COP4453 Robot Vision - Spring 2023
Homework 2 - Filtering

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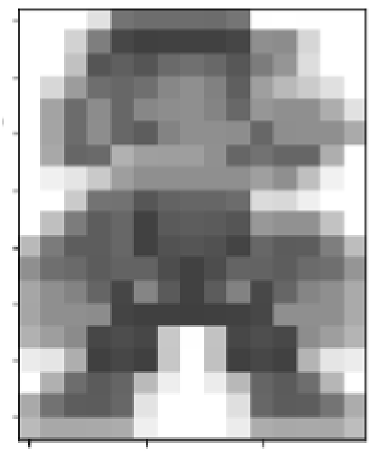
1. Consider the image below. What is the dimension of the matrix that represents the image? [10%]



```
[[255 255 255 225 115 109 109 109 108 117 254 255 254 255 255]
 [255 255 210 126  71  64  65  65  65  70 143 140 214 255 255]
 [255 255 193  85  95  85 106 112 104  87 124 149 221 255 255]
 [255 215 156 110 103 111 135 143 129  95 159 185 203 224 255]
 [255 163 108 140 103 135 141 143 132 104 150 143 143 173 225]
 [255 164 108 141 103  93 131 143 143 141 103 142 143 143 170]
 [255 168 102 106 177 159 158 158 144 102 114 102 102 174 255]
 [255 240 229 201 158 143 143 143 143 143 159 143 191 241 255]
 [255 255 202 115 114  86  99 102 102 108 219 216 239 255 255]
 [255 191 124  93 103  65  90  93  91  85 152 145 145 194 255]
 [185 123  93  93 103  65  82  85  81  68 103  92  93 126 189]
 [143 111 106  92  99  99  77  65  77 100  99  93 107 111 149]
 [166 143 131 101  70 133  90  65  92 130  72 102 133 143 170]
 [165 143 143 140  68  64  64  63  64  65  71 143 143 143 170]
 [177 158 139  71  77  71 200 254 194  71  78  72 141 158 180]
 [235 229 174  64  71  64 199 255 193  64  71  65 180 229 236]
 [255 176 109  91 101 186 238 255 236 184 101  91 110 182 255]
 [169 114  93  92 103 227 255 255 255 223 103  92  93 116 172]
 [186 168 168 169 172 239 255 255 255 236 172 168 169 168 189]]
```

Dimension of the matrix 19 x 15

2. If we filter using a 3x3 kernel and do not perform padding on the borders (convolution in the regions where filter and image are fully intersected), what will be the dimension of the previous image after filtering? [10%]

$(19 - 3 + 1)$ x $(15 - 3 + 1) = 17$ x 13

3. What will be the output dimension if the kernel is 5x5 (convolution in the regions where filter and image are fully intersected)? [10%]

$(19 - 5 + 1)$ x $(15 - 5 + 1) = 15$ x 11

4. Assuming the dimensions of the image are M x N. Can you come out with a general formula that tells you the dimension of the image after filtering if only considering 'valid' regions (where filter and image fully intersect)? [10%]

If the image has M rows and N columns, and the kernel has i rows and j columns, then the size of the image after filtering will be $(M - i + 1)$ x $(N - j + 1)$

5. Compute the output of applying the filter $\begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$ on the first 5 valid columns of row 2 of "mario" (row 2 is the one that starts with values 255 255 210 ..). Show your computations and write the obtained output. [10%]

```
[255 255 255 225 115 109 109 109 108
[255 255 210 126  71  64  65  65  65
[255 255 193  85  95  85 106 112 104
[255 215 156 110 103 111 135 143 129
[255 163 108 140 103 135 141 143 132
[255 164 108 141 103  93 131 143 143
[255 168 102 106 177 159 158 158 144
[255 240 229 201 158 143 143 143 143
[255 255 202 115 114  86  99 102 102
```

$255*1+255*2+255*1+255*0+255*0+210*0+255*-1+255*-2+193*-1=62$

```
[[255 255 255 225 115 109 109 109 108
 [255 255 210 126  71  64  65  65  65
 [255 255 193  85  95  85 106 112 104
 [255 215 156 110 103 111 135 143 129
 [255 163 108 140 103 135 141 143 132
 [255 164 108 141 103  93 131 143 143
 [255 168 102 106 177 159 158 158 144
 [255 240 229 201 158 143 143 143 143
 [255 255 202 115 114  86  99 102 102
```

$255*1+255*2+225*1+255*0+210*0+126*0+255*-1+193*-2+85*-1=264$

Replace value over 255 with 255

```
[[255 255 255 225 115 109 109 109 108
 [255 255 210 126  71  64  65  65  65
 [255 255 193  85  95  85 106 112 104
 [255 215 156 110 103 111 135 143 129
 [255 163 108 140 103 135 141 143 132
 [255 164 108 141 103  93 131 143 143
 [255 168 102 106 177 159 158 158 144
 [255 240 229 201 158 143 143 143 143
 [255 255 202 115 114  86  99 102 102
```

$255*1+225*2+115*1+210*0+126*0+71*0+193*-1+85*-2+95*-1=362$

Replace value over 255 with 255

```
[[255 255 255 225 115 109 109 109 108
 [255 255 210 126  71  64  65  65  65
 [255 255 193  85  95  85 106 112 104
 [255 215 156 110 103 111 135 143 129
 [255 163 108 140 103 135 141 143 132
 [255 164 108 141 103  93 131 143 143
 [255 168 102 106 177 159 158 158 144
 [255 240 229 201 158 143 143 143 143
 [255 255 202 115 114  86  99 102 102
```

