PHY473 Research Proposal: An Exploration of Quantum Algorithms for Fluid Dynamics & Navier-Stokes

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1 Introduction

This paper proposes the following research topic for PHY473: **an exploration on quantum and quantum-classical hybrid algorithms for solving Navier-Stokes equations efficiently**.

Motivation:

The Navier-Stokes equations are the holy grail of computational fluid dynamics , governing everything from aerospace flight vehicle design , weather forecasting , plasma magneto-hydrodynamics , and astrophysics. The problem is that **these nonlinear partial differential equations are computationally intractable for classical computers** when dealing with:

- turbulent flows at high Reynolds numbers
- multi-scale phenomena from molecular to macroscopic scales
- real-time applications like weather prediction or flight control

A quantum approach to finding flows of a Navier–Stokes fluid may prove to be less computationally intensive , providing the possibility for potential speedups , especially when used in conjunction with classical methods.

2 The Goals

There are many things to aim for in this research topic. Here is a brief overview of goals:

1. Develop a deep understanding of fluid mechanics and the Navier-Stokes equations , both in application and derivation.
2. Develop a deep understanding of quantum computing , and how quantum algorithms are made and used through both theory and application.

3. Develop a good understanding on the foundations of quantum mechanics and how they let quantum computers be possible.

4. Read existing scientific articles on the usage of quantum algorithms for solving Navier-Stokes flows and learn about these methods in depth.

5. Develop classical and quantum solvers for Navier-Stokes equations , compare different approaches , and investigate/make new classical-quantum hybrid algorithms.

6. Potentially publish a paper.

3 Existing Works

In addition to the works listed below , you can find many more interesting papers in the citations of said works.

🔗 Finding flows of a Navier–Stokes fluid through quantum computing

Gaitan et al. developed a quantum algorithm that provides quadratic speedup over classical random algorithms and exponential speedup over classical deterministic algorithms for rough/non-smooth flows.

The algorithm was validated using steady-state , inviscid compressible flow simulations in convergent divergent nozzles[1]. Such nozzles are critical components in propulsion systems , enabling the transition from subsonic to supersonic flow regimes[1].

For rough flows (the computationally difficult case) , there exists a regime where the speedup is quadratic over classical random algorithms and exponential over deterministic algorithms[1].

🔗 Quantum-inspired framework for computational fluid dynamics

Peddinti, R.D., Pisoni, S., Marini, A. et al. created a revolutionary approach to computational fluid dynamics (CFD) that doesn't actually use quantum computers , but instead borrows quantum mathematical techniques to achieve exponential speedups for classical fluid simulations[2].

The framework is based on matrix-product states, a compressed representation of quantum states, providing memory and run time scaling logarithmically in the "mesh" size. Traditionally , CFD scales $\mathcal{O}(N^3)$ for memory and operations at every time step (when dealing with 3D flows). With this framework's scaling , they achieved $\mathcal{O}(logN)$[2].

🔗 Quantum computing of fluid dynamics using the hydrodynamic Schrödinger equation

Meng et al. present a fundamentally new approach to quantum fluid dynamics that solves the core problem of applying quantum computing to the nonlinear , non-Hamiltonian Navier-Stokes equations.

The breakthrough is transforming fluid dynamics into quantum mechanics by using the Hydrodynamic Schrödinger Equation (HSE) , derived by generalizing the Madelung transform to compressible or incompressible flows with finite vorticity and dissipation[3]. Since the HSE is expressed as a unitary operator on a two-component wave function, it is more suitable than the NSE for quantum computing.

The flow governed by the HSE can resemble a turbulent flow consisting of tangled vortex tubes with the five-thirds scaling of energy spectrum. This means the quantum formulation naturally reproduces the Kolmogorov energy cascade , which is the hallmark of turbulence[3] This is profound because it suggests quantum mechanics and turbulent fluid dynamics share deep mathematical structures.

🔗 Solving incompressible Navier–Stokes via a hybrid quantum–classical scheme

This paper presents the first practical demonstration of solving fluid dynamics problems on actual noisy quantum hardware , a major breakthrough in making quantum CFD a reality.

Song et al. present a hybrid quantum–classical algorithm for the incompressible Navier–Stokes equations. A classical device performs nonlinear computations, and a quantum one uses a variational solver for the pressure Poisson equation[4].

While other approaches focus on future fault-tolerant quantum computers, this paper actually implements and tests the algorithm on IBM's current noisy superconducting quantum processors.

References

[1] Frank Gaitan. Finding flows of a navier–stokes fluid through quantum computing. *npj Quantum Information*, 6(1):61, 2020.

[2] Raghavendra Dheeraj Peddinti, Stefano Pisoni, Alessandro Marini, Philippe Lott, Henrique Argentieri, Egor Tiunov, and Leandro Aolita. Quantum-inspired framework for computational fluid dynamics. *Communications Physics*, 7(1):135, 2024.

[3] Zhaoyuan Meng and Yue Yang. Quantum computing of fluid dynamics using the hydrodynamic schrödinger equation. *Phys. Rev. Res.*, 5:033182, Sep 2023.

[4] Zhixin Song, Robert Deaton, Bryan Gard, and Spencer H. Bryngelson. Incompressible navier–stokes solve on noisy quantum hardware via a hybrid quantum–classical scheme. *Computers & Fluids*, 288:106507, 2025.