

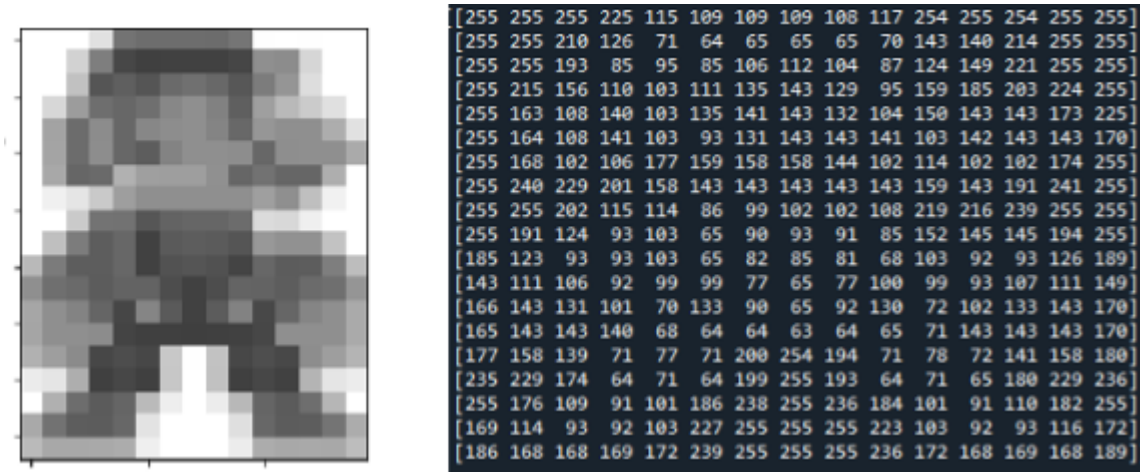
COP4453 Robot Vision - Spring 2023

Homework 2 - Filtering

Edelis Molina

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1. Consider the image below. What is the dimension of the matrix that represents the image? [10%]



Dimension of the matrix 19 x 15

2. If we filter using a 3x3 kernel and do not perform padding on the borders (convolution in the regions where filter and image are fully intersected), what will be the dimension of the previous image after filtering? [10%]

$$(19 - 3 + 1) \times (15 - 3 + 1) = 17 \times 13$$

3. What will be the output dimension if the kernel is 5x5 (convolution in the regions where filter and image are fully intersected)? [10%]

$$(19 - 5 + 1) \times (15 - 5 + 1) = 15 \times 11$$

4. Assuming the dimensions of the image are M x N. Can you come out with a general formula that tells you the dimension of the image after filtering if only considering 'valid' regions (where filter and image fully intersect)? [10%]

If the image has M rows and N columns, and the kernel has i rows and j columns, then the size of the image after filtering will be $(M - i + 1) \times (N - j + 1)$

5. Compute the output of applying the filter $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$ on the first 5 valid columns of row 2 of "mario" (row 2 is the one that starts with values 255 255 210 ..). Show your computations and write the obtained output. [10%]

[255	255	255	225	115	109	109	109	108
[255	255	210	126	71	64	65	65	65
[255	255	193	85	95	85	106	112	104
[255	215	156	110	103	111	135	143	129
[255	163	108	140	103	135	141	143	132
[255	164	108	141	103	93	131	143	143
[255	168	102	106	177	159	158	158	144
[255	240	229	201	158	143	143	143	143
[255	255	202	115	114	86	99	102	102

$$255 * 1 + 255 * 2 + 255 * 1 + 255 * 0 + 255 * 0 + 210 * 0 + 255 * -1 + 255 * -2 + 193 * -1 = 62$$

[255	255	255	225	115	109	109	109	108
[255	255	210	126	71	64	65	65	65
[255	255	193	85	95	85	106	112	104
[255	215	156	110	103	111	135	143	129
[255	163	108	140	103	135	141	143	132
[255	164	108	141	103	93	131	143	143
[255	168	102	106	177	159	158	158	144
[255	240	229	201	158	143	143	143	143
[255	255	202	115	114	86	99	102	102

$$255 * 1 + 255 * 2 + 225 * 1 + 255 * 0 + 210 * 0 + 126 * 0 + 255 * -1 + 193 * -2 + 85 * -1 = 264$$

Replace value over 255 with 255

[255	255	255	225	115	109	109	109	108
[255	255	210	126	71	64	65	65	65
[255	255	193	85	95	85	106	112	104
[255	215	156	110	103	111	135	143	129
[255	163	108	140	103	135	141	143	132
[255	164	108	141	103	93	131	143	143
[255	168	102	106	177	159	158	158	144
[255	240	229	201	158	143	143	143	143
[255	255	202	115	114	86	99	102	102

$$255 * 1 + 225 * 2 + 115 * 1 + 210 * 0 + 126 * 0 + 71 * 0 + 193 * -1 + 85 * -2 + 95 * -1 = 362$$

Replace value over 255 with 255

[255	255	255	225	115	109	109	109	108
[255	255	210	126	71	64	65	65	65
[255	255	193	85	95	85	106	112	104
[255	215	156	110	103	111	135	143	129
[255	163	108	140	103	135	141	143	132
[255	164	108	141	103	93	131	143	143
[255	168	102	106	177	159	158	158	144
[255	240	229	201	158	143	143	143	143
[255	255	202	115	114	86	99	102	102

$$225 * 1 + 115 * 2 + 109 * 1 + 126 * 0 + 71 * 0 + 64 * 0 + 85 * -1 + 95 * -2 + 85 * -1 = 204$$

