure. The balancing conditions are developed here for 3-dof spatial and planar open-loop kinematic chain mechanisms, whose topologies are composed of revolute and prismatic joints.

KEYWORDS: Dynamic balancing, serial mechanisms

Nomenclature

$a, b, \ldots$	Scalars, components of column-matrices, components of matrices or indexes
$A, B, \ldots$	Scalars, components of column-matrices or components of matrices
$\mathbb{a}, \mathbb{b}, \ldots$	Column-matrices
$\mathbb{A}, \mathbb{B}, \ldots$	Matrices
$\boldsymbol{a}, \boldsymbol{b}, \ldots$	Vectors
$\mathtt{A}, \mathtt{B}, \ldots$	Coordinate systems
$\mathscr{A}, \mathscr{B}, \ldots$	Sets or multibody mechanical systems
$\mathtt{B}_i$	Coordinate system fixed in the i$^{\text{th}}$ rigid body of the mechanical system
$\mathbb{C}$	Kinematic constraints matrix
$\mathsf{c}(.)$	Shorthand notation for $\mathsf{cos}(.)$
g	gravitational acceleration
$\mathbb{g}^{\#}$	Generalized gravitational forces column-matrix of a serial mechanism
$\mathbb{g}_i^{\#}$	Generalized gravitational forces column-matrix of a counter-rotating disc
$\mathbb{g}'$	Generalized uncoupled gravitational forces column-matrix of a serial mechanism coupled with counter-rotating discs
$\mathbb{g}'^{\#}$	Generalized gravitational forces column-matrix of a serial mechanism coupled with counter-rotating discs

$J_{x_i}, J_{y_i}, J_{z_i}$	Principal moments of inertia of the $\mathrm{i^{th}}$ rigid body of the mechanical system
l_i	Length of the $\mathrm{i^{th}}$ bar of a serial mechanism
l_{g_i}	Position of the mass center of the $\mathrm{i^{th}}$ bar relative to the $\mathrm{i^{th}}$ joint and of a serial mechanism
m_i	Mass of the $\mathrm{i^{th}}$ rigid body of the mechanical system
$\mathbb{M}^{\#}$	Generalized inertia matrix of a serial mechanism
$\mathbb{M}_i^{\#}$	Generalized inertia matrix of a counter-rotating disc
$\mathbb{M}'$	Generalized uncoupled inertia matrix of a serial mechanism coupled with counter-rotating discs
$\mathbb{M}'^{\#}$	Generalized inertia matrix of a serial mechanism coupled with counter-rotating discs
$\mathcal{N}$	Inertial reference frame
$\mathbb{p}^{\#}$	Independent quasi-velocities column-matrix
$\mathbb{p}^{\circ}$	Redundant quasi-velocities column-matrix
$\mathbb{p}$	Quasi-velocities column-matrix
q_i	Generalized coordinate
$\mathbb{q}^{\#}$	Independent generalized coordinates column-matrix
$\mathsf{s}(.)$	Shorthand notation for $\mathsf{sin}(.)$
u_i	Effort made by the $\mathrm{i^{th}}$ actuator of a serial mechanism
$\mathbb{u}$	Generalized actuators' efforts column-matrix
$\mathbb{v}^{\#}$	Generalized coupled gyroscopic inertia forces column-matrix of a serial mechanism
$\mathbb{v}_i^{\#}$	Generalized coupled gyroscopic inertia forces column-matrix of a counter-rotating disc
$\mathbb{v}'$	Generalized uncoupled gyroscopic inertia forces column-matrix of a serial mechanism coupled with counter-rotating discs
$\mathbb{v}'^{\#}$	Generalized coupled gyroscopic inertia forces column-matrix of a of a serial mechanism coupled with counter-rotating discs
$[\boldsymbol{\omega}_i]_{\mathtt{B}_j}$	Angular velocity of the $\mathrm{i^{th}}$ rigid body of the mechanical system measured relatively to a inertial reference frame $\mathcal{N}$, written in the basis of $\mathtt{B}_j$
ω_{x_i}	1^{st} component of $[\boldsymbol{\omega}_i]_{\mathtt{B}_i}$
ω_{y_i}	2^{nd} component of $[\boldsymbol{\omega}_i]_{\mathtt{B}_i}$
ω_{z_i}	3^{rd} component of $[\boldsymbol{\omega}_i]_{\mathtt{B}_i}$
$\mathbb{0}$	Null column-matrix or null matrix
$\mathbb{1}$	Identity matrix
$[\mathbb{1}]_{\mathtt{B}_i \mid \mathtt{B}_j}$	Change of basis matrix, i.e. $[\boldsymbol{v}]_{\mathtt{B}_i} = [\mathbb{1}]_{\mathtt{B}_i \mid \mathtt{B}_j} \cdot [\boldsymbol{v}]_{\mathtt{B}_j}$
$[\cdot]^{\mathsf{T}}$	Matrix transposition

1 Introduction and literature review

Balancing can be considered as an important issue related to the design of any kind of mechanical system in general, and also serial manipulators, in particular. As a matter of fact, the performance of open-loop kinematic chain mechanisms associated to specific applications depends on the choice of the balancing method, namely, either static [6] or dynamic [4], either passive [1, 4, 5, 6, 7, 8, 9, 10, 12, 13] or active [2, 3, 11, 14, 15, 16], whether it is valid for a given trajectory or even for any motion.

