A new approach for designing dynamic balanced serial mechanisms

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Abstract

Adaptive balancing means that the mechanical structure of the manipulator is modified in order to achieve the decoupling of dynamic equations. This work deals with a systematic methodology for the adaptive balancing. Basically, two balancing techniques are employed here: the addition of counterweight and counter-rotating disks coupled to the moving links. In addition, the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators is discussed regarding the achievement of such balancing and the complexity level of the modified mechanical structure. The balancing conditions are developed for 3-dof spatial and planar open loop-kinematic chain mechanisms, whose topologies are composed of revolute and prismatic joints.

KEYWORDS: Dynamic balancing, serial mechanisms

Nomenclature

$a, b, \ldots$	Scalars, components of column-matrices, components of matrices or indexes
$A, B, \ldots$	Scalars, components of column-matrices or components of matrices
$\mathbb{a}, \mathbb{b}, \ldots$	Column-matrices
$\mathbb{A}, \mathbb{B}, \ldots$	Matrices
$\boldsymbol{a}, \boldsymbol{b}, \ldots$	Vectors
$\mathsf{A}, \mathsf{B}, \ldots$	Coordinate systems
$\mathscr{A}, \mathscr{B}, \ldots$	Sets or multibody mechanical systems*
B_i	Coordinate system fixed in the i$^{\text{th}}$ rigid body of the mechanical system
$\mathbb{C}$	Kinematic constraints matrix
$\mathsf{c}(.)$	Shorthand notation for $\cos(.)$
g	gravitational acceleration
$\mathbb{g}^{\#}$	Generalized gravitational forces column-matrix of a serial mechanism
$\mathbb{g}_i^{\#}$	Generalized gravitational forces column-matrix of a counter-rotating disc
$\mathbb{g}'$	Generalized uncoupled gravitational forces column-matrix of a serial mechanism coupled with counter-rotating discs

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*A multibody mechanical system will be conceived as a set whose elements are material bodies, joints, actuators, energy storage, dissipation and transformation elements and a mathematical model (which includes physical parameters, model variables and constitutive, constraint and dynamic equations).

$\mathbb{g}'^{\#}$	Generalized gravitational forces column-matrix of a serial mechanism coupled with counter-rotating discs
$J_{x_i}, J_{y_i}, J_{z_i}$	Moments of inertia of the ith rigid body of the mechanical system
l_i	Length of the ith bar of a serial mechanism
l_{g_i}	Position of the mass center of the ith bar relative to the ith joint and of a serial mechanism
m_i	Mass of the ith rigid body of the mechanical system
$\mathbb{M}^{\#}$	Generalized inertia matrix of a serial mechanism
$\mathbb{M}_i^{\#}$	Generalized inertia matrix of a counter-rotating disc
$\mathbb{M}'$	Generalized uncoupled inertia matrix of a serial mechanism coupled with counter-rotating discs
$\mathbb{M}'^{\#}$	Generalized inertia matrix of a serial mechanism coupled with counter-rotating discs
$\mathbb{N}$	Inertial reference frame
$\mathbb{p}^{\#}$	Independent quasi-velocities column-matrix
$\mathbb{p}^{\circ}$	Redundant quasi-velocities column-matrix
$\mathbb{p}$	Quasi-velocities column-matrix
q_i	Generalized coordinate
$\mathbb{q}^{\#}$	Independent generalized coordinates column-matrix
$\mathsf{s}(.)$	Shorthand notation for $\sin(.)$
u_i	Effort made by the ith actuator of a serial mechanism
$\mathbb{u}$	Generalized actuators' efforts column-matrix
$\mathbb{v}^{\#}$	Generalized coupled gyroscopic inertia forces column-matrix of a serial mechanism
$\mathbb{v}_i^{\#}$	Generalized coupled gyroscopic inertia forces column-matrix of a counter-rotating disc
$\mathbb{v}'$	Generalized uncoupled gyroscopic inertia forces column-matrix of a serial mechanism coupled with counter-rotating discs
$\mathbb{v}'^{\#}$	Generalized coupled gyroscopic inertia forces column-matrix of a of a serial mechanism coupled with counter-rotating discs
$[\boldsymbol{\omega}_i]_{\text{B}_j}$	Angular velocity of the ith rigid body of the mechanical system measured relatively to a inertial reference frame $\mathbb{N}$, written in the basis of B_j
ω_{x_i}	1st component of $[\boldsymbol{\omega}_i]_{\text{B}_i}$
ω_{y_i}	2nd component of $[\boldsymbol{\omega}_i]_{\text{B}_i}$
ω_{z_i}	3rd component of $[\boldsymbol{\omega}_i]_{\text{B}_i}$
$\mathbb{O}$	Null column-matrix or null matrix
$\mathbb{1}$	Identity matrix
$[\mathbb{1}]_{\text{B}_i \mid \text{B}_j}$	Change of basis matrix, i.e. $[\boldsymbol{v}]_{\text{B}_i} = [\mathbb{1}]_{\text{B}_i \mid \text{B}_j} \cdot [\boldsymbol{v}]_{\text{B}_j}$
$[\cdot]^{\mathsf{T}}$	Matrix transposition

1   Introduction and literature review

Balancing is an important issue related to the design of mechanical systems in general, and also serial manipulators, in particular. In fact, the performance of serial mechanisms associated to specific applications depends on the choice of the balancing method, namely, either static or dynamic, either passive or active, whether it is valid for a given trajectory or even for any motion.

