Adaptative balancing techniques applied to parallel mechanisms

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SUMMARY

KEYWORDS:

1 Introduction and literature review

1.1 Dynamic Models

$\Re$ Modelo do mecanismo RR

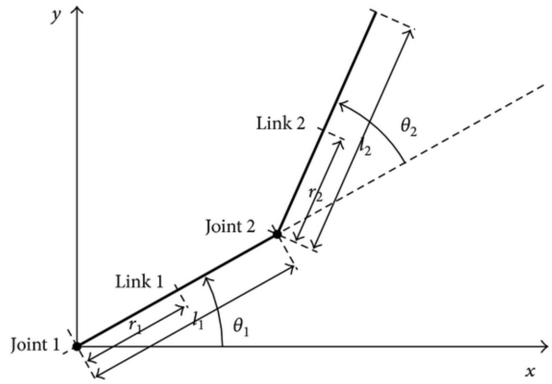

Figure 1: Robô RR

i) Primeiro definimos ν_q coordenadas $\mathbb{q}$. Estas podem ser subdivididas em $\nu^\#$ coordenadas independentes $\mathbb{q}^\#$ e ν_q° coordenadas redudantes $\mathbb{q}^\circ$.

$$\mathbb{q} = \begin{bmatrix} \mathbb{q}^\# \\ \mathbb{q}^\circ \end{bmatrix}$$

No caso do mecanismo *RR*, temos:

$$\mathbb{q}^\# = \begin{bmatrix} \theta_1 & \theta_2 \end{bmatrix}^T \tag{1}$$

$$\mathbb{q}^\circ = \begin{bmatrix} x_1 & y_1 & x_2 & y_2 \end{bmatrix}^T \tag{2}$$

Com $\nu^\# = 2$ e $\nu_q^\circ = 4$. Neste caso, as componentes de $\mathbb{q}^\circ$ são as coordenadas dos centros de massa das barras, escritas no referencial inercial O_{xy}.

ii) Depois definimos os vetores de velocidades absolutas:

$$\mathbb{w} = \begin{bmatrix} \mathbb{w}_\omega \\ \mathbb{w}_v \end{bmatrix}$$

$$\mathbb{w}_\omega = \begin{bmatrix} \omega_{z_1} & \omega_{z_2} \end{bmatrix}^T \tag{3}$$

$$\mathbb{w}_v = \begin{bmatrix} v_{x_1} & v_{y_1} & v_{x_2} & v_{y_2} \end{bmatrix}^T \tag{4}$$

Sendo $\mathbb{w}_v$ as componentes das velocidades absolutas dos centros de massa das barras, escritas nas bases presas s barras, e $\mathbb{w}_\omega$ as componentes das velocidades angulares absolutas, escritas nas bases presas s barras.

iii) Definimos ν_p coordenadas $\mathbb{p}$. Estas podem ser subdivididas em $\nu^\#$ coordenadas independentes $\mathbb{p}^\#$ e ν_p° coordenadas redudantes $\mathbb{p}^\circ$. As coordenadas $\mathbb{p}^\#$ podem ser subdividas em $\nu_\omega^\#$ velocidades angulares $\mathbb{\omega}^\#$ e $\nu_v^\#$ velocidades lineares $\mathbb{v}^\#$. As coordenadas $\mathbb{p}^\circ$ podem ser subdividas em ν_ω° velocidades angulares $\mathbb{\omega}^\circ$ e ν_v° velocidades lineares $\mathbb{v}^\circ$.

$$\mathbb{p} = \begin{bmatrix} \mathbb{p}^\# \\ \mathbb{p}^\circ \end{bmatrix} \qquad \mathbb{p}^\# = \begin{bmatrix} \mathbb{\omega}^\# \\ \mathbb{v}^\# \end{bmatrix} \qquad \mathbb{p}^\circ = \begin{bmatrix} \mathbb{\omega}^\circ \\ \mathbb{v}^\circ \end{bmatrix}$$

Como é conveniente que as velocidades generalizadas $\mathbb{p}$ sejam velocidades absolutas, escolhemos as componentes de $\mathbb{p}$ como sendo as mesmas componentes de $\mathbb{w}$, respeitando a ordenação indicada acima.

No caso do mecanismo <u>RR</u>, temos:

$$\mathbb{\omega}^\# = \begin{bmatrix} \omega_{z_1} \\ \omega_{z_2} \end{bmatrix} \tag{5}$$

$$\mathbb{v}^\# = \emptyset \tag{6}$$

$$\mathbb{\omega}^\circ = \emptyset \tag{7}$$

$$\mathbb{v}^\circ = \begin{bmatrix} v_{x_1} & v_{y_1} & v_{x_2} & v_{y_2} \end{bmatrix}^T \tag{8}$$

Com $\nu_\omega^\# = 2$, $\nu_v^\# = 0$, $\nu_\omega^\circ = 0$, $\nu_v^\circ = 4$ e $\nu_p^\circ = \nu_\omega^\circ + \nu_v^\circ = 4$.

iv) Realizamos a cinemtica de posição para os centros de massa das barras, de modo a relacionar as coordenadas $\mathbb{q}^{\circ}$ com as coordenadas $\mathbb{q}^{\#}$. Para isso, utilizamos matrizes de transformação homogênea.

$$[\boldsymbol{H}]_{\mathtt{B}_0|\mathtt{B}_1} = \begin{bmatrix} Rot(\theta_1, z_0) & [\overrightarrow{\mathtt{O}_0\mathtt{O}_1}]_{\mathtt{B}_0} \\ \mathbb{0}_{2x1} & 1 \end{bmatrix} = \begin{bmatrix} \mathtt{c}_1 & -\mathtt{s}_1 & 0 \\ \mathtt{s}_1 & \mathtt{c}_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \begin{bmatrix} [\overrightarrow{\mathtt{O}_1\mathtt{G}_1}]_{\mathtt{B}_1} \\ 1 \end{bmatrix} = \begin{bmatrix} l_{1g} \\ 0 \\ 1 \end{bmatrix}$$

