Active filters

Active filters are a special family of filters. They take their name from the fact that, aside from passive components, they also contain *active elements* such as transistors or operational amplifiers. Just like passive filters, depending on the design, they retain or eliminate a specific portion of a signal.

These types of filters have an advantage over passive filters, mainly because *bulky inductors at low frequencies can be avoided* and *higher quality factors can be obtained*.

Active filters can be implemented with different topologies:

1. Akerberg-Mossberg
2. Biquadratic
3. Dual Amplifier BandPass (DABP)
4. Fliege
5. Multiple feedback
6. Voltage-Controlled Voltage-Source (VCVS) and Sallen/Key
7. State variable
8. Wien

Active filters also come in different varieties:

1. Butterworth
2. Linkwitz-Riley
3. Bessel
4. Chebyshev (2 types)
5. Elliptic or Cauer
6. Synchronous
7. Gaussian
8. Legendre-Paupolis
9. Butterworth-Thomson or Linear phase
10. Transitional or Paynter

The Butterworth filter has the flattest response in the pass-band. The Chebyshev Type II filter has the steepest cutoff. The Linkwitz-Riley filter is often used in audio applications (crossovers).

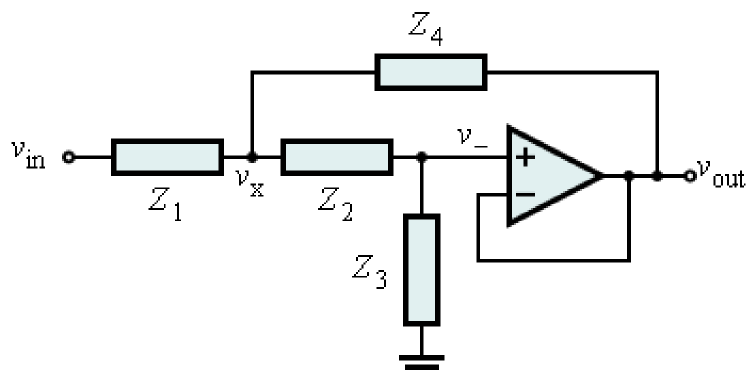
Note: Cauer is the name of an active filter but it's also the name of a passive topology. The two are different concepts.

Sallen/Key topology

The Sallen/Key topology was invented by R. P. Sallen and E. L. Key at MIT Lincoln Laboratory in 1955.

It is a degenerate form of a Voltage-Controlled Voltage-Source (VCVS) filter topology. It features an *extremely high input impedance* (practically infinite) and an *extremely low output impedance* (practically zero). These two characteristics are provided by the op-amp and they are often desired in circuit design for signal integrity.

The network for the Sallen/Key topology includes an op-amp, often in a buffer configuration, and a set of resistors and capacitors. The op-amp can sometimes be substituted by an emitter follower or a source follower circuit since both circuits produce unity gain. Cascading two or more stages will produce higher-order filters.



Sallen/Key generic configuration for 0dB gain (unity-gain)

Unlike RC passive filters, where only *one pole* is present in the circuit so that the gain drops by 6bB/octave or 20dB/decade past the cutoff frequency, the Sallen/Key circuit topology has *two poles* so the gain drops by 12bB/octave or 40dB/decade past the cutoff frequency.

Considering two signals with magnitudes A_1 and A_2, the definitions for octave and decade are

$$20\log_{10}\left(\frac{A_1}{A_2}\right) = 20\log_{10}\left(\frac{1}{2}\right) = -6dB/octave$$

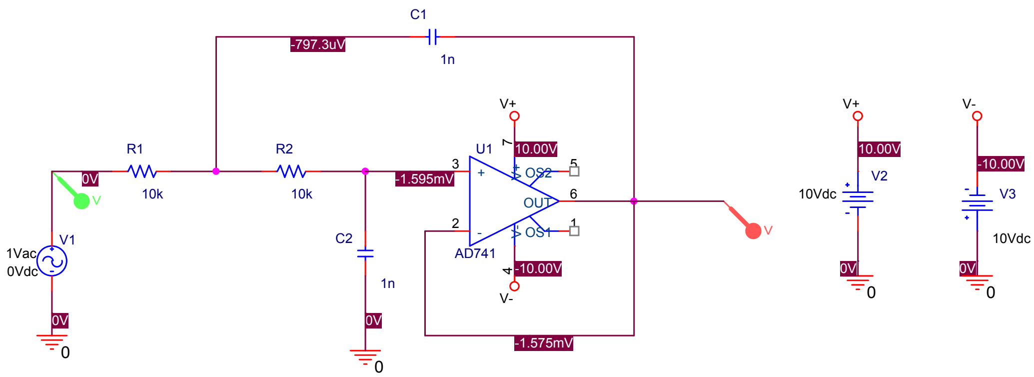
and

$$20\log_{10}\left(\frac{A_1}{A_2}\right) = 20\log_{10}\left(\frac{1}{10}\right) = -20dB/decade$$

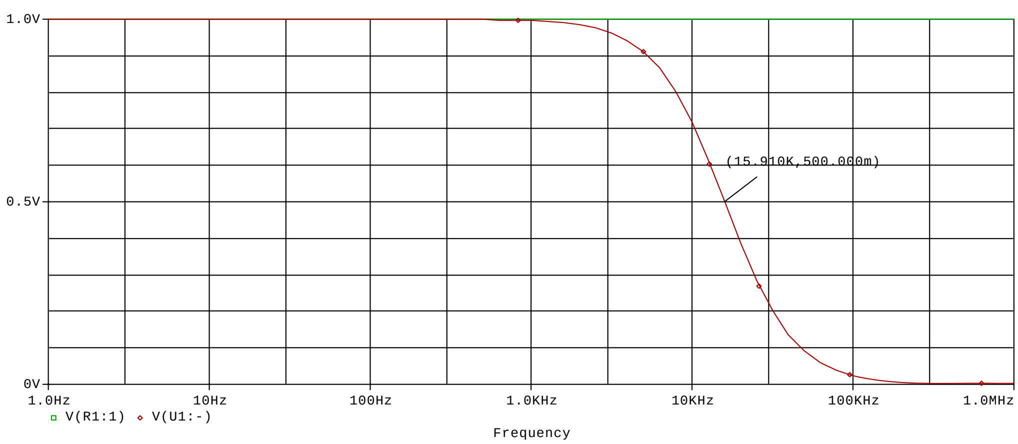
where *octave* means *twice* the magnitude and *decade* means *ten* times the magnitude.

Low-pass

The low-pass filter blocks high-frequency signals while leaving low-frequency signals untouched.



Sallen/Key low-pass filter



AC sweep from 1mHz to 1MHz

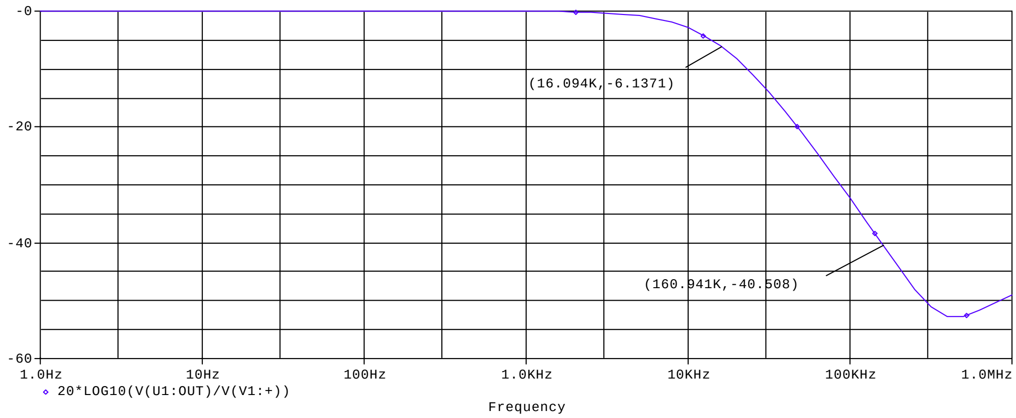
The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{10k\Omega \cdot 10k\Omega \cdot 1nF \cdot 1nF}} = 15.915kHz$$

The quality factor is

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)} = \frac{\sqrt{10k\Omega \cdot 10k\Omega \cdot 1nF \cdot 1nF}}{1nF \cdot (10k\Omega + 10k\Omega)} = 0.5$$

This circuit is critically damped.

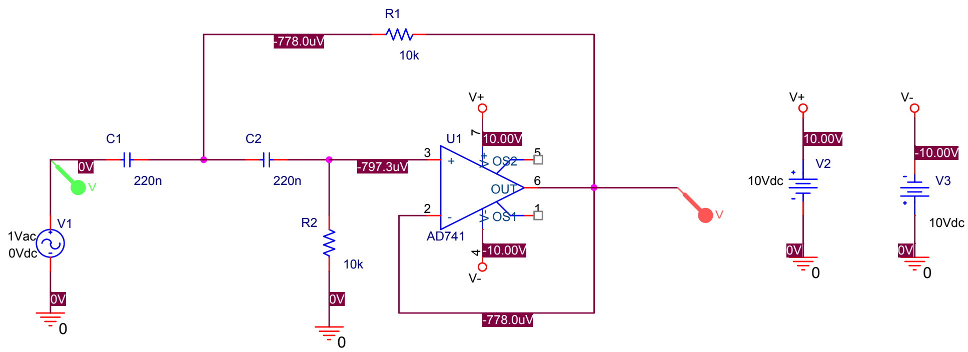


Bode plot from 1Hz to 1MHz

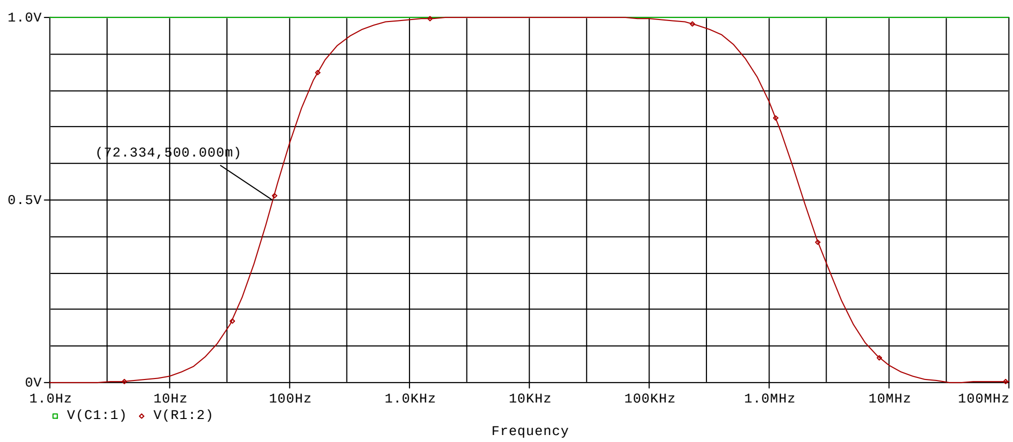
The gain drops to –6dB at 15.915kHz and it decreases by –40dB/decade.

High-pass

The high-pass filter blocks low-frequency signals while leaving high-frequency signals untouched.



Sallen/Key high-pass filter



AC sweep from 1mHz to 100MHz

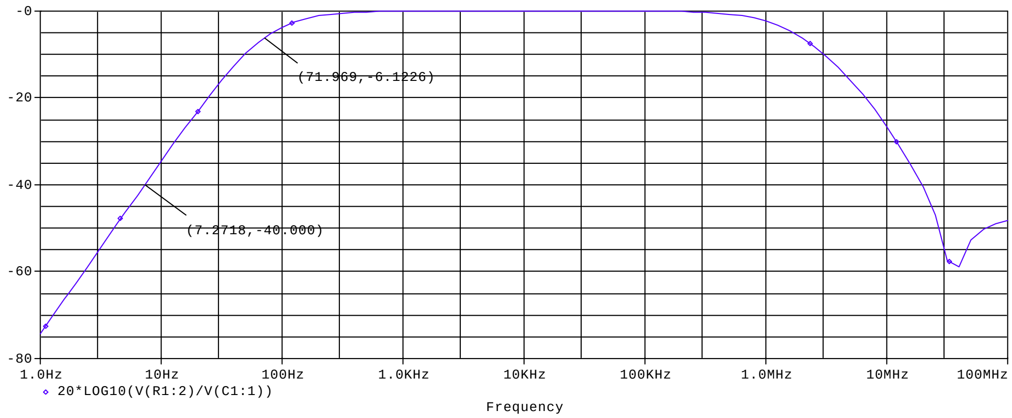
The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{10k\Omega \cdot 10k\Omega \cdot 220nF \cdot 220nF}} = 72.34Hz$$

The quality factor is

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)} = \frac{\sqrt{10k\Omega \cdot 10k\Omega \cdot 220nF \cdot 220nF}}{10k\Omega (220nF + 220nF)} = 0.5$$

This circuit is critically damped.



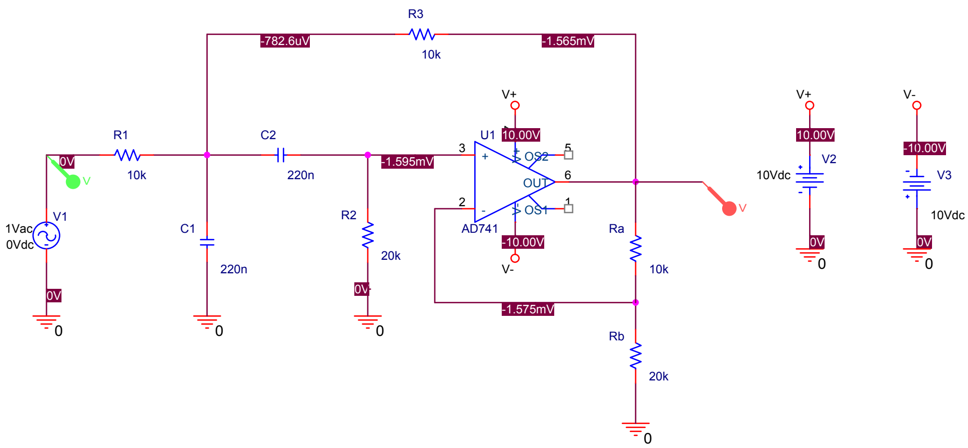
Bode plot from 1Hz to 100MHz

The gain drops to –6dB at 72Hz and it decreases by –40dB/decade.

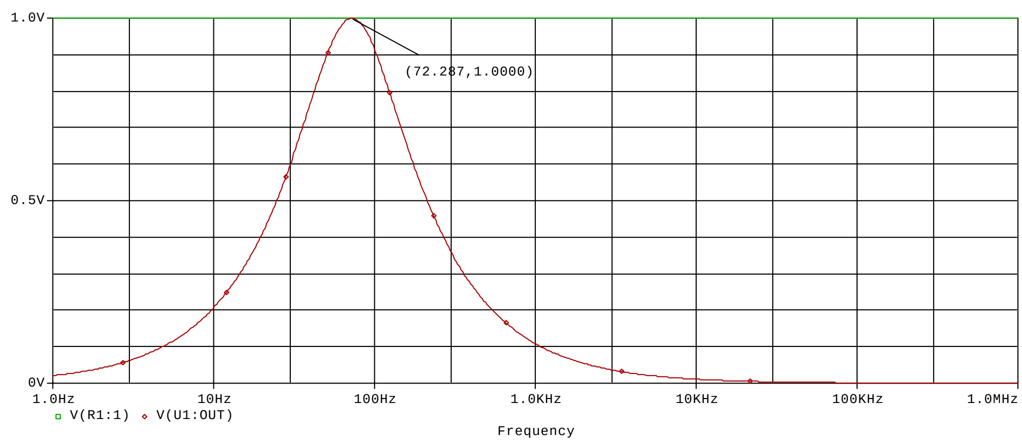
Note: the gain starts to roll off at 1MHz which is the bandwidth of the AD741 op-amp when it's connected in a buffer/non-inverting configuration.

Band-pass

The band-pass filter blocks low and high frequency signals. It peaks at the so-called *center frequency*. R_a and R_b provide gain. C_1-R_1 form a low-pass filter and C_2-R_2 form a high-pass filter.



Sallen/Key band-pass filter



AC sweep from 1mHz to 1MHz

The center frequency is

$$f_c = \frac{1}{2\pi}\sqrt{\frac{R_3 + R_1}{C_1 C_2 R_1 R_2 R_3}} = \frac{1}{2\pi}\sqrt{\frac{10k\Omega + 10k\Omega}{220nF \cdot 220nF \cdot 10k\Omega \cdot 20k\Omega \cdot 10k\Omega}} = 72.34Hz$$

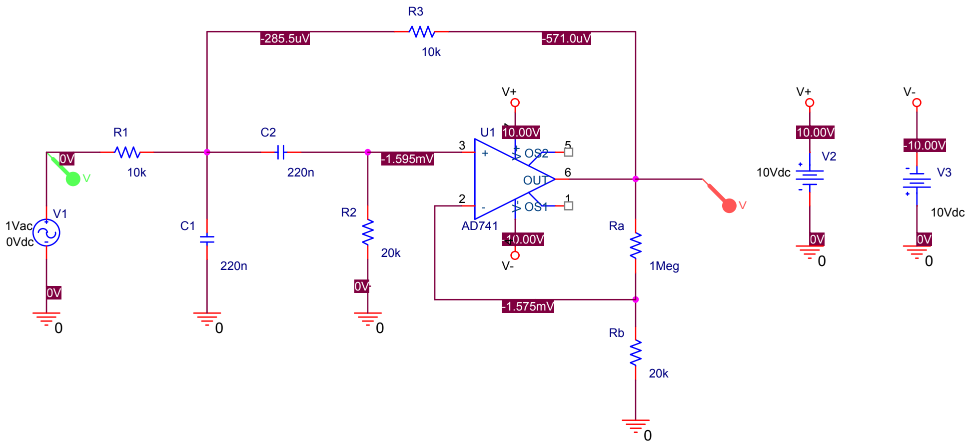
The gains are

$$G = 1 + \frac{R_a}{R_b} = 1 + \frac{10k\Omega}{20k\Omega} = 1 + 0.5 = 1.5 \qquad A = \frac{G}{3-G} = \frac{1.5}{3-1.5} = 1$$

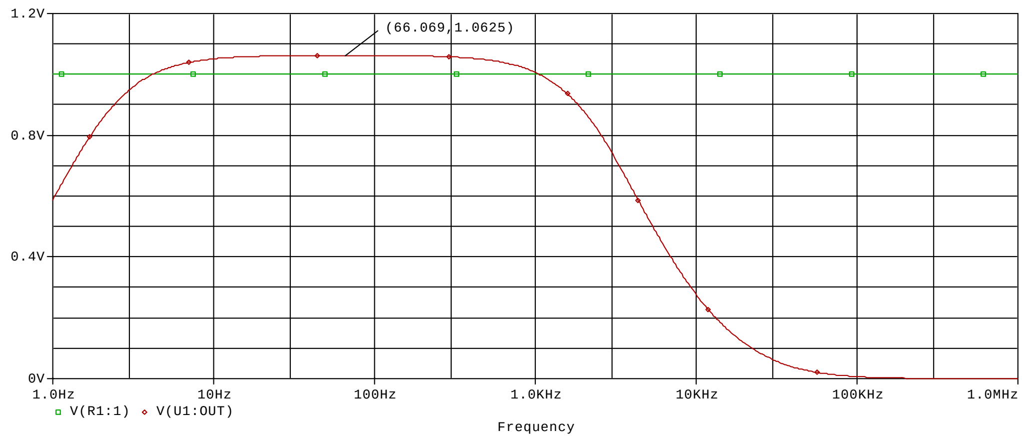
where G is the internal gain and A is the external gain.

The value of G should be below 3 to avoid oscillation.

The previous is a Sallen/Key circuit as long as the value of R_b is twice the value of R_a. If A is more or less than unity, the circuit provides amplification and becomes a VCVS filter.



VCVS band-pass filter I



AC sweep from 1mHz to 1MHz

The center frequency is

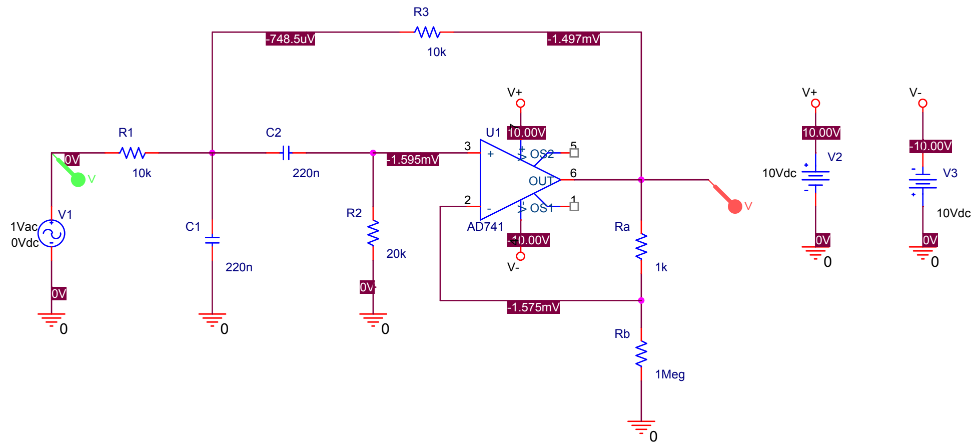
$$f_c = \frac{1}{2\pi}\sqrt{\frac{R_3 + R_1}{C_1 C_2 R_1 R_2 R_3}} = \frac{1}{2\pi}\sqrt{\frac{10k\Omega + 10k\Omega}{220nF \cdot 220nF \cdot 10k\Omega \cdot 20k\Omega \cdot 10k\Omega}} = 72.34Hz$$

However, if R_a>>R_b, the frequency response is rather flat.

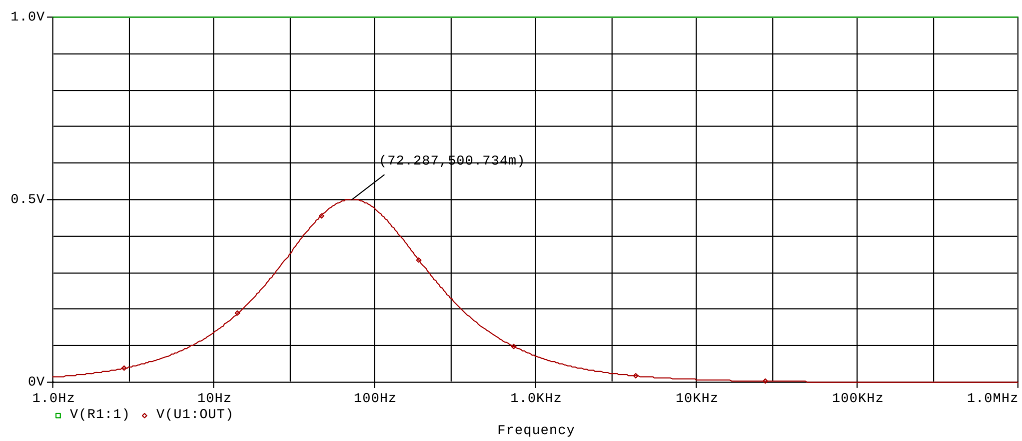
$$G = 1 + \frac{R_a}{R_b} = 1 + \frac{1M\Omega}{20k\Omega} = 1 + 50 = 51 \qquad A = \frac{G}{3-G} = \frac{51}{3-51} = -1.06$$

This circuit will oscillate because G is greater than 3.

Note: the magnitude of the output is 6% higher than the input and this is an indication of amplification.



VCVS band-pass filter II



AC sweep from 1mHz to 1MHz

The center frequency is

$$f_c = \frac{1}{2\pi}\sqrt{\frac{R_3 + R_1}{C_1 C_2 R_1 R_2 R_3}} = \frac{1}{2\pi}\sqrt{\frac{10k\Omega + 10k\Omega}{220nF \cdot 220nF \cdot 10k\Omega \cdot 20k\Omega \cdot 10k\Omega}} = 72.34Hz$$

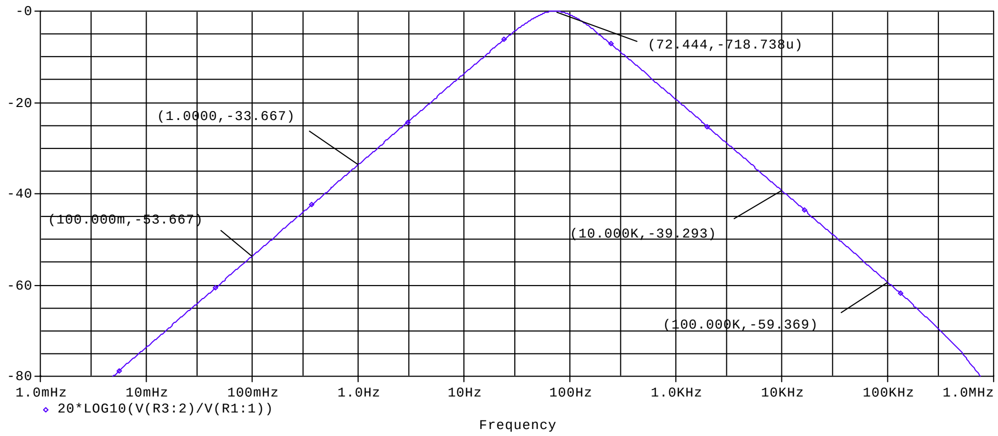
If R_a<<R_b, the gain drops to ½ at the center frequency.

$$G = 1 + \frac{R_a}{R_b} = 1 + \frac{1k\Omega}{1M\Omega} = 1 + 0 = 1 \qquad A = \frac{G}{3-G} = \frac{1}{3-1} = +0.5$$

The minimum gain attainable by this circuit is 0.5.

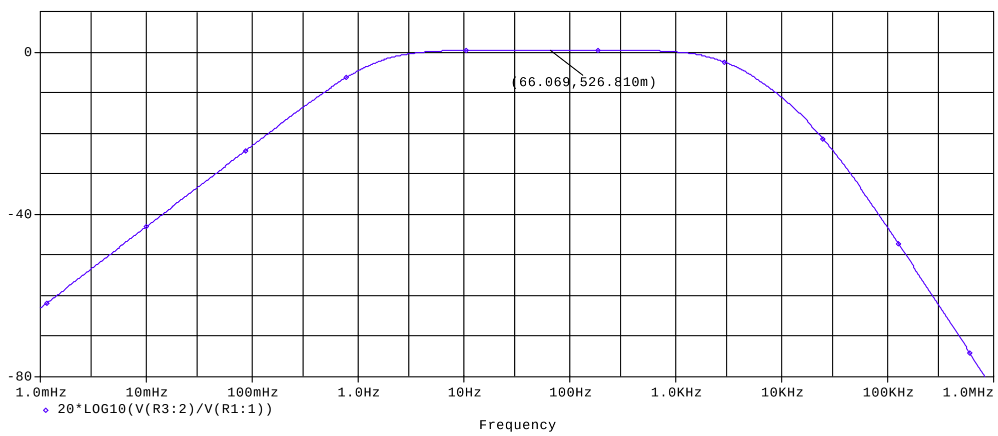
Connecting the op-amp in the buffer configuration would produce the same frequency response.

Note: the magnitude of the output at the center frequency is 50% lower than the input.



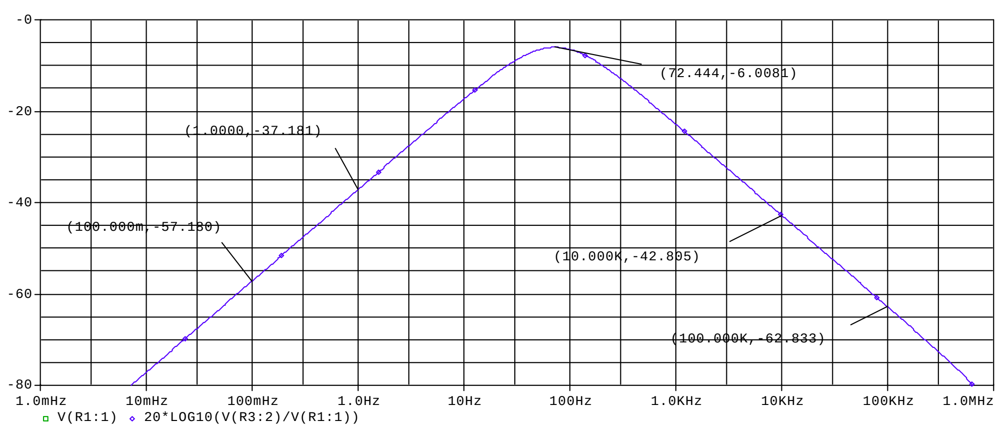
Sallen/Key band-pass filter: Bode plot from 1mHz to 1MHz

The center frequency is at 72Hz. The gain decreases by –20dB/decade to the left and right of the center frequency.



VCVS band-pass filter I: Bode plot from 1mHz to 1MHz

The response is rather flat.



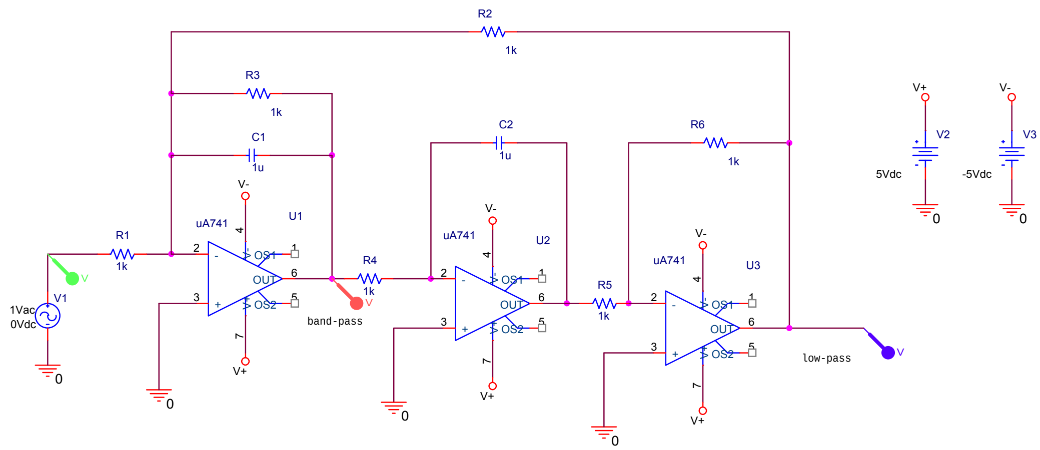
VCVS band-pass filter II: Bode plot from 1mHz to 1MHz

The center frequency is at 72Hz with –6dB. The gain decreases by –20dB/decade to the left and right of the center frequency.

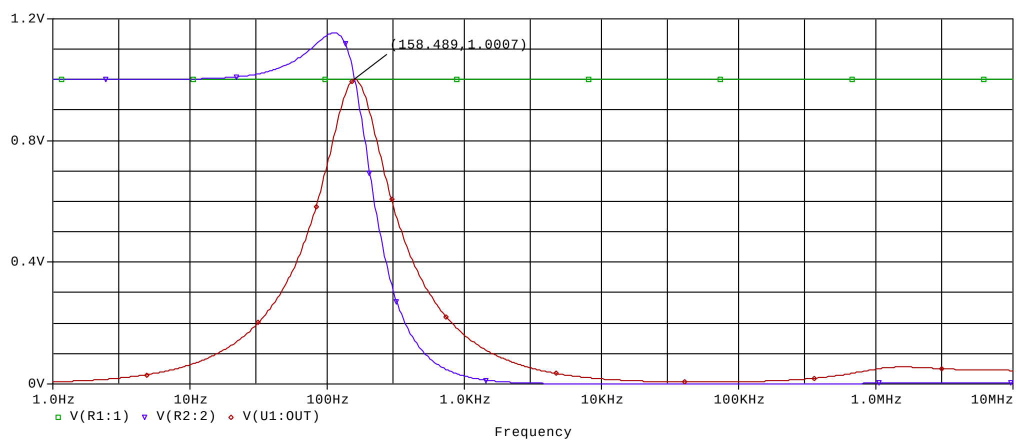
Biquadratic topology

The biquadratic topology takes its name from the fact its transfer function is the ratio of two quadratic functions. It can be implemented in two ways: single-amplifier biquadratic or two-integrator-loop.

An example of this filter topology is the so-called Tow-Thomas circuit. This circuit consists of three op-amps and it can be used as a low-pass or band-pass filter, depending on where its output is taken.



Biquadratic filter



AC sweep from 1Hz to 10MHz

The band-pass gain is $G_{BP} = -\frac{R_4}{R_2}$.

The low-pass gain is $G_{LP} = +\frac{R_2}{R_1}$.

The output for the band-pass option is shown in red.
The output for the low-pass option is shown in blue.

The center/cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_2 R_4 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 1\mu F \cdot 1\mu F}} = 159.15Hz$$

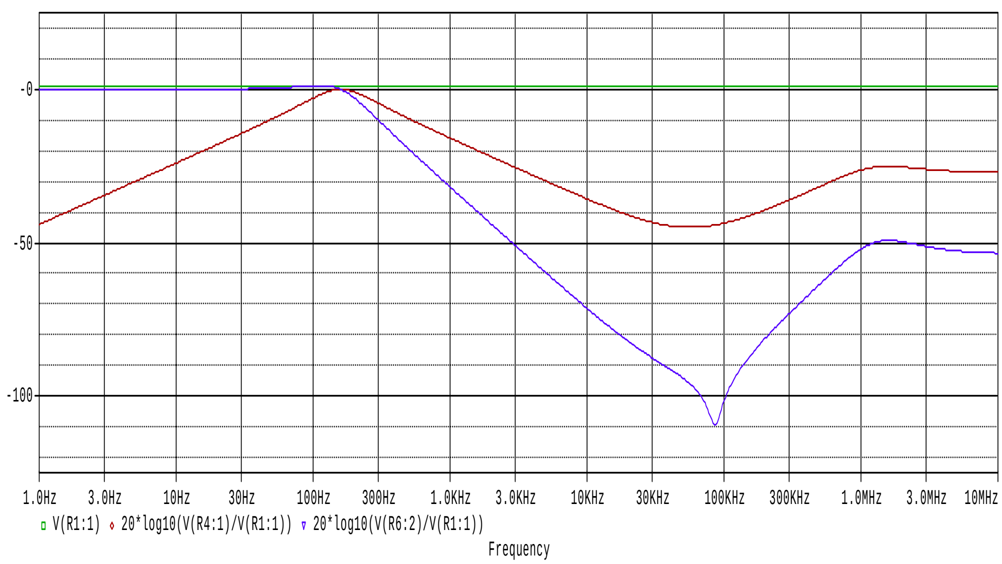
The quality factor is

$$Q = \sqrt{\frac{R_3^2 C_1}{R_2 R_4 C_2}} = \sqrt{\frac{1k\Omega^3 \cdot 1\mu F}{1k\Omega \cdot 1k\Omega \cdot 1\mu F}} = 0.03$$

The circuit is overdamped.

The bandwidth is approximately given by

$$BW \cong \frac{2\pi f_c}{Q} = \frac{2\pi \cdot 159.15Hz}{0.03} = 33.33kHz$$



Bode plots from 1Hz to 10MHz

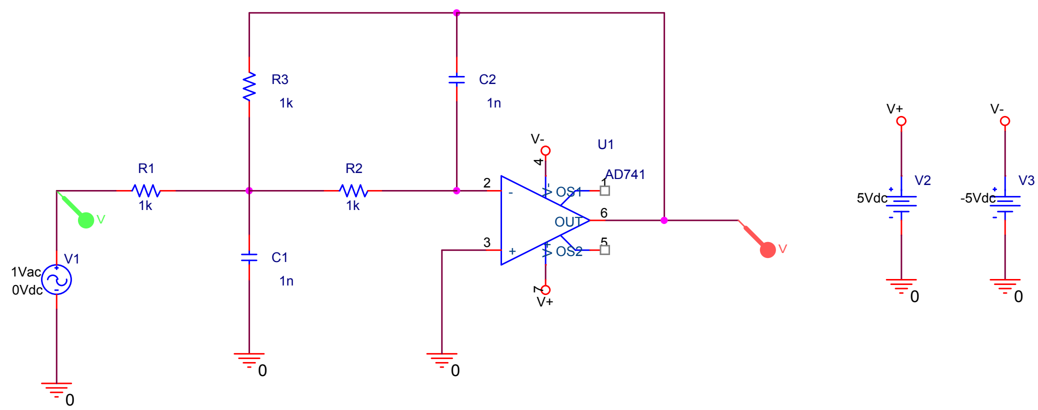
The center frequency for band-pass filter is at 159Hz and 0dB. The gain decreases by –20dB/decade to the left and right.

The cutoff frequency for low-pass filter is at 159Hz. The gain decreases linearly by –40dB/decade to the right of it.

