

A new approach for obtaining the dynamic balancing conditions in serial mechanisms

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Abstract

Adaptive balancing means that the mechanical structure of the manipulator is modified in order to achieve the decoupling of dynamic equations. This work deals with a systematic formulation for the adaptive balancing. Basically, two traditional balancing techniques are employed here: the addition of counterweight and counter-rotating disks coupled to the moving links. In addition, the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators is discussed regarding the achievement of such balancing and the complexity level of the modified mechanical structure. The balancing conditions are developed here for 3-dof spatial and planar open-loop kinematic chain mechanisms, whose topologies are composed of revolute and prismatic joints.

KEYWORDS: Adaptive balancing, dynamic balancing, serial mechanisms

Nomenclature

a, b, \dots	Scalars, components of column-matrices, components of matrices or indexes
A, B, \dots	Scalars, components of column-matrices or components of matrices
$\mathbf{a}, \mathbf{b}, \dots$	Column-matrices
$\mathbb{A}, \mathbb{B}, \dots$	Matrices
$\mathbf{a}, \mathbf{b}, \dots$	Vectors
$\mathbf{A}, \mathbf{B}, \dots$	Coordinate systems
$\mathcal{A}, \mathcal{B}, \dots$	Sets or multibody mechanical systems
\mathbf{B}_i	Coordinate system fixed in the i^{th} rigid body of the mechanical system
\mathbb{C}	Kinematic constraints matrix
$c(.)$	Shorthand notation for $\cos(.)$
g	Gravitational acceleration
$\mathbf{g}^{\#}$	Generalized gravitational forces column-matrix of a serial mechanism
$\mathbf{g}_i^{\#}$	Generalized gravitational forces column-matrix of a counter-rotating disc
\mathbf{g}'	Generalized uncoupled gravitational forces column-matrix of a serial mechanism coupled with counter-rotating discs
$\mathbf{g}'^{\#}$	Generalized gravitational forces column-matrix of a serial mechanism coupled with counter-rotating discs

$J_{x_i}, J_{y_i}, J_{z_i}$	Principal moments of inertia of the i^{th} rigid body of the mechanical system
l_i	Length of the i^{th} bar of a serial mechanism
l_{g_i}	Position of the mass center of the i^{th} bar relative to the i^{th} joint and of a serial mechanism
m_i	Mass of the i^{th} rigid body of the mechanical system
$\mathbb{M}^{\#}$	Generalized inertia matrix of a serial mechanism
$\mathbb{M}_i^{\#}$	Generalized inertia matrix of a counter-rotating disc
\mathbb{M}'	Generalized uncoupled inertia matrix of a serial mechanism coupled with counter-rotating discs
$\mathbb{M}'^{\#}$	Generalized inertia matrix of a serial mechanism coupled with counter-rotating discs
\mathcal{N}	Inertial reference frame
$\mathbb{p}^{\#}$	Independent quasi-velocities column-matrix
\mathbb{p}°	Redundant quasi-velocities column-matrix
\mathbb{p}	Quasi-velocities column-matrix
q_i	Generalized coordinate
$\mathbf{q}^{\#}$	Independent generalized coordinates column-matrix
$s(\cdot)$	Shorthand notation for $\sin(\cdot)$
u_i	Effort made by the i^{th} actuator of a serial mechanism
\mathbf{u}	Generalized actuators' efforts column-matrix
$\mathbf{v}^{\#}$	Generalized coupled gyroscopic inertia forces column-matrix of a serial mechanism
$\mathbf{v}_i^{\#}$	Generalized coupled gyroscopic inertia forces column-matrix of a counter-rotating disc
\mathbf{v}'	Generalized uncoupled gyroscopic inertia forces column-matrix of a serial mechanism coupled with counter-rotating discs
$\mathbf{v}'^{\#}$	Generalized coupled gyroscopic inertia forces column-matrix of a of a serial mechanism coupled with counter-rotating discs
$[\boldsymbol{\omega}_i]_{\mathcal{B}_j}$	Angular velocity of the i^{th} rigid body of the mechanical system measured relatively to a inertial reference frame \mathcal{N} , written in the basis of \mathcal{B}_j
ω_{x_i}	1 st component of $[\boldsymbol{\omega}_i]_{\mathcal{B}_i}$
ω_{y_i}	2 nd component of $[\boldsymbol{\omega}_i]_{\mathcal{B}_i}$
ω_{z_i}	3 rd component of $[\boldsymbol{\omega}_i]_{\mathcal{B}_i}$
$\mathbf{0}$	Null column-matrix or null matrix
$\mathbf{1}$	Identity matrix
$\mathbf{1}$	Identity tensor
$[\mathbf{1}]_{\mathcal{B}_i \mathcal{B}_j}$	Change of basis matrix, i.e. $[\mathbf{v}]_{\mathcal{B}_i} = [\mathbf{1}]_{\mathcal{B}_i \mathcal{B}_j} \cdot [\mathbf{v}]_{\mathcal{B}_j}$
$[\cdot]^{\text{T}}$	Matrix transposition

1 Introduction and literature review

Balancing can be considered as an important issue related to the design of any kind of mechanical system in general, and also serial manipulators, in particular. As a matter of fact, the performance of open-loop kinematic chain mechanisms associated to specific applications depends on the choice of the balancing method, namely, either static [6] or dynamic [4], either passive [1, 4, 5, 6, 7, 8, 9, 10, 12, 13] or active [2, 3, 11, 14, 15, 16].

