A new approach for designing dynamic balanced serial mechanisms

André Garnier Coutinho [a], Tarcisio Antonio Hess Coelho [b]

[a] *Department of Mechatronics and Mechanical Systems Engineering, Escola Politecnica, University of Sao Paulo, Brazil. E-mail: andre.garnier.coutinho@usp.br*
[b] *Department of Mechatronics and Mechanical Systems Engineering, Escola Politecnica, University of Sao Paulo, Brazil. E-mail: tarchess@usp.br*

Abstract

Balancing is an important issue related to the design of mechanical systems in general, and also parallel mechanisms, in particular. In fact, the performance of parallel mechanisms associated to specific applications depends on the choice of the balancing method, namely, either static or dynamic, either passive or active, whether it is valid for a given trajectory or even for any motion. The main contribution of this work is to highlight the importance of the dynamic modelling process in order to achieve the compensation conditions associated to the chosen balancing technique. Due to the fact that parallel mechanisms have highly complex structures, the use of dynamic formalisms that employ redundant generalized coordinates, in association with the successive coupling of additional balancing elements to the original system model, can bring remarkable benefits. Additionally, this book chapter also discusses the impact of the dynamic model, developed in accordance with the methodology shown here, for the control strategy of parallel mechanisms. Finally, the simulation results demonstrates how effective is the presented methodology for the planar 5-bar with revolute joints (5R).

KEYWORDS: Dynamic balancing, serial mechanisms

1 Introduction and literature review

2 Mothodology

2.1 Dynamic Model

O modelo dinâmico de um mecanismo serial pode ser escrito da seguinte maneira:

$$\mathbb{M}^{\#}(\mathbb{q}^{\#})\ddot{\mathbb{q}}^{\#} + \mathbb{v}^{\#}(\mathbb{q}^{\#}, \dot{\mathbb{q}}^{\#}) + \mathbb{g}^{\#}(\mathbb{q}^{\#}) = \mathbb{u} \tag{1}$$

Sendo $\mathbb{q}^{\#}$ um conjunto de coordenadas generalizadas independentes, cujos elementos são os deslocamentos relativos das juntas e $\mathbb{u}$ os esforços generalizados nas direções das quasi-velocidades independentes $\mathbb{p}^{\#} = \dot{\mathbb{q}}^{\#}$.

Para realizar o balanceamento dinâmico de um mecanismo serial, utilizando abordagem proposta, é necessário primeiro obter o modelo do mecanismo desbalanceado. Como em um mecanismo serial é possível de expressar todas as velocidades lineares absolutas dos centros de massa das barras e todas as velocidades angulares absolutas das barras em função de $\mathbb{q}^{\#}$ e $\dot{\mathbb{q}}^{\#}$, o modelo dinâmico pode ser obtido sem grandes dificuldades utilizando métodos de mecánica analítica, como Lagrange, Kane e Orsino, aliados a programas ou bibliotecas de linguagens de programação que são capazes de utilizar manipulação simbólica, como o Mathematica e o SymPy.

2.2 Static Balancing

Depois de obtido o modelo dinâmico, realiza-se o balanceamento estático encontrando as posições dos centros de massa das barras que fazem com que $\mathbb{g}^{\#} = \mathbb{0}$. Isso é possível para mecanismos com juntas apenas rotativas e mecanismos com juntas prismáticas cuja direções são ortogonais à gravidade. O posicionamento dos centros de massa é realizado mecanicamente com o prologamento das barras do mecanismo e adição de contra-pesos.

2.3 Dynamic Balancing

O balanceamento dinâmico é obtido acoplando discos girantes ao modelo do mecanismo estáticamente balanceado. Isso é feito utilizando a técnica de acoplamento de subsistemas do Método Orsino.

