Crack initiation from a plastically blunted notch

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Modelling the blunting of a crack tip due to plastic deformation and the initiation from this blunted crack tip using Leguillion's combined criterion [1]. Idea from the discussion of Aurélien and Martin on 2023-04-20 at the Arlberg Colloquium in Lech am Arlberg.

Can the initiation point of Figure 1 be predicted using Leguillion's combined criterion? Is it necessary or is an energy based criterion sufficient? Can something otherwise not clear from experiments be explained in that way?

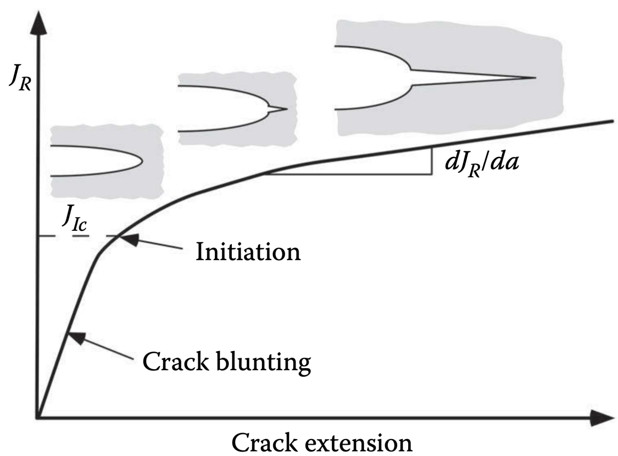


Figure 1: Schematic blunting curve in an elastic-plastic material [2]

1 The FE model

1.1 Geometry and crack opening

The model is shown in Figure 1. It consists of a region around a crack tip and uses symmetry in the y-direction. Loads are applied to the blue line using the displacements from the analytical formulae for the K_I displacement field. As long as the overall size of the model is much larger than the plastic zone at the crack tip, this loading is still accurate. The crack is opened by relaxing the y-symmetry conditions starting at the crack tip.

In order to use fine elements in the region of the crack tip in the possibly very large model, multiple partitions are introduced. Since it is not clear how often the mesh should be refined, these partition radii are introduced as a list `r_list`. The corresponding mesh sizes, along the finest mesh size, are defined in the list `mesh_sizes`.

The finest mesh size region x_0 has a height of $r_1/4$ and a width of $3r_1/4$, ranging from $x = -r_1/4$ to $x = r_1/2$. The Abaqus model can use plane strain or plane stress elements that are either quadratic (CPE8R or CPS8R) or bi-linear (CPE4R or CPS4R). An elastic-plastic material model with isotropic hardening is used with parameters E, ν, σ_{y0}, B and n. Note that the scripted model uses linear elastic material when $\sigma_{y0} = -1$ and ideal plastic material when B is set to 0.

