

# MathML in Browsers

Web Engines Hackfest - October, 2019

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

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<b>1977-1984</b> <b>Math typesetting rules</b>	 T <sub>E</sub> X typesetting system	The T <sub>E</sub> Xbook
<b>1993-1995</b> <b>Browser experiments</b>	 <MATH> Arena	 HTML
<b>1998-1999</b> <b>MathML standard</b>	 MathML Gecko	 MathML 1
<b>2002-2003</b> <b>Gecko/Mozilla</b>	 Mozilla 1.0	MathML 2
<b>2006-2007</b> <b>Presto/Opera, Microsoft Office</b>	 MathML Opera 9.5	 OpenType MATH
<b>2008</b> <b>MathML in HTML5!</b>	 (I) HISTORY	 HTML5
<b>2009-2011</b> <b>WebKit/Safari</b>	 MathML WebKit	MathML 3
<b>2012-2013</b> <b>WebKit/Chrome, Blink fork</b>	 MathML Chrome 24	 Blink Chrome 28
<b>2014</b> <b>OpenType MATH</b>	 OpenType WebKit/Gecko	MathML 3 2nd Edition
<b>2015-2016</b> <b>WebKit's refactoring</b>	 MathML WebKit	Implementation Note for MathML
<b>2019-</b> <b>Blink/Chrome, Edge switch</b>	 MathML Chromium	MathML Core & MathML 4



# THE <MATH> TAG

Consider the equation:

$$H(s) = \int_0^{\infty} e^{-st} h(t) dt$$

This can be represented as:

```
<math>
    H(s) = \int_{<sub>0</sub>}^{<sup>\infty</sup>} e^{<sup>-st</sup>} h(t) dt
</math>
```

The mathematical symbols are given with their standard ISO codes. The operators, the subscript/superscript text is centered over the symbol, and the integral limits are placed outside the symbol. This allows you to define more complex equations, as in:

$$C \frac{dV_{out}}{dt} = I_b \tanh\left(\frac{\kappa(V_{in} - V_{out})}{2}\right)$$

which is represented by:

```
<math>
    C \frac{dV_{<sub>out</sub>}}{dt} = I_{<sub>b</sub>} \tanh\left(\frac{\kappa(V_{<sub>in</sub>} - V_{<sub>out</sub>})}{2}\right)
</math>
```

# MATHML IN MOZILLA



A simple MathML example:  $\cos x = \frac{1}{\sqrt{1+\tan^2 x}}$

and more complex examples ...

$$\sigma_X = \sqrt{\frac{1}{n} \left\{ \sum_{i=1}^n X_i^2 - \frac{1}{n} \left( \sum_{i=1}^n X_i \right)^2 \right\}}$$
$$\overline{U}_\mu(\hat{x}) = \prod_{j=1}^n \left( \prod_{\substack{i=0 \\ i \neq \mu_j}}^k \frac{k \hat{x}_j - i}{\mu_j - i} \right)$$

*"One of the quietest open source achievers in Australia"* - [the SMH](#)

# MATHML IN HTML5

## MathML-in-HTML5

89 posts by 11 authors 



r...@maths.uq.edu.au

**Other recipients:** dev-tec...@lists.mozilla.org, dev-tec...@lists.mozilla.or

I am currently driving an effort to enable MathML-in-HTML (apart from MathML-in-XHTML that we already support). I have a patch that serves the dual purpose of showing where things are going and the issues to ponder about.

Here is a

[screenshot] <https://bugzilla.mozilla.org/attachment.cgi?id=239771>

which is a \_live\_ rendering of this testcase:

[mathml-in-html] <https://bugzilla.mozilla.org/attachment.cgi?id=239769>

Those interested in following this up can see bug 353926:

[https://bugzilla.mozilla.org/show\\_bug.cgi?id=353926](https://bugzilla.mozilla.org/show_bug.cgi?id=353926)

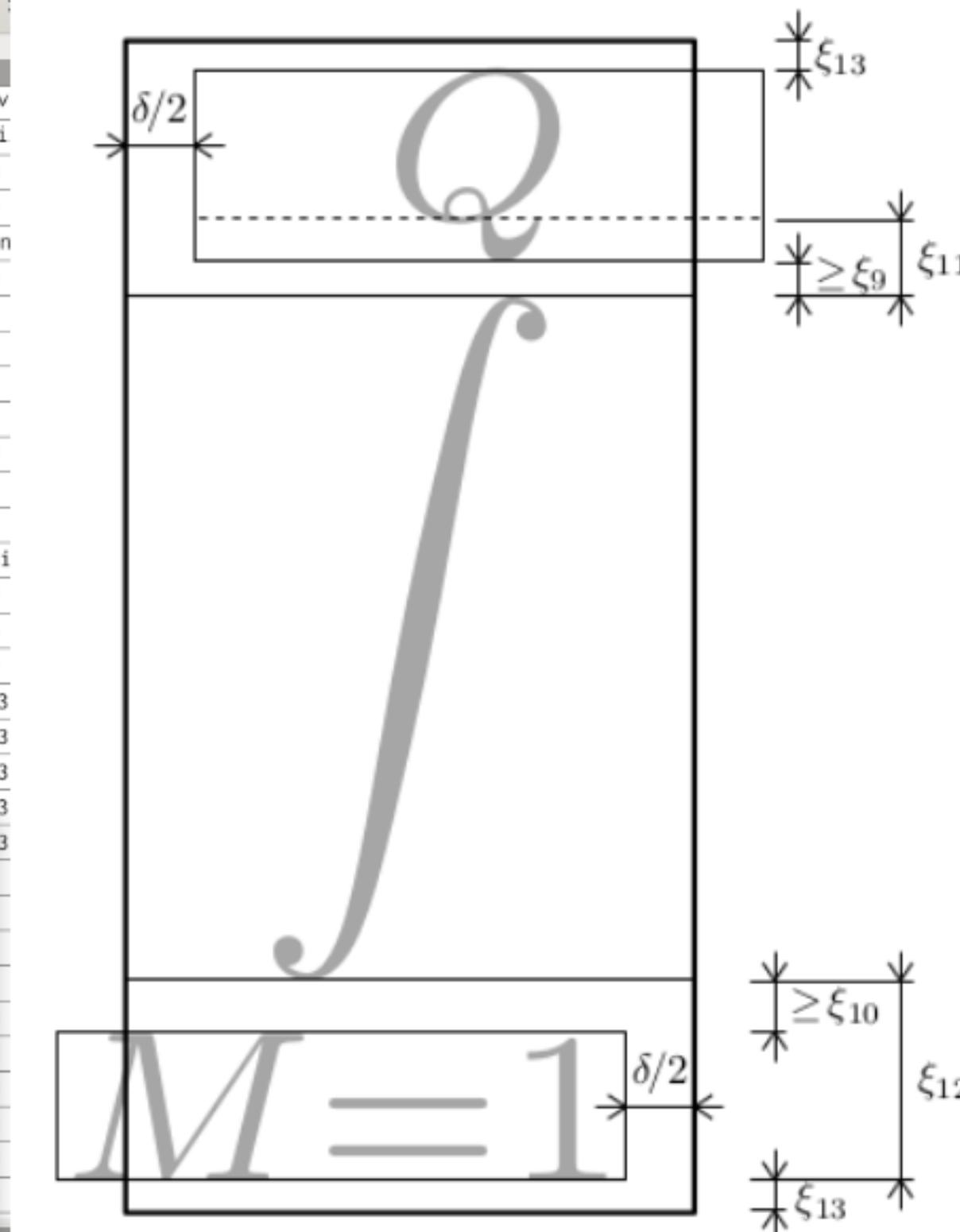
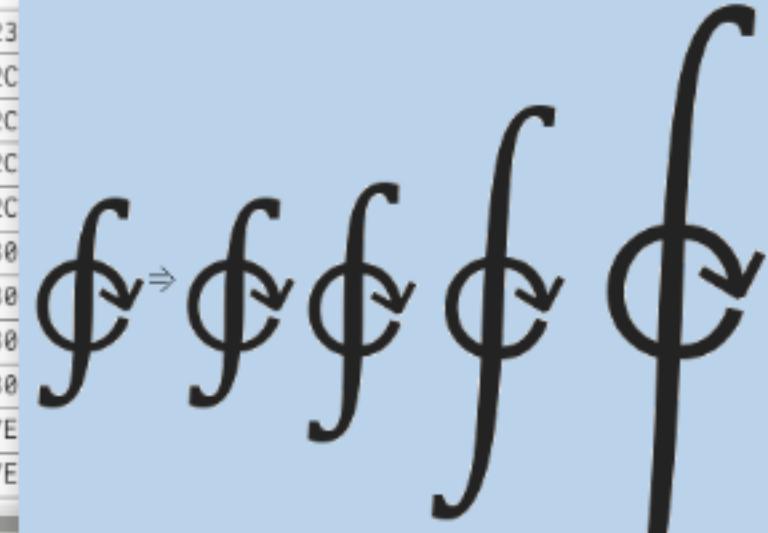
Quick background:

=====

At the Firefox engineering meeting in Mountain Views (last December 2005), I pleaded that we enable MathML in HTML5 to advance the cause

# MICROSOFT OFFICE 2007

MATH table	
Constants	
Sub/Superscript	
Limits	
Stacks	
Fractions	
Over/Underbars	
Radicals	
Connectors	
Exten Shapes	
Italic Correction	
Top Accent	
Math Kern	
Vert. Variants	
Vert. Construction	
Hor. Variants	
Hor. Construction	
uni2210(ʃ)	uni2210(ʃ) uni2210.vsize1 uni2210.vsize2 uni2210.vsize3
uni2223( )	uni2223( )
uni2225(  )	uni2225(  )
integral(ʃ)	integral(ʃ) integral.vsize1 integral.vsize2 integral.vsi
uni222C(ʃ)	uni222C(ʃ) uni222C.vsize1 uni222C.vsize2 uni222C.vsize3
uni222D(ʃ)	uni222D(ʃ) uni222D.vsize1 uni222D.vsize2 uni222D.vsize3
uni222E(ɸ)	uni222E(ɸ) uni222E.vsize1 uni222E.vsize2 uni222E.vsize3
uni222F(ɸ)	uni222F(ɸ) uni222F.vsize1 uni222F.vsize2 uni222F.vsize3
uni2230(ɸ)	uni2230(ɸ) uni2230.vsize1 uni2230.vsize2 uni2230.vsize3
uni2231(ɸ)	uni2231(ɸ) uni2231.vsize1 uni2231.vsize2 uni2231.vsize3
uni2232(ɸ)	uni2232(ɸ) uni2232.vsize1 uni2232.vsize2 uni2232.vsize3
uni2233(ɸ)	uni2233(ɸ)
uni22C0(ʌ)	uni22C0(ʌ)
uni22C1(ʌ)	uni22C1(ʌ)
uni22C2(ʌ)	uni22C2(ʌ)
uni22C3(ʌ)	uni22C3(ʌ)
uni2308(ʃ)	uni2308(ʃ)
uni2309( )	uni2309( )
uni230A( )	uni230A( )
uni230B( )	uni230B( )
uni27E6(ʃ)	uni27E6(ʃ)
uni27E7(ʃ)	uni27E7(ʃ)



# WEBKIT'S REFACTORING



"¡Qué horrible es esta implementación!" - alex@

# MATHML IN CHROMIUM

The screenshot shows the 'MathML' section of the Chrome Platform Status page. At the top, there's a navigation bar with a menu icon, the Chrome logo, and the text 'Chrome Platform Status'. Below this are four tabs: 'All features' (selected), 'Releases', 'Samples', and 'Stats'. To the right of the tabs are links for 'Misc', a bell icon for notifications, and a gear icon for settings. The main content area has a heading 'MathML' followed by a sub-heading 'A markup language for describing mathematical notation and capturing both its structure and content.' Below this are several links: 'Comments', 'Igalia is working on it: <https://mathml.igalia.com>', 'Specification', 'Working draft or equivalent', 'Status in Chromium', 'Blink components: Blink', and 'In development ([tracking bug](#))'.

DECEMBER 6, 2018 9:00 AM

## Microsoft Edge: Making the web better through more open source collaboration

By [Joe Belfiore](#) / Corporate Vice President, Windows

For the past few years, Microsoft has meaningfully increased participation in the open source software (OSS) community, becoming one of the world's largest supporters of OSS projects. Today we're announcing that we intend to adopt the Chromium open source project in the development of Microsoft Edge on the desktop to create better web compatibility for our customers and less fragmentation of the web for all web developers.

As part of this, we intend to become a significant contributor to the Chromium project, in a way that can make not just Microsoft Edge — but other browsers as well — better on both PCs and other devices.



2.1.1	HTML and SVG
2.1.2	CSS styling
2.1.3	DOM and Javascript
2.1.4	Text layout
2.1.5	Focus
2.2	Types for MathML Attribute Values
2.3	Global Attributes
2.3.1	Attributes common to HTML and MathML elements
2.3.1.1	Event Handler Content Attributes
2.3.2	Legacy MathML Style Attributes
2.3.3	The <code>mathvariant</code> attribute
2.3.4	The <code>displaystyle</code> and <code>scriptstyle</code> attributes
2.4	The Top-Level <code>&lt;math&gt;</code> Element