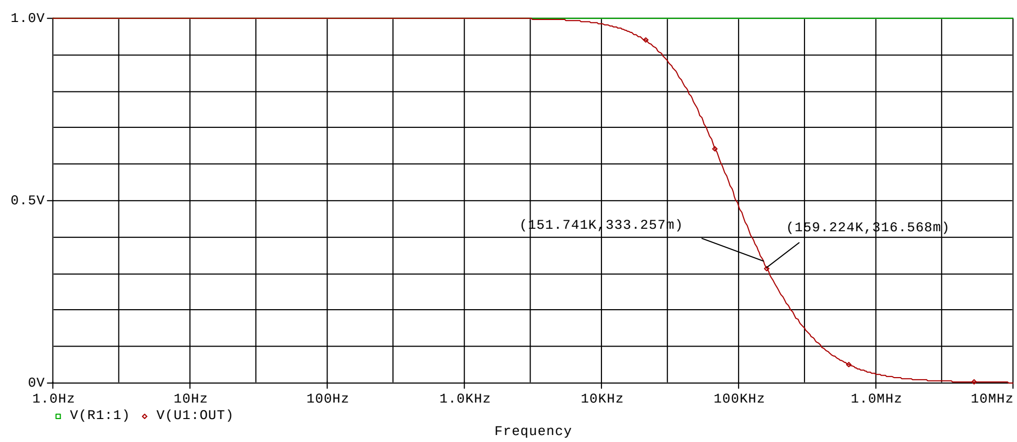
The bandwidth for both plots is about 33kHz.

Multiple feedback topology

The multiple feedback topology takes its name from the fact it has positive and negative feedback.



Multiple feedback filter



AC sweep from 1Hz to 10MHz

The transfer function for the circuit is

$$H(s) = \frac{V_o}{V_i} = -\frac{1}{As^2 + Bs + C} = \frac{K\omega_o^2}{s^2 + \frac{\omega_o}{Q}s + \omega_o^2}$$

where

$$A = R_1R_2C_1C_2 = 1k\Omega \cdot 1k\Omega \cdot 1nF \cdot 1nF = 1ps$$

$$B = R_2C_2 + R_1C_2 + \frac{R_1R_2C_2}{R_3} = 1k\Omega \cdot 1nF + 1k\Omega \cdot 1nF + \frac{1k\Omega \cdot 1k\Omega \cdot 1nF}{1k\Omega} = 3\mu s$$

$$C = \frac{R_1}{R_3} = \frac{1k\Omega}{1k\Omega} = 1$$

$$K = -\frac{R_3}{R_1} = -\frac{1k\Omega}{1k\Omega} = -1$$

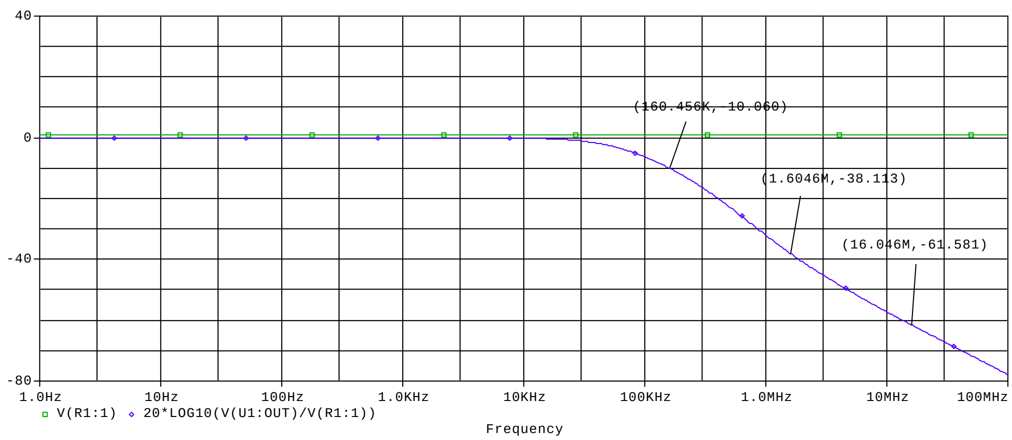
$$Q = \frac{\sqrt{R_2 R_3 C_1 C_2}}{\left(R_3 + R_2 + |K| R_2\right) \cdot C_2} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 1nF \cdot 1nF}}{\left(1k\Omega + 1k\Omega + |-1| \cdot 1k\Omega\right) \cdot 1nF} = 0.333$$

$$f_c = \frac{1}{2\pi\sqrt{R_2 R_3 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 1nF \cdot 1nF}} = 159.15kHz$$

$$\omega_o = 2\pi f_c = 2\pi \cdot 159.15kHz = 999.968kHz$$

where K is the DC voltage gain, Q is the quality factor, f_c is the cutoff frequency and ω_o is the angular frequency.

The circuit is overdamped.



Bode plot from 1Hz to 100MHz

The gain drops to –10.060dB at 160.456kHz and then it decreases in a *nonlinear* fashion to the right of the cutoff frequency.

Several stages can be *cascaded* to obtain high-order multiple feedback filters.

First-order filters

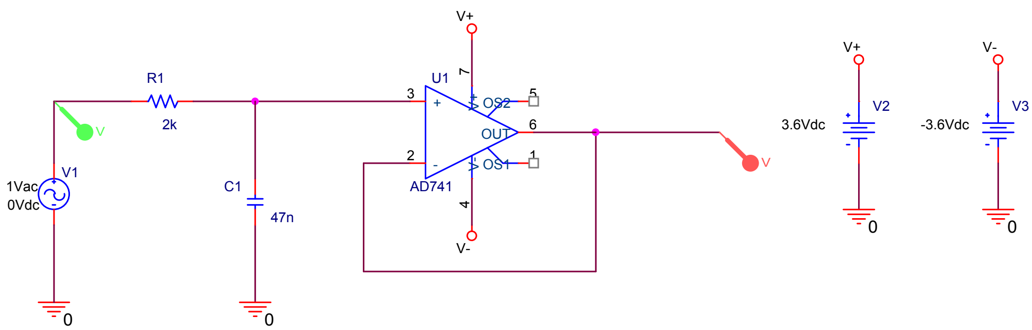
First-order filters are very simple. They have only one capacitor which in turn produces a single pole. The –3dB frequency is given by

$$f_{-3dB} = \frac{1}{2\pi RC}$$

First-older filters come in combinations of non-inverting, inverting, low-pass and high-pass. All the filters presented here have unity-gain.

Non-inverting low-pass

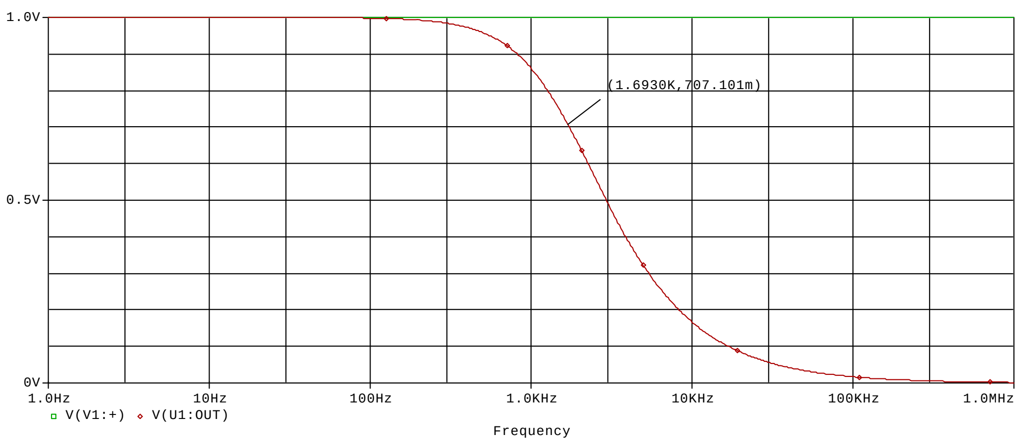
This circuit lets the low-frequency signals through without inverting the input.



First-order non-inverting low-pass filter

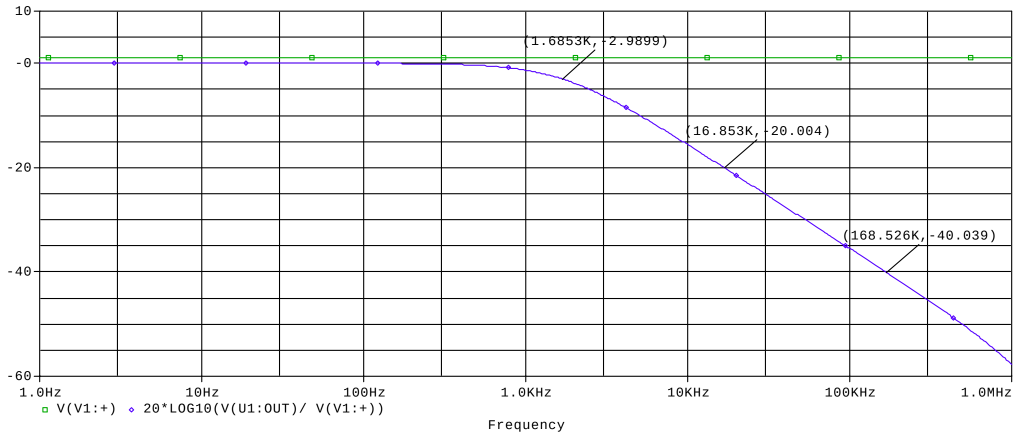
The –3dB frequency is given by

$$f_{-3dB} = \frac{1}{2\pi RC} = \frac{1}{2\pi \cdot 2k\Omega \cdot 47nF} = 1.693kHz$$



AC sweep from 1Hz to 1MHz

The –3dB frequency for the filter is at 1.693kHz.

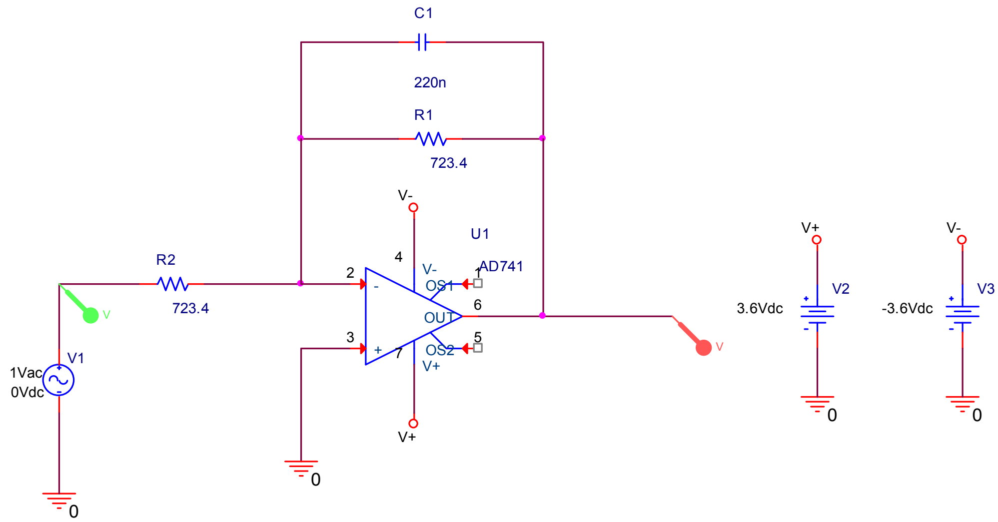


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1.693kHz and then it decreases by –20dB/decade.

Inverting low-pass

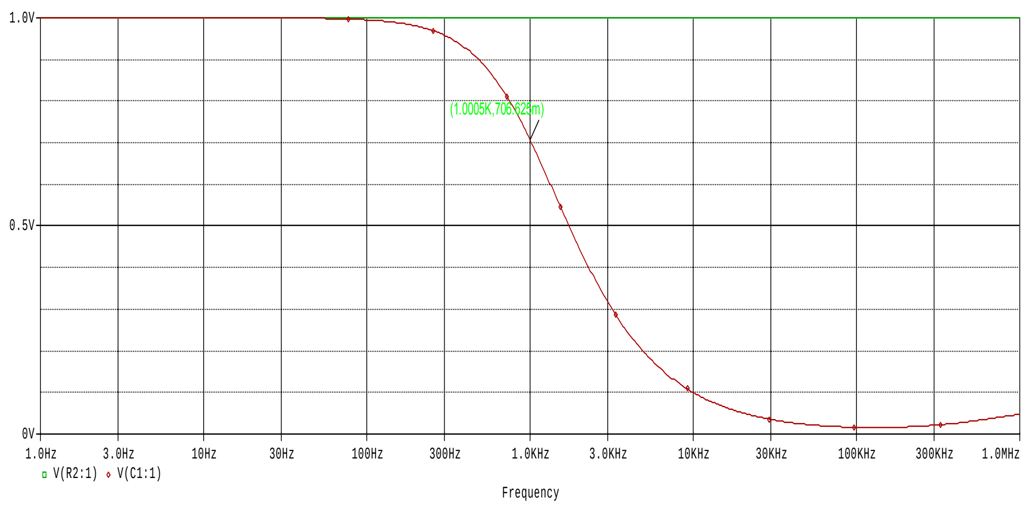
This circuit lets the low-frequency signals through while inverting the input.



First-order inverting low-pass filter

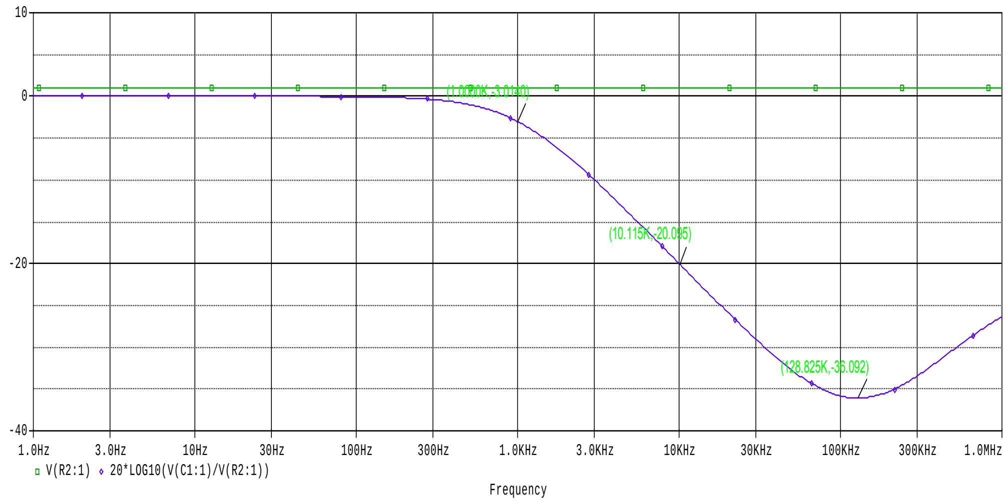
The –3dB frequency is given by

$$f_{-3dB} = \frac{1}{2\pi RC} = \frac{1}{2\pi \cdot 723.4\Omega \cdot 220nF} = 1kHz$$



AC sweep from 1Hz to 1MHz

The –3dB frequency for the filter is at 1kHz.

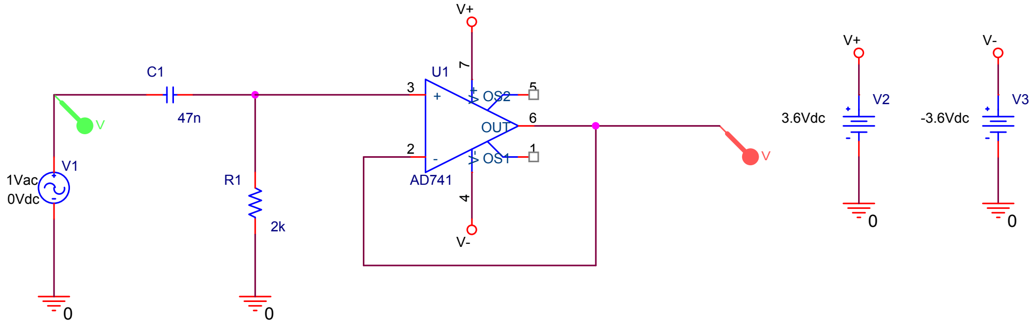


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases by –20dB/decade.

Non-inverting high-pass

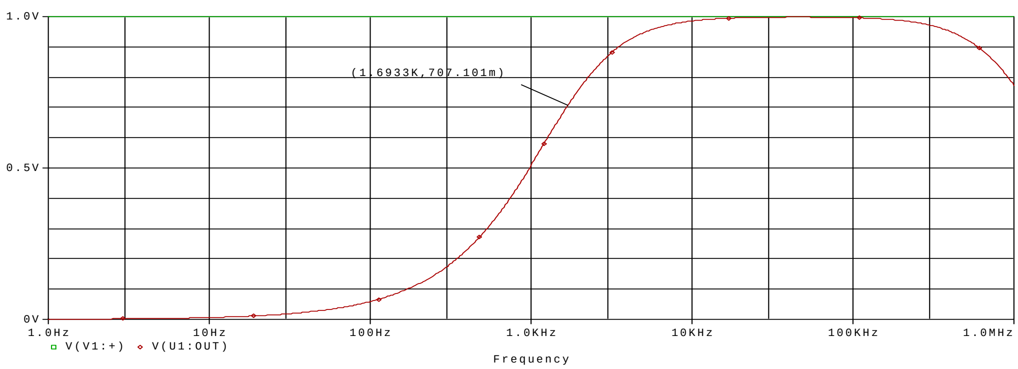
This circuit lets the high-frequency signals through without inverting the input.



First-order non-inverting high-pass filter

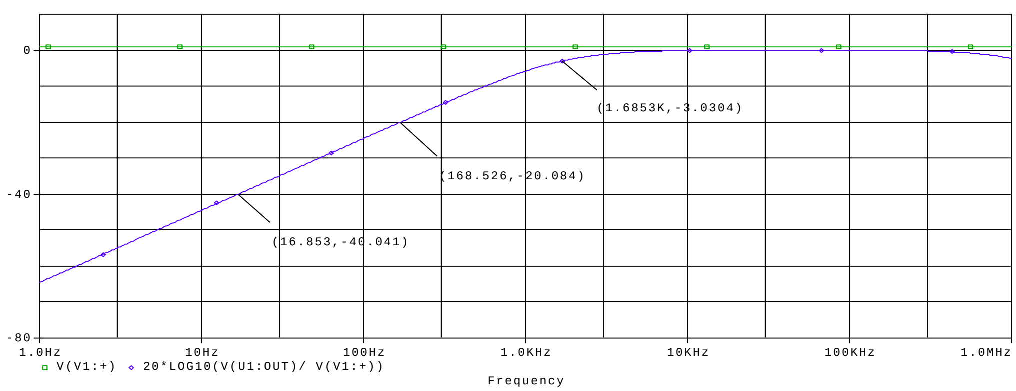
The –3dB frequency is given by

$$f_{-3dB} = \frac{1}{2\pi RC} = \frac{1}{2\pi \cdot 2k\Omega \cdot 47nF} = 1.693kHz$$



AC sweep from 1Hz to 1MHz

The –3dB frequency for the filter is at 1.693kHz.

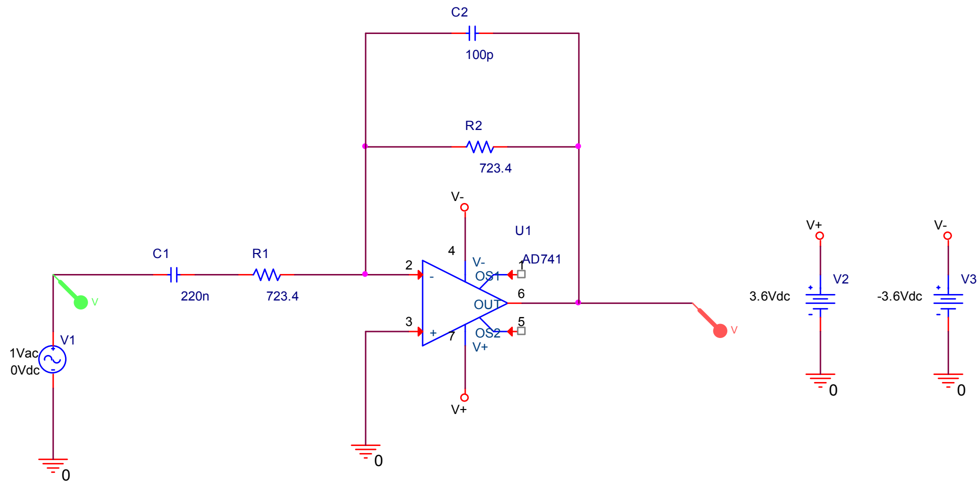


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1.693kHz and then it decreases by –20dB/decade.

Inverting high-pass

This circuit lets the high-frequency signals through while inverting the input.

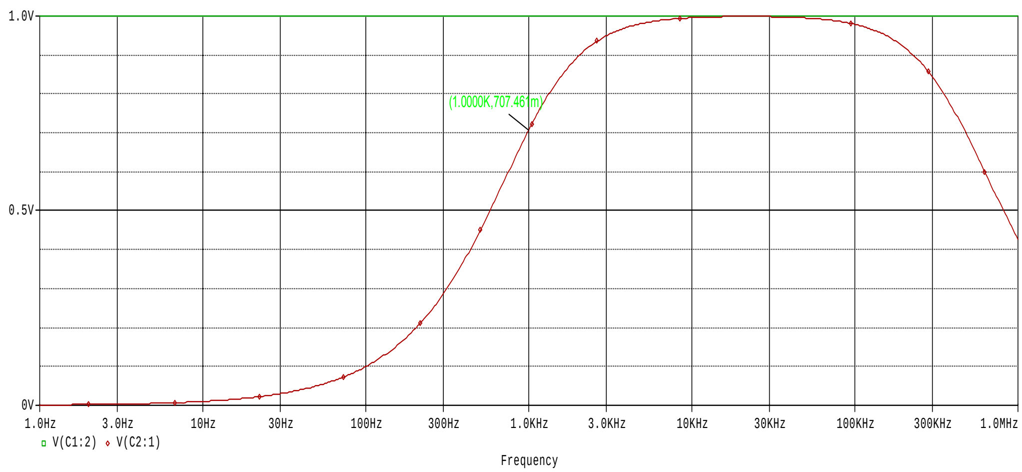


First-order inverting high-pass filter

The 100pF capacitor ensures stability at high frequency.

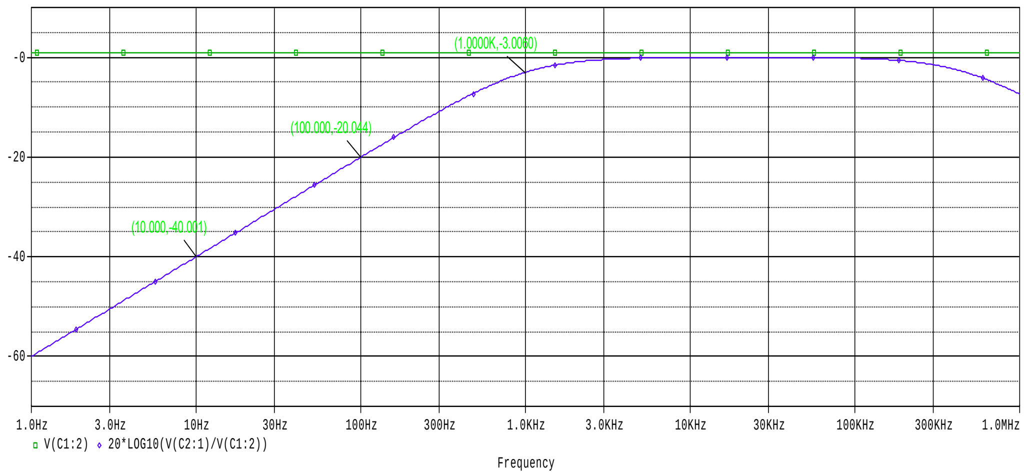
The –3dB frequency is given by

$$f_{-3dB} = \frac{1}{2\pi RC_1} = \frac{1}{2\pi \cdot 723.4\Omega \cdot 220nF} = 1kHz$$



AC sweep from 1Hz to 1MHz

The –3dB frequency for the filter is at 1kHz.



Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases by –20dB/decade.

Second-order filters

Second-order filters have two poles which are given by two capacitors.

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

Depending on low-pass or high-pass configurations, the quality factors are

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)} \qquad \text{low-pass}$$

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)} \qquad \text{high-pass}$$

where C_2 is the shunt capacitor for the low-pass configuration and R_1 is the bridging resistor in the high-pass configuration (consistency is important).

The quality factors for 2^{nd}-order and higher filters are summarized here:

Butterworth	0.7071
Linkwitz-Riley	0.5
Bessel	0.577
Chebyshev (0.5dB)	0.8637
Chebyshev (1dB)	0.9565
Chebyshev (2dB)	1.1286
Chebyshev (3dB)	1.3049
Transitional or Paynter	0.639
Butterworth-Thomson or Linear phase	0.6304

Every type of filter is designed to retain a portion of a signal for a specific range of frequencies while suppressing the same signal at other undesired frequencies. The major difference among filters is in the mathematical framework that lies behind them. Every filter has different ratios among resistors and capacitors and this produces a different response.

Filters of second and higher orders come in unity-gain or gain versions. If they are in the unity-gain configuration, the op-amp is in buffer mode. If gain is needed, then two additional resistors are used to provide the proper gain.

The trick behind designing 2^{nd}-order filters is to set up a *system of two equations (f_c and Q) in two unknowns (C and R)*. The first equation sets the frequency. The second equation sets the quality factor. It is necessary to choose specific values for frequency and quality factor while leaving two passive components unknowns. By using software like Maple it is possible to force a convergence and calculate the two unknowns (C and R).

Butterworth filter

The Butterworth filter was originally proposed by Stephen Butterworth in 1930.

This filter can be implemented with different orders. For every order, the gain of the filter will drop by –6dB/octave or –20dB/decade past the cutoff frequency. Increasing the order of the filter will produce a sharper cutoff.

The Butterworth filter has a very flat response and does not present ripples in the pass-band. It can be arranged for low-pass, high-pass, band-pass and band-stop/notch purposes.

A *band-pass* Butterworth filter is obtained by placing an inductor in *parallel* with each capacitor to form resonant circuits. The value of each additional component must be selected to resonate with the other component at the frequency of interest.

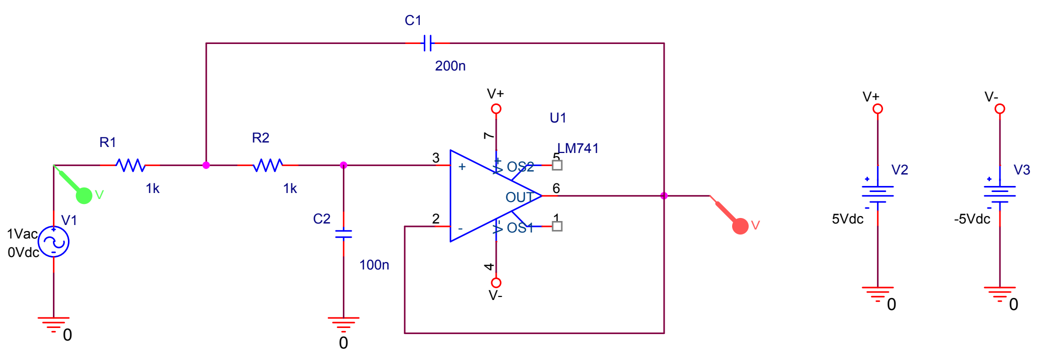
A *band-stop/notch* Butterworth filter is obtained by placing an inductor in *series* with each capacitor to form resonant circuits. The value of each additional component must be selected to resonate with the other component at the frequency to be rejected.

The Butterworth filter can be implemented with different topologies, including Cauer (passive) and Sallen/Key (active).

For a Butterworth filter, the quality factor must be 0.707.

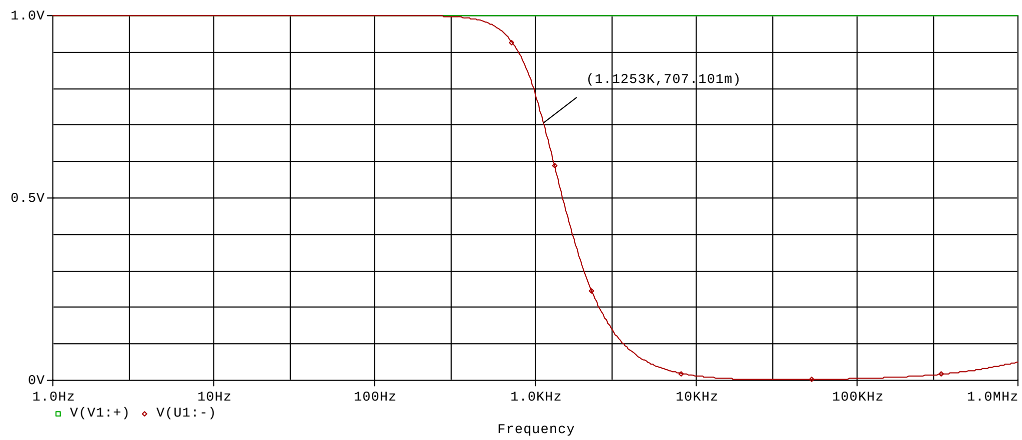
Second-order low-pass

In the following example, the circuit is implemented with the Sallen/Key topology. The capacitor values do not match here. This is a second-order filter because it has two capacitors.



Butterworth filter (low-pass) (2^{nd}-order)

Note: R_1=R_2 and C_1=2xC_2.



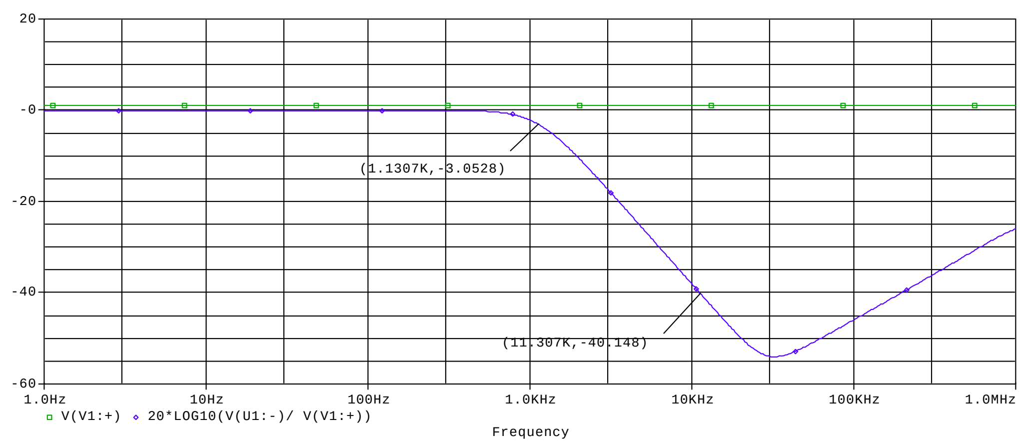
AC sweep from 1Hz to 1MHz

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 200nF \cdot 100nF}} = 1.125kHz$$

The quality factor is

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 200nF \cdot 100nF}}{100nF \cdot (1k\Omega + 1k\Omega)} = 0.707$$

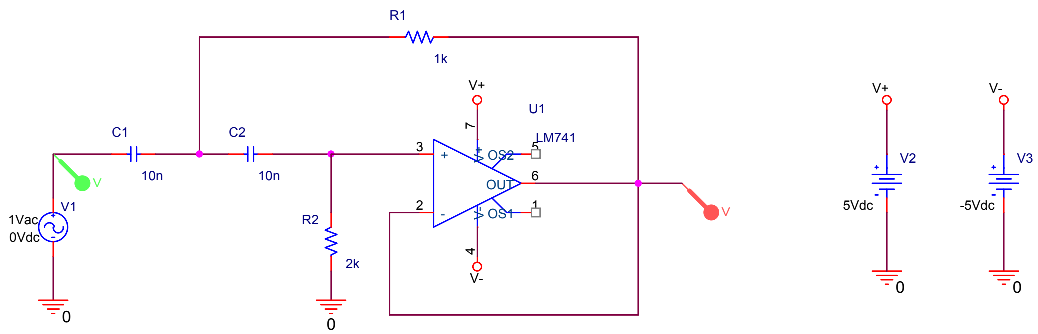


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1.125kHz and then it decreases by –40dB/decade.

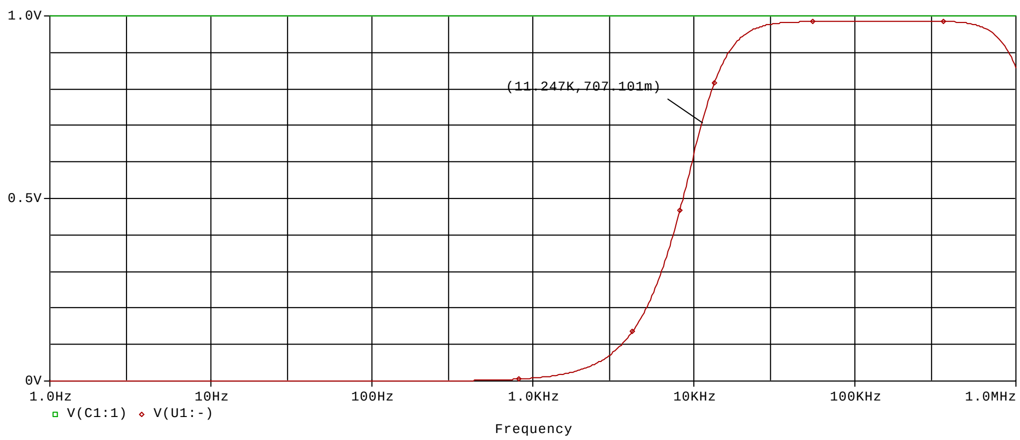
Second-order high-pass

In the following example, the circuit is implemented with the Sallen/Key topology. The resistor values do not match here. This is a second-order filter because it has two capacitors.



Butterworth filter (high-pass) (2nd-order)

Note: C_1=C_2 and R_2=2xR_1.



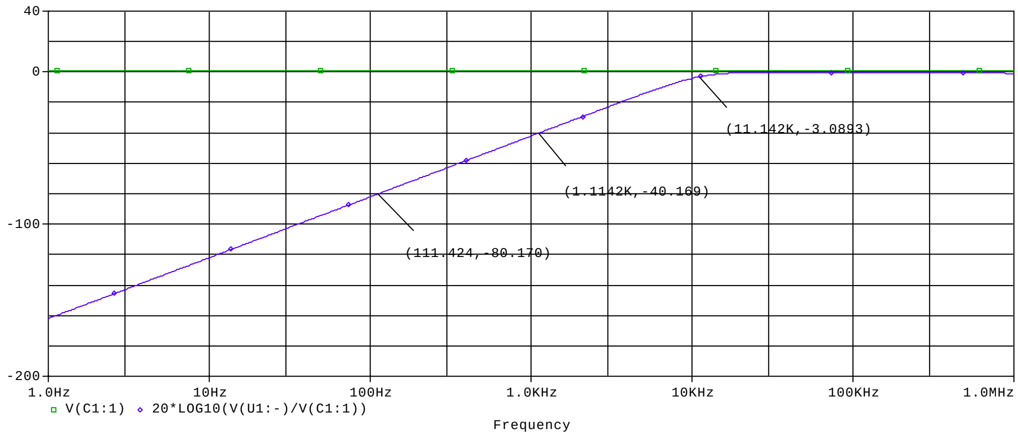
AC sweep from 1Hz to 1MHz

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 2k\Omega \cdot 10nF \cdot 10nF}} = 11.253kHz$$

The quality factor is

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1(C_1 + C_2)} = \frac{\sqrt{1k\Omega \cdot 2k\Omega \cdot 10nF \cdot 10nF}}{1k\Omega \cdot (10nF + 10nF)} = 0.707$$

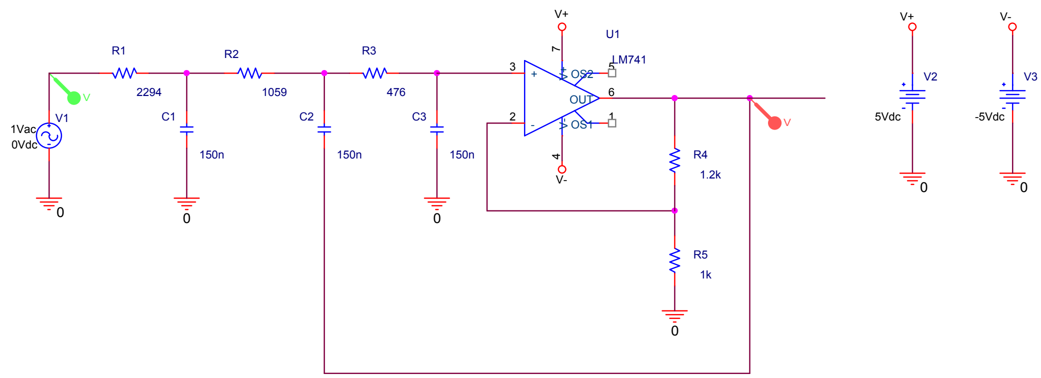


Bode plot from 10Hz to 1MHz

The gain drops to –3dB at about 11.253kHz and then it decreases by –40dB/decade.

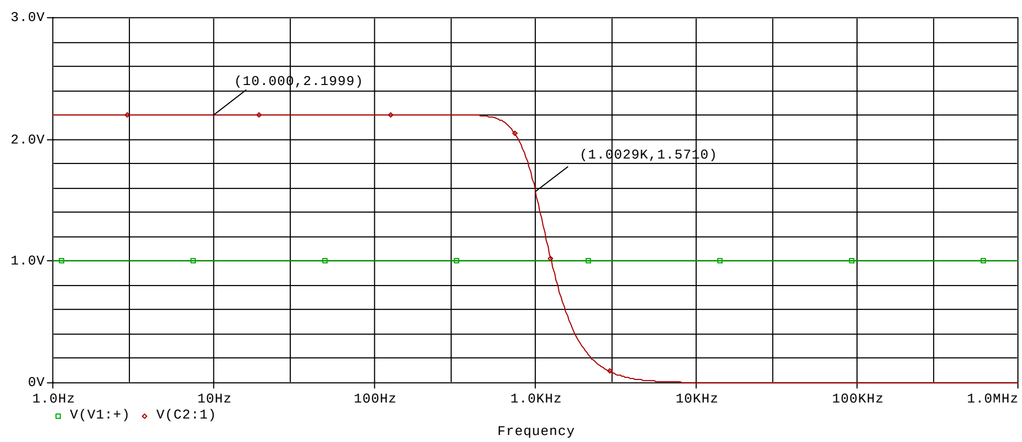
Third-order low-pass (same capacitor values)

In the following example, the circuit is implemented with the Sallen/Key topology. The capacitor values match here and the circuit provides gain by means of two additional resistors. This is a third-order filter because it has three capacitors.



