LM-Infinite: Simple On-the-Fly Length Generalization for Large Language Models

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ABSTRACT

In recent years, there have been remarkable advancements in the performance of Transformer-based Large Language Models (LLMs) across various domains. As these LLMs are deployed for increasingly complex tasks, they often face the needs to conduct longer reasoning processes or understanding larger contexts. In these situations, the length generalization failure of LLMs on long sequences become more prominent. Most pre-training schemes truncate training sequences to a fixed length (such as 2048 for LLaMa). LLMs often struggle to generate fluent texts, let alone carry out downstream tasks, after longer contexts, even with relative positional encoding which is designed to cope with this problem. Common solutions such as finetuning on longer corpora often involves daunting hardware and time costs and requires careful training process design. To more efficiently leverage the generation capacity of existing LLMs, we theoretically and empirically investigate the main out-of-distribution (OOD) factors contributing to this problem. Inspired by this diagnosis, we propose a simple yet effective solution for on-the-fly length generalization, LM-Infinite, which involves only a Λ -shaped attention mask and a distance limit while requiring no parameter updates or learning. We find it applicable to a variety of LLMs using relative-position encoding methods. LM-Infinite is computational efficient with O(n) time and space, and demonstrates consistent fluency and generation quality to as long as 32k tokens on ArXiv and OpenWebText2 datasets, with 2.72x decoding speedup. On downstream task such as passkey retrieval, it continues to work on inputs much longer than training lengths where vanilla models fail immediately.

1 Introduction

The evolution of Natural Language Generation (NLG) in recent years has been significantly driven by the progress of Large Language Models (LLMs) (Wei et al., 2022a; Kojima et al.; Wei et al., 2022b; Brown et al., 2020; Li et al., 2023b). LLMs have been successfully applied to a wide variety of tasks, demonstrating an impressive ability to understand and generate natural language across different contexts.

However, as LLMs are deployed for more complex tasks such as Document Understanding, Information Extraction and Cross-document Question Answering, they often face the challenge of conducting longer reasoning processes or handling larger volumes of information. This is often reflected in long text sequences, exceeding the typical length in pre-training. However, despite extensive explorations in smaller-scale models (Press et al., 2021; Sun et al., 2022; Chi et al., 2023), current SoTA LLMs still struggle to directly generalize to sequences of unseen lengths. When forced to generate after too long contexts, they either compromise the generation fluency. This challenge is known as *length generalization failures* on LLMs. In most pre-training schemes, to control the exploding time and economic costs with long text lengths, practitioners have to bound training sequences, such as 2048 tokens for LLaMA for 4096 for LLaMA-2. When there is a gap between training and inference lengths, LLMs fail to recognize the input and start to generate gibberish,

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despite the use of advanced techniques such as relative position encoding which were proposed to deal with this problem. Among the numerous relative position encoding techniques (RoPE(Su et al., 2021), NoPE(Razeghi et al., 2022), Alibi Press et al. (2021), XPos Sun et al. (2022)), RoPE and Alibi have been widely adopted by state-of-the-art LLMs including LLaMA series Touvron et al. (2023), MPT series (Team, 2023) and GPT-J series Wang & Komatsuzaki (2021). The main idea behind relative position encoding is that, instead of using absolute position information of tokens, the attention weight between two tokens rely on their distance in sequence. These designs are theoretically capable of running on unseen lengths, but on LLMs we still observe generalization failures or NaN values on inputs longer than training time (kaiokendev, 2023) (see also Sections 3 and 5).

In this paper, we undertake an empirical investigation of the main factors contributing to this generalization failure problem. In Section 3 through theoretical and empirical analysis, we identify 3 out-of-distribution (OOD) factors: unseen distances, unseen number of tokens under attention, and implicitly encoded positional information. Building upon these findings, we propose LM-Infinite, a surprisingly simple yet efficient solution compatible with a range of LLMs that use relative-position encodings. LM-Infinite introduces two innovative elements: a Λ -shaped attention mask and bounded distances during attention. As important advantages, it does not require any parameter updates for pre-trained LLMs, and only involves O(n) computational complexity. On length of 32k LM-Infinite provides a 3.16x speedup when encoding and 2.72x speedup when decoding.

Empirically, LM-Infinite demonstrates generalizability to sequences of much longer sequences, capable of maintaining consistent fluency and generation quality on documents as long as 32k in ArXiv and OpenWebText2 for a wide range of SoTA LLMs: LLaMA, LLaMA2, MPT-7B and GPT-J. It achieves performance superior or comparable to LLMs explicitly fine-tuned on long sequences, despite requiring no extra learning or parameter updates. In summary, our contributions in this work include:

- We analyse a behavioral model of LLMs regarding longer sequence through theoretical and empirical diagonsis, and explaining multiple factors which contributes to LLMs' generalization failures.
- We propose a simple on-the-fly decoding method, LM-Infinite, which brings computational efficiency as well as generalizability to unseen lengths. This saves researchers from the cost of fine-tuning or even training from scratch.
- We conduct experimental evaluation of LM-Infinite on downstream tasks. LLMs' the fluency and generation quality are consistently maintained over 32k-length sequences on ArXiv dataset, much longer than training time.

