ADAPTATIVE BALANCING - RR MECHANISM

DEFINITIONS

RR SUBSYSTEM

```
(*-Coordinates and quasivelocities-*)
𝕢"RR"[t_] =
  {q"RR",ℛ1[t], q"RR",ℛ2[t], q"RR",℘2,1[t], q"RR",℘2,2[t], q"RR",℘3,1[t], q"RR",℘3,2[t]};
ℙ"RR"[t_] = {p"RR",ℛ1[t], p"RR",ℛ2[t], p"RR",ℬ1,1[t], p"RR",ℬ1,2[t],
    p"RR",ℬ1,3[t], p"RR",ℬ2,1[t], p"RR",ℬ2,2[t], p"RR",ℬ2,3[t]};

(*-Inertia matrix-*)
𝕄"RR"[t_] =
  DiagonalMatrix[{0, 0, m"RR",ℬ1, m"RR",ℬ1, I"RR",ℬ1, m"RR",ℬ2, m"RR",ℬ2, I"RR",ℬ2}];

(*-Giroscopic forces-*)
𝕘"RR"[t_] = {0, 0, 0, 0, 0, 0, 0, 0};

(*-Active forces-*)
𝕗"RR"[t_] = {u"RR",1[t], u"RR",2[t], 0, -m"RR",ℬ1 g, 0, 0, -m"RR",ℬ2 g, 0};

(*-Constraint equations-*)
𝕔"RR"[t_] = Flatten[{
    p"RR",ℛ1[t] - q"RR",ℛ1'[t],
    p"RR",ℛ2[t] - q"RR",ℛ2'[t],
    Flatten[(p"RR",ℬ1,1[t]
             p"RR",ℬ1,2[t]) - (1 - γ"RR",ℬ1) (0
                                              0) - γ"RR",ℬ1 (q"RR",℘2,1'[t]
                                                             q"RR",℘2,2'[t])],
    Flatten[(p"RR",ℬ2,1[t]
             p"RR",ℬ2,2[t]) - (1 - γ"RR",ℬ2) (q"RR",℘2,1'[t]
                                              q"RR",℘2,2'[t]) - γ"RR",ℬ2 (q"RR",℘3,1'[t]
                                                                          q"RR",℘3,2'[t])],
    Flatten[(q"RR",℘2,1'[t]
             q"RR",℘2,2'[t]) - (0
                                0)] -
      Cross[Flatten[p"RR",ℬ1,3[t] ((q"RR",℘2,1[t]
                                    q"RR",℘2,2[t]) - (q"RR",℘1,1
                                                      q"RR",℘1,2))]],
    Flatten[(q"RR",℘3,1'[t]
             q"RR",℘3,2'[t]) - (q"RR",℘2,1'[t]
                                q"RR",℘2,2'[t])] -
      Cross[Flatten[p"RR",ℬ2,3[t] ((q"RR",℘3,1[t]
                                    q"RR",℘3,2[t]) - (q"RR",℘2,1[t]
                                                      q"RR",℘2,2[t]))]],
    p"RR",ℛ1[t] - p"RR",ℬ1,3[t],
    p"RR",ℛ1[t] + p"RR",ℛ2[t] - p"RR",ℬ2,3[t]
  }];
𝕢̇*"RR"[t_] = Flatten@
    Solve[(# == 0) & /@ (𝕔"RR"[t][[1 ;; #]] & @@ Dimensions[𝕢"RR"'[t]]), 𝕢"RR"'[t]];
𝕔*"RR"[t_] = If[# === {}, {0}, #] &@
    DeleteCases[FullSimplify[𝕔"RR"[t] /. 𝕢̇*"RR"[t]], 0];
```

```
(*-Constraint Matrix-*)
𝔸"RR"[t_] = FullSimplify[D[𝕔*"RR"[t], {𝕡"RR"[t]}]];
𝕓"RR"[t_] = FullSimplify[D[𝕔*"RR"[t], t] - 𝔸"RR"[t].𝕡"RR"'[t]];
ℂ"RR"[t_] = Transpose[FullSimplify@RowReduce[NullSpace[𝔸"RR"[t]]]];
ℂ"RR"[t] // MatrixForm
Norm[𝔸"RR"[t].ℂ"RR"[t] // FullSimplify]
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ \gamma_{RR,\mathcal{B}_1}\left(q_{RR,p_1,2} - q_{RR,p_2,2}[t]\right) & 0 \\ \gamma_{RR,\mathcal{B}_1}\left(-q_{RR,p_1,1} + q_{RR,p_2,1}[t]\right) & 0 \\ 1 & 0 \\ q_{RR,p_1,2} + (-1+\gamma_{RR,\mathcal{B}_2})\, q_{RR,p_2,2}[t] - \gamma_{RR,\mathcal{B}_2}\, q_{RR,p_3,2}[t] & \gamma_{RR,\mathcal{B}_2}\left(q_{RR,p_2,2}[t] - q_{RR,p_3,2}[t]\right) \\ -q_{RR,p_1,1} - (-1+\gamma_{RR,\mathcal{B}_2})\, q_{RR,p_2,1}[t] + \gamma_{RR,\mathcal{B}_2}\, q_{RR,p_3,1}[t] & \gamma_{RR,\mathcal{B}_2}\left(-q_{RR,p_2,1}[t] + q_{RR,p_3,1}[t]\right) \\ 1 & 1 \end{pmatrix}$$

0