3 Applying the technique

$$\mathbb{M}^{\#}(\mathbb{q}^{\#})\ddot{\mathbb{q}}^{\#} + \mathbb{v}^{\#}(\mathbb{q}^{\#}, \dot{\mathbb{q}}^{\#}) + \mathbb{g}^{\#}(\mathbb{q}^{\#}) = \mathbb{u} \tag{2}$$

For a 3-DOF serial mechanism:

$$\mathbb{M}^{\#} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} \tag{3}$$

$$\mathbb{v}^{\#} = \begin{bmatrix} D_{111} & D_{122} & D_{133} \\ D_{211} & D_{222} & D_{233} \\ D_{311} & D_{322} & D_{333} \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \\ \dot{q}_3^2 \end{bmatrix} + 2 \begin{bmatrix} D_{112} & D_{113} & D_{123} \\ D_{212} & D_{213} & D_{223} \\ D_{312} & D_{313} & D_{323} \end{bmatrix} \begin{bmatrix} \dot{q}_1\dot{q}_2 \\ \dot{q}_1\dot{q}_3 \\ \dot{q}_2\dot{q}_3 \end{bmatrix} \tag{4}$$

$$\mathbb{g}^{\#} = \begin{bmatrix} D_1 & D_2 & D_3 \end{bmatrix}^{\top} \tag{5}$$

$$\mathbb{q}^{\#} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^{\top} \tag{6}$$

$$\mathbb{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^{\top} \tag{7}$$

For a rotational joint we call $q_i = \theta_i$ and $u_i = \tau_i$, and for a prismatic joint we call $q_i = d_i$ and $u_i = f_i$.

3.1 3-DOF RRR planar serial mechanism

$$\begin{cases} D_1 = g[(m_1 l_{g_1} + m_2 l_1 + m_3 l_1)\mathsf{c}(\theta_1) + (m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_1 + \theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_1 + \theta_2 + \theta_3)] \\ D_2 = g[(m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_1 + \theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_1 + \theta_2 + \theta_3)] \\ D_3 = g[m_3 l_{g_3}\mathsf{c}(\theta_1 + \theta_2 + \theta_3)] \end{cases} \tag{8}$$

Static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g_2} = -\frac{l_2 m_3}{m_2} \\ l_{g_3} = 0 \end{cases} \tag{9}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{x_1} + J_{x_2} + J_{x_3} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ D_{22} = J_{x_2} + J_{x_3} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{x_3} \\ D_{12} = D_{22} \\ D_{13} = D_{23} = D_{33} \\ \mathbb{v}^{\#} = \mathbb{0} \\ \mathbb{g}^{\#} = \mathbb{0} \end{cases} \tag{10}$$

Coupling 4 discs:

Planar disc models:

$$\mathbb{M}_i^{\#} = \left[J_{x_{i+3}}\right] ; \ \mathbb{v}_i^{\#} = \left[0\right] ; \ \mathbb{g}_i^{\#} = \left[0\right] ; \ \mathbb{u}_i = \left[0\right] ; \ \mathbb{p}_i^{\#} = \left[\omega_{x_{i+3}}\right] , \ i = 1,2,3,4 \tag{11}$$

Quasi-velocities constraints:

$$\begin{cases} \omega_{x_4} = \omega_{x_1} + \dot{\theta}_2 \\ \omega_{x_5} = \omega_{x_1} + \beta\dot{\theta}_2 \\ \omega_{x_6} = \omega_{x_2} + \dot{\theta}_3 \\ \omega_{x_7} = \omega_{x_2} + \gamma\dot{\theta}_3 \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = \dot{\theta}_1 + \dot{\theta}_2 \\ \omega_{x_5} = \dot{\theta}_1 + \beta\dot{\theta}_2 \\ \omega_{x_6} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \\ \omega_{x_7} = \dot{\theta}_1 + \dot{\theta}_2 + \gamma\dot{\theta}_3 \end{cases} \quad \therefore \overline{\mathbb{p}} = \begin{bmatrix} \omega_{x_4} - \dot{\theta}_1 - \dot{\theta}_2 \\ \omega_{x_5} - \dot{\theta}_1 - \beta\dot{\theta}_2 \\ \omega_{x_6} - \dot{\theta}_1 - \dot{\theta}_2 - \dot{\theta}_3 \\ \omega_{x_7} - \dot{\theta}_1 - \dot{\theta}_2 - \gamma\dot{\theta}_3 \end{bmatrix} = \mathbb{0} \tag{12}$$