Butterworth filter (low-pass) (3rd-order)

Note: C_1=C_2=C_3.

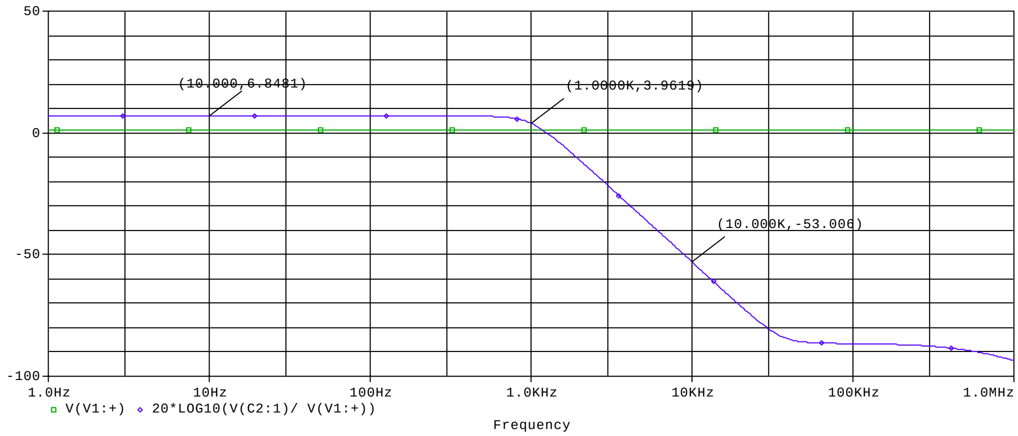


AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

The gain is given by the gain resistors:

$$A = \frac{R_4 + R_5}{R_5} = \frac{1.2k\Omega + 1k\Omega}{1k\Omega} = 2.2 \qquad \text{or} \qquad +6.848\text{dB}$$

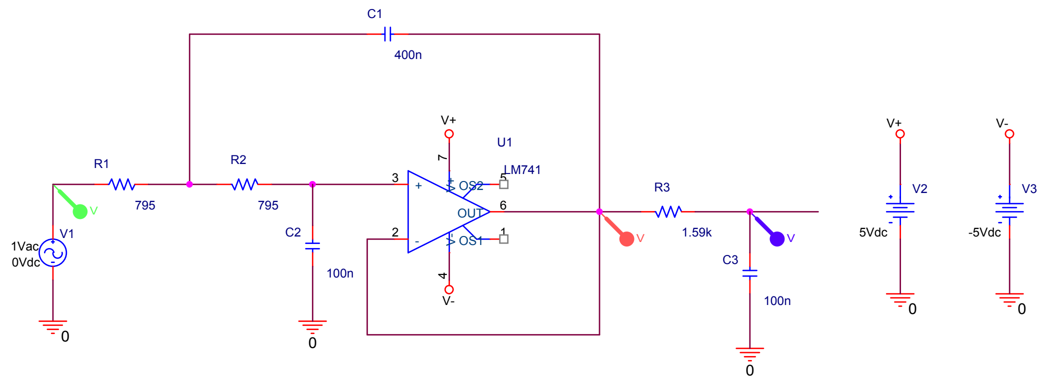


Bode plot from 1Hz to 1MHz

The gain is flat at +6.8481dB in the lower frequency range and then it drops to +3.9619dB at 1kHz. It eventually decreases to –53.006dB one decade later.

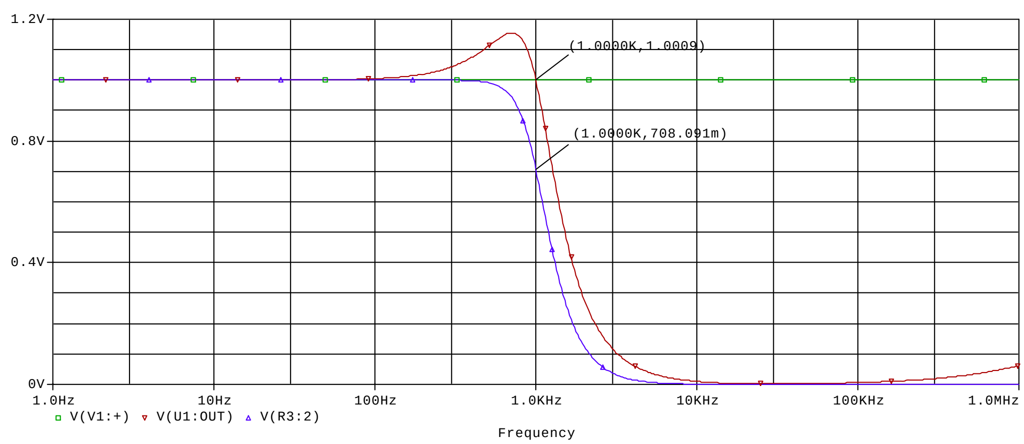
Third-order low-pass (cascaded)

This circuit is implemented with the Sallen/Key topology by *cascading* a simple low-pass filter after a second-order filter. For the second-order block the quality factor must be 1 whereas the quality factor for the low-pass filter is not defined. This is a third-order filter because it has three capacitors. The overall quality factor is 0.707.



Butterworth filter (low-pass) (3rd-order)

Note: R_1=R_2 and C_1=4xC_2.



AC sweep from 1Hz to 1MHz

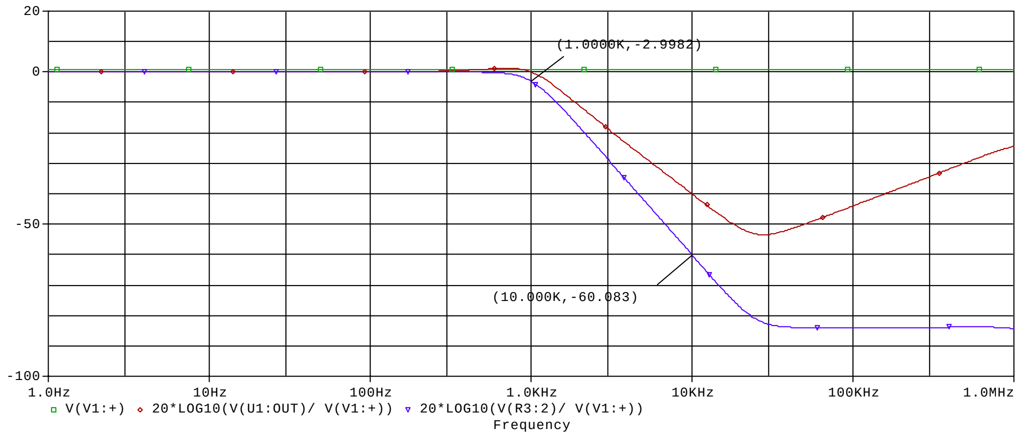
The cutoff frequencies are

$$f_{c1} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{795\Omega \cdot 795\Omega \cdot 400nF \cdot 100nF}} = 1kHz$$

$$f_{c2} = \frac{1}{2\pi R_3 C_3} = \frac{1}{2\pi \cdot 1.59k\Omega \cdot 100nF} = 1kHz$$

The quality factor of the second-order block is

$$Q_1 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)} = \frac{\sqrt{795\Omega \cdot 795\Omega \cdot 400nF \cdot 100nF}}{100nF \cdot (795\Omega + 795\Omega)} = 1$$

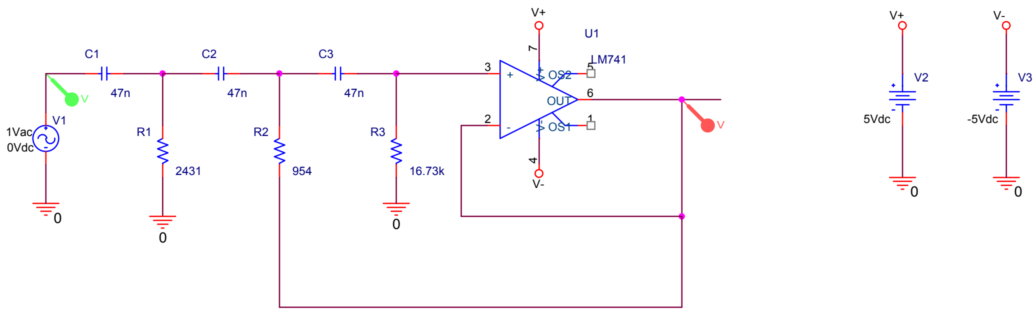


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases by –60dB/decade.

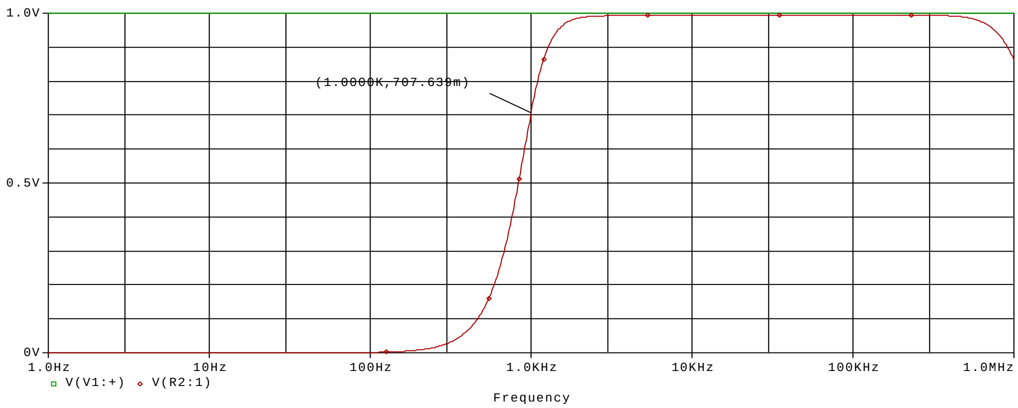
Third-order high-pass (same resistor values)

In the following example, the circuit is implemented with the Sallen/Key topology. This is a third-order filter because it has three capacitors.



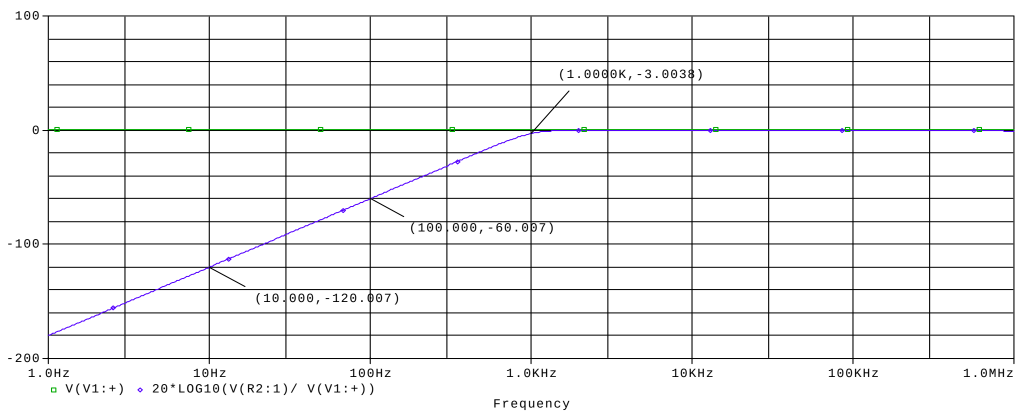
Butterworth filter (high-pass) (3rd-order)

Note: C_1=C_2=C_3.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

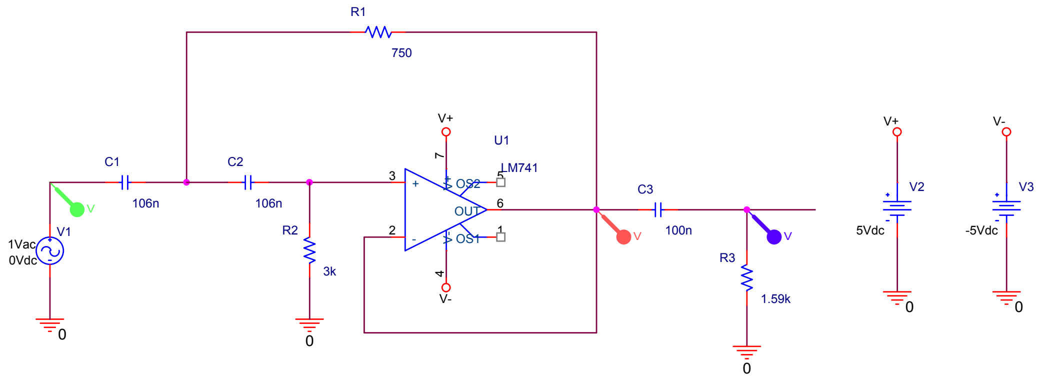


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases by –60dB/decade.

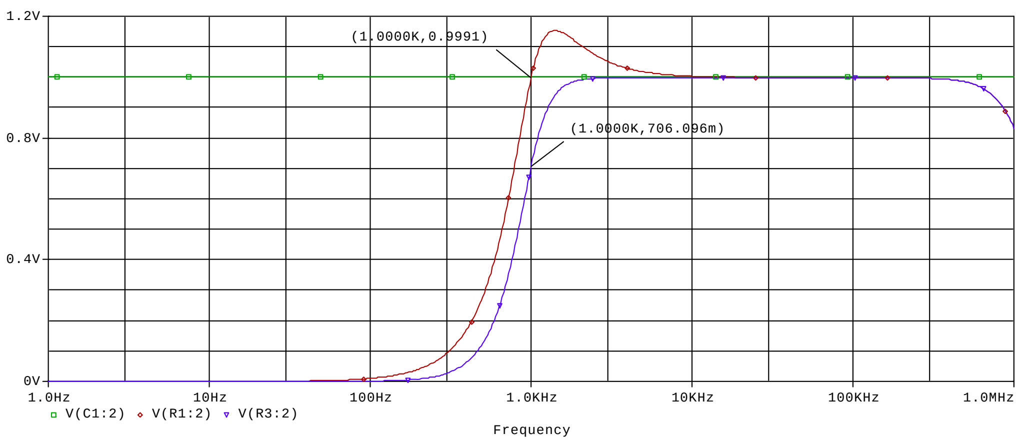
Third-order high-pass (cascaded)

This circuit is implemented with the Sallen/Key topology by *cascading* a simple high-pass filter after a second-order filter. For the second-order block the quality factor must be 1 whereas the quality factor for the low-pass filter is not defined. This is a third-order filter because it has three capacitors. The overall quality factor is 0.707.



Butterworth filter (high-pass) (3rd-order)

Note: C_1=C_2 and R_2=4xR_1.



AC sweep from 1Hz to 1MHz

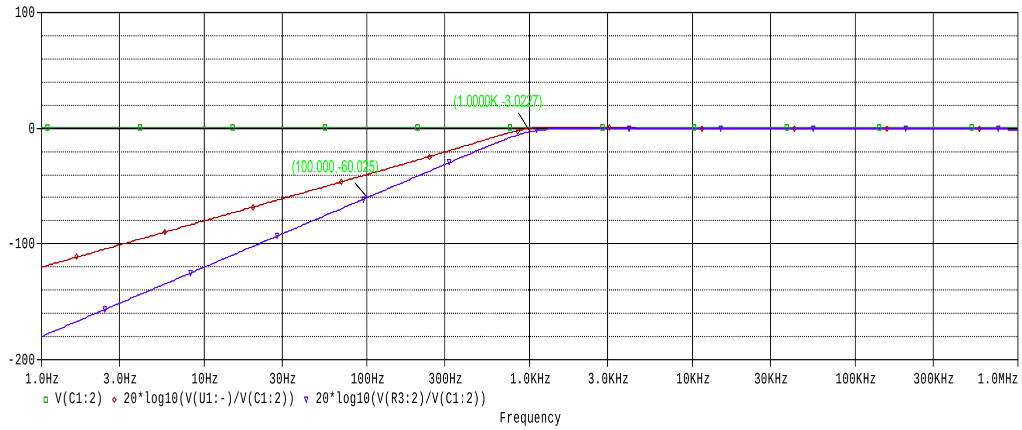
The cutoff frequencies are

$$f_{c1} = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} = \frac{1}{2\pi\sqrt{750\Omega\cdot 3k\Omega\cdot 106nF\cdot 106nF}} = 1kHz$$

$$f_{c2} = \frac{1}{2\pi R_3C_3} = \frac{1}{2\pi\cdot 1.59k\Omega\cdot 100nF} = 1kHz$$

The quality factor of the second-order block is

$$Q_1 = \frac{\sqrt{R_1R_2C_1C_2}}{R_1\left(C_1+C_2\right)} = \frac{\sqrt{750\Omega\cdot 3k\Omega\cdot 106nF\cdot 106nF}}{750\Omega\cdot\left(106nF+106nF\right)} = 1$$

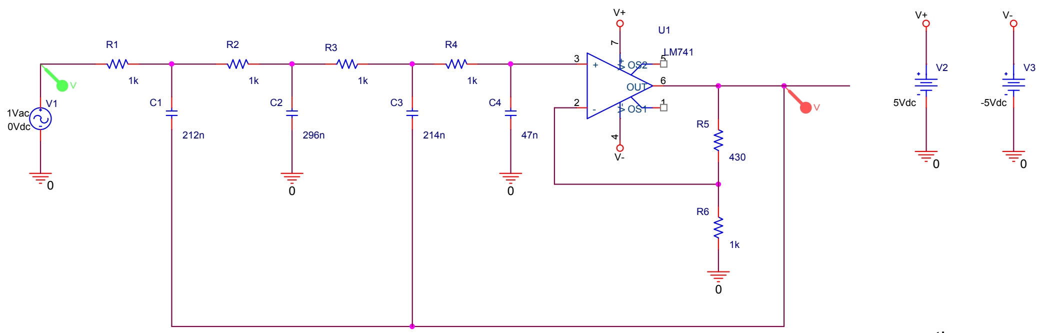


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases by –60dB/decade.

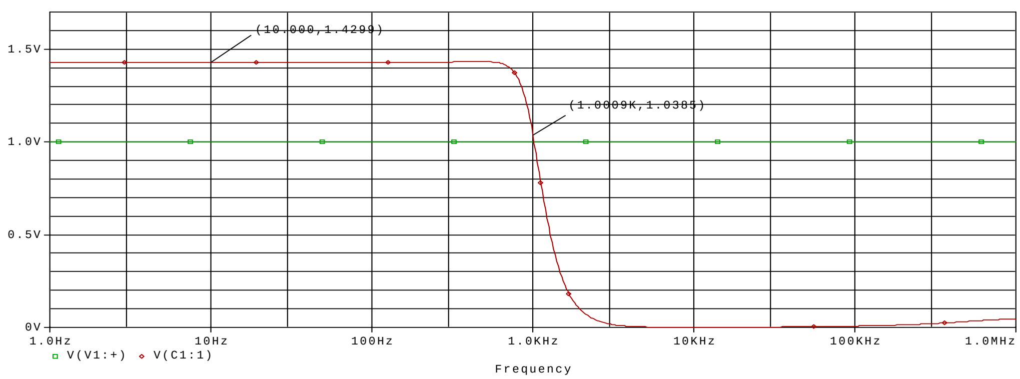
Fourth-order low-pass (same resistor values)

In the following example, the circuit is implemented with the Sallen/Key topology. The resistor values match here and the circuit provides gain by means of two additional resistors. This is a fourth-order filter because it has four capacitors.



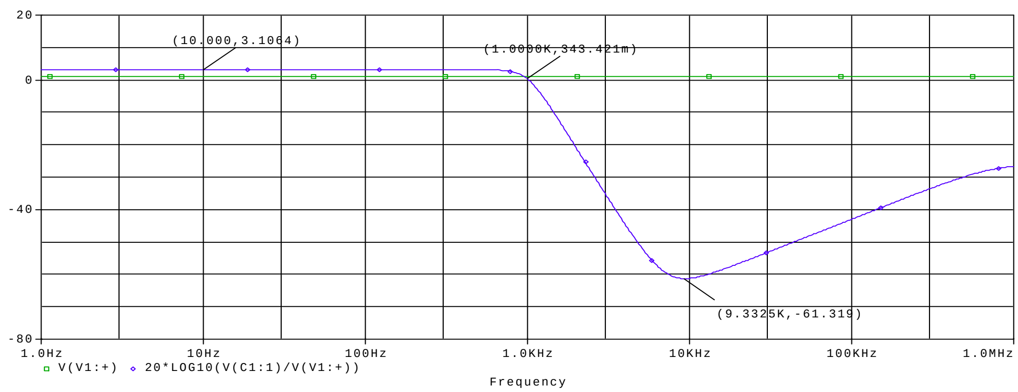
Butterworth filter (low-pass) (4^{th}-order)

Note: all resistor values match.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

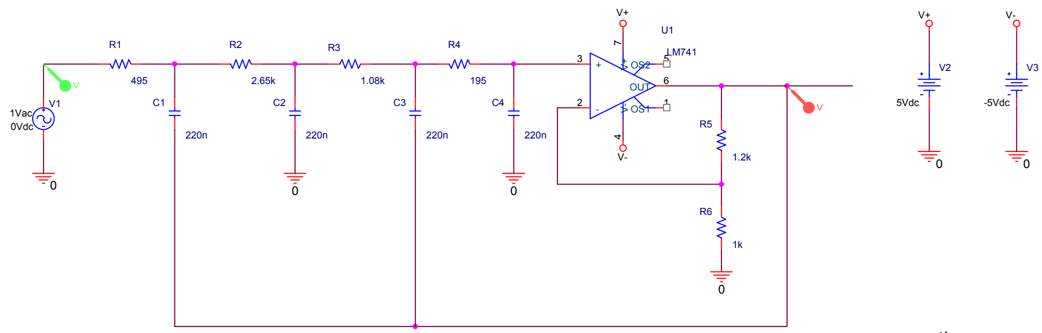


Bode plot from 1Hz to 1MHz

The gain is flat at +3.1064dB in the lower frequency range and then it drops to +0.343dB at 1kHz. It eventually decreases to –61.319dB at 9.3325kHz.

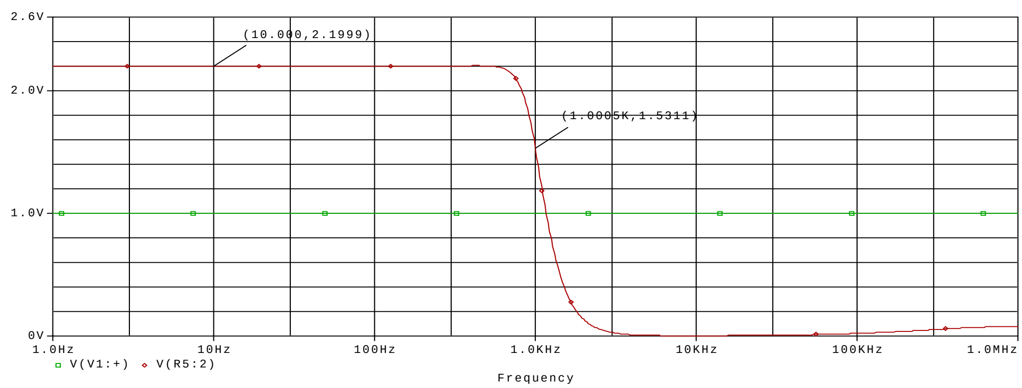
Fourth-order low-pass (same capacitor values)

In the following example, the circuit is implemented with the Sallen/Key topology. The capacitor values match here and the circuit provides gain by means of two additional resistors. This is a fourth-order filter because it has four capacitors.



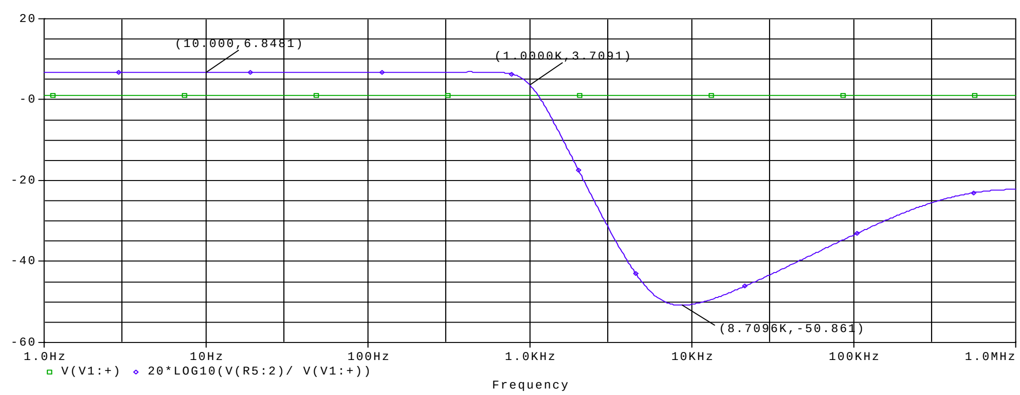
Butterworth filter (low-pass) (4th-order)

Note: all capacitor values match.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

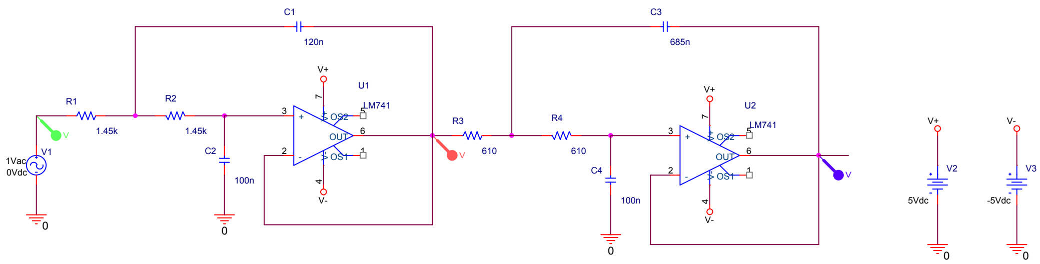


Bode plot from 1Hz to 1MHz

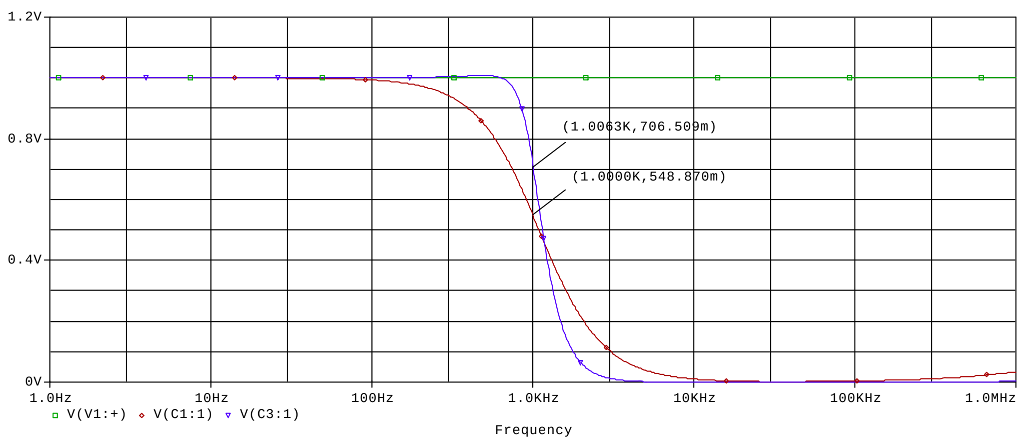
The gain is flat at +6.8481dB in the lower frequency range and then it drops to +3.7091dB at 1kHz. It eventually decreases to –50.861dB at 8.7096kHz.

Fourth-order low-pass (cascaded)

This circuit is implemented with the Sallen/Key topology by *cascading* two second-order filters. The quality factors for the first and the second block must be 0.5412 and 1.3065 respectively. This is a fourth-order filter because it has four capacitors. The overall quality factor is 0.707.



Butterworth filter (low-pass) (4th-order)



AC sweep from 1Hz to 1MHz

The cutoff frequencies are

$$f_{c1} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1.45k\Omega \cdot 1.45k\Omega \cdot 120nF \cdot 100nF}} = 1001Hz$$

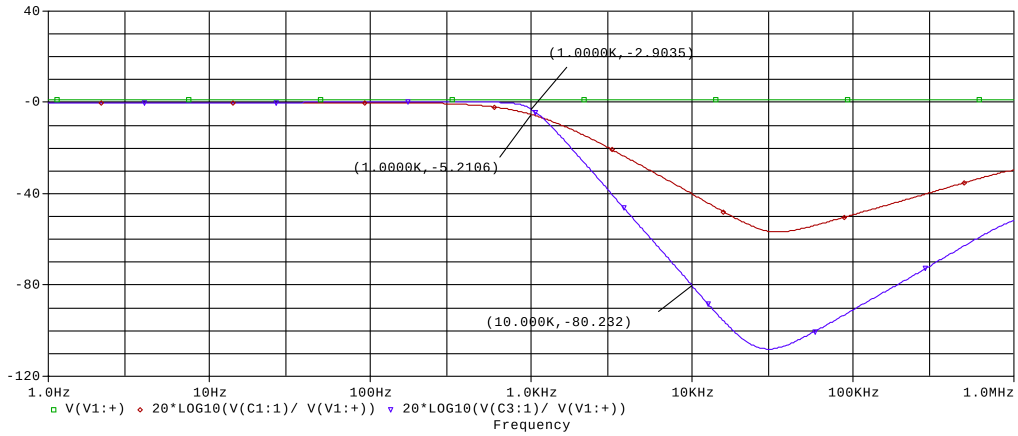
$$f_{c2} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{610\Omega \cdot 610\Omega \cdot 685nF \cdot 100nF}} = 997Hz$$

The quality factors are

$$Q_1 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)} = \frac{\sqrt{1.45k\Omega \cdot 1.45k\Omega \cdot 120nF \cdot 100nF}}{100nF \cdot (1.45k\Omega + 1.45k\Omega)} = 0.5477$$

$$Q_2 = \frac{\sqrt{R_3 R_4 C_3 C_4}}{C_4 (R_3 + R_4)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 685nF \cdot 100nF}}{100nF \cdot (1k\Omega + 1k\Omega)} = 1.3086$$

The quality factors are reasonably close to 0.5412 and 1.3065.

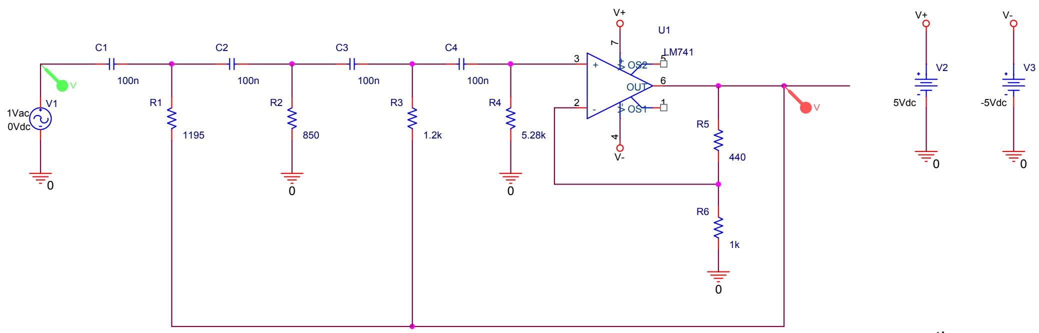


Bode plot from 1Hz to 1MHz

The gain drops to –2.9035dB at 1kHz and then it decreases by –80dB/decade.

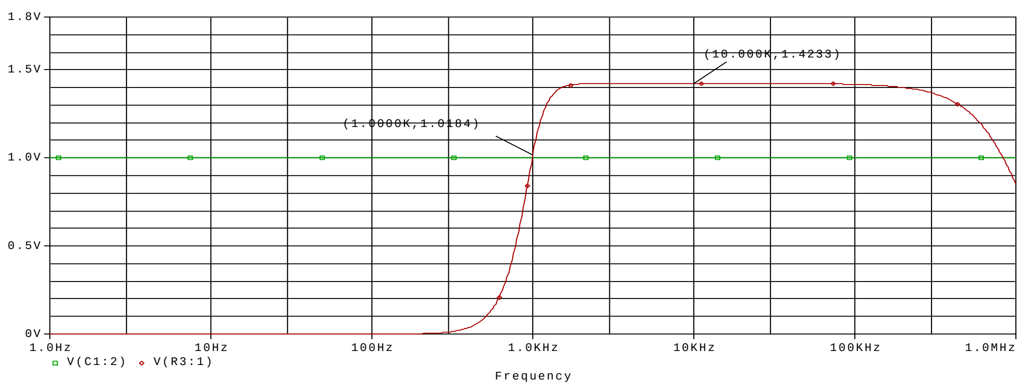
Fourth-order high-pass (same capacitor values)

In the following example, the circuit is implemented with the Sallen/Key topology. The capacitor values match here and the circuit provides gain by means of two additional resistors. This is a fourth-order filter because it has four capacitors.



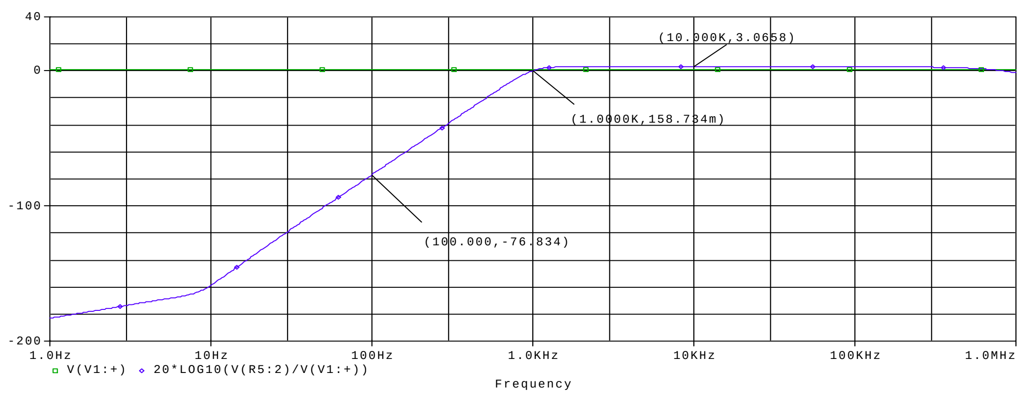
Butterworth filter (high-pass) (4^{th}-order)

Note: all capacitor values match.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

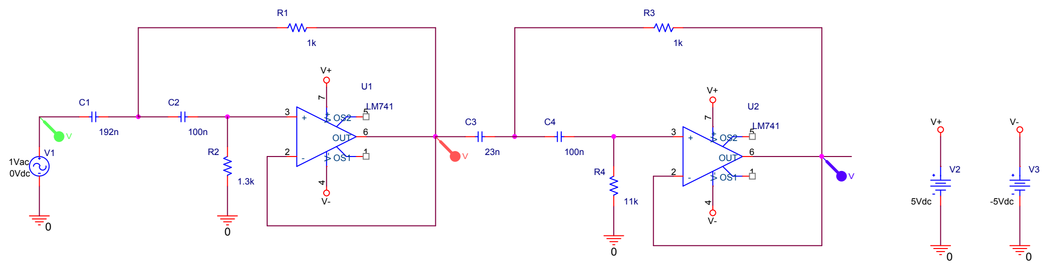


Bode plot from 1Hz to 1MHz

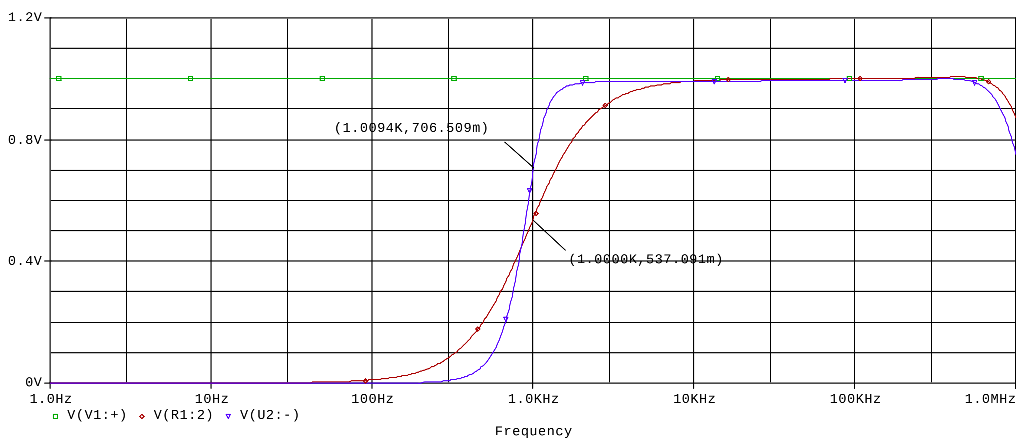
The gain is flat at +3.6058dB in the higher frequency range and then it drops to +0.158dB at 1kHz. It eventually decreases to –76.834 at 100Hz.