$$[\boldsymbol{H}]_{\mathtt{B}_1|\mathtt{B}_2} = \begin{bmatrix} Rot(\theta_2, z_1) & [\overrightarrow{\mathtt{O}_1\mathtt{O}_2}]_{\mathtt{B}_1} \\ \mathbb{0}_{2x1} & 1 \end{bmatrix} = \begin{bmatrix} \mathtt{c}_2 & -\mathtt{s}_2 & l_1 \\ \mathtt{s}_2 & \mathtt{c}_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \begin{bmatrix} [\overrightarrow{\mathtt{O}_2\mathtt{G}_2}]_{\mathtt{B}_2} \\ 1 \end{bmatrix} = \begin{bmatrix} l_{2g} \\ 0 \\ 1 \end{bmatrix}$$

$$[\boldsymbol{H}]_{\mathtt{B}_0|\mathtt{B}_2} = [\boldsymbol{H}]_{\mathtt{B}_0|\mathtt{B}_1}[\boldsymbol{H}]_{\mathtt{B}_1|\mathtt{B}_2} = \begin{bmatrix} \mathtt{c}_{1+2} & -\mathtt{s}_{1+2} & l_1\mathtt{c}_1 \\ \mathtt{s}_{1+2} & \mathtt{c}_{1+2} & l_1\mathtt{s}_1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = [\boldsymbol{H}]_{\mathtt{B}_0|\mathtt{B}_1} \begin{bmatrix} [\overrightarrow{\mathtt{O}_1\mathtt{G}_1}]_{\mathtt{B}_1} \\ 1 \end{bmatrix} = \begin{bmatrix} l_{1g}\mathtt{c}_1 \\ l_{1g}\mathtt{s}_1 \\ 1 \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = [\boldsymbol{H}]_{\mathtt{B}_0|\mathtt{B}_2} \begin{bmatrix} [\overrightarrow{\mathtt{O}_2\mathtt{G}_2}]_{\mathtt{B}_2} \\ 1 \end{bmatrix} = \begin{bmatrix} l_1\mathtt{c}_1 + l_{2g}\mathtt{c}_{1+2} \\ l_1\mathtt{s}_1 + l_{2g}\mathtt{s}_{1+2} \\ 1 \end{bmatrix} \tag{10}$$

Repare que a partir das matrizes de transformação homognea encontradas, encontramos tambm as seguintes matrizes de mudança de base:

$$\mathbb{R}_1 = [\mathbf{1}]_{\mathtt{B}_0|\mathtt{B}_1} = \begin{bmatrix} \mathtt{c}_1 & -\mathtt{s}_1 \\ \mathtt{s}_1 & \mathtt{c}_1 \end{bmatrix} \tag{11}$$

$$\mathbb{R}_2 = [\mathbf{1}]_{\mathtt{B}_0|\mathtt{B}_2} = \begin{bmatrix} \mathtt{c}_{1+2} & -\mathtt{s}_{1+2} \\ \mathtt{s}_{1+2} & \mathtt{c}_{1+2} \end{bmatrix} \tag{12}$$

Com a cinemtica de posição, conseguimos obter $\nu_q^{\circ} = 4$ equações vinculares de posição. Sendo assim, o vetor dos vínculos de posição dado por:

$$\mathbb{\phi}(\mathbb{q}) = \begin{bmatrix} x_1 - l_{1g}\mathtt{c}_1 \\ y_1 - l_{1g}\mathtt{s}_1 \\ x_2 - l_1\mathtt{c}_1 - l_{2g}\mathtt{c}_{1+2} \\ y_2 - l_1\mathtt{s}_1 - l_{2g}\mathtt{s}_{1+2} \end{bmatrix} \tag{13}$$

v) Utilizamos as matrizes de rotação para calcular as velocidades angulares em função de $\mathbb{q}^{\#}$ e $\dot{\mathbb{q}}^{\#}$:

$$[\boldsymbol{\omega}_1]^{\mathsf{S}}_{\mathtt{B}_1|\mathtt{B}_1} = \mathbb{R}_1^T\dot{\mathbb{R}}_1 = \begin{bmatrix} 0 & -\dot{\theta}_1 \\ \dot{\theta}_1 & 0 \end{bmatrix} \Rightarrow [\boldsymbol{\omega}_1]_{\mathtt{B}_1} = \dot{\theta}_1\hat{k} \tag{14}$$

$$[\boldsymbol{\omega}_2]^{\mathsf{S}}_{\mathtt{B}_2|\mathtt{B}_2} = \mathbb{R}_2^T\dot{\mathbb{R}}_2 = \begin{bmatrix} 0 & -\dot{\theta}_1 - \dot{\theta}_2 \\ \dot{\theta}_1 + \dot{\theta}_2 & 0 \end{bmatrix} \Rightarrow [\boldsymbol{\omega}_2]_{\mathtt{B}_2} = (\dot{\theta}_1 + \dot{\theta}_2)\hat{k} \tag{15}$$

vi) Derivamos as equações de posição ((29) e (30)) para encontrar as velocidades dos centros de massa:

$$[\boldsymbol{v}_1]_{\mathrm{B}_0} = \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \end{bmatrix} = \begin{bmatrix} -l_{1g}\mathsf{s}_1\dot{\theta}_1 \\ l_{1g}\mathsf{c}_1\dot{\theta}_1 \end{bmatrix} \tag{16}$$

$$[\boldsymbol{v}_2]_{\mathrm{B}_0} = \begin{bmatrix} \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} -l_1\mathsf{s}_1\dot{\theta}_1 - l_{2g}\mathsf{s}_{1+2}(\dot{\theta}_1 + \dot{\theta}_2) \\ l_1\mathsf{c}_1\dot{\theta}_1 + l_{2g}\mathsf{c}_{1+2}(\dot{\theta}_1 + \dot{\theta}_2) \end{bmatrix} \tag{17}$$

vii) Passamos as velocidades dos centros de massa para as bases presas nas barras:

$$[\boldsymbol{v}_1]_{\mathrm{B}_1} = [\mathbf{1}]_{\mathrm{B}_1 | \mathrm{B}_0}[\boldsymbol{v}_1]_{\mathrm{B}_0} = \mathbb{R}_1^T[\boldsymbol{v}_1]_{\mathrm{B}_0}$$
$$[\boldsymbol{v}_2]_{\mathrm{B}_2} = [\mathbf{1}]_{\mathrm{B}_2 | \mathrm{B}_0}[\boldsymbol{v}_2]_{\mathrm{B}_0} = \mathbb{R}_2^T[\boldsymbol{v}_2]_{\mathrm{B}_0}$$