2 RELATED WORK

2.1 Positional Encodings in Transformers

Since the advent of the Transformer (Vaswani et al., 2017), it along with its variants (generally named as Transformers) has become the most widely used architecture of modern LLMs, thanks to its performance and ability for parallel training. As the attention mechanism (the core component in Transformers) operates on a bag of token features regardless of their positions, Transformers usually rely on explicit designs to incorporate position information. These designs are called *positional* encodings, and can generally be categorized into two classes. The absolute positional encodings are those providing the absolute positions, usually with the help of a sequence of vectors called position embeddings. Examples of such include sinusoidal position embeddings added to the input token embeddings (Vaswani et al., 2017), or learned position embeddings in BERT (Kenton & Toutanova, 2019), or adding the dot product between two tokens' position embeddings on the attention logit (Ke et al., 2020). Recently, to overcome its drawback that Transformers become unfamiliar with unseen positions, relative positional encodings are proposed to instead use distance information between tokens. These information are usually applied in attention layers. Examples of such include a learnable attention logit bias as in T5 (Raffel et al., 2020), Transformer-XL Dai et al. (2019) and Sandwich (Chi et al., 2023); a fixed linear attention decay called Alibi Press et al. (2021); rotating query and key sequences based on distance such as RoPE (Su et al., 2021; Li et al., 2023a), CAPE Likhomanenko et al. (2021) and XPos (Sun et al., 2022; Ding et al., 2023). XPos and Longformer (Beltagy et al., 2020) also proposes a block-diagonal attention mask, which however relies on explicitly training to familiarize LLMs to, and is not a plug-and-play tool like ours. As an extreme example, NoPE (Kazemnejad et al., 2023) claims that Transformer can implicitly encode positional information so no positional encoding is needed. Despite theoretical promises and experimental verification on smaller scale experiments in these papers, length generalization failures are still prevalently observed when they are directly applied to large language models (kaiokendev, 2023). This gap motivates us to hypothesize that there still exists OOD factors in relative positional encoding. In Section 3 we identify such factors and demonstrate that, removing them allows relative positional encoding to have perfect length generalization on extremely long sequences.

2.2 Fine-Tuning on Longer Texts

In light of generalization failures observed in LLMs, one straightforward solution is to finetune them on longer text sequences, so that unseen positions can be exposed to LLMs for familiarity. Chen et al. (2023) interpolates positional encoding on longer sequences for finetuning. Tworkowski et al. (2023) adopts contrastive learning while finetuning on longer texts. Tao et al. (2023); Kiyono et al. (2021) uses padding and shifting for synthesizing long texts, respectively. These temporary remedies push the context length limit farther, but do not address the root causes of length generalization failures. They also require massive training resources due to the large sizes of LLMs. In contrast, our work aims at a on-the-fly solution by diagnosing the OOD factors preventing length generalization, greatly saving the resource costs.

2.3 OTHER EFFORTS TOWARDS LONG-CONTEXT LLMS

Besides directly addressing the length generalization problem, other solutions are proposed to grant LLMs with access to longer contexts without really reading them in full. For example, Recurrent-GPT (Zhou et al., 2023) prompts a LLM to recurrently generate texts, while at each iteration only read the most recent context and a summary of longer histories. Some other work introduces special mark-up tokens (Bueno et al., 2022) or landmark tokens (Mohtashami & Jaggi, 2023) that allows language models to access a subset of most informative tokens. Anil et al. (2022) proposes a prompting strategy that, when combined with pre-training and fine-tuning, is able to generalize to unseen lengths. Besides, Yang et al. (2023) uses a outliner and a controller for two-staged long story generation. Finally, augmenting LLMs with retrieval-based memories, such as in Wu et al. (2021); Guu et al. (2020); Borgeaud et al. (2022); Khandelwal et al. (2019); Kaiser et al. (2016); Yogatama et al. (2021) is another school of researches that lets LLMs only read retrieved information from a large database. These designs, however, usually need explicit finetuning, and are not directly compatible with the existing state-of-the-art LLMs. Our work, in contrast, aims at extending *existing* LLMs to longer texts on-the-fly which better leverages their impressive generalization power.

3 DIAGNOSING OOD FACTORS IN LLMS

In this section, we diagnose the out-of-distribution (OOD) factors contributing the length generalization failure. We analyze with both theoretical analysis and experimental verification.