### 3. Presentation Markup

#### 3.1 Visual formatting model

##### 3.1.1 Box Model

##### 3.1.2 Layout Algorithms

#### 3.2 Token Elements

##### 3.2.1 Text `<mtext>`

###### 3.2.1.1 Layout of `<mtext>`

###### 3.2.1.2 OpenType features

##### 3.2.2 Identifier `<mi>`

##### 3.2.3 Number `<mn>`

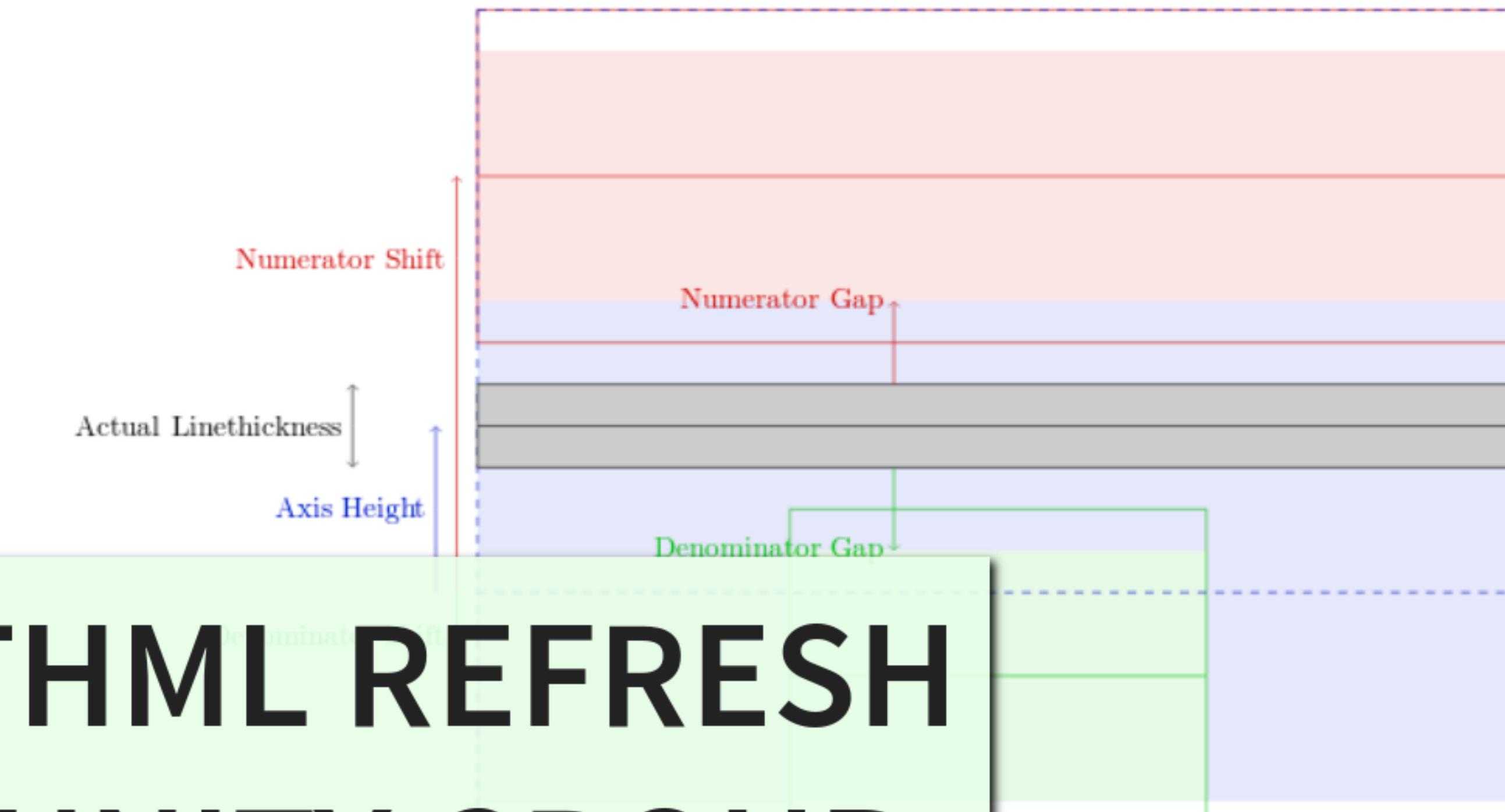
##### 3.2.4 Operator, Fence, Separator or Accent `<mo>`

###### 3.2.4.1 Embellished operators

###### 3.2.4.2 Dictionary-based attributes

###### 3.2.4.3 Layout of operators

###### 3.2.5 Other presentation elements



## (II) MATHML REFRESH COMMUNITY GROUP

The min-content (respectively max-content) inline size of content is the maximum between the min-content (respectively max-content) inline size of the numerator's margin box and the min-content (respectively max-content) of the denominator's margin box.

If there is an [inline stretch size constraint](#) or a [block stretch size constraint](#) then the numerator is also laid out with the same stretch size constraint otherwise it is laid out without any stretch size constraint. The denominator is always laid out without any stretch size constraint.

The inline size of the content is the maximum between the inline size of the numerator's margin box and the inline size of the denominator's margin box.

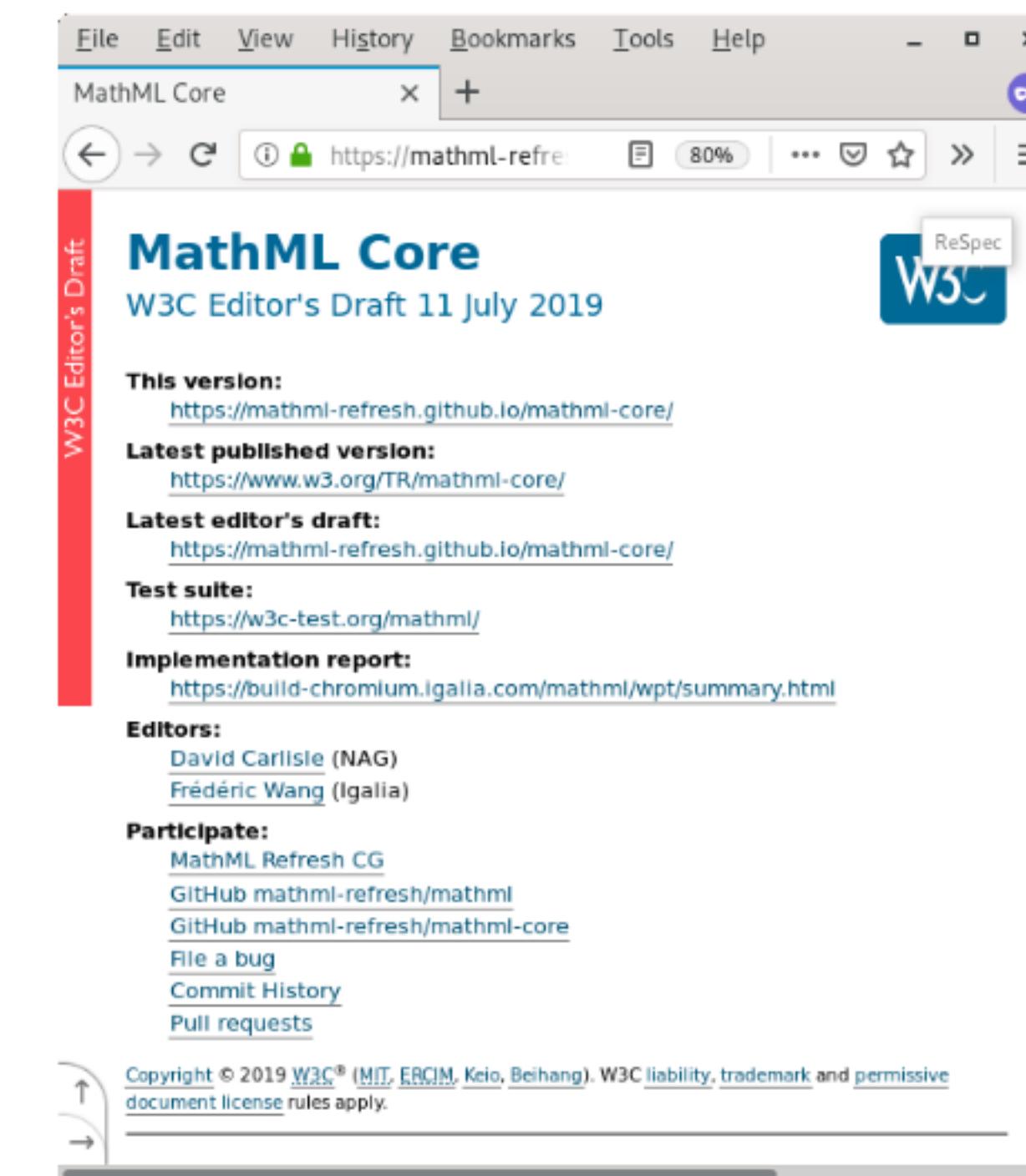
`NumeratorShift` is the maximum between:

`FractionNumeratorShiftUp` (respectively `FractionNumeratorDisplayStyleShiftUp`) if the `math`



# MATHML CORE

- Fundamental subset
  - Ask CG members
  - Analyze tools & use statistics
- Implementation details
  - TeX/OpenType layout
  - HTML5/CSS compatibility
- Browser-driven
  - Gecko/WebKit/Blink
  - Web Platform Tests!