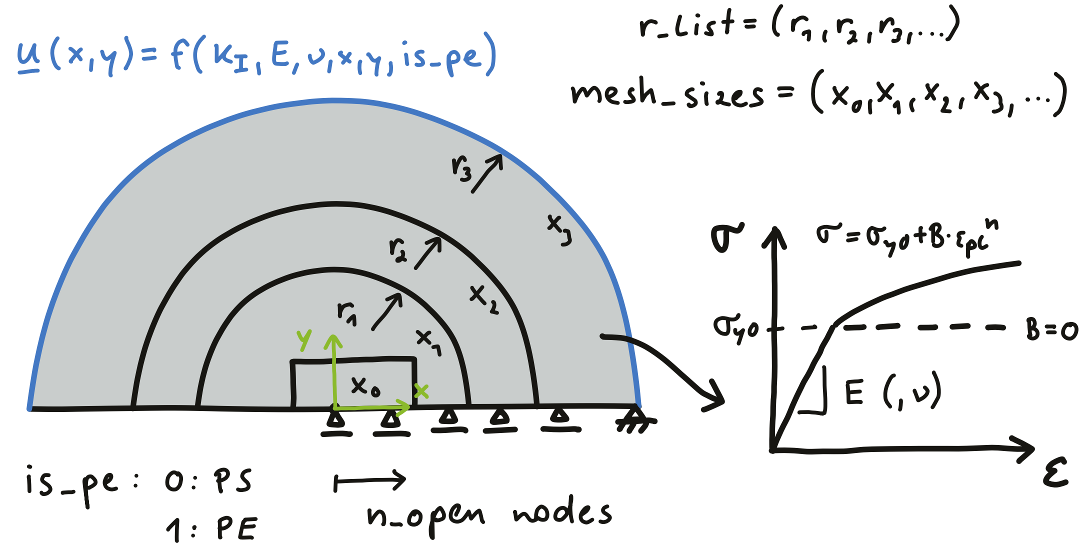


Figure 2: The model used to evaluate the initiation of a blunt crack including geometry, loads, boundary conditions and material model

Two nodes are opened at each step. The applied u loads are not changed according to the crack growth, assuming that the total crack growth length is much smaller than the radius of the model domain.

1.2 K controlled loads

The K_I displacement fields are applied to the boundaries. These are defined e.g. in Anderson's textbook [2] for Mode I loading:

$$u_x = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\cos(\theta/2)\left(\kappa - 1 + 2\sin^2(\theta/2)\right)$$

$$u_y = \frac{K_I}{2\mu}\sqrt{\frac{r}{2\pi}}\sin(\theta/2)\left(\kappa + 1 - 2\cos^2(\theta/2)\right)$$

where $\mu = E/(2(1+\nu))$ is the shear modulus and $\kappa = 3 - 4\nu$ (plane strain) or $\kappa = (3-\nu)/(1+\nu)$ (plane stress).

The radius of the plastic zone r_{pl} is $\frac{1}{6\pi}l_{ch}$ and $\frac{1}{2\pi}l_{ch}$ for plane strain and plane stress conditions respectively. Note that l_{ch} is Irwin's length which is defined as $\frac{K^2}{\sigma_y^2}$. See Table 1 for the r_{pl} of the load cases used. For reference, the globally applied G is shown in the results as $G = K_I^2/E'$, where $E' = E$ for plane stress and $E' = E/(1-\nu^2)$ for plane strain.

Table 1: Size of the plastic zone r_{pl} according to the analytical formulae and for a yield stress of 300 MPa.

K_I (MPa$\sqrt{m}$):	10	20	40
Plane stress	0.177 mm	0.707 mm	2.83 mm
Plane strain	0.0589 mm	0.236 mm	0.943 mm

1.3 Alternative model for small cracks

similar to [3]