Moreover, Coelho et al. [15], Moradi et al. [16] and Arakelian and Sargsyan [17] use the adaptive balancing to achieve the decoupling of dynamic equations for open-loop kinematic chain mechanisms. Consequently, this action simplifies the control of manipulators due to the fact that the actuators can be controlled independently. The necessary modifications comprise the addition of either counterweights, or counter-rotating disks or even both to the original kinematic chain of the manipulator. Consequently, the terms associated to gravitational, centripetal and Coriolis efforts are completely eliminated from the dynamic equations. As a matter of fact, the effective inertias for all the actuator axes are constant and the mathematical expressions of the driving torques/forces become rather simple. One of the main advantages of this approach concerns the reduction of computing time for a closed-loop control of manipulators. Such reduction is really significant and it constitutes in a great benefit for real-time applications.

The contributions of this work are the following: to present a systematic formulation to obtain the balancing conditions for the adaptive balancing, to discuss the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators, not only in terms of the possibility to achieve such balancing but also in terms of the increase in the complexity level of the modified mechanical structure. The analysed manipulators correspond to 3-dof spatial and planar open loop-kinematic chain, whose topologies are composed of revolute and prismatic joints.

This work is organized as follows. Section 2 describes the proposed formulation, while section 3 deals with the application of the formulation to 3 types of serial manipulators. Finally, the conclusions are drawn in section 4.

2 Formulation

2.1 Dynamic Model

The dynamic model of a serial mechanism can be written in this way:

$$\mathbb{M}^{\#}(\mathbb{q}^{\#})\ddot{\mathbb{q}}^{\#} + \mathbb{v}^{\#}(\mathbb{q}^{\#}, \dot{\mathbb{q}}^{\#}) + \mathbb{g}^{\#}(\mathbb{q}^{\#}) = \mathbb{u} \tag{1}$$

Being $\mathbb{q}^{\#}$ a column-matrix of independent generalized coordinates, whose entries are relative displacements of the joints, and $\mathbb{u}$ the generalized actuators' efforts in the directions of the independent quasi-velocities $\mathbb{p}^{\#} = \dot{\mathbb{q}}^{\#}$.

To perform the dynamic balancing of a serial mechanism, using the proposed approach, it is necessary to first obtain the dynamic model of the unbalanced mechanism. As in a serial mechanism it is possible to express all the absolute velocities of the links' centers of mass and all the absolute angular velocities of the links in function of $\mathbb{q}^{\#}$ and $\dot{\mathbb{q}}^{\#}$, the dynamic model can be obtained without major difficulties using analytical mechanics techniques, like Lagrange [18] and Kane [19] formalisms, Orsino's method [23], and Boltzmann-Hamel equations [20], allied to programs or libraries of programming languages that are capable of using symbolic manipulation, such as Mathematica and SymPy.

2.2 Static Balancing

After obtaining the dynamic model, the static balancing is performed finding the links' centers of mass positions that make $\mathbb{g}^{\#} = \mathbb{0}$. It is possible for mechanisms with only revolute joints and mechanisms with prismatic joints whose directions are orthogonal to the gravity. The positioning of the centers of mass is done mechanically extending the mechanism's bars and adding counterweights.

2.3 Dynamic Balancing

The dynamic balancing is performed coupling counter-rotating disks to the statically balanced mechanism model. This is done using the coupling subsystems technique of Orsino's method * [22, 23].

Let $\mathscr{M}_0$ be a mechanical subsystem composed by a statically balanced serial mechanism, whose equation of motion is given by (1), with $\mathbb{g}^\# = \mathbb{0}$. Let $\mathscr{M}_i$ be a mechanical subsystem composed by a counter-rotating disk that will be coupled to the mechanism, whose equation of motion is given by:

$$\mathbb{M}_i^\# \dot{\mathbb{p}}^\# + \mathbb{v}_i^\# + \mathbb{g}_i^\# = \mathbb{u}_i \tag{2}$$

Being $\mathbb{p}^\#$ a set of independent quasi-velocities, whose elements are non-null components of the absolute angular velocity vector of the disk, written in a basis fixed to the disk, and $\mathbb{v}_i^\# = \mathbb{g}_i^\# = \mathbb{u}_i = \mathbb{0}$. In this model, only the rotative inertias are considered, as the effects of the mass of the disk are considered in the mass and the inertia moments of the bar that the disk is coupled, also affecting the counterweights positioning calculus, so that the mechanism continues statically balanced.