Adaptive balancing means that the mechanical structure of the manipulator is modified in order to achieve the decoupling of dynamic equations. The modifications comprise the addition of either counter-weights, or counter-rotating disks or even both to the original kinematic chain of the manipulator. Consequently, the terms associated to gravitational, centripetal and Coriolis efforts are completely eliminated from the dynamic equations. As a matter of fact, the effective inertias for all the actuator axes are constant and the mathematical expressions of the driving torques/forces become rather simple. One of the main advantages of this approach concerns the reduction of computing time for a closed-loop control of manipulators. Such reduction is really significant and it constitutes in a great benefit for real-time applications.

In a statically balanced mechanism, the potential energy is invariant. Hence, the actuator torques/forces are null at any configuration [6].

On the other hand, in a dynamically balanced mechanism, the shaking forces and shaking moments at its frame are reduced or even eliminated. As a consequence, the mechanism components are less susceptible to vibration, wear and fatigue [4], improving its life.

Passive balancing means that the original mechanism architecture is modified by using some techniques. The most common techniques are the addition of counterweights [1, 2, 3, 4, 6, 7, 12, 15, 16], the use of counter-rotating inertias [2, 4, 15, 16] and the redistribution of masses [8, 9]. Alternatively, other works propose the addition of extra links [5, 12, 13]. However, for high speed manipulators, these techniques might cause the increase of the actuator torques and the size of links and joints.

Hence, in order to avoid such undesirable effects, some authors [6, 7, 8, 9, 10, 13] recommend the use of elastic springs attached to the mechanism.

For the active balancing [2, 3, 11, 14, 15, 16], more complex modifications are needed to implement it. For instance, the counterweights position in the moving links might be altered according to the end-effector load or the given trajectory. Hence, additional actuators/sensors and control are usually employed to reach satisfactory performance levels. Arakelian and Smith [2] employ a computer control of the motion of a inertia flywheel connected to the mechanism. Moreover, Coelho et al. [15], Moradi et al. [16] and [17] use the adaptive balancing to achieve the decoupling of dynamic equations for open-loop kinematic chain mechanisms. Consequently, this action simplifies the control of manipulators due to the fact that the actuators can be controlled independently.

The contributions of this work are the following: to present a systematic methodology for the adaptive balancing, to discuss the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators, not only in terms of the possibility to achieve such balancing but also in terms of the increase in the complexity level of the modified mechanical structure. The analysed manipulators correspond to 3-dof spatial and planar open loop-kinematic chain, whose topologies are composed of revolute and prismatic joints.

This work is organized as follows. Section 2 describes the proposed methodology, while section 3 deals with the application of the methodology to 3 types of serial manipulators. Finally, the conclusions are drawn in section 4.

2   Mothodology

2.1   Dynamic Model

The dynamic model of a serial mechanism can be written in this way:

$$\mathbb{M}^{\#}(\mathbb{q}^{\#})\ddot{\mathbb{q}}^{\#} + \mathbb{v}^{\#}(\mathbb{q}^{\#}, \dot{\mathbb{q}}^{\#}) + \mathbb{g}^{\#}(\mathbb{q}^{\#}) = \mathbb{u} \tag{1}$$

Being $\mathbb{q}^{\#}$ a set of independent generalized coordinates, whose elements are relative displacements of the joints, and $\mathbb{u}$ the generalized actuators' efforts in the directions of the independent quasi-velocities $\mathbb{p}^{\#} = \dot{\mathbb{q}}^{\#}$.

To perform the dynamic balancing of a serial mechanism, using the proposed approach, it is necessary to first obtain the dynamic model of the unbalanced mechanism. As in a serial mechanism it is possible to express all the absolute velocities of the links' centers of mass and all the absolute angular velocities of the links in function of $\mathbb{q}^\#$ and $\dot{\mathbb{q}}^\#$, the dynamic model can be obtained without major difficulties using analytical mechanics techniques, like Lagrange and Kane formalisms and Orsino's method, allied to programs or libraries of programming languages that are capable of using symbolic manipulation, such as Mathematica and SymPy.

2.2 Static Balancing

After obtaining the dynamic model, the static balancing is performed finding the links' centers of mass positions that make $\mathbb{g}^\# = \mathbb{0}$. Is is possible for mechanisms with only revolute joints and mechanisms with prismatic joints whose directions are orthogonal to the gravity. The positioning of the centers of mass is done mechanically extending the mechanism's bars and adding counterweights.

2.3 Dynamic Balancing

The dynamic balancing is performed coupling counter-rotating disks to the statically balanced mechanism model. This is done using the coupling subsystems technique of Orsino's method.

