Crash Course Mathematics

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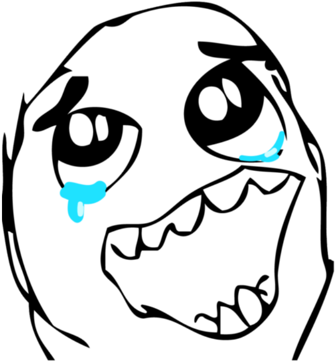
Before starting ... Where do I find this slides?



Essentials of Set Theory

What is a set?

A set may be seen as a collection of elements. The following

$$S = \{a, b, c\}$$

is said to be a set. In particualr we are talking about the set S with elements a b and c.

Be carfull

The order is not important meaninig that given two vectors:

$$A = \{1, 2, 3\} \quad B = \{1, 3, 2, 1\}$$

$$A = B$$

Essentials of Set Theory II

Let's elaborate it a little more...

The notation we used previously is usefull with short sets but what about big sets and infinite sets? We need to use a different notation defining the property of the set:

$$B = \{(x, y) : px + qy \leq m, x \geq 0, y \geq 0\}$$

Set Membership

$$x \in S \quad x \notin S$$

$$A \subseteq B$$

Essentials of Set Theory III

Exercise

Try to state the properties of the **even** number set.

Essentials of Set Theory IV

Set operations

Notation	The set that consists of the elements that:
$A \cup B$	belong to at least one of the sets A and B
$A \cap B$	belong to both A and B
$A \setminus B$	belong to set A, but not to B
A^c	do not belong to set A

Tips and tricks

For a small number of sets you can use Venn diagramms as a visual help!

Essentials of Set Theory V (Exercises)

1.1.5 Determine which of the following formulas are true. If any is false, find a counter example to demostrate this.

1. $A \setminus B = B \setminus A$
2. $A \cap (B \cup C) \subseteq (A \cap B) \cup C$
3. $A \cup (B \cap C) \setminus (A \cup B) \cap C$
4. $A \setminus (B \setminus C) = (A \setminus B) \setminus C$

1.1.6 Use the Venn diagrams to prove that:

1. $(A \cup B)^c = A^c \cap B^c$
2. $(A \cap B)^c = A^c \cup B^c$

Some Aspects of Logic

Propositions

Are assertions that are either true or false.
Eg: all individuals who breathe are alive **TRUE proposition**
Eg: all individuals who breathe are healthy **FALSE proposition**

Implications

Implications are used to keep track of the logical chain reasoning.
Eg: $P \implies Q$ **is read as if** P **then** Q
Eg: $P \iff Q$ **is read as** P **if and only if** Q

Some Aspects of Logic II

Necessary and Sufficient Conditions

Taking the last example

$$P \implies Q$$

P is a **sufficient condition** for Q
Q is a **necessary condition** for P

$$\overbrace{P}^{\text{Living in Paris}} \underbrace{\implies}_{\text{If then}} \overbrace{Q}^{\text{Living in France}}$$

1.2.2 Determine which of the following formulas are true. I any formula is false, find a counter a counter example to demonstrate this.

1. $A \subseteq B \iff A \cup B = B$
2. $A \subseteq B \iff A \cap B = A$
3. $A \cap B = A \cap C \implies B = C$
4. $A \cup B = A \cup C \implies B = C$
5. $A = B \iff (x \in A \iff x \in B)$

Some Aspects of Logic IV (Exercises)

1.2.5 For each of the following proposition, state the negation as simply as possible.

1. $x \geq andy \geq 0$
2. All x satisfy $x \geq a$
3. Neither x nor y is less than 5
4. For each $\epsilon \geq 0$, there exists a $\delta > 0$ such that B is satisfied
5. No one can help liking cats
6. Everyone loves somebody some of the time

Mathematical Proofs

Characteristics

Looking on what argued by Thomas Tymoczko, three are the main features of a mathematical proof:

1. Convincingness
2. Surveyability
3. Formalizability

Direct proof

We start with the premises P and work forward to Q

Indirect proof

We are going backwards supposing Q is false and proving the falseness of P

Mathematical induction

Direct and indirect proof are a both examples of deductive reasoning (we use the rules of logic to prove something). Inductive reasoning moves from a different assumption meaning that we can establish certain conclusions by observations.

Algebra

The Real Numbers

- Natural numbers: $\mathbb{N} = \{0, 1, 2, ...\}$
- Integers: $\mathbb{Z} = \mathbb{N} \cup \{..., -2, -1\}$
- Rational numbers: $\mathbb{Q} = \{x : x = \frac{a}{b}, a \in N, b \in N\}$
- Irrational numbers: $\mathbb{I} = \mathbb{R}\ \mathbb{Q}$
- Real numbers: $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$

2.1.1 Explain why the infinite decimal expansion below is not a rational number

1.01001000100001...

Integer Powers

Definition

$$a^n = \overbrace{a \cdot a ... a}^{n}$$

$$a^{-n} = \overbrace{\frac{1}{a} \cdot \frac{1}{a} ... \frac{1}{a}}^{n}$$

And ...

$$a^0 = 1 \quad \forall a \neq 0$$

$$a^0 = undefined \quad if \quad a = 0$$

Integer Powers II

Properties

For any real number a, and any integer numbers r and s:

$$a^r \cdot a^s = a^{r+s}$$

$$(a^r)^s = a^{rs}$$

Integer Powers III

Rules of Algebra

Fractions

Fractional Power

Inequalities

Intervals and Absolute Values

Summation

Rules for Sums

Newton’s Binomial Formula

Duble Summs

END OF CH 2