Fourth-order high-pass (cascaded)

This circuit is implemented with the Sallen/Key topology by *cascading* two second-order filters. The quality factors for the first and the second block must be 0.5412 and 1.3065 respectively. This is a fourth-order filter because it has four capacitors. The overall quality factor is 0.707.



Butterworth filter (high-pass) (4th-order)



AC sweep from 1Hz to 1MHz

The cutoff frequencies are

$$f_{c1} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1.3k\Omega \cdot 192nF \cdot 100nF}} = 1007Hz$$

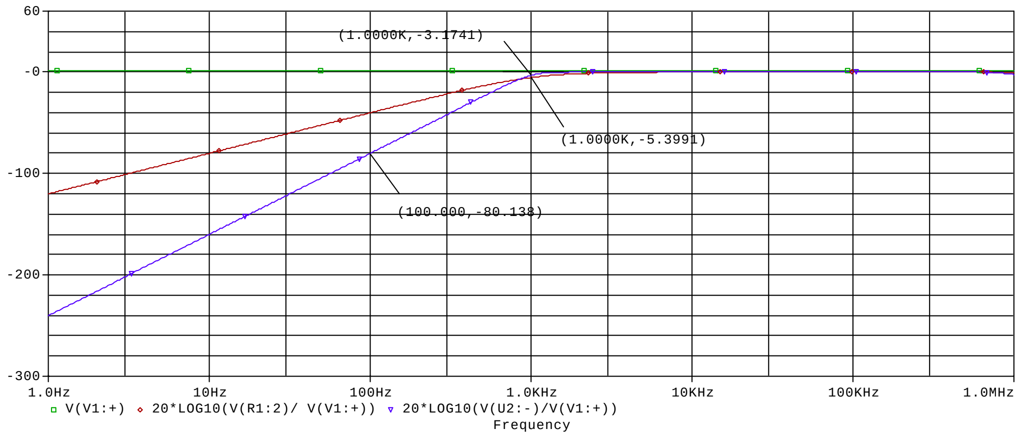
$$f_{c2} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 11k\Omega \cdot 23nF \cdot 100nF}} = 1kHz$$

The quality factors are

$$Q_1 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)} = \frac{\sqrt{1k\Omega \cdot 1.3k\Omega \cdot 192nF \cdot 100nF}}{1k\Omega \cdot (192nF + 100nF)} = 0.5411$$

$$Q_2 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)} = \frac{\sqrt{1k\Omega \cdot 11k\Omega \cdot 23nF \cdot 100nF}}{1k\Omega \cdot (23nF + 100nF)} = 1.2932$$

The quality factors are reasonably close to 0.5412 and 1.3065.

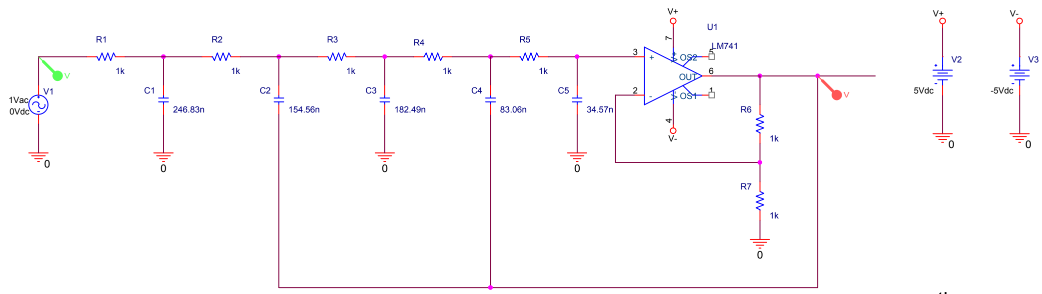


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases by –80dB/decade.

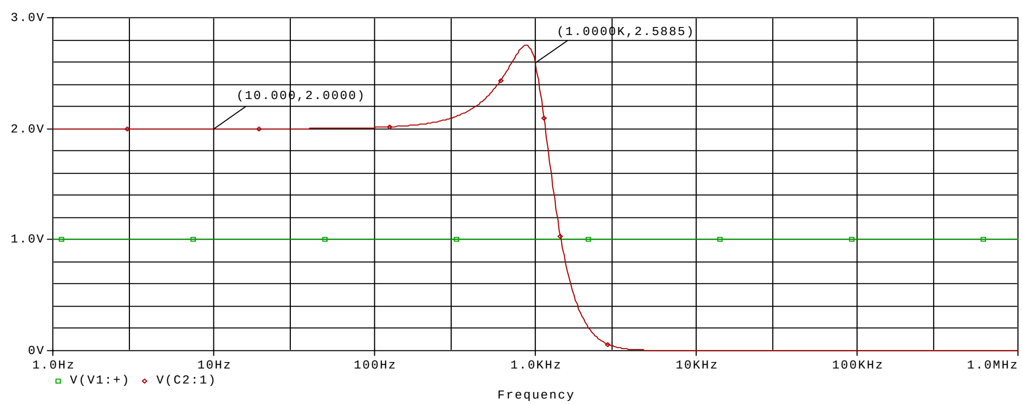
Fifth-order low-pass (same resistor values)

In the following example, the circuit is implemented with the Sallen/Key topology. The resistor values match here and the circuit provides gain by means of two additional resistors. This is a fifth-order filter because it has five capacitors.



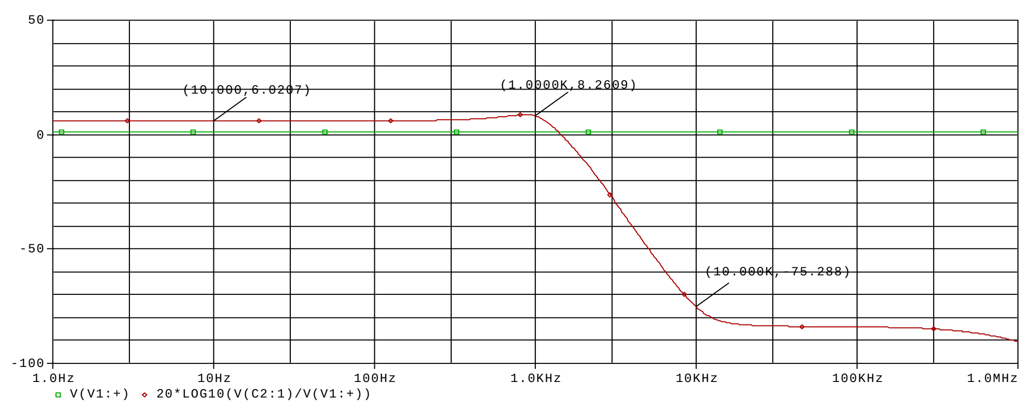
Butterworth filter (low-pass) (5th-order)

Note: all resistor values match.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

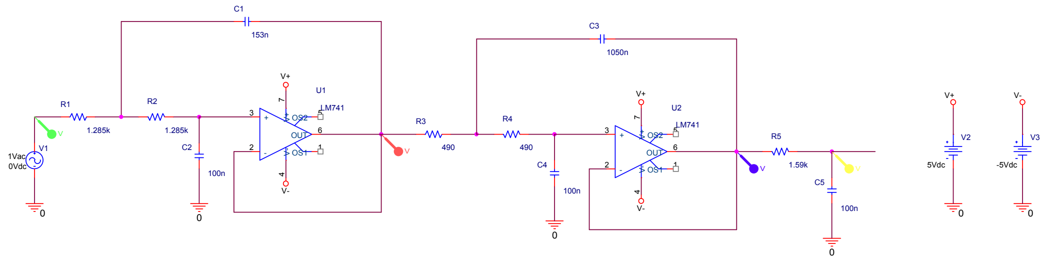


Bode plot from 1Hz to 1MHz

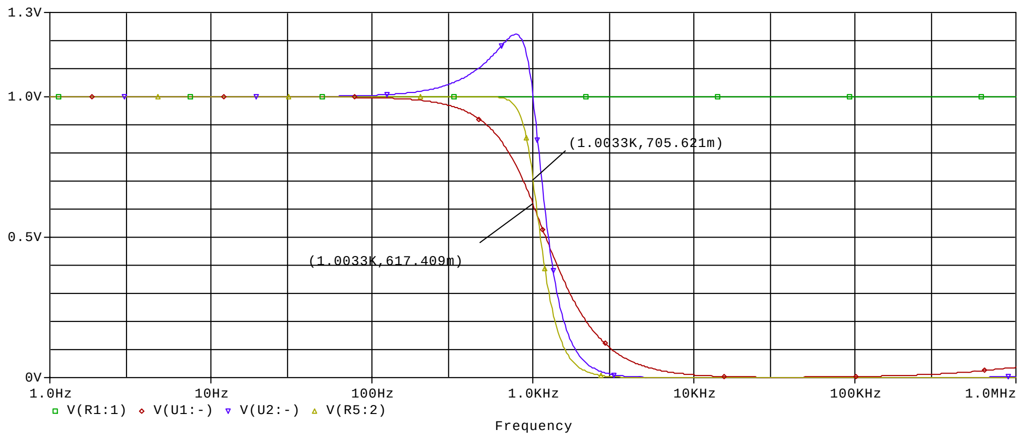
The gain is flat at +6dB in the lower frequency range and then it peaks to +8.2609dB at 1kHz. It eventually decreases to –75.288dB at 10kHz.

Fifth-order low-pass (cascaded)

This circuit is implemented with the Sallen/Key topology by *cascading* a simple low-pass filter after two second-order filters. The quality factors for the first and the second block must be 0.6180 and 1.6181 whereas the quality factor for the low-pass filter is not defined. This is a fifth-order filter because it has five capacitors. The overall quality factor is 0.707.



Butterworth filter (low-pass) (5th-order)



AC sweep from 1Hz to 1MHz

The cutoff frequencies are

$$f_{c1} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1.285k\Omega \cdot 1.285k\Omega \cdot 153nF \cdot 100nF}} = 1001Hz$$

$$f_{c2} = \frac{1}{2\pi\sqrt{R_3 R_4 C_3 C_4}} = \frac{1}{2\pi\sqrt{490\Omega \cdot 490\Omega \cdot 1050nF \cdot 100nF}} = 1002Hz$$

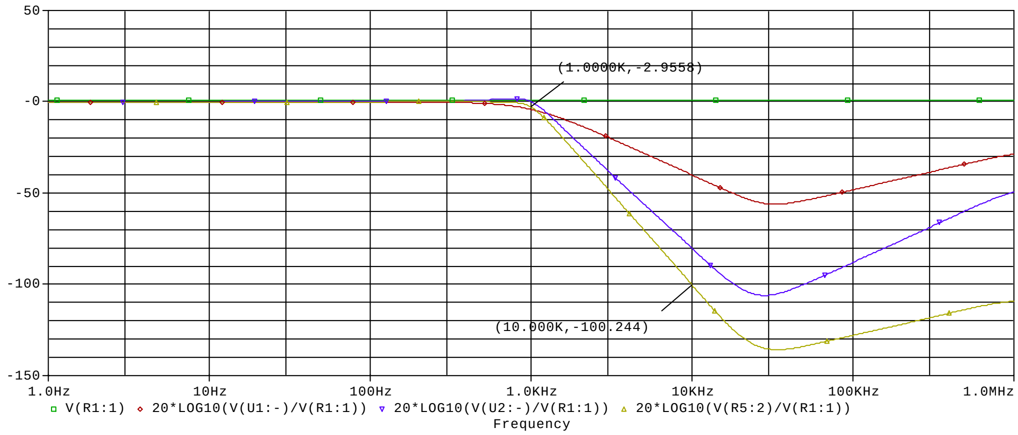
$$f_{c3} = \frac{1}{2\pi R_5 C_5} = \frac{1}{2\pi \cdot 1.59k\Omega \cdot 100nF} = 1kHz$$

The quality factors are

$$Q_1 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)} = \frac{\sqrt{1.285k\Omega \cdot 1.285k\Omega \cdot 153nF \cdot 100nF}}{100nF \cdot (1.285k\Omega + 1.285k\Omega)} = 0.6185$$

$$Q_2 = \frac{\sqrt{R_3 R_4 C_3 C_4}}{C_4 (R_3 + R_4)} = \frac{\sqrt{490\Omega \cdot 490\Omega \cdot 1050nF \cdot 100nF}}{100nF \cdot (490\Omega + 490\Omega)} = 1.6202$$

The quality factors are reasonably close to 0.6180 and 1.6181.

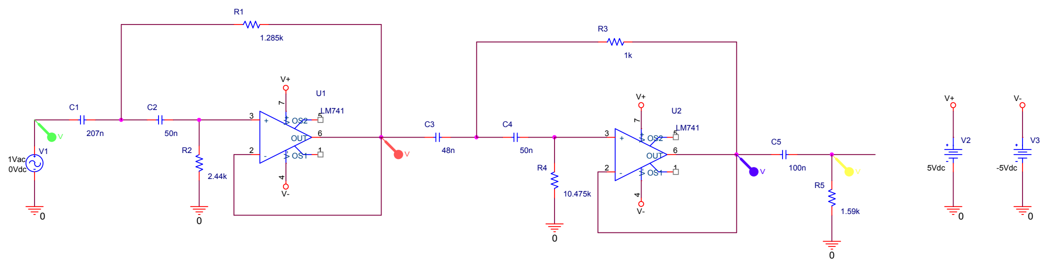


Bode plot from 1Hz to 1MHz

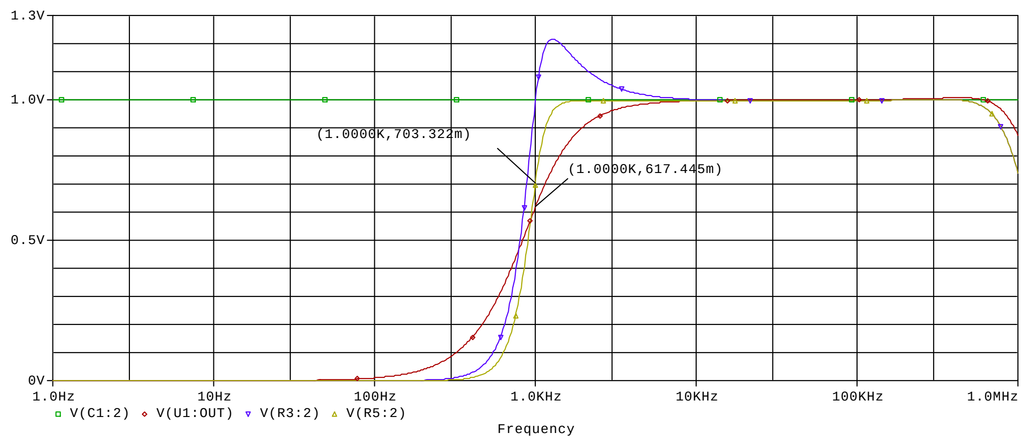
The gain drops to –3dB at 1kHz and then it decreases by –100dB/decade.

Fifth-order high-pass (cascaded)

This circuit is implemented with the Sallen/Key topology by *cascading* a simple low-pass filter after two second-order filters. The quality factors for the first and the second block must be 0.6180 and 1.6181 whereas the quality factor for the high-pass filter is not defined. This is a fifth-order filter because it has five capacitors. The overall quality factor is 0.707.



Butterworth filter (high-pass) (5th-order)



AC sweep from 1Hz to 1MHz

The cutoff frequencies are

$$f_{c1} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 2.44k\Omega \cdot 207nF \cdot 50nF}} = 1001Hz$$

$$f_{c2} = \frac{1}{2\pi\sqrt{R_3 R_4 C_3 C_4}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 10.475k\Omega \cdot 48nF \cdot 50nF}} = 1003Hz$$

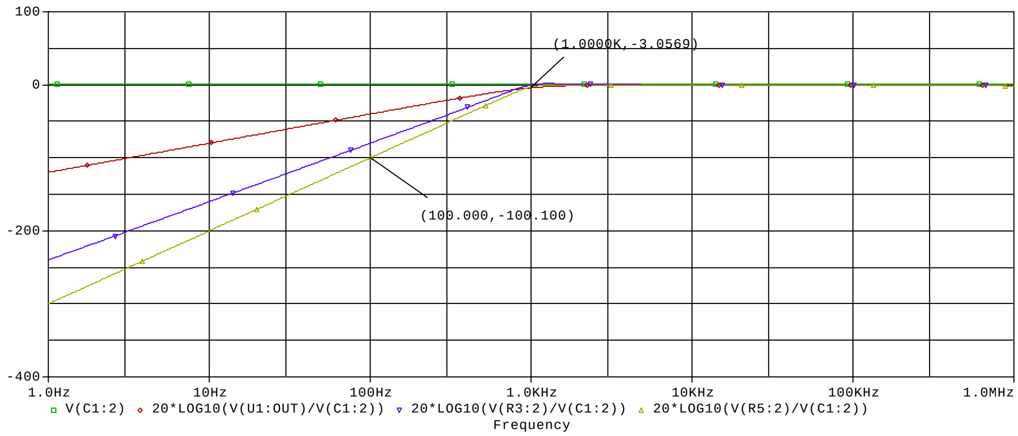
$$f_{c3} = \frac{1}{2\pi R_5 C_5} = \frac{1}{2\pi \cdot 1.59k\Omega \cdot 100nF} = 1kHz$$

The quality factors are

$$Q_1 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1(C_1 + C_2)} = \frac{\sqrt{1k\Omega \cdot 2.44k\Omega \cdot 207nF \cdot 50nF}}{1k\Omega \cdot (207nF + 50nF)} = 0.6183$$

$$Q_2 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1(C_1 + C_2)} = \frac{\sqrt{1k\Omega \cdot 10.475k\Omega \cdot 48nF \cdot 50nF}}{1k\Omega \cdot (48nF + 50nF)} = 1.6179$$

The quality factors are reasonably close to 0.6180 and 1.6181.

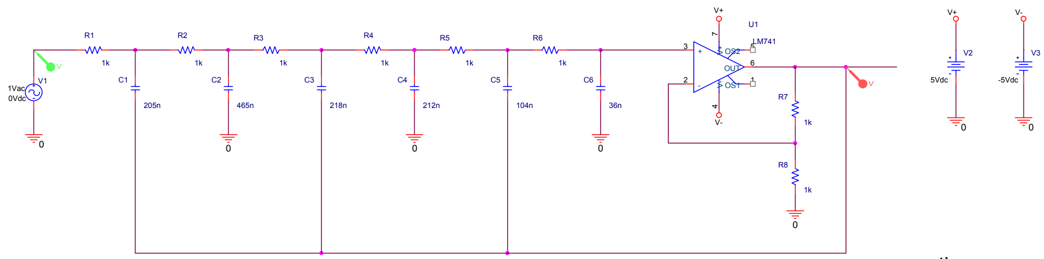


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases by –100dB/decade.

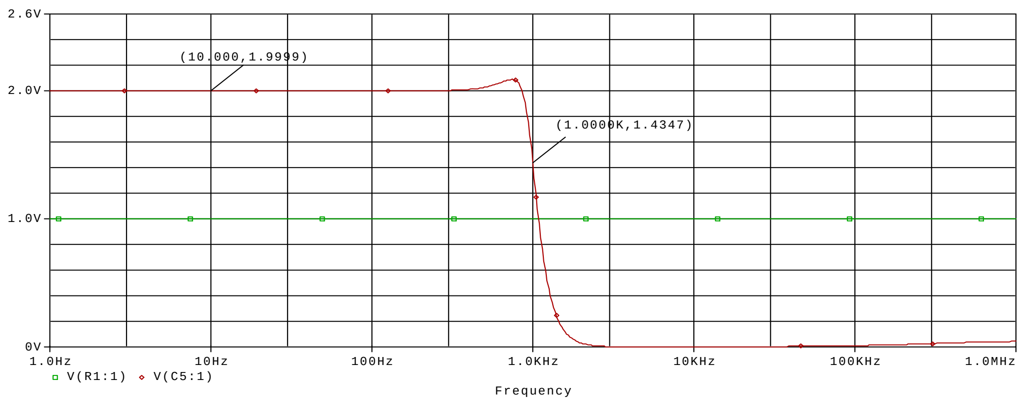
Sixth-order low-pass (same resistor values)

In the following example, the circuit is implemented with the Sallen/Key topology. The resistor values match here and the circuit provides gain by means of two additional resistors. This is a sixth-order filter because it has six capacitors.



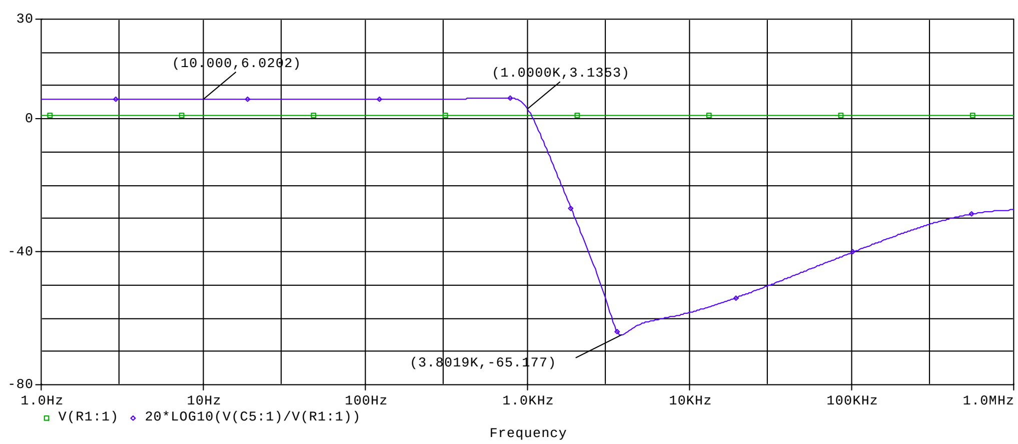
Butterworth filter (low-pass) (6th-order)

Note: all resistor values match.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

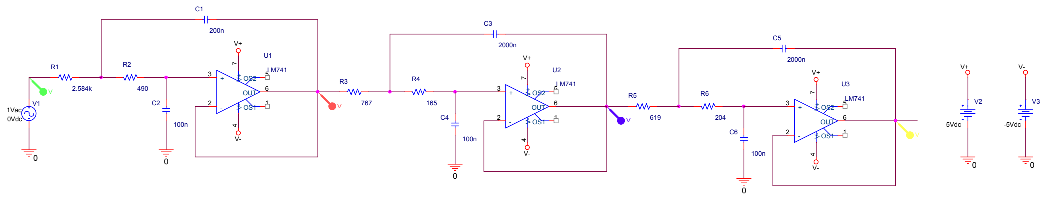


Bode plot from 1Hz to 1MHz

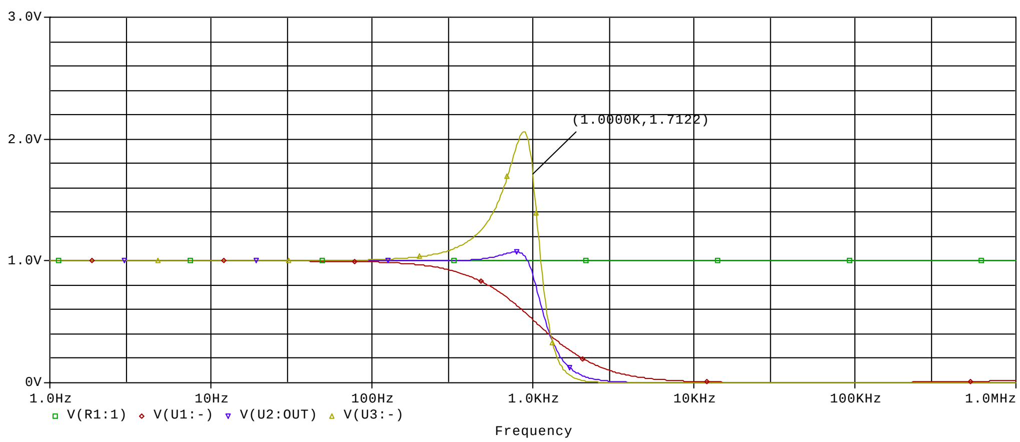
The gain is flat at +6dB in the lower frequency range and then it drops to +3.1353dB at 1kHz. It eventually decreases to –65.177dB at 3.8019kHz.

Sixth-order low-pass (cascaded)

For sake of illustration, a sixth-order low-pass Butterworth filter is shown below. The circuit is nothing more than a *cascade* of 3 second-order blocks like the ones previously shown. The stages must have Q_1=0.5177, Q_2=1.7071 and Q_3=1.9320.



Butterworth filter (low-pass) (6th-order)



AC sweep from 1Hz to 1MHz

The plot shown above clearly demonstrates that increasing the order of the filter will produce a sharper response. Increasing the order of the filter will produce a roughly vertical drop at the cutoff frequency.

The cutoff frequencies are

$$f_{c1}=\frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}=\frac{1}{2\pi\sqrt{2.584k\Omega\cdot 490\Omega\cdot 200nF\cdot 100nF}}=1kHz$$

$$f_{c2}=\frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}=\frac{1}{2\pi\sqrt{767\Omega\cdot 165\Omega\cdot 2000nF\cdot 100nF}}=1kHz$$

$$f_{c3}=\frac{1}{2\pi\sqrt{R_1R_2C_1C_2}}=\frac{1}{2\pi\sqrt{619\Omega\cdot 204\Omega\cdot 2000nF\cdot 100nF}}=1.001kHz$$

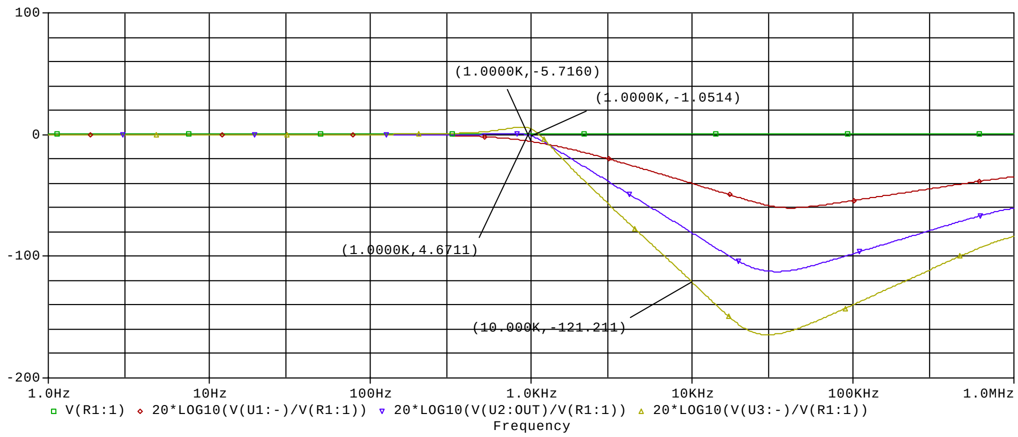
The quality factors are

$$Q_1 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 \left(R_1 + R_2\right)} = \frac{\sqrt{2.584k\Omega \cdot 490\Omega \cdot 200nF \cdot 100nF}}{100nF \cdot \left(2.584k\Omega + 490\Omega\right)} = 0.5177$$

$$Q_2 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 \left(R_1 + R_2\right)} = \frac{\sqrt{767\Omega \cdot 165\Omega \cdot 2000nF \cdot 100nF}}{100nF \cdot \left(767\Omega + 165\Omega\right)} = 1.7070$$

$$Q_3 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 \left(R_1 + R_2\right)} = \frac{\sqrt{619\Omega \cdot 204\Omega \cdot 2000nF \cdot 100nF}}{100nF \cdot \left(619\Omega + 204\Omega\right)} = 1.9310$$

The quality factors are reasonably close to 0.5177, 1.7071 and 1.9320.

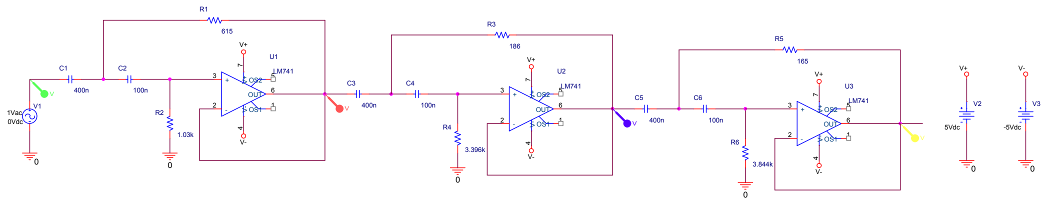


Bode plot from 1Hz to 1MHz

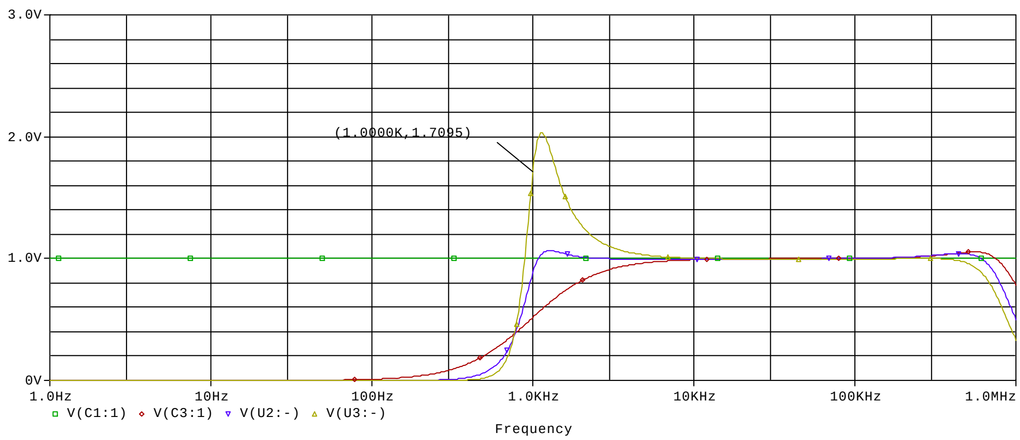
The gain peaks to +4.6711dB at 1 kHz and then it decreases by about –120dB/decade.

Sixth-order high-pass (cascaded)

For sake of illustration, a sixth-order high-pass Butterworth filter is shown below. The circuit is nothing more than a *cascade* of 3 second-order blocks like the ones previously shown. The stages must have Q_1=0.5177, Q_2=1.7071 and Q_3=1.9320 respectively.



Butterworth filter (high-pass) (6th-order)



AC sweep from 1Hz to 1MHz

The plot shown above clearly demonstrates that increasing the order of the filter will produce a sharper response. Increasing the order of the filter will produce a roughly vertical drop at the cutoff frequency.

The cutoff frequencies are

$$f_{c1} = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} = \frac{1}{2\pi\sqrt{615\Omega \cdot 1.03k\Omega \cdot 400nF \cdot 100nF}} = 999.9Hz$$

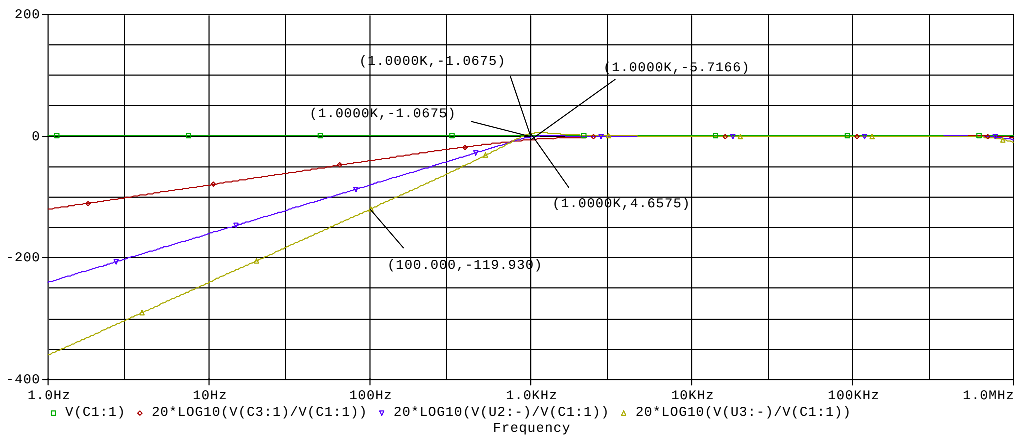
$$f_{c2} = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} = \frac{1}{2\pi\sqrt{186\Omega \cdot 3.396k\Omega \cdot 400nF \cdot 100nF}} = 1001Hz$$

$$f_{c3} = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} = \frac{1}{2\pi\sqrt{165\Omega \cdot 3.844k\Omega \cdot 400nF \cdot 100nF}} = 999.2Hz$$

The quality factors are

$$Q_1 = \frac{\sqrt{R_1R_2C_1C_2}}{R_1(C_1+C_2)} = \frac{\sqrt{615\Omega\cdot 1.03k\Omega\cdot 400nF\cdot 100nF}}{615\Omega\cdot(400nF+100nF)} = 0.5177$$
$$Q_2 = \frac{\sqrt{R_1R_2C_1C_2}}{R_1(C_1+C_2)} = \frac{\sqrt{186\Omega\cdot 3.396k\Omega\cdot 400nF\cdot 100nF}}{186\Omega\cdot(400nF+100nF)} = 1.7092$$
$$Q_3 = \frac{\sqrt{R_1R_2C_1C_2}}{R_1(C_1+C_2)} = \frac{\sqrt{165\Omega\cdot 3.844k\Omega\cdot 400nF\cdot 100nF}}{165\Omega\cdot(400nF+100nF)} = 1.9307$$

The quality factors are reasonably close to 0.5177, 1.7071 and 1.9320.



Bode plot from 1Hz to 1MHz

The gain peaks at +4.6575dB at 1kHz and then it decreases by about –120dB/decade.

Linkwitz-Riley filter

The Linkwitz-Riley filter was invented by Siegfried Linkwitz and Russ Riley in 1978. This filter is alternatively called *Butterworth squared filter* (squared because for the Linkwitz-Riley filter Q=0.5, for the Butterworth filter Q=0.707 and the square of 0.707 is 0.5). This filter is used in audio crossovers.

The Linkwitz-Riley filter can be implemented with different orders. For every order, the gain of the filter will drop by –6dB/octave or –20dB/decade past the cutoff frequency. Increasing the order of the filter will produce a sharper cutoff.

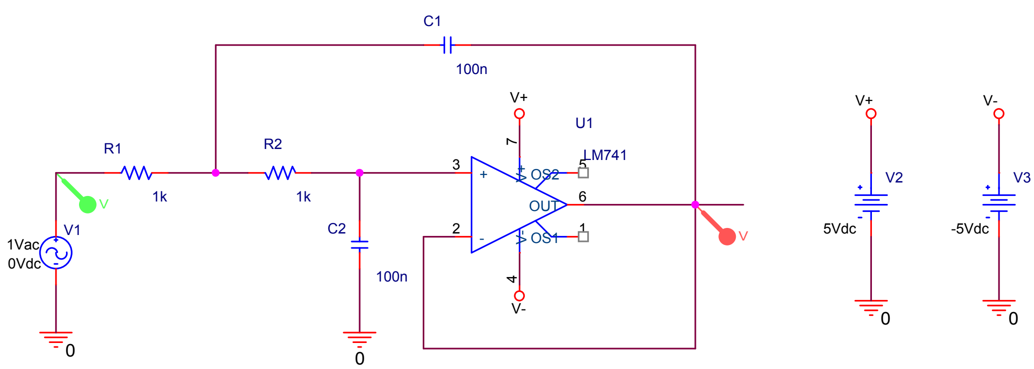
A $2n^{th}$-order Linkwitz-Riley filter can be obtained from *cascading* 2 n^{th}-order Butterworth filters (2 2^{nd}-order Butterworth filters will produce a 4^{th}-order Linkwitz-Riley filter).

In a way, the Linkwitz-Riley filter is a superset of the Butterworth filter which in turn exploits the Sallen/Key topology.

For a Linkwitz-Riley filter, the quality factor must be 0.5.

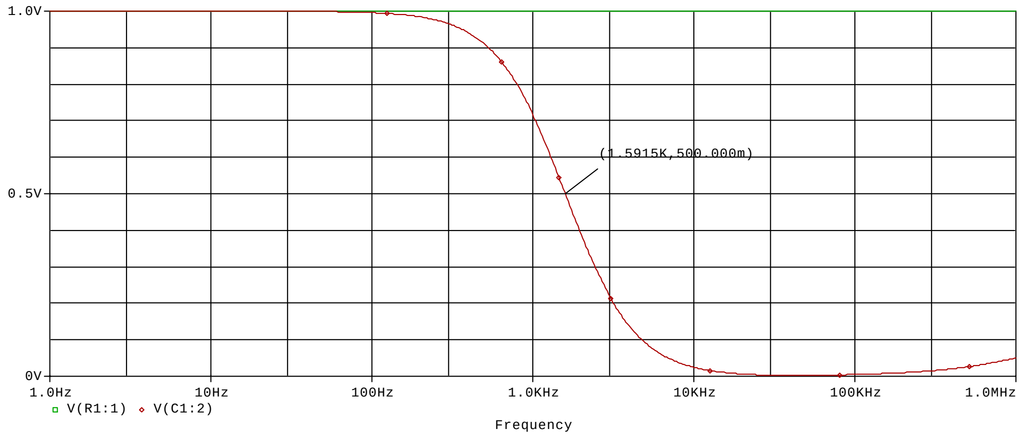
Second-order low-pass

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors.



Linkwitz-Riley filter (low-pass) (2^{nd}-order)

Note: R_1=R_2 and C_1=C_2.



