Optimization Problems, Lecture 2, Segment 2

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Search Tree Algorithm

- Gave us a better answer than any of the greedy solutions
- Finished quickly
- But 2^8 is not a large number
- Let's look at what happens when we have a more extensive menu to choose from

Code to Try Larger Examples

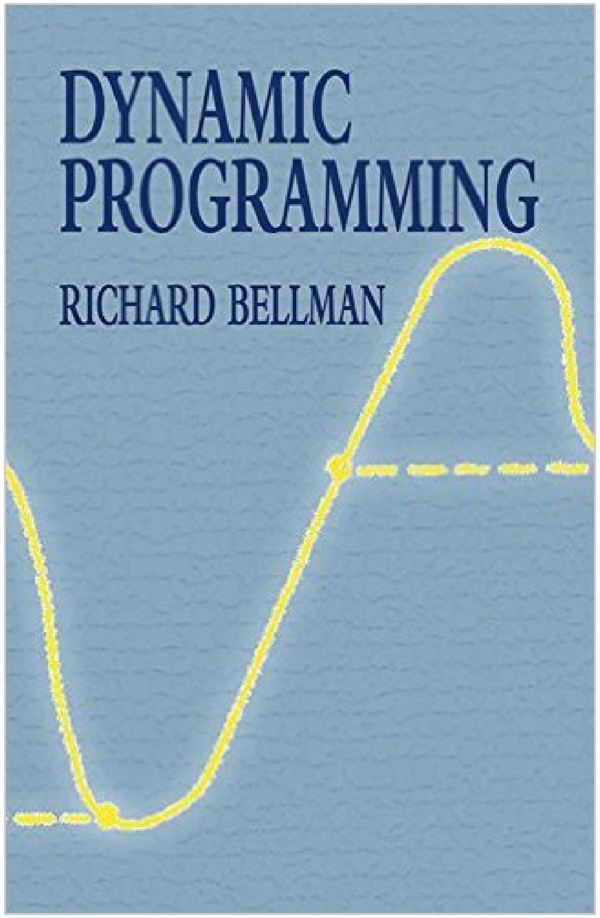
```
import random

def buildLargeMenu(numItems, maxVal, maxCost):
    items = []
    for i in range(numItems):
        items.append(Food(str(i),
                          random.randint(1, maxVal),
                          random.randint(1, maxCost)))
    return items


for numItems in (5, 10, 15, 20, 25, 30, 35, 40, 45):
    items = buildLargeMenu(numItems, 90, 250)
    testMaxVal(items, 750, False)
```

Is It Hopeless?

- In theory, yes
- In practice, no!
- Dynamic programming to the rescue



Dynamic Programming?

Sometimes a name is just a name

“The 1950s were not good years for mathematical research… I felt I had to do something to shield Wilson and the Air Force from the fact that I was really doing mathematics... What title, what name, could I choose? ... It's impossible to use the word dynamic in a pejorative sense. Try thinking of some combination that will possibly give it a pejorative meaning. It's impossible. Thus, I thought dynamic programming was a good name. It was something not even a Congressman could object to. So I used it as an umbrella for my activities.