Moreover, Coelho et al. [15], Moradi et al. [16] and Arakelian and Sargsyan [17] use the adaptive balancing to achieve the decoupling of dynamic equations for open-loop kinematic chain mechanisms. Consequently, this action simplifies the control of manipulators due to the fact that the actuators can be controlled independently. The necessary modifications comprise the addition of either counterweights, or counter-rotating disks or even both to the original kinematic chain of the manipulator. Consequently, the terms associated to gravitational, centripetal and Coriolis efforts are completely eliminated from the dynamic equations. As a matter of fact, the effective inertias for all the actuator axes are constant and the mathematical expressions of the driving torques/forces become rather simple. One of the main advantages of this approach concerns the reduction of computing time for a closed-loop control of manipulators. Such reduction is really significant and it constitutes in a great benefit for real-time applications.

The contributions of this work are the following: to present a systematic formulation to obtain the balancing conditions for the adaptive balancing, to discuss the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators, not only in terms of the possibility to achieve such balancing but also regarding the increase of the complexity level of the modified mechanical structure. The analysed manipulators correspond to 3-dof spatial and planar open-loop kinematic chain, whose topologies are composed of revolute and prismatic joints.

This work is organized as follows. Section 2 describes the proposed formulation, while section 3 deals with the application of the formulation to 3 types of serial manipulators. Finally, the conclusions are drawn in section 4.

2 Formulation

2.1 Dynamic Model

The dynamic model of a serial mechanism can be written as follows:

$$\mathbb{M}^{\#}(\mathbf{q}^{\#})\ddot{\mathbf{q}}^{\#} + \mathbf{v}^{\#}(\mathbf{q}^{\#}, \dot{\mathbf{q}}^{\#}) + \mathbf{g}^{\#}(\mathbf{q}^{\#}) = \mathbf{u} \quad (1)$$

where $\mathbf{q}^{\#}$ is a column-matrix of independent generalized coordinates, whose entries are relative displacements of the joints, and \mathbf{u} is a column-matrix of the generalized actuators' efforts in the directions of the independent quasi-velocities $\mathbf{p}^{\#} = \dot{\mathbf{q}}^{\#}$.

In order to perform the dynamic balancing of a serial mechanism, it is required to obtain the dynamic model of the unbalanced mechanism. Once in a serial mechanism it is possible to express all the absolute velocities of the links' centres of mass and all the absolute angular velocities of the links as functions of $\mathbf{q}^{\#}$ and $\dot{\mathbf{q}}^{\#}$, the dynamic model can be obtained without major difficulties by using analytical mechanics techniques, like Lagrange [18] and Kane [19] formalisms, Orsino's method [24], and Boltzmann-Hamel equations [21], allied to programs or libraries of programming languages that are capable of using symbolic manipulation, such as Mathematica and SymPy.

2.2 Static Balancing

After obtaining the dynamic model, the static balancing is performed by determining the links' centres of mass positions that annul the term $\mathbf{g}^{\#}$. That can be achieved in mechanisms whose revolute joints axes are in any direction and whose prismatic joints axes are orthogonal to the gravity. The positioning of the centres of mass is done mechanically by extending the mechanism's bars and adding counterweights.

2.3 Dynamic Balancing

The dynamic balancing is performed by attaching counter-rotating disks to the statically balanced mechanism. One can take into account this structural modification in the dynamic model by using the coupling subsystems technique of Orsino's method * [23, 24].

Let \mathcal{M}_0 be a mechanical subsystem composed of a statically balanced serial mechanism, whose equation of motion is given by (1), with $\mathfrak{g}^\# = \mathbb{0}$. Let \mathcal{M}_i be a mechanical subsystem composed of a counter-rotating disk that will be coupled to the mechanism, whose equation of motion is given by:

$$\mathbb{M}_i^\# \dot{\mathfrak{p}}_i^\# + \mathfrak{v}_i^\# + \mathfrak{g}_i^\# = \mathfrak{u}_i \quad (2)$$

where $\mathfrak{p}_i^\#$ is a set of independent quasi-velocities, whose elements are non-null components of the absolute angular velocity vector of the disk, expressed in a reference frame fixed to the disk, and $\mathfrak{v}_i^\# = \mathfrak{g}_i^\# = \mathfrak{u}_i = \mathbb{0}$. In this model, only the rotational effect of the disk inertias is considered. The translational effects of the disk inertias are taken into account in the model of bar where the disk is attached.

