

# Universal Turing Machine Search

## EECS 600: Ruby on Rails

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## 1 Abstract

We propose to investigate Darwin's Theory of Natural Selection, in abstract, by analyzing the results of a Genetic Algorithm optimization on a population of Turing Machines. Using the Turing machines as analogs for simple organisms, the goal of the research is to perform an optimization where the result is a population which contains some Universal Turing Machines. We take Universal Turing Machines to represent more complex organisms on an evolutionary path. We will include a web based visualization environment, written in Ruby on Rails, to help visualize the results of the optimizations.

## 2 Introduction

In 1861 Charles Darwin published his theory of natural selection as a means by which living creatures evolved to best meet the challenges presented to them in their environment. (Darwin 1861) According to Darwin, species undergo changes because certain members of a population possess traits which allow them to reproduce more successfully than others. Those organisms offspring retain the advantageous traits and hence continue to reproduce more successfully themselves. It is an interesting thought experiment to consider Darwin's theory in the extreme, where life exists as very simple organisms and progressively evolves into more complex ones eventually resulting in mammals and then primates and then humans. When examined in the extreme, Darwin's theory seems almost impossible.

We have developed a method to examine this extreme view of Darwin's theory, in an abstract manner, by applying a global optimization process to

Turing Machines (TM). TMs, initially conceived as a thought experiment by Alan Turing in 1937, "are simple abstract computational devices intended to help investigate the extent and limitations of what can be computed". (Barker-Plummer 2004) These simple machines can be constructed so that they are capable of computations that range in complexity from essentially useless to practically unlimited. In their most complex form, TMs are given a special designation: Universal Turing Machines (UTM). A UTM can reproduce the output of any other TM (even other UTM's), given the same input. We employ special capability of a UTM as an analogue to the complexity that exists in modern living organisms. By using TMs as the subjects of a global optimization, and carefully constructing the function to evaluate the fitness of each TM, an evaluation of the possibility that "complex" TMs can evolve from effectively random "simple" TMs is performed.

### 3 Project Plan

Genetic algorithm optimization is designed to mimic the evolutionary process described by Darwin's Theory. In this genetic algorithm, potential solutions are encoded as strings, the fittest of which survive to produce more solutions in later generations. The key to a successful optimization using the Genetic Algorithm is the selection of the fitness function with which to evaluate the subject of the optimization. However, determining the fitness test was not the only step required to execute the optimization. A method of encoding a TM into a "genome" was required along with some method of reproduction between parent TMs.

#### 3.1 Turing Machine Encoding

The encoding translates a binary string into an actual TM. Binary string encoding was chosen for its simplicity and well documented use in genetic algorithms. To define the encoding scheme, first it was necessary to define a set of constants for the TM population:

- NUM.STATES: Total number of states represented in each state table
- GENE.LENGTH: The number of bits being used to represent a single entry in the state table

The size of GENE.LENGTH is required to be at least large enough to represent NUM.STATES while reserving two bits at the end for the **write** bit and **movement** bit(left = 0, right = 1). However, it is allowed for there to be many more bits than necessary to accommodate future growth in NUM.STATES without changing the alignment in all previous TMs. To handle this issue, the modulus operator was employed to ensure that all possible state values fall within a valid range.

current state	read bit	next state	write bit	movement
1	0	11	1	1
1	1	10	1	0
2	0	10	0	1
2	1	10	0	1
3	0	01	0	1
3	1	01	1	1

Table 1: State transition table for notional 3-state TM where the next state has been represented in an appropriate length binary string

Take the example encoding: '11110101001100101000110', with NUM\_STATES=3 and GENE\_LENGTH=4. This is broken up into segments which are GENE\_LENGTH bits long: 1111,1010,1001,1001,0100,0110. Each gene segment then represents a single entry in a state transition table. The resulting state transition table can be seen in Table 1. An alternate, visual representation of the turing machine is shown in Figure 1

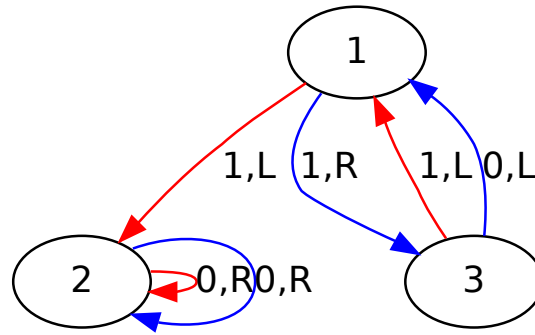


Figure 1: State transition diagram for notional 3-state TM.