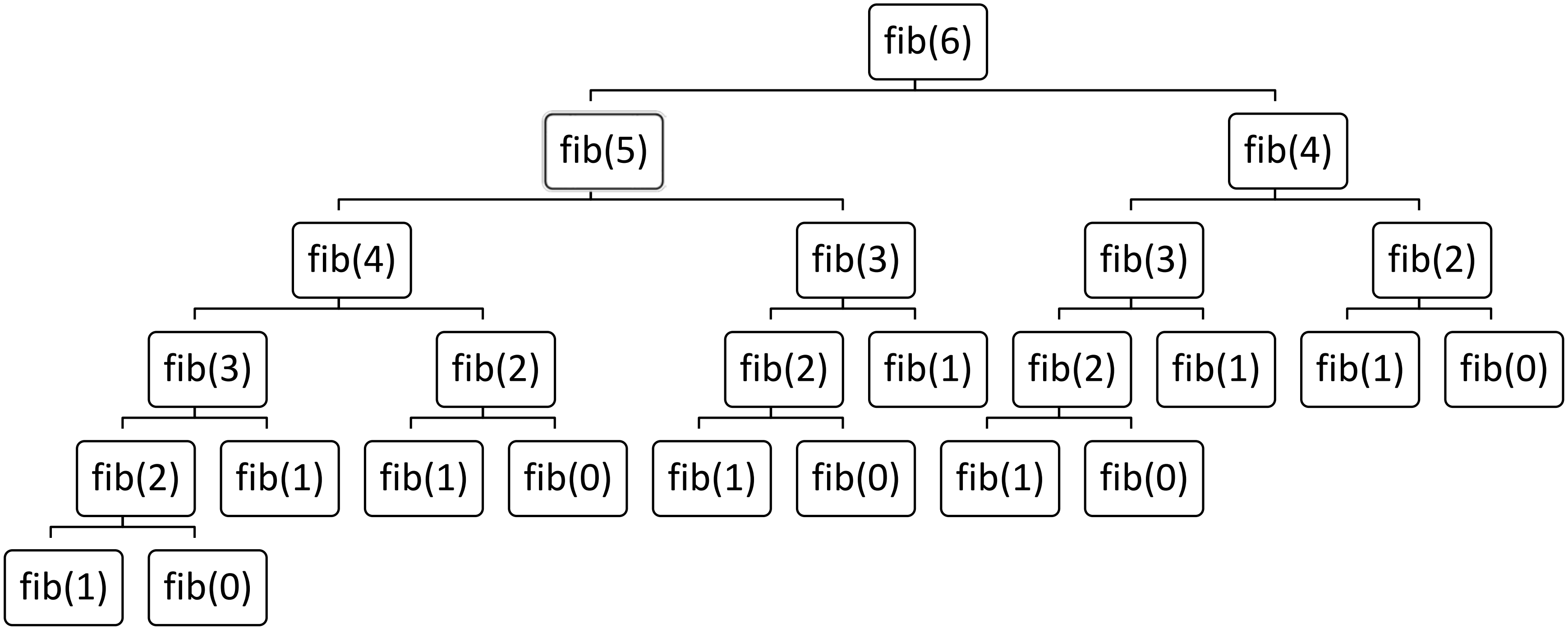
-- Richard Bellman

Recursive Implementation of Fibonnaci

```
def fib(n):
    if n == 0 or n == 1:
        return 1
    else:
        return fib(n - 1) + fib(n - 2)
```

```
fib(120) = 8,670,007,398,507,948,658,051,921
```

Call Tree for Recursive Fibonnaci(6) = 13



Clearly a Bad Idea to Repeat Work

- Trade a time for space
- Create a table to record what we've done
  - Before computing fib(x), check if value of fib(x) already stored in the table
    - If so, look it up
    - If not, compute it and then add it to table
  - Called memoization

Using a Memo to Compute Fibonnaci

```
def fastFib(n, memo = {}):
    """Assumes n is an int >= 0, memo used only by
         recursive calls
       Returns Fibonacci of n"""
    if n == 0 or n == 1:
        return 1
    try:
        return memo[n]
    except KeyError:
        result = fastFib(n-1, memo) +\
                 fastFib(n-2, memo)
        memo[n] = result
        return result
```

When Does It Work?

- Optimal substructure: a globally optimal solution can be found by combining optimal solutions to local subproblems
  - For $x > 1$, $fib(x) = fib(x - 1) + fib(x – 2)$

- Overlapping subproblems: finding an optimal solution involves solving the same problem multiple times
  - Compute fib(x) or many times

What About 0/1 Knapsack Problem?

- Do these conditions hold?