Let $\mathscr{M}_0$ be a mechanical subsystem composed by a statically balanced serial mechanism, whose equation of motion is given by (1), with $\mathbb{g}^\# = \mathbb{0}$. Let $\mathscr{M}_i$ be a mechanical subsystem composed by a counter-rotating disk that will be coupled to the mechanism, whose equation of motion is given by:

$$\mathbb{M}_i^\# \dot{\mathbb{p}}^\# + \mathbb{v}_i^\# + \mathbb{g}_i^\# = \mathbb{u}_i \tag{2}$$

Being $\mathbb{p}^\#$ a set of independent quasi-velocities, whose elements are non-null components of the absolute angular velocity vector of the disk, written in a basis fixed to the disk, and $\mathbb{v}_i^\# = \mathbb{g}_i^\# = \mathbb{u}_i = \mathbb{0}$. In this model, only the rotative inertias are considered, as the effects of the disk's mass are considered in the mass and the inertia moments of the bar that the disk is coupled, also affecting the counterweights positioning calculus, so that the mechanism continues statically balanced.

Assuming that n counter-rotating disks will be coupled to the mechanism, the following definitions are made:

$$\mathbb{M}' = \begin{bmatrix} \mathbb{M}^\# & \mathbb{0} & \dots & \mathbb{0} \\ \mathbb{0} & \mathbb{M}_1^\# & \dots & \mathbb{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{0} & \mathbb{0} & \dots & \mathbb{M}_n^\# \end{bmatrix} \tag{3}$$

$$\mathbb{v}' = \begin{bmatrix} \mathbb{v}^{\#\mathsf{T}} & \mathbb{v}_1^{\#\mathsf{T}} & \dots & \mathbb{v}_n^{\#\mathsf{T}} \end{bmatrix}^\mathsf{T} \tag{4}$$

$$\mathbb{g}' = \begin{bmatrix} \mathbb{g}^{\#\mathsf{T}} & \mathbb{g}_1^{\#\mathsf{T}} & \dots & \mathbb{g}_n^{\#\mathsf{T}} \end{bmatrix}^\mathsf{T} \tag{5}$$

$$\mathbb{p}^\circ = \begin{bmatrix} \mathbb{p}_1^{\#\mathsf{T}} & \dots & \mathbb{p}_n^{\#\mathsf{T}} \end{bmatrix}^\mathsf{T} \tag{6}$$

$$\mathbb{p} = \begin{bmatrix} \mathbb{p}^{\#\mathsf{T}} & \mathbb{p}^{\circ\mathsf{T}} \end{bmatrix}^\mathsf{T} \tag{7}$$

Let $\underline{\mathbb{p}}^\circ$ be the vector $\mathbb{p}^\circ$ written in function of $\mathbb{q}^\#$ e $\mathbb{p}^\#$, i.e.:

$$\mathbb{p}^\circ = \underline{\mathbb{p}}^\circ(\mathbb{q}^\#, \mathbb{p}^\#) \tag{8}$$

The kinematic constraints matrix is defined:

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \dfrac{\partial \underline{\mathbb{p}}^\circ}{\partial \mathbb{p}^\#} \end{bmatrix} \tag{9}$$

The dynamic model of the serial mechanism coupled with counter-rotating disk is given by:

$$\mathbb{M}'^\#(\mathbb{q}^\#)\ddot{\mathbb{q}}^\# + \mathbb{v}'^\#(\mathbb{q}^\#, \dot{\mathbb{q}}^\#) + \mathbb{g}'^\#(\mathbb{q}^\#) = \mathbb{u} \tag{10}$$

Being:

$$\mathbb{M}'^\# = \mathbb{C}^\mathsf{T}\mathbb{M}'\mathbb{C} \tag{11}$$

$$\mathbb{v}'^\# = \mathbb{C}^\mathsf{T}(\mathbb{M}'\dot{\mathbb{C}}\dot{\mathbb{q}}^\# + \mathbb{v}') \tag{12}$$

$$\mathbb{g}'^\# = \mathbb{C}^\mathsf{T}\mathbb{g}' \tag{13}$$

The dynamic balancing is achieved by finding the relationships between system parameters that make diagonal $\mathbb{M}'^\#$ diagonal and $\mathbb{v}'^\#$ null.

3 Applying the technique

In this section, the proposed methodology will be applied in three different 3-dof serial mechanisms. To do this, first some definitions valid for these mechanisms will be made:

$$\mathbb{M}^\# = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} \tag{14}$$

$$\mathbb{v}^\# = \begin{bmatrix} D_{111} & D_{122} & D_{133} \\ D_{211} & D_{222} & D_{233} \\ D_{311} & D_{322} & D_{333} \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \\ \dot{q}_3^2 \end{bmatrix} + 2 \begin{bmatrix} D_{112} & D_{113} & D_{123} \\ D_{212} & D_{213} & D_{223} \\ D_{312} & D_{313} & D_{323} \end{bmatrix} \begin{bmatrix} \dot{q}_1\dot{q}_2 \\ \dot{q}_1\dot{q}_3 \\ \dot{q}_2\dot{q}_3 \end{bmatrix} \tag{15}$$

$$\mathbb{g}^\# = \begin{bmatrix} D_1 & D_2 & D_3 \end{bmatrix}^\top \tag{16}$$

$$\mathbb{q}^\# = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^\top \tag{17}$$