We are mainly inspired by the hypothesis that relative positional encodings in pre-trained LLMs already capture ability of dealing with relative positions. However, when applied to longer sequences, the internal features (such as attention weights and hidden states) become "unfaimilar" to LLMs, i.e., out of the training distribution. Upon removal of these factors, we can shift internal features back to the training distribution, which are "comfort zones" to LLMs. Therefore LLMs will be able to generate with their original quality. In this section we search for such factors. The intuition is to look for internal features that might be OOD and verify their existence. This analysis motivates the development of LM-Infinite in Section 4

3.1 OOD FACTOR 1: UNSEEN DISTANCES

Recall that, in relative positional encoding, the attention weight between two tokens depends on their distance. It is intuitive to realize that, if texts become too long, some distances will increase to an unseen large number, eventually exceeding those seen in pre-training. In the following we

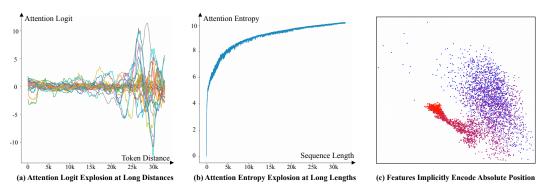


Figure 1: Diagnosis of OOD factors in LLMs.

will demonstrate formally and empirically that, as length increases, the attention logits will have to explode to infinity for the relative position encoding to recognize new distances.

Let us denote the attention function in a relative position encoding as $w(\mathbf{q}, \mathbf{k}, d) \in \mathbb{R}$. Here $w(\cdot, \cdot, \cdot)$ takes query vector \mathbf{q} , key vector \mathbf{k} and their distance d and returns a scalar as attention logit. The final attention weights are usually calculated by a softmax operation. Specifically, if there are n tokens with indices $(1, \dots, n)$, the attentions paid by the last token on a preceding token i is calculated as

$$Attn(token_n, token_i) = \frac{e^{w(\mathbf{q}_n, \mathbf{k}_i, n-i)}}{\sum_{j=1}^n e^{w(\mathbf{q}_n, \mathbf{k}_j, n-j)}}$$
(1)

Theorem 1. (Long-Distance Attention Logit Explosion) Let \mathbf{q} and \mathbf{k} be random vectors from distributions $\mathcal{D}_{\mathbf{q}}$ and $\mathcal{D}_{\mathbf{k}}$, respectively. We use the pseudo-dimension $\dim_P(\cdot)$ defined in Pollard (1990). Assume that the set of distance-based logit functions $\mathcal{H} = \{w(\cdot,\cdot,d)|d\in\mathbb{N}\}$ has bounded pseudo-dimension $\dim_P(\mathcal{H}) = r^1$. Let us also define the distinguish-ability of two distances d and d' under w as follows: $\mu_w(d,d') = \mathbb{E}_{\mathbf{q}\sim\mathcal{D}_{\mathbf{q}},\mathbf{k}\sim\mathcal{D}_{\mathbf{k}}}(w(\mathbf{q},\mathbf{k},d)-w(\mathbf{q},\mathbf{k},d'))^2$. We assume that w will not recognize only a finite group of distances, otherwise all distances longer than a threshold will become almost the same as shorter distances. Formally, for any n there is a partition of $\{0,1,\cdots,n\}$ into $\alpha(n)$ groups so that, $\mu_w(d,d') \geq \epsilon$ for any d, d' from different groups. $\alpha(n) \in \mathbb{N}$ is non-decreasing and unbounded function. Then we have:

$$\sup_{\mathbf{q},\mathbf{k},d\leq n} |w(\mathbf{q},\mathbf{k},d)| \geq \left(\frac{\alpha(n)}{2}\right)^{\frac{1}{2r}} \frac{\epsilon}{4e}$$

The proof can be found in Appendix A. We also empirically verify this on LLaMA on the longest sequence ArXiv dataset, truncated down to 32k tokens. We select the 0-th attention head in each Transformer layer for clarity of visualization, and plot the attention weights from the 32k-th token to all preceding tokens in Figure 1(a). We can see that at long distances, the absolute value of attention logits oscillate to significantly larger values than those within the training length of 4k.

The takeaway message is that, either the relative positional encoding fails to recognize the unseen distances, or the logits will increase to infinite values. The latter case will create OOD logits, which are "unfamiliar" to LLMs, and potentiall lead to irregular results. Even if the former case is true, in the next Section we show that it will cause another type of OOD factor. To alleviate the current factor, we conjecture that *one needs to limit the distance values during attention*.

3.2 OOD FACTOR 2: UNSEEN NUMBER OF TOKENS

Another factor we notice that potentially becomes out-of-distribution is the number of tokens to be attended to. When texts become longer, later tokens will need to attend to more and tokens. This might dilute the attention weights and make the attention distribution more flattened, losing information in the attention. Here we study the entropy of attentions, which is the theoretical metric for measuring the informativeness of a distribution. In the next proposition we formally demonstrate

¹This is true for most current techniques. See discussions in Appendix C

that, unless the logits explode, the entropy of attention weights will increase to infinity. In other words, there is a dilemma between the OOD factor 1 and 2.

