PHY 1101: Assignment 01

Fall 2024

Mrenal Kanti Das Mridul
Department of Computer Science & Engineering (DAY)
Batch: 65, Section: A
ID: 02724205101001

Instructor:
Dr. Ranojit Kumar Dutta
Associate Professor & Coordinator
Department of General Education
City University
Savar, Dhaka 1340, Bangladesh.

Date: *September* 26, 2024

ASSIGNMENT

Two small particles of mass m_1 and mass m_2 attract each other with a force that varies inversely with the cube of their separation (Fig.1). At time t_0, m_1 has a velocity of magnitude v_0, directed towards m_2, which is at rest a distance d away. At time t_1, the particles collide (Fig. 1). Calculate L, the distance travelled by particle 1 during the time interval $t_1 - t_0$. Express your answer using some or all of the following variables: m_1, m_2, t_0, t_1, v_0, and d.

THEORETICAL CONSTRUCTION

One takes an arbitrary force C, dependent on the system of our framework, and that such is acting on the variables we will relate it with.

Let F be the expression of the force between the two particles that varies inversely with the cube of their separation r.

$$F = \frac{C}{r^3}$$

Since the particle m_1 is moving; the force is being acted upon it, moving towards m_2 which is initially at rest, we will need acceleration of the first. We tend to Sir Newton's second law;

$$F = m_1 a_1 = \frac{C}{r^3}$$

where a_1 is the acceleration of the particle m_1, which by rearranging one gets;

$$a_1 = \frac{C}{m_1 r^3}$$

For the system we chose to work with, there is no mention of other outward or external force. Hence the momentum will be conserved. Let us for the sake of assumption have m_2 to be stationary as m_1 is headed towards it. The force is dependent on the distance, and in an ideal system like such that we have, the total energy will be conserved. We will have the kinetic energy and the potential energy due to the acting force. Denoting the total energy to be E, and the potential to be $U(d)$ at the initial distance d, we will have;

$$E_{total} = \frac{1}{2} m_1 v(t)^2 + U(d)$$

We represent the expressions as functions since the total energy of a given (ideal) system will stay the same, as per the law of conservation. Better to note that the two forms of energy, meaning kinetic and potential are variable, even though the amount or quantity remains constant. But the potential will decrease as the particle m_1 gains kinetic energy as it leads toward m_2. Mathematically the potential;

$$F = -\frac{dU(r)}{dr} = \frac{C}{r^3}$$

Integrating the expression, we have the collective potential;

$$\Sigma F = U(t) = -\frac{C}{2r^2}$$

For the kinetic energy, the velocity is expressed by;

$$v_1 = \frac{dr}{dt}$$

and the kinetic energy as;

$$E_{kinetic} = \frac{1}{2}mv(t)^2$$

One can express the total energy as;

$$E_{constant} = \frac{1}{2}m_1 v_1^2 - \frac{C}{2r^2}$$

We now have our final expression of the total energy of the given system. We will have two expressions for t_0 and t_1. Now, since we have all the variables at place; one can proceed to writing;

$$\frac{1}{2}m_1 v_1^2 - \frac{C}{2r^2} = \frac{1}{2}m_1 v_0^2 - \frac{C}{2d^2}$$

Solving for v_1 one would have;

$$v_1^2 = v_0^2 + \frac{C}{m_1}\left(\frac{1}{r^2} - \frac{1}{d^2}\right) \Rightarrow v_1 = \sqrt{v_0^2 + \frac{C}{m_1}\left(\frac{1}{r^2} - \frac{1}{d^2}\right)}$$

We know $v_1 = dr/dt$, thus;

$$\frac{dr}{dt} = \sqrt{v_0^2 + \frac{C}{m_1}\left(\frac{1}{r^2} - \frac{1}{d^2}\right)}$$

By some rearrangement, we find the time dependence;

$$dt = \frac{dr}{v_1} = \frac{dr}{\sqrt{v_0^2 + \frac{C}{m_1}\left(\frac{1}{r^2} - \frac{1}{d^2}\right)}}$$

If one integrates the above expression, substituting the value of v_1,we will have the distance travelled by the first particle. We denote such by L (not a Laplacian), and proceed;

$$L = \int_{t_0}^{t_1} v_1 \; dt$$

and then one substitutes the value of v_1;

$$L = \int_{r=d}^{r=0} \frac{dr}{\sqrt{v_0^2 + \frac{C}{m_1}\left(\frac{1}{r^2} - \frac{1}{d^2}\right)}}$$

NOTE

Dear Sir Dutta,

The calculations are kept to a minimum and only the results of the final lines are shown. Despite the desperate greed to input the gravitational constant G into the equation, alongside the electric charges and the other constants, I realized that I do not know much of relativity and electromagnetism to work with such framework. My classmates have shown different results, each distinct than the others, somewhat making me conclude that my result is too much absurd. Most probably because of my interest in mathematics and approaching a problem axiomatically which is a norm in pure mathematics. I yet not know the correct answer, but an insight alongside thrashing feedback would be very much appreciated.

Humbly and Sincerely,
Mrenal