Optimization Problems, Lecture 2, Segment 3

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Dynamic Programming

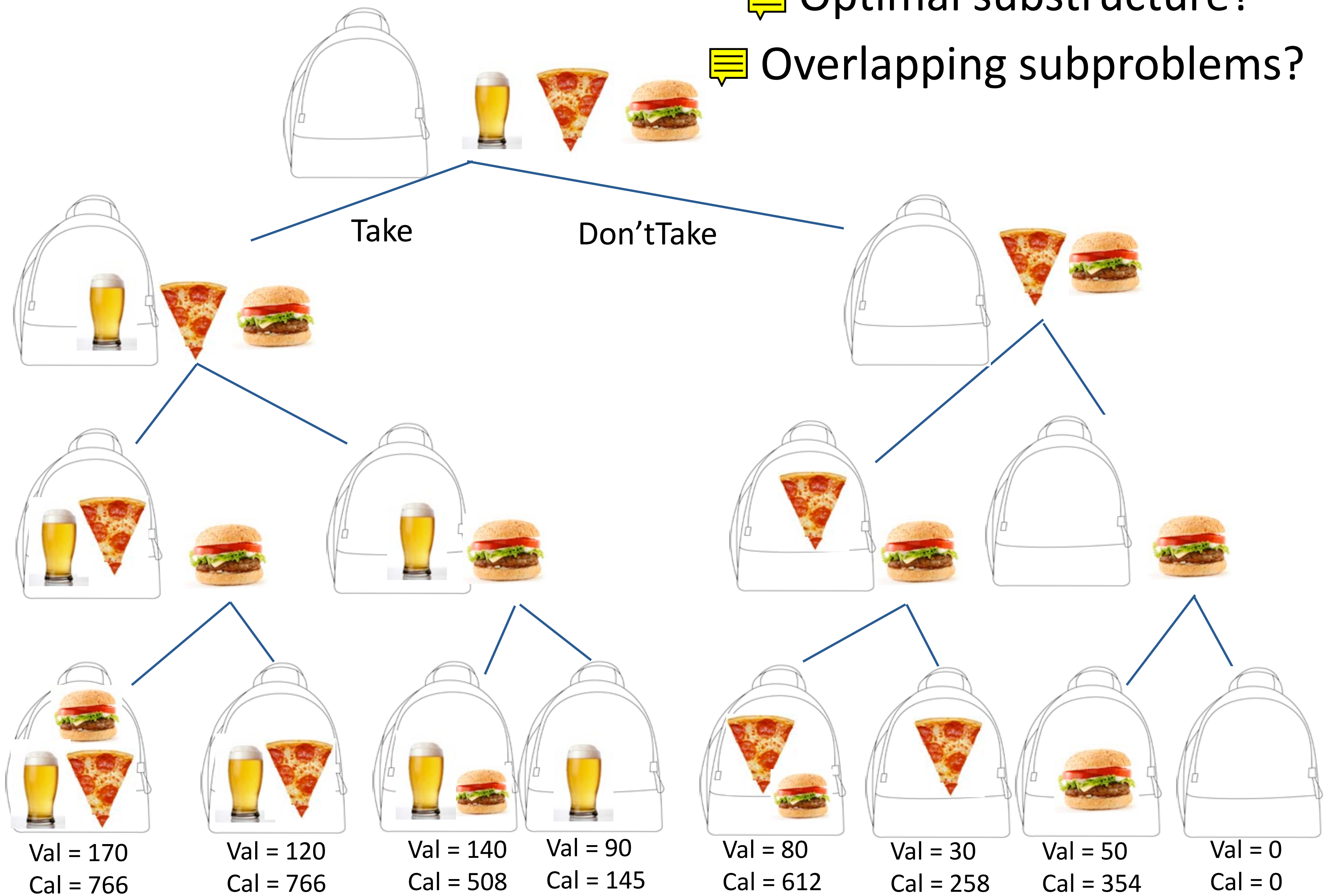
- Optimal substructure: a globally optimal solution can be found by combining optimal solutions to local subproblems
  - For x > 1, fib(x) = fib(x - 1) + fib(x – 2)

- Overlapping subproblems: finding an optimal solution involves solving the same problem multiple times
  - Compute fib(x) or many times