Proposition 1. (Attention Entropy Explosion) Let $w_1, w_2, \dots, w_n \in [-B, B]$ be a sequence of attention logits that do not explode. Then the entropy of the distribution defined by attention weights will increase to infinity:

Entropy
$$\left(\left(\frac{e^{w_i}}{\sum_{j=1}^n e^{w_j}} \middle| 1 \le i \le n \right) \right) = \Omega(\ln n)$$

The proof is provided in Section B. We go on to empirically verify it this is the case in practice. We follow the setting in Section 3.1 and plot the attention entropy against context length in Figure 1(b). The curve shows an ever increasing attention entropy in accordance with the prediction.

This finding suggests us to limit the number of tokens to be attended to, so that LLMs can operate on a familiar sub-space of features. After analyses of these two factors, one might be tempted to propose an easy solution: forcing each token only to attend to the nearest few tokens and to ignore farther tokens during attention. This is similar to the block-diagonal attention mask used in XPos (Sun et al., 2022) and Longformer (Beltagy et al., 2020). However, we find that this does not work and LLMs' performance actually degrades on shorter texts. This means that XPos' extrapolation ability heavily relies on explicit training on it, and is not directly applicable to other LLMs. This phenomenon indicates the existence of another OOD factor, which we analyse in the following section.

3.3 OOD FACTOR 3: IMPLICITLY-ENCODED ABSOLUTE POSITION

In this section, we are going to demonstrate a counter-intuitive phenomenon. Even if absolute position information is not explicitly encoded in the computation graph, the attention mechanism is still able to implicitly encode it. We conjecture that this is actually happening in Transformers with relative positional encodings. The following theorem from Kazemnejad et al. (2023) proves this fact:

Theorem 2. (Implicitly Encoded Position) Let x be an input sequence of length T+1 without positional encoding. Then there exist a parameterization for a vanilla self-attention layer such that its output features is able to recover absolute positions [1, ..., T+1].

In the construction provided in Kazemnejad et al. (2023), the initial tokens' signals are stronger and easier to distinguish than tailing tokens. If this is true, then it constitutes another OOD factor. When the length is short, the LLM implicitly encodes positional information of initial tokens. However when the length exceeds training corpus, initial tokens are mishandled due to OOD factors 1 and 2, and their absolute position information become distorted or missing. However, the theorem is existential: it only proves that such implicitly encoding of absolute positions is *possible*, but does not guarantee that this is actually *happening* in real LLMs. As an empirical verification, we take the hidden state outputted by the first layer of LLaMA and plot its PCA projection into a 2-d plane in Figure 1(c). The dots corresponds to the first 4096 tokens in a sequence, with blue ones corresponding to initial tokens and red tokens being tail ones. In the plot we see that tokens at different positions do occupy distinct sub-spaces in the features space, even without explicit implementation to encode absolute position information. This provides an explanation why the simple solution mentioned in the end of Section 3.2 fails: when the sequence becomes long, directly limiting the attention window will eliminate initial tokens, so that the feature sub-space will become invisible. We conjecture that keeping these starting few tokens is important for LLMs to normally function.

After identifying thesee OOD factors, we claim that we have found the missing pieces behind the length generalization problem. In the following we propose our solution LM-Infinite in Section 4 and picture a conceptual model depicting how the relative position encoding works.

4 LM-Infinite

4.1 GENERAL PRINCIPLES

Based on the analyses in the preceding subsections, we propose our solution, LM-Infinite, which is a simple on-the-fly technique for length generalization on Transformer-based LLMs with relative

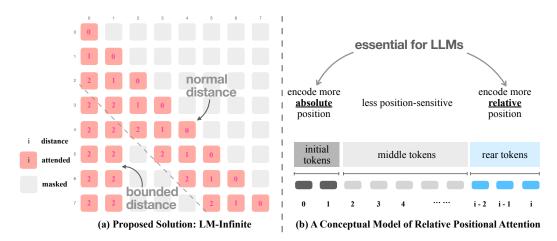


Figure 2: (a) The proposed approach LM-Infinite consists of a Λ -shaped mask and a distance constraint during attention, a plug-and-play solution for various LLMs. (b) We also provide a conceptual model for understanding how relative position encoding works.

positional encodings. Note that these encodings have different implementations, so LM-Infinite provides a set of high-level principles which is not limited to one single LLM.