$$\mathbb{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^\top \tag{18}$$

For revolute joints is defined $q_i = \theta_i$ and $u_i = \tau_i$, and for prismatic joints is defined $q_i = d_i$ and $u_i = f_i$, to conform with the more traditional notations of this area.
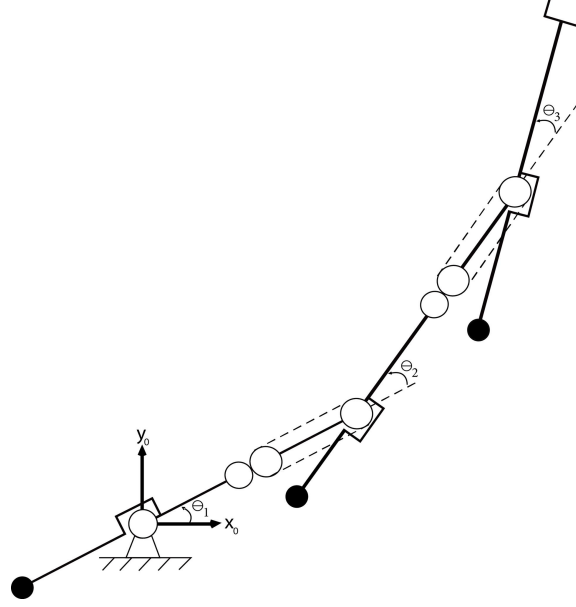
3.1 3-DOF RRR planar serial mechanism

Figure 1: Mecanismo RRR plano balanceado dinamicamente

The $\mathbb{g}^{\#}$ components for the unbalanced mechanism are given by:

$$\begin{cases} D_1 = g[(m_1 l_{g_1} + m_2 l_1 + m_3 l_1)\mathsf{c}(\theta_1) + (m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_1 + \theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_1 + \theta_2 + \theta_3)] \\ D_2 = g[(m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_1 + \theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_1 + \theta_2 + \theta_3)] \\ D_3 = g[m_3 l_{g_3}\mathsf{c}(\theta_1 + \theta_2 + \theta_3)] \end{cases} \tag{19}$$

Performing the static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g_2} = -\frac{l_2 m_3}{m_2} \\ l_{g_3} = 0 \end{cases} \tag{20}$$

Substituting (20) in the mechanism model, the dynamic model of the statically balanced mechanism is obtained:

$$\begin{cases} D_{11} = J_{z_1} + J_{z_2} + J_{z_3} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ D_{22} = J_{z_2} + J_{z_3} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{z_3} \\ D_{12} = D_{22} \\ D_{13} = D_{23} = D_{33} \\ \mathbb{v}^{\#} = \mathbb{0} \\ \mathbb{g}^{\#} = \mathbb{0} \end{cases} \tag{21}$$

To perform the dynamic balancing, 4 counter-rotating disks are coupled to the mechanism, as shown in figure 1. As the disks rotate in only one plane, the following dynamic models are used for them:

$$\mathbb{M}_i^{\#} = \left[J_{z_{i+3}} \right]; \ \mathbb{p}_i^{\#} = \left[\omega_{z_{i+3}} \right], \ i = 1, 2, 3, 4 \tag{22}$$

Os discos 1 e 2 estão acoplados à barra 1, sendo que o disco 1 apresenta um giro de θ_2 relativo à barra 1, devido à transmissão por correia do giro do motor 2, enquanto o disco 2 apresenta um giro de $\beta\theta_2$, com $\beta < 0$, relativo á barra 1, devido á transmissão por engrenagem do giro do disco 1.

Os discos 3 e 4 estão acoplados à barra 2, sendo que o disco 3 apresenta um giro de θ_3 relativo à barra 2, devido à transmissão por correia do giro do motor 3, enquanto o disco 4 apresenta um giro de $\gamma\theta_3$, com $\gamma < 0$, relativo á barra 2, devido á transmissão por engrenagem do giro do disco 3.

Sendo assim, obtemos os seguintes vínculos de quasi-velocidades:

$$
\begin{cases} \omega_{z_4} = \omega_{z_1} + \dot{\theta}_2 \\ \omega_{z_5} = \omega_{z_1} + \beta\dot{\theta}_2 \\ \omega_{z_6} = \omega_{z_2} + \dot{\theta}_3 \\ \omega_{z_7} = \omega_{z_2} + \gamma\dot{\theta}_3 \end{cases}
\Rightarrow
\begin{cases} \omega_{z_4} = \dot{\theta}_1 + \dot{\theta}_2 \\ \omega_{z_5} = \dot{\theta}_1 + \beta\dot{\theta}_2 \\ \omega_{z_6} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \\ \omega_{z_7} = \dot{\theta}_1 + \dot{\theta}_2 + \gamma\dot{\theta}_3 \end{cases}
\Rightarrow
\mathbb{p}^\circ = \begin{bmatrix} \dot{\theta}_1 + \dot{\theta}_2 \\ \dot{\theta}_1 + \beta\dot{\theta}_2 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \\ \dot{\theta}_1 + \dot{\theta}_2 + \gamma\dot{\theta}_3 \end{bmatrix}
\tag{23}
$$

$$
\therefore \mathbb{C} = \begin{bmatrix} \dfrac{\mathbb{1}}{\partial \underline{\mathbb{p}}^\circ} \\ \dfrac{\partial \underline{\mathbb{p}}^\circ}{\partial \underline{\mathbb{p}}^{\#}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \\ 1 & 1 & 1 \\ 1 & 1 & \gamma \end{bmatrix}
\tag{24}
$$