It should be noted that all TM using this encoding assume that the 0 state is the halt state. It is not required that any TM utilize the halt state at all. The example TM above did not use the halt state.

### 3.2 Turing Machine Fitness Evaluation

The function of the fitness evaluation in a Genetic Algorithm is to provide the means of scoring one population candidate against another. The particular

formulation that is chosen for a fitness evaluation will effect the entire outcome of any optimization. For this research, the purpose is to investigate the likelihood that an evolutionary process can produce very "complex" organisms from "simple" ones. Taken TMs as analogs for organisms, the best possible result we can hope for is a population of UTM's. So the fitness function employed must grade TMs that are most like a UTM higher than ones that are less like a UTM.

There is not simple test for that can be applied to a TM to determine if it is in fact a UTM. If there were, you would simply apply that test to every member of a population and grade them accordingly. Lacking such a test, it is necessary to establish certain measurable qualities of a UTM that could be tested for on a candidate. The fundamental problem with this approach is that when a specific candidate has been shown to share a lot of the qualities that you have associated with a UTM, there is no single logical path that can extend that information to show that the candidate is actually a UTM (that's why there is no test!).

Instead of testing directly for qualities of candidate TMs that are similar to those of a UTM, a more simple fitness evaluation was used, which follows more directly with the stated hypothesis. TMs are scored based on the complexity of their output. This complexity is measured by applying a compression algorithm to the output string of the turing machine, and then using the resulting size of the compressed data as the fitness of the TM. The DEFLATE compression algorithm was selected because it is a lossless compression algorithm and is part of the ruby standard library.

Since the output of a TM is dependent on the input given to it, the outcome of this fitness evaluation is highly dependent on the input given to each candidate TM as well. Three interesting input types were evaluated:

1. Fixed input string for all generations
2. Randomly generated input string for each generation
3. Input string with some mutation between generations

The first option tests the outcome of an optimization where each generation is being given effectively the same exact test as all the others. TMs which are good at making complex output for this specific input will thrive. The second option basically generated a random new test for each generation, and only TMs that can produce complex output for arbitrary input will thrive. These TMs could be considered very adaptable. The third option presents a hybrid approach to the previous two; here each successive generation is operating on an input string which is similar to the last (how similar depends on the mutation rate) but not identical. Over a large number of generations however the input string is effectively appears random. For this research, the when using the third input string option, a mutation rate of 50% was used.

### 3.3 Results Visualization

As part of our project, we will produce a visualization system for the results of our process. Specifically, we will use the Ruby on Rails web framework to create

a web application that displays Turing Machines generated by our algorithm, including genome, structure, and sample output.

## 4 Project Timeline

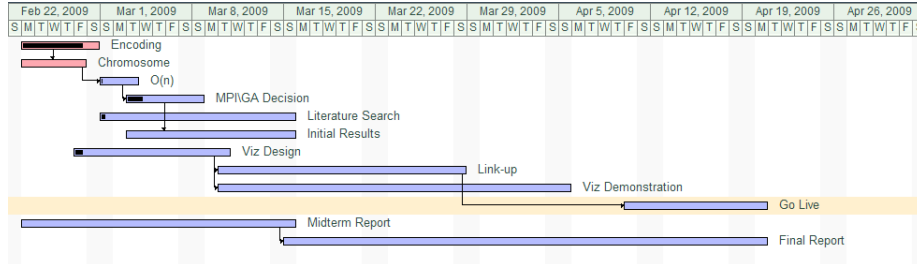


Figure 2: Project timeline of major milestones

Figure 2 shows a detailed timeline for the completion of the project. The major milestones for all aspects of the project present, including necessary work for the creation of an optimization scheme to search for UTMs as well as the creation of a web based visualization tool to facilitate the examination of the project results. The key to the task names used in Figure 2 is presented below:

- **Encoding:** Definition of the encoding method for Turing Machines completed
- **Chromosome:** Implementation of the chromosome class for the AI4R Genetic Algorithm package completed.
- **O(n):** Computational cost for fitness evaluation of a Turing Machine is determined.
- **MPI/GA Decision:** Decision made regarding necessity of use for MPI along with AI4R Genetic Algorithm to reduce computation time.
- **Literature Search:** Literature search to generate a list of known UTMs
- **Initial Results:** Initial Turing Machine Optimization run completed. Data analyzed to ensure the viability of optimization scheme
- **Viz Design:** Visualization web-app design complete
- **Link-up:** Link between genetic algorithm generation data and visualization database completed.
- **Viz Demonstration:** Ruby on Rails visualization application implementation completed.

- **Go Live:** Ruby on Rails visualization application goes live with link to genetic algorithm running perpetually.
- **Midterm Report:** Midterm Report Delivery
- **Final Report:** Final Report Delivery
- **Presentation:** Final Presentation

## 5 Organizational Structure

### 5.1 Project division

A first priority will be to define a standard format for representing a Turing Machine; this format will be used pervasively in the entire application. From there, several independent modules fall naturally out of the problem specification:

- **Execution:** Given a TM and its input, returning successive states of the tape at points in the future. (Josh Lee)
- **Evaluation:** Determining a heuristic for universality, ranking TMs, and producing stochastic test data. (Justin Gray, Steven Dee)
- **Breeding and mutating:** The GA's selection and combination processes. (Neil Sandberg)
- **Web based visualization tool,** using Ruby on Rails. (Justin Gray, Neil Sandberg)

It is our expectation that, since these modules have well-defined interfaces and can be tested independently of each other, we will be able to work on them individually or in pairs, depending on the size of the task. The responsible parties are listed next to each module

### 5.2 Communication

We will use a mailing list to coordinate communication for the group. Justin Gray will serve as the point of contact for the group as a whole. Github will be used as a source repository to enable distributed collaboration.

## 6 Risk Analysis

Our hypothesis is that applying our selected fitness evaluation methods to a set of Turing Machines will lead a genetic algorithm optimization to an optimum that is a set of UTMs. It is possible, using a single fitness evaluation, that we could get no UTMs. In that case, the data will have shown that our particular fitness evaluation does not lead to universality. This however would not be a

wholesale disproof of the research method. To deal with this possibility, we will employ a set of fitness evaluation methods and compare their results. This way, a slightly broader dataset can be generated. Though both methods still can not conclusively disprove the research method, together they provide a much stronger conclusion in either direction.

At this point, the amount of time necessary for the optimizations to run is unknown. We assume that the amount of time it takes to evaluate a single member of the candidate population is trivial. If this does not turn out to be the case, then it is possible that the optimization could benefit greatly from the usage of parallel evaluation of population members. We have identified an implementation of the Message Passing Interface (MPI) for Ruby which could be used to implement such a parallel scheme. By March 8th, we will have evaluated the computational requirements for fitness evaluation and if necessary can begin work on implementing a parallel optimization algorithm.

Even if the computational requirements for evaluation of a candidate Turing Machine are modest, the optimization algorithm could move toward an optimum slowly. Genetic Algorithms are known for their slow convergence rates. We have provided ample time to allow for this potential bottleneck, by starting a test of our optimizations on March 3rd. That will provide plenty of time for a slow optimization to run, leaving time to analyze the results.

## References

- Barker-Plummer, D. (2004), 'Turing machines'. Stanford Encyclopedia of Philosophy.
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