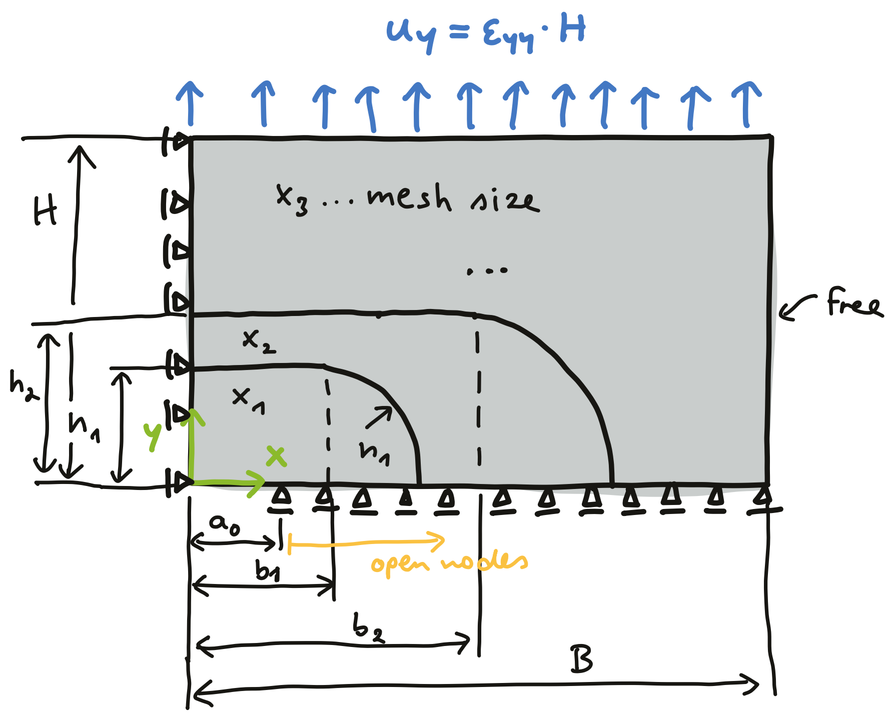


Figure 3: The model used to study small cracks and for what sizes energy-driven and stress-driven failure occurs

1.4 Evaluating σ, G_{inc}, and PEEQ

A path is defined in Abaqus from the initial crack tip to the right. The vertical stress σ_{yy} is evaluated in this path (including all intersections with elements). For the accumulated equivalent plastic strains (PEEQ), the interpolation of the results caused problems for this path. Therefore, another path half an element edge length in the positive y direction is defined and used for the PEEQ evaluation. For a final evaluation I will evaluate the integration point values and use the row closest to the x-axis.

The energy balance for crack propagation can be defined by setting the change of the work of external forces ΔW_{ext} equal to the change in elastic strain energy ΔU_{el} plus the change in plastic dissipation ΔW_{pl} plus the separation work of the crack ΔW_{sep}:

$$\Delta W_{ext} = \Delta U_{el} + \Delta W_{pl} + \Delta W_{sep}$$

For assessing crack propagation, these energies are first divided by the crack area change $\Delta A = \Delta a\, t$ with t as the thickness for a 2D setup. Then, they are separated into the load on the crack face G and and the resistance of the crack to its propagation G_c. A crack grows once $G \geq G_c$.

It is obvious that ΔW_{sep} belongs into the crack resistance G_c, which is a material parameter. For the plastic dissipation ΔW_{pl}, however, it is not clear if that belongs to G or to G_c. Hutchinson mentions in [4] that Orowan and Irwin argued that the critical energy release rate of a material for small scale yielding should include ΔW_{pl}:

$$G = \frac{\Delta W_{ext} - \Delta U_{el}}{\Delta a\, t}, \qquad G_c = \frac{\Delta W_{sep} + \Delta W_{pl}}{\Delta a\, t}$$

This makes sense as long there is only small plastic deformation contained in the vicinity of the crack tip. Also, it assumes that during crack growth, the same amount of ΔW_{pl} per crack face is dissipated. Also, ΔW_{pl} is not accessible through experimental curves such that the contribution of crack face separation and plastic deformation cannot be separated. Note that applying a u-field for K loading, the local $G = K_I^2/E$ for plane stress is the G of this formulation.

For bigger regions of plastic deformation, the plastic dissipation ΔW_{pl} can be considered in the energy release rate G. That means that in a FEM model, the plastic dissipation due to crack propagation is calculated and reduces the crack driving force. The crack growth resistance G_c then only contains the remaining ΔW_{sep}. We refer to this alternative formulation using an asterisk:

$$G^* = \frac{\Delta W_{ext} - \Delta U_{el} - \Delta W_{pl}}{\Delta a\, t}, \qquad G_c^* = \frac{\Delta W_{sep}}{\Delta a\, t}$$

Note that ΔU_{el} is usually negative for a growing crack (energy is released) and $\Delta W_{pl} \geq 0$. Therefore, $G^* \leq G$ and $G_c^* \geq G_c$. For no elastic deformations, G and G_c of the two formulations coincide.

In Abaqus, U_{el} and W_{pl} are evaluated as the history output of the total strain energy (`ALLSE`)and the plastic dissipation (`ALLPD`), respectively.

