Wyprowadzenie sformułowania wariacyjnego

Potencjał elektromagnetyczny

$$\frac{d\varphi^2}{d^2x} = -\frac{\rho}{\varepsilon_r} \quad \text{dla} \quad \Omega = \langle 0, 3\rangle$$

$$\rho = 1$$

$$\varepsilon_r(x) = \begin{cases} 10 & x \in \langle 0, 1\rangle \\ 5 & x \in (1, 2\rangle \\ 1 & x \in (2, 3\rangle \end{cases}$$

$$\begin{cases} \varphi'(0) + \varphi(0) = 5 \\ \varphi(3) = 2 \end{cases}$$

$$\varphi'' = -\frac{\rho}{\varepsilon_r}$$

Funkcja testowa: $v(x) \in V$, $V = \{f \in H^1 : f(3) = 0\}$

$$\varphi'' v = -\frac{\rho}{\varepsilon_r} v$$

$$\int_0^3 \varphi'' v \, dx = \int_0^3 -\frac{\rho}{\varepsilon_r} v \, dx$$

$$[\varphi' v]_0^3 - \int_0^3 \varphi' v' \, dx = \int_0^3 -\frac{\rho}{\varepsilon_r} v \, dx$$

$$\varphi'(3)v(3) - \varphi'(0)v(0) - \int_0^3 \varphi' v' \, dx = \int_0^3 -\frac{\rho}{\varepsilon_r} v \, dx$$

$$\varphi'(0) + \varphi(0) = 5 \Rightarrow \varphi'(0) = 5 - \varphi(0)$$

$$(\varphi(0) - 5)\, v(0) - \int_0^3 \varphi' v' \, dx = -\int_0^3 \frac{\rho}{\varepsilon_r} v \, dx$$

Shift: $\varphi = w + \bar{\varphi}$

$$\bar{\varphi}(x) = 2 \quad \Rightarrow \quad \varphi = w + 2 \quad \Rightarrow \varphi' = w'$$

$$(w(0) - 3)v(0) - \int_0^3 w' v' \, dx = -\int_0^3 \frac{\rho}{\varepsilon_r} v \, dx$$

$$\underbrace{w(0)v(0) - \int_0^3 w' v' \, dx}_{B(w,v)} = \underbrace{3v(0) - \int_0^3 \frac{\rho}{\varepsilon_r} v \, dx}_{L(v)}$$