Assuming that n counter-rotating disks will be coupled to the mechanism, the following definitions are made:

$$\mathbb{M}' = \begin{bmatrix} \mathbb{M}^\# & \mathbb{0} & \dots & \mathbb{0} \\ \mathbb{0} & \mathbb{M}_1^\# & \dots & \mathbb{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{0} & \mathbb{0} & \dots & \mathbb{M}_n^\# \end{bmatrix} \tag{3}$$

$$\mathbb{v}' = \begin{bmatrix} \mathbb{v}^{\#\mathsf{T}} & \mathbb{v}_1^{\#\mathsf{T}} & \dots & \mathbb{v}_n^{\#\mathsf{T}} \end{bmatrix}^\mathsf{T} \tag{4}$$

$$\mathbb{g}' = \begin{bmatrix} \mathbb{g}^{\#\mathsf{T}} & \mathbb{g}_1^{\#\mathsf{T}} & \dots & \mathbb{g}_n^{\#\mathsf{T}} \end{bmatrix}^\mathsf{T} \tag{5}$$

$$\mathbb{p}^\circ = \begin{bmatrix} \mathbb{p}_1^{\#\mathsf{T}} & \dots & \mathbb{p}_n^{\#\mathsf{T}} \end{bmatrix}^\mathsf{T} \tag{6}$$

$$\mathbb{p} = \begin{bmatrix} \mathbb{p}^{\#\mathsf{T}} & \mathbb{p}^{\circ\mathsf{T}} \end{bmatrix}^\mathsf{T} \tag{7}$$

Let $\underline{\mathbb{p}}^\circ$ be the vector $\mathbb{p}^\circ$ written in function of $\mathbb{q}^\#$ e $\mathbb{p}^\#$, i.e.:

$$\mathbb{p}^\circ = \underline{\mathbb{p}}^\circ(\mathbb{q}^\#, \mathbb{p}^\#) \tag{8}$$

The kinematic constraints matrix is defined:

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \dfrac{\partial \underline{\mathbb{p}}^\circ}{\partial \mathbb{p}^\#} \end{bmatrix} \tag{9}$$

The dynamic model of the serial mechanism coupled with counter-rotating disk is given by:

$$\mathbb{M}'^\#(\mathbb{q}^\#)\ddot{\mathbb{q}}^\# + \mathbb{v}'^\#(\mathbb{q}^\#, \dot{\mathbb{q}}^\#) + \mathbb{g}'^\#(\mathbb{q}^\#) = \mathbb{u} \tag{10}$$

Being:

$$\mathbb{M}'^\# = \mathbb{C}^\mathsf{T}\mathbb{M}'\mathbb{C} \tag{11}$$

*Similar modelling approaches are presented in the works by Altuzarra et al. [20] and Orsino at al. [21]

$$\mathbb{v}'^{\#} = \mathbb{C}^{\mathsf{T}}(\mathbb{M}'\dot{\mathbb{C}}\dot{\mathbb{q}}^{\#} + \mathbb{v}') \tag{12}$$

$$\mathbb{g}'^{\#} = \mathbb{C}^{\mathsf{T}}\mathbb{g}' \tag{13}$$

The dynamic balancing is achieved by finding the relations between system parameters that make diagonal $\mathbb{M}'^{\#}$ diagonal and $\mathbb{v}'^{\#}$ null.

3 Applying the technique

In this section, the proposed formulation will be applied in three different 3-dof serial mechanisms. In order to do this, first some definitions valid for these mechanisms will be made:

$$\mathbb{M}^{\#} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} \tag{14}$$

$$\mathbb{v}^{\#} = \begin{bmatrix} D_{111} & D_{122} & D_{133} \\ D_{211} & D_{222} & D_{233} \\ D_{311} & D_{322} & D_{333} \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \\ \dot{q}_3^2 \end{bmatrix} + 2 \begin{bmatrix} D_{112} & D_{113} & D_{123} \\ D_{212} & D_{213} & D_{223} \\ D_{312} & D_{313} & D_{323} \end{bmatrix} \begin{bmatrix} \dot{q}_1\dot{q}_2 \\ \dot{q}_1\dot{q}_3 \\ \dot{q}_2\dot{q}_3 \end{bmatrix} \tag{15}$$

$$\mathbb{g}^{\#} = \begin{bmatrix} D_1 & D_2 & D_3 \end{bmatrix}^{\top} \tag{16}$$

$$\mathbb{q}^{\#} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^{\top} \tag{17}$$

$$\mathbb{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^{\top} \tag{18}$$

For revolute joints is defined $q_i = \theta_i$ and $u_i = \tau_i$, and for prismatic joints is defined $q_i = d_i$ and $u_i = f_i$, to conform with the more traditional notations of this area.

Denavit-Hartenberg convention will be used for defining the coordinate system fixed to the links of the mechanism and to enumerate the links and joints of the mechanism.