```
(*-Dynamic equations-*)
𝕕"RR"[t_] =
  FullSimplify[Transpose[ℂ"RR"[t]].(-𝕄"RR"[t].𝕡"RR"'[t] + 𝕘"RR"[t] + 𝕗"RR"[t])];
(*𝕕"RR"[t]//TableForm*)
```

PAYLOAD SUBSYSTEM / ROTOR SUBSYSTEM

```
(*-Coordinates and quasivelocities-*)
ℚ"PL"[t_] = {q"PL",p1,1[t], q"PL",p1,2[t]};
𝕡"PL"[t_] = {p"PL",ℬ1,1[t], p"PL",ℬ1,2[t], p"PL",ℬ1,3[t]};

(*-Inertia matrix-*)
𝕄"PL"[t_] = DiagonalMatrix[{m"PL",ℬ1, m"PL",ℬ1, I"PL",ℬ1}];

(*-Giroscopic forces-*)
𝕘"PL"[t_] = {0, 0, 0};

(*-Active forces-*)
𝕗"PL"[t_] = {0, -m"PL",ℬ1 g, 0};

(*-Constraint equations-*)
𝕔"PL"[t_] = Flatten[{
     p"PL",ℬ1,1[t] - q"PL",p1,1'[t],
     p"PL",ℬ1,2[t] - q"PL",p1,2'[t]
    }];
ℚ̇*"PL"[t_] = Flatten@
    Solve[(# == 0) & /@ (𝕔"PL"[t][[1 ;; #]] & @@ Dimensions[ℚ"PL"'[t]]), ℚ"PL"'[t]];
𝕔*"PL"[t_] = If[# === {}, {0}, #] &@
    DeleteCases[FullSimplify[𝕔"PL"[t] /. ℚ̇*"PL"[t]], 0];
```

```
(*-Constraint Matrix-*)
𝔸"PL"[t_] = FullSimplify[D[𝕔*"PL"[t], {ℙ"PL"[t]}]];
𝕓"PL"[t_] = FullSimplify[D[𝕔*"PL"[t], t] - 𝔸"PL"[t].ℙ"PL"'[t]];
ℂ"PL"[t_] = Transpose[FullSimplify@RowReduce[NullSpace[𝔸"PL"[t]]]];
ℂ"PL"[t] // MatrixForm
Norm[𝔸"PL"[t].ℂ"PL"[t] // FullSimplify]
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
0
```

```
(*-Dynamic equations-*)
𝕕"PL"[t_] =
  FullSimplify[Transpose[ℂ"PL"[t]].(-𝕄"PL"[t].ℙ"PL"'[t] + 𝕘"PL"[t] + 𝕗"PL"[t])];
(*𝕕"PL"[t]//TableForm*)
```

COMPENSATION INERTIA SUBSYSTEM

```
(*-Coordinates and quasivelocities-*)
ℚ"BI"[t_] = {q"BI",p1,1[t], q"BI",p1,2[t]};
ℙ"BI"[t_] = {p"BI",ℬ1,1[t], p"BI",ℬ1,2[t]};

(*-Inertia matrix-*)
𝕄"BI"[t_] = DiagonalMatrix[{m"BI",ℬ1, m"BI",ℬ1}];

(*-Giroscopic forces-*)
𝕘"BI"[t_] = {0, 0};

(*-Active forces-*)
𝕗"BI"[t_] = {0, -m"BI",ℬ1 g};

(*-Constraint equations-*)
𝕔"BI"[t_] = Flatten[{
     p"BI",ℬ1,1[t] - q"BI",p1,1'[t],
     p"BI",ℬ1,2[t] - q"BI",p1,2'[t]
    }];
𝕔̇*"BI"[t_] = Flatten@
    Solve[(# == 0) & /@ (𝕔"BI"[t][[1 ;; #]] & @@ Dimensions[ℚ"BI"'[t]]), ℚ"BI"'[t]];
𝕔*"BI"[t_] = If[# === {}, {0}, #] &@
    DeleteCases[FullSimplify[𝕔"BI"[t] /. 𝕔̇*"BI"[t]], 0];

(*-Constraint Matrix-*)
𝔸"BI"[t_] = FullSimplify[D[𝕔*"BI"[t], {ℙ"BI"[t]}]];
𝕓"BI"[t_] = FullSimplify[D[𝕔*"BI"[t], t] - 𝔸"BI"[t].ℙ"BI"'[t]];
ℂ"BI"[t_] = Transpose[FullSimplify@RowReduce[NullSpace[𝔸"BI"[t]]]];
ℂ"BI"[t] // MatrixForm
Norm[𝔸"BI"[t].ℂ"BI"[t] // FullSimplify]
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
0
```