1.5 The used load cases

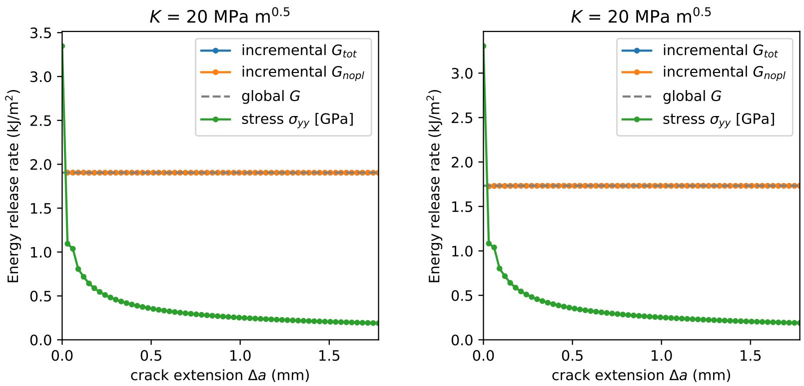
A simple material with a yield stress of 300 MPa is used and a bunch of hardening options: ideal plastic, linear elastic, and three types of hardening curves with n of 0.7, 1, and 3.

```
r_list = (3.5,6,25,70,200,500,1200,3500)
mesh_sizes = (0.03,0.1,0.8,4,8,25,50,100,200)

n_open = 100
E, nu = 210000., 0.3

is_pe = 0
k_list = (10.,20.,40.)

# list of (sig_y0, B, n), for sig_y = -1: lin.el.
load_cases = ((-1, E/10., 1), (300., 0, 1.), (300., E/10., 1),
              (300., E/20., 3), (300., E/100., 0.7))
```

2 First results

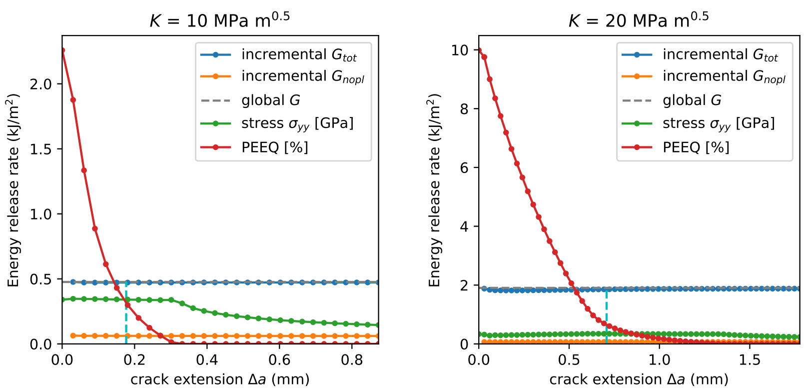
2.1 Linear elastic cases

Only one K_I, because results can be scaled. Plane stress and plane strain results:

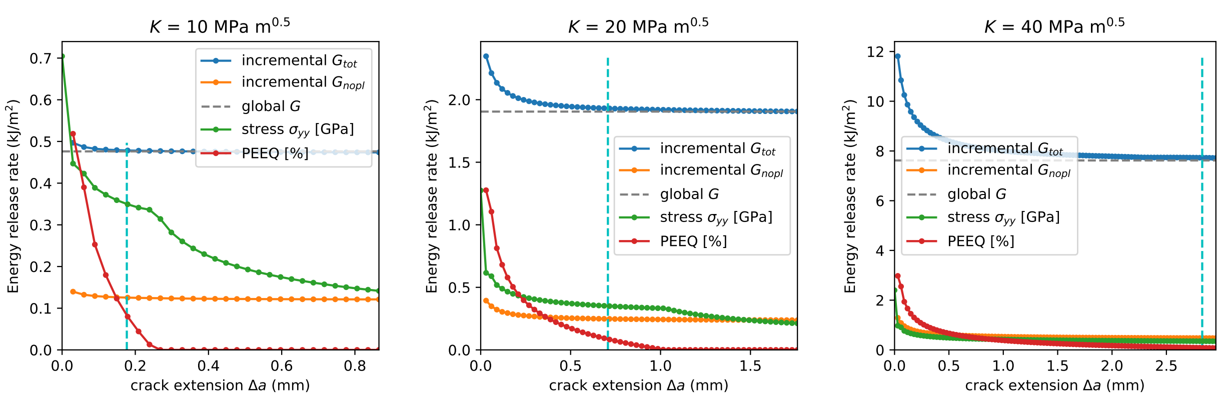


2.2 Plane stress cases

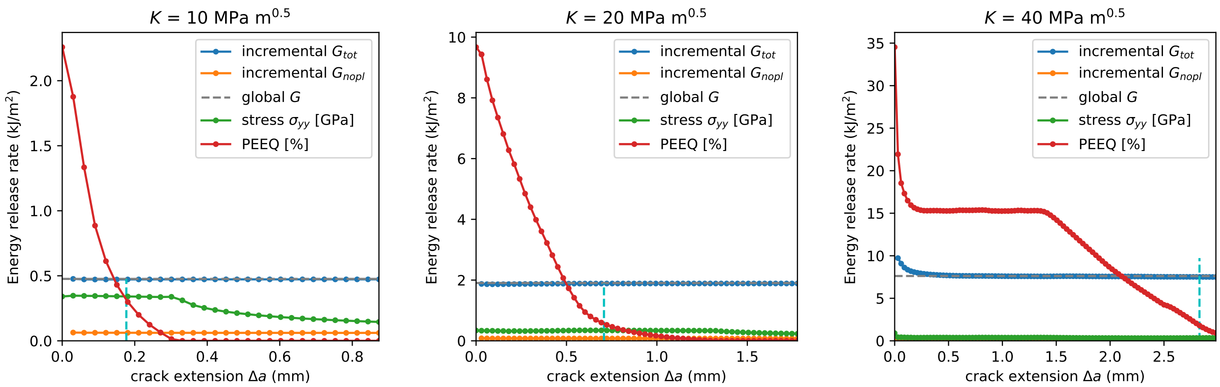
2.2.1 elastic-plastic material (σ_y= 300 MPa, ideal plastic)



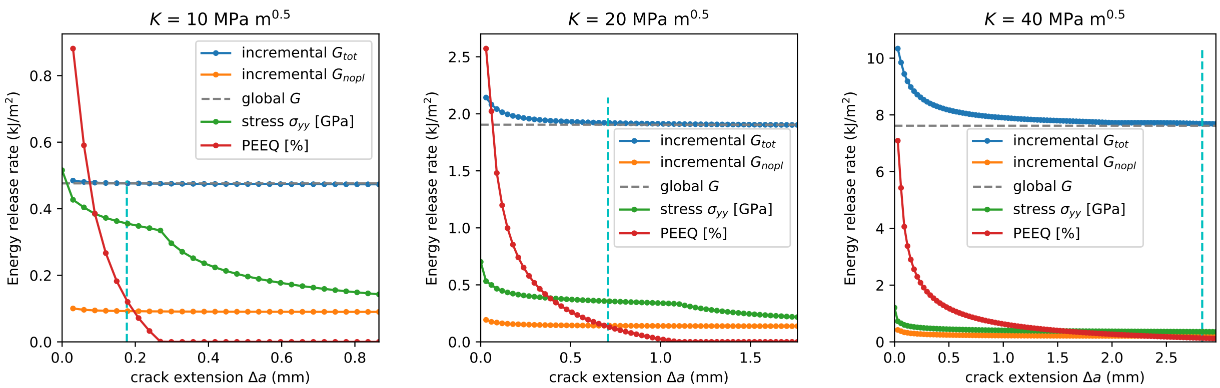
2.2.2 elastic-plastic material (σ_y= 300 MPa, $E_{pl} = E/10$, n = 1)



2.2.3 elastic-plastic material (σ_y= 300 MPa, $E_{pl} = E/20$, n = 3)