An overview of LM-Infinite is illustrated in Figure2(a). This simple solution consists of two components: a Λ -shaped attention mask and a distance limit. As visualized in the figure, the Λ -shaped attention mask has two branches: a global branch at the left and a local branch at the right. The global branch allows each token to attend to the initial n_{global} tokens if they appear before the current token. The local branch allow each token to attend to preceding tokens within n_{local} distance. Any other token outside these two branches are ignored during attention. Here we heuristically set $n_{local} = L_{pretrain}$ as equal to training length limit, such as 2048 for LLaMA and 4096 for Llama-2. This choice includes the "comfort zone" of LLMs into attention. The selection of n_{global} has less effect on model performance, and we find that the range [10,100] is generally okay. This design is based on the OOD factors 2 and 3 above, where we aim to control the number of tokens to be attended to, while also ensuring the inclusion of initial tokens. Theoretically, LM-Infinite can access information from as long as a $n_{layer}L_{pretrain}$ context, because each deeper layer allows the attention to span $L_{pretrain}$ farther than the layer above.

The **distance limit** involves bounding the "effective distance" d within $L_{\rm pretrain}$. This only affects tokens that are in global branch. In specific, recall that in relative positional encoding, the attention logit is originally formulated as $w(\mathbf{q},\mathbf{k},d)$, where d is the distance between two tokens. Now we modify it as $w(\mathbf{q},\mathbf{k},\min(d,L_{\rm pretrain}))$. This design is motivated by the OOD factor 1, and ensures that LLMs are not exposed to distances that are unseen during pre-training.

4.2 IMPLEMENTATION DETAILS

The principles in LM-Infinite is applicable to most relative positional encodings. As this work is focused on addressing the length generalization problem of LLMs, we will evaluate LM-Infinite on 3 families of SoTA open-sourced LLMs in Section 5: LLaMA series (LLaMA and Llama-2), MPT-7B series and GPT-J series. Both LLaMA and GPT-J use RoPE encoding, and MPT-7B uses Alibi encoding. The principles can be easily generalized to other relative positional encodings.

RoPE (Rotary Position Embedding) Su et al. (2021) proposes to rotate the key and query vectors based on position before computing inner product. Specifically, each vector \mathbf{x} (either key or query) is split into pairs of elements $\{(x_0,x_1),(x_2,x_3),\cdots\}$, with each pair interpreted as a 2-d vector. RoPE then rotates the vector (x_a,x_{a+1}) of token i with angle $\theta_{a,i}=i\omega_a$, where ω_a is the rotating speed associated with dimension pair (a,a+1). After rotation, the 2-d vector becomes $\begin{pmatrix} \cos i\omega_a & -\sin i\theta_a \\ \sin i\theta_a & \cos i\theta_a \end{pmatrix} \begin{pmatrix} x_i \\ x_{i+1} \end{pmatrix}$. They show that the inner product between rotated \mathbf{q}_i and rotated \mathbf{k}_j is solely determined by values of \mathbf{q}_i , \mathbf{k}_j and distance i-j. In LM-Infinite, the Λ -shaped mask is

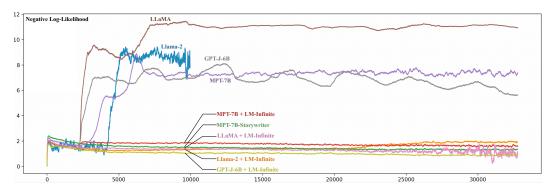


Figure 3: LM-Infinite flattens the NLL (negative log-likelihood, the logarithm of perplexity) curve of various LLMs on ArXiv dataset without any paramter updates, demonstrating similar trends to MPT-7B-Storywriter, an explicitly fine-tuned LLM. Llama-2 outputs NaN on too long sequences so the curve is relatively shorter.

straightforward to implement in RoPE. For the limited distance principle, the local branch follows original calculation. On the global branch (excluding the overlap with the local branch), we keep all ${\bf k}$ vectors unrotated, and rotate all ${\bf q}$ vectors to a fixed distance $L_{\rm pretrain}$. Then the two branches are composed together before attention masking.

AliBi (Press et al., 2021) proposes to offset all attention logits between tokens i, j by a linear term -|m(i-j)| and become $\mathbf{q}_i^{\top}\mathbf{k}_j - |m(i-j)|$. The MPT-7B codes use an offset matrix as an additive term in attention logits. To augment with LM-Infinite, we simply clip the offset matrix with a minimum value of $-|mL_{\text{pretrain}}|$.

4.3 A CONCEPTUAL MODEL FOR RELATIVE POSITION ATTENTION

In this section we describe a conceptual model on how relative positional encoding functions in Figure 2(b), based on the OOD factor diagnosis and LM-Infinite designing principles. The figure illustrates the view when *generating one next token*, that is, each token paying attention to all preceding tokens. The last token then gathers information via self-attention, computes output features and predict the next token. In this explanation, a long context can be roughly partitioned into 3 parts. Note how this conceptual model correlates with LM-Infinite's design.

- 1. The **initial tokens** encodes predominantly their absolute position information as explained in Section 3.3. They are essential components for attention layers because their features occupy a region in the feature space (e.g., upper-right in Figure 1(c)). If this region is missing or attended to using an unseen large distance, this will create the OOD factor 3.
- 2. The **rear tokens** which are closest to the final token. Here the relative positions are more important. Rear tokens are essential for the attention layer to correctly function.
- 3. The **middle tokens** encodes less position-sensitive information. As analyzed in Section 3.1 and 3.2, this region will either have exploding attention logits or too high attention entropy (OOD factor 1 and 2). Thus it does more harm than good for length generalization, so we remove them in LM-Infinite on sequences longer than training.