```
(*-Dynamic equations-*)
𝕕ₙ"BI"[t_] =
  FullSimplify[Transpose[ℂ"BI"[t]].(-𝕄"BI"[t].ℙ"BI"'[t] + 𝕘"BI"[t] + 𝕗"BI"[t])];
(*𝕕"BI"[t]//TableForm*)
```

AUXILIARY FUNCTIONS

```
Coeff = {◇u, ◇f, ◇p} ↦
    (# → Module[{◇a, ◇b}, ◇a = FullSimplify[Normal@CoefficientArrays[
              # /. ◇f, D[◇p, t], "Symmetric" → True]];
         ◇b = FullSimplify[Normal@CoefficientArrays[Flatten[◇a[[1]]],
              ◇p, "Symmetric" → True]];
         Join[Flatten@{◇a[[2]]}, Flatten@{◇b[[3]]}, Flatten@{◇b[[2]]},
          Flatten@{◇b[[1]]}]]) & /@ ◇u;

Subsystem = {◇system, ◇number, ◇type} ↦ (
     𝕢ₙ◇system,◇number[t_] = 𝕢ₙ◇type[t] /. {◇type → ◇number};
     ℙ◇system,◇number[t_] = ℙ◇type[t] /. {◇type → ◇number};
     𝕄◇system,◇number[t_] = 𝕄◇type[t] /. {◇type → ◇number};
     𝕘◇system,◇number[t_] = 𝕘◇type[t] /. {◇type → ◇number};
     𝕗◇system,◇number[t_] = 𝕗◇type[t] /. {◇type → ◇number};
     ℂ*◇system,◇number[t_] = ℂ*◇type[t] /. {◇type → ◇number};
     𝕢̇*◇system,◇number[t_] = 𝕢̇*◇type[t] /. {◇type → ◇number};
     𝔸◇system,◇number[t_] = 𝔸◇type[t] /. {◇type → ◇number};
     𝕓◇system,◇number[t_] = 𝕓◇type[t] /. {◇type → ◇number};
     ℂ◇system,◇number[t_] = ℂ◇type[t] /. {◇type → ◇number};
     𝕕ₙ◇system,◇number[t_] = 𝕕ₙ◇type[t] /. {◇type → ◇number};);
```

MODELING

UNBALANCED MODEL (MoU)

```
(*-Subsystems-*)
Σ"MoU" = {1, 2};
Subsystem["MoU", 1, "RR"] (*Subsystem 1: RR*)
Subsystem["MoU", 2, "PL"] (*Subsystem 2: Payload*)

(*-System Variables-*)
𝕢ₙ"MoU"[t_] = Join @@ (𝕢ₙ"MoU",#[t] & /@ Σ"MoU");
ℙ"MoU"[t_] = Join @@ (ℙ"MoU",#[t] & /@ Σ"MoU");
```

```
(*-Additional constraints-*)
q̇I*"MoU"[t_] = Join @@ (q̇I*"MoU",#[t] & /@ Σ"MoU");
c⊕"MoU"[t_] = ({q1,p3,1'[t] - q2,p1,1'[t],
      q1,p3,2'[t] - q2,p1,2'[t], p1,B2,3[t] - p2,B1,3[t]} /. q̇I*"MoU"[t]);
c*"MoU"[t_] = DeleteCases[Join[Join @@ (c*"MoU",#[t] & /@ Σ"MoU"), c⊕"MoU"[t]], 0];

(*-Additional constraints matrix-*)
B"MoU"[t_] = Transpose@(Join @@
      (Transpose@FullSimplify[D[c⊕"MoU"[t], {P"MoU",#[t]}].C"MoU",#[t]] & /@ Σ"MoU"));
C"MoU"[t_] = Transpose[FullSimplify@RowReduce[NullSpace[B"MoU"[t]]]];
C"MoU"[t] // MatrixForm
Norm[B"MoU"[t].C"MoU"[t] // FullSimplify]
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ q_{1,p_1,2} - q_{1,p_3,2}[t] & q_{1,p_2,2}[t] - q_{1,p_3,2}[t] \\ -q_{1,p_1,1} + q_{1,p_3,1}[t] & -q_{1,p_2,1}[t] + q_{1,p_3,1}[t] \\ 1 & 1 \end{pmatrix}$$

```
0

(*-Dynamic equations-*)
d"MoU"[t_] = FullSimplify[Transpose[C"MoU"[t]].Join @@ (d"MoU",#[t] & /@ Σ"MoU")];

(*-Inverse dynamics solution-*)
P#"MoU"[t_] = {p1,R1[t], p1,R2[t]};
u#"MoU"[t_] = {u1,1[t], u1,2[t]};
P*"MoU"[t_] = FullSimplify[Join[#, D[#, t] /. q̇I*"MoU"[t] /. #] &@
     Flatten@Solve[(# == 0) & /@ c*"MoU"[t], Complement[P"MoU"[t], P#"MoU"[t]]]];
q*"MoU"[t_] = {q1,p1,1 → 0, q1,p1,2 → 0, q1,p2,1[t] → a1 Cos[q1,R1[t]], q1,p2,2[t] →
     a1 Sin[q1,R1[t]], q1,p3,1[t] → a1 Cos[q1,R1[t]] + a2 Cos[q1,R1[t] + q1,R2[t]],
    q1,p3,2[t] → a1 Sin[q1,R1[t]] + a2 Sin[q1,R1[t] + q1,R2[t]]};
u*"MoU"[t_] = Flatten[Solve[(# == 0) & /@
      FullSimplify[d"MoU"[t] /. P*"MoU"[t] /. q*"MoU"[t]], u#"MoU"[t]]];
u*©"MoU"[t_] = Coeff[u#"MoU"[t], u*"MoU"[t], P#"MoU"[t]];
(Print[StringForm["*-- `` Coefficients --*\n", #],
      # /. u*©"MoU"[t] // TableForm, "\n"]) & /@ u#"MoU"[t];
```

```
*-- u1,1[t] Coefficients --*
```