Aplicando (21), (22) e (24) em (11), (12) e (13), obtemos o modelo do mecanismo estaticamente balanceado com os discos acoplados:

$$
\begin{cases} D'_{11} = D_{11} + J_{z_4} + J_{z_5} + J_{z_6} + J_{z_7} \\ D'_{22} = D_{22} + J_{z_4} + J_{z_5}\beta^2 + J_{z_6} + J_{z_7} \\ D'_{33} = D_{33} + J_{z_6} + J_{z_7}\gamma^2 \\ D'_{12} = D_{12} + J_{z_4} + J_{z_5}\beta + J_{z_6} + J_{z_7} \\ D'_{13} = D_{13} + J_{z_6} + J_{z_7}\gamma \\ D'_{23} = D'_{13} \\ \mathbb{v}'^{\#} = \mathbb{0} \end{cases}
\tag{25}
$$

Para realizar o balanceamento dinâmico, encontramos os valores de β e γ em função dos parâmetros do mecanismo que tornam a matriz $\mathbb{M}'^{\#}$ diagonal. Assim, temos:

$$
\begin{cases} D'_{12} = 0 \\ D'_{13} = 0 \end{cases}
\Rightarrow
\begin{cases} \beta = -\dfrac{J_{z_2} + J_{z_3} + J_{z_4} + J_{z_6} + J_{z_7} + m_3 l_2^2 + \frac{m_3^2 l_2^2}{m_2}}{J_{z_5}} \\ \gamma = -\dfrac{J_{z_3} + J_{z_6}}{J_{z_7}} \end{cases}
\tag{26}
$$

Substituindo (26) em (25), obtemos o modelo do mecanismo dinamicamente balanceado:

$$
\begin{cases} \tau_1 = k_1\ddot{\theta}_1 \\ \tau_2 = k_2\ddot{\theta}_2 \\ \tau_3 = k_3\ddot{\theta}_3 \end{cases}
\tag{27}
$$

Sendo:

$$
\begin{cases} k_1 = J_{z_1} + J_{z_2} + J_{z_3} + J_{z_4} + J_{z_5} + J_{z_6} + J_{z_7} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ k_2 = J_{z_2} + J_{z_3} + J_{z_4} + J_{z_6} + J_{z_7} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} + \frac{\left(J_{z_2}+J_{z_3}+J_{z_4}+J_{z_6}+J_{z_7}+m_3 l_2^2 + \frac{m_3^2 l_2^2}{m_2}\right)^2}{J_{z_5}} \\ k_3 = \frac{(J_{z_3}+J_{z_6})(J_{z_3}+J_{z_6}+J_{z_7})}{J_{z_7}} \end{cases}
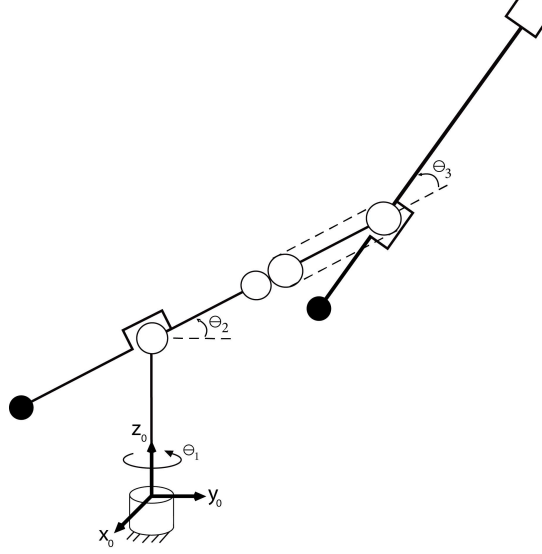\tag{28}
$$

Figure 2: Mecanismo RRR espacial balanceado dinamicamente

As componentes de $\mathfrak{g}^{\#}$ para o mecanismo em questão desbalanceado são dadas por:

$$\begin{cases} D_1 = 0 \\ D_2 = g[(m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_2 + \theta_3)] \\ D_3 = g[m_3 l_{g_3}\mathsf{c}(\theta_2 + \theta_3)] \end{cases} \tag{29}$$

Realizando o balanceamento estático, temos:

$$\begin{cases} D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_2} = -\frac{l_2 m_3}{m_2} \\ l_{g_3} = 0 \end{cases} \tag{30}$$

Substituindo (30) no modelo do mecanismo, obtemos o modelo dinâmico do mecanismo estaticamente balanceado:

$$\begin{cases} D_{11} = J_{x_2}\mathsf{s}^2(\theta_2) + J_{x_3}\mathsf{s}^2(\theta_2 + \theta_3) + J_{z_1} + J_{y_2}\mathsf{c}^2(\theta_2) + J_{y_3}\mathsf{c}^2(\theta_2 + \theta_3) + m_3(l_1 + l_2\mathsf{c}(\theta_2))^2 + \frac{(m_2 l_1 - m_3 l_2 \mathsf{c}(\theta_2))^2}{m_2} \\ D_{22} = J_{z_2} + J_{z_3} + m_2 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{z_3} \\ D_{12} = D_{13} = 0 \\ D_{23} = D_{33} \\ D_{211} = -\frac{1}{2}\left((J_{x_2} - J_{y_2})\mathsf{s}(2\theta_2) + (J_{x_3} - J_{y_3} - m_3 l_2^2(1 + \frac{m_3}{m_2}))\mathsf{s}(2\theta_2 + 2\theta_3) \right) \\ D_{311} = \frac{1}{2}\left((J_{y_3} - J_{x_3})\mathsf{s}(2\theta_2 + 2\theta_3) \right) \\ D_{111} = D_{122} = D_{133} = D_{222} = D_{233} = D_{322} = D_{333} = 0 \\ D_{112} = -D_{211} \\ D_{113} = -D_{311} \\ D_{123} = D_{212} = D_{213} = D_{223} = D_{312} = D_{313} = D_{323} = 0 \\ \mathfrak{g} = \mathbb{0} \end{cases} \tag{31}$$