$$\mathbb{A}^{\#} = \frac{\partial\overline{\mathbb{p}}}{\partial\mathbb{p}^{\#}} = -\begin{bmatrix} 1 & 1 & 0 \\ 1 & \beta & 0 \\ 1 & 1 & 1 \\ 1 & 1 & \gamma \end{bmatrix} ; \ \mathbb{A}^{\circ} = \frac{\partial\overline{\mathbb{p}}}{\partial\mathbb{p}^{\circ}} = \mathbb{1}; \ \mathbb{C} = \begin{bmatrix} \mathbb{1} \\ -(\mathbb{A}^{\circ})^{-1}\mathbb{A}^{\#} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \\ 1 & 1 & 1 \\ 1 & 1 & \gamma \end{bmatrix} \tag{13}$$

$$\begin{cases} D'_{11} = D_{11} + J_{x_4} + J_{x_5} + J_{x_6} + J_{x_7} \\ D'_{22} = D_{22} + J_{x_4} + J_{x_5}\beta^2 + J_{x_6} + J_{x_7} \\ D'_{33} = D_{33} + J_{x_6} + J_{x_7}\gamma^2 \\ D'_{12} = D_{12} + J_{x_4} + J_{x_5}\beta + J_{x_6} + J_{x_7} \\ D'_{13} = D_{13} + J_{x_6} + J_{x_7}\gamma \\ D'_{23} = D'_{13} \\ \mathbb{v}'^{\#} = \mathbb{0} \end{cases} \tag{14}$$

Dynamic balancing:

$$\begin{cases} D'_{12} = 0 \\ D'_{13} = 0 \end{cases} \Rightarrow \begin{cases} \beta = -\frac{J_{x_2}+J_{x_3}+J_{x_4}+J_{x_6}+J_{x_7}+m_3 l_2^2+\frac{m_3^2 l_2^2}{m_2}}{J_{x_5}} \\ \gamma = -\frac{J_{x_3}+J_{x_6}}{J_{x_7}} \end{cases} \tag{15}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1 = k_1\ddot{\theta}_1 \\ \tau_2 = k_2\ddot{\theta}_2 \\ \tau_3 = k_3\ddot{\theta}_3 \end{cases} \tag{16}$$

Being:

$$\begin{cases} k_1 = J_{x_1} + J_{x_2} + J_{x_3} + J_{x_4} + J_{x_5} + J_{x_6} + J_{x_7} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ k_2 = J_{x_2} + J_{x_3} + J_{x_4} + J_{x_6} + J_{x_7} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} + \frac{\left(J_{x_2}+J_{x_3}+J_{x_4}+J_{x_6}+J_{x_7}+m_3 l_2^2+\frac{m_3^2 l_2^2}{m_2}\right)^2}{J_{x_5}} \\ k_3 = \frac{(J_{x_3}+J_{x_6})(J_{x_3}+J_{x_6}+J_{x_7})}{J_{x_7}} \end{cases} \tag{17}$$

3.2 3-DOF RRR spatial serial mechanism

$$\begin{cases} D_1 = 0 \\ D_2 = g[(m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_2+\theta_3)] \\ D_3 = g[m_3 l_{g_3}\mathsf{c}(\theta_2+\theta_3)] \end{cases} \tag{18}$$

Static balancing:

