A new approach for designing dynamic balanced serial mechanisms

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Abstract

Balancing is an important issue related to the design of mechanical systems in general, and also parallel mechanisms, in particular. In fact, the performance of parallel mechanisms associated to specific applications depends on the choice of the balancing method, namely, either static or dynamic, either passive or active, whether it is valid for a given trajectory or even for any motion. The main contribution of this work is to highlight the importance of the dynamic modelling process in order to achieve the compensation conditions associated to the chosen balancing technique. Due to the fact that parallel mechanisms have highly complex structures, the use of dynamic formalisms that employ redundant generalized coordinates, in association with the successive coupling of additional balancing elements to the original system model, can bring remarkable benefits. Additionally, this book chapter also discusses the impact of the dynamic model, developed in accordance with the methodology shown here, for the control strategy of parallel mechanisms. Finally, the simulation results demonstrates how effective is the presented methodology for the planar 5-bar with revolute joints (5R).

KEYWORDS: Dynamic balancing, serial mechanisms

1 Introduction and literature review

2 Mothodology

2.1 Static Balancing

O modelo dinmico de um mecanismo serial pode ser escrito da seguinte maneira:

$$\mathbb{M}^{\#}(\mathbb{q}^{\#})\ddot{\mathbb{q}}^{\#} + \mathbb{v}^{\#}(\mathbb{q}^{\#}, \dot{\mathbb{q}}^{\#}) + \mathbb{g}^{\#}(\mathbb{q}^{\#}) = \mathbb{u} \tag{1}$$

Sendo $\mathbb{q}^{\#}$ um conjunto de coordenadas generalizadas independentes, cujos elementos são os deslocamentos relativos das juntas (lineares para as prismáticas e angulares para as rotativas) e $\mathbb{u}$ os esforços generalizados nas direções das quasi-velocidades independentes $\mathbb{p}^{\#} = \dot{\mathbb{q}}^{\#}$.

O modelo dinâmico completo de um mecanismo serial, apesar de muitas vezes ser de alta complexidade, não é algo muito difícil de se obter, pois, além de ser possível de espressar todas as velocidades lineares absolutas dos centros de massa das barras e todas as velocidades angulares absolutas em função de $\mathbb{q}^{\#}$ e $\dot{\mathbb{q}}^{\#}$, o que possibilita a obtenção dos modelos por Kane ou Lagrange, atualmente existem vários programas e bibliotecas de linguagens de programação que são capazes de utilizar manipulação simbólica, como o Mathematica e SymPy.

2.2 Dynamic Balancing

3 Applying the technique

$$\mathbb{M}^{\#}(\mathbb{q}^{\#})\ddot{\mathbb{q}}^{\#} + \mathbb{v}^{\#}(\mathbb{q}^{\#}, \dot{\mathbb{q}}^{\#}) + \mathbb{g}^{\#}(\mathbb{q}^{\#}) = \mathbb{u} \tag{2}$$

For a 3-DOF serial mechanism:

$$\mathbb{M}^{\#} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} \tag{3}$$

$$\mathbb{v}^{\#} = \begin{bmatrix} D_{111} & D_{122} & D_{133} \\ D_{211} & D_{222} & D_{233} \\ D_{311} & D_{322} & D_{333} \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \\ \dot{q}_3^2 \end{bmatrix} + 2 \begin{bmatrix} D_{112} & D_{113} & D_{123} \\ D_{212} & D_{213} & D_{223} \\ D_{312} & D_{313} & D_{323} \end{bmatrix} \begin{bmatrix} \dot{q}_1\dot{q}_2 \\ \dot{q}_1\dot{q}_3 \\ \dot{q}_2\dot{q}_3 \end{bmatrix} \tag{4}$$

$$\mathbb{g}^{\#} = \begin{bmatrix} D_1 & D_2 & D_3 \end{bmatrix}^\top \tag{5}$$

$$\mathbb{q}^{\#} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^\top \tag{6}$$

$$\mathbb{u} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^\top \tag{7}$$

For a rotational joint we call $q_i = \theta_i$ and $u_i = \tau_i$, and for a prismatic joint we call $q_i = d_i$ and $u_i = f_i$.