Definindo:

$$\mathbb{R}_\Diamond = \begin{bmatrix} \mathbb{R}_1 & \mathbb{O}_{2x2} \\ \mathbb{O}_{2x2} & \mathbb{R}_2 \end{bmatrix} \tag{18}$$

Temos:

$$\mathbb{w}_v = \mathbb{R}_\Diamond^T \dot{\mathbb{q}}^\circ \tag{19}$$

$$\therefore \begin{bmatrix} v_{x_1} \\ v_{y_1} \\ v_{x_2} \\ v_{y_2} \end{bmatrix} = \begin{bmatrix} \mathsf{c}_1 & -\mathsf{s}_1 & 0 & 0 \\ \mathsf{s}_1 & \mathsf{c}_1 & 0 & 0 \\ 0 & 0 & \mathsf{c}_{1+2} & -\mathsf{s}_{1+2} \\ 0 & 0 & \mathsf{s}_{1+2} & \mathsf{c}_{1+2} \end{bmatrix}^T \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{x}_2 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ l_{1g}\dot{\theta}_1 \\ l_1\mathsf{s}_2\dot{\theta}_1 \\ (l_1\mathsf{c}_2 + l_{2g})\dot{\theta}_1 + l_{2g}\dot{\theta}_2 \end{bmatrix} \tag{20}$$

viii) Montamos os vetores $\mathbb{p}^\#$ e $\mathbb{p}^\circ$ em função de $\mathbb{q}^\#$ e $\dot{\mathbb{q}}^\#$:

$$\mathbb{p}^\# = \begin{bmatrix} \omega_{z_1} \\ \omega_{z_2} \end{bmatrix} = \mathbb{p}_\star^\#(\mathbb{q}^\#, \dot{\mathbb{q}}^\#) = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix} \tag{21}$$

$$\mathbb{p}^\circ = \begin{bmatrix} v_{x_1} \\ v_{y_1} \\ v_{x_2} \\ v_{y_2} \end{bmatrix} = \mathbb{p}_\star^\circ(\mathbb{q}^\#, \dot{\mathbb{q}}^\#) = \begin{bmatrix} 0 \\ l_{1g}\dot{\theta}_1 \\ l_1\mathsf{s}_2\dot{\theta}_1 \\ (l_1\mathsf{c}_2 + l_{2g})\dot{\theta}_1 + l_{2g}\dot{\theta}_2 \end{bmatrix} \tag{22}$$

ix) Utilizando o fato de que $\mathbb{p}_\star^\#(\mathbb{q}^\#, \dot{\mathbb{q}}^\#)$ e $\mathbb{p}_\star^\circ(\mathbb{q}^\#, \dot{\mathbb{q}}^\#)$ são lineares em $\dot{\mathbb{q}}^\#$, encontramos as transformações lineares $\Psi(\mathbb{q})$ e $\Upsilon(\mathbb{q})$ e o vetor dos vínculos de velocidades $\mathbb{A}(\mathbb{q}, \mathbb{p})$:

$$\mathbb{p}^\# = \mathbb{p}_\star^\#(\mathbb{q}^\#, \dot{\mathbb{q}}^\#) = \frac{\partial \mathbb{p}_\star^\#}{\partial \dot{\mathbb{q}}^\#}\dot{\mathbb{q}}^\# = \Psi\dot{\mathbb{q}}^\# \tag{23}$$

$$\mathbb{p}^\circ = \mathbb{p}^\circ_\star(\mathbb{q}^\#, \dot{\mathbb{q}}^\#) = \frac{\partial \mathbb{p}^\circ_\star}{\partial \dot{\mathbb{q}}^\#}\dot{\mathbb{q}}^\# = \Upsilon \dot{\mathbb{q}}^\# \tag{24}$$

No caso do mecanismo $\underline{RR}$, temos:

$$\Psi = \frac{\partial \mathbb{p}^\#_\star}{\partial \dot{\mathbb{q}}^\#} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{25}$$

$$\Upsilon = \frac{\partial \mathbb{p}^\circ_\star}{\partial \dot{\mathbb{q}}^\#} = \begin{bmatrix} 0 & 0 \\ l_{1g} & 0 \\ l_1 \mathsf{s}_2 & 0 \\ l_1 \mathsf{c}_2 + l_{2g} & l_{2g} \end{bmatrix} \tag{26}$$

Como $\mathbb{p}^\#$ e $\dot{\mathbb{q}}^\#$ são independentes e tem o mesmo tamanho:

$$\dot{\mathbb{q}}^\# = \Psi^{-1}\mathbb{p}^\#$$
$$\Rightarrow \mathbb{p}^\circ = \Upsilon\Psi^{-1}\mathbb{p}^\#$$

$$\therefore \mathbb{A}(\mathbb{q}, \mathbb{p}) = \Upsilon\Psi^{-1}\mathbb{p}^\# - \mathbb{p}^\circ = 0 \tag{27}$$

x) A partir de $\mathbb{A}(\mathbb{q}, \mathbb{p})$, encontramos o jacobiano $\mathbb{A}$ dos vínculos de velocidade e a matriz $\mathbb{C}$ dos vínculos cinemáticos:

$$\mathbb{A}(\mathbb{q}, \mathbb{p}) = \begin{bmatrix} \Upsilon\Psi^{-1} & -\mathbb{1} \end{bmatrix} \mathbb{p}$$

$$\mathbb{A} = \frac{\partial \mathbb{A}}{\partial \mathbb{p}} = \begin{bmatrix} \Upsilon\Psi^{-1} & -\mathbb{1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 \\ l_{1g} & 0 & 0 & -1 & 0 & 0 \\ l_1 s_2 & 0 & 0 & 0 & -1 & 0 \\ l_1 c_2 & l_{2g} & 0 & 0 & 0 & -1 \end{bmatrix} \tag{28}$$