3.1 3-dof RRR planar serial mechanism

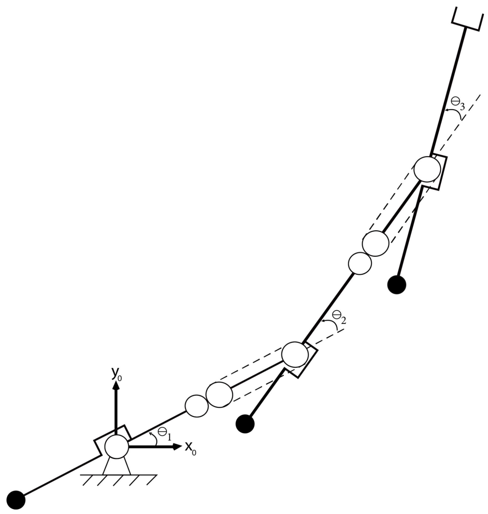

Figure 1: Dynamically balanced RRR planar serial mechanism

The entries of $\mathbb{g}^{\#}$ for the unbalanced mechanism are given by:

$$\begin{cases} D_1 = g[(m_1 l_{g_1} + m_2 l_1 + m_3 l_1)\mathsf{c}(\theta_1) + (m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_1+\theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_1+\theta_2+\theta_3)] \\ D_2 = g[(m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_1+\theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_1+\theta_2+\theta_3)] \\ D_3 = g[m_3 l_{g_3}\mathsf{c}(\theta_1+\theta_2+\theta_3)] \end{cases} \tag{19}$$

Performing the static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g_2} = -\frac{l_2 m_3}{m_2} \\ l_{g_3} = 0 \end{cases} \tag{20}$$

Substituting (20) in the mechanism model, the dynamic model of the statically balanced mechanism is obtained:

$$\begin{cases} D_{11} = J_{z_1} + J_{z_2} + J_{z_3} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ D_{22} = J_{z_2} + J_{z_3} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{z_3} \\ D_{12} = D_{22} \\ D_{13} = D_{23} = D_{33} \\ \mathbb{v}^{\#} = \mathbb{0} \\ \mathbb{g}^{\#} = \mathbb{0} \end{cases} \tag{21}$$

To perform the dynamic balancing, 4 counter-rotating disks are coupled to the mechanism, as shown in figure 1. As the disks rotate in only one plane, the following dynamic models are used for them:

$$\mathbb{M}_i^{\#} = \left[J_{z_{i+3}}\right]\,;\; \mathbb{p}_i^{\#} = \left[\omega_{z_{i+3}}\right]\,,\; i = 1,2,3,4 \tag{22}$$

The counter-rotating disks 1 and 2 (rigid bodies 4 and 5) are coupled to link 1, being that disk 1 has an angular displacement of θ_2 relative to link 1, due to the belt transmission of the spin of motor 2, while disk 2 has an angular displacement of $\beta\theta_2$, with $\beta < 0$, relative to link 1, due to the gear transmission of the spin of disk 1.

The counter-rotating disks 3 and 4 (rigid bodies 6 and 7) are coupled to link 2, being that disk 3 has an angular displacement of θ_3 relative to link 2, due to the belt transmission of the spin of motor 3, while disk 4 has an angular displacement of $\gamma\theta_3$, with $\gamma < 0$, relative to link 2, due to the gear transmission of the spin of disk 3.

Thus, the following quasi-velocities constraints are obtained:

$$\begin{cases} \omega_{z_4} = \omega_{z_1} + \dot\theta_2 \\ \omega_{z_5} = \omega_{z_1} + \beta\dot\theta_2 \\ \omega_{z_6} = \omega_{z_2} + \dot\theta_3 \\ \omega_{z_7} = \omega_{z_2} + \gamma\dot\theta_3 \end{cases} \Rightarrow \begin{cases} \omega_{z_4} = \dot\theta_1 + \dot\theta_2 \\ \omega_{z_5} = \dot\theta_1 + \beta\dot\theta_2 \\ \omega_{z_6} = \dot\theta_1 + \dot\theta_2 + \dot\theta_3 \\ \omega_{z_7} = \dot\theta_1 + \dot\theta_2 + \gamma\dot\theta_3 \end{cases} \Rightarrow \underline{\mathbb{p}}^{\circ} = \begin{bmatrix} \dot\theta_1 + \dot\theta_2 \\ \dot\theta_1 + \beta\dot\theta_2 \\ \dot\theta_1 + \dot\theta_2 + \dot\theta_3 \\ \dot\theta_1 + \dot\theta_2 + \gamma\dot\theta_3 \end{bmatrix} \tag{23}$$

$$\therefore \mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \dfrac{\partial \underline{\mathbb{p}}^{\circ}}{\partial \mathbb{p}^{\#}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \\ 1 & 1 & 1 \\ 1 & 1 & \gamma \end{bmatrix} \tag{24}$$

Applying (21), (22) and (24) in (11), (12) and (13), the mechanism's statically balanced model coupled with the counter-rotating disks is obtained:

$$
\begin{cases}
D'_{11} = D_{11} + J_{z_4} + J_{z_5} + J_{z_6} + J_{z_7} \\
D'_{22} = D_{22} + J_{z_4} + J_{z_5}\beta^2 + J_{z_6} + J_{z_7} \\
D'_{33} = D_{33} + J_{z_6} + J_{z_7}\gamma^2 \\
D'_{12} = D_{12} + J_{z_4} + J_{z_5}\beta + J_{z_6} + J_{z_7} \\
D'_{13} = D_{13} + J_{z_6} + J_{z_7}\gamma \\
D'_{23} = D'_{13} \\
\mathbb{v}'^{\#} = \mathbb{0}
\end{cases}
\tag{25}
$$

