CH 3 Determinants

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0.1 Introduction to Determinants

0.1.1 Recursive Def of Determinant

for $n \geq 2$ the det of nxn is

$$\sum_{j=1}^{n} (-1)^{1+j} a_{1j} \text{det} A_{1j}$$

0.1.2 Fundamental Cofactor Expansion Theorem

det of nxn can be computed by cofactor expansion across any row or down any col.

$$\text{det}A = \cdots + a_{in}C_{in}$$
$$\text{det}A = \cdots + a_{nj}C_{nj}$$

(i, j)-cofactor depends on position of a_{ij} independent of sign of a_{ij}
SEE EXAMPLE 2
cofactor is just the subdeterminant in a 3x3 det (the coefficient of i, j, k, etc)

0.1.3 Diagonal Product Theorem

if A is a TMX (triangular) then det A is the product of the entries on the main diag of A

0.2 Properties of Determinants

0.2.1 Determinant Row Operations Theorem

let A be square
a. if multiple of one row of A is added to another row to produce a matrix B, then det B = det A
b. if two rows of A are swapped to produce B, then det B = - det A (remember properties of cross product)
c. if one row A is multiplied by k to produce B, then det B = k det A.
if A invertible, then

$$\det A = (-1)^r \cdot \text{(product of pivots in U)}$$

if A is not invertible then det A = 0
THEOREM: A SMX (square mx) is invertible IFF det $A = 0$
det A = 0 when rows of A are linearly dependent
SEE INVERTIBLE MATRIX THEOREM

0.2.2 Column Operations

CO have same affect of det as RO.

0.2.3 Transpose Determinant Theorem

if nxn MX then $\det A^T = \det A$

0.2.4 Multiplicative Property of Determinants Theorem

If A and B are nxn MX, then

$$\det AB = (\det A)(\det B)$$

*Think geometrically (two transforms)

0.2.5 Linearity Property of Determinant

det A is a linear function of a vector variable if the rest of the col vecs are fixed.
SEE 175

0.3 Cramer's Rule, Volume, and Linear Transformations

0.4 Cramer's Rule Theorem (Inefficient)

Let A be invertible nxn MX. For any $\boldsymbol{b} \in \mathbb{R}^n$, the unique solution $\boldsymbol{x}$ of $A\boldsymbol{x} = \boldsymbol{b}$ has the entries:

$$x_i = \frac{\det A_i(\boldsymbol{b})}{\det A}, i = 1, 2, ..., n$$

0.4.1 Application in Engineering

Cramer's Rule, LODE, Laplace Transforms

0.4.2 Inverse A Formula

Matrix of cofactors transposed is adjugate of A. (adjont)
Inverse Formula for A an invertible nxn:

$$A^{-1} = \frac{1}{\det A}\text{adj}A$$

0.5 Det as Area or Volume

area parallelogram determined by cols of A is det A
volume of parallelepiped determeined by cols of A is det A
See 3brown1blue.
EQUAL: $\boldsymbol{a_1}, \boldsymbol{a_2} \neq 0$ then for any c the area deteremined by a1,a2 is determined by a1 and $\boldsymbol{a_2} + c\boldsymbol{a_1}$
SEE FIG 2, FIG 4: col interchanges have no effect on volume.

0.5.1 Linear Transformations

Let $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ be the LT determined by 2x2 or 3x3 MX A. If S is a parallelogram/parallelepiped in $\mathbb{R}^2$ then

$$\{\text{area of} T(S)\} = |\det A| \cdot \{\text{area of} A\}$$
$$\{\text{vol of} T(S)\} = |\det A| \cdot \{\text{vol of} A\}$$