$$\begin{cases} D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_2} = -\frac{l_2 m_3}{m_2} \\ l_{g_3} = 0 \end{cases} \tag{19}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{y_2}\mathsf{s}^2(\theta_2) + J_{y_3}\mathsf{s}^2(\theta_2+\theta_3) + J_{z_1} + J_{z_2}\mathsf{c}^2(\theta_2) + J_{z_3}\mathsf{c}^2(\theta_2+\theta_3) + m_3(l_1 + l_2\mathsf{c}(\theta_2))^2 + \frac{(m_2 l_1 - m_3 l_2 \mathsf{c}(\theta_2))^2}{m_2} \\ D_{22} = J_{x_2} + J_{x_3} + m_2 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{x_3} \\ D_{12} = D_{13} = 0 \\ D_{23} = D_{33} \\ D_{211} = -\frac{1}{2}\Big(\big(J_{y_2} - J_{z_2}\big)\mathsf{s}(2\theta_2) + \big(J_{y_3} - J_{z_3} - m_3 l_2^2(1 + \frac{m_3}{m_2})\big)\mathsf{s}(2\theta_2 + 2\theta_3)\Big) \\ D_{311} = \frac{1}{2}\Big(\big(J_{z_3} - J_{y_3}\big)\mathsf{s}(2\theta_2 + 2\theta_3)\Big) \\ D_{111} = D_{122} = D_{133} = D_{222} = D_{233} = D_{322} = D_{333} = 0 \\ D_{112} = -D_{211} \\ D_{113} = -D_{311} \\ D_{123} = D_{212} = D_{213} = D_{223} = D_{312} = D_{313} = D_{323} = 0 \\ \mathbb{g} = \mathbb{0} \end{cases} \tag{20}$$

Coupling 2 discs:

Spatial disc models:

$$\mathbb{M}_i^{\#} = \begin{bmatrix} J_{x_{i+3}} & 0 & 0 \\ 0 & J_{y_{i+3}} & 0 \\ 0 & 0 & J_{z_{i+3}} \end{bmatrix} ; \ \mathbb{v}_i^{\#} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \ \mathbb{g}_i^{\#} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \ \mathbb{u}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \ \mathbb{p}_i^{\#} = \begin{bmatrix} \omega_{x_{i+3}} \\ \omega_{y_{i+3}} \\ \omega_{z_{i+3}} \end{bmatrix}, \ i = 1, 2 \tag{21}$$

Quasi-velocities constraints:

$$\begin{cases} [\boldsymbol{\omega}_4]_{\mathsf{B}_4} = [\mathbb{1}]_{\mathsf{B}_4 \mid \mathsf{B}_2}[\boldsymbol{\omega}_2]_{\mathsf{B}_2} + \begin{bmatrix} \dot\theta_3 \\ 0 \\ 0 \end{bmatrix} \\ [\boldsymbol{\omega}_5]_{\mathsf{B}_5} = [\mathbb{1}]_{\mathsf{B}_5 \mid \mathsf{B}_2}[\boldsymbol{\omega}_2]_{\mathsf{B}_2} + \begin{bmatrix} \beta\dot\theta_3 \\ 0 \\ 0 \end{bmatrix} \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = \dot\theta_2 + \dot\theta_3 \\ \omega_{y_4} = (\dot\theta_1 \mathsf{s}(\theta_2))\mathsf{c}(\theta_3) + (\dot\theta_1\mathsf{c}(\theta_2))\mathsf{s}(\theta_3) \\ \omega_{z_4} = -(\dot\theta_1\mathsf{s}(\theta_2))\mathsf{s}(\theta_3) + (\dot\theta_1\mathsf{c}(\theta_2))\mathsf{c}(\theta_3) \\ \omega_{x_5} = \dot\theta_2 + \beta\dot\theta_3 \\ \omega_{y_5} = (\dot\theta_1\mathsf{s}(\theta_2))\mathsf{c}(\beta\theta_3) + (\dot\theta_1\mathsf{c}(\theta_2))\mathsf{s}(\beta\theta_3) \\ \omega_{z_5} = -(\dot\theta_1\mathsf{s}(\theta_2))\mathsf{s}(\beta\theta_3) + (\dot\theta_1\mathsf{c}(\theta_2))\mathsf{c}(\beta\theta_3) \end{cases} \tag{22}$$

$$\therefore \overline{\mathbb{p}} = \begin{bmatrix} \omega_{x_4} - \dot\theta_2 - \dot\theta_3 \\ \omega_{y_4} - \dot\theta_1\mathsf{s}(\theta_2+\theta_3) \\ \omega_{z_4} - \dot\theta_1\mathsf{c}(\theta_2+\theta_3) \\ \omega_{x_5} - \dot\theta_2 - \beta\dot\theta_3 \\ \omega_{y_5} - \dot\theta_1\mathsf{s}(\theta_2+\beta\theta_3) \\ \omega_{z_5} - \dot\theta_1\mathsf{c}(\theta_2+\beta\theta_3) \end{bmatrix} = \mathbb{0} \tag{23}$$