3.1 3-DOF RRR planar serial mechanism

$$\begin{cases} D_1 = g[(m_1 l_{g_1} + m_2 l_1 + m_3 l_1)\mathsf{c}(\theta_1) + (m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_1+\theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_1+\theta_2+\theta_3)] \\ D_2 = g[(m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_1+\theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_1+\theta_2+\theta_3)] \\ D_3 = g[m_3 l_{g_3}\mathsf{c}(\theta_1+\theta_2+\theta_3)] \end{cases} \tag{8}$$

Static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g_2} = -\frac{l_2 m_3}{m_2} \\ l_{g_3} = 0 \end{cases} \tag{9}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{x_1} + J_{x_2} + J_{x_3} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ D_{22} = J_{x_2} + J_{x_3} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{x_3} \\ D_{12} = D_{22} \\ D_{13} = D_{23} = D_{33} \\ \mathbb{v}^{\#} = \mathbb{0} \\ \mathbb{g}^{\#} = \mathbb{0} \end{cases} \tag{10}$$

Coupling 4 discs:

Planar disc models:

$$\mathbb{M}_i^{\#} = \begin{bmatrix} J_{x_{i+3}} \end{bmatrix};\ \mathbb{v}_i^{\#} = \begin{bmatrix} 0 \end{bmatrix};\ \mathbb{g}_i^{\#} = \begin{bmatrix} 0 \end{bmatrix};\ \mathbb{u}_i = \begin{bmatrix} 0 \end{bmatrix};\ \mathbb{p}_i^{\#} = \begin{bmatrix} \omega_{x_{i+3}} \end{bmatrix},\ i = 1,2,3,4 \tag{11}$$

Quasi-velocities constraints:

$$\begin{cases} \omega_{x_4} = \omega_{x_1} + \dot{\theta}_2 \\ \omega_{x_5} = \omega_{x_1} + \beta\dot{\theta}_2 \\ \omega_{x_6} = \omega_{x_2} + \dot{\theta}_3 \\ \omega_{x_7} = \omega_{x_2} + \gamma\dot{\theta}_3 \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = \dot{\theta}_1 + \dot{\theta}_2 \\ \omega_{x_5} = \dot{\theta}_1 + \beta\dot{\theta}_2 \\ \omega_{x_6} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \\ \omega_{x_7} = \dot{\theta}_1 + \dot{\theta}_2 + \gamma\dot{\theta}_3 \end{cases} \quad \therefore \overline{\mathbb{p}} = \begin{bmatrix} \omega_{x_4} - \dot{\theta}_1 - \dot{\theta}_2 \\ \omega_{x_5} - \dot{\theta}_1 - \beta\dot{\theta}_2 \\ \omega_{x_6} - \dot{\theta}_1 - \dot{\theta}_2 - \dot{\theta}_3 \\ \omega_{x_7} - \dot{\theta}_1 - \dot{\theta}_2 - \gamma\dot{\theta}_3 \end{bmatrix} = \mathbb{0} \tag{12}$$

$$\mathbb{A}^{\#} = \frac{\partial\overline{\mathbb{p}}}{\partial\mathbb{p}^{\#}} = -\begin{bmatrix} 1 & 1 & 0 \\ 1 & \beta & 0 \\ 1 & 1 & 1 \\ 1 & 1 & \gamma \end{bmatrix};\ \mathbb{A}^{\circ} = \frac{\partial\overline{\mathbb{p}}}{\partial\mathbb{p}^{\circ}} = \mathbb{1};\ \mathbb{C} = \begin{bmatrix} \mathbb{1} \\ -(\mathbb{A}^{\circ})^{-1}\mathbb{A}^{\#} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \\ 1 & 1 & 1 \\ 1 & 1 & \gamma \end{bmatrix} \tag{13}$$

$$\begin{cases} D'_{11} = D_{11} + J_{x_4} + J_{x_5} + J_{x_6} + J_{x_7} \\ D'_{22} = D_{22} + J_{x_4} + J_{x_5}\beta^2 + J_{x_6} + J_{x_7} \\ D'_{33} = D_{33} + J_{x_6} + J_{x_7}\gamma^2 \\ D'_{12} = D_{12} + J_{x_4} + J_{x_5}\beta + J_{x_6} + J_{x_7} \\ D'_{13} = D_{13} + J_{x_6} + J_{x_7}\gamma \\ D'_{23} = D'_{13} \\ \mathbb{v}'^{\#} = \mathbb{0} \end{cases} \tag{14}$$