5 EVALUATION

In this section we empirically evaluate LM-Infinite's performance. We select ArXiv and OpenWeb-Text2 corpora from the Pile dataset (Gao et al., 2020), which consists of preprint papers from ArXiv and Reddit submissions from 2005 up until April 2020, respectively. For LLMs to evaluate, we use LLaMA-7B, Llama-2-7b-Chat, MPT-7B and GPT-J-6B. MPT-7B-Storywriter (fine-tuned on long sequences) is used as one of the baselines.

		ArXiv				OpenWebText2				
Model	Setting	2k	4k	8k	16k	32k	2k	4k	8k	16k
Sandwich	Train	5.02	5.15	5.28	-	-	23.3	23.8	24.7	-
XPos	Train	21.6	20.73	-	-	-	-	-	-	-
LongLLaMA	Fine-tune	8.17	7.44	-	6.94	-	-	-	-	-
MPT-7B-SW	Fine-tune	6.46	5.43	4.31	4.36	3.61	9.77	10.92	6.59	5.12
MPT-7B	Vanilla	5.49	247.6	1122	1672	1601	8.26	128.9	190.6	132.5
LLaMA	Vanilla	3.84	10k	60k	68k	49k	6.16	6636	456k	44k
GPT-J-6B	Vanilla	3.90	1285	1011	1617	278	8.83	746	1348	1803
Llama-2	Vanilla	3.37	3.76	8461	NaN	NaN	6.18	5.76	6507	NaN
MPT-7B	LM-Infinite	5.69	6.76	5.79	5.98	4.60	8.46	12.25	8.54	8.93
LLaMA	LM-Infinite	4.38	4.54	3.68	4.20	1.02	6.33	6.08	9.53	7.03
GPT-J-6B	LM-Infinite	3.84	3.13	3.00	3.06	2.14	8.83	8.49	6.49	7.39
Llama-2	LM-Infinite	4.33	3.63	3.33	4.18	6.49	6.13	5.32	8.28	8.15

Table 1: Perplexity scores comparison on ArXiv and OpenWebText2 dataset. LLMs with LM-Infinite achieve SoTA perplexity on 7 out of 9 columns while requiring no parameter updates.

5.1 Fluency

We first evaluate the fluency LM-Infinite using the widely adopted perplexity metric. Formally, when evaluating the quality of a probabilistic model M on modelling a distribution \mathcal{D} , perplexity is defined as the exponentiation of average negative log-likelihood (NLL): $PPL(\mathcal{D}, M) = \exp{-\mathbb{E}_{x\in\mathcal{D}}\ln{M(x)}}$. We plot the NLL curve in Figure 3 on ArXiv dataset. Note that Llama-2 outputs NaN on too long sequences so the curve is relatively shorter. We see that LM-Infinite successfully flattens the perplexity curve to lengths much longer than their training input lengths. This suggests a consistent and unharmed fluency on long sequences.

We also numerically log the perplexity scores at a few milestone lengths (2k, 4k, 8k, 16k and 32k) on ArXiv and OpenWebText2 in Table 1, which shows a similar trend. OpenWebText2 has very few data over length 32k so we omit the column. Note that with the help of LM-Infinite, LLMs successfully accomplish length generalization and achieves SoTA scores on 7 out of 9 columns. This is an encouraging result considering that LM-Infinite does not require any parameter updates in contrast to numerous strong baselines. As a direct comparison, MPT-7B+LM-Infinite achieves slightly only inferior scores than its fine-tuned cousin, MPT-7B-Storywriter. This suggests that LM-Infinite is an efficient counterpart to resource-consuming fine-tuning.

5.2 GENERATION PERFORMANCE

As perplexity is an internal metric for LLMs, we evaluate LM-Infinite's advantage on generation quality on ArXiv and OpenWebText2, with BLEU Papineni et al. (2002) and ROUGE (Lin, 2004) (ROUGE-LSum to be specific) as metrics. In simple words, both metrics evaluate the overlap on n-grams between generated text and reference text, with BLEU emphasizing on precision and ROUGE focused on recall. We let the LLMs generate 100 tokens after each milestone length, and use the following 100 tokens in original texts as reference. As generation takes a long time we sample 100 long sequences of evaluation in each dataset. The results are listed in Table 2. With a similar trend to the last section, LM-Infinite successfully allows LLMs to extend their generation quality to lengths longer than training, comparable to the effect of fine-tuning while without parameter updates. Note that LM-Infinite has slightly different effects on different LLMs. For example, on LLaMA and GPT-J-6B the quality is better maintained at longer positions, while on Llama-2 the quality is better at nearer positions. We also evaluate the computation efficiency on length of 32k in Appendix D, where LM-Infinite demonstrates 3.16x speedup when encoding and 2.72x speedup when decoding.