$$\mathbb{A}(\mathbb{q}, \mathbb{p}) = 0 \Rightarrow \mathbb{p}^\circ = \Upsilon\Psi^{-1}\mathbb{p}^\# \Rightarrow \mathbb{p} = \begin{bmatrix} \mathbb{1} \\ \Upsilon\Psi^{-1} \end{bmatrix} \mathbb{p}^\#$$

$$\therefore \mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \Upsilon\Psi^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ l_{1g} & 0 \\ l_1 s_2 & 0 \\ l_1 c_2 & l_{2g} \end{bmatrix} \tag{29}$$

xi) Como (19) e (23) são transformações inversíveis, encontramos a transformação linear $\mathbb{\Gamma}(\mathbb{q})$:

$$\dot{\mathbb{q}} = \begin{bmatrix}\dot{\mathbb{q}}^{\#}\\ \dot{\mathbb{q}}^{\circ}\end{bmatrix} = \dot{\mathbb{q}}_{\star}(\mathbb{q},\mathbb{p}) = \begin{bmatrix}\mathbb{\Psi}^{-1}(\mathbb{q})\mathbb{p}^{\#}\\ \mathbb{R}_{\Diamond}(\mathbb{q})\mathbb{w}_{v}(\mathbb{p})\end{bmatrix} = \begin{bmatrix}\omega_{z_1}\\ \omega_{z_2}-\omega_{z_1}\\ v_{x_1}\,\mathsf{c}_1 - v_{y_1}\,\mathsf{s}_1\\ v_{x_1}\,\mathsf{s}_1 + v_{y_1}\,\mathsf{c}_1\\ v_{x_2}\,\mathsf{c}_{1+2} - v_{y_2}\,\mathsf{s}_{1+2}\\ v_{x_2}\,\mathsf{s}_{1+2} + v_{y_2}\,\mathsf{c}_{1+2}\end{bmatrix} \tag{30}$$

$$\mathbb{\Gamma}(\mathbb{q}) = \frac{\partial\dot{\mathbb{q}}_{\star}}{\partial\mathbb{p}} = \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ -1 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & \mathsf{c}_1 & -\mathsf{s}_1 & 0 & 0\\ 0 & 0 & \mathsf{s}_1 & \mathsf{c}_1 & 0 & 0\\ 0 & 0 & 0 & 0 & \mathsf{c}_{1+2} & -\mathsf{s}_{1+2}\\ 0 & 0 & 0 & 0 & \mathsf{s}_{1+2} & \mathsf{c}_{1+2}\end{bmatrix} \tag{31}$$

xii) Aplicamos o mtodos de Gibbs-Appel extendido:

O mtodo de Gibbs-Appell apresenta certa simularidade com o mtodo de Lagrange, pois utiliza derivadas de uma função energia para encontrar a equações de movimento do sistema. Porém, a função energia utilizada no a energia cintica, mas sim a energia de acelerações. A energia de acelerações para um corpo rígido é dada pela seguinte expressão:

$$\mathcal{S} = \frac{1}{2}m(\boldsymbol{a}_G\cdot\boldsymbol{a}_G) + \frac{1}{2}(\dot{\boldsymbol{\omega}}\cdot\boldsymbol{I}\dot{\boldsymbol{\omega}} + 2\dot{\boldsymbol{\omega}}(\boldsymbol{\omega}\wedge\boldsymbol{I}\boldsymbol{\omega}))$$

Sendo m a massa do corpo rígido, $\boldsymbol{I}$ seu tensor de inércia, $\boldsymbol{a}_G$ o vetor aceleração absoluta de seu centro de massa e $\boldsymbol{\omega}$ o vetor velocidade angular absoluta.

O modelo dinmico utilizando o mtodo de Gibbs-Appel extendido, dado pela seguinte expresso:

$$\mathbb{C}(q)^T(\mathbb{M}(\mathbb{q})\dot{\mathbb{p}} + \mathbb{v}(\mathbb{q},\mathbb{p}) + \mathbb{g}(\mathbb{q})) = (\mathbb{\Psi}^T)^{-1}\mathbb{f}_{\dot{\mathbb{q}}^{\#}} \tag{32}$$

Com:

$$\mathbb{M}(\mathbb{q}) = \frac{\partial^2\mathcal{S}}{\partial\dot{\mathbb{p}}^2} \tag{33}$$

$$\mathbb{v}(\mathbb{q},\mathbb{p}) = \frac{\partial\mathcal{S}}{\partial\dot{\mathbb{p}}} - \frac{\partial^2\mathcal{S}}{\partial\dot{\mathbb{p}}^2}\dot{\mathbb{p}} \tag{34}$$

$$\mathbb{g}(\mathbb{q}) = \mathbb{\Gamma}^T\frac{\partial E_p}{\partial\mathbb{q}} \tag{35}$$

Sendo E_p a energia potencial do sistema e $\mathbb{f}_{\dot{\mathbb{q}}^{\#}}$ os esforços nas direções de $\dot{\mathbb{q}}^{\#}$.

Como já calculamos os vetores de velocidades absolutas dos centros de massa e de velocidades angulares absolutas, escritos em bases presas s barras, as acelerações absolutas dos centros de massa so dadas por:

$$[\boldsymbol{a}_i]_{\mathtt{B}_i} = \frac{\mathsf{d}}{\mathsf{d}t}[\boldsymbol{v}_i]_{\mathtt{B}_i} + [\boldsymbol{\omega}_i]^{\mathsf{S}}_{\mathtt{B}_i \mid \mathtt{B}_i}[\boldsymbol{v}_i]_{\mathtt{B}_i}$$

Como o mecanismo plano:

$$[\boldsymbol{a}_i]_{\mathtt{B}_i} = \begin{bmatrix} \dot{v}_{x_i} \\ \dot{v}_{y_i} \end{bmatrix} + \begin{bmatrix} 0 & -\omega_{z_i} \\ \omega_{z_i} & 0 \end{bmatrix} \begin{bmatrix} v_{x_i} \\ v_{y_i} \end{bmatrix} = \begin{bmatrix} \dot{v}_{x_i} - \omega_{z_i} v_{y_i} \\ \dot{v}_{y_i} + \omega_{z_i} v_{x_i} \end{bmatrix}$$