By assuming that n counter-rotating disks will be attached to the mechanism, the following definitions are made:

$$\mathbb{M}' = \begin{bmatrix} \mathbb{M}^\# & \mathbb{0} & \dots & \mathbb{0} \\ \mathbb{0} & \mathbb{M}_1^\# & \dots & \mathbb{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{0} & \mathbb{0} & \dots & \mathbb{M}_n^\# \end{bmatrix} \quad (3)$$

$$\mathfrak{v}' = \begin{bmatrix} \mathfrak{v}^{\#T} & \mathfrak{v}_1^{\#T} & \dots & \mathfrak{v}_n^{\#T} \end{bmatrix}^T \quad (4)$$

$$\mathfrak{g}' = \begin{bmatrix} \mathfrak{g}^{\#T} & \mathfrak{g}_1^{\#T} & \dots & \mathfrak{g}_n^{\#T} \end{bmatrix}^T \quad (5)$$

$$\mathfrak{p}^\circ = \begin{bmatrix} \mathfrak{p}_1^{\#T} & \dots & \mathfrak{p}_n^{\#T} \end{bmatrix}^T \quad (6)$$

$$\mathfrak{p} = \begin{bmatrix} \mathfrak{p}^{\#T} & \mathfrak{p}^{\circ T} \end{bmatrix}^T \quad (7)$$

Let $\underline{\mathfrak{p}}^\circ$ be the vector \mathfrak{p}° expressed as function of $\mathfrak{q}^\#$ e $\mathfrak{p}^\#$, i.e.,

$$\mathfrak{p}^\circ = \underline{\mathfrak{p}}^\circ(\mathfrak{q}^\#, \mathfrak{p}^\#) \quad (8)$$

The kinematic constraints matrix is defined as

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \frac{\partial \underline{\mathfrak{p}}^\circ}{\partial \mathfrak{p}^\#} \end{bmatrix} \quad (9)$$

The dynamic model of the serial mechanism coupled with the counter-rotating disks is given by

$$\mathbb{M}'^\#(\mathfrak{q}^\#) \ddot{\mathfrak{q}}^\# + \mathfrak{v}'^\#(\mathfrak{q}^\#, \dot{\mathfrak{q}}^\#) + \mathfrak{g}'^\#(\mathfrak{q}^\#) = \mathfrak{u} \quad (10)$$

where

$$\mathbb{M}'^\# = \mathbb{C}^T \mathbb{M}' \mathbb{C} \quad (11)$$

* Similar modelling approaches are presented in the works by Cheng and Yiu [20], Altuzarra et al. [21] and Orsino et al. [22]

$$\mathbf{v}'^\# = \mathbb{C}^\top (\mathbb{M}' \dot{\mathbb{C}} \dot{\mathbf{q}}^\# + \mathbf{v}') \quad (12)$$

$$\mathbf{g}'^\# = \mathbb{C}^\top \mathbf{g}' \quad (13)$$

The dynamic balancing conditions are achieved by determining the system parameters that make $\mathbb{M}'^\#$ diagonal and $\mathbf{v}'^\#$ null.

3 Applying the technique

In this section, the proposed formulation will be applied in three different 3-dof serial mechanisms. First, some definitions, valid for these mechanisms, are required:

$$\mathbb{M}^\# = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{bmatrix} \quad (14)$$

$$\mathbf{v}^\# = \begin{bmatrix} D_{111} & D_{122} & D_{133} \\ D_{211} & D_{222} & D_{233} \\ D_{311} & D_{322} & D_{333} \end{bmatrix} \begin{bmatrix} \dot{q}_1^2 \\ \dot{q}_2^2 \\ \dot{q}_3^2 \end{bmatrix} + 2 \begin{bmatrix} D_{112} & D_{113} & D_{123} \\ D_{212} & D_{213} & D_{223} \\ D_{312} & D_{313} & D_{323} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \dot{q}_2 \\ \dot{q}_1 \dot{q}_3 \\ \dot{q}_2 \dot{q}_3 \end{bmatrix} \quad (15)$$

$$\mathbf{g}^\# = [D_1 \quad D_2 \quad D_3]^\top \quad (16)$$

$$\mathbf{q}^\# = [q_1 \quad q_2 \quad q_3]^\top \quad (17)$$

$$\mathbf{u} = [u_1 \quad u_2 \quad u_3]^\top \quad (18)$$

In order to employ the traditional notation, $q_i = \theta_i$ and $u_i = \tau_i$ for revolute joints, and $q_i = d_i$ and $u_i = f_i$ for prismatic joints.

The Denavit-Hartenberg convention is used here not only to define the coordinate system in each link, but also to enumerate the links and joints of the mechanism.