$225*1+115*2+109*1+126*0+71*0+64*0+85*-1+95*-2+85*-1=204$

```
[[255 255 255 225 115 109 109 109 108
 [255 255 210 126  71  64  65  65  65
 [255 255 193  85  95  85 106 112 104
 [255 215 156 110 103 111 135 143 129
 [255 163 108 140 103 135 141 143 132
 [255 164 108 141 103  93 131 143 143
 [255 168 102 106 177 159 158 158 144
 [255 240 229 201 158 143 143 143 143
 [255 255 202 115 114  86  99 102 102
```

$115*1+109*2+109*1+71*0+64*0+65*0+95*-1+85*-2+106*-1=71$

Obtained values for the first 5 columns

255	255	255	255	115	109	109	...
255	62	264	362	204	71	65	...
255	255	193	85	95	85	106	...

6. Use the formula of 1D Gaussian function to find coefficients of a kernel of size 7 when $\sigma = 1.4$ Hint: x is evaluated in the interval [-3 -2 -1 0 1 2 3] [10%]

$$G(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$$

$$\begin{bmatrix} 0.029 & 0.103 & 0.221 & 0.285 & 0.221 & 0.103 & 0.029 \end{bmatrix}$$

7. The size of a gaussian kernel is usually chosen to have values in the order of 2 or 3 sigmas, since after that the values of the function are almost zero. In the extreme parts of this kernel (when x is either -3 or 3) how many sigmas it corresponds to? Is the chosen size of 7 a good value? [10%]

It corresponds to $\frac{3}{1.4} = 2.14\sigma$

The chosen size of 7 of the Gaussian kernel is a good value given that represents values in the order of 2σ

8. Approximate the obtained kernel as a fraction of integer numbers. Hint: use 64 as the denominator. [10%]

$$\frac{1}{64} \begin{bmatrix} 2 & 7 & 14 & 18 & 14 & 7 & 2 \end{bmatrix}$$

9. Compute a 7x7 Gaussian kernel using the 1D estimated kernel you estimated in the previous exercise. Remember, this is a separable filter and can be obtained using matrix multiplication. [10%]

$G = K_{7x1} * K_{1x7}$

$$G = \frac{1}{64} \begin{bmatrix} 2 \\ 7 \\ 14 \\ 18 \\ 14 \\ 7 \\ 2 \end{bmatrix} \frac{1}{64} \begin{bmatrix} 2 & 7 & 14 & 18 & 14 & 7 & 2 \end{bmatrix} = \frac{1}{4096} \begin{bmatrix} 4 & 14 & 28 & 36 & 28 & 14 & 4 \\ 14 & 49 & 98 & 126 & 98 & 49 & 14 \\ 28 & 98 & 196 & 252 & 196 & 98 & 28 \\ 36 & 126 & 252 & 324 & 252 & 126 & 36 \\ 28 & 98 & 196 & 252 & 196 & 98 & 28 \\ 14 & 49 & 98 & 126 & 98 & 49 & 14 \\ 4 & 14 & 28 & 36 & 28 & 14 & 4 \end{bmatrix}$$

10. In class we build a sharpen filter as the sum of the original filter + detail. The detail part was built with the original function and a box filter. Create a new kernel for sharpening but this time uses a gaussian filter. [10%]

To sharpen an image

- Blur image using Box, Tent, Gaussian
- Subtract blurred image from the original image to get high-frequency details
- Add high-frequency details to the original image

original + (original - gaussian) = 2 * original - gaussian

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} - \frac{1}{4096} \begin{bmatrix} 4 & 14 & 28 & 36 & 28 & 14 & 4 \\ 14 & 49 & 98 & 126 & 98 & 49 & 14 \\ 28 & 98 & 196 & 252 & 196 & 98 & 28 \\ 36 & 126 & 252 & 324 & 252 & 126 & 36 \\ 28 & 98 & 196 & 252 & 196 & 98 & 28 \\ 14 & 49 & 98 & 126 & 98 & 49 & 14 \\ 4 & 14 & 28 & 36 & 28 & 14 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} -1.000e-03 & -3.000e-03 & -7.000e-03 & -9.000e-03 & -7.000e-03 & -3.000e-03 & -1.000e-03 \\ -3.000e-03 & -1.200e-02 & -2.400e-02 & -3.100e-02 & -2.400e-02 & -1.200e-02 & -3.000e-03 \\ -7.000e-03 & -2.400e-02 & -4.800e-02 & -6.200e-02 & -4.800e-02 & -2.400e-02 & -7.000e-03 \\ -9.000e-03 & -3.100e-02 & -6.200e-02 & 1.921e+00 & -6.200e-02 & -3.100e-02 & -9.000e-03 \\ -7.000e-03 & -2.400e-02 & -4.800e-02 & -6.200e-02 & -4.800e-02 & -2.400e-02 & -7.000e-03 \\ -3.000e-03 & -1.200e-02 & -2.400e-02 & -3.100e-02 & -2.400e-02 & -1.200e-02 & -3.000e-03 \\ -1.000e-03 & -3.000e-03 & -7.000e-03 & -9.000e-03 & -7.000e-03 & -3.000e-03 & -1.000e-03 \end{bmatrix}$$

Rounded values