No caso do mecanismo <u>*RR*</u>, temos:

$$\mathcal{S} = \frac{1}{2}\Big(m_1((\dot{v}_{x_1} - \omega_{z_1} v_{y_1})^2 + (\dot{v}_{y_1} + \omega_{z_1} v_{x_1})^2) + m_2((\dot{v}_{x_2} - \omega_{z_2} v_{y_2})^2 + (\dot{v}_{y_2} + \omega_{z_2} v_{x_2})^2) + J_{z_1}\dot{\omega}_{z_1}^2 + J_{z_2}\dot{\omega}_{z_2}^2 \Big) \tag{36}$$

$$E_p = m_1 g y_1 + m_2 g y_2 \tag{37}$$

Calculando as derivadas:

$$\mathbb{M}(\mathbb{q}) = \begin{bmatrix} J_{z_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & J_{z_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_2 \end{bmatrix} \tag{38}$$

$$\mathbb{v}(\mathbb{q}, \mathbb{p}) = \begin{bmatrix} 0 \\ 0 \\ -m_1\omega_{z_1} v_{y_1} \\ m_1 \omega_{z_1} v_{x_1} \\ -m_2 \omega_{z_2} v_{y_2} \\ m_2 \omega_{z_2} v_{x_2} \end{bmatrix} \tag{39}$$

$$\mathbb{g} = g \begin{bmatrix} 0 \\ 0 \\ m_1 \mathsf{s}_1 \\ m_1 \mathsf{c}_1 \\ m_2 \mathsf{s}_{1+2} \\ m_2 \mathsf{c}_{1+2} \end{bmatrix} \tag{40}$$

Sendo assim, o modelo dinmico para o mecanismo <u>RR</u> dado por:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ l_{1g} & 0 \\ l_1 \mathsf{s}_2 & 0 \\ l_1 \mathsf{c}_2 & l_{2g} \end{bmatrix}^T \left\{ \begin{bmatrix} J_{z_1}\dot{\omega}_{z_1} \\ J_{z_2}\dot{\omega}_{z_2} \\ m_1 \dot{v}_{x_1} \\ m_1 \dot{v}_{y_1} \\ m_2 \dot{v}_{x_2} \\ m_2 \dot{v}_{y_2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -m_1\omega_{z_1} v_{y_1} \\ m_1 \omega_{z_1} v_{x_1} \\ -m_2 \omega_{z_2} v_{y_2} \\ m_2 \omega_{z_2} v_{x_2} \end{bmatrix} + g \begin{bmatrix} 0 \\ 0 \\ m_1 \mathsf{s}_1 \\ m_1 \mathsf{c}_1 \\ m_1 \mathsf{s}_{1+2} \\ m_1 \mathsf{c}_{1+2} \end{bmatrix} \right\} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{41}$$

Repare que o modelo no depende das coordenadas $\mathbb{q}^{\circ}$. Elas foram uteis para a dedução do modelo, mas com o modelo deduzido elas no tem mais utilidade.

- Massa pontual:

$$\mathbb{q}_n = \begin{bmatrix} x_n \\ y_n \end{bmatrix} = \begin{bmatrix} q_{n,1} \\ q_{n,2} \end{bmatrix} \tag{42}$$

$$\mathbb{p}_n = \begin{bmatrix} p_{n,1} \\ p_{n,2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_{n,1} \\ \dot{q}_{n,2} \end{bmatrix} \tag{43}$$

$$\begin{cases} \begin{bmatrix} \dot{q}_{n,1} \\ \dot{q}_{n,2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_{n,1} \\ p_{n,2} \end{bmatrix} \\ \begin{bmatrix} M_n \dot{p}_{n,1} \\ M_n \dot{p}_{n,2} \end{bmatrix} + g \begin{bmatrix} 0 \\ M_n \end{bmatrix} = \begin{bmatrix} f_{n,1} \\ f_{n,2} \end{bmatrix} \end{cases} \tag{44}$$

Que pode ser reescrito como:

$$\begin{bmatrix} M_n & 0 \\ 0 & M_n \end{bmatrix} \begin{bmatrix} \ddot{q}_{n,1} \\ \ddot{q}_{n,2} \end{bmatrix} + g \begin{bmatrix} 0 \\ M_n \end{bmatrix} = \begin{bmatrix} f_{n,1} \\ f_{n,2} \end{bmatrix}$$

- RR:

$$\mathbb{q}_n = \begin{bmatrix} \theta_{1\,n} \\ \theta_{2\,n} \end{bmatrix} = \begin{bmatrix} q_{n,1} \\ q_{n,2} \end{bmatrix} \tag{45}$$

$$\mathbb{p}_n = \begin{bmatrix} p_{n,1} \\ p_{n,2} \\ p_{n,3} \\ p_{n,4} \\ p_{n,5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ l_{g1} & 0 \\ l_1 \mathsf{s}_{n,2} & 0 \\ l_{g2} + l_1 \mathsf{c}_{n,2} & l_{g2} \end{bmatrix} \begin{bmatrix} \dot{q}_{n,1} \\ \dot{q}_{n,2} \end{bmatrix} \tag{46}$$

$$\begin{cases} \begin{bmatrix} \dot{q}_{n,1} \\ \dot{q}_{n,2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} p_{n,1} \\ p_{n,2} \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ l_{g1} & 0 \\ l_1 \mathsf{s}_{n,2} & 0 \\ l_1 \mathsf{c}_{n,2} & l_{g2} \end{bmatrix}^T \left\{ \begin{bmatrix} J_{z1} \dot{p}_{n,1} \\ J_{z2} \dot{p}_{n,2} \\ m_1 \dot{p}_{n,3} \\ m_2 \dot{p}_{n,4} \\ m_2 \dot{p}_{n,5} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -m_1 p_{n,2} p_{n,5} \\ m_1 p_{n,2} p_{n,4} \end{bmatrix} + g \begin{bmatrix} 0 \\ 0 \\ m_1 \mathsf{c}_{n,1} \\ m_2 \mathsf{s}_{n,1+2} \\ m_2 \mathsf{c}_{n,1+2} \end{bmatrix} \right\} = \begin{bmatrix} u_{n,1} \\ u_{n,2} \end{bmatrix} \\ \begin{bmatrix} l_{g1} & 0 & -1 & 0 & 0 \\ l_1 \mathsf{s}_{n,2} & 0 & 0 & -1 & 0 \\ l_1 \mathsf{c}_{n,2} & l_{g2} & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \dot{p}_{n,1} \\ \dot{p}_{n,2} \\ \dot{p}_{n,3} \\ \dot{p}_{n,4} \\ \dot{p}_{n,5} \end{bmatrix} = - \begin{bmatrix} 0 \\ l_1 \mathsf{c}_{n,2} p_{n,1} (-p_{n,1} + p_{n,2}) \\ l_1 \mathsf{s}_{n,2} p_{n,1} (p_{n,1} - p_{n,2}) \end{bmatrix} \end{cases} \tag{47}$$