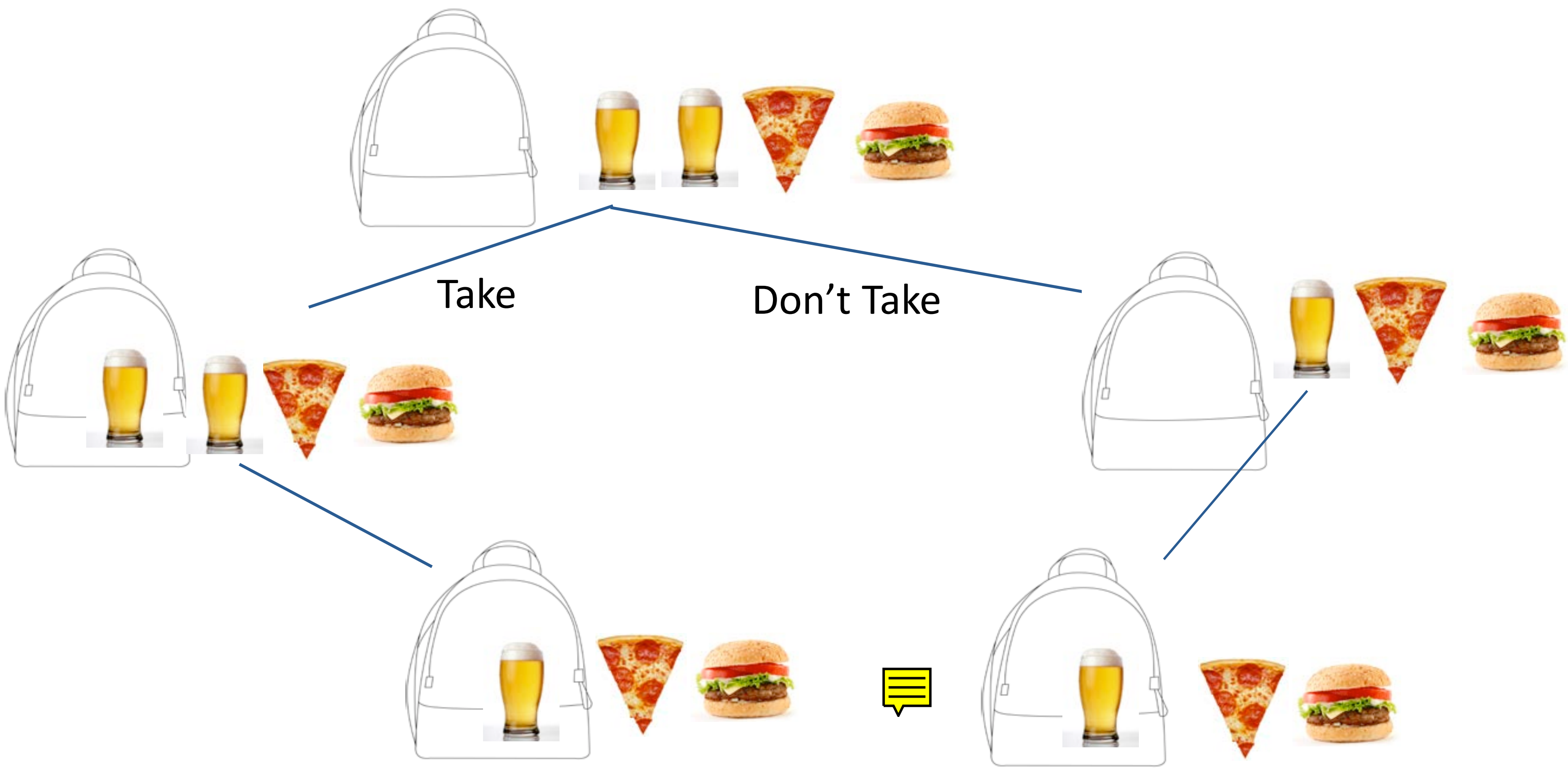
Search Tree

Optimal substructure?

Overlapping subproblems?