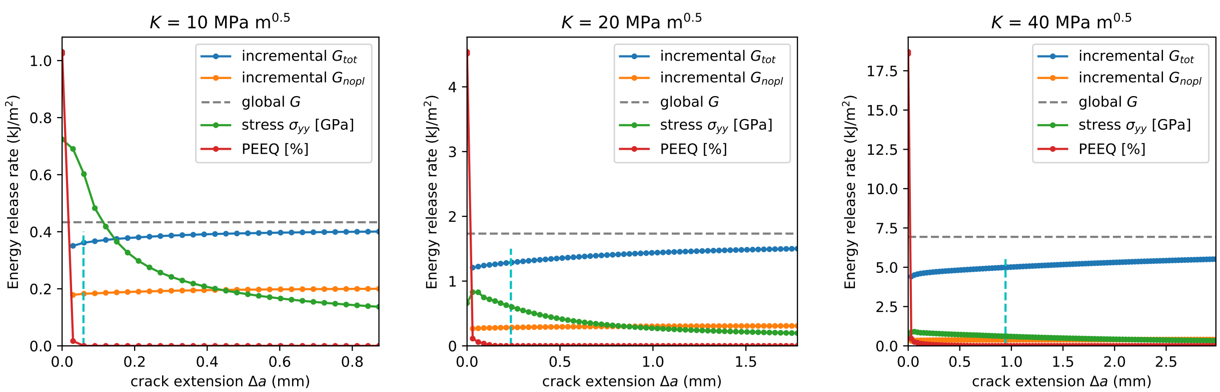


2.2.4 elastic-plastic material (σ_y= 300 MPa, $E_{pl} = E/100$, n = 0.7)

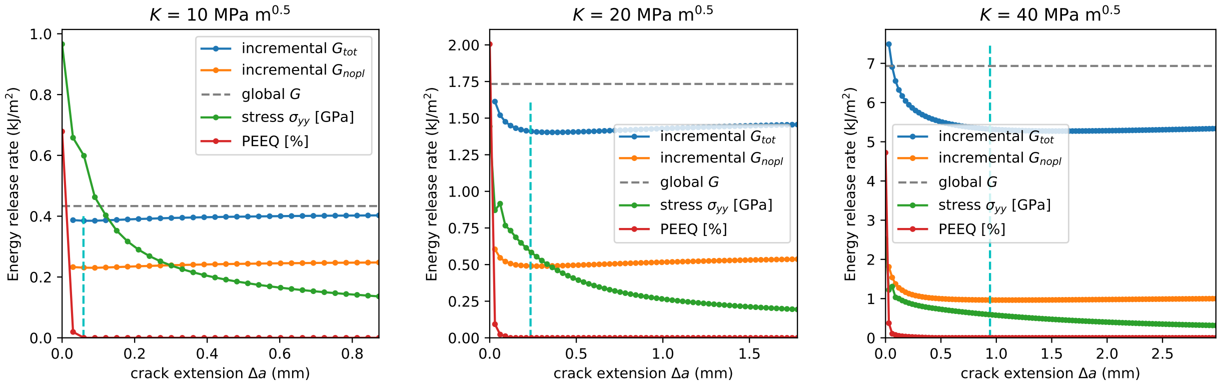


2.3 Plane stain cases

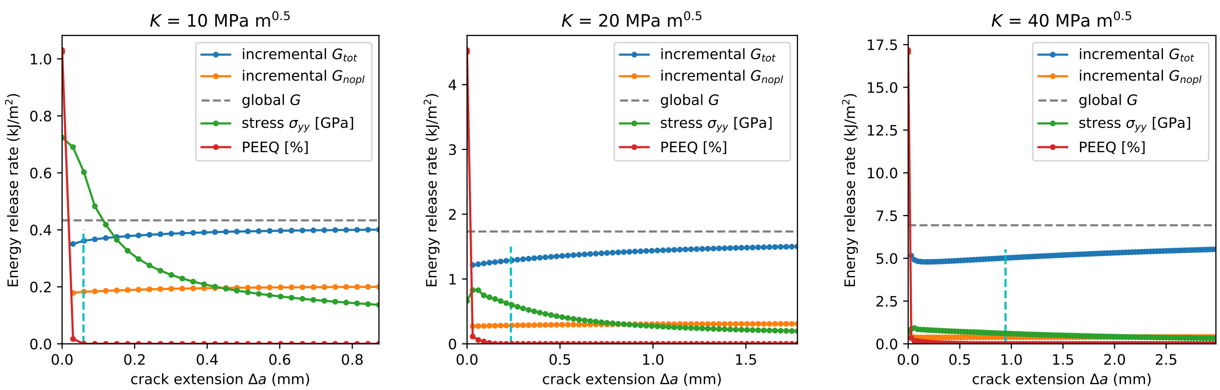
2.3.1 elastic-plastic material (σ_y= 300 MPa, ideal plastic)