# EXTENSIBILITY

- MathMLElement IDL (1)
  - \*EventHandlers
  - ElementCSSInlineStyle
  - HTMLOrForeignElement
- CSS/Houdini
  - CSS properties? (2)
  - Layout constraints? (3)
  - Font APIs? (4)
- New MathML elements
  - Overridable display (5)
  - Element.shadowDOM? (6)
  - Custom element? (7)

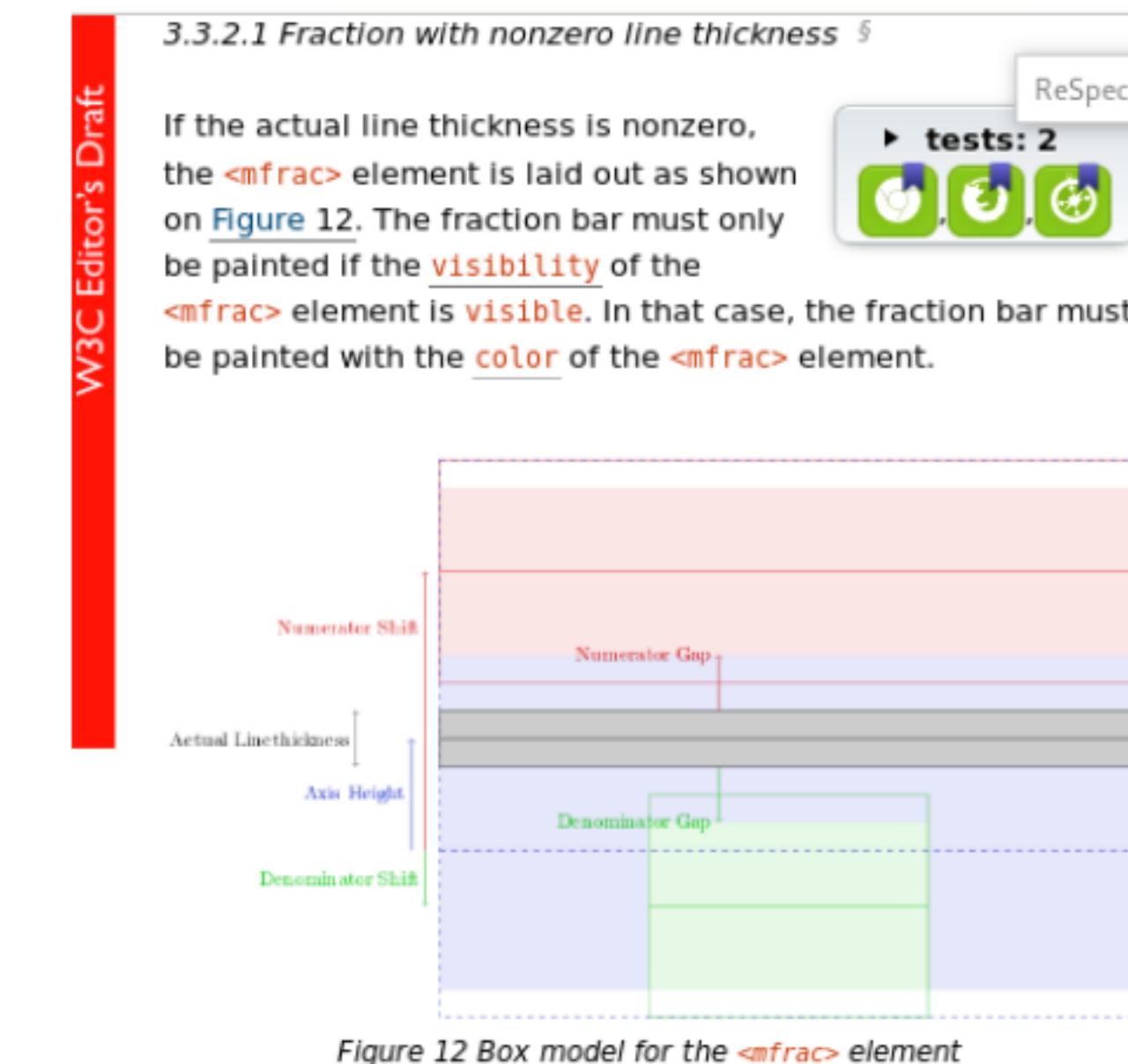
$$\frac{1+A}{2-B} \text{ VS } \frac{1+\mathfrak{A}}{2-\mathcal{B}}$$

$$\frac{1+A}{2-B} \text{ VS } \frac{1+A}{2-B}$$

```
<math>
  <mfenced open="{" close="}" separators=", ;">
    #shadowRoot
    <mrow>
      <mo>{</mo>
      <mi>a</mi>
      <mo>,</mo>
      <mi>b</mi>
      <mo>;</mo>
      <mi>c</mi>
      <mo>}</mo>
    </mrow>
  </mfenced>
</math>
```

# CSS COMPATIBILITY

- CSS layout
  - Visual box model?
  - Supported features?
  - Interpretation?
- Math-specific
  - Invalid markup
  - New CSS properties
  - OpenType parameters
  - Special painting
  - Text metrics
  - Operator Stretching



The min-content (respectively max-content) inline size of content is the maximum between the min-content (respectively max-content) inline size of the numerator's margin box and the min-content (respectively max-content) of the denominator's margin box.

If there is an [inline stretch size constraint](#) or a [block stretch size constraint](#) then the numerator is also laid out with the same stretch size constraint otherwise it is laid out without any stretch size constraint. The denominator is always laid out without any stretch size constraint.

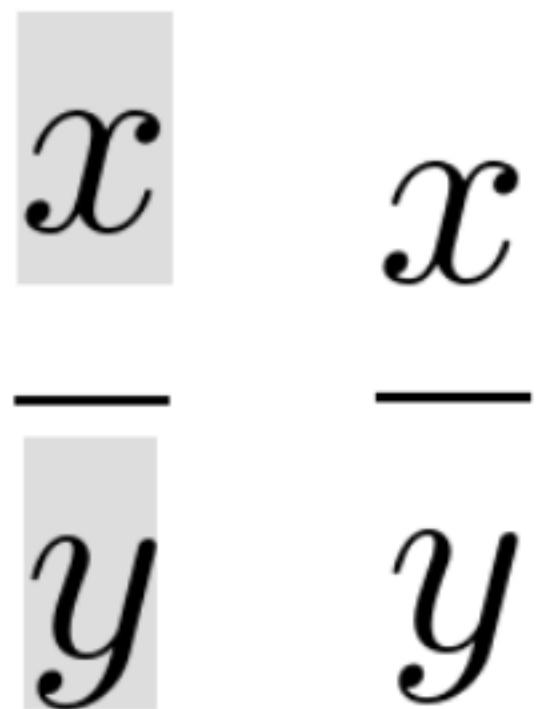
The inline size of the content is the maximum between the inline size of the numerator's margin box and the inline size of the denominator's margin box.