$$a_1^2 \left(m_{1,\mathcal{B}_2} + m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_1} \gamma_{1,\mathcal{B}_1}^2\right) + 2 \cos[q_{1,\mathcal{R}_2}[t]]\, a_1\, a_2\, (m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2}) + a_2^2 \left(m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2}^2\right) + \mathcal{I}_{1,\mathcal{B}_1} + \mathcal{I}_{}$$

$$\cos[q_{1,\mathcal{R}_2}[t]]\, a_1\, a_2\, (m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2}) + a_2^2 \left(m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2}^2\right) + \mathcal{I}_{1,\mathcal{B}_2} + \mathcal{I}_{2,\mathcal{B}_1}$$

$$0$$

$$-\sin[q_{1,\mathcal{R}_2}[t]]\, a_1\, a_2\, (m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2})$$

$$-\sin[q_{1,\mathcal{R}_2}[t]]\, a_1\, a_2\, (m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2})$$

$$-\sin[q_{1,\mathcal{R}_2}[t]]\, a_1\, a_2\, (m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2})$$

$$0$$

$$0$$

$$g \cos[q_{1,\mathcal{R}_1}[t]]\, a_1\, (m_{1,\mathcal{B}_2} + m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_1} \gamma_{1,\mathcal{B}_1}) + g \cos[q_{1,\mathcal{R}_1}[t] + q_{1,\mathcal{R}_2}[t]]\, a_2\, (m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2})$$

```
*-- u1,2[t] Coefficients --*
```

$$\cos[q_{1,\mathcal{R}_2}[t]]\, a_1\, a_2\, (m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2}) + a_2^2 \left(m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2}^2\right) + \mathcal{I}_{1,\mathcal{B}_2} + \mathcal{I}_{2,\mathcal{B}_1}$$

$$a_2^2 \left(m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2}^2\right) + \mathcal{I}_{1,\mathcal{B}_2} + \mathcal{I}_{2,\mathcal{B}_1}$$

$$\sin[q_{1,\mathcal{R}_2}[t]]\, a_1\, a_2\, (m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2})$$

$$0$$

$$0$$

$$0$$

$$0$$

$$0$$

$$g \cos[q_{1,\mathcal{R}_1}[t] + q_{1,\mathcal{R}_2}[t]]\, a_2\, (m_{2,\mathcal{B}_1} + m_{1,\mathcal{B}_2} \gamma_{1,\mathcal{B}_2})$$

