$$f(t) = f'(t)W(t) = P(t) = \sum_{m=-\infty}^{\infty} F(\omega)e^{i\omega t} \tag{1}$$

where $f'(t)$, $W(t)$, $f(t)$, and $P(t)$ are respectively the raw function, the window function, the tapered (windowed) function and the approximated function by the complex Fourier series. In Eq. (1) $\omega = m\omega_0$.

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \tag{2}$$

If $f(t) \neq 0$ only when $0 < t < T$ then for the normalized Fourier transform over a definite time interval T we have

$$F(\omega) = \frac{1}{\sqrt{T}} \int_{0}^{T} f(t)e^{-i\omega t}dt \tag{3}$$

Using discrete Fourier transform for the above equation we get

$$F(\omega) = \frac{\Delta t}{\sqrt{T}} \sum_{n=1}^{N} f_n e^{-i\omega n \Delta t} \tag{4}$$

where $T = N\Delta t$ is the total pulse duration. The power spectrum $S(\omega)$ is then computed by

$$S(\omega) = |F(\omega)|^2 = \frac{\Delta t^2}{T} \left| \sum_{n=1}^{N} f_n e^{-i\omega n \Delta t} \right|^2 \tag{5}$$

In order to compute the amplitude and the phase spectra one could simply use

$$A(\omega) = \sqrt{S(\omega)} \tag{6}$$

and

$$\phi(\omega) = \arctan\left(\frac{\mathrm{Im}(F(\omega))}{\mathrm{Re}(F(\omega))}\right) \tag{7}$$

Using the fast Fourier transform algorithm (CooleyTukey FFT), the number of multiplications that should be done in order to compute $F(\omega)$ drops from N^2 (for DFT) to $N\log_2(N)$ (for FFT). In the FFT packages such as $FFTW3.0$ or $dfti - MKL$, the frequency resolution is equal to $d\nu = \frac{1}{T} = \frac{1}{N\Delta t}$. Giving a N points time domain array to these packages, only the first half of the output frequency domain (positive frequencies up to the Nyquist frequency $(\frac{N}{2})$) is useful. The second half is negative. The first output would also be the DC frequency.

The resolution of FFT could be increased by padding a number of zeros to the end of the function array. Doing the so called Zero-padding, $T = T + T_{Zero-padded}$ would be the new total time. The data should be windowed before zero-padding. The point of the windowing process is to smooth out the end-points of the data prior to taking the FFT, so that the spectral leakage is reduced. Otherwise, it seems that all the added zeros are considered a part of the data, which is incorrect.

Filtering the computed Fourier transform of the windowed function in a desired frequency interval (harmonics $m = \frac{\omega}{\omega_0}$) and suppressing the other frequencies (harmonics), the new time domain function would be computed using the inverse Fourier transform

$$f_{filtered}(t) = \sum_{m=m_1}^{m_2} F(\omega)e^{i\omega t}. \tag{8}$$

The power spectrum of the filtered inverse Fourier transform $S_{filtered}(t) = |f_{filtered}(t)|^2$ is usually plotted. The time-frequency spectrum is also obtainable via a short time Fourier transform method (STFT). One of such transformations is the Gabor transform:

$$G(\omega, t) = \int dt' f(t')e^{-i\omega t'} e^{\left[-\frac{(t-t')^2}{2\tau^2}\right]} \tag{9}$$

where τ is the width of the time window. The time-frequency spectrum is then calculated using

$$S(G(\omega, t)) = |G(\omega, t)|^2 \tag{10}$$