Que pode ser reescrito como:

$$\begin{bmatrix} J_{z1}+J_{z2}+m_1 l_{g1}^2 + m_2(l_1^2+2l_1l_{g2}\mathsf{c}_{n,2}+l_{g2}^2) & J_{z2}+m_2 l_{g2}(l_1\mathsf{c}_{n,2}+l_{g2}) \\ J_{z2}+m_2l_{g2}(l_1\mathsf{c}_{n,2}+l_{g2}) & J_{z2}+m_2l_{g2}^2\end{bmatrix}\begin{bmatrix}\ddot q_{n,1}\\ \ddot q_{n,2}\end{bmatrix}$$
$$+ \begin{bmatrix} -m_2l_1l_{g2}\mathsf{s}_{n,2}\dot q_{n,2}^2 - 2m_2l_1l_{g2}\mathsf{s}_{n,2}\dot q_{n,1}\dot q_{n,2} \\ m_2l_1l_{g2}\mathsf{s}_{n,2}\dot q_{n,1}^2\end{bmatrix} + g\begin{bmatrix} m_1l_{g1}\mathsf{c}_{n,1}+m_2(l_{g2}\mathsf{c}_{n,1+2}+l_1\mathsf{c}_{n,1}) \\ m_2l_{g2}\mathsf{c}_{n,1+2}\end{bmatrix} = \begin{bmatrix}u_{n,1}\\u_{n,2}\end{bmatrix}$$

- RR (0) com 2 massas acopladas (1 e 2):

$$\begin{cases} \begin{bmatrix}\dot q_{0,1}\\ \dot q_{0,2}\end{bmatrix} = \begin{bmatrix}1 & 0\\ -1 & 1\end{bmatrix}\begin{bmatrix}p_{0,1}\\ p_{0,2}\end{bmatrix} \\ \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ l_{g1} & 0 \\ l_1\mathsf{s}_{0,2} & 0 \\ l_1\mathsf{c}_{0,2} & l_{g2} \\ -L_1\mathsf{s}_{0,1} & 0 \\ L_1\mathsf{c}_{0,1} & 0 \\ -l_1\mathsf{s}_{0,1} & -L_2\mathsf{s}_{0,1} \\ l_1\mathsf{c}_{0,1} & L_2\mathsf{c}_{0,1}\end{bmatrix}^T \left\{ \begin{bmatrix} J_{z1}\dot p_{0,1} \\ J_{z2}\dot p_{0,2} \\ m_1\dot p_{0,3} \\ m_2\dot p_{0,4} \\ m_2\dot p_{0,5} \\ M_1\dot p_{1,1} \\ M_1\dot p_{1,2} \\ M_2\dot p_{2,1} \\ M_2\dot p_{2,1}\end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -m_1p_{0,2}p_{0,5} \\ m_1p_{0,2}p_{0,4} \\ 0 \\ 0 \\ 0 \\ 0\end{bmatrix} + g\begin{bmatrix} 0 \\ 0 \\ m_1\mathsf{c}_{0,1} \\ m_2\mathsf{s}_{0,1+2} \\ m_2\mathsf{c}_{0,1+2} \\ 0 \\ M_1 \\ 0 \\ M_2\end{bmatrix}\right\} = \begin{bmatrix}u_{0,1}\\ u_{0,2}\end{bmatrix} \\ \begin{bmatrix} l_{g1} & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_1\mathsf{s}_{i,2} & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ l_1\mathsf{c}_{i,2} & l_{g2} & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ L_1\mathsf{s}_{0,1} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ -L_1\mathsf{c}_{0,1} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ l_1\mathsf{s}_{0,1} & L_2\mathsf{s}_{0,1+2} & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -l_1\mathsf{c}_{0,1} & -L_2\mathsf{c}_{0,1+2} & 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix} \begin{bmatrix}\dot p_{0,1}\\ \dot p_{0,2}\\ \dot p_{0,3}\\ \dot p_{0,4}\\ \dot p_{0,5}\\ \dot p_{1,1}\\ \dot p_{1,2}\\ \dot p_{2,1}\\ \dot p_{2,1}\end{bmatrix} = -\begin{bmatrix} 0 \\ l_1\mathsf{c}_{0,2}p_{0,1}(-p_{0,1}+p_{0,2}) \\ l_1\mathsf{s}_{0,2}p_{0,1}(p_{0,1}-p_{0,2}) \\ L_1\mathsf{c}_{0,1}p_{0,1}^2 \\ L_1\mathsf{s}_{0,1}p_{0,1}^2 \\ l_1\mathsf{c}_{0,1}p_{0,1}^2 + L_1\mathsf{c}_{0,1+2}p_{0,2}^2 \\ l_1\mathsf{s}_{0,1}p_{0,1}^2 + L_1\mathsf{s}_{0,1+2}p_{0,2}^2\end{bmatrix}\end{cases} \tag{48}$$

Que pode ser reescrito como:

$$\mathbb{M}^{\#}\ddot{\mathbb{q}}_0 + \mathbb{v}^{\#} + \mathbb{g}^{\#} = \mathbb{u}_0 \tag{49}$$