To perform the dynamic balancing, the values of β and γ in function of the mechanism's parameters that makes $\mathbb{M}'^{\#}$ diagonal are found. Thus:

$$
\begin{cases}
D'_{12} = 0 \\
D'_{13} = 0
\end{cases}
\Rightarrow
\begin{cases}
\beta = -\frac{J_{z_2}+J_{z_3}+J_{z_4}+J_{z_6}+J_{z_7}+m_3 l_2^2+\frac{m_3^2 l_2^2}{m_2}}{J_{z_5}} \\
\gamma = -\frac{J_{z_3}+J_{z_6}}{J_{z_7}}
\end{cases}
\tag{26}
$$

Applying (26) in (25), the machanism's dynamic balanced model is obtained:

$$
\begin{cases}
\tau_1 = k_1\ddot{\theta}_1 \\
\tau_2 = k_2\ddot{\theta}_2 \\
\tau_3 = k_3\ddot{\theta}_3
\end{cases}
\tag{27}
$$

Being:

$$
\begin{cases}
k_1 = J_{z_1} + J_{z_2} + J_{z_3} + J_{z_4} + J_{z_5} + J_{z_6} + J_{z_7} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\
k_2 = J_{z_2} + J_{z_3} + J_{z_4} + J_{z_6} + J_{z_7} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} + \frac{\left(J_{z_2}+J_{z_3}+J_{z_4}+J_{z_6}+J_{z_7}+m_3 l_2^2+\frac{m_3^2 l_2^2}{m_2}\right)^2}{J_{z_5}} \\
k_3 = \frac{(J_{z_3}+J_{z_6})(J_{z_3}+J_{z_6}+J_{z_7})}{J_{z_7}}
\end{cases}
\tag{28}
$$

3.2 3-dof RRR spatial serial mechanism

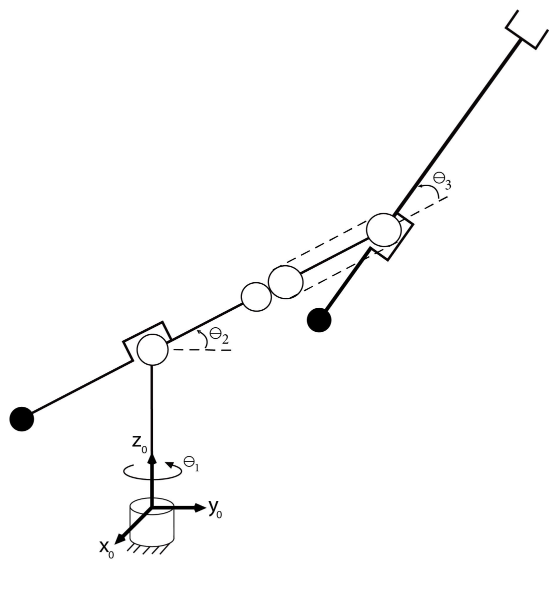

Figure 2: Dynamically balanced RRR spatial serial mechanism

The entries of $\mathbb{g}^{\#}$ for the unbalanced mechanism are given by:

$$\begin{cases} D_1 = 0 \\ D_2 = g[(m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_2+\theta_3)] \\ D_3 = g[m_3 l_{g_3}\mathsf{c}(\theta_2+\theta_3)] \end{cases} \tag{29}$$

Performing the static balancing:

$$\begin{cases} D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_2} = -\frac{l_2 m_3}{m_2} \\ l_{g_3} = 0 \end{cases} \tag{30}$$

Substituting (30) in the mechanism model, the dynamic model of the statically balanced mechanism is obtained:

$$\begin{cases} D_{11} = J_{x_2}\mathsf{s}^2(\theta_2) + J_{x_3}\mathsf{s}^2(\theta_2+\theta_3) + J_{z_1} + J_{y_2}\mathsf{c}^2(\theta_2) + J_{y_3}\mathsf{c}^2(\theta_2+\theta_3) + m_3(l_1 + l_2\mathsf{c}(\theta_2))^2 + \frac{(m_2 l_1 - m_3 l_2 \mathsf{c}(\theta_2))^2}{m_2} \\ D_{22} = J_{z_2} + J_{z_3} + m_2 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{z_3} \\ D_{12} = D_{13} = 0 \\ D_{23} = D_{33} \\ D_{211} = -\frac{1}{2}\Big(\big(J_{x_2} - J_{y_2}\big)\mathsf{s}(2\theta_2) + \big(J_{x_3} - J_{y_3} - m_3 l_2^2(1 + \frac{m_3}{m_2})\big)\mathsf{s}(2\theta_2 + 2\theta_3)\Big) \\ D_{311} = \frac{1}{2}\Big(\big(J_{y_3} - J_{x_3}\big)\mathsf{s}(2\theta_2 + 2\theta_3)\Big) \\ D_{111} = D_{122} = D_{133} = D_{222} = D_{233} = D_{322} = D_{333} = 0 \\ D_{112} = -D_{211} \\ D_{113} = -D_{311} \\ D_{123} = D_{212} = D_{213} = D_{223} = D_{312} = D_{313} = D_{323} = 0 \\ \mathbb{g}^{\#} = \mathbb{0} \end{cases} \tag{31}$$