Para realizar o balanceamento dinâmico, acoplamos 2 discos ao mecanismo, assim como é mostrado na figura 2. Utilizamos os seguintes modelos dinâmicos para eles:

$$
\mathbb{M}_i^{\#} = \begin{bmatrix} J_{x_{i+3}} & 0 & 0 \\ 0 & J_{y_{i+3}} & 0 \\ 0 & 0 & J_{z_{i+3}} \end{bmatrix} ; \quad \mathbb{p}_i^{\#} = \begin{bmatrix} \omega_{x_{i+3}} \\ \omega_{y_{i+3}} \\ \omega_{z_{i+3}} \end{bmatrix}, \ i = 1,2 \tag{32}
$$

Os discos 1 e 2 estão acoplados à barra 2, sendo que o disco 1 apresenta um giro de θ_3 relativo à barra 2, devido à transmissão por correia do giro do motor 3, enquanto o disco 2 apresenta um giro de $\beta\theta_3$, com $\beta < 0$, relativo á barra 2, devido à transmissão por engrenagem do giro do disco 2.

Sendo assim, obtemos os seguintes vínculos de quasi-velocidades:

$$
\begin{cases} [\boldsymbol{\omega}_4]_{\mathsf{B}_4} = [\mathbb{1}]_{\mathsf{B}_4 \mid \mathsf{B}_2}[\boldsymbol{\omega}_2]_{\mathsf{B}_2} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} \\[1em] [\boldsymbol{\omega}_5]_{\mathsf{B}_5} = [\mathbb{1}]_{\mathsf{B}_5 \mid \mathsf{B}_2}[\boldsymbol{\omega}_2]_{\mathsf{B}_2} + \begin{bmatrix} 0 \\ 0 \\ \beta\dot{\theta}_3 \end{bmatrix} \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = (\dot{\theta}_1\mathsf{s}(\theta_2))\mathsf{c}(\theta_3) + (\dot{\theta}_1\mathsf{c}(\theta_2))\mathsf{s}(\theta_3) \\ \omega_{y_4} = -(\dot{\theta}_1\mathsf{s}(\theta_2))\mathsf{s}(\theta_3) + (\dot{\theta}_1\mathsf{c}(\theta_2))\mathsf{c}(\theta_3) \\ \omega_{z_4} = \dot{\theta}_2 + \dot{\theta}_3 \\ \omega_{x_5} = (\dot{\theta}_1\mathsf{s}(\theta_2))\mathsf{c}(\beta\theta_3) + (\dot{\theta}_1\mathsf{c}(\theta_2))\mathsf{s}(\beta\theta_3) \\ \omega_{y_5} = -(\dot{\theta}_1\mathsf{s}(\theta_2))\mathsf{s}(\beta\theta_3) + (\dot{\theta}_1\mathsf{c}(\theta_2))\mathsf{c}(\beta\theta_3) \\ \omega_{z_5} = \dot{\theta}_2 + \beta\dot{\theta}_3 \end{cases} \tag{33}
$$

$$
\Rightarrow \mathbb{p}^{\circ} = \begin{bmatrix} \dot{\theta}_1\mathsf{s}(\theta_2 + \theta_3) \\ \dot{\theta}_1\mathsf{c}(\theta_2 + \theta_3) \\ \dot{\theta}_2 + \dot{\theta}_3 \\ \dot{\theta}_1\mathsf{s}(\theta_2 + \beta\theta_3) \\ \dot{\theta}_1\mathsf{c}(\theta_2 + \beta\theta_3) \\ \dot{\theta}_2 + \beta\dot{\theta}_3 \end{bmatrix} \tag{34}
$$

$$
\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \dfrac{\partial \mathbb{p}^{\circ}}{\partial \mathbb{p}^{\#}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \mathsf{s}(\theta_2 + \theta_3) & 0 & 0 \\ \mathsf{c}(\theta_2 + \theta_3) & 0 & 0 \\ 0 & 1 & 1 \\ \mathsf{s}(\theta_2 + \beta\theta_3) & 0 & 0 \\ \mathsf{c}(\theta_2 + \beta\theta_3) & 0 & 0 \\ 0 & 1 & \beta \end{bmatrix} \tag{35}
$$