A Different Menu

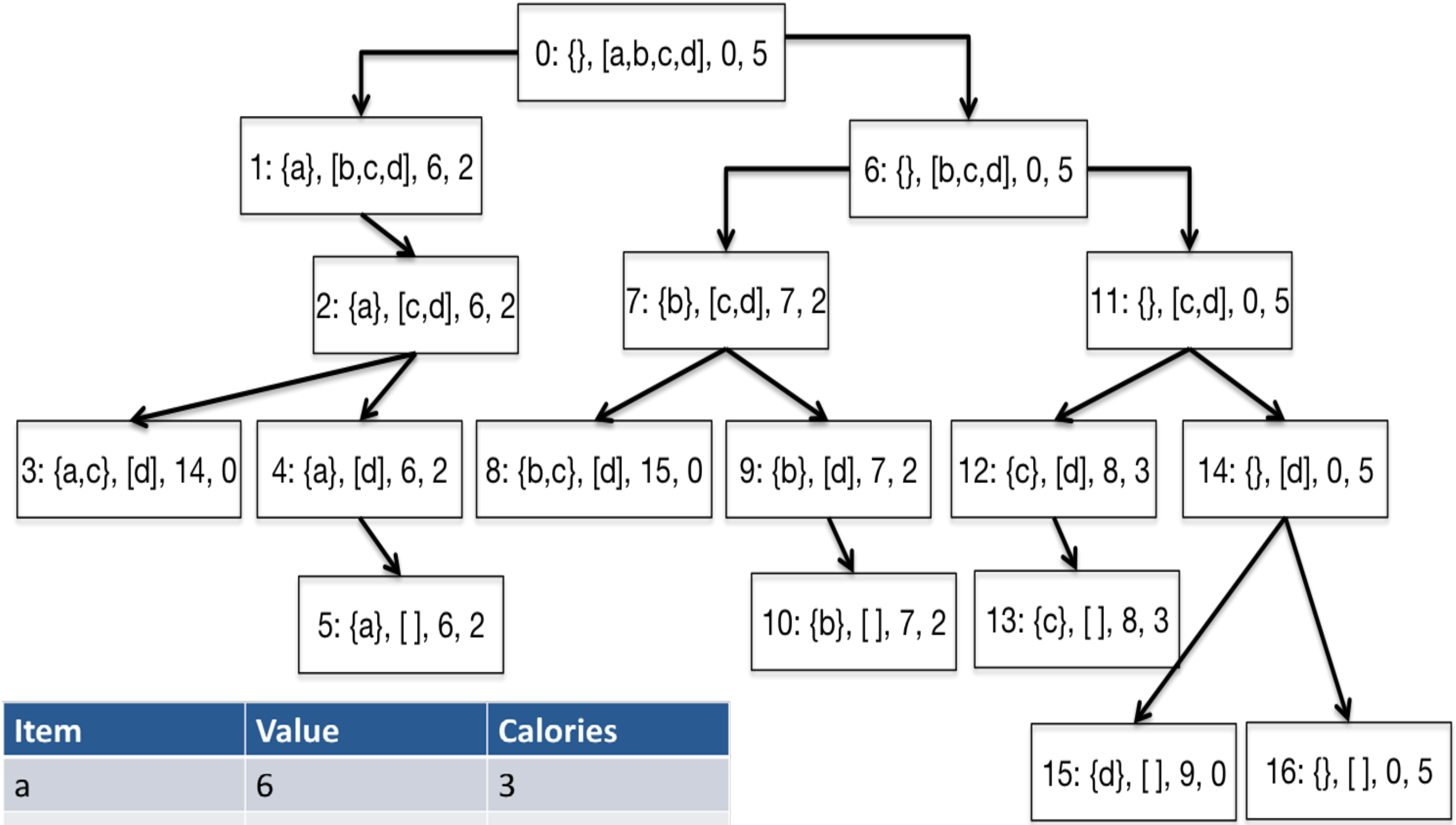


Need Not Have Copies of Items

Item	Value	Calories
a	6	3
b	7	3
c	8	2
d	9	5

Search Tree

- Each node = <taken, left, value, remaining calories>



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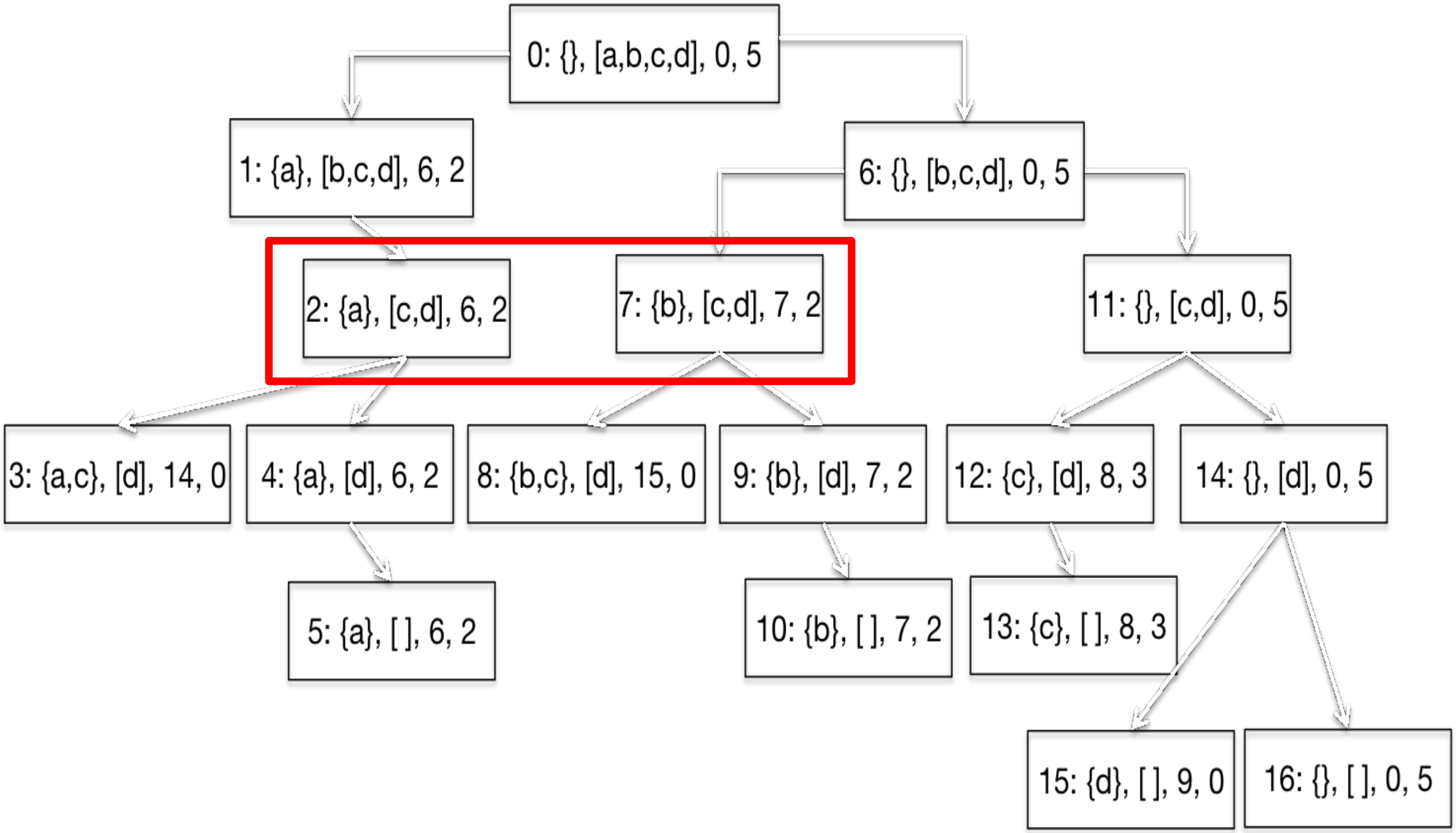
What Problem is Solved at Each Node?

- Given remaining weight, maximize value by choosing among remaining items
- Set of previously chosen items, or even value of that set, doesn’t matter!

Overlapping Subproblems



Modify maxVal to Use a Memo

- Add memo as a third argument
  - `def fastMaxVal(toConsider, avail, memo = {}):`
- Key of memo is a tuple
  - (items left to be considered, available weight)
  - Items left to be considered represented by `len(toConsider)`
- First thing body of function does is check whether the optimal choice of items given the the available weight is already in the memo
- Last thing body of function does is update the memo

Performance

len(items)	2**len(items)	Number of calls
2	4	7
4	16	25
8	256	427
16	65,536	5,191
32	4,294,967,296	22,701
64	18,446,744,073,709,551,616	42,569
128	Big	83,319
256	Really Big	176,614
512	Ridiculously big	351,230
1024	Absurdly big	703,802

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- Problem is exponential
- Have we overturned the laws of the universe?
- Is dynamic programming a miracle?

How Can This Be?

- Problem is exponential
- Have we overturned the laws of the universe?
- Is dynamic programming a miracle?
- No, but computational complexity can be subtle
- Running time of `fastMaxVal` is governed by number of distinct pairs, `<toConsider, avail>`
  - Number of possible values of `toConsider` bounded by `len(items)`
  - Possible values of `avail` a bit harder to characterize
    - Bounded by number of distinct sums of weights
  - Covered in more detail in assigned reading

Summary of Lectures 1-2

- Many problems of practical importance can be formulated as optimization problems
- Greedy algorithms often provide adequate (though not necessarily optimal) solutions
- Finding an optimal solution is usually exponentially hard
- But dynamic programming often yields good performance for a subclass of optimization problems—those with optimal substructure and overlapping subproblems
  - Solution always correct
  - Fast under the right circumstances