To perform the dynamic balancing, 2 counter-rotating disks are coupled to the mechanism, as shown in figure 2. The following dynamic models are used for them:

$$\mathbb{M}_i^{\#} = \begin{bmatrix} J_{x_{i+3}} & 0 & 0 \\ 0 & J_{y_{i+3}} & 0 \\ 0 & 0 & J_{z_{i+3}} \end{bmatrix} ; \ \mathbb{p}_i^{\#} = \begin{bmatrix} \omega_{x_{i+3}} \\ \omega_{y_{i+3}} \\ \omega_{z_{i+3}} \end{bmatrix} , \ i = 1, 2 \tag{32}$$

The counter-rotating disks 1 and 2 (rigid bodies 4 and 5) are coupled to link 2, being that disk 1 has an angular displacement of θ_3 relative to link 2, due to the belt transmission of the spin of motor 3, while disk 2 has an angular displacement of $\beta\theta_3$, with $\beta < 0$, relative to link 2, due to the gear transmission of the spin of disk 1.

Thus, the following quasi-velocities constraints are obtained:

$$\begin{cases} [\boldsymbol{\omega}_4]_{\mathsf{B}_4} = [\mathbb{1}]_{\mathsf{B}_4 \mid \mathsf{B}_2}[\boldsymbol{\omega}_2]_{\mathsf{B}_2} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} \\ [\boldsymbol{\omega}_5]_{\mathsf{B}_5} = [\mathbb{1}]_{\mathsf{B}_5 \mid \mathsf{B}_2}[\boldsymbol{\omega}_2]_{\mathsf{B}_2} + \begin{bmatrix} 0 \\ 0 \\ \beta\dot{\theta}_3 \end{bmatrix} \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = (\dot{\theta}_1\mathsf{s}(\theta_2))\mathsf{c}(\theta_3) + (\dot{\theta}_1\mathsf{c}(\theta_2))\mathsf{s}(\theta_3) \\ \omega_{y_4} = -(\dot{\theta}_1\mathsf{s}(\theta_2))\mathsf{s}(\theta_3) + (\dot{\theta}_1\mathsf{c}(\theta_2))\mathsf{c}(\theta_3) \\ \omega_{z_4} = \dot{\theta}_2 + \dot{\theta}_3 \\ \omega_{x_5} = (\dot{\theta}_1\mathsf{s}(\theta_2))\mathsf{c}(\beta\theta_3) + (\dot{\theta}_1\mathsf{c}(\theta_2))\mathsf{s}(\beta\theta_3) \\ \omega_{y_5} = -(\dot{\theta}_1\mathsf{s}(\theta_2))\mathsf{s}(\beta\theta_3) + (\dot{\theta}_1\mathsf{c}(\theta_2))\mathsf{c}(\beta\theta_3) \\ \omega_{z_5} = \dot{\theta}_2 + \beta\dot{\theta}_3 \end{cases} \tag{33}$$

$$\Rightarrow \underline{\mathbb{p}}^{\circ} = \begin{bmatrix} \dot{\theta}_1 \mathsf{s}(\theta_2+\theta_3) \\ \dot{\theta}_1 \mathsf{c}(\theta_2+\theta_3) \\ \dot{\theta}_2+\dot{\theta}_3 \\ \dot{\theta}_1 \mathsf{s}(\theta_2+\beta\theta_3) \\ \dot{\theta}_1 \mathsf{c}(\theta_2+\beta\theta_3) \\ \dot{\theta}_2+\beta\dot{\theta}_3 \end{bmatrix} \tag{34}$$

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \dfrac{\partial \underline{\mathbb{p}}^{\circ}}{\partial \mathbb{p}^{\#}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \mathsf{s}(\theta_2+\theta_3) & 0 & 0 \\ \mathsf{c}(\theta_2+\theta_3) & 0 & 0 \\ 0 & 1 & 1 \\ \mathsf{s}(\theta_2+\beta\theta_3) & 0 & 0 \\ \mathsf{c}(\theta_2+\beta\theta_3) & 0 & 0 \\ 0 & 1 & \beta \end{bmatrix} \tag{35}$$

Applying (31), (32) and (35) in (11), (12) and (13), the mechanism's statically balanced model coupled with the counter-rotating disks is obtained:

$$\begin{cases} D'_{11} = D_{11} + J_{x_4}\mathsf{s}^2(\theta_2+\theta_2) + J_{x_5}\mathsf{s}^2(\beta\theta_2+\theta_2) + J_{y_4}\mathsf{c}^2(\theta_2+\theta_2) + J_{y_5}\mathsf{c}^2(\beta\theta_2+\theta_2) \\ D'_{22} = D_{22} + J_{z_4} + J_{z_5} \\ D'_{33} = D_{33} + J_{z_4} + J_{z_5}\beta^2 \\ D'_{12} = D'_{13} = 0 \\ D'_{23} = D_{23} + J_{z_4} + J_{z_5}\beta \\ D'_{211} = D_{211} \\ D'_{311} = D_{311} \\ D'_{111} = D'_{122} = D'_{133} = D'_{222} = D'_{233} = D'_{322} = D'_{333} = 0 \\ D'_{112} = D_{112} + \frac{1}{4}\Big(\big(J_{x_4} - J_{y_4}\big)\mathsf{s}(2\theta_2+2\theta_3) + \big(J_{x_5} - J_{y_5}\big)\mathsf{s}(2\beta\theta_2+2\theta_3)\Big) \\ D'_{113} = D_{113} + \frac{1}{4}\Big((J_{x_4} - J_{y_4})\mathsf{s}(2\theta_2+2\theta_3) + (J_{x_5} - J_{y_5})\mathsf{s}(2\beta\theta_2+2\theta_3)\Big) \\ D'_{123} = D'_{212} = D'_{213} = D'_{223} = D'_{312} = D'_{313} = D'_{323} = 0 \end{cases} \tag{36}$$