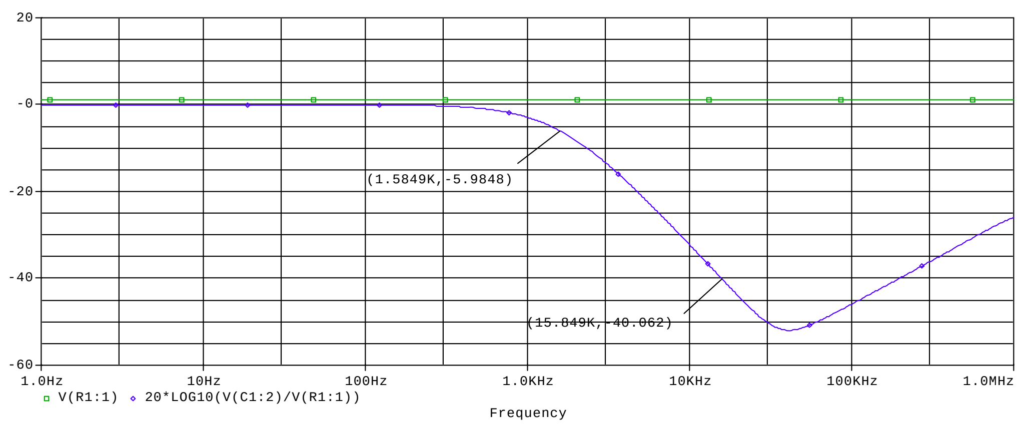
AC sweep from 1Hz to 1MHz

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 100nF \cdot 100nF}} = 1.5915kHz$$

The quality factor is

$$Q = \frac{\sqrt{R_1R_2C_1C_2}}{C_2\left(R_1 + R_2\right)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 100nF \cdot 100nF}}{100nF \cdot \left(1k\Omega + 1k\Omega\right)} = 0.5$$

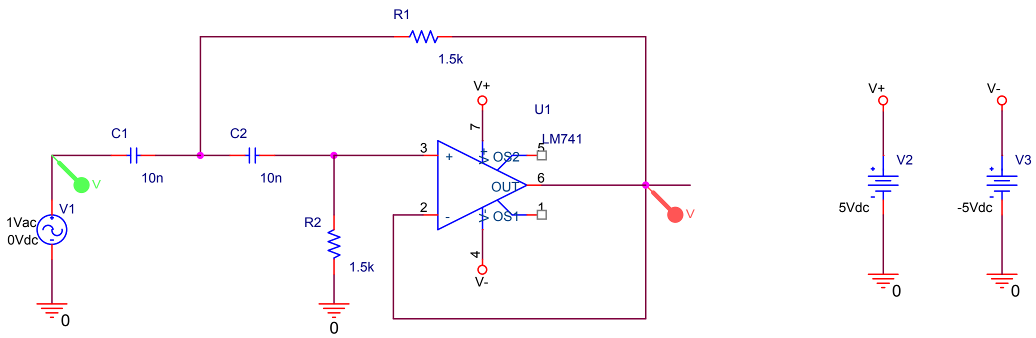


Bode plot from 1Hz to 1MHz

The gain drops to –6dB at 1.5915kHz and then it decreases by –40dB/decade.

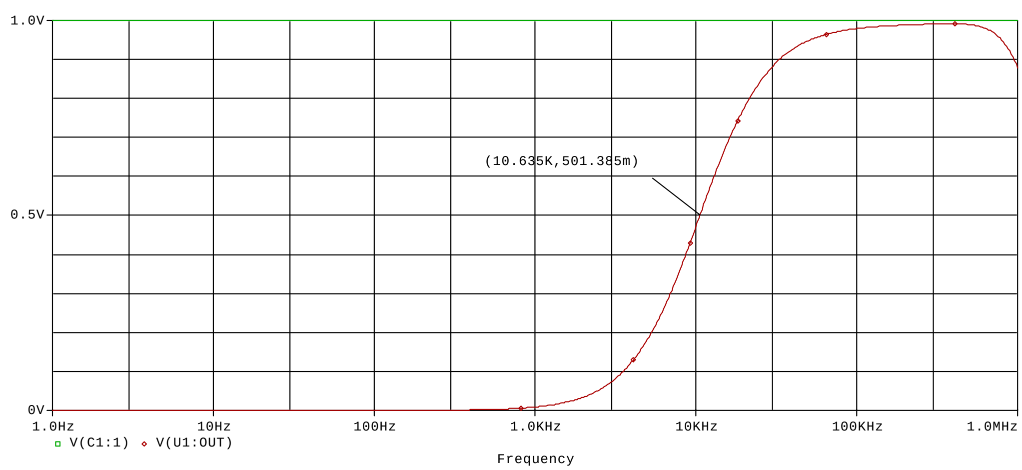
Second-order high-pass

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors.



Linkwitz-Riley filter (high-pass) (2nd-order)

Note: C_1=C_2 and R_1=R_2.



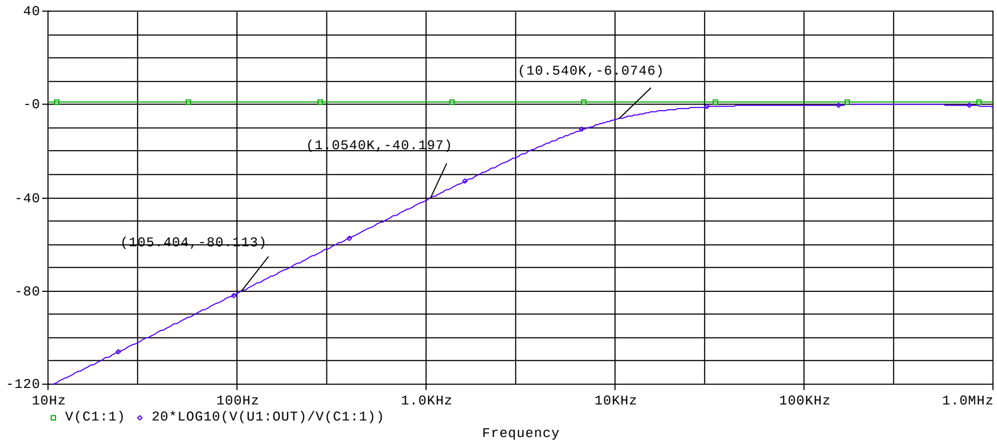
AC sweep from 1Hz to 1MHz

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1.55k\Omega \cdot 1.65k\Omega \cdot 10nF \cdot 10nF}} = 10.61kHz$$

The quality factor is

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1\left(C_1 + C_2\right)} = \frac{\sqrt{1.5k\Omega \cdot 1.5k\Omega \cdot 10nF \cdot 10nF}}{1.5k\Omega \cdot \left(10nF + 10nF\right)} = 0.5$$

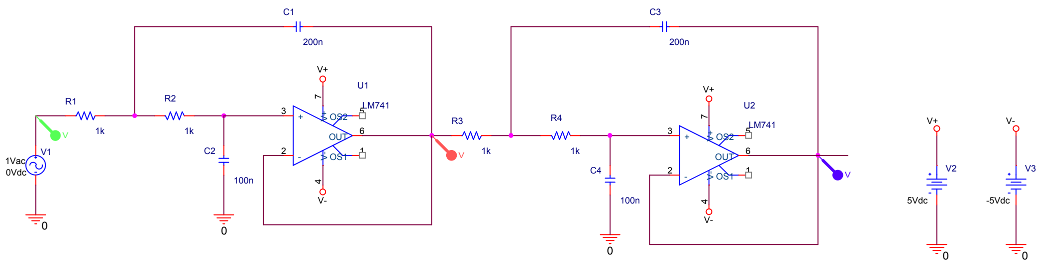


Bode plot from 10Hz to 1MHz

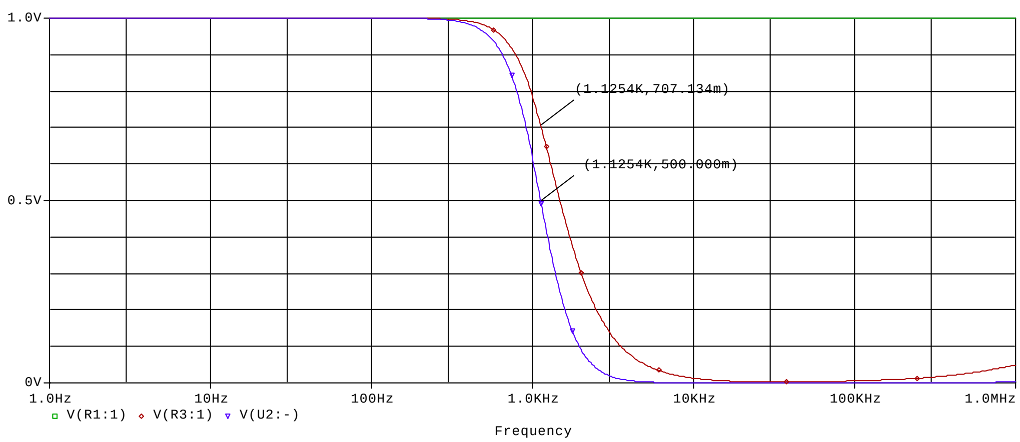
The gain drops to –6dB at about 10.61kHz and then it decreases by –40dB/decade.

Fourth-order low-pass (cascaded)

This circuit is implemented with the Sallen/Key topology by *cascading* two identical second-order filters. The quality factors for the blocks must be 0.707 (equivalent to two cascaded Butterworth stages). This is a fourth-order filter because it has four capacitors. The overall quality factor is 0.5.



Linkwitz-Riley filter (low-pass) (4th-order)



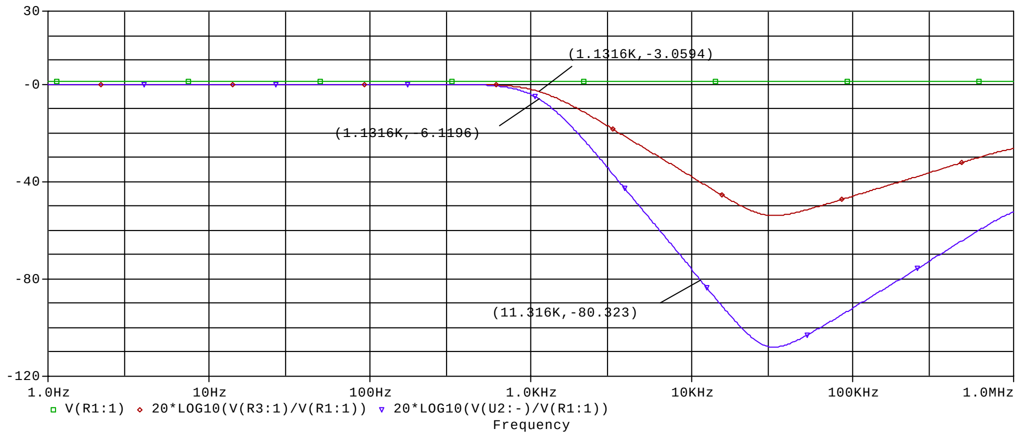
AC sweep from 1Hz to 1MHz

The cutoff frequencies are

$$f_{c1} = f_{c2} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 200nF \cdot 100nF}} = 1.125kHz$$

The quality factors are

$$Q_1 = Q_2 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2\left(R_1 + R_2\right)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 200nF \cdot 100nF}}{100nF \cdot \left(1k\Omega + 1k\Omega\right)} = 0.707$$

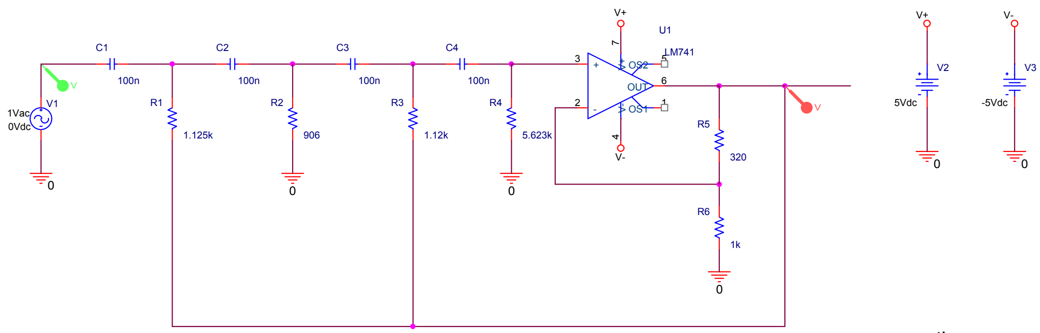


Bode plot from 1Hz to 1MHz

The gain drops to –6dB at 1.125kHz and then it decreases by –80dB/decade.

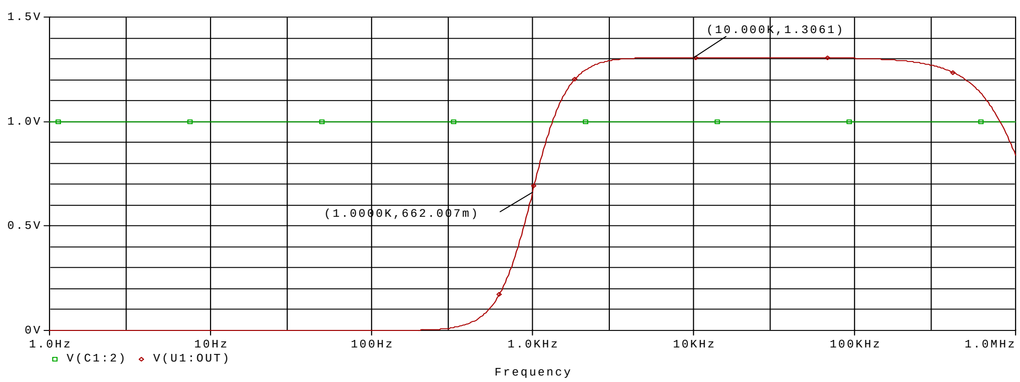
Fourth-order high-pass (same capacitor values)

In the following example, the circuit is implemented with the Sallen/Key topology. The capacitor values match here and the circuit provides gain by means of two additional resistors. This is a fourth-order filter because it has four capacitors.



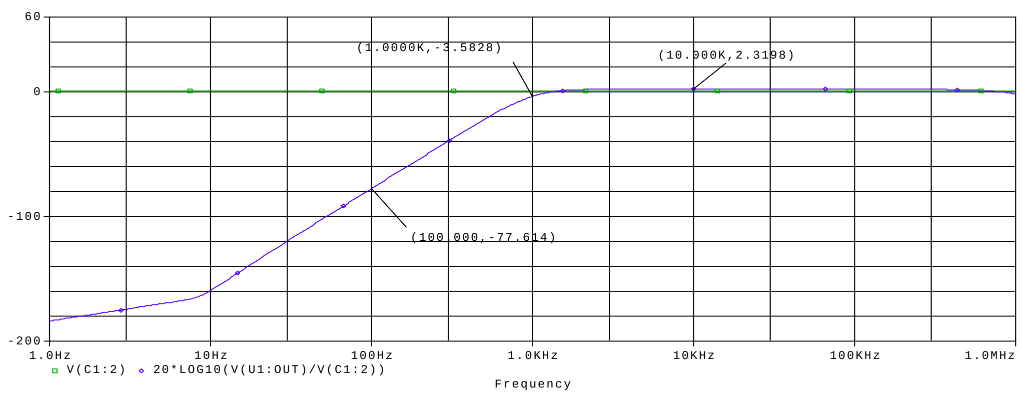
Linkwitz-Riley filter (high-pass) (4th-order)

Note: all capacitor values match.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

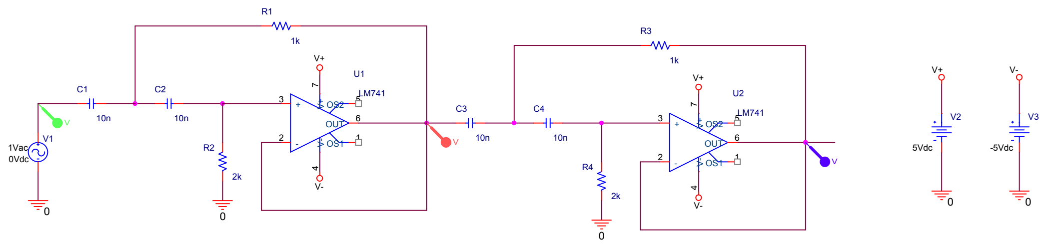


Bode plot from 1Hz to 1MHz

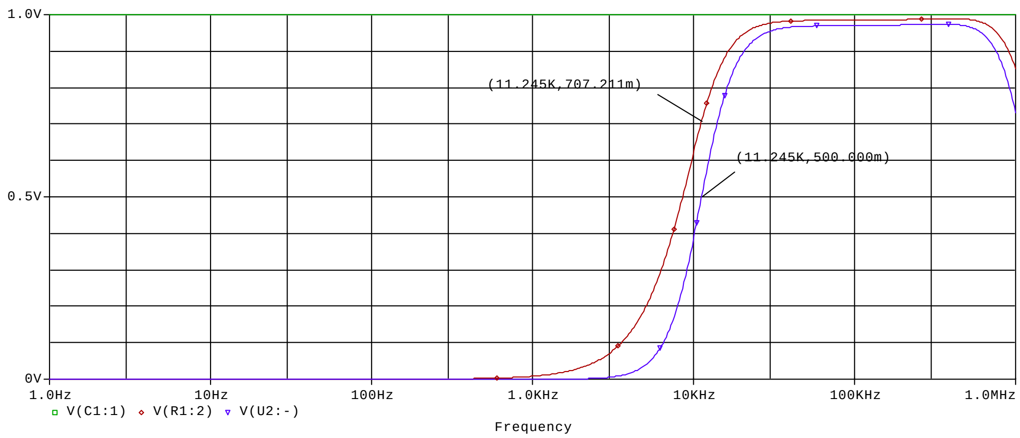
The gain is flat at +2.3198dB in the higher frequency range and then it drops to –3.5828dB at 1kHz. It eventually decreases by –80dB/decade.

Fourth-order high-pass (cascaded)

This circuit is implemented with the Sallen/Key topology by *cascading* two identical second-order filters. The quality factors for the blocks must be 0.707 (equivalent to two cascaded Butterworth stages). This is a fourth-order filter because it has four capacitors. The overall quality factor is 0.5.



Linkwitz-Riley filter (high-pass) (4th-order)



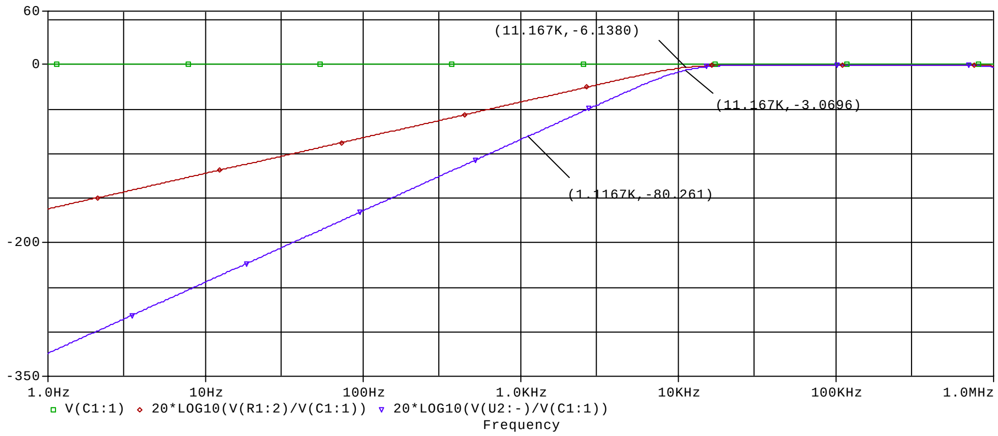
AC sweep from 1Hz to 1MHz

The cutoff frequencies are

$$f_{c1} = f_{c2} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 2k\Omega \cdot 10nF \cdot 10nF}} = 11.253kHz$$

The quality factors are

$$Q_1 = Q_2 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1\left(C_1 + C_2\right)} = \frac{\sqrt{1k\Omega \cdot 2k\Omega \cdot 10nF \cdot 10nF}}{1k\Omega \cdot \left(10nF + 10nF\right)} = 0.707$$

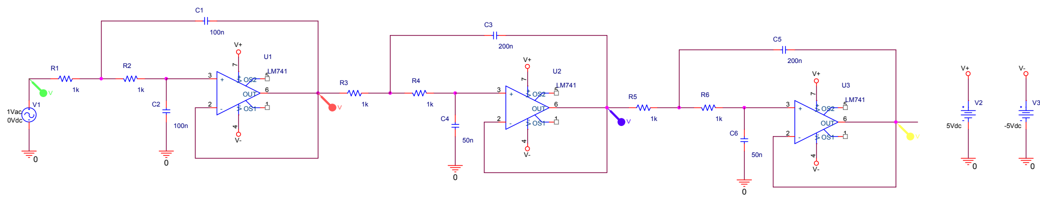


Bode plot from 1Hz to 1MHz

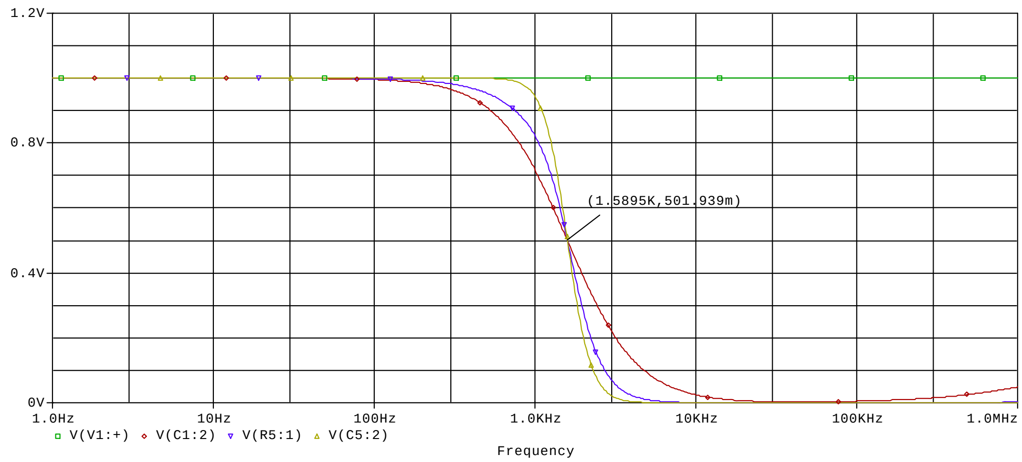
The gain drops to –6dB at 11.253kHz and then it decreases by –80dB/decade.

Sixth-order low-pass (cascaded)

For sake of illustration, a 6th-order low-pass Linkwitz-Riley filter is shown below. The circuit is nothing more than a *cascade* of 3 2nd-order blocks like the ones previously shown. All blocks cut off at the same frequency. However, the first stage must have Q=0.5, the second and the third stages must have Q=1.



Linkwitz-Riley filter (low-pass) (6th-order)



AC sweep from 1Hz to 1MHz

The plot shown above clearly demonstrates that increasing the order of the filter will produce a sharper response. Increasing the order of the filter will produce a roughly vertical drop at the cutoff frequency.

The cutoff frequencies are

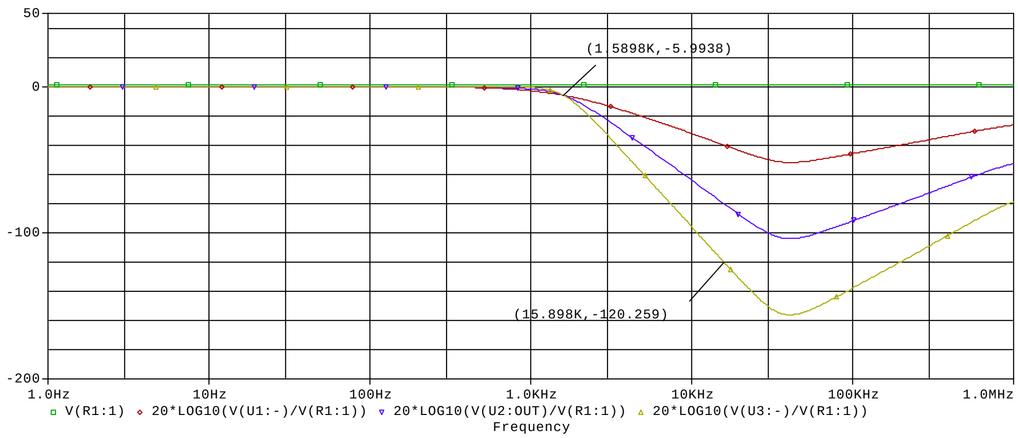
$$f_{c1} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 100nF \cdot 100nF}} = 1.591kHz$$

$$f_{c2} = f_{c3} = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 200nF \cdot 50nF}} = 1.591kHz$$

The quality factors are

$$Q_1 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 100nF \cdot 100nF}}{100nF \cdot (1k\Omega + 1k\Omega)} = 0.5$$

$$Q_2 = Q_3 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 200nF \cdot 50nF}}{50nF \cdot (1k\Omega + 1k\Omega)} = 1$$

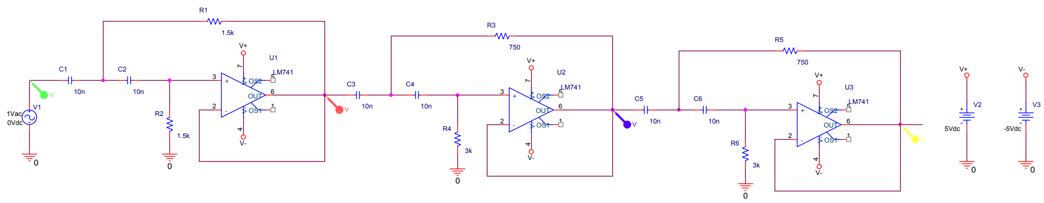


Bode plot from 1Hz to 1MHz

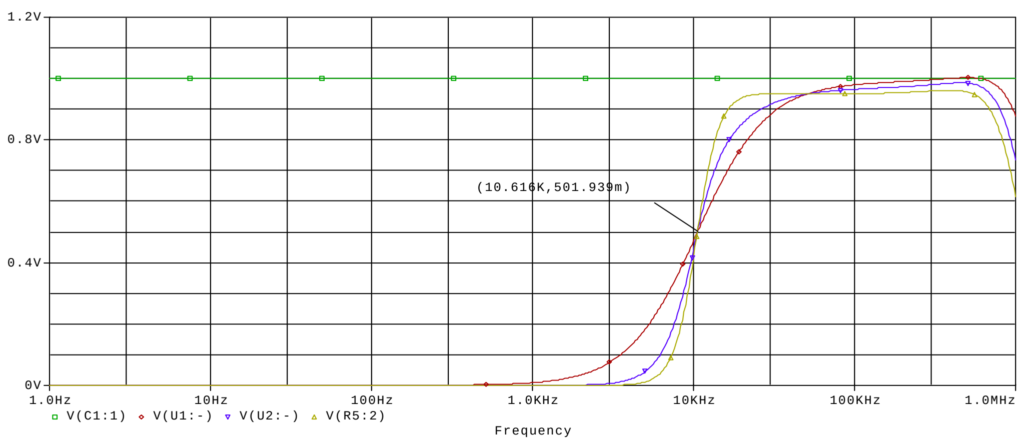
The gain drops to –6dB at 1.591kHz and then it decreases by –120dB/decade.

Sixth-order high-pass (cascaded)

For sake of illustration, a 6th-order high-pass Linkwitz-Riley filter is shown below. The circuit is nothing more than a *cascade* of 3 2nd-order blocks like the ones previously shown. All blocks cut off at the same frequency. However, the first stage must have Q=0.5, the second and the third stages must have Q=1.



Linkwitz-Riley filter (high-pass) (6th-order)



AC sweep from 1Hz to 1MHz

The plot shown above clearly demonstrates that increasing the order of the filter will produce a sharper response. Increasing the order of the filter will produce a roughly vertical drop at the cutoff frequency.

The cutoff frequencies are

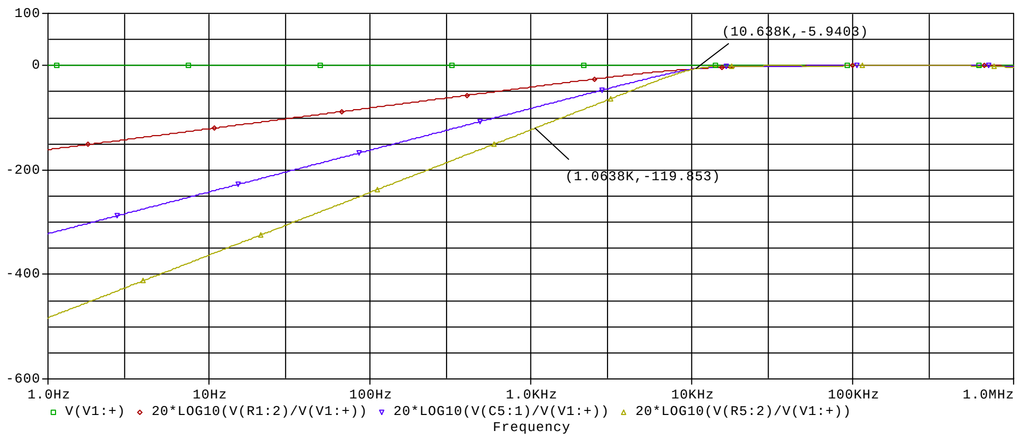
$$f_{c1} = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} = \frac{1}{2\pi\sqrt{1.5k\Omega \cdot 1.5k\Omega \cdot 10nF \cdot 10nF}} = 10.61kHz$$

$$f_{c2} = f_{c3} = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} = \frac{1}{2\pi\sqrt{750\Omega \cdot 3k\Omega \cdot 10nF \cdot 10nF}} = 10.61kHz$$

The quality factors are

$$Q_1 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 \left(C_1 + C_2\right)} = \frac{\sqrt{1.5k\Omega \cdot 1.5k\Omega \cdot 10nF \cdot 10nF}}{1.5k\Omega \cdot \left(10nF + 10nF\right)} = 0.5$$

$$Q_2 = Q_3 = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 \left(C_1 + C_2\right)} = \frac{\sqrt{750\Omega \cdot 3k\Omega \cdot 10nF \cdot 10nF}}{750\Omega \cdot \left(10nF + 10nF\right)} = 1$$



Bode plot from 1Hz to 1MHz

The gain drops to –6dB at 10.61kHz and then it decreases by –120dB/decade.

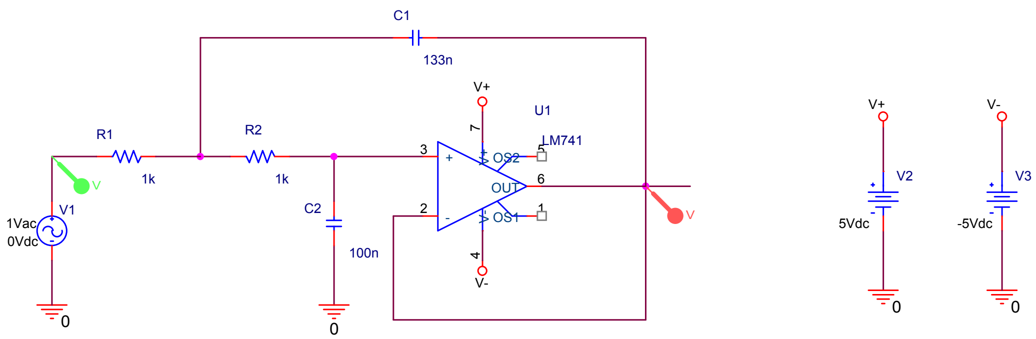
Bessel filter

The Bessel filter is named after Friedrich Bessel, a German mathematician who studied the mathematics behind the filter before it was implemented. This is also known as Bessel-Thomson filter. W. E. Thomson was responsible for actually using the theory and putting it to work.

For a second-order Bessel filter, the quality factor must be 0.577.

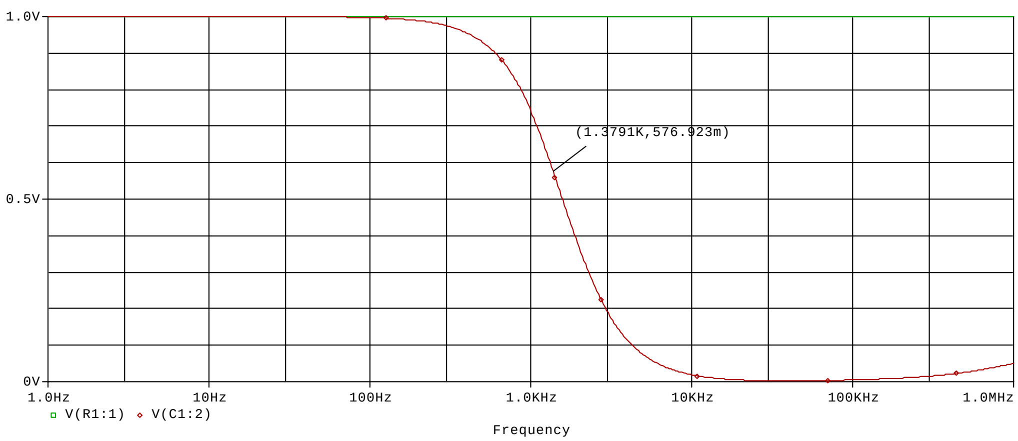
Second-order low-pass

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors.



Bessel filter (low-pass) (2^{nd}-order)

Note: R_1=R_2 and C_1=1.336xC_2.



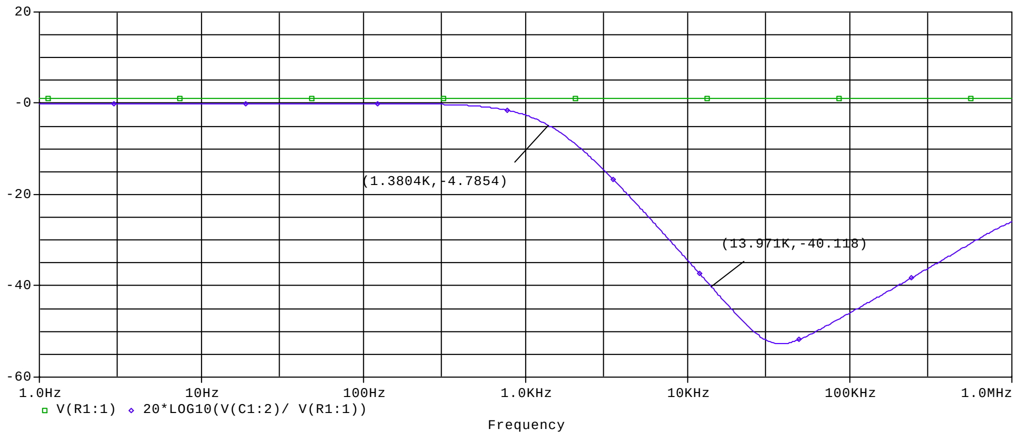
AC sweep from 1Hz to 1MHz

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 133nF \cdot 100nF}} = 1.38kHz$$

The quality factor is

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 133nF \cdot 100nF}}{100nF \cdot (1k\Omega + 1k\Omega)} = 0.577$$

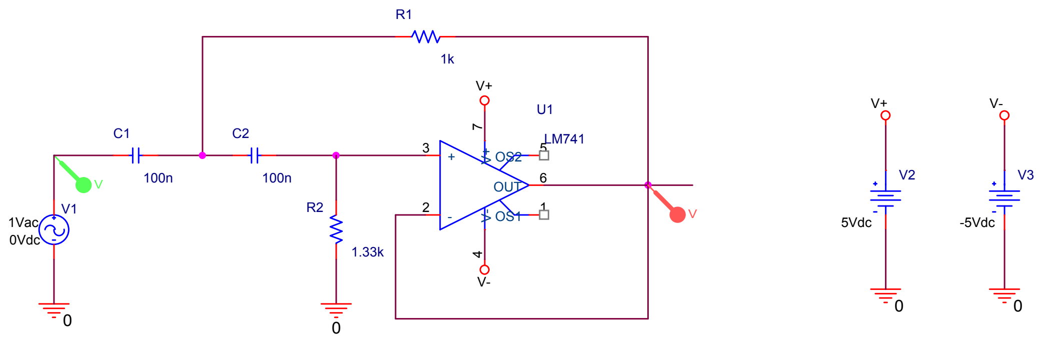


Bode plot from 1Hz to 1MHz

The gain drops to –4.78dB at 1.38kHz and then it decreases by –40dB/decade.

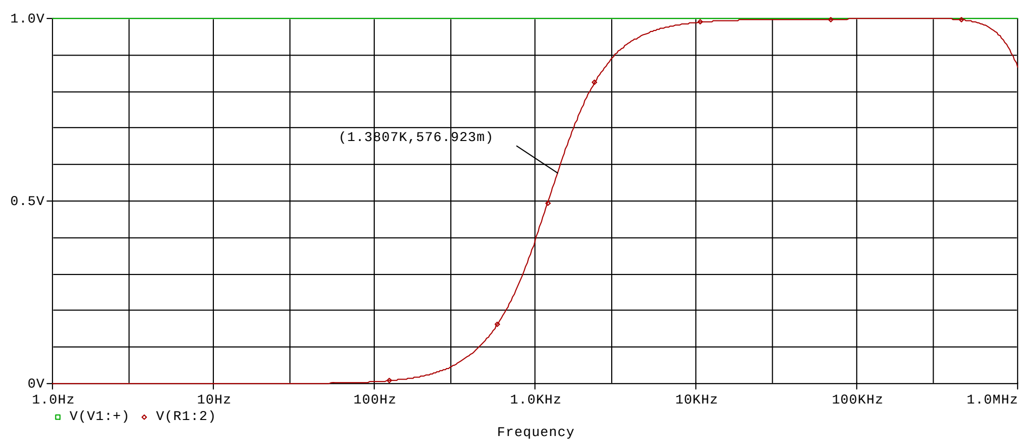
Second-order high-pass

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors.



