Exercise 7: Harmonic Oscillator Absorption

1. Calculate the (squared) matrix elements $\|\mu_{n0}\|^2$ between the harmonic-oscillator ground state and the first two excited states (i.e., n = 1 and 2).

2. Calculate the frequency difference ω_{n0} between the ground state (n=0) and the first two excited states (i.e., n=1 and 2).

3. Write down an explicit expression for $R^{(1)}(\tau_1)$ for the harmonic oscillator, considering only the first three states (n=0, 1, and 2). (You can ignore polarization indices, assuming an isotropic distribution of dipole orientations.)

4. Given this result, what will the absorption spectrum for a harmonic oscillator look like? (You can either describe it in words or give a mathematical expression.)

5. Does your expression depend on $\hbar$? What does this suggest about the "quantumness" of the result?

Exercise 7: Extra Credit

In our calculation, we made a "low-temperature" assumption, that the population of excited states was negligible. A more complete expression is

$$R^{(1)}(\tau_1) = \frac{2}{3\hbar\pi^2}\Theta(\tau_1) \sum_{m,n} \rho_{mm}\|\mu_{nm}\|^2 \sin(\omega_{nm}\tau_1),$$

where the index m runs over the *initial state* of the system (before excitation), and the index n runs over the *final* state (after excitation). The density matrix elements ρ_{mm} are given (at equilibrium) by

$$\rho_{mm} = \frac{e^{-\beta H_{mm}}}{\sum_k e^{-\beta H_{kk}}},$$

where the index k runs over all system states.

Assignment: For the harmonic oscillator, write down an explicit expression for ρ_{mm} that depends only on ω and m. (Include only the *first three* states in the sum over k.) Calculate numerical values for ρ_{11} with T = 300 K and both $\omega/(2\pi c) = 100$ cm^{-1} and $\omega/(2\pi c) = 1600$ cm^{-1}. In each case, how good is the "low-temperature" approximation?