NumeratorShift is the maximum between:

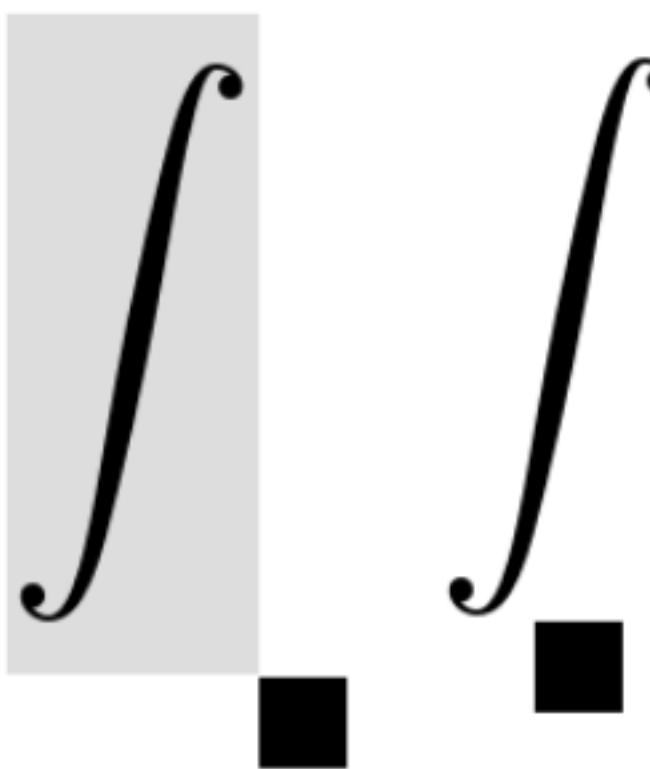


# TEXT METRICS

- Ink block metrics (1, 2)


$$\begin{array}{c} x \quad x \\ \hline y \quad y \end{array}$$

- Italic correction (3)



# OPERATOR STRETCHING

- Stretch constraints (1)

Horizontal stretch size

Vertical stretch size

above/below baseline

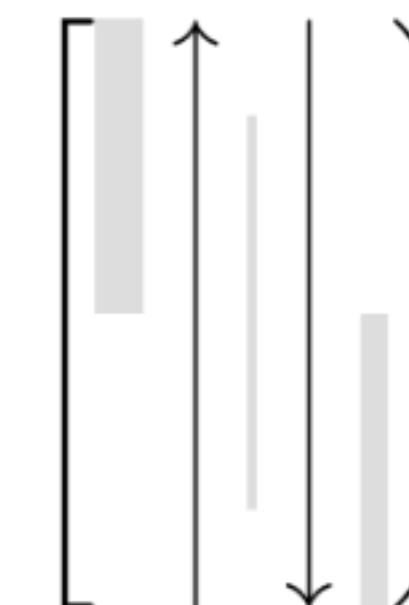


- Parent layout (2, 3)

Layout non-stretchy children

Calculate stretch constraint

Layout stretchy children



- Real-life operators

~~Distort glyph~~

Upper estimate of min/max (4)



work in progress. The reported results do not necessarily reflect the true capabilities of each web browser, so they should not be used to evaluate or port.

## Test Results

## Interoperability

# (III) BROWSER INTEROPERABILITY

les, like 'cors/allow-headers.htm', then

press <Enter>

669 tests (1673955 subtests) from the latest master test runs for chrome[experimental], edge[experimental], firefox[experimental],  
mental]

For information on the search syntax, [view the syntax](#)

LINK

### Path

 Dev  
Chrome 78  
Linux 18.04  
 [8488b4e](#)  
Sep 24, 2019

 DEV  
Edge 78  
Windows 10.0  
 [8488b4e](#)  
Sep 24, 2019

   
Firefox 71  
Linux 18.04  
 [8488b4e](#)  
Sep 24, 2019

   
Safari 82 pre  
macOS 10.  
 [8488b4e](#)  
Sep 24, 2019

### Non-markup/

37 / 477  
84 / 1502

37 / 477  
84 / 1502

386 / 477  
762 / 1502

348 / 477  
723 / 1503



# WEB PLATFORM TESTS

- ~2000 tests ([1](#), [2](#), [3](#))
  - Math layout
  - Removed features
  - DOM/CSS interaction
- Results ([4](#))
  - Igalia's Blink: ~99%
  - Gecko: ~73%
  - WebKit: ~72%

## MathML Core - Implementation Report

Blink-MathML  1944/1971 (98.63%) [+](#)

Development build of [Igalia's Chromium fork](#).

Blink  17/1971 (0.86%) [+](#)

Development build of [Igalia's Chromium fork](#) with the LayoutNGMathML runtime feature disabled.

*These results might contain false positives.*

Gecko  1434/1971 (72.75%) [+](#)

[Nightly build](#) of Firefox.

WebKit  1418/1971 (71.94%) [+](#)

Development build of [WebKitGTK](#).

# PAST IMPLEMENTATIONS

Firefox/Gecko  
2008

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

---

Chrome/WebKit  
2013

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

---

Opera/Presto  
2007

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

# 2019 IMPLEMENTATIONS

**Firefox/Gecko**  
July release

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

---

**Epiphany/WebKit**  
Build r249360

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

---

**Chromium/Blink**  
Igalia's Branch

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

# GECKO AND WEBKIT (1, 2)

- Standardize browser behaviors
  - MathML3 interpretations
  - "Hacks" for legacy content
- Unship features
  - Counters & Deprecation (3)
  - Runtime flag
- Tests
  - Convert & synchronize
  - More "pass" / Less "flaky"
- Enhancement and bug fixes
  - Math DOM (4, 5)
  - Math layout (6, 7, 8, ...)
  - CSS compatibility



# Chromium builder



NAVIGATION

Home



Grid View



Waterfall View



Console View



&gt; Builds



Builders

Last Changes

Build Masters

Schedulers

Workers

About



Settings



2	⌚ install_build_deps	11 s './build/install-build-deps.sh --no-arm ...'
3	⌚ gclient_sync	19 s 'gclient sync'
4	⌚ git_cl_format	19 s 'git cl ...'
5	⌚ git_cl_presubmit	1:13 'git config ...'
6	⌚ icecc_config	10 s 'BUILD_ENV=\$(pwd)\$(/usr/lib/icecc/icecc-create-env --clang ...'
7	⌚ Release_gn_config	34 s 'gn gen ...'
8	⌚ Release_base_build	27 s 'rm -f ...'
9	⌚ Release_copy_sandbox	0 s 'sudo cp ...'
10	⌚ Release_mathml_enabled_tests	39 s 'python third_party/blink/tools/run_web_tests.py ...'
11	⌚ Release_mathml_enabled_results	2 s 'RESULTS=\$(date +%Y-%m-%d_%T)-\$(git ...'
12	⌚ Release_mathml_disabled_tests	39 s 'python third_party/blink/tools/run_web_tests.py ...'
13	⌚ Release_mathml_disabled_results	2 s 'RESULTS=\$(date +%Y-%m-%d_%T)-\$(git ...'
14	⌚ Debug_gn_config	8 s 'gn gen ...'
15	⌚ Debug_base_build	32 s 'rm -f ...'
16	⌚ Debug_copy_sandbox	0 s 'sudo cp ...'
17	⌚ Debug_mathml_enabled_tests	4:11 'python third_party/blink/tools/run_web_tests.py ...'
18	⌚ Debug_mathml_enabled_results	2 s 'RESULTS=\$(date +%Y-%m-%d_%T)-\$(git ...'
19	⌚ Debug_mathml_disabled_tests	17 s 'python third_party/blink/tools/run_web_tests.py ...'
20	⌚ Debug_mathml_disabled_results	2 s 'RESULTS=\$(date +%Y-%m-%d_%T)-\$(git ...'
21	⌚ Release_chrome_build	41 s 'rm -f ...'
22	⌚ linux_packages	KAGES=out/Release/chromium-mathml-unstable*; chmod ...'
23	⌚ torture_test_screenshot	/www/mathml/torture-test/\$(date +%Y-%m-%d_%T)-\$(git ...'