To perform the dynamic balancing, the values of β in function of the mechanism's parameters that makes $\mathbb{M}'^{\#}$ diagonal and the relationships between the mechanism's parameters the make $\mathbb{v}'^{\#}$ null are found. Thus:

$$\begin{cases} D'_{23} = 0 \\ D'_{211} = 0 \\ D'_{311} = 0 \\ D'_{112} = 0 \\ D'_{113} = 0 \end{cases} \Rightarrow \begin{cases} \beta = -\frac{J_{z_3}+J_{z_4}}{J_{z_5}} \\ J_{x_2} = J_{y_2} \\ J_{x_3} = J_{y_3} + m_3 l_2^2 (1 + \frac{m_3}{m_2}) \\ J_{x_4} = J_{y_4} \\ J_{x_5} = J_{y_5} \end{cases} \tag{37}$$

Applying (37) in (36), the machanism's dynamic balanced model is obtained:

$$\begin{cases} \tau_1 = k_1\ddot{\theta}_1 \\ \tau_2 = k_2\ddot{\theta}_2 \\ f_3 = k_3\ddot{d}_3 \end{cases} \tag{38}$$

Being:

$$\begin{cases} k_1 = J_{z_1} + J_{y_2} + J_{y_3} + J_{y_4} + J_{y_5} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2 m_3^2}{m_2} \\ k_2 = J_{z_2} + J_{z_3} + J_{z_4} + J_{z_5} + m_3 l_2^2 + \frac{l_1^2 m_3^2}{m_2} \\ k_3 = \frac{(J_{z_3}+J_{z_4})(J_{z_3}+J_{z_4}+J_{z_5})}{J_{z_5}} \end{cases} \tag{39}$$

Note that the necessary conditions for the dynamic balancing of this mechanism require very high longitudinal moments of inertia for the bars 2 and 3, which is not convenient for an industrial manipulator. Among the five conditions of (37), let's consider the following two:

$$\begin{cases} J_{x_2} = J_{y_2} \\ J_{x_3} = J_{y_3} + m_3 l_2^2 (1 + \frac{m_3}{m_2}) \end{cases} \tag{40}$$

According Denavit-Hartenberg convention, the x-axis is the longitudinal direction while the y-axis and the z-axis are the corresponding transversal directions, for the bars 2 and 3. In this case, tipically, the J_x moment of inertia is quite low in a comparison with J_y and J_z. In order to satisfy the balancing conditions, the values of J_x should be equal J_y for bar 2 and higher than J_y for bar 3. Consequently, the link section should be increased according, which would lead to an inconvenient extremely large cross section.

The next example of application will be in another 3-dof spatial serial mechanism, in which such inconvenience does not occur.

3.3 3-dof <u>RRP</u> spatial serial mechanism (SCARA)

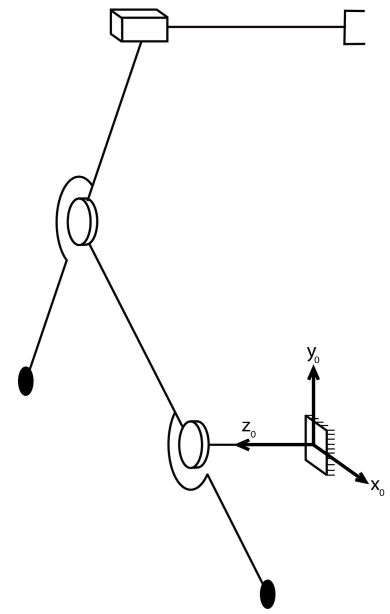

Figure 3: Statically balanced <u>RRP</u> spatial serial mechanism

The entries of $\mathbb{g}^{\#}$ for the unbalanced mechanism are given by:

$$\begin{cases} D_1 = g[(m_1 l_{g_1} + m_2 l_1 + m_3 l_1)\mathsf{c}(\theta_1) + m_2 l_{g_2}\mathsf{c}(\theta_1 + \theta_2)] \\ D_2 = g[m_2 l_{g_2}\mathsf{c}(\theta_1 + \theta_2)] \\ D_3 = 0 \end{cases} \tag{41}$$

Performing the static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g_2} = 0 \end{cases} \tag{42}$$

Substituting (42) in the mechanism model, the dynamic model of the statically balanced mechanism is obtained:

$$\begin{cases} D_{11} = J_{z_1} + J_{z_2} + J_{z_3} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2(m_2+m_3)^2}{m_1} \\ D_{22} = J_{z_2} + J_{z_3} \\ D_{33} = m_3 \\ D_{12} = D_{22} \\ D_{13} = D_{23} = 0 \\ \mathbb{v}^\# = \mathbb{0} \\ \mathbb{g}^\# = \mathbb{0} \end{cases} \tag{43}$$