Bessel filter (high-pass) (2^{nd}-order)

Note: $C_1=C_2$ and $R_2=R_1$x1.336.



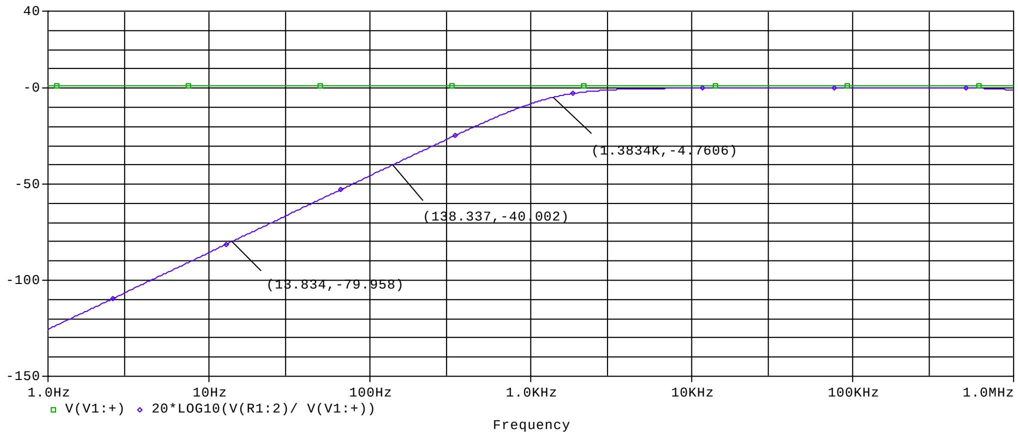
AC sweep from 1Hz to 1MHz

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1.33k\Omega \cdot 100nF \cdot 100nF}} = 1.38kHz$$

The quality factor is

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1\left(C_1 + C_2\right)} = \frac{\sqrt{1k\Omega \cdot 1.33k\Omega \cdot 100nF \cdot 100nF}}{1k\Omega \cdot \left(100nF + 100nF\right)} = 0.577$$

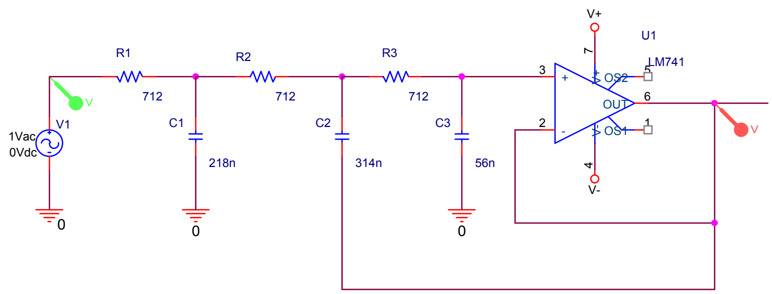


Bode plot from 1Hz to 1MHz

The gain drops to –4.78dB at 1.38kHz and then it decreases by –40dB/decade.

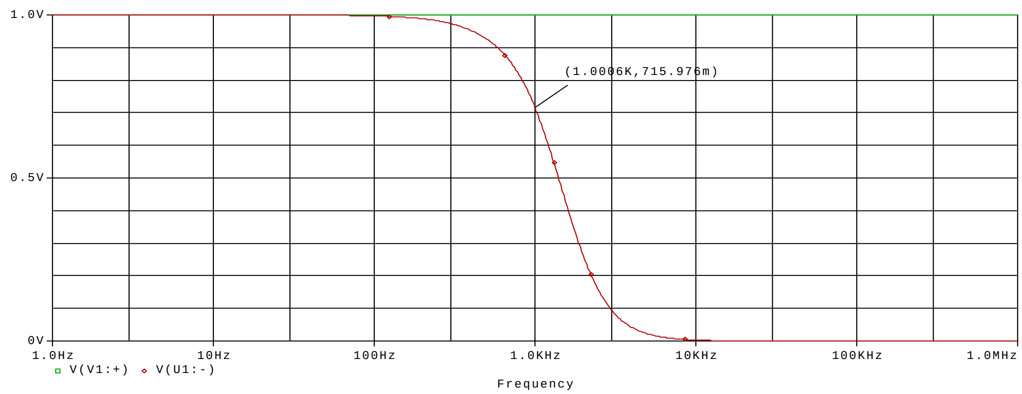
Third-order low-pass (same resistor values)

In the following example, the circuit is implemented with the Sallen/Key topology. The resistor values match here. This is a third-order filter because it has three capacitors.



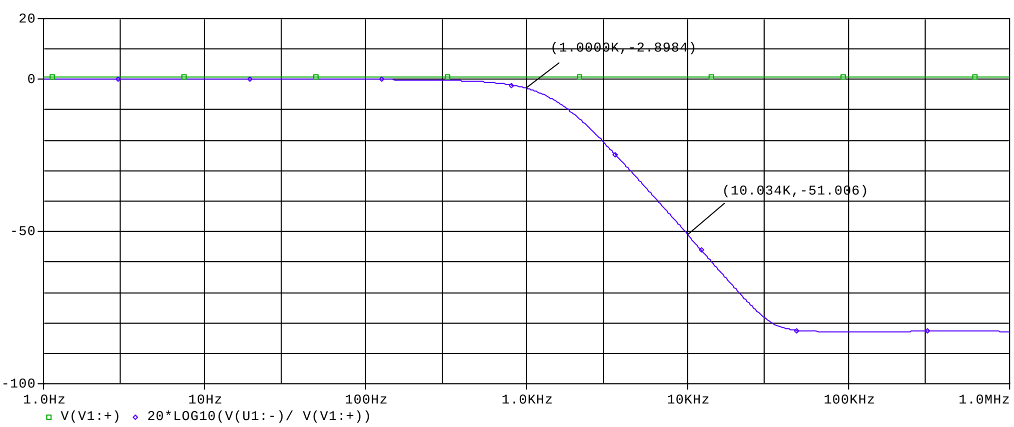
Bessel filter (low-pass) (3rd-order)

Note: R_1=R_2=R_3.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

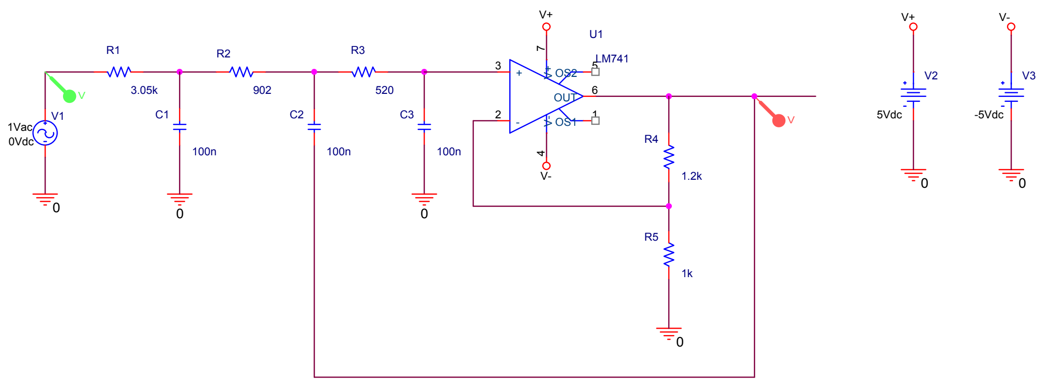


Bode plot from 1Hz to 1MHz

The gain drops to –2.8984dB at 1kHz and then it decreases to –51.006dB a decade later.

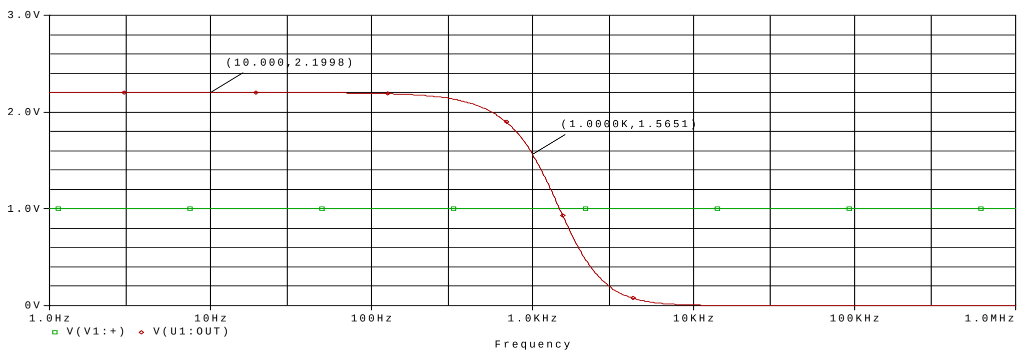
Third-order low-pass (same capacitor values)

In the following example, the circuit is implemented with the Sallen/Key topology. The capacitor values match here and the circuit provides gain by means of two additional resistors. This is a third-order filter because it has three capacitors.



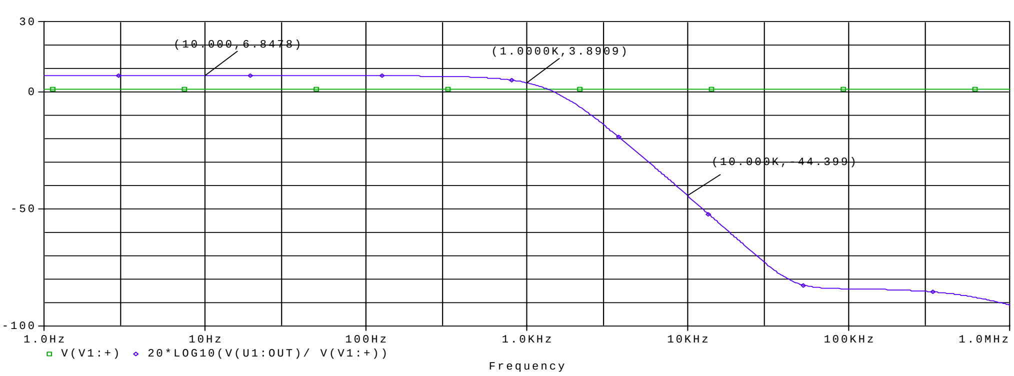
Bessel filter (low-pass) (3rd-order)

Note: C_1=C_2=C_3.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

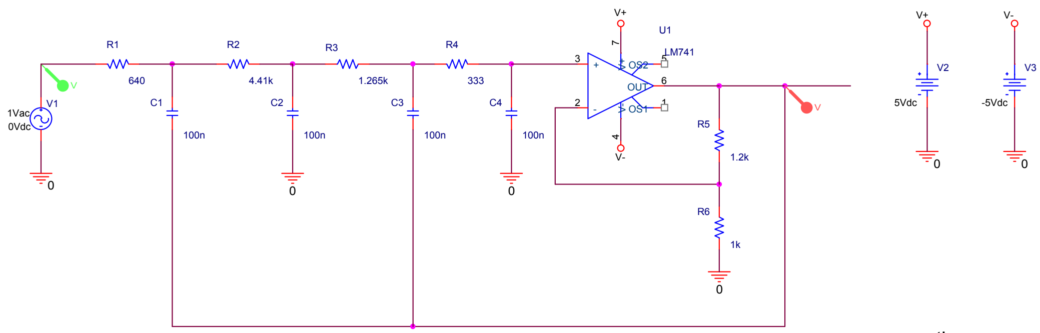


Bode plot from 1Hz to 1MHz

The gain is flat at +6.8478dB in the lower frequency range and then it drops to +3.8909dB at 1kHz. It eventually decreases to –44.399dB one decade later.

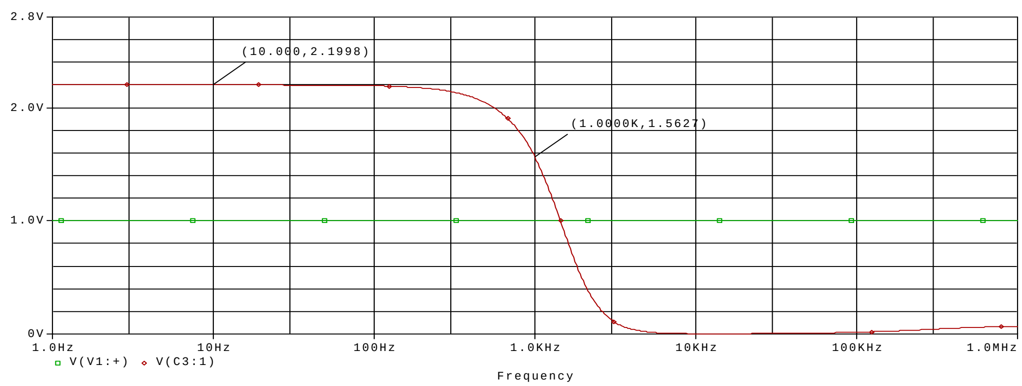
Fourth-order low-pass (same capacitor values)

In the following example, the circuit is implemented with the Sallen/Key topology. The capacitor values match here and the circuit provides gain by means of two additional resistors. This is a fourth-order filter because it has four capacitors.



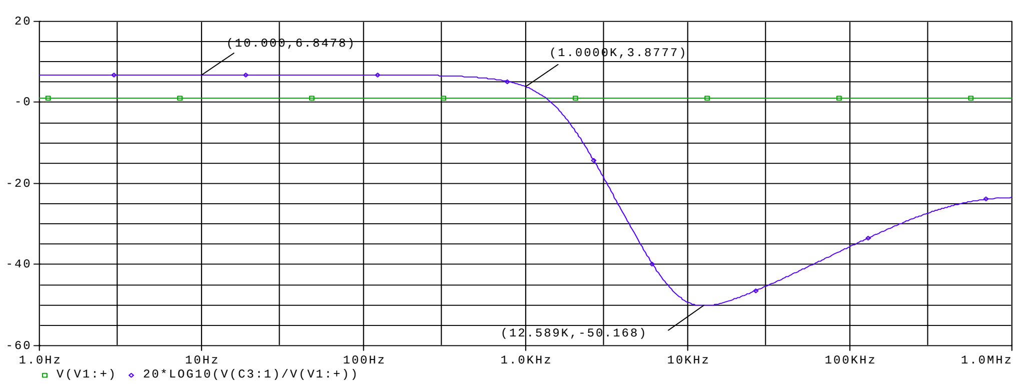
Bessel filter (low-pass) (4^{th}-order)

Note: all capacitor values match.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.



Bode plot from 1Hz to 1MHz

The gain is flat at +6.8478dB in the lower frequency range and then it drops to +3.8777dB at 1kHz. It eventually decreases to –50.168dB at 12.689kHz.

Chebyshev filter

The Chebyshev filter bears the name of Pafnuty Chebyshev, a Russian mathematician who developed the theory behind the Chebyshev polynomials.

Chebyshev filters come in two variants: if the ripple is present in the *passband*, they are called *Type I* otherwise, if the ripple is present in the *stopband*, they are called *Type II* (also known as Inverted).

The filter can be designed to have different ripples that can vary from 0.5dB to 3dB and intermediate values (typically in 0.5dB increments).

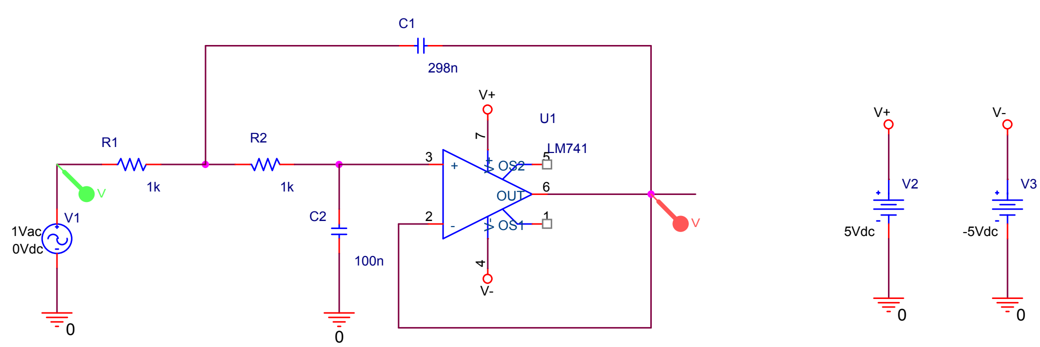
Type I

This is the Chebyshev filter with the ripple in the passband. It's probably the most common version of the filter.

For a second-order Chebyshev Type I filter, the quality factors must be 0.864, 0.956, 1.129 and 1.305 for 0.5dB, 1dB, 2dB and 3dB versions of the filter respectively.

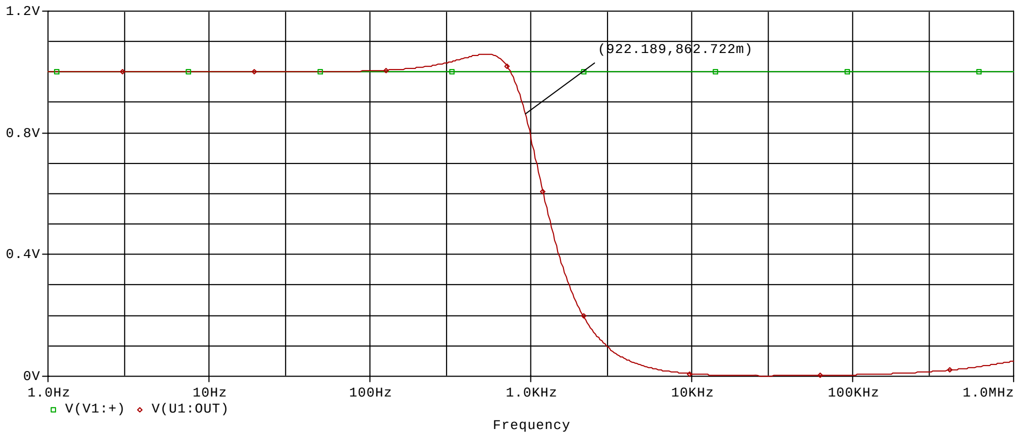
Second-order low-pass (0.5dB ripple)

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors. This specific filter is designed to have a 0.5dB ripple in the passband.



Chebyshev 0.5dB ripple filter (low-pass) (2nd-order)

Note: for a 0.5dB ripple, $R_1=R_2$ *and* $C_1=2.986xC_2$*.*



AC sweep from 1Hz to 1MHz

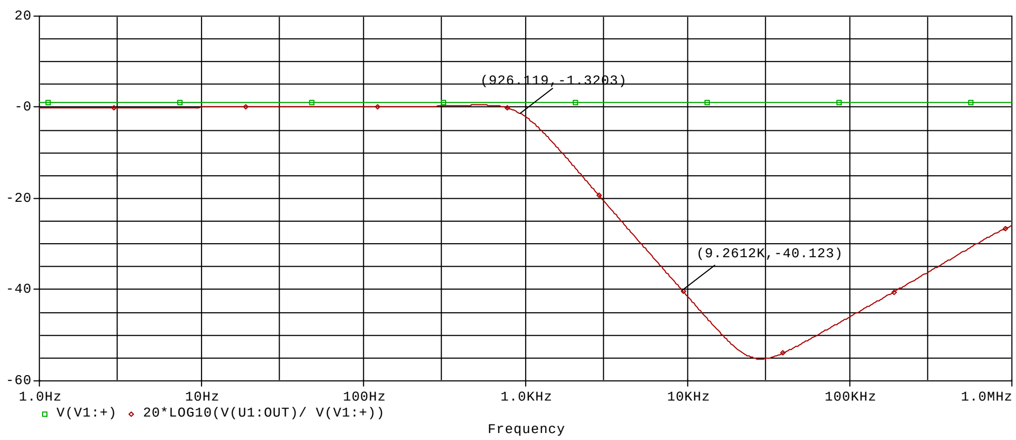
The ripple in the passband is noticeable.

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 298nF \cdot 100nF}} = 921.96Hz$$

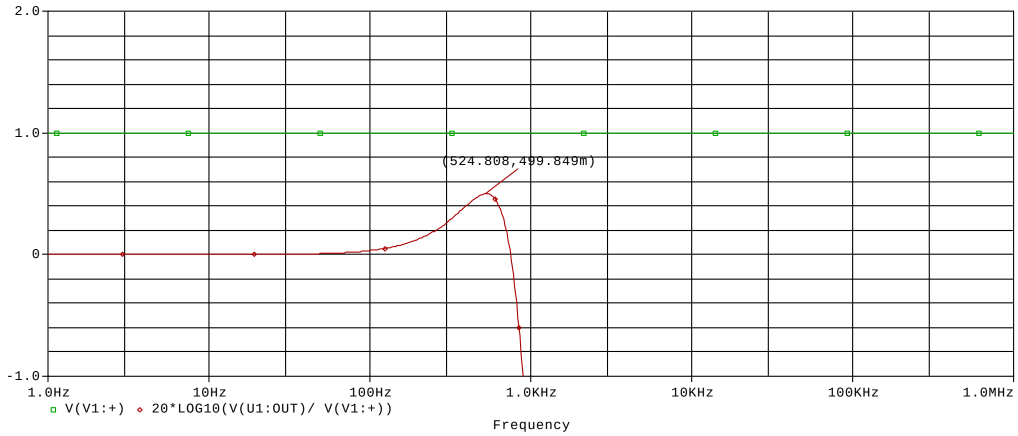
The quality factor is

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2\left(R_1 + R_2\right)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 298nF \cdot 100nF}}{100nF \cdot \left(1k\Omega + 1k\Omega\right)} = 0.863$$



Bode plot from 1Hz to 1MHz

The gain drops to –1.3203dB at 926Hz and then it decreases by –40dB/decade.

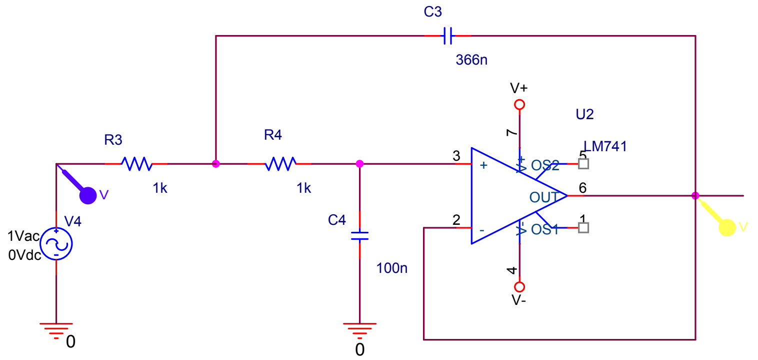


Close-up of 0.5dB ripple

The frequency response of the circuit peaks at 524Hz with a 0.5dB overshoot and then it decays rapidly.

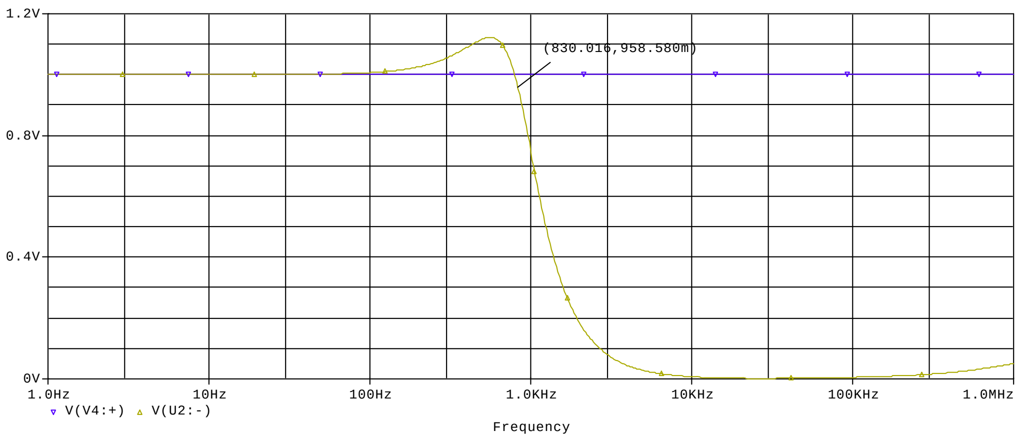
Second-order low-pass (1dB ripple)

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors. This specific filter is designed to have a 1dB ripple in the passband.



Chebyshev 1dB ripple filter (low-pass) (2^{nd}-order)

Note: for a 1dB ripple, R_3=R_4 and C_3=3.663xC_4.



AC sweep from 1Hz to 1MHz

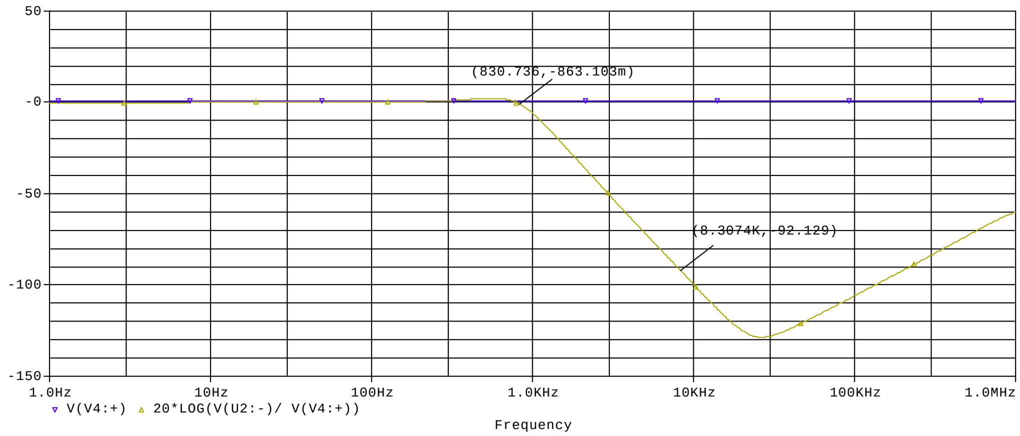
The ripple in the passband is noticeable.

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_2 R_3 C_2 C_3}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 366nF \cdot 100nF}} = 831.92Hz$$

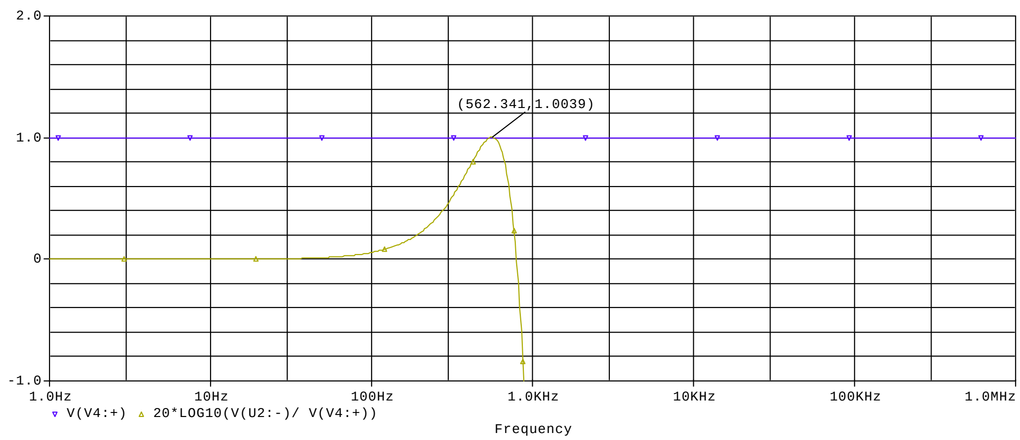
The quality factor is

$$Q = \frac{\sqrt{R_2 R_3 C_2 C_3}}{C_3\left(R_2 + R_3\right)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 366nF \cdot 100nF}}{100nF \cdot \left(1k\Omega + 1k\Omega\right)} = 0.957$$



Bode plot from 1Hz to 1MHz

The gain drops to –0.863dB at 830Hz and then it decreases to –92dB a decade past the cutoff frequency (much faster roll-off).

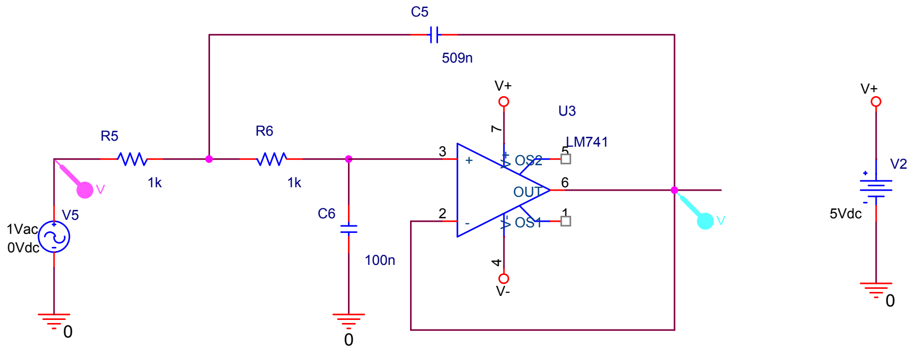


Close-up of 1dB ripple

The frequency response of the circuit peaks at 562Hz with a 1dB overshoot and then it decays rapidly.

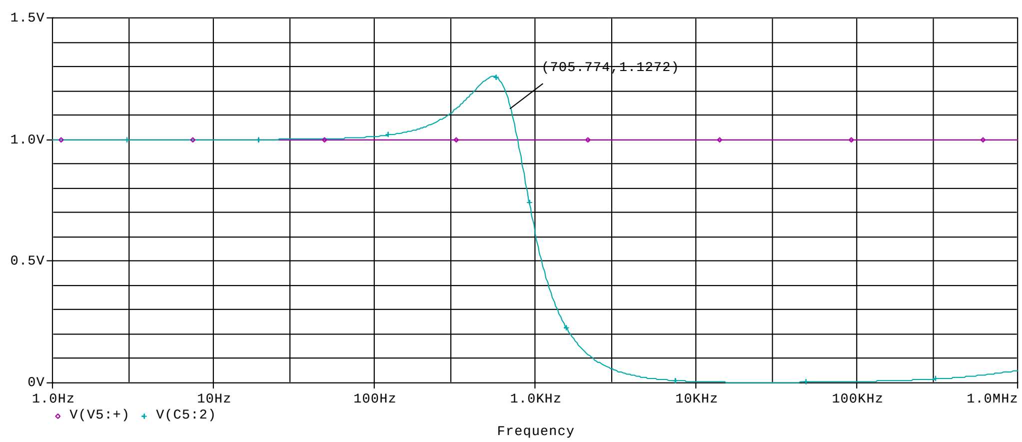
Second-order low-pass (2dB ripple)

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors. This specific filter is designed to have a 2dB ripple in the passband.



Chebyshev 2dB ripple filter (low-pass) (2nd-order)

Note: for a 1dB ripple, R_5=R_6 and C_5=5.098xC_6.



AC sweep from 1Hz to 1MHz

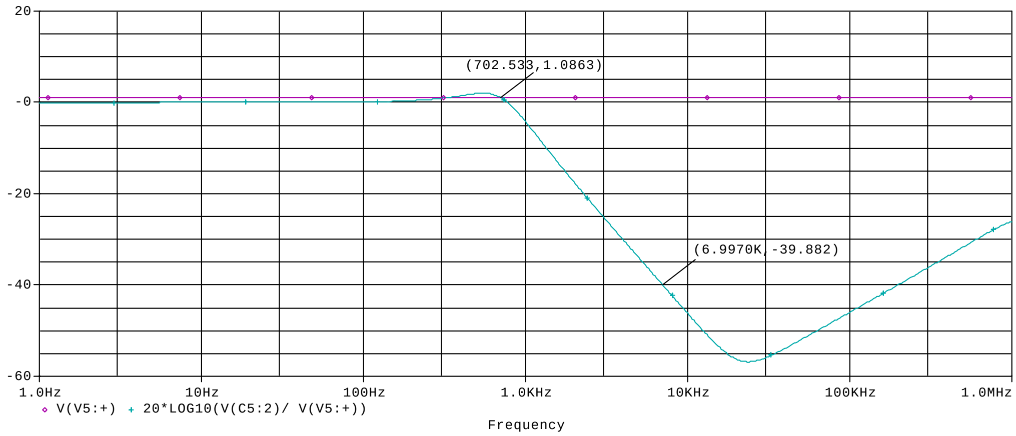
The ripple in the passband is noticeable.

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_5 R_6 C_5 C_6}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 509nF \cdot 100nF}} = 705.44Hz$$

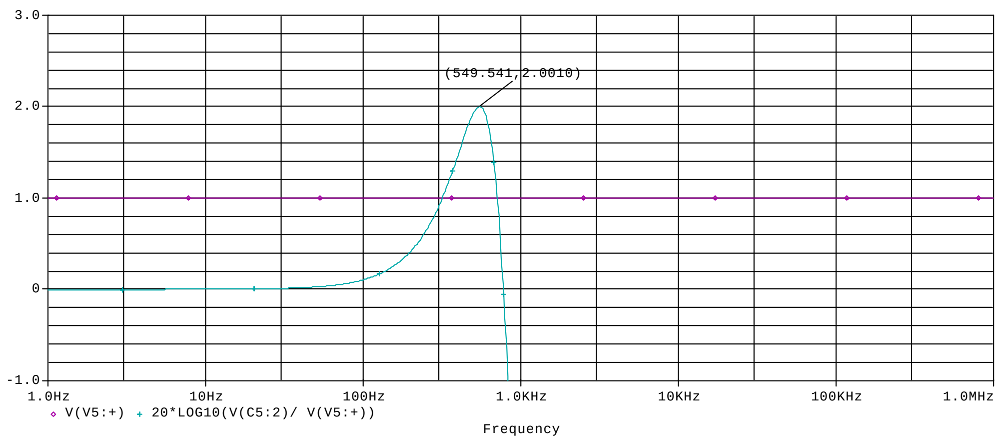
The quality factor is

$$Q = \frac{\sqrt{R_5 R_6 C_5 C_6}}{C_6\left(R_5 + R_6\right)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 509nF \cdot 100nF}}{100nF \cdot \left(1k\Omega + 1k\Omega\right)} = 1.128$$



Bode plot from 1Hz to 1MHz

The gain is +1.0863dB at 702Hz and then it decreases by –40dB/decade.

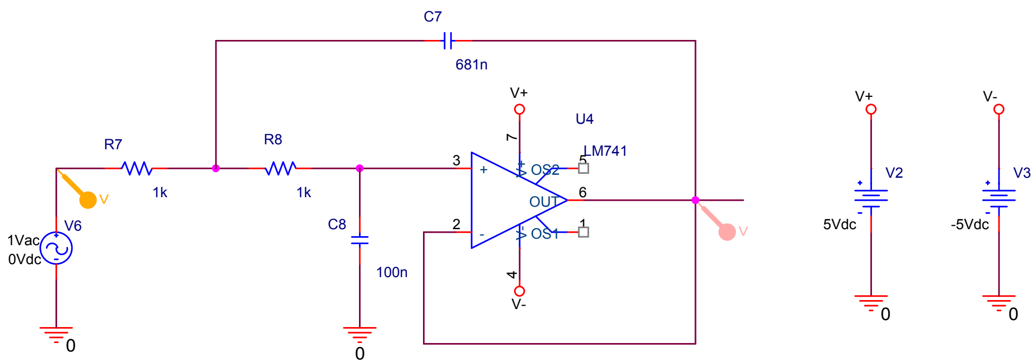


Close-up of 2dB ripple

The frequency response of the circuit peaks at 549Hz with a 2dB overshoot and then it decays rapidly.

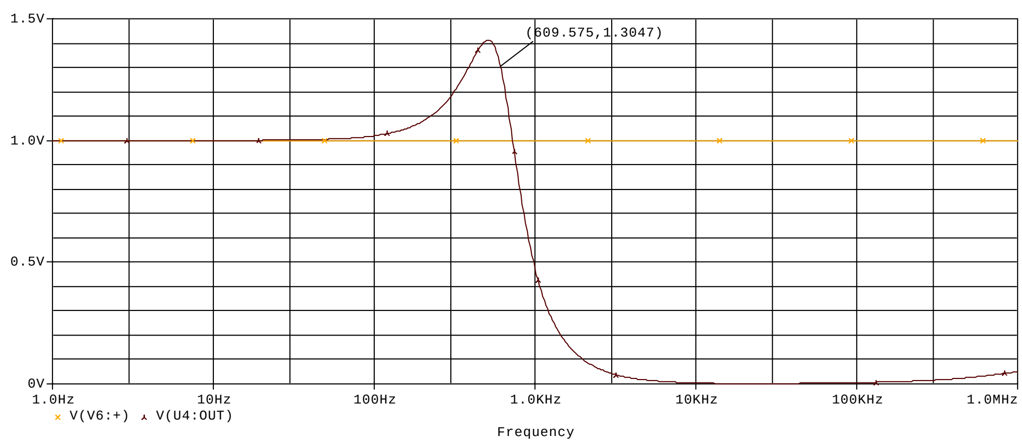
Second-order low-pass (3dB ripple)

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors. This specific filter is designed to have a 3dB ripple in the passband.



Chebyshev 3dB ripple filter (low-pass) (2nd-order)

Note: for a 1dB ripple, $R_7=R_8$ and $C_7=6.812xC_8$.