Sendo:

$$\mathbb{M}^{\#}_{1,1} = J_{z1} + J_{z2} + M_1L_1^2 + M_2L_2^2 + m_1l_{g1}^2 + m_2l_{g2}^2 + (M_2+m_2)l_1^2 + 2l_1\mathsf{c}_{0,2}(L_2M_2 + m_2l_{g2}) \tag{50}$$

$$\mathbb{M}^{\#}_{1,2} = \mathbb{M}^{\#}_{2,1} = J_{z2} + M_2L_2(l_1\mathsf{c}_{0,2} + L_2) + m_2l_{g2}(l_1\mathsf{c}_{0,2} + l_{g2}) \tag{51}$$

$$\mathbb{M}^{\#}_{2,2} = J_{z2} + M_2L_2^2 + m_2l_{g2}^2 \tag{52}$$

$$\mathbb{v}_1^\# = -(M_2L_2 + m_2l_{g2})l_1\mathsf{s}_{0,2}\dot{q}_{0,2}(2\dot{q}_{0,1} + \dot{q}_{0,2}) \tag{53}$$

$$\mathbb{v}_2^\# = (M_2L_2 + m_2l_{g2})l_1\mathsf{s}_{0,2}\dot{q}_{0,1}^2 \tag{54}$$

$$\mathbb{g}_1^\# = g(M_1L_1\mathsf{c}_{0,1} + m_1l_{g1}\mathsf{c}_{0,1} + (M_2 + m_2)l_1\mathsf{c}_{0,1} + (M_2L_2 + m_2l_{g2})\mathsf{c}_{0,1+2}) \tag{55}$$

$$\mathbb{g}_2^\# = g(M_2L_2 + m_2l_{g2})\mathsf{c}_{0,1+2} \tag{56}$$

- RR balanceado:

  Escolhendo L_1 e L_2 de modo que $\mathbb{g}^\# = \mathbb{0}$:

$$\begin{cases} L_1 = -\frac{M_2l_1 + m_1l_{g1} + m_2l_2}{M_1} \\ L_2 = -\frac{m_2l_{g2}}{M_2} \end{cases} \tag{57}$$

  Obtemos o seguinte sistema:

$$\mathbb{M}_{1,1}^\# = J_{z1} + J_{z2} + m_1l_{g2}^2 + l_1^2(M_2 + m_2) + \frac{m_2^2l_{g2}^2}{M_2} + \frac{(m_1l_{g1} + (M_2 + m_2)l_1)^2}{M_1} \tag{58}$$

$$\mathbb{M}_{1,2}^\# = \mathbb{M}_{2,1}^\# = \mathbb{M}_{2,2}^\# = J_{z2} + \frac{m_2(M_2 + m_2)l_{g2}^2}{M_2} \tag{59}$$

$$\mathbb{v}_1^\# = 0 \tag{60}$$

$$\mathbb{v}_2^\# = 0 \tag{61}$$

$$\mathbb{g}_1^\# = 0 \tag{62}$$

$$\mathbb{g}_2^\# = 0 \tag{63}$$

  Ou seja:

$$\begin{bmatrix} J_{z1} + J_{z2} + m_1l_{g2}^2 + l_1^2(M_2 + m_2) + \frac{m_2^2l_{g2}^2}{M_2} + \frac{(m_1l_{g1} + (M_2+m_2)l_1)^2}{M_1} & J_{z2} + \frac{m_2(M_2+m_2)l_{g2}^2}{M_2} \\ J_{z2} + \frac{m_2(M_2+m_2)l_{g2}^2}{M_2} & J_{z2} + \frac{m_2(M_2+m_2)l_{g2}^2}{M_2} \end{bmatrix} \begin{bmatrix} \ddot{q}_{n,1} \\ \ddot{q}_{n,2} \end{bmatrix} = \begin{bmatrix} u_{n,1} \\ u_{n,2} \end{bmatrix} \tag{64}$$

- 5R balanceado:

$$
\begin{cases}
\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\mathsf{c}_{1,1+2}}{l_1\mathsf{s}_{1,2}} & \frac{\mathsf{s}_{1,1+2}}{l_1\mathsf{s}_{1,2}} \\ -\frac{l_1\mathsf{c}_{1,1}+l_2\mathsf{c}_{1,1+2}}{l_1l_2\mathsf{s}_{1,2}} & -\frac{l_1\mathsf{s}_{1,1}+l_2\mathsf{s}_{1,1+2}}{l_1l_2\mathsf{s}_{1,2}} \\ \frac{\mathsf{c}_{2,1+2}}{l_1\mathsf{s}_{2,2}} & \frac{\mathsf{s}_{2,1+2}}{l_1\mathsf{s}_{2,2}} \\ -\frac{l_1\mathsf{c}_{2,1}+l_2\mathsf{c}_{2,1+2}}{l_1l_2\mathsf{s}_{2,2}} & -\frac{l_1\mathsf{s}_{2,1}+l_2\mathsf{s}_{2,1+2}}{l_1l_2\mathsf{s}_{2,2}} \end{bmatrix}^T
\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbb{M}^{\#}_{1,1} & \mathbb{M}^{\#}_{1,2} & 0 & 0 \\ 0 & 0 & \mathbb{M}^{\#}_{1,2} & \mathbb{M}^{\#}_{1,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbb{M}^{\#}_{1,1} & \mathbb{M}^{\#}_{1,2} \\ 0 & 0 & 0 & 0 & \mathbb{M}^{\#}_{1,2} & \mathbb{M}^{\#}_{1,2} \end{bmatrix}
\begin{bmatrix} \ddot{q}_{0,1} \\ \ddot{q}_{0,2} \\ \ddot{q}_{1,1} \\ \ddot{q}_{1,2} \\ \ddot{q}_{2,1} \\ \ddot{q}_{2,2} \end{bmatrix}
= \begin{bmatrix} \frac{\mathsf{c}_{1,1+2}}{l_1\mathsf{s}_{1,2}} & \frac{\mathsf{s}_{1,1+2}}{l_1\mathsf{s}_{1,2}} \\ \frac{\mathsf{c}_{2,1+2}}{l_1\mathsf{s}_{2,2}} & \frac{\mathsf{s}_{2,1+2}}{l_1\mathsf{s}_{2,2}} \end{bmatrix}
\begin{bmatrix} u_{1,1} \\ u_{2,1} \end{bmatrix} \\
\begin{bmatrix} 1 & 0 & l_1\mathsf{s}_{1,1}+l_2\mathsf{s}_{1,1+2} & l_2\mathsf{s}_{1,1+2} & 0 & 0 \\ 0 & 1 & -l_1\mathsf{c}_{1,1}-l_2\mathsf{c}_{1,1+2} & -l_2\mathsf{c}_{1,1+2} & 0 & 0 \\ 1 & 0 & 0 & 0 & l_1\mathsf{s}_{2,1}+l_2\mathsf{s}_{2,1+2} & l_2\mathsf{s}_{2,1+2} \\ 0 & 1 & 0 & 0 & -l_1\mathsf{c}_{2,1}-l_2\mathsf{c}_{2,1+2} & -l_2\mathsf{c}_{2,1+2} \end{bmatrix}
\begin{bmatrix} \ddot{q}_{0,1} \\ \ddot{q}_{0,2} \\ \ddot{q}_{1,1} \\ \ddot{q}_{1,2} \\ \ddot{q}_{2,1} \\ \ddot{q}_{2,2} \end{bmatrix} = \\
-\begin{bmatrix} l_1\mathsf{c}_{1,1}\dot{q}_{1,1}^2 + l_2\mathsf{c}_{1,1+2}(\dot{q}_{1,1}+\dot{q}_{1,2})^2 \\ l_1\mathsf{s}_{1,1}\dot{q}_{1,1}^2 + l_2\mathsf{s}_{1,1+2}(\dot{q}_{1,1}+\dot{q}_{1,2})^2 \\ l_1\mathsf{c}_{2,1}\dot{q}_{2,1}^2 + l_2\mathsf{c}_{2,1+2}(\dot{q}_{2,1}+\dot{q}_{2,2})^2 \\ l_1\mathsf{s}_{2,1}\dot{q}_{2,1}^2 + l_2\mathsf{s}_{2,1+2}(\dot{q}_{2,1}+\dot{q}_{2,2})^2 \end{bmatrix}
\end{cases}
\tag{65}
$$