To perform the dynamic balancing, 2 counter-rotating disks are coupled to the mechanism. As the disks rotate in only one plane, the following dynamic models are used for them:

$$\mathbb{M}_i^\# = \left[J_{z_{i+3}}\right];\ \mathbb{p}_i^\# = \left[\omega_{z_{i+3}}\right],\ i = 1, 2 \tag{44}$$

The counter-rotating disks 1 and 2 (rigid bodies 4 and 5) are coupled to link 1, being that disk 1 has an angular displacement of θ_2 relative to link 1, due to the belt transmission of the spin of motor 2, while disk 2 has an angular displacement of $\beta\theta_2$, with $\beta < 0$, relative to link 1, due to the gear transmission of the spin of disk 1.

Thus, the following quasi-velocities constraints are obtained:

$$\begin{cases} \omega_{z_4} = \omega_{z_1} + \dot{\theta}_2 \\ \omega_{z_5} = \omega_{z_1} + \beta\dot{\theta}_2 \end{cases} \Rightarrow \begin{cases} \omega_{z_4} = \dot{\theta}_1 + \dot{\theta}_2 \\ \omega_{z_5} = \dot{\theta}_1 + \beta\dot{\theta}_2 \end{cases} \Rightarrow \underline{\mathbb{p}}^\circ = \begin{bmatrix} \dot{\theta}_1 + \dot{\theta}_2 \\ \dot{\theta}_1 + \beta\dot{\theta}_2 \end{bmatrix} \tag{45}$$

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \dfrac{\partial \underline{\mathbb{p}}^\circ}{\partial \mathbb{p}^\#} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \end{bmatrix} \tag{46}$$

Applying (43), (44) and (46) in (11), (12) and (13), the mechanism's statically balanced model coupled with the counter-rotating disks is obtained:

$$\begin{cases} D'_{11} = D_{11} + J_{z_4} + J_{z_5} \\ D'_{22} = D_{22} + J_{z_4} + J_{z_5}\beta^2 \\ D'_{33} = D_{33} \\ D'_{12} = D_{12} + J_{z_4} + J_{z_5}\beta \\ D'_{13} = 0 \\ D'_{23} = 0 \\ \mathbb{v}'^\# = \mathbb{0} \end{cases} \tag{47}$$

To perform the dynamic balancing, the values of β in function of the mechanism's parameters that makes $\mathbb{M}'^\#$ diagonal are found. Thus:

$$D'_{12} = 0 \Rightarrow \beta = -\frac{J_{z_2} + J_{z_3} + J_{z_4}}{J_{z_5}} \tag{48}$$

Applying (48) in (47), the machanism's dynamic balanced model is obtained:

$$\begin{cases} \tau_1 = k_1\ddot{\theta}_1 \\ \tau_2 = k_2\ddot{\theta}_2 \\ f_3 = k_3\ddot{d}_3 \end{cases} \tag{49}$$

Being:

$$\begin{cases} k_1 = J_{z_1} + J_{z_2} + J_{z_3} + J_{z_4} + J_{z_5} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2 (m_2+m_3)^2}{m_1} \\ k_2 = \frac{(J_{z_2}+J_{z_3}+J_{z_4})(J_{z_2}+J_{z_3}+J_{z_4}+J_{z_5})}{J_{z_5}} \\ k_3 = m_3 \end{cases} \tag{50}$$

Note that the necessary conditions for the dynamic balancing of this mechanism does not require restrictions on the mechanism inertia parameters, as in the previous example. Thus, as the mechanism in the previous example and the mechanism of this example are both spatial serial mechanisms that perform effector translations in 3 axes, it can be said that the mechanism in question is a good alternative to section 3.2 mechanism in applications in which the dynamic balancing is advantageous to the system.

4 Conclusions

This work dealt with a systematic formulation for the adaptive balancing. This special formulation uses the dynamic coupling between subsystems in order to derive the equations of motion of the whole system in a explicit form. Consequently, this feature allows the automatic generation of the adaptive balancing conditions. Two traditional balancing techniques were employed here: the addition of counterweight and counter-rotating disks coupled to the moving links. In addition, the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators was discussed regarding the achievement of such balancing and the complexity level of the modified mechanical structure. The balancing conditions were developed for 3-dof spatial and planar open loop-kinematic chain mechanisms, whose topologies are composed of revolute and prismatic joints. By analysing the necessary conditions, one can notice that the adaptive balancing brings great benefits for the planar <u>RRR</u> and the spatial <u>RRP</u>. However, for the spatial <u>RRR</u>, in spite of the achievement of the adaptive balancing, the modifications in the mechanical structure require very high longitudinal moments of inertia for the second and third bars of the mechanism, which would lead to bars with extremely large cross sections. Consequently, the authors believe that the discussion provided here might help the designer to choose an adequate topology for a specific application taking advantage of the adaptive balancing whenever it brings no further consequences in terms of the added inertias.

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