## (IV) MATHML IN CHROMIUM

```
emacs@igalia-fwang
```

```
File Edit Options Buffers Tools C++ Help
AddOutOfFlowChildren();
```

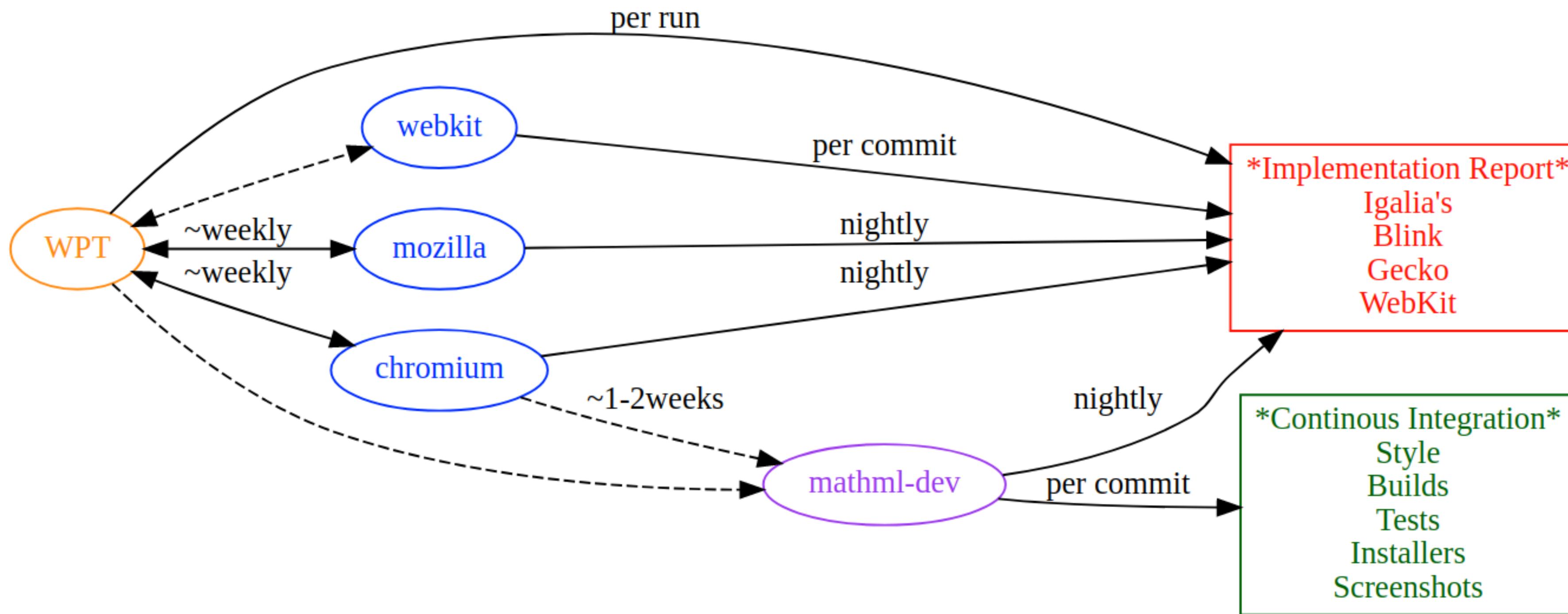
```
NCBlockNode numerator = To<NCBlockNode>(FirstChildInFlow(Node()));
DCHECK(numerator);
NGBlockNode denominator = To<NGBlockNode>(NextSiblingInFlow(numerator));
DCHECK(denominator);
IGConstraintSpace constraint_space = CreateConstraintSpaceForChild(
    constraint_space(), numeratorCreatesNewFormattingContext(), numerator);
scoped_refptr<const NGLayoutResult> numerator_layout_result =
    numerator.Layout(constraint_space);
auto numerator_margins =
    ComputeMarginsForSelf(constraint_space, numerator.Style());
constraint_space = CreateConstraintSpaceForChild(
    ConstraintSpace(), denominatorCreatesNewFormattingContext(),
    denominator);
NGMathRowLayoutAlgorithm::ChildrenAndOffsets denominatorChildren;
LayoutSize max_row_size;
LayoutUnit max_row_block_baseline;
fallback_.LayoutRowItems(denominatorChildren, ConstraintSpace(),
    InkBounds::Yes, max_row_block_baseline, max_row_size,
    denominator);
auto denominator_margins =
    ComputeMarginsForSelf(constraint_space, denominator.Style());
LogicalOffset numerator_offset;
LogicalOffset denominator_offset;
```

```
NGFragment numerator_fragment(
    numerator.Style().GetWritingMode(),
    To<NGPhysicalBoxFragment>(numerator_layout_result->PhysicalFragment()));

LayoutUnit content_inline_size =
    std::max(numerator_fragmentInlineSize() + numerator_marginsInlineSum(),
    -1);
```

```
ng_math_fraction_layout_algorithm.cc 14% L61 Git-mathml-dev (C++/l Abi
```

# PROJECT WORKFLOW



- Upstream mathml-dev!
- Non-Linux platforms?
- Automated WPT sync for WebKit 😊

# IMPLEMENTATION ROADMAP

- Basic setup ✓
- Basic layout ✓
- Operator Dictionary ✓
- Stretchy operators: ✓
- Advanced style ✓
- HTML5 Compatibility •
- Upstreamed & shipped ✘

$$\lim_{n \rightarrow +\infty} \frac{\sqrt{2\pi n}}{n!} \left(\frac{n}{e}\right)^n = 1$$

$$\det(A) = \sum_{\sigma \in S_n} \epsilon(\sigma) \prod_{i=1}^n a_{i,\sigma_i}$$