AC sweep from 1Hz to 1MHz

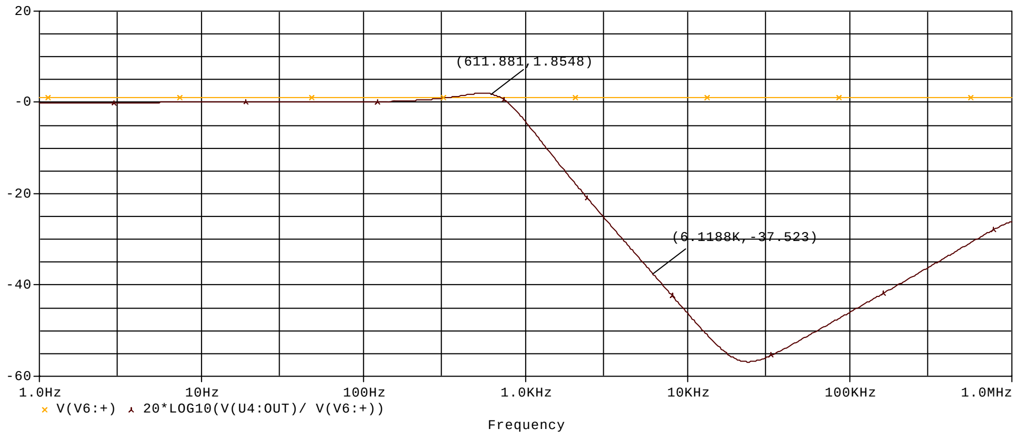
The ripple in the passband is noticeable.

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_7 R_8 C_7 C_8}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 1k\Omega \cdot 681nF \cdot 100nF}} = 609.88Hz$$

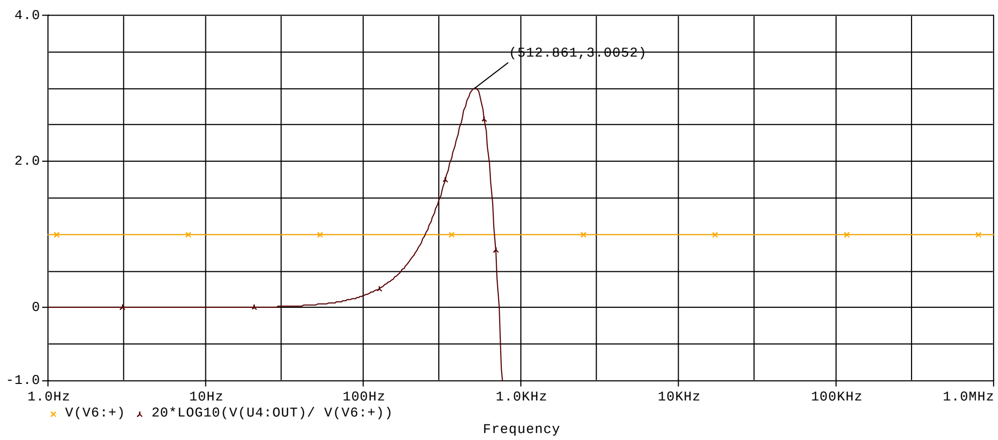
The quality factor is

$$Q = \frac{\sqrt{R_7 R_8 C_7 C_8}}{C_8\left(R_7 + R_8\right)} = \frac{\sqrt{1k\Omega \cdot 1k\Omega \cdot 681nF \cdot 100nF}}{100nF \cdot \left(1k\Omega + 1k\Omega\right)} = 1.305$$



Bode plot from 1Hz to 1MHz

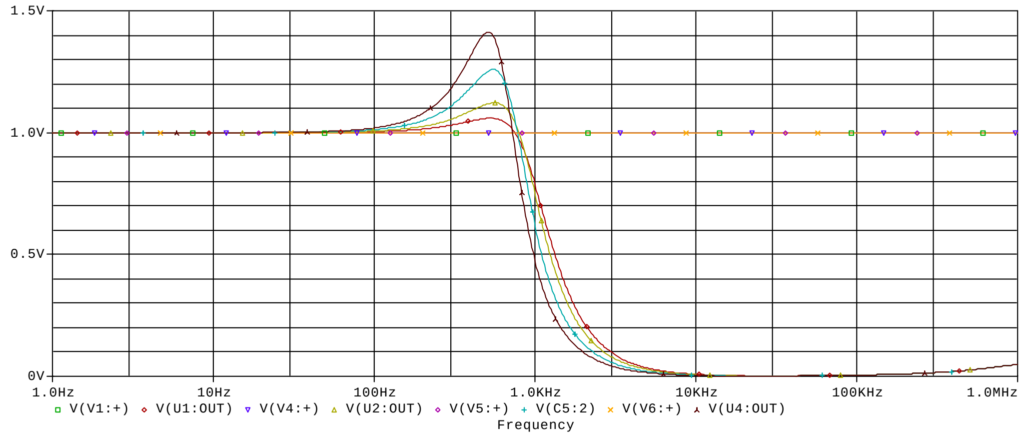
The gain is +1.8548dB at 611Hz and then it decreases by –40dB/decade.



Close-up of 3dB ripple

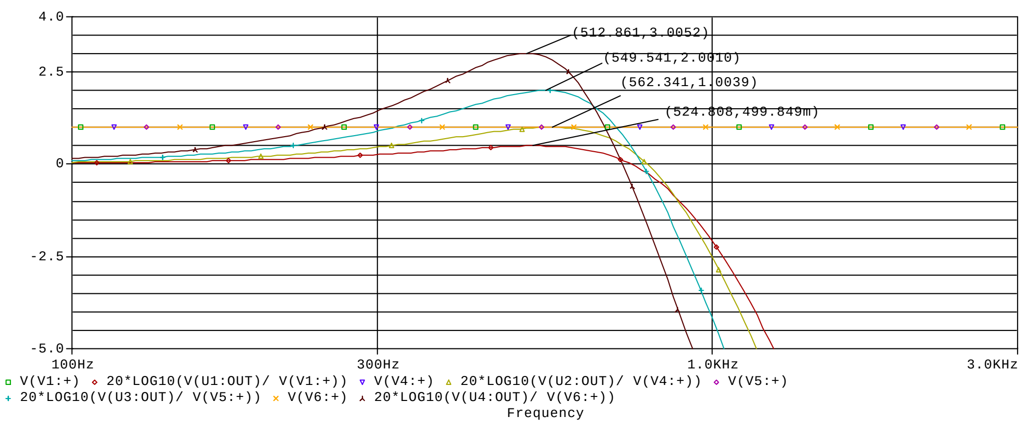
The frequency response of the circuit peaks at 512Hz with a 3dB overshoot and then it decays rapidly.

Second-order low-pass ripple comparison



0.5dB, 1dB, 2dB and 3dB circuit response comparison

Above is a comparison of the response for the low-pass circuits previously presented. The peaks in the response are clearly not matching because the circuits have been designed with the same resistor values so the cutoff frequency moves to the left of the plot as the magnitude of the ripple increases.

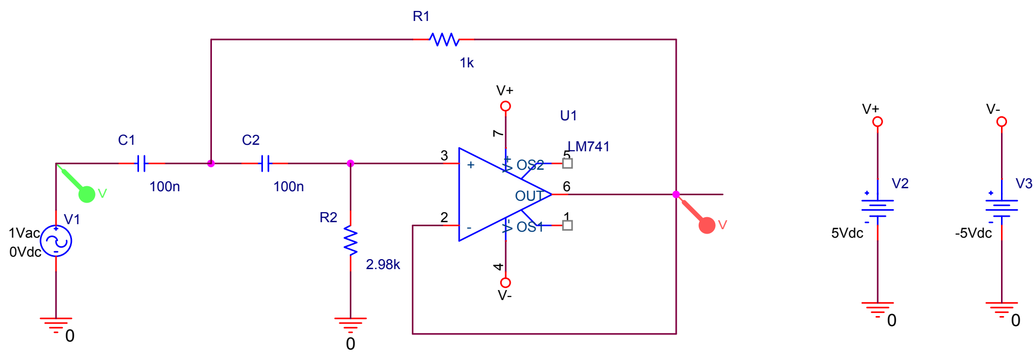


0.5dB, 1dB, 2dB and 3dB ripple comparison

The plot shown above compares the ripples for every circuit previously presented. It is clear that a higher amount of ripple will produce a sharper cutoff.

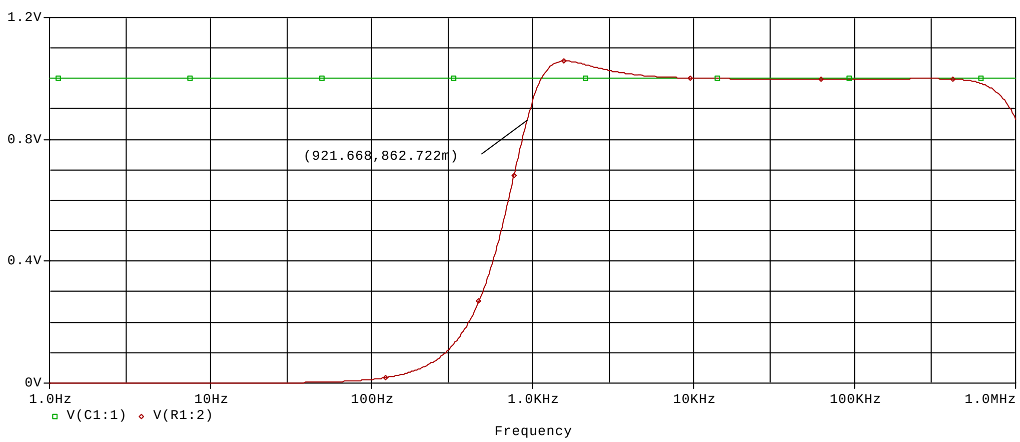
Second-order high-pass (0.5dB ripple)

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors. This specific filter is designed to have a 0.5dB ripple in the passband.



Chebyshev 0.5dB ripple filter (high-pass) (2^{nd}-order)

Note: for a 0.5dB ripple, C_1=C_2 and R_2=2.986xR_1.



AC sweep from 1Hz to 1MHz

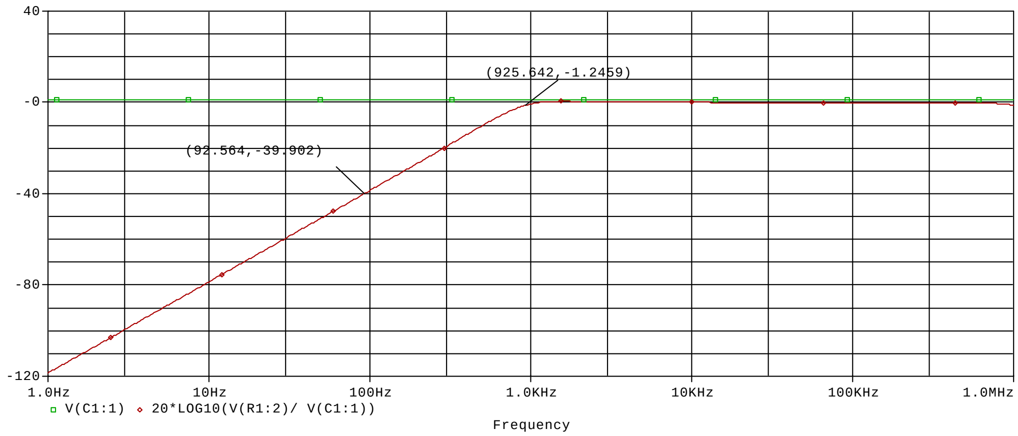
The ripple in the passband is noticeable.

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 2.98k\Omega \cdot 100nF \cdot 100nF}} = 921.96Hz$$

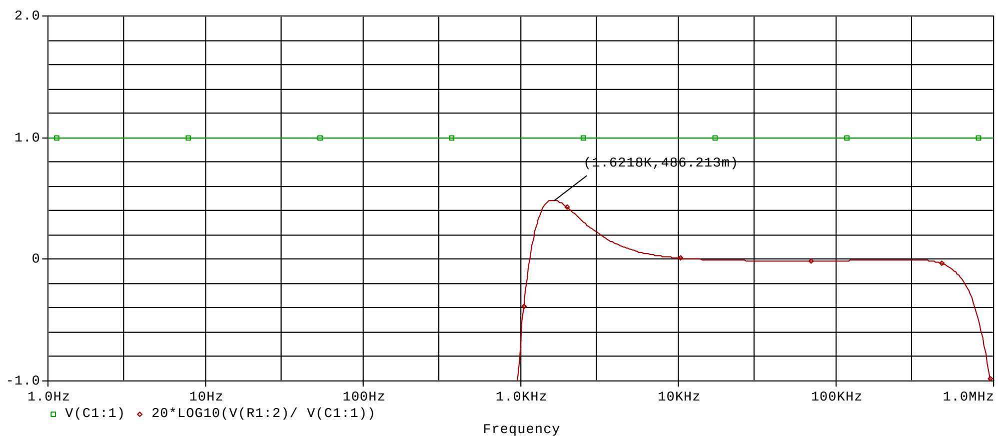
The quality factor is

$$Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1(C_1+C_2)} = \frac{\sqrt{1k\Omega \cdot 2.98k\Omega \cdot 100nF \cdot 100nF}}{1k\Omega \cdot (100nF + 100nF)} = 0.863$$



Bode plot from 1Hz to 1MHz

The gain drops to –1.2459dB at 925Hz and then it decreases by –40dB/decade.

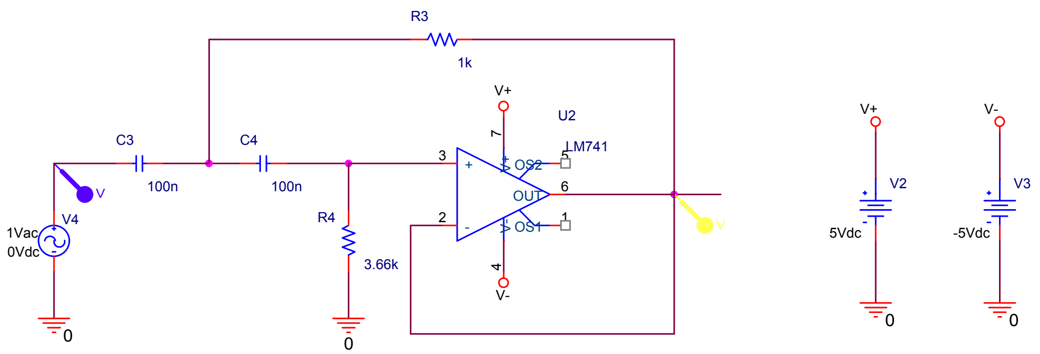


Close-up of 0.5dB ripple

The frequency response of the circuit peaks at 1.62kHz with a 0.5dB overshoot and then it decays rapidly.

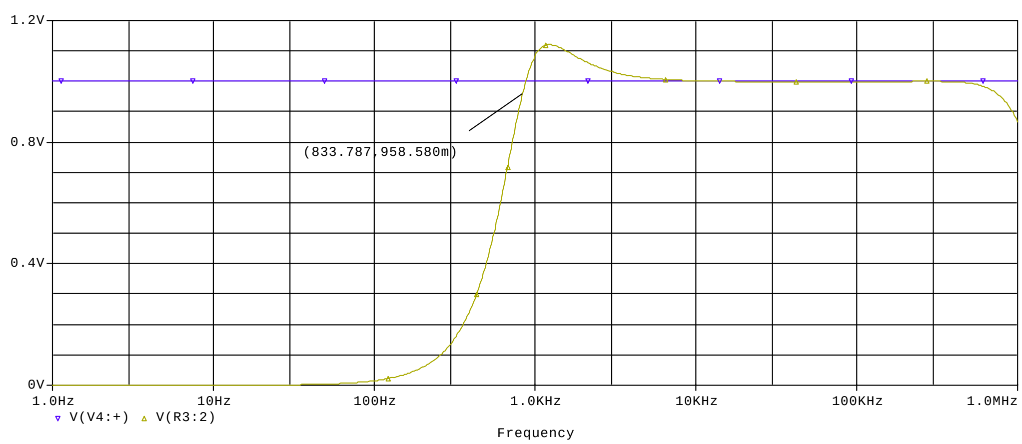
Second-order high-pass (1dB ripple)

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors. This specific filter is designed to have a 1dB ripple in the passband.



Chebyshev 1dB ripple filter (high-pass) (2nd-order)

Note: for a 1dB ripple, C_3=C_4 and R_4=3.663xR_3.



AC sweep from 1Hz to 1MHz

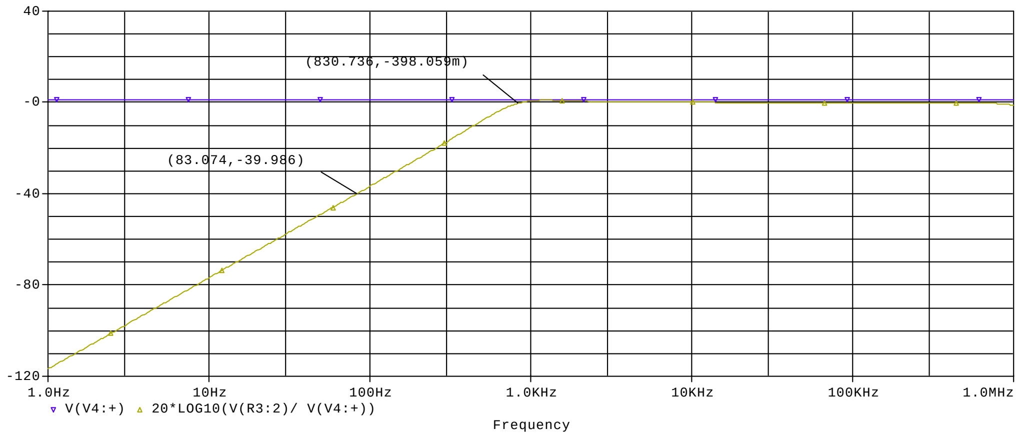
The ripple in the passband is noticeable.

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_3 R_4 C_3 C_4}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 3.66k\Omega \cdot 100nF \cdot 100nF}} = 831.92Hz$$

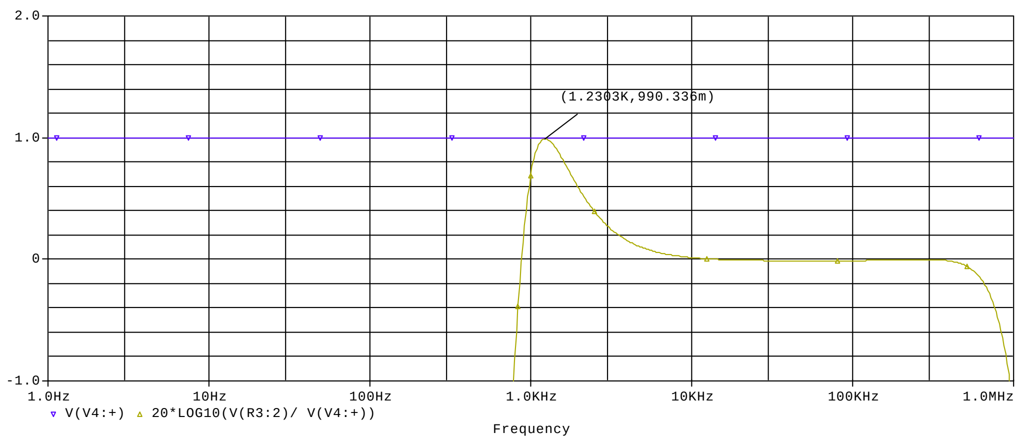
The quality factor is

$$Q = \frac{\sqrt{R_3 R_4 C_3 C_4}}{R_3\left(C_3 + C_4\right)} = \frac{\sqrt{1k\Omega \cdot 3.66k\Omega \cdot 100nF \cdot 100nF}}{1k\Omega \cdot \left(100nF + 100nF\right)} = 0.957$$



Bode plot from 1Hz to 1MHz

The gain drops to –0.398dB at 830Hz and then it decreases by –40dB/decade.

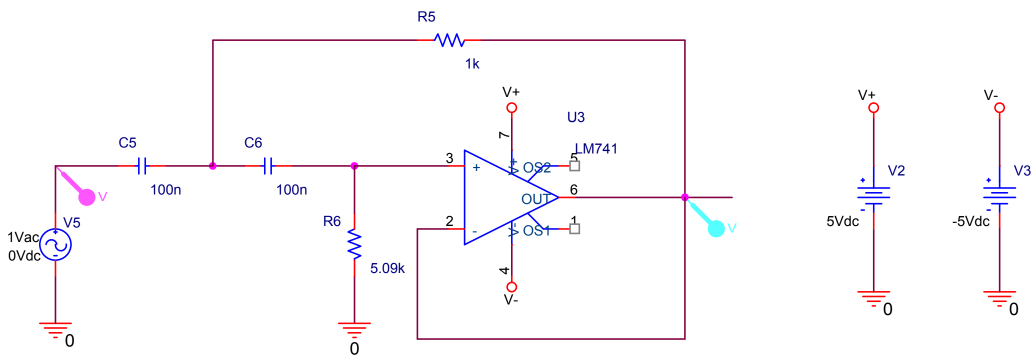


Close-up of 1dB ripple

The frequency response of the circuit peaks at 1.23kHz with a 1dB overshoot and then it decays rapidly.

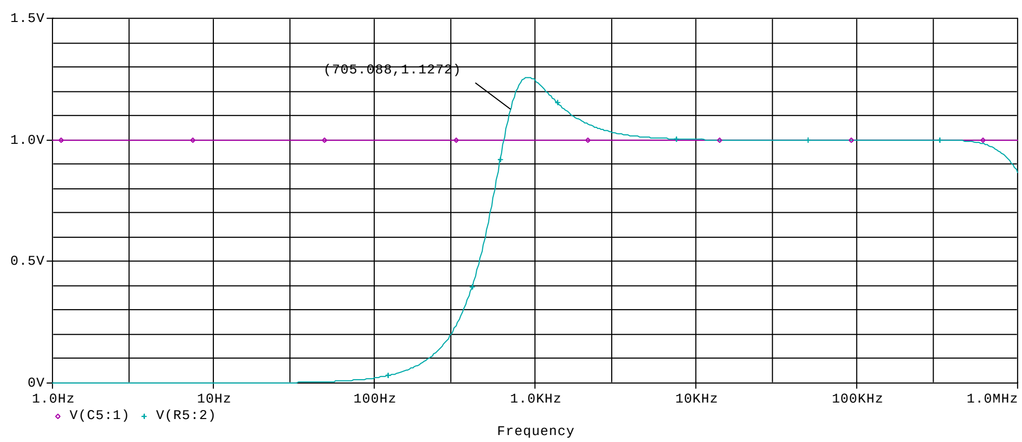
Second-order high-pass (2dB ripple)

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors. This specific filter is designed to have a 2dB ripple in the passband.



Chebyshev 2dB ripple filter (high-pass) (2^{nd}-order)

Note: for a 2dB ripple, C_5=C_6 and R_6=5.098xR_5.



AC sweep from 1Hz to 1MHz

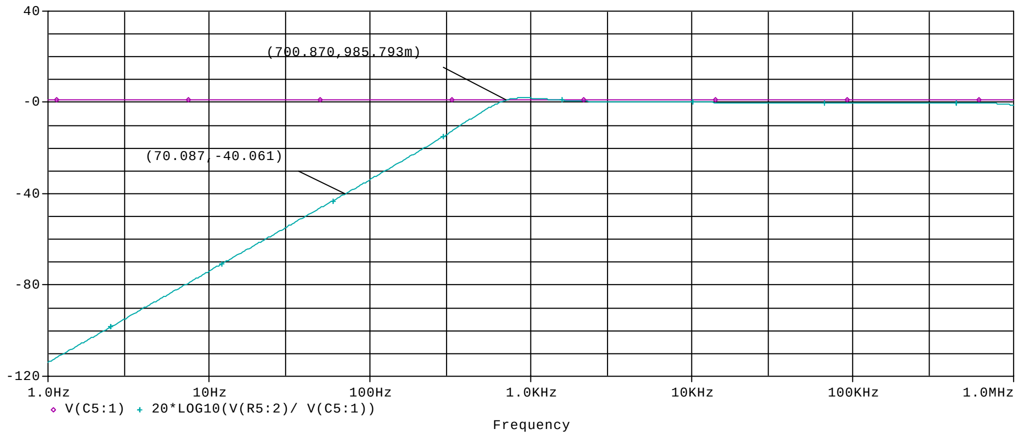
The ripple in the passband is noticeable.

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_5 R_6 C_5 C_6}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 5.09k\Omega \cdot 100nF \cdot 100nF}} = 705.44Hz$$

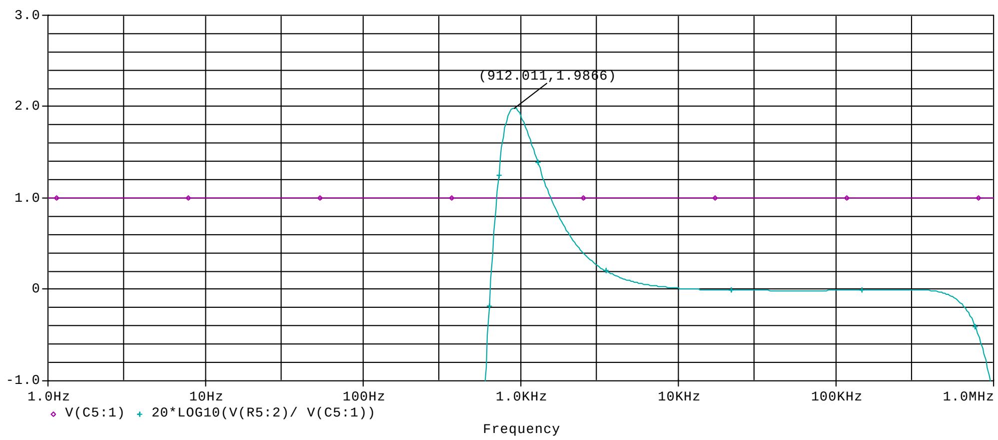
The quality factor is

$$Q = \frac{\sqrt{R_5 R_6 C_5 C_6}}{R_5 (C_5 + C_6)} = \frac{\sqrt{1k\Omega \cdot 5.09k\Omega \cdot 100nF \cdot 100nF}}{1k\Omega \cdot (100nF + 100nF)} = 1.128$$



Bode plot from 1Hz to 1MHz

The gain is +0.985dB at 700Hz and then it decreases by –40dB/decade.

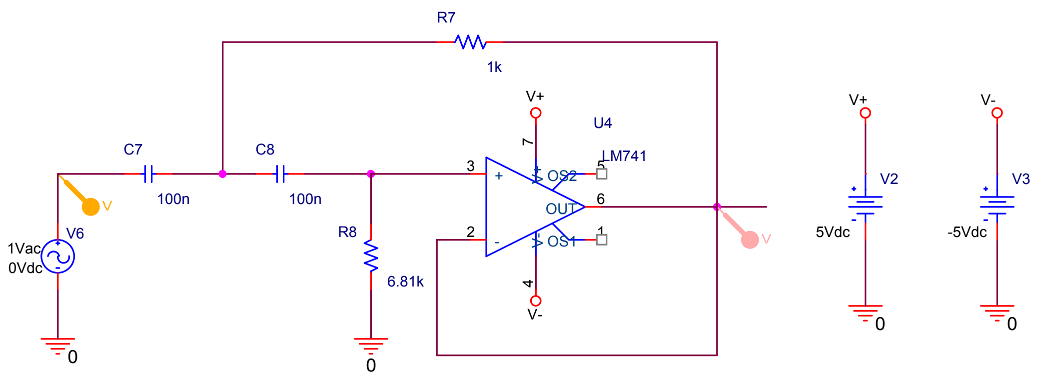


Close-up of 2dB ripple

The frequency response of the circuit peaks at 912Hz with a 2dB overshoot and then it decays rapidly.

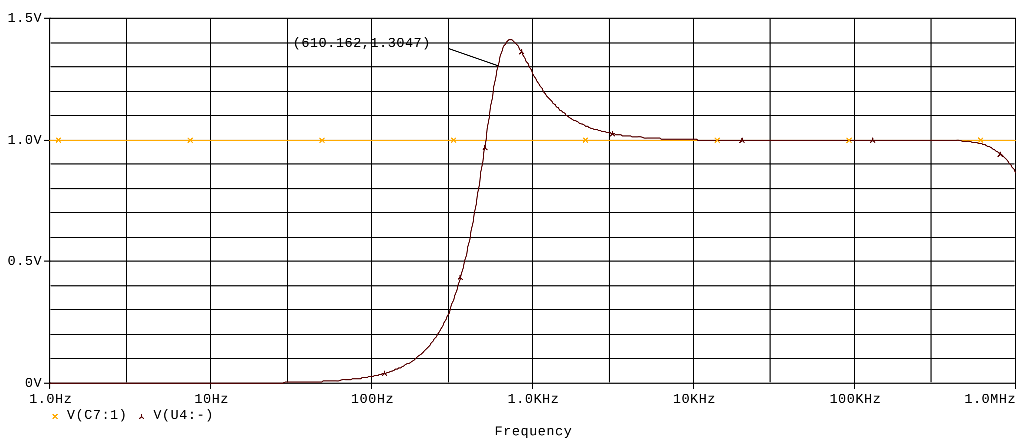
Second-order high-pass (3dB ripple)

In the following example, the circuit is implemented with the Sallen/Key topology. This is a second-order filter because it has two capacitors. This specific filter is designed to have a 3dB ripple in the passband.



Chebyshev 3dB ripple filter (high-pass) (2nd-order)

Note: for a 3dB ripple, C_7=C_8 and R_8=6.812xR_7.



AC sweep from 1Hz to 1MHz

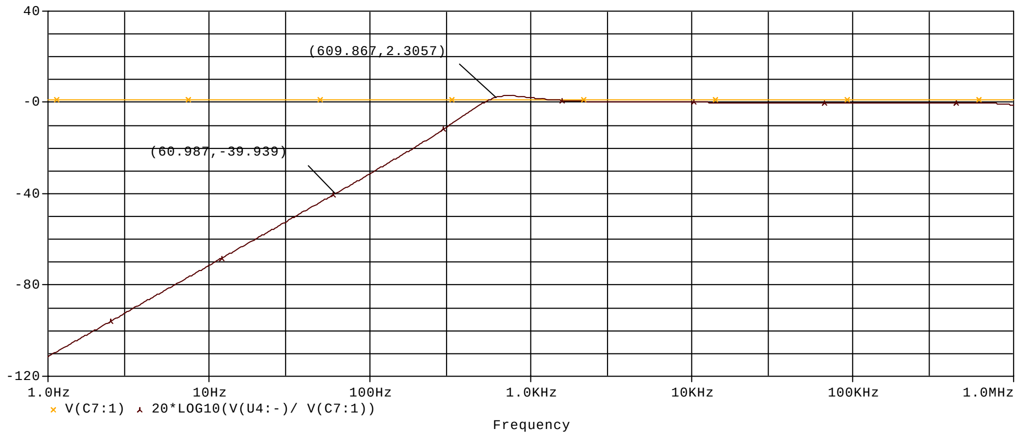
The ripple in the passband is noticeable.

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_7 R_8 C_7 C_8}} = \frac{1}{2\pi\sqrt{1k\Omega \cdot 6.81k\Omega \cdot 100nF \cdot 100nF}} = 609.88Hz$$

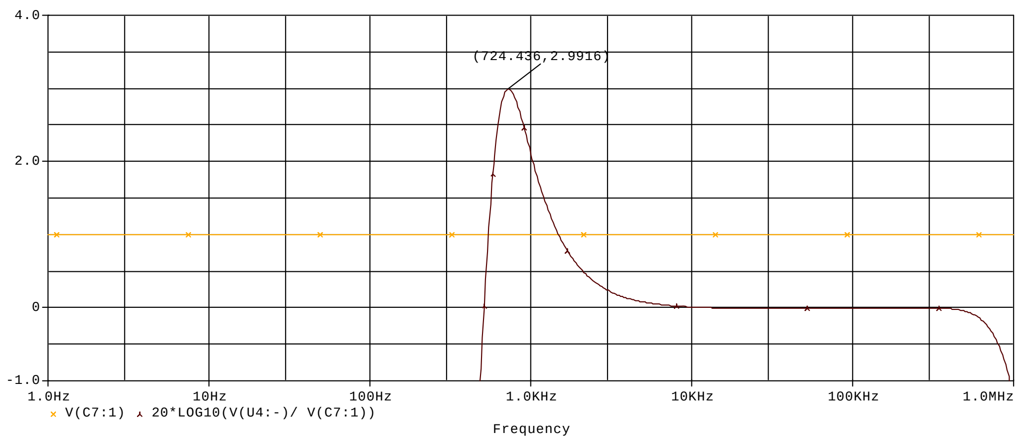
The quality factor is

$$Q = \frac{\sqrt{R_7 R_8 C_7 C_8}}{R_7\left(C_7 + C_8\right)} = \frac{\sqrt{1k\Omega \cdot 6.81k\Omega \cdot 100nF \cdot 100nF}}{1k\Omega \cdot \left(100nF + 100nF\right)} = 1.305$$



Bode plot from 1Hz to 1MHz

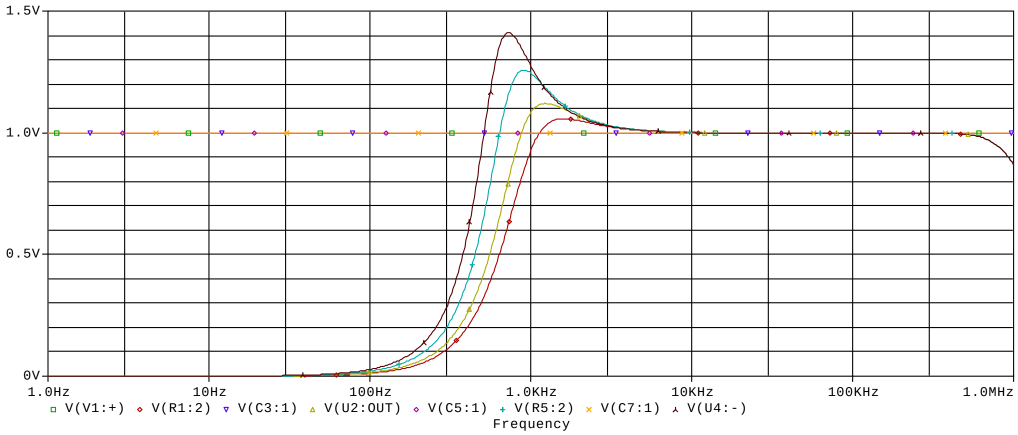
The gain is +2.3057dB at 609Hz and then it decreases by –40dB/decade.



Close-up of 3dB ripple

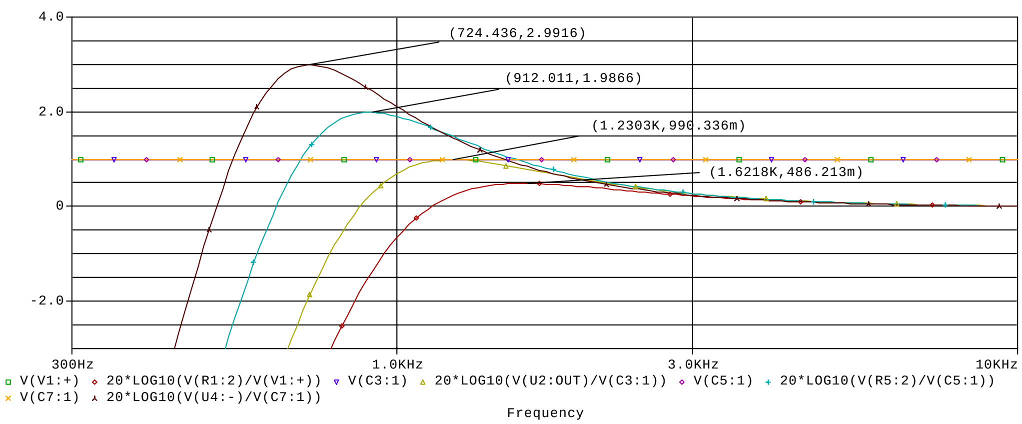
The frequency response of the circuit peaks at 724Hz with a 3dB overshoot and then it decays rapidly.

Second-order high-pass ripple comparison



0.5dB, 1dB, 2dB and 3dB circuit response comparison

Above is a comparison of the response for the high-pass circuits previously presented. The peaks in the response are clearly not matching because the circuits have been designed with the same capacitor value so the cutoff frequency moves to the left of the plot as the magnitude of the ripple increases.



0.5dB, 1dB, 2dB and 3dB ripple comparison

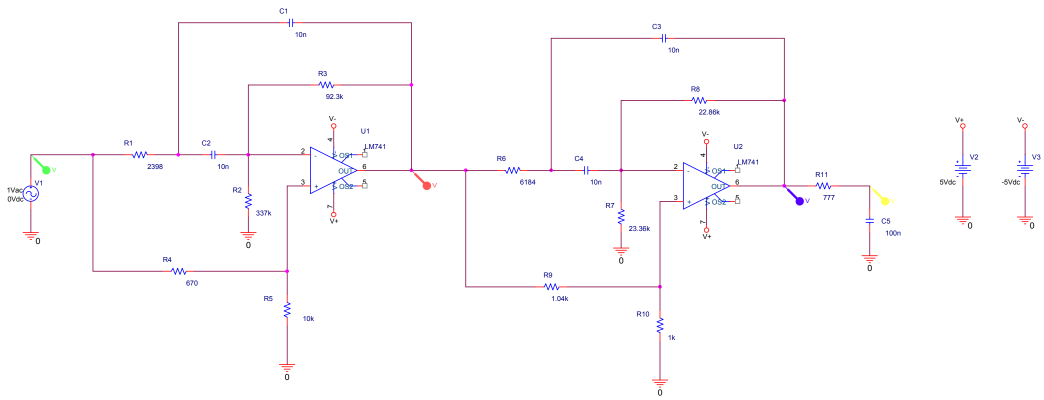
The plot shown above compares the ripples for every circuit previously presented. It is clear that a higher amount of ripple will produce a sharper cutoff.