1.2 Control

$$\begin{bmatrix} \mathbb{C}^T\mathbb{M} \\ \mathbb{A} \end{bmatrix} \ddot{\mathbb{q}} = \begin{bmatrix} \varsigma\mathbb{u} \\ -\mathbb{b} \end{bmatrix}$$

$$\mathbb{u} = \varsigma^{-1}\mathbb{C}^T\mathbb{M}\mathbb{u}'$$

$$\mathbb{u}' = \ddot{\mathbb{q}}_d + \lambda\dot{\mathbb{e}} - k\,\mathsf{sign}(\mathbb{s})$$

$$\mathbb{s} = -\dot{\mathbb{e}} - \lambda\mathbb{e}$$

$$\dot{\mathbb{s}} = -\ddot{\mathbb{e}} - \lambda\dot{\mathbb{e}} = \ddot{\mathbb{q}} - \ddot{\mathbb{q}}_d - \lambda\dot{\mathbb{e}}$$

$$\dot{\mathbb{s}} = \begin{bmatrix} \mathbb{C}^T\mathbb{M} \\ \mathbb{A} \end{bmatrix}^{-1} \begin{bmatrix} \varsigma\mathbb{u} \\ -\mathbb{b} \end{bmatrix} - \ddot{\mathbb{q}}_d - \lambda\dot{\mathbb{e}}$$

Aplicando a lei de controle:

$$\dot{\mathbb{s}} = \begin{bmatrix} \mathbb{C}^T\mathbb{M} \\ \mathbb{A} \end{bmatrix}^{-1} \begin{bmatrix} \mathbb{C}^T\mathbb{M}(\ddot{\mathbb{q}}_d + \lambda\dot{\mathbb{e}} - k\,\mathsf{sign}(\mathbb{s})) \\ -\mathbb{b} \end{bmatrix} - \ddot{\mathbb{q}}_d - \lambda\dot{\mathbb{e}}$$

$$\dot{\mathbb{s}} = \begin{bmatrix} \mathbb{C}^T\mathbb{M} \\ \mathbb{A} \end{bmatrix}^{-1} \begin{bmatrix} \mathbb{C}^T\mathbb{M}(\ddot{\mathbb{q}}_d + \lambda\dot{\mathbb{e}} - k\,\mathsf{sign}(\mathbb{s})) - \mathbb{C}^T\mathbb{M}(\ddot{\mathbb{q}}_d + \lambda\dot{\mathbb{e}}) \\ -\mathbb{b} - \mathbb{A}(\ddot{\mathbb{q}}_d + \lambda\dot{\mathbb{e}}) \end{bmatrix}$$

$$\dot{\mathbb{s}} = \begin{bmatrix} \mathbb{C}^T\mathbb{M} \\ \mathbb{A} \end{bmatrix}^{-1} \begin{bmatrix} -\mathbb{C}^T\mathbb{M}k\,\mathsf{sign}(\mathbb{s}) \\ -\mathbb{b} - \mathbb{A}(\ddot{\mathbb{q}}_d + \lambda\dot{\mathbb{e}}) \end{bmatrix}$$

Definindo:

$$\begin{bmatrix} \mathbb{C}^T\mathbb{M} \\ \mathbb{A} \end{bmatrix}^{-1} = \begin{bmatrix} (\mathbb{C}^T\mathbb{M})^\dagger & \mathbb{A}^\dagger \end{bmatrix}$$

Temos:

$$\dot{\mathbb{s}} = -(\mathbb{C}^T\mathbb{M})^\dagger \mathbb{C}^T\mathbb{M} k \,\mathsf{sign}(\mathbb{s}) - \mathbb{A}^\dagger \mathbb{b} - \mathbb{A}^\dagger \mathbb{A}(\ddot{\mathbb{q}}_d + \lambda \dot{\mathbb{e}})$$

Sendo assim, se a seguinte inequao for respeitada para pelo menos $\nu^\#$ componentes de $\dot{\mathbb{s}}$, o erro vai a zero:

$$\dot{\mathbb{s}} = -(\mathbb{C}^T\mathbb{M})^\dagger \mathbb{C}^T\mathbb{M} k \,\mathsf{sign}(\mathbb{s}) - \mathbb{A}^\dagger \mathbb{b} - \mathbb{A}^\dagger \mathbb{A}(\ddot{\mathbb{q}}_d + \lambda \dot{\mathbb{e}}) \leq \mathbb{0}$$

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