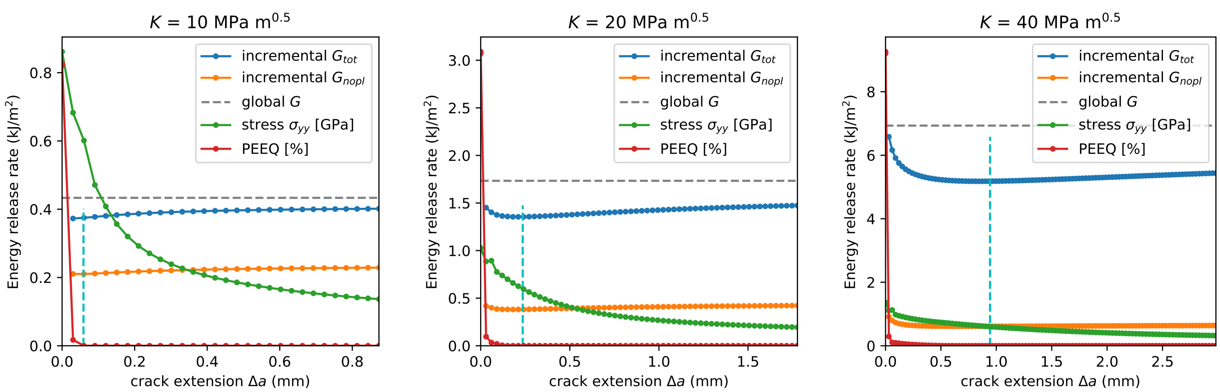
2.3.2 elastic-plastic material (σ_y= 300 MPa, $E_{pl} = E/10$, n = 1)



2.3.3 elastic-plastic material (σ_y= 300 MPa, $E_{pl} = E/20$, n = 3)



2.3.4 elastic-plastic material (σ_y= 300 MPa, $E_{pl} = E/100$, n = 0.7)



3 Next steps

- ☐ Is it just an energy-based problem because G_{inc} is usually highest close to crack tip?
- ☐ Should I check what diffrence *kinematic* instead of *isotropic* hardening would make?

☐ Should the local G be higher and lower than the globally applied G for plane stress and plane strain, respectively?

☐ Should the G without plastic dissipation be used for assessing crack initiation, but compared to G_c without its plastic contribution?

☐ Also evaluate *Configurational Forces* (CFs) [5] to also obtain the crack-driving force at the initial crack tip? Also evaluate CFs for the growing crack and evaluate CFs with and without taking away the plastic part of the strain energy?

Literature

[1] D. Leguillon, Strength or toughness? A criterion for crack onset at a notch, European Journal of Mechanics - A/Solids. 21 (2002) 61–72. https://doi.org/10.1016/S0997-7538(01)01184-6.

[2] T.L. Anderson, T.L. Anderson, Fracture mechanics: Fundamentals and applications, third edition, Taylor & Francis, 2005. https://doi.org/10.1201/9781420058215.

[3] G. Molnár, A. Doitrand, R. Estevez, A. Gravouil, Toughness or strength? Regularization in phase-field fracture explained by the coupled criterion, Theoretical and Applied Fracture Mechanics. 109 (2020) 102736. https://doi.org/https://doi.org/10.1016/j.tafmec.2020.102736.

[4] J.W. Hutchinson, D. tekniske højskole, A course on nonlinear fracture mechanics, Technical University of Denmark, 1979. https://books.google.at/books?id=UXG5nQEACAAJ.

[5] O. Kolednik, R. Schöngrundner, F.D. Fischer, A new view on j-integrals in elastic–plastic materials, Int J Fract. 187 (2014) 77–107. https://doi.org/10.1007/s10704-013-9920-6.