STATICALLY BALANCED MODEL (MoS)

```
(*-Subsystems-*)
Σ"MoS" = {1, 2, 3, 4};
Subsystem["MoS", 1, "RR"] (*Subsystem 1: RR*)
Subsystem["MoS", 2, "PL"] (*Subsystem 2: Payload*)
Subsystem["MoS", 3, "BI"] (*Subsystem 3: Compensation inertia*)
Subsystem["MoS", 4, "BI"] (*Subsystem 4: Compensation inertia*)

(*-System Variables-*)
q"MoS"[t_] = Join @@ (q"MoS",#[t] & /@ Σ"MoS");
ℙ"MoS"[t_] = Join @@ (ℙ"MoS",#[t] & /@ Σ"MoS");
```

```
(*-Additional constraints-*)
q̇I*"MoS"[t_] = Join @@ (q̇I*"MoS",#[t] & /@ Σ"MoS");
c⊕"MoS"[t_] =
  ({q1,p3,1'[t] - q2,p1,1'[t], q1,p3,2'[t] - q2,p1,2'[t], p1,B2,3[t] - p2,B1,3[t],
    q3,p1,1'[t] + γ3,B1 q1,p2,1'[t], q3,p1,2'[t] + γ3,B1 q1,p2,2'[t],
    (q4,p1,1'[t] - q1,p2,1'[t]) + γ4,B1 (q1,p3,1'[t] - q1,p2,1'[t]),
    (q4,p1,2'[t] - q1,p2,2'[t]) + γ4,B1 (q1,p3,2'[t] - q1,p2,2'[t])} /. q̇I*"MoS"[t]);
c*"MoS"[t_] = DeleteCases[Join[Join @@ (c*"MoS",#[t] & /@ Σ"MoS"), c⊕"MoS"[t]], 0];

(*-Additional constraints matrix-*)
B"MoS"[t_] = Transpose@(Join @@
    (Transpose@FullSimplify[D[c⊕"MoS"[t], {P"MoS",#[t]}].C"MoS",#[t]] & /@ Σ"MoS"));
C"MoS"[t_] = Transpose[FullSimplify@RowReduce[NullSpace[B"MoS"[t]]]];
C"MoS"[t] // MatrixForm
Norm[B"MoS"[t].C"MoS"[t] // FullSimplify]
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ q_{1,p_1,2} - q_{1,p_3,2}[t] & q_{1,p_2,2}[t] - q_{1,p_3,2}[t] \\ -q_{1,p_1,1} + q_{1,p_3,1}[t] & -q_{1,p_2,1}[t] + q_{1,p_3,1}[t] \\ 1 & 1 \\ \gamma_{3,\mathcal{B}_1}\left(-q_{1,p_1,2} + q_{1,p_2,2}[t]\right) & 0 \\ \gamma_{3,\mathcal{B}_1}\left(q_{1,p_1,1} - q_{1,p_2,1}[t]\right) & 0 \\ q_{1,p_1,2} - (1+\gamma_{4,\mathcal{B}_1})\, q_{1,p_2,2}[t] + \gamma_{4,\mathcal{B}_1} q_{1,p_3,2}[t] & \gamma_{4,\mathcal{B}_1}\left(-q_{1,p_2,2}[t] + q_{1,p_3,2}[t]\right) \\ -q_{1,p_1,1} + (1+\gamma_{4,\mathcal{B}_1})\, q_{1,p_2,1}[t] - \gamma_{4,\mathcal{B}_1} q_{1,p_3,1}[t] & \gamma_{4,\mathcal{B}_1}\left(q_{1,p_2,1}[t] - q_{1,p_3,1}[t]\right) \end{pmatrix}$$

```
0

(*-Dynamic equations-*)
dI"MoS"[t_] = FullSimplify[Transpose[C"MoS"[t]].Join @@ (dI"MoS",#[t] & /@ Σ"MoS")];

(*-Inverse dynamics solution-*)
P#"MoS"[t_] = {p1,R1[t], p1,R2[t]};
u#"MoS"[t_] = {u1,1[t], u1,2[t]};
P*"MoS"[t_] = FullSimplify[Join[#, D[#, t] /. q̇I*"MoS"[t] /. #] &@
    Flatten@Solve[(# == 0) & /@ c*"MoS"[t], Complement[P"MoS"[t], P#"MoS"[t]]]];
qI*"MoS"[t_] = {q1,p1,1 → 0, q1,p1,2 → 0, q1,p2,1[t] → a1 Cos[q1,R1[t]], q1,p2,2[t] →
    a1 Sin[q1,R1[t]], q1,p3,1[t] → a1 Cos[q1,R1[t]] + a2 Cos[q1,R1[t] + q1,R2[t]],
   q1,p3,2[t] → a1 Sin[q1,R1[t]] + a2 Sin[q1,R1[t] + q1,R2[t]]};
u*"MoS"[t_] = Flatten[Solve[(# == 0) & /@
    FullSimplify[dI"MoS"[t] /. P*"MoS"[t] /. qI*"MoS"[t]], u#"MoS"[t]]];
u*©"MoS"[t_] = Coeff[u#"MoS"[t], u*"MoS"[t], P#"MoS"[t]];
(Print[StringForm["*-- `` Coefficients --*\n", #],
    # /. u*©"MoS"[t] // TableForm, "\n"]) & /@ u#"MoS"[t];
```

-- $u_{1,1}$[t] Coefficients --

$a_1^2\left(m_{1,\mathcal{B}_2}+m_{2,\mathcal{B}_1}+m_{4,\mathcal{B}_1}+m_{1,\mathcal{B}_1}\gamma_{1,\mathcal{B}_1}^2+m_{3,\mathcal{B}_1}\gamma_{3,\mathcal{B}_1}^2\right)+2\,\mathrm{Cos}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}-m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1})+a_2^2$

$\mathrm{Cos}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}-m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1})+a_2^2\left(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}^2+m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1}^2\right)+I_{1,\mathcal{B}_2}+I_{2,\mathcal{B}_1}$

0

$-\mathrm{Sin}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}-m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1})$

$-\mathrm{Sin}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}-m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1})$

$-\mathrm{Sin}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}-m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1})$

0

0

$g\,\mathrm{Cos}[q_{1,\mathcal{R}_1}[t]]\,a_1\,(m_{1,\mathcal{B}_2}+m_{2,\mathcal{B}_1}+m_{4,\mathcal{B}_1}+m_{1,\mathcal{B}_1}\gamma_{1,\mathcal{B}_1}-m_{3,\mathcal{B}_1}\gamma_{3,\mathcal{B}_1})+g\,\mathrm{Cos}[q_{1,\mathcal{R}_1}[t]+q_{1,\mathcal{R}_2}[t]]\,a_2\,(m_{2,\mathcal{B}_1}+m_1$

-- $u_{1,2}$[t] Coefficients --

$\mathrm{Cos}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}-m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1})+a_2^2\left(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}^2+m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1}^2\right)+I_{1,\mathcal{B}_2}+I_{2,\mathcal{B}_1}$

$a_2^2\left(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}^2+m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1}^2\right)+I_{1,\mathcal{B}_2}+I_{2,\mathcal{B}_1}$

$\mathrm{Sin}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}-m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1})$

0

0

0

0

0

$g\,\mathrm{Cos}[q_{1,\mathcal{R}_1}[t]+q_{1,\mathcal{R}_2}[t]]\,a_2\,(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}-m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1})$