Dynamic balancing:

$$\begin{cases} D'_{12} = 0 \\ D'_{13} = 0 \end{cases} \Rightarrow \begin{cases} \beta = -\frac{J_{x_2}+J_{x_3}+J_{x_4}+J_{x_6}+J_{x_7}+m_3 l_2^2+\frac{m_3^2 l_2^2}{m_2}}{J_{x_5}} \\ \gamma = -\frac{J_{x_3}+J_{x_6}}{J_{x_7}} \end{cases} \tag{15}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1 = k_1\ddot{\theta}_1 \\ \tau_2 = k_2\ddot{\theta}_2 \\ \tau_3 = k_3\ddot{\theta}_3 \end{cases} \tag{16}$$

Being:

$$\begin{cases} k_1 = J_{x_1} + J_{x_2} + J_{x_3} + J_{x_4} + J_{x_5} + J_{x_6} + J_{x_7} + m_2 l_1^2 + m_3(l_1^2 + l_2^2) + \frac{l_1^2(m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ k_2 = J_{x_2} + J_{x_3} + J_{x_4} + J_{x_6} + J_{x_7} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} + \frac{\left(J_{x_2}+J_{x_3}+J_{x_4}+J_{x_6}+J_{x_7}+m_3 l_2^2+\frac{m_3^2 l_2^2}{m_2}\right)^2}{J_{x_5}} \\ k_3 = \frac{(J_{x_3}+J_{x_6})(J_{x_3}+J_{x_6}+J_{x_7})}{J_{x_7}} \end{cases} \tag{17}$$

3.2 3-DOF RRR spatial serial mechanism

$$\begin{cases} D_1 = 0 \\ D_2 = g[(m_2 l_{g_2} + m_3 l_2)\mathsf{c}(\theta_2) + m_3 l_{g_3}\mathsf{c}(\theta_2 + \theta_3)] \\ D_3 = g[m_3 l_{g_3}\mathsf{c}(\theta_2 + \theta_3)] \end{cases} \tag{18}$$

Static balancing:

$$\begin{cases} D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_2} = -\frac{l_2 m_3}{m_2} \\ l_{g_3} = 0 \end{cases} \tag{19}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{y_2}\mathsf{s}^2(\theta_2) + J_{y_3}\mathsf{s}^2(\theta_2+\theta_3) + J_{z_1} + J_{z_2}\mathsf{c}^2(\theta_2) + J_{z_3}\mathsf{c}^2(\theta_2+\theta_3) + m_3(l_1 + l_2\mathsf{c}(\theta_2))^2 + \frac{(m_2 l_1 - m_3 l_2 \mathsf{c}(\theta_2))^2}{m_2} \\ D_{22} = J_{x_2} + J_{x_3} + m_2 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{x_3} \\ D_{12} = D_{13} = 0 \\ D_{23} = D_{33} \\ D_{211} = -\frac{1}{2}\Big(\big(J_{y_2} - J_{z_2}\big)\mathsf{s}(2\theta_2) + \big(J_{y_3} - J_{z_3} - m_3 l_2^2(1+\frac{m_3}{m_2})\big)\mathsf{s}(2\theta_2 + 2\theta_3)\Big) \\ D_{311} = \frac{1}{2}\Big(\big(J_{z_3} - J_{y_3}\big)\mathsf{s}(2\theta_2 + 2\theta_3)\Big) \\ D_{111} = D_{122} = D_{133} = D_{222} = D_{233} = D_{322} = D_{333} = 0 \\ D_{112} = -D_{211} \\ D_{113} = -D_{311} \\ D_{123} = D_{212} = D_{213} = D_{223} = D_{312} = D_{313} = D_{323} = 0 \\ \mathbb{g} = \mathbb{0} \end{cases} \tag{20}$$