Aplicando (31), (32) e (35) em (11), (12) e (13), obtemos o modelo do mecanismo estaticamente balanceado com os discos acoplados:

$$
\begin{cases} D_{11}' = D_{11} + J_{x_4}\mathsf{s}^2(\theta_2 + \theta_2) + J_{x_5}\mathsf{s}^2(\beta\theta_2 + \theta_2) + J_{y_4}\mathsf{c}^2(\theta_2 + \theta_2) + J_{y_5}\mathsf{c}^2(\beta\theta_2 + \theta_2) \\ D_{22}' = D_{22} + J_{z_4} + J_{z_5} \\ D_{33}' = D_{33} + J_{z_4} + J_{z_5}\beta^2 \\ D_{12}' = D_{13}' = 0 \\ D_{23}' = D_{23} + J_{z_4} + J_{z_5}\beta \\ D_{211}' = D_{211} \\ D_{311}' = D_{311} \\ D_{111}' = D_{122}' = D_{133}' = D_{222}' = D_{233}' = D_{322}' = D_{333}' = 0 \\ D_{112}' = D_{112} + \frac{1}{4}\Big((J_{x_4} - J_{y_4})\mathsf{s}(2\theta_2 + 2\theta_3) + (J_{x_5} - J_{y_5})\mathsf{s}(2\beta\theta_2 + 2\theta_3)\Big) \\ D_{113}' = D_{113} + \frac{1}{4}\Big((J_{x_4} - J_{y_4})\mathsf{s}(2\theta_2 + 2\theta_3) + (J_{x_5} - J_{y_5})\mathsf{s}(2\beta\theta_2 + 2\theta_3)\Big) \\ D_{123}' = D_{212}' = D_{213}' = D_{223}' = D_{312}' = D_{313}' = D_{323}' = 0 \end{cases} \tag{36}
$$

9

Para realizar o balanceamento dinâmico, encontramos os valores de β em função dos parâmetros do mecanismo que tornam a matriz $\mathbb{M}'^{\#}$ diagonal e as relações entre os parâmetros do mecanismos que zeram $v'^{\#}$. Assim, temos:

$$\begin{cases} D'_{23} = 0 \\ D'_{211} = 0 \\ D'_{311} = 0 \\ D'_{112} = 0 \\ D'_{113} = 0 \end{cases} \Rightarrow \begin{cases} \beta = -\frac{J_{z_3}+J_{z_4}}{J_{z_5}} \\ J_{x_2} = J_{y_2} \\ J_{x_3} = J_{y_3} + m_3 l_2^2 (1 + \frac{m_3}{m_2}) \\ J_{x_4} = J_{y_4} \\ J_{x_5} = J_{y_5} \end{cases} \tag{37}$$

Substituindo (37) em (36), obtemos o modelo do mecanismo dinamicamente balanceado:

$$\begin{cases} \tau_1 = k_1 \ddot{\theta}_1 \\ \tau_2 = k_2 \ddot{\theta}_2 \\ f_3 = k_3 \ddot{d}_3 \end{cases} \tag{38}$$

Sendo:

$$\begin{cases} k_1 = J_{z_1} + J_{y_2} + J_{y_3} + J_{y_4} + J_{y_5} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2 m_3^2}{m_2} \\ k_2 = J_{z_2} + J_{z_3} + J_{z_4} + J_{z_5} + m_3 l_2^2 + \frac{l_1^2 m_3^2}{m_2} \\ k_3 = \frac{(J_{z_3}+J_{z_4})(J_{z_3}+J_{z_4}+J_{z_5})}{J_{z_5}} \end{cases} \tag{39}$$

Repare que as condições necessárias para o balanceamento dinâmico deste mecanismo requerem momentos de inércia transversais muito elevados para as barras 2 e 3, o que não é nada conveniente para um manipulador industrial. O próximo exemplo de aplicação será em outro mecanismo serial espacial de 3 graus de liberdade, em que esse tipo de inconveniente não acontece.

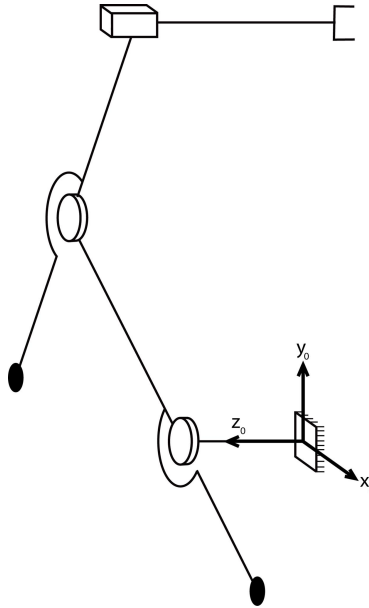*3.3   3-DOF RRP spatial serial mechanism (SCARA)*

Figure 3: Mecanismo RRP balanceado estaticamente

As componentes de $\mathfrak{g}^\#$ para o mecanismo em questão desbalanceado são dadas por:

$$\begin{cases} D_1 = g[(m_1 l_{g_1} + m_2 l_1 + m_3 l_1)\mathsf{c}(\theta_1) + m_2 l_{g_2}\mathsf{c}(\theta_1 + \theta_2)] \\ D_2 = g[m_2 l_{g_2}\mathsf{c}(\theta_1 + \theta_2)] \\ D_3 = 0 \end{cases} \tag{40}$$

Realizando o balanceamento estático, temos:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g_2} = 0 \end{cases} \tag{41}$$