```
(*-Static balancing-*)
a"MoS"[t_] = {((*m2,B1+*)m1,B2 γ1,B2 - m4,B1 γ4,B1) → 0,
    ((*m2,B1+*)m1,B2 + m4,B1 + m1,B1 γ1,B1 - m3,B1 γ3,B1) → 0};
(Print[StringForm["*-- `` Coefficients --*\n", #],
      # /. (u*©"MoS"[t] //. a"MoS"[t]) // TableForm, "\n"]) & /@ u#"MoS"[t];
```

-- $u_{1,1}$[t] Coefficients --

$2\,\mathrm{Cos}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,m_{2,\mathcal{B}_1}+a_1^2\left(m_{1,\mathcal{B}_2}+m_{2,\mathcal{B}_1}+m_{4,\mathcal{B}_1}+m_{1,\mathcal{B}_1}\gamma_{1,\mathcal{B}_1}^2+m_{3,\mathcal{B}_1}\gamma_{3,\mathcal{B}_1}^2\right)+a_2^2\left(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}^2+m_{4,\mathcal{B}_1}\right.$

$\mathrm{Cos}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,m_{2,\mathcal{B}_1}+a_2^2\left(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}^2+m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1}^2\right)+I_{1,\mathcal{B}_2}+I_{2,\mathcal{B}_1}$

0

$-\mathrm{Sin}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,m_{2,\mathcal{B}_1}$

$-\mathrm{Sin}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,m_{2,\mathcal{B}_1}$

$-\mathrm{Sin}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,m_{2,\mathcal{B}_1}$

0

0

$g\,\mathrm{Cos}[q_{1,\mathcal{R}_1}[t]]\,a_1\,m_{2,\mathcal{B}_1}+g\,\mathrm{Cos}[q_{1,\mathcal{R}_1}[t]+q_{1,\mathcal{R}_2}[t]]\,a_2\,m_{2,\mathcal{B}_1}$

-- $u_{1,2}$[t] Coefficients --

$\mathrm{Cos}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,m_{2,\mathcal{B}_1}+a_2^2\left(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}^2+m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1}^2\right)+I_{1,\mathcal{B}_2}+I_{2,\mathcal{B}_1}$

$a_2^2\left(m_{2,\mathcal{B}_1}+m_{1,\mathcal{B}_2}\gamma_{1,\mathcal{B}_2}^2+m_{4,\mathcal{B}_1}\gamma_{4,\mathcal{B}_1}^2\right)+I_{1,\mathcal{B}_2}+I_{2,\mathcal{B}_1}$

$\mathrm{Sin}[q_{1,\mathcal{R}_2}[t]]\,a_1\,a_2\,m_{2,\mathcal{B}_1}$

0

0

0

0

0

$g\,\mathrm{Cos}[q_{1,\mathcal{R}_1}[t]+q_{1,\mathcal{R}_2}[t]]\,a_2\,m_{2,\mathcal{B}_1}$

DYNAMICALLY BALANCED MODEL (MoD)

```
(*-Subsystems-*)
Σ"MoD" = {1, 2, 3, 4, 5, 6};
Subsystem["MoD", 1, "RR"] (*Subsystem 1: RR*)
Subsystem["MoD", 2, "PL"] (*Subsystem 2: Payload*)
Subsystem["MoD", 3, "BI"] (*Subsystem 3: Compensation inertia*)
Subsystem["MoD", 4, "BI"] (*Subsystem 4: Compensation inertia*)
Subsystem["MoD", 5, "PL"] (*Subsystem 5: Rotor*)
Subsystem["MoD", 6, "PL"] (*Subsystem 6: Rotor*)