$$\mathbb{A}^{\#}=\frac{\partial\overline{\mathbb{p}}}{\partial\mathbb{p}^{\#}}=-\begin{bmatrix}0&1&1\\ \mathsf{s}(\theta_2+\theta_3)&0&0\\ \mathsf{c}(\theta_2+\theta_3)&0&0\\ 0&1&\beta\\ \mathsf{s}(\theta_2+\beta\theta_3)&0&0\\ \mathsf{c}(\theta_2+\beta\theta_3)&0&0\end{bmatrix};\ \mathbb{A}^{\circ}=\frac{\partial\overline{\mathbb{p}}}{\partial\mathbb{p}^{\circ}}=\mathbb{1};\ \mathbb{C}=\begin{bmatrix}\mathbb{1}\\ -(\mathbb{A}^{\circ})^{-1}\mathbb{A}^{\#}\end{bmatrix}=\begin{bmatrix}1&0&0\\ 0&1&0\\ 0&0&1\\ 0&1&1\\ \mathsf{s}(\theta_2+\theta_3)&0&0\\ \mathsf{c}(\theta_2+\theta_3)&0&0\\ 0&1&\beta\\ \mathsf{s}(\theta_2+\beta\theta_3)&0&0\\ \mathsf{c}(\theta_2+\beta\theta_3)&0&0\end{bmatrix} \tag{24}$$

$$\begin{cases} D'_{11}=D_{11}+J_{y_4}\mathsf{s}^2(\theta_2+\theta_2)+J_{y_5}\mathsf{s}^2(\beta\theta_2+\theta_2)+J_{z_4}\mathsf{c}^2(\theta_2+\theta_2)+J_{z_5}\mathsf{c}^2(\beta\theta_2+\theta_2)\\ D'_{22}=D_{22}+J_{x_4}+J_{x_5}\\ D'_{33}=D_{33}+J_{x_4}+J_{x_5}\beta^2\\ D'_{12}=D'_{13}=0\\ D'_{23}=D_{23}+J_{x_4}+J_{x_5}\beta\\ D'_{211}=D_{211}\\ D'_{311}=D_{311}\\ D'_{111}=D'_{122}=D'_{133}=D'_{222}=D'_{233}=D'_{322}=D'_{333}=0\\ D'_{112}=D_{112}+\frac{1}{4}\Big((J_{y_4}-J_{z_4})\mathsf{s}(2\theta_2+2\theta_3)+\big(J_{y_5}-J_{z_5}\big)\mathsf{s}(2\beta\theta_2+2\theta_3)\Big)\\ D'_{113}=D_{113}+\frac{1}{4}\Big((J_{y_4}-J_{z_4})\mathsf{s}(2\theta_2+2\theta_3)+(J_{y_5}-J_{z_5})\mathsf{s}(2\beta\theta_2+2\theta_3)\Big)\\ D'_{123}=D'_{212}=D'_{213}=D'_{223}=D'_{312}=D'_{313}=D'_{323}=0 \end{cases} \tag{25}$$

Dynamic balancing:

$$\begin{cases} D'_{23}=0\\ D'_{211}=0\\ D'_{311}=0\\ D'_{112}=0\\ D'_{113}=0 \end{cases} \Rightarrow \begin{cases} \beta=-\frac{J_{x_3}+J_{x_4}}{J_{x_5}}\\ J_{y_2}=J_{z_2}\\ J_{y_3}=J_{z_3}+m_3l_2^2(1+\frac{m_3}{m_2})\\ J_{y_4}=J_{z_4}\\ J_{y_5}=J_{z_5} \end{cases} \tag{26}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1=k_1\ddot{\theta}_1\\ \tau_2=k_2\ddot{\theta}_2\\ f_3=k_3\ddot{d}_3 \end{cases} \tag{27}$$