Substituindo (41) no modelo do mecanismo, obtemos o modelo dinâmico do mecanismo estaticamente balanceado:

$$\begin{cases} D_{11} = J_{z_1} + J_{z_2} + J_{z_3} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2(m_2+m_3)^2}{m_1} \\ D_{22} = J_{z_2} + J_{z_3} \\ D_{33} = m_3 \\ D_{12} = D_{22} \\ D_{13} = D_{23} = 0 \\ \mathsf{v}^\# = \mathbb{0} \\ \mathfrak{g}^\# = \mathbb{0} \end{cases} \tag{42}$$

Para realizar o balanceamento dinâmico, acoplamos 2 discos ao mecanismo. Como os discos giram em apenas um plano, utilizamos os seguintes modelos dinâmicos para eles:

$$\mathbb{M}_i^\# = \begin{bmatrix} J_{z_{i+3}} \end{bmatrix}; \ \mathbb{p}_i^\# = \begin{bmatrix} \omega_{z_{i+3}} \end{bmatrix}, \ i = 1,2 \tag{43}$$

Os discos 1 e 2 estão acoplados à barra 1, sendo que o disco 1 apresenta um giro de θ_2 relativo à barra 1, devido à transmissão por correia do giro do motor 2, enquanto o disco 2 apresenta um giro de $\beta\theta_2$, com $\beta < 0$, relativo á barra 1, devido á transmissão por engrenagem do giro do disco 1.

Sendo assim, obtemos os seguintes vínculos de quasi-velocidades:

$$\begin{cases} \omega_{z_4} = \omega_{z_1} + \dot{\theta}_2 \\ \omega_{z_5} = \omega_{z_1} + \beta\dot{\theta}_2 \end{cases} \Rightarrow \begin{cases} \omega_{z_4} = \dot{\theta}_1 + \dot{\theta}_2 \\ \omega_{z_5} = \dot{\theta}_1 + \beta\dot{\theta}_2 \end{cases} \Rightarrow \overline{\mathbb{p}} = \begin{bmatrix} \dot{\theta}_1 + \dot{\theta}_2 \\ \dot{\theta}_1 + \beta\dot{\theta}_2 \end{bmatrix} \tag{44}$$

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \frac{\partial \mathbb{p}^\circ}{\partial \mathbb{p}^\#} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \end{bmatrix} \tag{45}$$

Aplicando (42), (43) e (45) em (11), (12) e (13), obtemos o modelo do mecanismo estaticamente balanceado com os discos acoplados:

$$\begin{cases} D'_{11} = D_{11} + J_{z_4} + J_{z_5} \\ D'_{22} = D_{22} + J_{z_4} + J_{z_5}\beta^2 \\ D'_{33} = D_{33} \\ D'_{12} = D_{12} + J_{z_4} + J_{z_5}\beta \\ D'_{13} = 0 \\ D'_{23} = 0 \\ \mathsf{v}'^\# = \mathbb{0} \end{cases} \tag{46}$$

Para realizar o balanceamento dinâmico, encontramos os valores de β em função dos parâmetros do mecanismo que tornam a matriz $\mathbb{M}'^{\#}$ diagonal. Assim, temos:

$$D'_{12} = 0 \Rightarrow \beta = -\frac{J_{z_2} + J_{z_3} + J_{z_4}}{J_{z_5}} \tag{47}$$

Substituindo (47) em (46), obtemos o modelo do mecanismo dinamicamente balanceado:

$$\begin{cases} \tau_1 = k_1 \ddot{\theta}_1 \\ \tau_2 = k_2 \ddot{\theta}_2 \\ f_3 = k_3 \ddot{d}_3 \end{cases} \tag{48}$$

Sendo:

$$\begin{cases} k_1 = J_{z_1} + J_{z_2} + J_{z_3} + J_{z_4} + J_{z_5} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2(m_2+m_3)^2}{m_1} \\ k_2 = \frac{(J_{z_2}+J_{z_3}+J_{z_4})(J_{z_2}+J_{z_3}+J_{z_4}+J_{z_5})}{J_{z_5}} \\ k_3 = m_3 \end{cases} \tag{49}$$

Repare que as condições necessárias para o balanceamento dinâmico deste mecanismo não requerem restrições nos parâmetros de inércia do mecanismo, como acontece no exemplo anterior. Sendo assim, como o mecanismo do exemplo anterior e o do exemplo em questão são ambos mecanismos seriais espaciais que realizam translações do efetuador em 3 eixos, pode-se dizer que o mecanismo em questão é uma boa alternativa ao mecanismo da seção 3.2 em aplicações em que o balanceamento dinâmico é vantajoso ao sistema.

4   Conclusions

This work dealt with a systematic methodology for the adaptive balancing. Two balancing techniques were employed here: the addition of counterweight and counter-rotating disks coupled to the moving links. In addition, the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators was discussed regarding the achievement of such balancing and the complexity level of the modified mechanical structure. The balancing conditions were developed for 3-dof spatial and planar open loop-kinematic chain mechanisms, whose topologies are composed of revolute and prismatic joints. By analysing the necessary conditions, one can notice that the adaptive balancing brings great benefits for the planar RRR and the spatial RRP. However, for the spatial RRR, in spite of the achievement of the adaptive balancing, the modifications in the mechanical structure cause the increase of the link inertias and the actuator torques, accordingly. Consequently, the authors believe that the discussion provided here might help the designer to choose an adequate topology for a specific application taking advantage of the adaptive balancing whenever it brings no further consequences in terms of the added inertias.

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