[255	255	255	225	115	109	109	109	108
[255	255	210	126	71	64	65	65	65
[255	255	193	85	95	85	106	112	104
[255	215	156	110	103	111	135	143	129
[255	163	108	140	103	135	141	143	132
[255	164	108	141	103	93	131	143	143
[255	168	102	106	177	159	158	158	144
[255	240	229	201	158	143	143	143	143
[255	255	202	115	114	86	99	102	102

$$115 * 1 + 109 * 2 + 109 * 1 + 71 * 0 + 64 * 0 + 65 * 0 + 95 * -1 + 85 * -2 + 106 * -1 = 71$$

Obtained values for the first 5 columns

```
255 255 255 255 115 109 109 ...
255 62 264 362 204 71 65 ...
255 255 193 85 95 85 106 ...
```

6. Use the formula of 1D Gaussian function to find coefficients of a kernel of size 7 when $\sigma = 1.4$ Hint: x is evaluated in the interval $[-3 -2 -1 0 1 2 3]$ [10%]

$$G(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}}$$

[0.029 0.103 0.221 0.285 0.221 0.103 0.029]

7. The size of a gaussian kernel is usually chosen to have values in the order of 2 or 3 sigmas, since after that the values of the function are almost zero. In the extreme parts of this kernel (when x is either -3 or 3) how many sigmas it corresponds to? Is the chosen size of 7 a good value? [10%]

It corresponds to $\frac{3}{1.4} = 2.14\sigma$

The chosen size of 7 of the Gaussian kernel is a good value given that represents values in the order of 2σ

8. Approximate the obtained kernel as a fraction of integer numbers. Hint: use 64 as the denominator. [10%]

$\frac{1}{64} [2 \ 7 \ 14 \ 18 \ 14 \ 7 \ 2]$

9. Compute a 7x7 Gaussian kernel using the 1D estimated kernel you estimated in the previous exercise. Remember, this is a separable filter and can be obtained using matrix multiplication. [10%]

$$G = K_{7 \times 1} * K_{1 \times 7}$$

$$G = \frac{1}{64} \begin{bmatrix} 2 \\ 7 \\ 14 \\ 18 \\ 14 \\ 7 \\ 2 \end{bmatrix} \frac{1}{64} [2 \ 7 \ 14 \ 18 \ 14 \ 7 \ 2] = \frac{1}{4096} \begin{bmatrix} 4 & 14 & 28 & 36 & 28 & 14 & 4 \\ 14 & 49 & 98 & 126 & 98 & 49 & 14 \\ 28 & 98 & 196 & 252 & 196 & 98 & 28 \\ 36 & 126 & 252 & 324 & 252 & 126 & 36 \\ 28 & 98 & 196 & 252 & 196 & 98 & 28 \\ 14 & 49 & 98 & 126 & 98 & 49 & 14 \\ 4 & 14 & 28 & 36 & 28 & 14 & 4 \end{bmatrix}$$

10. In class we build a sharpen filter as the sum of the original filter + detail. The detail part was built with the original function and a box filter. Create a new kernel for sharpening but this time uses a gaussian filter. [10%]

To sharpen an image

- Blur image using Box, Tent, Gaussian
- Subtract blurred image from the original image to get high-frequency details
- Add high-frequency details to the original image

$$\text{original} + (\text{original} - \text{gaussian}) = 2 * \text{original} - \text{gaussian}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} - \frac{1}{4096} \begin{bmatrix} 4 & 14 & 28 & 36 & 28 & 14 & 4 \\ 14 & 49 & 98 & 126 & 98 & 49 & 14 \\ 28 & 98 & 196 & 252 & 196 & 98 & 28 \\ 36 & 126 & 252 & 324 & 252 & 126 & 36 \\ 28 & 98 & 196 & 252 & 196 & 98 & 28 \\ 14 & 49 & 98 & 126 & 98 & 49 & 14 \\ 4 & 14 & 28 & 36 & 28 & 14 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} -1.000e-03 & -3.000e-03 & -7.000e-03 & -9.000e-03 & -7.000e-03 & -3.000e-03 & -1.000e-03 \\ -3.000e-03 & -1.200e-02 & -2.400e-02 & -3.100e-02 & -2.400e-02 & -1.200e-02 & -3.000e-03 \\ -7.000e-03 & -2.400e-02 & -4.800e-02 & -6.200e-02 & -4.800e-02 & -2.400e-02 & -7.000e-03 \\ -9.000e-03 & -3.100e-02 & -6.200e-02 & 1.921e+00 & -6.200e-02 & -3.100e-02 & -9.000e-03 \\ -7.000e-03 & -2.400e-02 & -4.800e-02 & -6.200e-02 & -4.800e-02 & -2.400e-02 & -7.000e-03 \\ -3.000e-03 & -1.200e-02 & -2.400e-02 & -3.100e-02 & -2.400e-02 & -1.200e-02 & -3.000e-03 \\ -1.000e-03 & -3.000e-03 & -7.000e-03 & -9.000e-03 & -7.000e-03 & -3.000e-03 & -1.000e-03 \end{bmatrix}$$

Rounded values