3.1 3-dof RRR planar serial mechanism

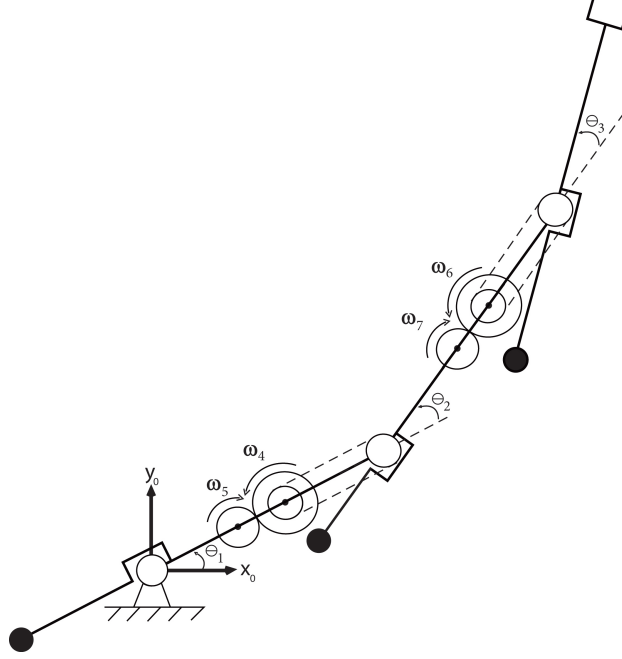


Figure 1: Dynamically balanced RRR planar serial mechanism

The entries of $\mathfrak{g}^\#$ for the unbalanced mechanism are given by

$$\begin{cases} D_1 = g[(m_1 l_{g1} + m_2 l_1 + m_3 l_1) c(\theta_1) + (m_2 l_{g2} + m_3 l_2) c(\theta_1 + \theta_2) + m_3 l_{g3} c(\theta_1 + \theta_2 + \theta_3)] \\ D_2 = g[(m_2 l_{g2} + m_3 l_2) c(\theta_1 + \theta_2) + m_3 l_{g3} c(\theta_1 + \theta_2 + \theta_3)] \\ D_3 = g[m_3 l_{g3} c(\theta_1 + \theta_2 + \theta_3)] \end{cases} \quad (19)$$

By performing the static balancing,

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g1} = -\frac{l_1(m_2+m_3)}{m_1} \\ l_{g2} = -\frac{l_2 m_3}{m_2} \\ l_{g3} = 0 \end{cases} \quad (20)$$

Substituting (20) in the mechanism model, the terms of the dynamic model of the statically balanced mechanism are obtained

$$\begin{cases} D_{11} = J_{z_1} + J_{z_2} + J_{z_3} + m_2 l_1^2 + m_3 (l_1^2 + l_2^2) + \frac{l_1^2 (m_2+m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ D_{22} = J_{z_2} + J_{z_3} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{z_3} \\ D_{12} = D_{22} \\ D_{13} = D_{23} = D_{33} \\ \mathfrak{v}^\# = \mathbb{0} \\ \mathfrak{g}^\# = \mathbb{0} \end{cases} \quad (21)$$

To perform the dynamic balancing, four counter-rotating disks are coupled to the mechanism, as shown in figure 1. Once the disks rotate in a single plane, their corresponding dynamic models are as follows

$$\mathbb{M}_i^\# = [J_{z_{i+3}}]; \quad \mathbb{P}_i^\# = [\omega_{z_{i+3}}], \quad i = 1, 2, 3, 4 \quad (22)$$

The counter-rotating disks 1 and 2 (rigid bodies 4 and 5) are coupled to link 1. The angular displacement of disk 1 with respect to link 1 is θ_2 , due to the belt transmission of the motor 2 motion, while the angular displacement of disk 2 with respect to link 1 is $\beta\theta_2$, with $\beta < 0$, due to the gear transmission of disk 1 motion.

The counter-rotating disks 3 and 4 (rigid bodies 6 and 7) are coupled to link 2. The angular displacement of disk 3 with respect to link 2 is θ_3 , due to the belt transmission of the motor 3 motion, while the angular displacement of disk 4 with respect to link 2 is $\gamma\theta_3$, with $\gamma < 0$, due to the gear transmission of disk 3 motion.

Thus, the following quasi-velocities constraints are obtained:

$$\begin{cases} \omega_{z_4} = \omega_{z_1} + \dot{\theta}_2 \\ \omega_{z_5} = \omega_{z_1} + \beta\dot{\theta}_2 \\ \omega_{z_6} = \omega_{z_2} + \dot{\theta}_3 \\ \omega_{z_7} = \omega_{z_2} + \gamma\dot{\theta}_3 \end{cases} \Rightarrow \begin{cases} \omega_{z_4} = \dot{\theta}_1 + \dot{\theta}_2 \\ \omega_{z_5} = \dot{\theta}_1 + \beta\dot{\theta}_2 \\ \omega_{z_6} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \\ \omega_{z_7} = \dot{\theta}_1 + \dot{\theta}_2 + \gamma\dot{\theta}_3 \end{cases} \Rightarrow \underline{\mathbb{P}}^\circ = \begin{bmatrix} \dot{\theta}_1 + \dot{\theta}_2 \\ \dot{\theta}_1 + \beta\dot{\theta}_2 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \\ \dot{\theta}_1 + \dot{\theta}_2 + \gamma\dot{\theta}_3 \end{bmatrix} \quad (23)$$

$$\therefore \mathbb{C} = \begin{bmatrix} \underline{\mathbb{1}} \\ \frac{\partial \underline{\mathbb{P}}^\circ}{\partial \underline{\mathbb{P}}^\#} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \\ 1 & 1 & 1 \\ 1 & 1 & \gamma \end{bmatrix} \quad (24)$$

By applying (21), (22) and (24) in (11), (12) and (13), the mechanism's statically balanced model coupled with the counter-rotating disks is obtained:

$$\begin{cases} D'_{11} = D_{11} + J_{z_4} + J_{z_5} + J_{z_6} + J_{z_7} \\ D'_{22} = D_{22} + J_{z_4} + J_{z_5}\beta^2 + J_{z_6} + J_{z_7} \\ D'_{33} = D_{33} + J_{z_6} + J_{z_7}\gamma^2 \\ D'_{12} = D_{12} + J_{z_4} + J_{z_5}\beta + J_{z_6} + J_{z_7} \\ D'_{13} = D_{13} + J_{z_6} + J_{z_7}\gamma \\ D'_{23} = D'_{13} \\ \mathbb{V}^\# = \underline{\mathbb{0}} \end{cases} \quad (25)$$

