CH 8 - Induction

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1 Principle of Mathematical Induction

Suppose that for each positive integer n we have a statement $P(n)$. If we prove the following two things:
$(a)P(1)$ is true;
(b) for all n, if $P(n)$ is true then $P(n+1)$ is also true;
then $P(n)$ is true for all positive integers n

2 Principle of Mathematical Induction 2

Let k be an integer. Suppose that for each integer $n \geq k$ we have a statement $P(n)$. If we prove that following two things:
(a) $P(k)$ is true;
(b) for all $n \geq k$, if $P(n)$ is true then $P(n+1)$ is also true;
then $P(n)$ is true for all integers $n \geq k$.

3 Factorial

n factorial is defined as:
$n! = n(n-1)(n-2)...3 \cdot 2 \cdot 1$
$0! = 1$

4 Guesswork

Some problems have to use guesswork to identify a pattern first.

5 Summation Notation

If f1-n are numbers we abbreviate the sum of all of them by

$$f1 + ... + fn = \sum_{r=1}^{n} f_r$$

Some summation algebra:

$$\sum_{r=1}^{n}(af_r + bg_r + c) = a\sum_{r=1}^{n} f_r + b\sum_{r=1}^{n} g_r + cn$$

6 Geometric Examples

see later

7 Prime Factorization

Definition: a prime number is a positive integer p such that $p \geq 2$ and the only positive integers diving p are 1 and p.
Proposition 8.1 (Prime facorization):
Every positive int greater than 1 is equal to a product of prime numbers.

8 Principle of Strong Mathematical Induction

Suppose that for each integer $n \geq k$ we have a statement $P(n)$. If we prove the following two things:
(a) $P(k)$ is true;
(b) for all n, if $P(k), P(k+1), ..., P(n)$ are all true, then $P(n+1)$ is also true
then P(n) is true for all $n \geq k$.

9 Cauchy's Inequality: Proposition 8.2

Let n be a positive integer. Then for any real numbers a1-n and b1-n:

$$suma_1b_1toa_nb_n \leq \sqrt{a_1^2 + ... + a_n^2}\sqrt{b_1^2 + ... + b_n^2}$$