(*-System Variables-*)
q"MoD"[t_] = Join @@ (q"MoD",#[t] & /@ Σ"MoD");
p"MoD"[t_] = Join @@ (p"MoD",#[t] & /@ Σ"MoD");

(*-Additional constraints-*)
q̇*"MoD"[t_] = Join @@ (q̇*"MoD",#[t] & /@ Σ"MoD");
c⊕"MoD" [t_] =
   ({q1,p3,1'[t] - q2,p1,1'[t], q1,p3,2'[t] - q2,p1,2'[t], p1,B2,3[t] - p2,B1,3[t],
      q3,p1,1'[t] + γ3,B1 q1,p2,1'[t], q3,p1,2'[t] + γ3,B1 q1,p2,2'[t],
      (q4,p1,1'[t] - q1,p2,1'[t]) + γ4,B1 (q1,p3,1'[t] - q1,p2,1'[t]),
      (q4,p1,2'[t] - q1,p2,2'[t]) + γ4,B1 (q1,p3,2'[t] - q1,p2,2'[t]),
      q5,p1,1'[t] - γ5,B1 q1,p2,1'[t], q5,p1,2'[t] - γ5,B1 q1,p2,2'[t], p5,B1,3[t] - p1,B1,3[t] +
       ρ5,B1 p1,R2[t], q6,p1,1'[t], q6,p1,2'[t], p6,B1,3[t] + ρ6,B1 p1,R1[t]} /. q̇*"MoD"[t]);
c*"MoD"[t_] = DeleteCases[Join[Join @@ (c*"MoD",#[t] & /@ Σ"MoD"), c⊕"MoD" [t]], 0];

(*-Additional constraints matrix-*)
B"MoD"[t_] = Transpose@(Join @@
      (Transpose@FullSimplify[D[c⊕"MoD" [t], {p"MoD",#[t]}].C"MoD",#[t]] & /@ Σ"MoD"));
C"MoD"[t_] = Transpose[FullSimplify@RowReduce[NullSpace[B"MoD"[t]]]];
C"MoD"[t] // MatrixForm
Norm[B"MoD"[t].C"MoD"[t] // FullSimplify]
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ q_{1,p_1,2} - q_{1,p_3,2}[t] & q_{1,p_2,2}[t] - q_{1,p_3,2}[t] \\ -q_{1,p_1,1} + q_{1,p_3,1}[t] & -q_{1,p_2,1}[t] + q_{1,p_3,1}[t] \\ 1 & 1 \\ \gamma_{3,\mathcal{B}_1}\left(-q_{1,p_1,2} + q_{1,p_2,2}[t]\right) & 0 \\ \gamma_{3,\mathcal{B}_1}\left(q_{1,p_1,1} - q_{1,p_2,1}[t]\right) & 0 \\ q_{1,p_1,2} - (1+\gamma_{4,\mathcal{B}_1})\, q_{1,p_2,2}[t] + \gamma_{4,\mathcal{B}_1} q_{1,p_3,2}[t] & \gamma_{4,\mathcal{B}_1}\left(-q_{1,p_2,2}[t] + q_{1,p_3,2}[t]\right) \\ -q_{1,p_1,1} + (1+\gamma_{4,\mathcal{B}_1})\, q_{1,p_2,1}[t] - \gamma_{4,\mathcal{B}_1} q_{1,p_3,1}[t] & \gamma_{4,\mathcal{B}_1}\left(q_{1,p_2,1}[t] - q_{1,p_3,1}[t]\right) \\ \gamma_{5,\mathcal{B}_1}\left(q_{1,p_1,2} - q_{1,p_2,2}[t]\right) & 0 \\ \gamma_{5,\mathcal{B}_1}\left(-q_{1,p_1,1} + q_{1,p_2,1}[t]\right) & 0 \\ 1 & -\rho_{5,\mathcal{B}_1} \\ 0 & 0 \\ 0 & 0 \\ -\rho_{6,\mathcal{B}_1} & 0 \end{pmatrix}$$