To perform the dynamic balancing, the values of β and γ as functions of the mechanism's parameters that makes $\mathbb{M}^\#$ diagonal are found. Thus,

$$\begin{cases} D'_{12} = 0 \\ D'_{13} = 0 \end{cases} \Rightarrow \begin{cases} \beta = -\frac{J_{z_2} + J_{z_3} + J_{z_4} + J_{z_6} + J_{z_7} + m_3 l_2^2 + \frac{m_3^2 l_2^2}{m_2}}{J_{z_5}} \\ \gamma = -\frac{J_{z_3} + J_{z_6}}{J_{z_7}} \end{cases} \quad (26)$$

By applying (26) in (25), the mechanism's dynamic balanced model is as follows

$$\begin{cases} \tau_1 = k_1 \ddot{\theta}_1 \\ \tau_2 = k_2 \ddot{\theta}_2 \\ \tau_3 = k_3 \ddot{\theta}_3 \end{cases} \quad (27)$$

where

$$\begin{cases} k_1 = J_{z_1} + J_{z_2} + J_{z_3} + J_{z_4} + J_{z_5} + J_{z_6} + J_{z_7} + m_2 l_1^2 + m_3 (l_1^2 + l_2^2) + \frac{l_1^2 (m_2 + m_3)^2}{m_1} + \frac{l_2^2 m_3^2}{m_2} \\ k_2 = J_{z_2} + J_{z_3} + J_{z_4} + J_{z_6} + J_{z_7} + m_3 l_2^2 + \frac{l_2^2 m_3^2}{m_2} + \frac{(J_{z_2} + J_{z_3} + J_{z_4} + J_{z_6} + J_{z_7} + m_3 l_2^2 + \frac{m_3^2 l_2^2}{m_2})^2}{J_{z_5}} \\ k_3 = \frac{(J_{z_3} + J_{z_6})(J_{z_3} + J_{z_6} + J_{z_7})}{J_{z_7}} \end{cases} \quad (28)$$

3.2 3-dof RRR spatial serial mechanism

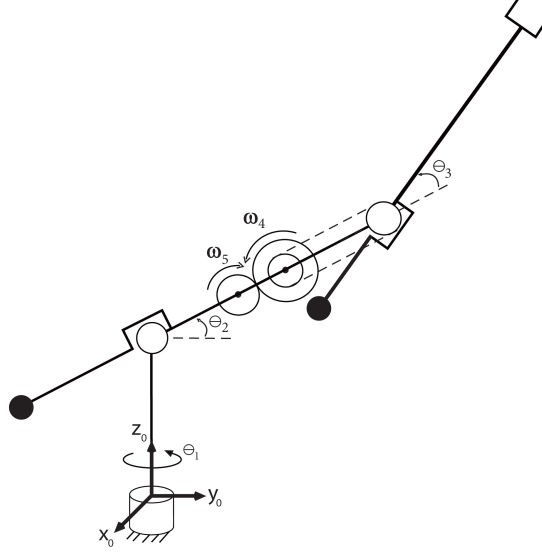


Figure 2: Dynamically balanced RRR spatial serial mechanism

The entries of $\mathfrak{g}^\#$ for the unbalanced mechanism are given by

$$\begin{cases} D_1 = 0 \\ D_2 = g[(m_2 l_{g2} + m_3 l_2) \cos(\theta_2) + m_3 l_{g3} \cos(\theta_2 + \theta_3)] \\ D_3 = g[m_3 l_{g3} \cos(\theta_2 + \theta_3)] \end{cases} \quad (29)$$

By performing the static balancing,

$$\begin{cases} D_2 = 0 \\ D_3 = 0 \end{cases} \Rightarrow \begin{cases} l_{g2} = -\frac{l_2 m_3}{m_2} \\ l_{g3} = 0 \end{cases} \quad (30)$$

Substituting (30) in the mechanism model, the terms of the dynamic model of the statically balanced mechanism are obtained

$$\begin{cases} D_{11} = J_{x_2} s^2(\theta_2) + J_{x_3} s^2(\theta_2 + \theta_3) + J_{z_1} + J_{y_2} c^2(\theta_2) + J_{y_3} c^2(\theta_2 + \theta_3) + m_3 (l_1 + l_2 c(\theta_2))^2 + \frac{(m_2 l_1 - m_3 l_2 c(\theta_2))^2}{m_2} \\ D_{22} = J_{z_2} + J_{z_3} + m_2 l_2^2 + \frac{l_2^2 m_3^2}{m_2} \\ D_{33} = J_{z_3} \\ D_{12} = D_{13} = 0 \\ D_{23} = D_{33} \\ D_{211} = -\frac{1}{2} \left((J_{x_2} - J_{y_2}) s(2\theta_2) + (J_{x_3} - J_{y_3} - m_3 l_2^2 (1 + \frac{m_3}{m_2})) s(2\theta_2 + 2\theta_3) \right) \\ D_{311} = \frac{1}{2} \left((J_{y_3} - J_{x_3}) s(2\theta_2 + 2\theta_3) \right) \\ D_{111} = D_{122} = D_{133} = D_{222} = D_{233} = D_{322} = D_{333} = 0 \\ D_{112} = -D_{211} \\ D_{113} = -D_{311} \\ D_{123} = D_{212} = D_{213} = D_{223} = D_{312} = D_{313} = D_{323} = 0 \\ \mathfrak{g}^\# = 0 \end{cases} \quad (31)$$