Coupling 2 discs:

Spatial disc models:

$$\mathbb{M}_i^{\#} = \begin{bmatrix} J_{x_{i+3}} & 0 & 0 \\ 0 & J_{y_{i+3}} & 0 \\ 0 & 0 & J_{z_{i+3}} \end{bmatrix} ;\ \mathbb{v}_i^{\#} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ;\ \mathbb{g}_i^{\#} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ;\ \mathbb{u}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ;\ \mathbb{p}_i^{\#} = \begin{bmatrix} \omega_{x_{i+3}} \\ \omega_{y_{i+3}} \\ \omega_{z_{i+3}} \end{bmatrix},\ i = 1, 2 \tag{21}$$

Quasi-velocities constraints:

$$\begin{cases} [\boldsymbol{\omega}_4]_{\mathsf{B}_4} = [\mathbb{1}]_{\mathsf{B}_4 | \mathsf{B}_2} [\boldsymbol{\omega}_2]_{\mathsf{B}_2} + \begin{bmatrix} \dot\theta_3 \\ 0 \\ 0 \end{bmatrix} \\ [\boldsymbol{\omega}_5]_{\mathsf{B}_5} = [\mathbb{1}]_{\mathsf{B}_5 | \mathsf{B}_2} [\boldsymbol{\omega}_2]_{\mathsf{B}_2} + \begin{bmatrix} \beta\dot\theta_3 \\ 0 \\ 0 \end{bmatrix} \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = \dot\theta_2 + \dot\theta_3 \\ \omega_{y_4} = (\dot\theta_1 \mathsf{s}(\theta_2))\mathsf{c}(\theta_3) + (\dot\theta_1 \mathsf{c}(\theta_2))\mathsf{s}(\theta_3) \\ \omega_{z_4} = -(\dot\theta_1 \mathsf{s}(\theta_2))\mathsf{s}(\theta_3) + (\dot\theta_1 \mathsf{c}(\theta_2))\mathsf{c}(\theta_3) \\ \omega_{x_5} = \dot\theta_2 + \beta\dot\theta_3 \\ \omega_{y_5} = (\dot\theta_1 \mathsf{s}(\theta_2))\mathsf{c}(\beta\theta_3) + (\dot\theta_1 \mathsf{c}(\theta_2))\mathsf{s}(\beta\theta_3) \\ \omega_{z_5} = -(\dot\theta_1 \mathsf{s}(\theta_2))\mathsf{s}(\beta\theta_3) + (\dot\theta_1 \mathsf{c}(\theta_2))\mathsf{c}(\beta\theta_3) \end{cases} \tag{22}$$

$$\therefore \overline{\mathbb{p}} = \begin{bmatrix} \omega_{x_4} - \dot\theta_2 - \dot\theta_3 \\ \omega_{y_4} - \dot\theta_1 \mathsf{s}(\theta_2 + \theta_3) \\ \omega_{z_4} - \dot\theta_1 \mathsf{c}(\theta_2 + \theta_3) \\ \omega_{x_5} - \dot\theta_2 - \beta\dot\theta_3 \\ \omega_{y_5} - \dot\theta_1 \mathsf{s}(\theta_2 + \beta\theta_3) \\ \omega_{z_5} - \dot\theta_1 \mathsf{c}(\theta_2 + \beta\theta_3) \end{bmatrix} = \mathbb{0} \tag{23}$$