0

```
(*-Dynamic equations-*)
d"MoD"[t_] = FullSimplify[Transpose[C"MoD"[t]].Join @@ (d"MoD",#[t] & /@ Σ"MoD")];
```

```
(*-Inverse dynamics solution-*)
ℙ#"MoD"[t_] = {p1,ℛ1[t], p1,ℛ2[t]};
u#"MoD"[t_] = {u1,1[t], u1,2[t]};
ℙ⋆"MoD"[t_] = FullSimplify[Join[#, D[#, t] /. q̇⋆"MoD"[t] /. #] &@
    Flatten@Solve[(# == 0) & /@ c⋆"MoD"[t], Complement[ℙ"MoD"[t], ℙ#"MoD"[t]]]];
q⋆"MoD"[t_] = {q1,p1,1 → 0, q1,p1,2 → 0, q1,p2,1[t] → a1 Cos[q1,ℛ1[t]], q1,p2,2[t] →
    a1 Sin[q1,ℛ1[t]], q1,p3,1[t] → a1 Cos[q1,ℛ1[t]] + a2 Cos[q1,ℛ1[t] + q1,ℛ2[t]],
   q1,p3,2[t] → a1 Sin[q1,ℛ1[t]] + a2 Sin[q1,ℛ1[t] + q1,ℛ2[t]]};
u⋆"MoD"[t_] = Flatten[Solve[(# == 0) & /@
     FullSimplify[d"MoD"[t] /. ℙ⋆"MoD"[t] /. q⋆"MoD"[t]], u#"MoD"[t]]];
u⋆©"MoD"[t_] = Coeff[u#"MoD"[t], u⋆"MoD"[t], ℙ#"MoD"[t]];
(Print[StringForm["*-- `` Coefficients --*\n", #],
     # /. u⋆©"MoD"[t] // TableForm, "\n"]) & /@ u#"MoD"[t];
```

```
*-- u1,1[t] Coefficients --*

2 Cos[q1,ℛ2[t]] a1 a2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2 - m4,ℬ1 γ4,ℬ1) + a2^2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2^2 + m4,ℬ1 γ4,ℬ1^2) + a1^2 (m1,ℬ2 + m2,ℬ1
Cos[q1,ℛ2[t]] a1 a2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2 - m4,ℬ1 γ4,ℬ1) + a2^2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2^2 + m4,ℬ1 γ4,ℬ1^2) + I1,ℬ2 + I2,ℬ1 - I5,
0
-Sin[q1,ℛ2[t]] a1 a2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2 - m4,ℬ1 γ4,ℬ1)
-Sin[q1,ℛ2[t]] a1 a2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2 - m4,ℬ1 γ4,ℬ1)
-Sin[q1,ℛ2[t]] a1 a2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2 - m4,ℬ1 γ4,ℬ1)
0
0
g Cos[q1,ℛ1[t] + q1,ℛ2[t]] a2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2 - m4,ℬ1 γ4,ℬ1) + g Cos[q1,ℛ1[t]] a1 (m1,ℬ2 + m2,ℬ1 + m4,ℬ1 + m1


*-- u1,2[t] Coefficients --*

Cos[q1,ℛ2[t]] a1 a2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2 - m4,ℬ1 γ4,ℬ1) + a2^2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2^2 + m4,ℬ1 γ4,ℬ1^2) + I1,ℬ2 + I2,ℬ1 - I5,
a2^2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2^2 + m4,ℬ1 γ4,ℬ1^2) + I1,ℬ2 + I2,ℬ1 + I5,ℬ1 ρ5,ℬ1^2
Sin[q1,ℛ2[t]] a1 a2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2 - m4,ℬ1 γ4,ℬ1)
0
0
0
0
0
g Cos[q1,ℛ1[t] + q1,ℛ2[t]] a2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2 - m4,ℬ1 γ4,ℬ1)
```

```
(*-Dynamic balancing-*)
a"MoD"[t_] = {(m2,ℬ1 + m1,ℬ2 γ1,ℬ2 - m4,ℬ1 γ4,ℬ1) → 0,
    (m2,ℬ1 + m1,ℬ2 + m4,ℬ1 + m1,ℬ1 γ1,ℬ1 - m3,ℬ1 γ3,ℬ1 + m5,ℬ1 γ5,ℬ1) → 0,
    (a2^2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2^2 + m4,ℬ1 γ4,ℬ1^2) + I1,ℬ2 + I2,ℬ1 - I5,ℬ1 ρ5,ℬ1^2) → 0,
    (a2^2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2^2 + m4,ℬ1 γ4,ℬ1^2) + a1^2
        (m1,ℬ2 + m2,ℬ1 + m4,ℬ1 + m1,ℬ1 γ1,ℬ1^2 + m3,ℬ1 γ3,ℬ1^2 + m5,ℬ1 γ5,ℬ1^2) + I1,ℬ1 + I1,ℬ2 + I2,ℬ1 + I5,ℬ1) →
     I6,ℬ1 ρ6,ℬ1, (a2^2 (m2,ℬ1 + m1,ℬ2 γ1,ℬ2^2 + m4,ℬ1 γ4,ℬ1^2) + I1,ℬ2 + I2,ℬ1) → I5,ℬ1 ρ5,ℬ1};
(Print[StringForm["*-- `` Coefficients --*\n", #],
     # /. FullSimplify[u⋆©"MoD"[t] //. a"MoD"[t]] // TableForm, "\n"]) & /@ u#"MoD"[t];
```

-- $u_{1,1}[t]$ Coefficients --

$\mathbb{I}_{6,\mathcal{B}_1} \rho_{6,\mathcal{B}_1} (1 + \rho_{6,\mathcal{B}_1})$

0

0

0

0

0

0

0

0

-- $u_{1,2}[t]$ Coefficients --

0

$\mathbb{I}_{5,\mathcal{B}_1} \rho_{5,\mathcal{B}_1} (1 + \rho_{5,\mathcal{B}_1})$

0

0

0

0

0

0

0