5.3 TASK SOLVING

Finally, as LLMs are often deployed in solving downstream tasks, we evaluate on how LM-Infinite performs on long-input tasks. We follow Mohtashami & Jaggi (2023) and use the passkey retrieval task. It buries a passkey at a random position in a long distraction text, and in the end asks what

	2	2k	4k		8k		16k		32k	
ArXiv	bleu	rouge								
MPT-7B-SW	16.6	26.5	21.5	30.1	15.2	26.6	18.9	27.4	14.8	27.0
MPT-7B	0.0	5.6	0.2	3.6	0.0	5.9	0.0	1.7	0.4	1.4
MPT-7B + LM-Infinite	16.1	23.8	20.2	24.9	12.6	24.1	23.9	29.0	19.7	26.6
Llama-2	26.6	31.4	0	0.2	0.0	0.0	0.0	0.0	0.0	0.0
Llama-2 + LM-Infinite	26.9	31.8	23.6	30.9	23.9	28.2	24.8	29.2	18.4	20.4
OpenWebText2	bleu	rouge	bleu	rouge	bleu	rouge	bleu	rouge		
MPT-7B-SW	8.4	21.0	6.1	19.3	7.5	18.5	8.4	22.0		
MPT-7B	0.9	7.5	0.9	6.6	1.0	6.4	1.0	6.8		
MPT-7B + LM-Infinite	5.0	16.6	4.1	15.4	5.1	16.2	2.8	16.0		
Llama-2	8.8	22.4	0.0	0.2	0.0	0.0	0.0	0.0		
Llama-2 + LM-Infinite	9.0	21.9	7.2	21.2	9.7	19.6	9.6	19.6		

Table 2: Evaluation on text generation on ArXiv and OpenWebText2 corpora. LM-Infinite consistently generalizes the generation quality to extreme lengths, outperforming or similar to the fine-tuned LLM, MPT-7B-Storywriter.

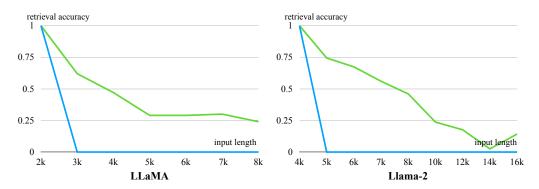


Figure 4: LM-Infinite extends passkey retrieval accuracy to longer inputs for LLaMA and Llama-2.

the passkey is. As an synthetic dataset, we have more flexible control on the input length to have fine-grained analysis. We plot the answer accuracy in Figure 4. We can see that LM-Infinite allows LLMs to keep slower decaying accuracy on lengths longer than trainig, compared to vanilla models which fail immediately. As passkey retrieval is an information-sensitive task, the curve shows that LM-Infinite does not perfectly maintain information perception capability, even though LM-Infinite theoretically can access information from as long as $n_{\rm layer}L_{\rm pretrain}$ context. We attribute this to the Λ -mask which trades off information for fluency. We leave the problem of maintaining higher information sensitivity to future work.

6 CONCLUSIONS AND FUTURE WORK

In this paper, we provide an explanation and a simple on-the-fly solution to the length generalization problem in Transformer-based LLMs with relative positional encodings. We start with theoretical and empirical analysis of OOD (out-of-distribution) factors that might contribute to the length generalization failures. Based on these intuitions we propose LM-Infinite, an plug-and-play cure without any parameter updates. Our empirical evaluations show that we can let multiple open-sourced SoTA LLMs maintain their original generation quality, similar to performance after explicit fine-tuning. LM-Infinite also extends task solving ability to sequences much longer than training samples. Future work can explore how to let LM-Infinite better perceive information in the masked out attention region. We hope that LM-Infinite's computational efficiency and ease of use allow researchers without enormous computational resources to also use LLMs on long sequences.

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A PROOF OF THEOREM 1

We first borrow a lemma from Haussler (2018), which we paste below. Note that a cover size $\mathcal{N}(\epsilon, \mathcal{H}, \mu)$ is defined as the minimum cardinal of a cover-set \mathcal{H}' so that any element of $h \in \mathcal{H}$ will be within ϵ distance to at least one element $h' \in \mathcal{H}'$.

Lemma 3. Let \mathcal{H} be a function family mapping from space X to range [0,B], and its pseudo-dimension $\dim_P(\mathcal{H}) = r$. Then for any probabilistic measure P on X, and $\epsilon \in [0,B]$, we have that the ϵ cover of \mathcal{H} under metric $\mu(h_1,h_2) = \mathbb{E}_{x \sim P}(h_1(x) - h_2(x))^2$ is bounded by:

$$\mathcal{N}_P(\epsilon, \mathcal{H}, \mu) \le 2 \left(\frac{2eB}{\epsilon} \ln \frac{2eB}{\epsilon} \right)^r$$

With this lemma we can go on to prove Theorem 1 as follows.