# CHANGESETS

## Base Setup

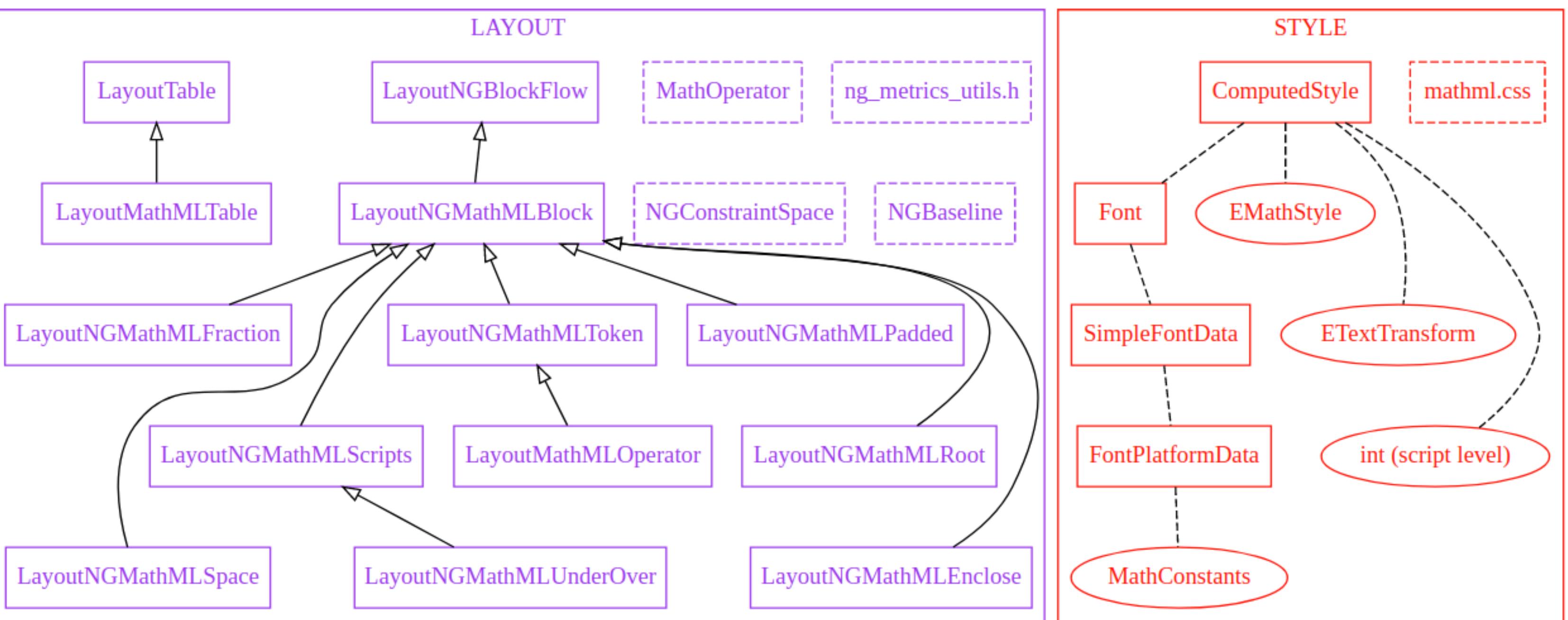
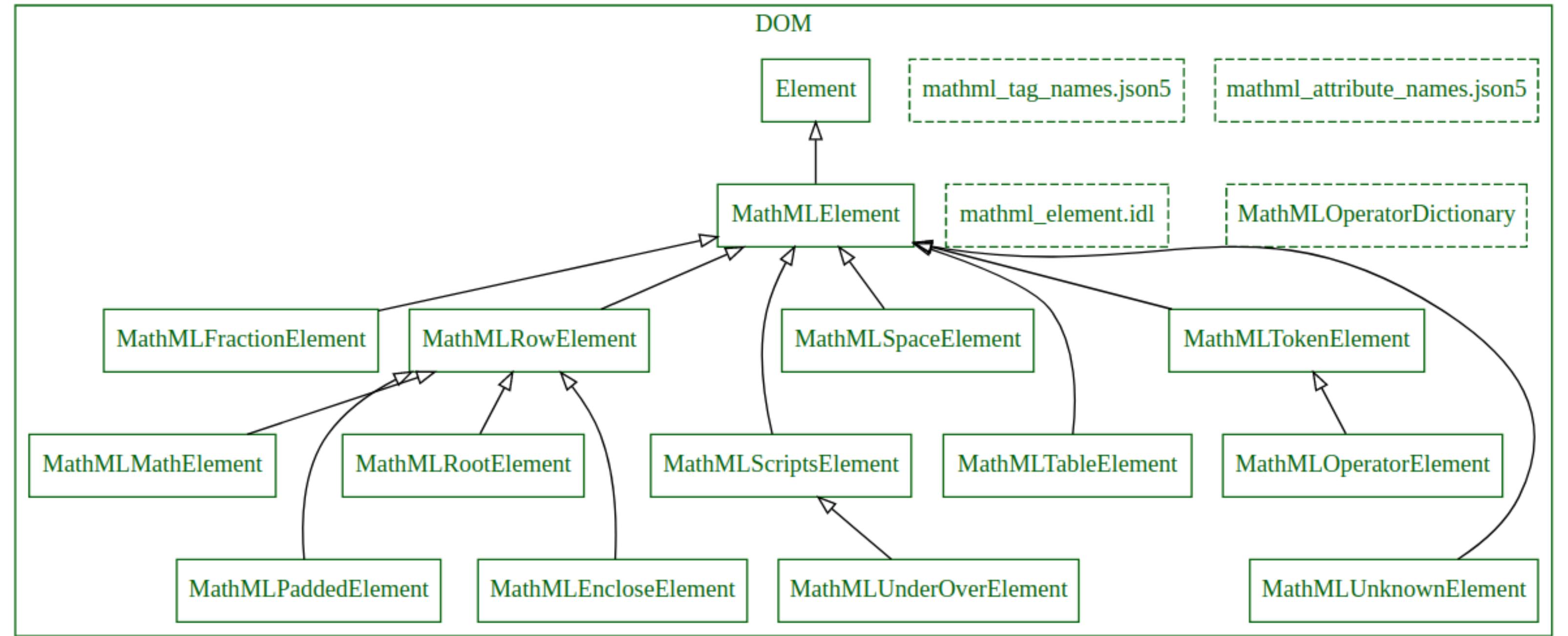
- Runtime flag
- `mathml.css`
- Element classes
- New CSS
- New DOM/IDL

## Low-level API

- Font parameters
- Glyph stretching
- Layout constraints
- Ink baselines

## MathML

- Dictionary
- Attributes
- Painters
- Layout utils
- Layout classes



# SIZE IMPACT

## CHROME

```
is_debug=0  
is_official_build=1  
enable_linux_installer=1
```

- Binary:  $\Delta < 320\text{KB}$
- Archive:
  - deb:  $\Delta < 45\text{KB}$
  - rpm:  $\Delta < 26\text{KB}$

## CODE

Folders	#Lines
core/layout/ng/mathml	4977
core/mathml	3279
core/css, core/style	940
core/paint	285
platform/fonts, platform/graphics	268
core/layout, core/layout/ng	164
core/dom, core/html, core/svg	44
Total	$\Delta < 10\text{k}$

Current SVG code size (only for core/svg,  
core/layout/svg) is  $> 62\text{k}$

# LAUNCHING FEATURES

- **Idea Phase:** explainer, [intent-to-implement](#)
- **Design Phase:** design doc & spec, TAG review,  
[chromestatus entry](#), chrome launch review
- **Implementation Phase:** implementation & tests, intent-to-ship, 3 LGTMs
- **Post Launch:** crashes, regressions, bug fixes, interop, doc, cleanup...

Consider

$$\int_0^{+\infty} (x^2 + x + 1)e^{-3x} dx$$

We consider  $f(x) = x^2 + x + 1$  and  $g(x) = \frac{e^{-3x}}{-3}$  that is  $f'(x) = 2x + 1$  and  $g'(x) = e^{-3x}$ . The integral can be written  $\int fg' dx$  and hence we get

$$\left[ (x^2 + x + 1) \frac{e^{-3x}}{-3} \right]_0^{+\infty} - \int_0^{+\infty} (2x + 1) \frac{e^{-3x}}{-3} dx = \frac{1}{3} + \frac{1}{3} \int_0^{+\infty} (2x + 1)e^{-3x} dx$$

We now consider  $h(x) = 2x + 1$ ,  $h'(x) = 2$  the second integral can be written  $\int hg' dx$  and hence

$$\int_0^{+\infty} (2x + 1)e^{-3x} dx = \left[ (2x + 1) \frac{e^{-3x}}{-3} \right]_0^{+\infty} - \int_0^{+\infty} 2 \frac{e^{-3x}}{-3} dx = \frac{1}{3} + \frac{2}{3} \int_0^{+\infty} e^{-3x} dx$$

$$\int_0^{+\infty} e^{-3x} dx = \left[ \frac{e^{-3x}}{-3} \right]_0^{+\infty} = \frac{1}{3}$$