$$\mathbb{A}^{\#}=\frac{\partial\overline{\mathbb{p}}}{\partial\mathbb{p}^{\#}}=-\begin{bmatrix}0 & 1 & 1\\ \mathsf{s}(\theta_2+\theta_3) & 0 & 0\\ \mathsf{c}(\theta_2+\theta_3) & 0 & 0\\ 0 & 1 & \beta\\ \mathsf{s}(\theta_2+\beta\theta_3) & 0 & 0\\ \mathsf{c}(\theta_2+\beta\theta_3) & 0 & 0\end{bmatrix};\ \mathbb{A}^{\circ}=\frac{\partial\overline{\mathbb{p}}}{\partial\mathbb{p}^{\circ}}=\mathbb{1};\ \mathbb{C}=\begin{bmatrix}\mathbb{1}\\ -(\mathbb{A}^{\circ})^{-1}\mathbb{A}^{\#}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 1 & 1\\ \mathsf{s}(\theta_2+\theta_3) & 0 & 0\\ \mathsf{c}(\theta_2+\theta_3) & 0 & 0\\ 0 & 1 & \beta\\ \mathsf{s}(\theta_2+\beta\theta_3) & 0 & 0\\ \mathsf{c}(\theta_2+\beta\theta_3) & 0 & 0\end{bmatrix} \tag{24}$$

$$\begin{cases}D'_{11}=D_{11}+J_{y_4}\mathsf{s}^2(\theta_2+\theta_2)+J_{y_5}\mathsf{s}^2(\beta\theta_2+\theta_2)+J_{z_4}\mathsf{c}^2(\theta_2+\theta_2)+J_{z_5}\mathsf{c}^2(\beta\theta_2+\theta_2)\\ D'_{22}=D_{22}+J_{x_4}+J_{x_5}\\ D'_{33}=D_{33}+J_{x_4}+J_{x_5}\beta^2\\ D'_{12}=D'_{13}=0\\ D'_{23}=D_{23}+J_{x_4}+J_{x_5}\beta\\ D'_{211}=D_{211}\\ D'_{311}=D_{311}\\ D'_{111}=D'_{122}=D'_{133}=D'_{222}=D'_{233}=D'_{322}=D'_{333}=0\\ D'_{112}=D_{112}+\frac{1}{4}\Big(\big(J_{y_4}-J_{z_4}\big)\mathsf{s}(2\theta_2+2\theta_3)+\big(J_{y_5}-J_{z_5}\big)\mathsf{s}(2\beta\theta_2+2\theta_3)\Big)\\ D'_{113}=D_{113}+\frac{1}{4}\Big((J_{y_4}-J_{z_4})\mathsf{s}(2\theta_2+2\theta_3)+(J_{y_5}-J_{z_5})\mathsf{s}(2\beta\theta_2+2\theta_3)\Big)\\ D'_{123}=D'_{212}=D'_{213}=D'_{223}=D'_{312}=D'_{313}=D'_{323}=0\end{cases} \tag{25}$$

Dynamic balancing:

$$\begin{cases}D'_{23}=0\\ D'_{211}=0\\ D'_{311}=0\\ D'_{112}=0\\ D'_{113}=0\end{cases}\Rightarrow\begin{cases}\beta=-\frac{J_{x_3}+J_{x_4}}{J_{x_5}}\\ J_{y_2}=J_{z_2}\\ J_{y_3}=J_{z_3}+m_3l_2^2(1+\frac{m_3}{m_2})\\ J_{y_4}=J_{z_4}\\ J_{y_5}=J_{z_5}\end{cases} \tag{26}$$

Dynamic balanced mechanism:

$$\begin{cases}\tau_1=k_1\ddot{\theta}_1\\ \tau_2=k_2\ddot{\theta}_2\\ f_3=k_3\ddot{d}_3\end{cases} \tag{27}$$

Being:

$$\begin{cases}k_1=J_{z_1}+J_{z_2}+J_{z_3}+J_{z_4}+J_{z_5}+m_2l_1^2+m_3(l_1^2+l_2^2)+\frac{l_1^2m_3^2}{m_2}\\ k_2=J_{x_2}+J_{x_3}+J_{x_4}+J_{x_5}+m_3l_2^2+\frac{l_1^2m_3^2}{m_2}\\ k_3=\frac{(J_{x_3}+J_{x_4})(J_{x_3}+J_{x_4}+J_{x_5})}{J_{x_5}}\end{cases} \tag{28}$$

3.3 3-DOF RRP spatial serial mechanism (SCARA)

$$\begin{cases}D_1=g[(m_1l_{g_1}+m_2l_1+m_3l_1)\mathsf{c}(\theta_1)+m_2l_{g_2}\mathsf{c}(\theta_1+\theta_2)]\\ D_2=g[m_2l_{g_2}\mathsf{c}(\theta_1+\theta_2)]\\ D_3=0\end{cases} \tag{29}$$

