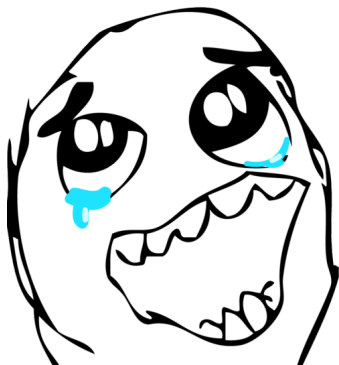


Crash Course Mathematics

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Before starting ... Where do I find this slides?



Essentials of Set Theory

What is a set?

A set may be seen as a collection of elements. The following

$$S = \{a, b, c\}$$

is said to be a set. In particular we are talking about the set S with elements a , b and c .

Be careful

The order is not important meaning that given two vectors:

$$A = \{1, 2, 3\} \quad B = \{1, 3, 2, 1\}$$

$$A = B$$

Essentials of Set Theory II

Let's elaborate it a little more...

The notation we used previously is useful with short sets but what about big sets and infinite sets? We need to use a different notation defining the property of the set:

$$B = \{(x, y) : px + qy \leq m, x \geq 0, y \geq 0\}$$

Set Membership

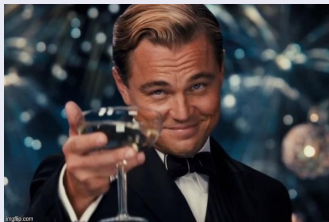
$$x \in S \quad x \notin S$$

$$A \subseteq B$$

Essentials of Set Theory III

Exercise

Try to state the properties of the **even** number set.



Essentials of Set Theory IV

Set operations

Notation	The set that consists of the elements that:
$A \cup B$	belong to at least one of the sets A and B
$A \cap B$	belong to both A and B
$A \setminus B$	belong to set A , but not to B
A^c	do not belong to set A

Tips and tricks

For a small number of sets you can use Venn diagrams as a visual help!

Essentials of Set Theory V (Exercises)

1.1.5 Determine which of the following formulas are true. If any is false, find a counter example to demonstrate this.

- ① $A \setminus B = B \setminus A$
- ② $A \cap (B \cup C) \subseteq (A \cap B) \cup C$
- ③ $A \cup (B \cap C) \setminus (A \cup B) \cap C$
- ④ $A \setminus (B \setminus C) = (A \setminus B) \setminus C$

1.1.6 Use the Venn diagrams to prove that:

- ① $(A \cup B)^c = A^c \cap B^c$
- ② $(A \cap B)^c = A^c \cup B^c$

Some Aspects of Logic

Propositions

Are assertions that are either true or false.

Eg: all individuals who breathe are alive **TRUE proposition**

Eg: all individuals who breathe are healthy **FALSE proposition**

Implications

Implications are used to keep track of the logical chain reasoning.

Eg: $P \implies Q$ is read as if P then Q

Eg: $P \iff Q$ is read as P if and only if Q

Some Aspects of Logic II

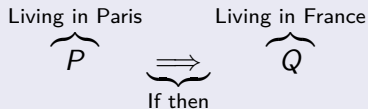
Necessary and Sufficient Conditions

Taking the last example

$$P \implies Q$$

P is a **sufficient condition** for Q

Q is a **necessary condition** for P



Some Aspects of Logic III (Exercises)

1.2.2 Determine which of the following formulas are true. If any formula is false, find a counter example to demonstrate this.

- ① $A \subseteq B \iff A \cup B = B$
- ② $A \subseteq B \iff A \cap B = A$
- ③ $A \cap B = A \cap C \implies B = C$
- ④ $A \cup B = A \cup C \implies B = C$
- ⑤ $A = B \iff (x \in A \iff x \in B)$

Some Aspects of Logic IV (Exercises)

1.2.5 For each of the following proposition, state the negation as simply as possible.

- ① $x \geq \text{andy} \geq 0$
- ② All x satisfy $x \geq a$
- ③ Neither x nor y is less than 5
- ④ For each $\epsilon \geq 0$, there exists a $\delta > 0$ such that B is satisfied
- ⑤ No one can help liking cats
- ⑥ Everyone loves somebody some of the time

Mathematical Proofs

Characteristics

Looking on what argued by Thomas Tymoczko, three are the main features of a mathematical proof:

- 1 Convincingness
- 2 Surveyability
- 3 Formalizability

Direct proof

We start with the premises P and work forward to Q

Indirect proof

We are going backwards supposing Q is false and proving the falseness of P

Mathematical induction

Direct and indirect proof are both examples of deductive reasoning (we use the rules of logic to prove something). Inductive reasoning moves from a different assumption meaning that we can establish certain conclusions by observations.

The Real Numbers

- Natural numbers $N = \dots, -2, -1, 0, 1, 2, \dots$
- Rational numbers $Q = \{x : x = \frac{a}{b}, a \in N, b \in N\}$
- Irrational numbers

2.1.1 Explain why the infinite decimal expansion below is not a rational number

1.01001000100001...

Integer Powers

Definition

$$a^n = \overbrace{a \cdot a \dots a}^n$$

$$a^{-n} = \frac{1}{a} \cdot \frac{1}{a} \dots \frac{1}{a}$$

Rules of Algebra

- Turn off alarm
- Get out of bed

Fractions

- Eat eggs
- Drink coffee

Fractional Power

- Eat eggs
- Drink coffee

Inequalities

- Eat eggs
- Drink coffee

Intervals and Absolute Values

- Eat eggs
- Drink coffee

Summation

- Turn off alarm
- Get out of bed

Rules for Sums

- Eat eggs
- Drink coffee

Newton's Binomial Formula

- Eat eggs
- Drink coffee

Duble Summs

- Eat eggs
- Drink coffee

END OF CH 2

- Eat eggs
- Drink coffee

Dinner

- Eat spaghetti
- Drink wine

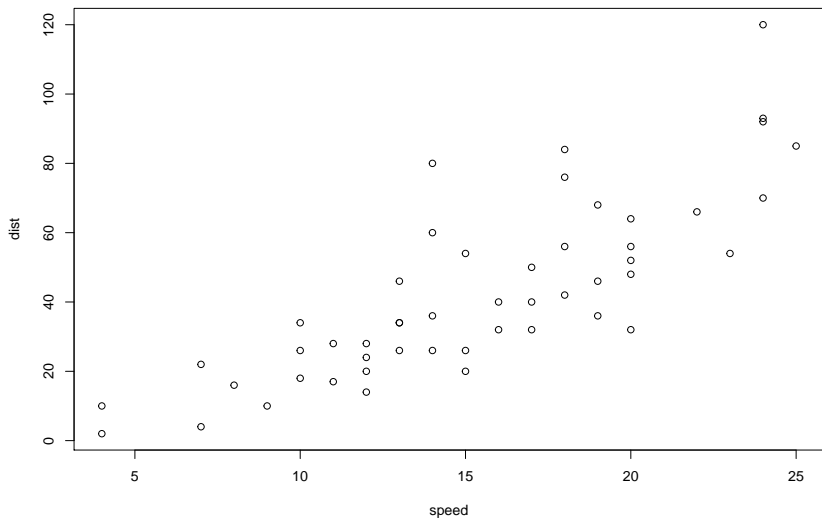


Figure 1: A scatterplot.