Finally,

$$\int_0^{+\infty} (x^2 + x + 1)e^{-3x} dx = \frac{1}{3} + \frac{1}{3} \times \left( \frac{1}{3} + \frac{2}{3} \times \frac{1}{3} \right) = \frac{14}{27}$$

## Example 2

$$\int_1^2 x^3 (2\ln x + 7\arctan x) dx$$

We let  $f(x) = 2\ln x + 7\arctan x$  and  $g(x) = \frac{x^3}{3}$ . Hence  $f'(x) = \frac{2}{x} + \frac{7}{1+x^2}$  and  $g'(x) = x^2$ . The integration by parts gives:

# (V) MATHML DEMOS

```
\int_0^{+\infty} (2x+1) \frac{e^{-3x}}{-3} dx
= \left[ (2x+1) \frac{e^{-3x}}{-3} \right]_0^{+\infty} - \int_0^{+\infty} 2 \frac{e^{-3x}}{-3} dx
= \left[ \frac{e^{-3x}}{-3} \right]_0^{+\infty} = \frac{1}{3}
```

html body la-tex #shadow-root math

Styles Computed Event Listeners DOM Breakpoints >

Filter

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	As rendered by TeX	As rendered by your browser
1	$x^2y^2$	$x^2y^2$
2	${}_2F_3$	${}_2F_3$
3	$\frac{x+y^2}{k+1}$	$\frac{x+y^2}{k+1}$
4	$x+y^{\frac{2}{k+1}}$	$x+y^{\frac{2}{k+1}}$
5	$\frac{a}{b/2}$	$\frac{a}{b/2}$
6	$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4}}}}$	$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4}}}}$
7	$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4}}}}$	$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4}}}}$
8	$\binom{n}{k/2}$	$\binom{n}{k/2}$
9	$\binom{p}{2}x^2y^{p-2} - \frac{1}{1-x}\frac{1}{1-x^2}$	$\binom{p}{2}x^2y^{p-2} - \frac{1}{1-x}\frac{1}{1-x^2}$
10	$\sum_{\substack{0 \leq i \leq m \\ 0 < j < n}} P(i,j)$	$\sum_{\substack{0 \leq i \leq m \\ 0 < j < n}} P(i,j)$
11	$x^{2y}$	$x^{2y}$
12	$\sum_{i=1}^p \sum_{j=1}^q \sum_{k=1}^r a_{ij} b_{jk} c_{ki}$	$\sum_{i=1}^p \sum_{j=1}^q \sum_{k=1}^r a_{ij} b_{jk} c_{ki}$
13	$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+x}}}}}}$	$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+x}}}}}}}$
14	$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)  \varphi(x+iy) ^2 = 0$	$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)  \varphi(x+\textcolor{brown}{iy}) ^2 = 0$
15	$2^{2^{2^x}}$	$2^{2^x}$
	$\int^x dt$	$\int^x dt$



# <la-tex> Custom Element

## Basic Integration

### Ideas

If  $F$  is a primitive of  $f$  then  $\int_a^b f(x)dx = F(b) - F(a)$ , provided the integrand and integral have no singularities on the path of integration. As a consequence, we can use a table of derivatives:

	Function	Derivative
Linearity	$af + bg$	$a\dot{f} + b\dot{g}$
Leibniz rule	$fg$	$\dot{f}g + g\dot{f}$
Reciprocal rule	$\frac{1}{f}$	$-\frac{\dot{f}}{f^2}$
Chain Rule	$f \circ g$	$(\dot{f} \circ g)g'$
Inverse function rule	$f^{-1}$	$\frac{1}{f \circ f^{-1}}$
Elementary power rule	$x^n$	$nx^{n-1}$
Generalized power rule	$f^g = e^{g \ln(f)}$	$f^g \left( \dot{f} \frac{g}{f} + g' \ln(f) \right)$
Exponential	$\exp x$	$\exp x$
Logarithm	$\ln x$	$\frac{1}{x}$
Sine	$\sin x$	$\cos x$
Cosine	$\cos x$	$-\sin x$
Tangent	$\tan x$	$\frac{1}{\cos^2 x} = 1 + \tan^2 x$

### Examples

Using linearity and elementary power rule:

$$\int_0^1 u^4 - 2u^3 + 5u^2 + 4du = \left[ \frac{u^5}{5} - 2\frac{u^4}{4} + 5\frac{u^3}{3} + 4u \right]_{u=0}^1 = \frac{161}{30}$$

Using linearity and sine/cosine:

$$\int_0^\pi 2\cos(\theta) - 3\sin(\theta)d\theta = [2\sin(\theta) + 3\cos(\theta)]_{\theta=0}^\pi = -6$$

Using Leibniz rule, Chain rule and Exponential/Power:

$$\int_0^2 2xe^{-3x} - 3x^2e^{-3x}dx = [x^2e^{-3x}]_0^2 = \frac{4}{e^{12}}$$

Using the inverse function rule and tangent:

$$\int_0^1 \frac{1}{1+x^2}dx = \int_0^1 \frac{1}{\tan'(\arctan(x))}dx = [\arctan(x)]_0^1 = \frac{\pi}{4}$$

Using linearity, reciprocal rule and logarithm:

$$\int_2^3 \frac{1}{v(\ln(v))^2}dv = - \int_2^3 -\frac{\frac{1}{v}}{(\ln(v))^2}dv = - \left[ \frac{1}{\ln(v)} \right]_2^3 = \frac{1}{\ln 2} - \frac{1}{\ln 3}$$

# LaTeX-to-MathML converter

Display Mode ▾ ; RTL ▾ ; Riemann zeta function at even positive integer ▾

Font: Latin Modern ▾

```
{\zeta(2k)} =  
{\sum_{n=1}^{\infty} \frac{1}{n^{2k}}} =  
{\left(\frac{(-1)^{k+1}}{(2\pi)^{2k}}\right) \frac{2(2k)!}{\left(\sum_{m=0}^{2k} (-1)^m \binom{m}{l} \frac{l^{2k}}{(m+1)!}\right)}}
```

$$\left( \frac{k2l}{1+m} \binom{m}{l} l! (1-) \sum_{0=l}^m \sum_{0=m}^{k2} \right) \frac{k2(\pi 2)_1 + k}{!(k2)2} (1-) = \frac{1}{k2n} \sum_{1=n}^{\infty} = (k2)\zeta$$

## Padding, border and margin

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

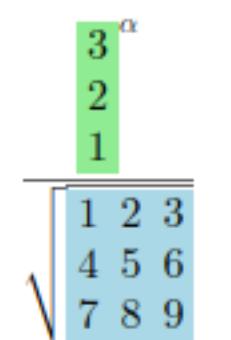
$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

$$\Gamma(t) = \int_0^{+\infty} x^{t-1} e^{-x} dx = \frac{1}{t} \prod_{n=1}^{\infty} \frac{\left(1 + \frac{1}{n}\right)^t}{1 + \frac{t}{n}} \sim \sqrt{\frac{2\pi}{t}} \left(\frac{t}{e}\right)^t$$

# Custom display values

## Flexbox and Grid



## CSS Layout API

Click a B to toggle between multiscripts and custom layout:

$$\frac{1}{2} + \boxed{B_{135}^{246}} - \boxed{B_1^{32}} + \sqrt{7^f + k}$$

**(VI) ¿QUESTIONS?**