Static balancing:

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \end{cases} \Rightarrow \begin{cases} l_{g_1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g_2} = 0 \end{cases} \tag{30}$$

Static balanced mechanism:

$$\begin{cases} D_{11} = J_{x_1} + J_{x_2} + J_{x_3} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2(m_2+m_3)^2}{m_1} \\ D_{22} = J_{x_2} + J_{x_3} \\ D_{33} = m_3 \\ D_{12} = D_{22} \\ D_{13} = D_{23} = 0 \\ \mathbb{v} = \mathbb{0} \\ \mathbb{g} = \mathbb{0} \end{cases} \tag{31}$$

Coupling 2 discs:

Planar disc models:

$$\mathbb{M}_i^{\#} = \left[J_{x_{i+3}}\right]; \ \mathbb{v}_i^{\#} = \left[0\right]; \ \mathbb{g}_i^{\#} = \left[0\right]; \ \mathbb{u}_i = \left[0\right]; \ \mathbb{p}_i^{\#} = \left[\omega_{x_{i+3}}\right], \ i = 1, 2 \tag{32}$$

Quasi-velocities constraints:

$$\begin{cases} \omega_{x_4} = \omega_{x_1} + \dot{\theta}_2 \\ \omega_{x_5} = \omega_{x_1} + \beta\dot{\theta}_2 \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = \dot{\theta}_1 + \dot{\theta}_2 \\ \omega_{x_5} = \dot{\theta}_1 + \beta\dot{\theta}_2 \end{cases} \quad \therefore \overline{\mathbb{p}} = \begin{bmatrix} \omega_{x_4} - \dot{\theta}_1 - \dot{\theta}_2 \\ \omega_{x_5} - \dot{\theta}_1 - \beta\dot{\theta}_2 \end{bmatrix} = \mathbb{0} \tag{33}$$

$$\mathbb{A}^{\#} = \frac{\partial \overline{\mathbb{p}}}{\partial \mathbb{p}^{\#}} = -\begin{bmatrix} 1 & 1 & 0 \\ 1 & \beta & 0 \end{bmatrix}; \ \mathbb{A}^{\circ} = \frac{\partial \overline{\mathbb{p}}}{\partial \mathbb{p}^{\circ}} = \mathbb{1}; \ \mathbb{C} = \begin{bmatrix} \mathbb{1} \\ -(\mathbb{A}^{\circ})^{-1}\mathbb{A}^{\#} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \end{bmatrix} \tag{34}$$

$$\begin{cases} D'_{11} = D_{11} + J_{x_4} + J_{x_5} \\ D'_{22} = D_{22} + J_{x_4} + J_{x_5}\beta^2 \\ D'_{33} = D_{33} \\ D'_{12} = D_{12} + J_{x_4} + J_{x_5}\beta \\ D'_{13} = 0 \\ D'_{23} = 0 \\ \mathbb{v}' = \mathbb{0} \end{cases} \tag{35}$$

Dynamic balancing:

$$D'_{12} = 0 \Rightarrow \beta = -\frac{J_{x_2} + J_{x_3} + J_{x_4}}{J_{x_5}} \tag{36}$$

Dynamic balanced mechanism:

$$\begin{cases} \tau_1 = k_1\ddot{\theta}_1 \\ \tau_2 = k_2\ddot{\theta}_2 \\ f_3 = k_3\ddot{d}_3 \end{cases} \tag{37}$$

Being:

$$\begin{cases} k_1 = J_{x_1} + J_{x_2} + J_{x_3} + J_{x_4} + J_{x_5} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2 (m_2+m_3)^2}{m_1} \\ k_2 = \frac{(J_{x_2}+J_{x_3}+J_{x_4})(J_{x_2}+J_{x_3}+J_{x_4}+J_{x_5})}{J_{x_5}} \\ k_3 = m_3 \end{cases} \tag{38}$$

4 Conclusions

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