Question 1 [6 points]

TechEssentials Inc. is a prominent player in the technology industry, renowned for its diverse range of laptop computers. The company's portfolio is segmented into three primary product lines: EvoTech Pro Series, InfiniteEdge Notebooks, and FusionBook Elite Series. Each of these lines encompasses three versions – Basic, Advanced, and Premium – designed to cater to varying consumer needs and preferences. TechEssentials Inc.'s key objective is to strategically set the prices for these products to achieve maximum revenue. This pricing strategy takes into account the unique market dynamics, including the sensitivity of consumer demand to price changes and production limitations.

Each product has a linear price response relationship with demand. This means that as the price of a product increases, the demand tends to decrease and vice versa. Moreover, an interesting aspect of TechEssentials Inc.'s product lines is the cross-elasticity of demand observed within each line. The price set for one version of a product, such as the Basic EvoTech Laptop, can influence demand for its other product lines and versions. This interconnected demand pattern necessitates a careful consideration of pricing strategies across all products to avoid cannibalization of sales.

Given the data set of price response functions, formulate and solve a nonlinear optimization model to determine the optimal price points for each of the products and product lines.

(a) When TechEssentials initially launched 15 years ago, it had just one product line (the EvoTech Pro Series computers) with two versions: Basic and Advanced. Apply the KKT conditions to find the optimal prices of each of the two computers within this series, taking into account that the Advanced version should have a higher price as compared to the Basic one. That is,

$$\text{Maximize } p_1 (a_1 - b_1 p_1) + p_2 (a_1 - b_1 p_2)$$

for some prices $p_i \geq 0$ and linear price response function coefficients $a_i \geq 0$ and $b_i \geq 0$ where $i = \{1, 2\}$ subject to price ordering and demand non-negativity constraints.

(b) Contrast the solution from the previous question with the solution obtained from the application of a projected gradient descent algorithm. Note that the constraint set is linear, so the *projection* step can be solved as a quadratic program with Gurobi. Start with all prices initialized to zero, a step size of 0.001, and a stopping criterion of 10^{-6}, what do you get as the optimal prices?

(c) Now consider all three product lines and the three versions within each product line. Assuming that the Basic, Advanced, and Premium versions of the product should be increasing in price within each product line, formulate and solve a quadratic optimization problem in Gurobi. What is the optimal revenue suggested by the model?

(d) Considering that prices should increase (i) within each product line (Basic, Advanced, Premium), and (ii) within the same version (e.g., a FusionBook Elite should be more expensive than an InfiniteEdge Notebook which should be more expensive than an EvoTech Pro), modify the previous quadratic optimization problem by adding new constraints to ensure this logic is captured in the model. What is the optimal revenue TechEssentials can anticipate now?

(e) What set of prices, from (c) or (d), do you think makes the most sense to implement in practice?

(f) Do you believe something is missing such that the model does not faithfully represent reality.

Question 2 [6 points]

Víctor Lapeña, the head coach of the Canadian women's national team, is currently involved in the midst of a rigorous recruitment process. He is meticulously working to finalize the list of 21 training camp invitations to solidify the team for the upcoming FIBA Olympic Qualifying tournament. To aid in this decision-making process, Lapeña's coaching staff has conducted a thorough assessment of each player that may be invited, evaluating their skills in ball handling, shooting, rebounding, defense, athletic ability, toughness, and mental acuity. Each player has been assigned a rating, with a score of 3 representing excellence, 2 indicating marginal performance, and 1 denoting mediocrity.

- At least 30% of the invitations should go to players that can play the guard position (G, G/F) and at least 40% should go to players that can play Forward/Center (F, C, F/C).
- For each of the seven skills, the average value of all invited players should exceed 2.05.
- If any player from 20-24 (inclusive) is invited, all players from 72-78 (inclusive) cannot be.
- If any player from 105-114 (inclusive) is invited, at least one player from 45-49 (inclusive) and 65-69 (inclusive) must be invited.
- At least one player must be invited from: $1-10$, $11-20$, $21-30$, ..., $131-140$, $141-50$.

Detailed information on each player has been provided in a data file. Formulate and solve a mathematical program that will help Víctor determine which players to invite to the training camp.

(a) What types of decision variables are needed to solve the problem?

(b) How many decision variables must be included in the model?

(c) What is the objective function? Defend why this is an appropriate choice.

(d) Write down the following constraint in mathematical notation but only for player 72: *If any player from 20-24 (inclusive) is invited, all players from 72-78 (inclusive) cannot be.*

(e) Write down the following constraint in mathematical notation: *At least 30% of the invitations should go to players that can play the guard position (G, G/F).*

(f) What is the optimal objective function value?

(g) How many guards (G, G/F) are invited to the training camp?

(h) What is the smallest number of training camp invitations that can be sent before the model yields an infeasible solution? What constraint(s) cannot be satisfied?

(i) Describe (do not implement) the challenge of modifying your solution approach to ensure that players with a total score of 12 or under would not be invited to training camp.

(j) What do you perceive as a problem with Victor's approach of choosing participants?