Being:

$$\begin{cases} k_1=J_{z_1}+J_{z_2}+J_{z_3}+J_{z_4}+J_{z_5}+m_2l_1^2+m_3(l_1^2+l_2^2)+\frac{l_1^2m_3^2}{m_2}\\ k_2=J_{x_2}+J_{x_3}+J_{x_4}+J_{x_5}+m_3l_2^2+\frac{l_1^2m_3^2}{m_2}\\ k_3=\frac{(J_{x_3}+J_{x_4})(J_{x_3}+J_{x_4}+J_{x_5})}{J_{x_5}} \end{cases} \tag{28}$$

3.3 3-DOF RRP spatial serial mechanism (SCARA)

$$\begin{cases} D_1=g[(m_1l_{g_1}+m_2l_1+m_3l_1)\mathsf{c}(\theta_1)+m_2l_{g_2}\mathsf{c}(\theta_1+\theta_2)]\\ D_2=g[m_2l_{g_2}\mathsf{c}(\theta_1+\theta_2)]\\ D_3=0 \end{cases} \tag{29}$$

Static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g_2} = 0 \end{cases} \tag{30}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{x_1} + J_{x_2} + J_{x_3} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2(m_2+m_3)^2}{m_1} \\ D_{22} = J_{x_2} + J_{x_3} \\ D_{33} = m_3 \\ D_{12} = D_{22} \\ D_{13} = D_{23} = 0 \\ \mathbb{v} = \mathbb{0} \\ \mathbb{g} = \mathbb{0} \end{cases} \tag{31}$$

Coupling 2 discs:

Planar disc models:

$$\mathbb{M}_i^{\#} = \left[J_{x_{i+3}}\right];\ \mathbb{v}_i^{\#} = \left[0\right];\ \mathbb{g}_i^{\#} = \left[0\right];\ \mathbb{u}_i = \left[0\right];\ \mathbb{p}_i^{\#} = \left[\omega_{x_{i+3}}\right],\ i = 1,2 \tag{32}$$

Quasi-velocities constraints:

$$\begin{cases} \omega_{x_4} = \omega_{x_1} + \dot{\theta}_2 \\ \omega_{x_5} = \omega_{x_1} + \beta\dot{\theta}_2 \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = \dot{\theta}_1 + \dot{\theta}_2 \\ \omega_{x_5} = \dot{\theta}_1 + \beta\dot{\theta}_2 \end{cases} \quad \therefore \overline{\mathbb{p}} = \begin{bmatrix} \omega_{x_4} - \dot{\theta}_1 - \dot{\theta}_2 \\ \omega_{x_5} - \dot{\theta}_1 - \beta\dot{\theta}_2 \end{bmatrix} = \mathbb{0} \tag{33}$$

$$\mathbb{A}^{\#} = \frac{\partial \overline{\mathbb{p}}}{\partial \mathbb{p}^{\#}} = -\begin{bmatrix} 1 & 1 & 0 \\ 1 & \beta & 0 \end{bmatrix};\ \mathbb{A}^{\circ} = \frac{\partial \overline{\mathbb{p}}}{\partial \mathbb{p}^{\circ}} = \mathbb{1};\ \mathbb{C} = \begin{bmatrix} \mathbb{1} \\ -(\mathbb{A}^{\circ})^{-1}\mathbb{A}^{\#} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \end{bmatrix} \tag{34}$$

$$\begin{cases} D'_{11} = D_{11} + J_{x_4} + J_{x_5} \\ D'_{22} = D_{22} + J_{x_4} + J_{x_5}\beta^2 \\ D'_{33} = D_{33} \\ D'_{12} = D_{12} + J_{x_4} + J_{x_5}\beta \\ D'_{13} = 0 \\ D'_{23} = 0 \\ \mathbb{v}' = \mathbb{0} \end{cases} \tag{35}$$

Dynamic balancing:

$$D'_{12} = 0 \Rightarrow \beta = -\frac{J_{x_2} + J_{x_3} + J_{x_4}}{J_{x_5}} \tag{36}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1 = k_1\ddot{\theta}_1 \\ \tau_2 = k_2\ddot{\theta}_2 \\ f_3 = k_3\ddot{d}_3 \end{cases} \tag{37}$$