To perform the dynamic balancing, two counter-rotating disks are coupled to the mechanism, as shown in figure 2. Their respective dynamic models are as follows

$$\mathbb{M}_i^\# = \begin{bmatrix} J_{x_{i+3}} & 0 & 0 \\ 0 & J_{y_{i+3}} & 0 \\ 0 & 0 & J_{z_{i+3}} \end{bmatrix}; \quad \mathbb{p}_i^\# = \begin{bmatrix} \omega_{x_{i+3}} \\ \omega_{y_{i+3}} \\ \omega_{z_{i+3}} \end{bmatrix}, \quad i = 1, 2 \quad (32)$$

The counter-rotating disks 1 and 2 (rigid bodies 4 and 5) are coupled to link 2. The angular displacement of disk 1 with respect to link 2 is θ_3 , due to the belt transmission of the motor 3 motion, while the angular displacement of disk 2 with respect to link 2 is $\beta\theta_3$, with $\beta < 0$, due to the gear transmission of disk 1 motion.

Thus, the following quasi-velocities constraints are obtained:

$$\begin{cases} [\boldsymbol{\omega}_4]_{B_4} = [\mathbf{1}]_{B_4|B_2} [\boldsymbol{\omega}_2]_{B_2} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_3 \end{bmatrix} \\ [\boldsymbol{\omega}_5]_{B_5} = [\mathbf{1}]_{B_5|B_2} [\boldsymbol{\omega}_2]_{B_2} + \begin{bmatrix} 0 \\ 0 \\ \beta\dot{\theta}_3 \end{bmatrix} \end{cases} \Rightarrow \begin{cases} \omega_{x_4} = (\dot{\theta}_1 \mathbf{s}(\theta_2)) \mathbf{c}(\theta_3) + (\dot{\theta}_1 \mathbf{c}(\theta_2)) \mathbf{s}(\theta_3) \\ \omega_{y_4} = -(\dot{\theta}_1 \mathbf{s}(\theta_2)) \mathbf{s}(\theta_3) + (\dot{\theta}_1 \mathbf{c}(\theta_2)) \mathbf{c}(\theta_3) \\ \omega_{z_4} = \dot{\theta}_2 + \dot{\theta}_3 \\ \omega_{x_5} = (\dot{\theta}_1 \mathbf{s}(\theta_2)) \mathbf{c}(\beta\theta_3) + (\dot{\theta}_1 \mathbf{c}(\theta_2)) \mathbf{s}(\beta\theta_3) \\ \omega_{y_5} = -(\dot{\theta}_1 \mathbf{s}(\theta_2)) \mathbf{s}(\beta\theta_3) + (\dot{\theta}_1 \mathbf{c}(\theta_2)) \mathbf{c}(\beta\theta_3) \\ \omega_{z_5} = \dot{\theta}_2 + \beta\dot{\theta}_3 \end{cases} \quad (33)$$

$$\Rightarrow \underline{\mathbb{p}}^\circ = \begin{bmatrix} \dot{\theta}_1 \mathbf{s}(\theta_2 + \theta_3) \\ \dot{\theta}_1 \mathbf{c}(\theta_2 + \theta_3) \\ \dot{\theta}_2 + \dot{\theta}_3 \\ \dot{\theta}_1 \mathbf{s}(\theta_2 + \beta\theta_3) \\ \dot{\theta}_1 \mathbf{c}(\theta_2 + \beta\theta_3) \\ \dot{\theta}_2 + \beta\dot{\theta}_3 \end{bmatrix} \quad (34)$$

$$\mathbb{C} = \begin{bmatrix} \mathbf{1} \\ \frac{\partial \underline{\mathbb{p}}^\circ}{\partial \mathbb{p}^\#} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \mathbf{s}(\theta_2 + \theta_3) & 0 & 0 \\ \mathbf{c}(\theta_2 + \theta_3) & 0 & 0 \\ 0 & 1 & 1 \\ \mathbf{s}(\theta_2 + \beta\theta_3) & 0 & 0 \\ \mathbf{c}(\theta_2 + \beta\theta_3) & 0 & 0 \\ 0 & 1 & \beta \end{bmatrix} \quad (35)$$

By applying (31), (32) and (35) in (11), (12) and (13), the mechanism's statically balanced model coupled with the counter-rotating disks is obtained:

$$\begin{cases} D'_{11} = D_{11} + J_{x_4} \mathbf{s}^2(\theta_2 + \theta_2) + J_{x_5} \mathbf{s}^2(\beta\theta_2 + \theta_2) + J_{y_4} \mathbf{c}^2(\theta_2 + \theta_2) + J_{y_5} \mathbf{c}^2(\beta\theta_2 + \theta_2) \\ D'_{22} = D_{22} + J_{z_4} + J_{z_5} \\ D'_{33} = D_{33} + J_{z_4} + J_{z_5} \beta^2 \\ D'_{12} = D'_{13} = 0 \\ D'_{23} = D_{23} + J_{z_4} + J_{z_5} \beta \\ D'_{211} = D_{211} \\ D'_{311} = D_{311} \\ D'_{111} = D'_{122} = D'_{133} = D'_{222} = D'_{233} = D'_{322} = D'_{333} = 0 \\ D'_{112} = D_{112} + \frac{1}{4} \left((J_{x_4} - J_{y_4}) \mathbf{s}(2\theta_2 + 2\theta_3) + (J_{x_5} - J_{y_5}) \mathbf{s}(2\beta\theta_2 + 2\theta_3) \right) \\ D'_{113} = D_{113} + \frac{1}{4} \left((J_{x_4} - J_{y_4}) \mathbf{s}(2\theta_2 + 2\theta_3) + (J_{x_5} - J_{y_5}) \mathbf{s}(2\beta\theta_2 + 2\theta_3) \right) \\ D'_{123} = D'_{212} = D'_{213} = D'_{223} = D'_{312} = D'_{313} = D'_{323} = 0 \end{cases} \quad (36)$$

