Options trading is a form of derivative trading where the trader has the right, but not the obligation, to buy or sell an underlying asset at a predetermined price within a specific period. The two main types of options are calls (which give the right to buy) and puts (which give the right to sell).

Options Trading

Key Concepts:

1. Underlying Asset: The asset on which the option is based (e.g., stocks, indices, commodities).
2. Strike Price: The price at which the option holder can buy or sell the underlying asset.
3. Expiration Date: The date on which the option expires.
4. Premium: The price paid for the option.
5. Call Option: Grants the holder the right to buy the underlying asset at the strike price.
6. Put Option: Grants the holder the right to sell the underlying asset at the strike price.

Strategies:

- Long Call: Buying a call option to profit from a price increase in the underlying asset.
- Long Put: Buying a put option to profit from a price decrease in the underlying asset.
- Covered Call: Owning the underlying asset while selling a call option on the same asset to generate income.
- Protective Put: Buying a put option while owning the underlying asset to hedge against a price decline.

Binomial Options Pricing Model

The Binomial Options Pricing Model (BOPM) is a discrete-time model used to price options. It involves the following steps:

1. Construct a Binomial Tree: A tree representing possible future prices of the underlying asset at different time steps.
2. Calculate Option Values at Expiration: Determine the payoff of the option at each final node of the tree.
3. Work Backwards through the Tree: Use the risk-neutral probability to calculate the expected value of the option at each preceding node.

Formula:

For a single period, the price of a call option C is given by:

$$ C = e^{-r \Delta t} [p \cdot C_u + (1 - p) \cdot C_d] $$

where:

- $e^{-r \Delta t}$ is the discount factor.
- p is the risk-neutral probability.
- C_u and C_d are the option values in the up and down states.

Steps:

1. Set Parameters: Risk-free rate r, volatility σ, time to expiration T, number of steps N.
2. Calculate Up and Down Factors: $u = e^{\sigma \sqrt{\Delta t}}$, $d = e^{-\sigma \sqrt{\Delta t}}$.
3. Calculate Risk-neutral Probability: $p = \frac{e^{r \Delta t} - d}{u - d}$.
4. Build the Price Tree.
5. Calculate Payoffs and Discount Back.

Black-Scholes Model

The Black-Scholes Model (BSM) is a continuous-time model used to price European options. It provides a closed-form solution for the price of call and put options.

Assumptions:

- The stock price follows a geometric Brownian motion with constant volatility and drift.
- No dividends are paid.
- Markets are frictionless (no transaction costs or taxes).
- The risk-free interest rate is constant.
- The option can only be exercised at expiration (European style).

Black-Scholes Formula:

For a European call option:

$$ C = S_0 N(d_1) - X e^{-rT} N(d_2) $$

For a European put option:

$$ P = X e^{-rT} N(-d_2) - S_0 N(-d_1) $$

where:

$$ d_1 = \frac{\ln(S_0 / X) + (r + \sigma^2 / 2) T}{\sigma \sqrt{T}} $$

$$ d_2 = d_1 - \sigma \sqrt{T} $$

- S_0 = Current stock price
- X = Strike price
- T = Time to expiration
- r = Risk-free interest rate
- σ = Volatility of the underlying asset
- $N(\cdot)$ = Cumulative distribution function of the standard normal distribution

Greeks:

- Delta (Δ): Sensitivity of the option price to changes in the price of the underlying asset.
- Gamma (Γ): Sensitivity of delta to changes in the price of the underlying asset.
- Theta (Θ): Sensitivity of the option price to the passage of time.
- Vega (ν): Sensitivity of the option price to changes in volatility.
- Rho (ρ): Sensitivity of the option price to changes in the risk-free interest rate.

Summary

- Options Trading: Buying and selling options to manage risk or speculate.
- Binomial Model: Uses a discrete-time approach to model option pricing through binomial trees.

- Black-Scholes Model: Provides a closed-form solution for European options using a continuous-time approach.

Both models are foundational in the field of financial derivatives and help traders and risk managers to price options and understand their risk characteristics.