Being:

$$
\begin{cases}
k_1 = J_{x_1} + J_{x_2} + J_{x_3} + J_{x_4} + J_{x_5} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2(m_2+m_3)^2}{m_1} \\
k_2 = \frac{(J_{x_2}+J_{x_3}+J_{x_4})(J_{x_2}+J_{x_3}+J_{x_4}+J_{x_5})}{J_{x_5}} \\
k_3 = m_3
\end{cases}
\tag{38}
$$

4 Conclusions

References

[1] V. Van der Wijk, Shaking moment balancing of mechanisms with principal vectors and moments *Front. Mech. Eng.*, 8(1): 10–16, 2013.

[2] V. H. Arakelian V. , M. R. Smith, Design of planar 3-dof 3-RRR reactionless parallel manipulators *Mechatronics*, 18: 601–606, 2008.

[3] J.-T. Seo, J. H. Woo, H. Lim, J. Chung, W. K. Kim, and B.-J. Yi, Design of an Antagonistically Counter-Balancing Parallel Mechanism *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Tokyo, November 3-7: 2882–2887, 2013.

[4] Y. Wu, C. M. Gosselin, Design of reactionless 3-dof and 6-dof parallel manipulators using parallelepiped mechanisms *IEEE Transactions on Robotics*, 21(5): 821–833, 2005.

[5] C. M. Gosselin, F. Vollmer, G. Ct, Y. Wu, Synthesis and design of reactionless three-degree-of-freedom parallel mechanisms *IEEE Transactions on Robotics and Automation*, 20(2): 191–199, 2004.

[6] J. Wang, C. M. Gosselin, Static balancing of spatial four-degree-of-freedom parallel mechanisms *Mech. Mach. Theory*, 35: 563–592, 2000.

[7] J. Wang, C. M. Gosselin, Static balancing of spatial three-degree-of-freedom parallel mechanisms *Mech. Mach. Theory*, 34: 437–452, 1999.

[8] G. Alici, B. Shirinzadeh, Optimum Force Balancing with Mass Distribution and a Single Elastic Element for a Five-bar Parallel Manipulator *Proceedings of the IEEE International Conference on Robotics and Automation*,Taipei, September 14-19: 3666–3671, 2003.

[9] G. Alici, B. Shirinzadeh, Optimum dynamic balancing of planar parallel manipulators based on sensitivity analysis *Mech. Mach. Theory*, 41: 1520–1532, 2006.

[10] M. B. Dehkordi, A. Frisoli, E. Sotgiu, M. Bergamasco, Modelling and Experimental Evaluation of a Static Balancing Technique for a new Horizontally Mounted 3-UPU Parallel Mechanism *International Journal of Advanced Robotic Systems*, 9: 193–205, 2012.

[11] K. Wang, M. Luo, T. Mei, J. Zhao, Y. Cao, Dynamics Analysis of a Three-DOF Planar Serial-Parallel Mechanism for Active Dynamic Balancing with Respect to a Given Trajectory *International Journal of Advanced Robotic Systems*, 10: 23–33, 2013.

[12] A. Russo, R. Sinatra, F. Xi, Static balancing of parallel robots *Mech. Mach. Theory*, 40: 191–202, 2005.

[13] S. K. Agrawal, A. Fattah, Gravity-balancing of spatial robotic manipulators *Mech. Mach. Theory*, 39: 1331–1344, 2004.

[14] S. Briot, V. Arakelian, J.-P. Le Baron, Shaking force minimization of high-speed robots via centre of mass acceleration control *Mech. Mach. Theory*, 57: 1–12, 2012.

[15] T. A. H. Coelho, L. Yong, V. F. A. Alves, Decoupling of dynamic equations by means of adaptive balancing of 2-dof open-loop mechanisms *Mech. Mach. Theory*, 39: 871–881, 2004.

[16] M. Moradi, A. Nikoobin, S. Azadi, Adaptive Decoupling for Open Chain Planar Robots *Transaction B: Mechanical Engineering*, 17(5): 376–386, 2010.