To perform the dynamic balancing, $\mathbb{M}^\#$ becomes diagonal for a specific value of β and $\mathbb{V}^\#$ is null for a given set of the mechanism's parameters. Thus,

$$\begin{cases} D'_{23} = 0 \\ D'_{211} = 0 \\ D'_{311} = 0 \\ D'_{112} = 0 \\ D'_{113} = 0 \end{cases} \Rightarrow \begin{cases} \beta = -\frac{J_{z3}+J_{z4}}{J_{z5}} \\ J_{x2} = J_{y2} \\ J_{x3} = J_{y3} + m_3 l_2^2 (1 + \frac{m_3}{m_2}) \\ J_{x4} = J_{y4} \\ J_{x5} = J_{y5} \end{cases} \quad (37)$$

By applying (37) in (36), the mechanisms dynamic balanced model is as follows

$$\begin{cases} \tau_1 = k_1 \ddot{\theta}_1 \\ \tau_2 = k_2 \ddot{\theta}_2 \\ f_3 = k_3 \ddot{d}_3 \end{cases} \quad (38)$$

where

$$\begin{cases} k_1 = J_{z1} + J_{y2} + J_{y3} + J_{y4} + J_{y5} + m_2 l_1^2 + m_3 (l_1^2 + l_2^2) + \frac{l_1^2 m_3^2}{m_2} \\ k_2 = J_{z2} + J_{z3} + J_{z4} + J_{z5} + m_3 l_2^2 + \frac{l_1^2 m_3^2}{m_2} \\ k_3 = \frac{(J_{z3}+J_{z4})(J_{z3}+J_{z4}+J_{z5})}{J_{z5}} \end{cases} \quad (39)$$

Note that the necessary conditions for the dynamic balancing of this mechanism require very high longitudinal moments of inertia for the bars 2 and 3, which is not practically feasible for an industrial manipulator. Among the five conditions of (37), let us consider the following two:

$$\begin{cases} J_{x2} = J_{y2} \\ J_{x3} = J_{y3} + m_3 l_2^2 (1 + \frac{m_3}{m_2}) \end{cases} \quad (40)$$

According to Denavit-Hartenberg convention, the x-axis is the longitudinal direction while the y-axis and the z-axis are the corresponding transversal directions, for the bars 2 and 3. In this case, typically, the J_x moment of inertia is quite low in a comparison with J_y and J_z . In order to satisfy the balancing conditions, the values of J_x should be equal to J_y for bar 2 and higher than J_y for bar 3. Consequently, the link section should be increased according, which would lead to an inconvenient extremely large cross section area.

In the following example, which corresponds to another 3-dof spatial serial mechanism, such inconvenience will not occur.

3.3 3-dof RRP spatial serial mechanism (SCARA)

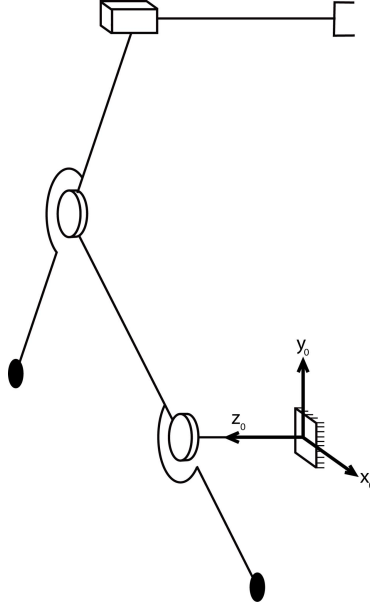


Figure 3: Statically balanced RRP spatial serial mechanism

The entries of $\mathfrak{g}^\#$ for the unbalanced mechanism are given by

$$\begin{cases} D_1 = g[(m_1 l_{g1} + m_2 l_1 + m_3 l_1) \cos(\theta_1) + m_2 l_{g2} \cos(\theta_1 + \theta_2)] \\ D_2 = g[m_2 l_{g2} \cos(\theta_1 + \theta_2)] \\ D_3 = 0 \end{cases} \quad (41)$$

By performing the static balancing,

$$\begin{cases} D_1 = 0 \\ D_2 = 0 \end{cases} \Rightarrow \begin{cases} l_{g1} = -\frac{l_1(m_2 + m_3)}{m_1} \\ l_{g2} = 0 \end{cases} \quad (42)$$

Substituting (42) in the mechanism model, the terms of the dynamic model of the statically balanced mechanism are obtained

$$\begin{cases} D_{11} = J_{z_1} + J_{z_2} + J_{z_3} + m_2 l_1^2 + m_3 l_1^2 + \frac{l_1^2(m_2 + m_3)^2}{m_1} \\ D_{22} = J_{z_2} + J_{z_3} \\ D_{33} = m_3 \\ D_{12} = D_{22} \\ D_{13} = D_{23} = 0 \\ \mathfrak{v}^\# = \mathbb{0} \\ \mathfrak{g}^\# = \mathbb{0} \end{cases} \quad (43)$$