Type II

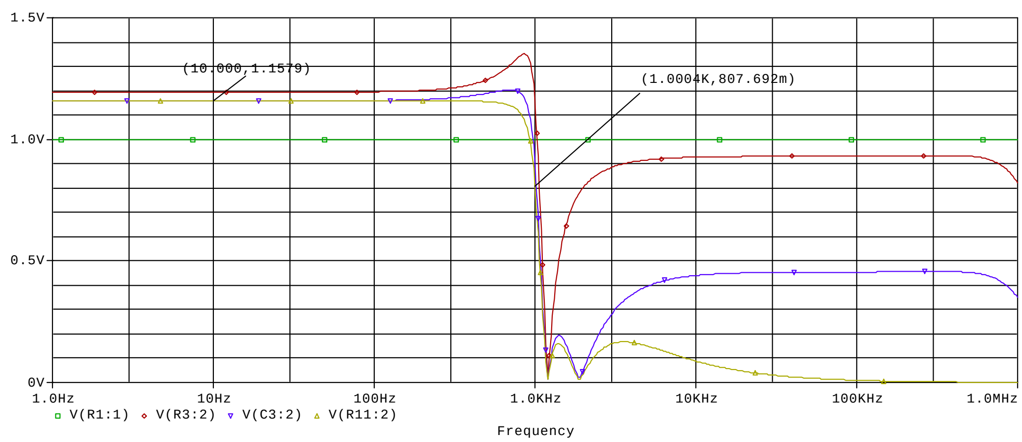
This is the Chebyshev filter with the ripple in the stopband. This filter has a very sharp cutoff.

Fifth-order low-pass

This circuit is implemented with two notch filter blocks and a simple RC filter. This is a fifth-order filter because the circuit contains five capacitors.

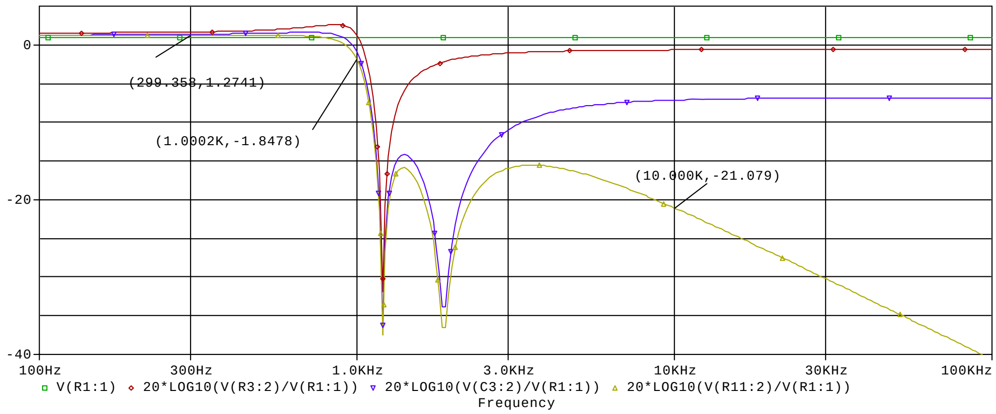


Inverted Chebyshev filter (low-pass) (5^{th}-order)



AC sweep from 1Hz to 1MHz

The gain in the passband boosts the input from 1V to 1.15V. The filter drops rapidly right before 1kHz. The ripple in the stopband is noticeable.



Bode plot from 100Hz to 100kHz

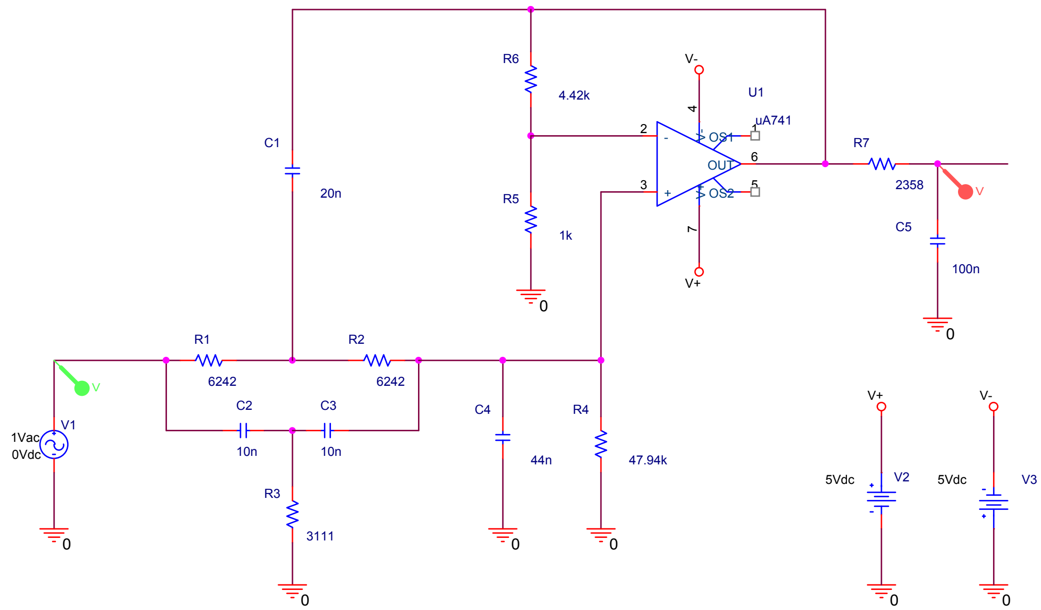
The gain is +1.2741dB in the passband and –1.8478dB at 1kHz. Then it drops to –21.079dB a decade later. The ripple in the stopband is noticeable.

Elliptic filter

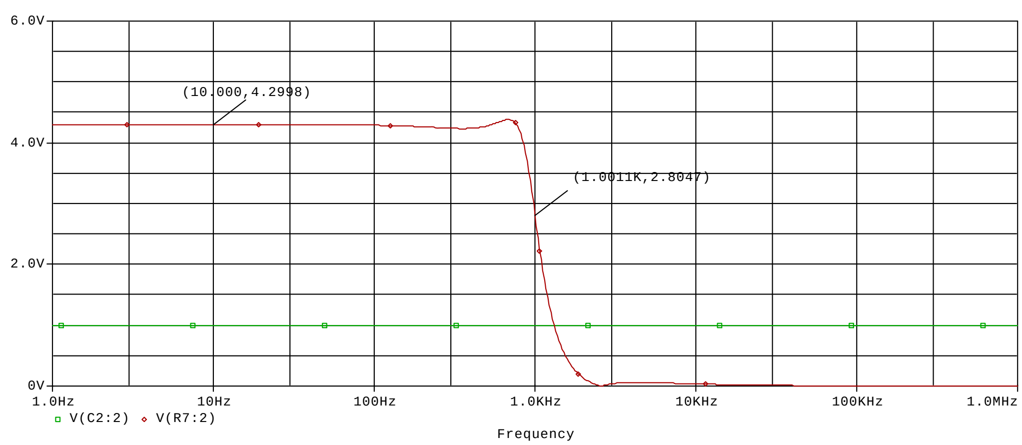
The Elliptic filter, also known as Cauer filter, has a sharp cutoff. This filter is named after Wilheld Cauer, a German mathematician who developed the theory behind the filter.

Third-order low-pass

This circuit is implemented with an asymmetrical twin-T notch filter (R_1, R_2, R_3, C_2, C_3, C_4). This is a third-order filter.

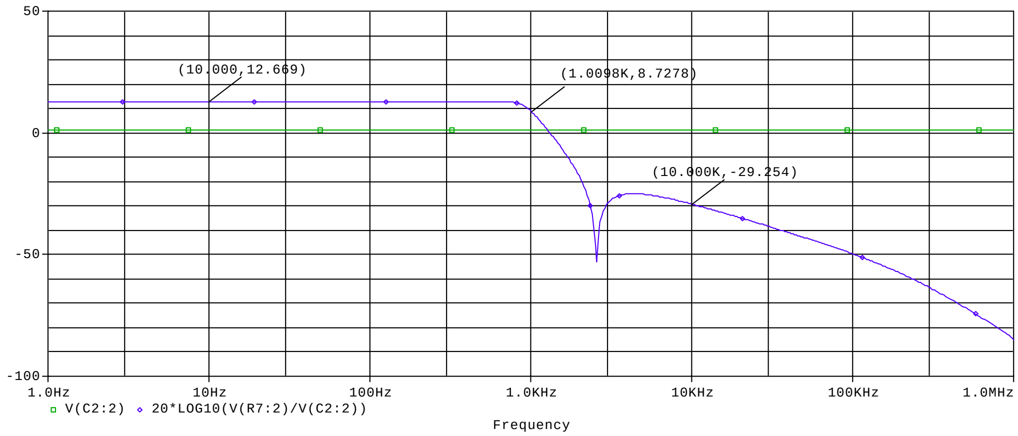


Elliptic filter (low-pass) (3^{rd}-order)



AC sweep from 1Hz to 1MHz

The gain in the passband boosts the input from 1V to 4.3V. The ripple in the passband is barely noticeable. The gain drops rapidly right before 1kHz.



Bode plot from 1Hz to 1MHz

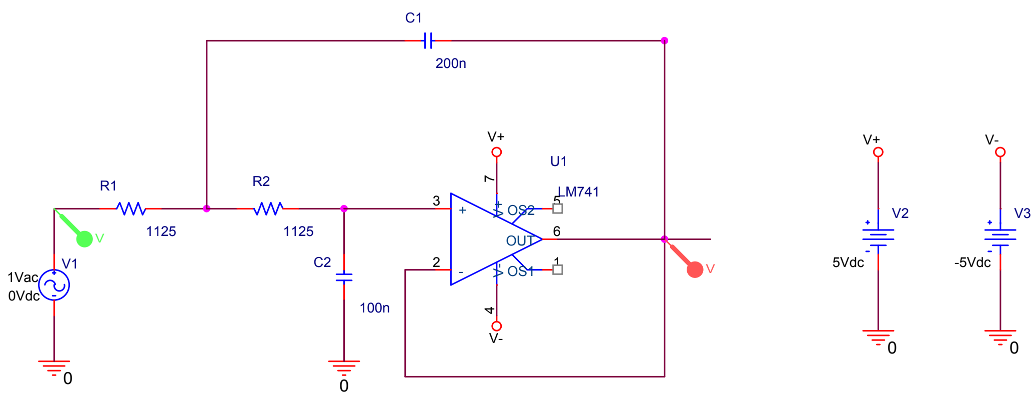
The gain is +12.669dB in the passband and +8.7278dB at 1kHz. Then it drops to –29.254dB a decade later.

Synchronous filter

Synchronous filters are made up by a series of filters *cascaded* after each other. Each capacitor introduces a pole. Resistors and capacitors usually have all the same values and they are tuned to a specific cutoff frequency.

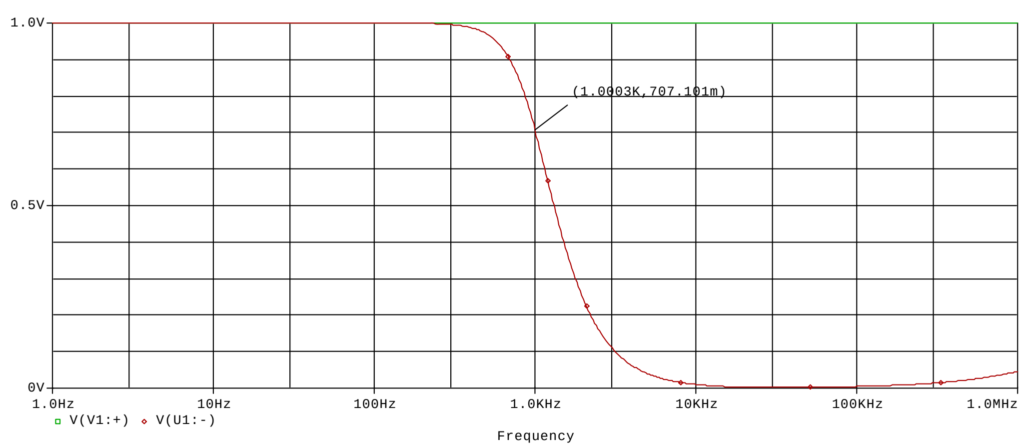
Second-order low-pass

In the following example, the circuit is implemented with the Sallen/Key topology. This circuit is equivalent to the Butterworth circuit discussed previously (Q=0.707). This is a second-order filter because it has two capacitors.



Synchronous filter (low-pass) (2^{nd}-order)

Note: $R_1=R_2$ *and* $C_1=2xC_2$.



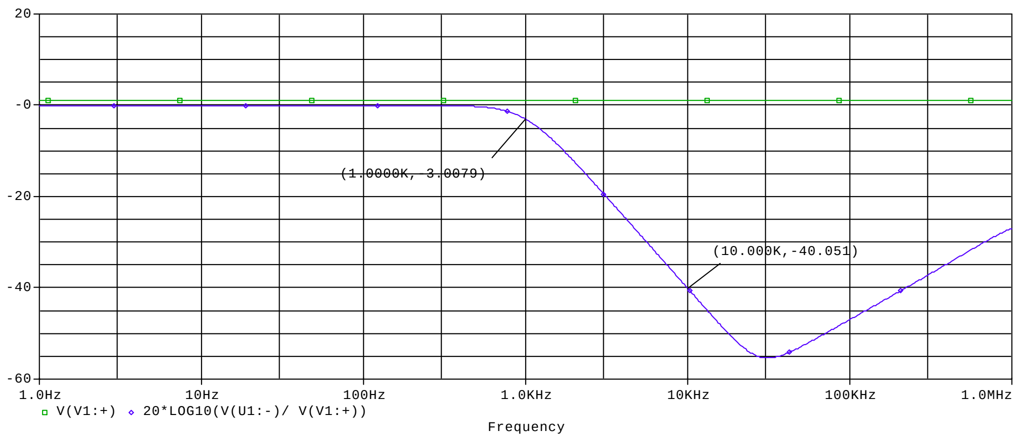
AC sweep from 1Hz to 1MHz

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1125\Omega \cdot 1125\Omega \cdot 200nF \cdot 100nF}} = 1kHz$$

The quality factor is

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{C_2 (R_1 + R_2)} = \frac{\sqrt{1125\Omega \cdot 1125\Omega \cdot 200nF \cdot 100nF}}{100nF \cdot (1125\Omega + 1125\Omega)} = 0.707$$

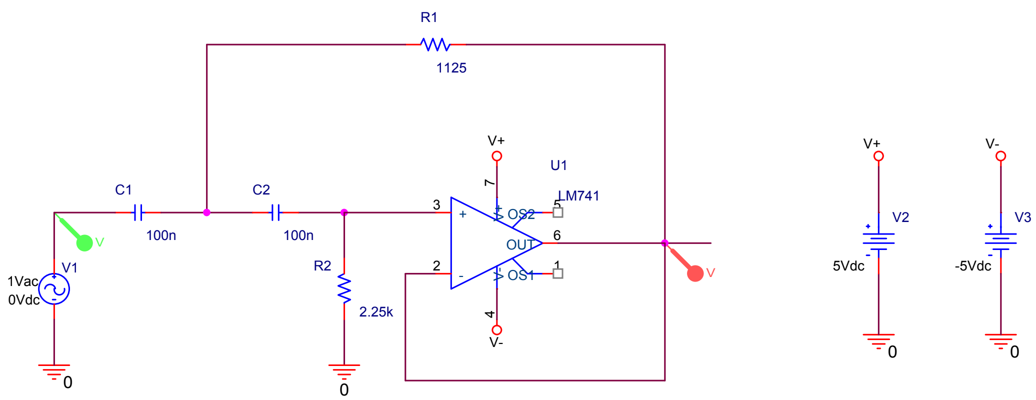


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases by –40dB/decade.

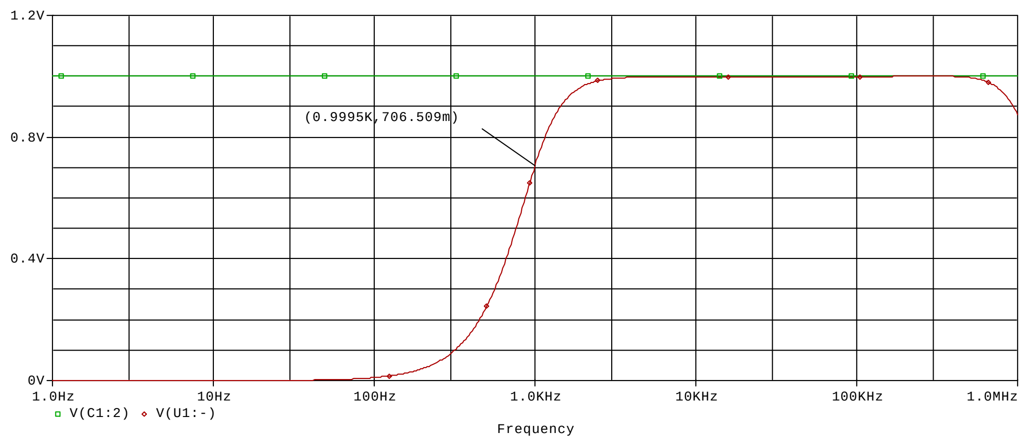
Second-order high-pass

In the following example, the circuit is implemented with the Sallen/Key topology. This circuit is equivalent to the Butterworth circuit discussed previously (Q=0.707). This is a second-order filter because it has two capacitors.



Synchronous filter (high-pass) (2^{nd}-order)

Note: C_1=C_2 and R_2=2xR_1.



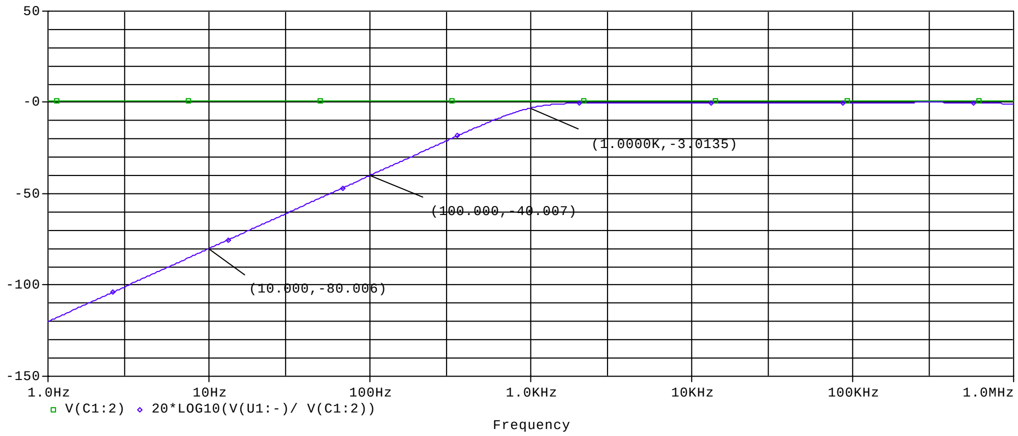
AC sweep from 1Hz to 1MHz

The cutoff frequency is

$$f_c = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \frac{1}{2\pi\sqrt{1125\Omega \cdot 2.25k\Omega \cdot 100nF \cdot 100nF}} = 1kHz$$

The quality factor is

$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)} = \frac{\sqrt{1125\Omega \cdot 2.25k\Omega \cdot 100nF \cdot 100nF}}{1125\Omega \cdot (100nF + 100nF)} = 0.707$$

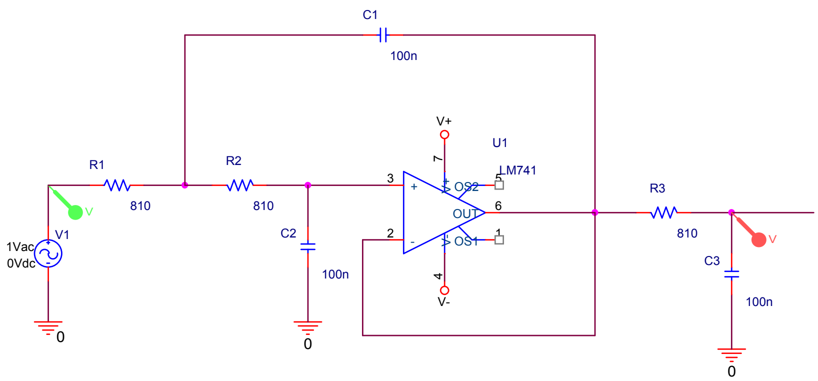


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases by –40dB/decade.

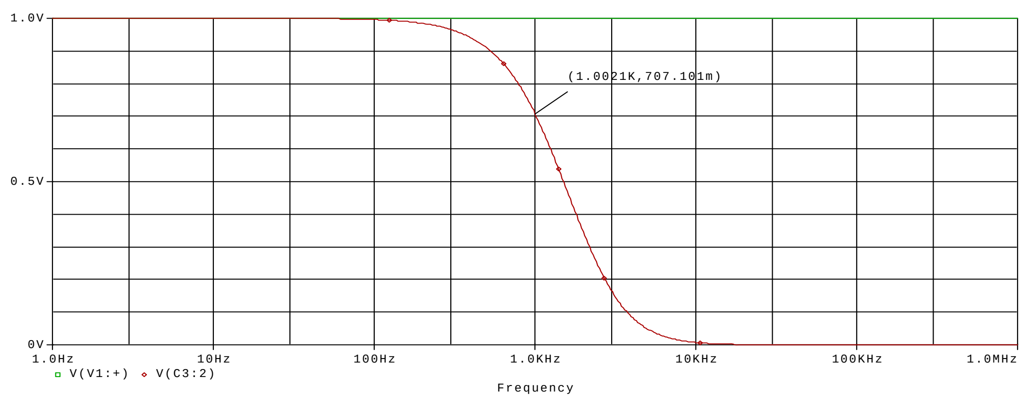
Third-order low-pass

In the following example, the circuit is implemented with the Sallen/Key topology by *cascading* a simple low-pass filter after a second-order filter. This is a third-order filter because it has three capacitors.



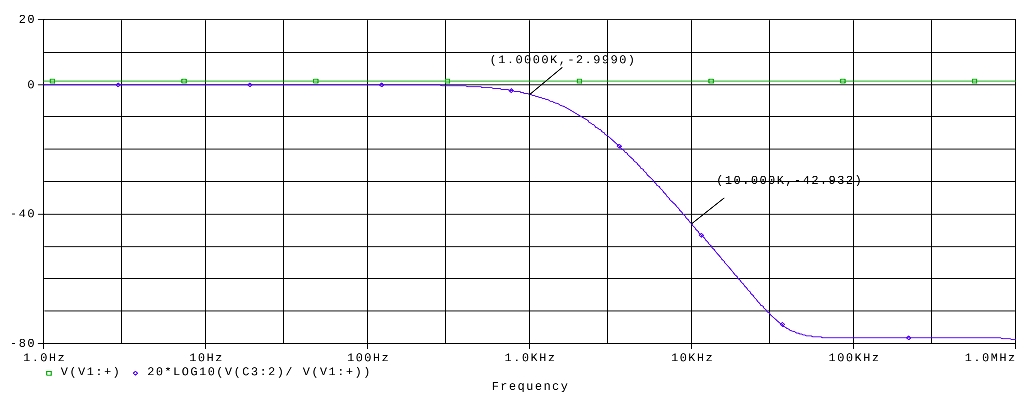
Synchronous filter (low-pass) (3rd-order)

Note: all resistor and capacitor values match.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.

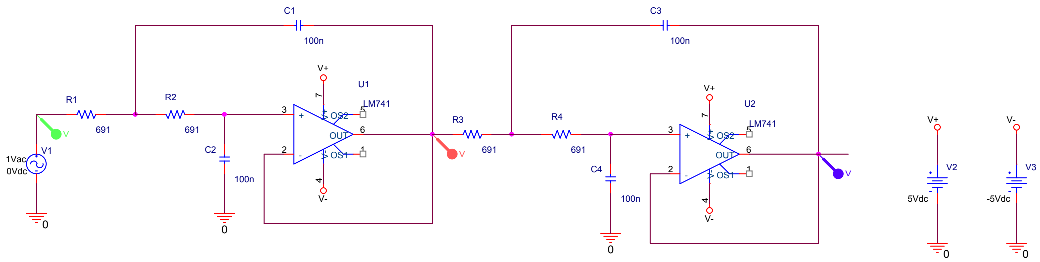


Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases by –40dB/decade.

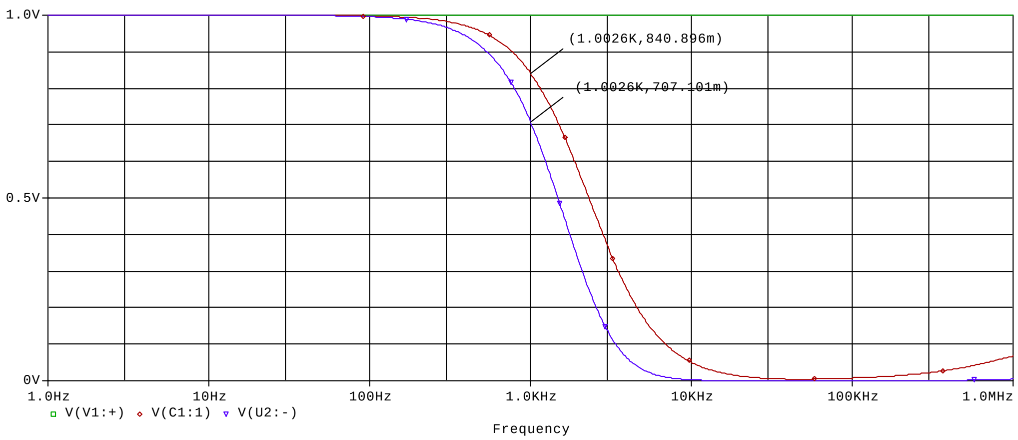
Fourth-order low-pass

In the following example, the circuit is implemented with the Sallen/Key topology by *cascading* two identical second-order filters. This is a fourth-order filter because it has four capacitors.



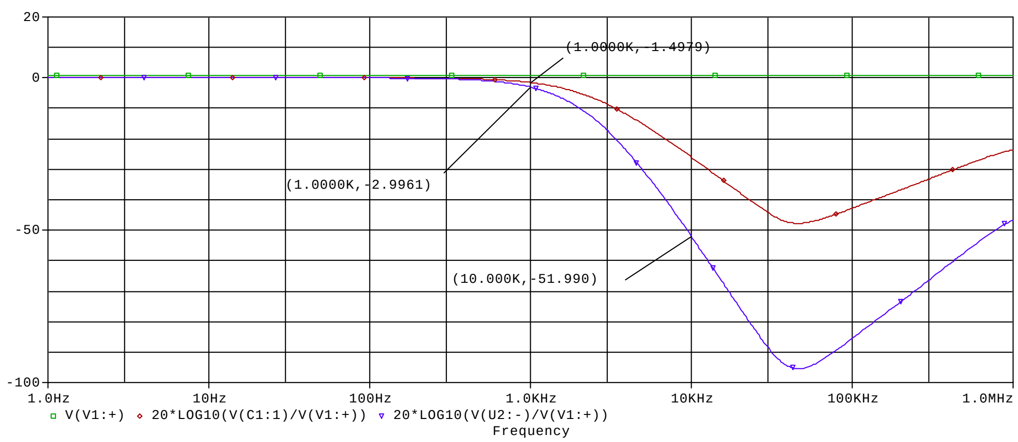
Synchronous filter (low-pass) (4^{th}-order)

Note: all resistor and capacitor values match.



AC sweep from 1Hz to 1MHz

The cutoff frequency is 1kHz.



Bode plot from 1Hz to 1MHz

The gain drops to –3dB at 1kHz and then it decreases to –51.99dB a decade later.

Linkwitz-Riley crossover

The Linkwitz-Riley crossover is an audio application that stems from the work of Linkwitz and Riley.

The crossover can be implemented with different orders. For every order, the gain of the filter will drop by –6dB/octave or –20dB/decade past the cutoff frequency. Increasing the order of the filter will produce a sharper cutoff.

The crossover can be designed to split the audible spectrum in 2, 3 or 4 ways. A 2-way audio crossover splits the audible spectrum in two parts, it has a single cutoff frequency and it’s implemented by *cascoding* two Butterworth filters (low-pass and high-pass). For a 3-way crossover, there will be three regions with a low cutoff frequency and a high cutoff frequency. This is arguably the most popular crossover configuration in the market. The reason why the audible spectrum is divided into 3 sections is explained by the need for audio systems to handle each section more effectively through speakers for proper sound reproduction. A 3-way system has 6 speakers (2 for each channel).

A 3-way 4th-order Linkwitz-Riley crossover can be designed with the following expression:

$$f = \frac{1}{2\pi\sqrt{2}RC}$$

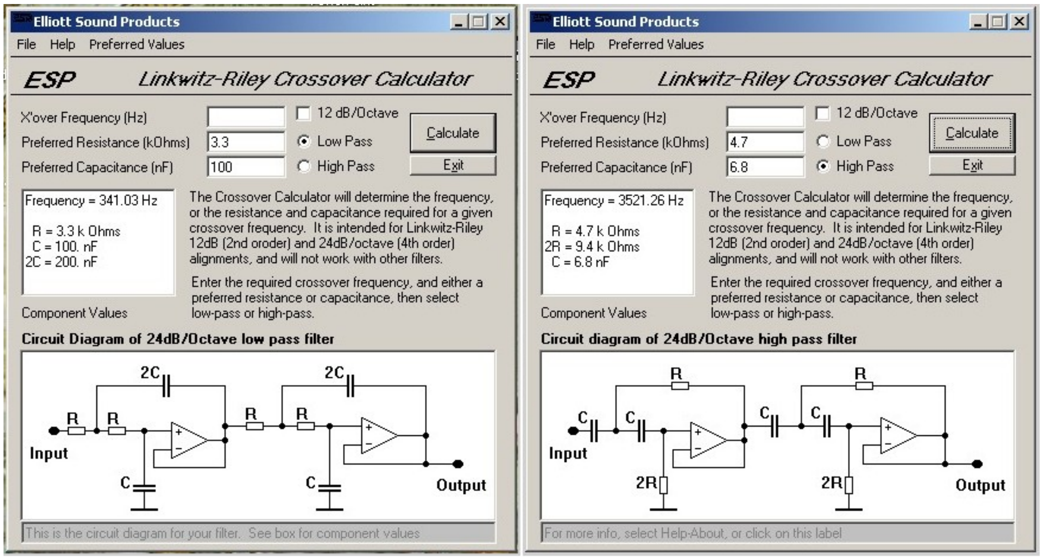
First of all the designer needs to choose cutoff frequencies for the specific regions of the spectrum. At that point, with a set frequency, a value for capacitance (C) or resistance (R) is chosen and the other one is derived.

Assuming that the desired low cutoff frequency is 340Hz then C can be chosen to be 100nF and R can be chosen to be 3.3kΩ.

$$f_L = \frac{1}{2\pi\sqrt{2}RC} = \frac{1}{2\pi\sqrt{2}\cdot 3.3k\Omega \cdot 100nF} = 341.029Hz$$

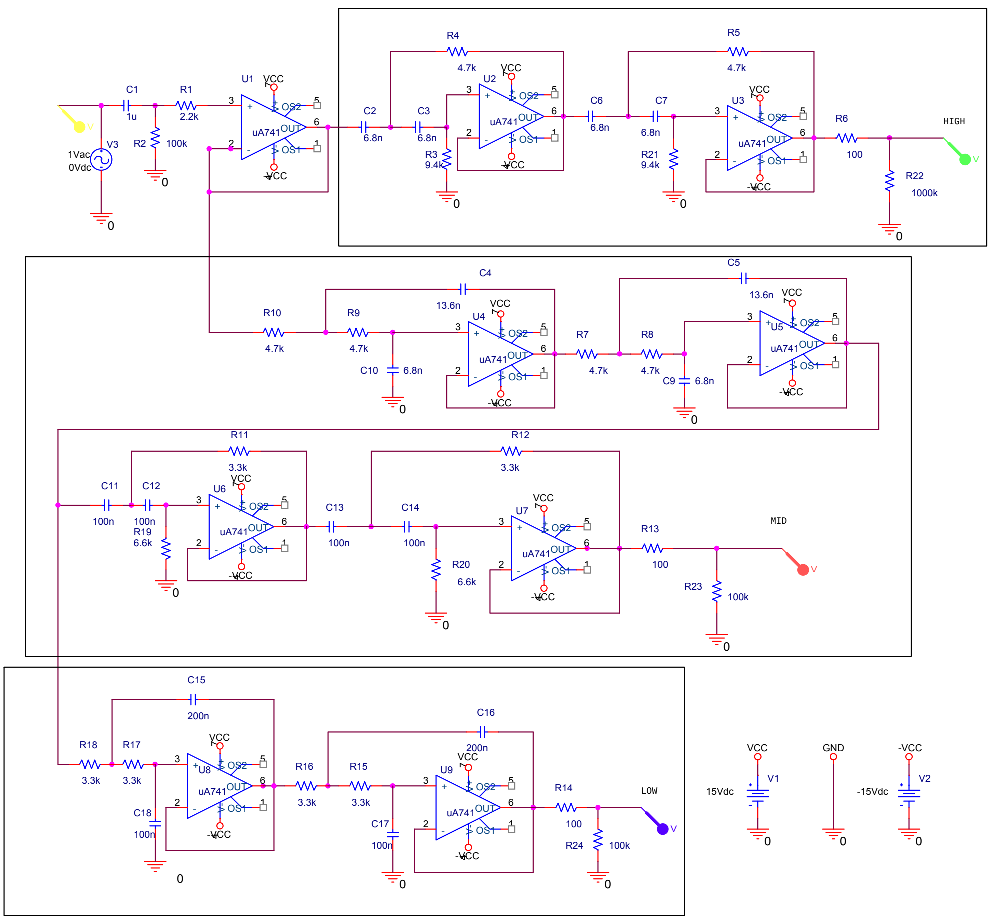
Assuming that the desired high cutoff frequency is 3.5kHz then C can be chosen to be 6.8nF and R can be chosen to be 4.7kΩ.

$$f_H = \frac{1}{2\pi\sqrt{2}RC} = \frac{1}{2\pi\sqrt{2}\cdot 4.7k\Omega \cdot 6.8nF} = 3.521kHz$$



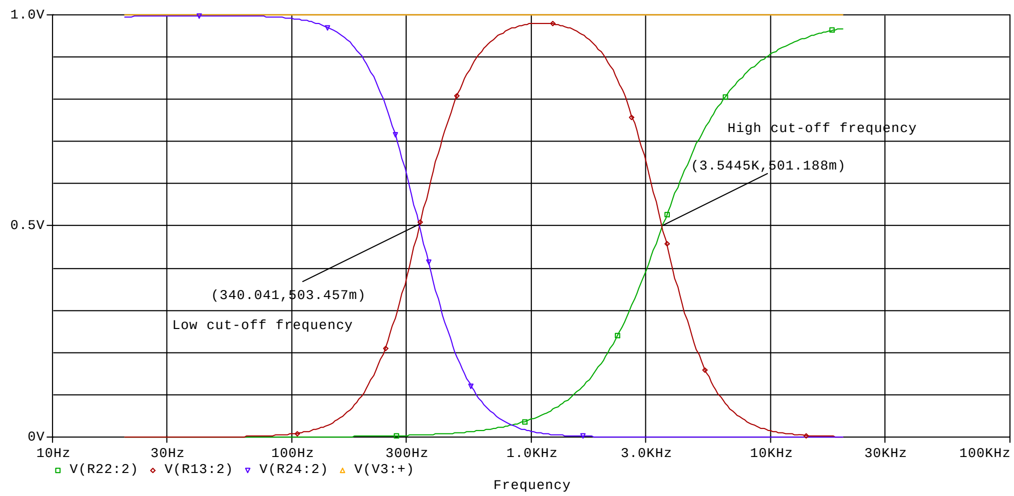
E.S.P. Linkwitz-Riley Crossover Calculator screenshots for low pass and high pass

The values for the two sections of the crossover need to be arranged just like shown above. The values of capacitance or resistance double depending on the configuration of the specific section of the filter.



3-way Linkwitz-Riley crossover

The circuit previously shown is a *cascode* of 3 sections. The top provides the high frequencies, the bottom provides the low frequencies and the central part provides the mid frequencies. High and low sections are made up by a *cascade* of 2 2^{nd}-order Butterworth filters. The middle section is a high-pass section followed by a low-pass section.



AC sweep from 20Hz to 20kHz

The frequency response for the 3-way Linkwitz-Riley crossover is shown above. The low cutoff frequency is 340Hz. The high cutoff frequency is 3.5kHz.

A 3-way 4^{th}-order crossover's gain will drop by –24dB/octave or –80dB/decade past the cutoff frequency.