Proof. We prove by contradiction. Assume that $\sup_{\mathbf{q},\mathbf{k},d\leq n}|w(\mathbf{q},\mathbf{k},d)|< a=\left(\frac{\alpha(n)}{2}\right)^{\frac{1}{2r}}\frac{\epsilon}{4e}$. Without loss of generality we can shift all the values to range [0,2a]. Then the function family $\mathcal{H}=\{w(\cdot,\cdot,d)|d\in\mathbb{N}\}$ will have cover size $\mathcal{N}_P(\epsilon,\mathcal{H},\mu)\leq 2\left(\frac{4\epsilon a}{\epsilon}\ln\frac{4\epsilon a}{\epsilon}\right)^r<\alpha(n)$.

However, this is smaller than the number of different distances and relative weight attentions \mathcal{H} , which means that at least 2 functions will be close to each other $(w(\cdot,\cdot,d),w(\cdot,\cdot,d'))^2<\epsilon$. This constitutes a contradiction with the distinguish-ability condition.

B PROOF OF PROPOSITION 1

Proof. Note that entropy on a discrete distribution is defined as $\text{Entropy}(P) = -\sum_i p_i \ln p_i$. Then the attention entropy determined by attention logits $\{w_i | 1 \le i \le n\}$ is

$$\begin{aligned} \text{Entropy}(\text{Attention}) &= -\sum_{i} \frac{e^{w_i}}{\sum_{j} e^{w_j}} \ln \frac{e^{w_i}}{\sum_{j} e^{w_j}} \\ &= -\sum_{i} \frac{e^{w_i}}{\sum_{j} e^{w_j}} \left(w_i - \ln \sum_{j} e^{w_j} \right) \\ &= -\sum_{i} \frac{e^{w_i}}{\sum_{j} e^{w_j}} w_i + \ln \sum_{j} e^{w_j} \\ &\geq -\max_{i} w_i + \ln(ne^{-B}) \\ &\geq \ln n - 2B \\ &= \Omega(\ln n) \end{aligned}$$

C PSEUDO-DIMENSION ASSUMPTION ON ATTENTION LOGIT FUNCTIONS

In Theorem 1, we assumed that the family of distance-based logit functions $\mathcal{H}=\{w(\cdot,\cdot,d)|d\in\mathbb{N}\}$ has a finite pseud-dimension. In this section, we demonstrate that most current implementations of relative positional encodings do have a finite pseudo-dimension. Let us discuss a few examples in the following:

T5-Bias and Alibi It is easy to see that, the difference between any two logit functions is uniform: $w(\cdot, \cdot, d_1) - w(\cdot, \cdot, d_2) = \text{bias}(d_1) - \text{bias}(d_2)$ regardless of the input. Therefore this family cannot shatter any set larger than 2, so the pseudo-dimension is 1.

Windowed Attention This operation is equivalent to limiting the family to a finite size $|\mathcal{H}| = d_{\text{max}} + 1$, where d_{max} is the size of the window. Therefore $\dim_P(\mathcal{H}) \leq d_{\text{max}} + 1$.

NoPE As there is no explicit positional encoding implemented, all distances are equivalent. The pseudo-dimension is 1.

RoPE, CAPE and XPos For RoPE, the logit function $w(\mathbf{q}, \mathbf{k}, d)$ is the weighted sum of finite fixed sinusoidal functions $\{\sin(\omega_i d), \cos(\omega_i d)\}$. The size of this set is equivalent to the feature dimension number k. We know that $\dim_P(\mathcal{H}_1 + \mathcal{H}_1) \leq \dim_P(\mathcal{H}_1) + \dim_P(\mathcal{H}_2)$. Also, the scaling of a single function can only have pseudo-dimension of 2. Therefore, the whole family has a bounded pseudo-dimension $\dim_P(\mathcal{H}) \leq 2k$. The analysis on CAPE and XPos is similar.

D COMPUTATIONAL EFFICIENCY EVALUATION

To evaluate the computational efficiency of LM-Infinite, we run Llama-2-7B model on 100 sequences of 32k length in ArXiv dataset. The hardware is a single A100 GPU with 80GB GPU memory. As the memory is not enough for hosting the whole computation graph during decoding, we use DeepSpeed (Rasley et al., 2020) with Zero3 optimization. We also have to modify the computation code in order to further reduce GPU memory usages and prevent out-of-memory error.

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With that in mind, the vanilla Llama-2-7B model encodes with an average speed of 48.19s per sequence, while LM-Infinite encodes with average 15.26s per sequences, a 3.16x speedup. The vanilla Llama-2-7B model decodes with 7.34s per token, while LM-Infinite decodes with 2.70s per token, a 2.72x speedup.