To perform the dynamic balancing, two counter-rotating disks are coupled to the mechanism. Once the disks rotate in a single plane, their corresponding dynamic models are as follows

$$\mathbb{M}_i^\# = [J_{z_{i+3}}]; \quad \mathbb{P}_i^\# = [\omega_{z_{i+3}}], \quad i = 1, 2 \quad (44)$$

The counter-rotating disks 1 and 2 (rigid bodies 4 and 5) are coupled to link 1. The angular displacement of disk 1 with respect to link 1 is θ_2 , due to the belt transmission of the motor 2 motion, while the angular

displacement of disk 2 with respect to link 1 is $\beta\theta_2$, with $\beta < 0$, due to the gear transmission of disk 1 motion.

Thus, the following quasi-velocities constraints are obtained:

$$\begin{cases} \omega_{z_4} = \omega_{z_1} + \dot{\theta}_2 \\ \omega_{z_5} = \omega_{z_1} + \beta\dot{\theta}_2 \end{cases} \Rightarrow \begin{cases} \omega_{z_4} = \dot{\theta}_1 + \dot{\theta}_2 \\ \omega_{z_5} = \dot{\theta}_1 + \beta\dot{\theta}_2 \end{cases} \Rightarrow \underline{\mathbb{P}}^\circ = \begin{bmatrix} \dot{\theta}_1 + \dot{\theta}_2 \\ \dot{\theta}_1 + \beta\dot{\theta}_2 \end{bmatrix} \quad (45)$$

$$\mathbb{C} = \begin{bmatrix} \mathbb{1} \\ \frac{\partial \underline{\mathbb{P}}^\circ}{\partial \underline{\mathbb{P}}^\#} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & \beta & 0 \end{bmatrix} \quad (46)$$

By applying (43), (44) and (46) in (11), (12) and (13), the mechanism's statically balanced model coupled with the counter-rotating disks is obtained:

$$\begin{cases} D'_{11} = D_{11} + J_{z_4} + J_{z_5} \\ D'_{22} = D_{22} + J_{z_4} + J_{z_5}\beta^2 \\ D'_{33} = D_{33} \\ D'_{12} = D_{12} + J_{z_4} + J_{z_5}\beta \\ D'_{13} = 0 \\ D'_{23} = 0 \\ \mathbb{V}'^\# = 0 \end{cases} \quad (47)$$

To perform the dynamic balancing, the values of β as function of the mechanism's parameters that makes $\mathbb{M}'^\#$ diagonal are found. Thus,

$$D'_{12} = 0 \Rightarrow \beta = -\frac{J_{z_2} + J_{z_3} + J_{z_4}}{J_{z_5}} \quad (48)$$

By applying (48) in (47), the mechanisms dynamic balanced model is as follows

$$\begin{cases} \tau_1 = k_1\ddot{\theta}_1 \\ \tau_2 = k_2\ddot{\theta}_2 \\ f_3 = k_3\ddot{d}_3 \end{cases} \quad (49)$$

where

$$\begin{cases} k_1 = J_{z_1} + J_{z_2} + J_{z_3} + J_{z_4} + J_{z_5} + m_2l_1^2 + m_3l_1^2 + \frac{l_1^2(m_2+m_3)^2}{m_1} \\ k_2 = \frac{(J_{z_2}+J_{z_3}+J_{z_4})(J_{z_2}+J_{z_3}+J_{z_4}+J_{z_5})}{J_{z_5}} \\ k_3 = m_3 \end{cases} \quad (50)$$

Note that the necessary conditions for the dynamic balancing of this mechanism does not require restrictions on the mechanism inertia parameters, as in the previous example. Thus, as the mechanism in the previous example and the mechanism of this example are both spatial serial mechanisms that perform end-effector translations in 3 axes, it can be said that the mechanism in question is a good alternative to section 3.2 mechanism in applications in which the dynamic balancing is advantageous to the system.

4 Conclusions

This work dealt with a systematic formulation for the adaptive balancing. This special formulation uses the dynamic coupling between subsystems in order to derive the equations of motion of the whole system in an explicit form. Consequently, this feature allows the automatic generation of the adaptive balancing conditions. Two traditional balancing techniques were employed here: the addition of counterweight and counter-rotating disks coupled to the moving links. In addition, the feasibility of the dynamic decoupling for 3 distinct types of serial manipulators was discussed regarding the achievement of such balancing and the complexity level of the modified mechanical structure. The balancing conditions were developed for 3-dof spatial and planar open-loop kinematic chain mechanisms, whose topologies are composed of revolute and prismatic joints. By analysing the necessary conditions, one can notice that the adaptive balancing brings great benefits for the planar RRR and the spatial RRP. However, for the spatial RRR, in spite of the achievement of the adaptive balancing, the modifications in the mechanical structure require very high longitudinal moments of inertia for the second and third bars of the mechanism, which would lead to bars with extremely large cross section areas. Consequently, the authors believe that the discussion provided here might help the designer to choose an adequate topology for a specific application taking advantage of the adaptive balancing whenever it brings no further consequences in terms of the added inertias.

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