CH 5 Multivariate Probability Distribution

mrevanisworking

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0.1 Intro

multivariate probability distributions useful when need to check for the intersections of two or more events.

0.2 Bivariate and Multivariate Probability Distributions

think multiple dice

0.2.1 Definition of Joint or Bivariate Probability Fxn

Let Y1 and Y2 be DRV. The joint probability fxn (JPF, BPF) for Y1 and Y2:

$$p(y_1, y_2) = P(Y_1 = y_1, Y_2 = y_2), -\infty < y_1 < \infty, -\infty < y_2 < \infty$$

0.2.2 Properties of JPF

If Y1 and Y2 are DRV w/ JPF $p(y_1, y_2)$ 1. $p(y_1, y_2) \geq 0$ for all y1, y2
2. $\sum_{y_1, y_2} p(y_1, y_2) = 1$ where the sum is over all values (y_1, y_2) that are assigned NZ P
Sometimes called the Joint probability mass fxn because it sepcifies P (mass) associated w/ each of the possible pairs of values for RV.

0.2.3 Definition of Joint Distribution Fxn

Joint CDF
For any RV Y1, Y2 the joint (bivariate) distribution function is $F(y_1, y_2) = P(Y_1 \leq y_1, Y_2 \leq y_2)$, $-\infty < y_1 < inf, -\infty < y_2 < \infty$

0.2.4 Definition of Join PDF

Let Y1 and Y2 be CRV w/ JCDF $F(y_1, y_2)$. If there exists a nonnegative fxn $f(y_1, y_2)$such that

$$F(y_1, y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f(t_1, t_2) dt_2 dt_1$$

for all $-\infty < y_1 < \infty, -\infty < y_2 < \infty$, then Y1 and Y2 are said to be Jointly CRV. The fxn $f(y_1, y_2)$ is the joint PDF.

0.2.5 Properties of JCDF Theorem

If Y1 and Y2 are RV w/ JCDF $F(y_1, y_2)$, then: 1.$F(-\infty, -\infty) = F(-\infty, y_2) = F(y_1, -\infty) = 0$
2.$F(\infty, \infty) = 1$
3. If $y_1* \geq y_1$ and $y_2* \geq y_2$, then

$$F(y_1^*, y_2^*) - F(y_1^*, y_2) - F(y_1, y_2^*) + F(y_1, y_2) \geq 0$$

0.2.6 Properties of JPDF Theorem

If Y1 and Y2 are JCRV w/ JPDF $f(y_1, y_2)$, then 1.$f(y_1, y_2) \geq 0$ for all y1, y2
2.$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(y_1, y_2) dy_1 dy_2 = 1$
SEE FIG 5.2 on 252

0.3 Marginal an Conditional PD

distinct values assumed by DRV represent ME E_i. same for bivariate

0.3.1 Definition of Marginal P Fxn and MPDF

a. Let Y1 and Y2 be jointly DRV w/ PF:$p(y_1, y_2)$. Then the marginal probability functions of Y1 and Y2 are:

$$p_1(y_1) = \sum_{\text{all } y_2} p(y_1, y_2) \text{ and } p_2(y_2) = \sum_{\text{all}}^{y_2} p(y_1, y_2)$$

b. Let Y1 and Y2 be joinly CRV w/ join density function $f(y_1, y_2)$. Then the margina density function (MPDF):

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_2 \text{ and } f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_1$$

Marginal: the probabilities on the y_1 axis (margin).

0.3.2 Conditional Discrete P Fxn

If Y1 and Y2 are Joinly DRV w/ joint P Fxn $p(y_1, y_2)$ and the MDF $p_1(y_1), p_2(y_2)$, then the conditional discrete P fxn of Y1 given Y2 is

$$p(y_1|y_2) = P(Y_1 = y_1|Y_2 = y_2) = \frac{P(Y_1 = y_1|Y_2 = y_2)}{P(Y_2 = y_2)} = \frac{p(y_1, y_2)}{p_2(y_2)}$$

provided that $p_2(y_2)$ (undefined if p2y2 is 0)

0.3.3 Definition of Joint CRV Conditional CDF

IF Y1 and Y2 are jointly CRV w/ JPDF $f(y_1, y_2)$ then the CCDF of Y! give $Y_2 = y_2$ is

$$F(y_1|y_2) = P(Y_1 \leq y_1|Y_2 = y_2)$$

0.3.4 Definition of Conditional Density

Let Y1 and Y2 be JCRV w/ JPDF and marginal densities $f_1(y_1), f_2(y_2)$. For any $y_2$such that $f_2(y_2) > 0$, the conditional density of Y1 given $Y_2 = 2$

$$f(y_1|y_2) = \frac{f(y_1, y_2)}{f_2(y_2)}$$

and for any y_1 such that $f_1(y_1) > 0$, the conditional density of Y_2 given $Y_1 = y_1$

$$f(y_2|y_1) = \frac{f(y_1, y_2)}{f_1(y_1)}$$

undefined when the bottom is zero

0.4 Independent Random Variables

0.4.1 Definition of Independent RV

Let Y1 have a CDF $F_1(y_1), Y_2$have a CDF $F_2(y_2)$ and Y1 and Y2 have a JCDF $F(y_1, y_2)$. Then Y1 and Y2 are independent IFF

$$F(y_1, y_2) = F_1(y_1)F_2(y_2)$$

for every pair of real num (y_1, y_2) If Y1 and Y2 are not independent, they are dependent

0.4.2 Marginal Independence Theorem

If Y1 and Y2 are DRV w/ JPDF $p(y_1, y_2)$ and the MPDF $p_1(y_1), p_2(y_2)$, then Y1 and Y2 are independent IFF

$$p(y_1, y_2) = p_1(y_1)p_2(y_2)$$

for all pairs of real num (y_1, y_2)
If Y1 and Y2 are CRV w/ JPDF $p(y_1, y_2)$ and the MPDF $f_1(y_1), f_2(y_2)$, then Y1 and Y2 are independent IFF

$$f(y_1, y_2) = f_1(y_1)f_2(y_2)$$

for all pairs of real numbers y_1, y_2

0.4.3 JPDF Fxns Independent Theorem

Let Y1 and Y2 be a JPDF $f(y_1, y_2)$ that is positive IFF $a \leq y_1 \leq b, c \leq y_2 \leq d$, for abcd; and$f(y_1, y_2) = 0$ otherwise. Then Y1 and Y2 are IRV IFF

$$f(y_1, y_2) =; g(y_1)h(y_2)$$

where $g(y_1)$ is a NN fxn of y_1 alone and $h(y_2)$ is a NN fxn of y_2 alone
Key benefit of this theorem: do not need to derive the marginal densities

0.5 The Expected Value of a Fxn of RV

0.5.1 Definition of Expected Value of Fxn of RV

Let g(Y1toK) be a fxn of DRV, Y1tok, which have a P fxn p(y1tok). Then the EV of g(Y1tok) is

$$E[g(Y1tok)] = \sum_{\text{all}yk to 1} g(y1tok)p(y1tok)$$

If Y1tok are CRV w/ JPDF f(y1tok) then:

$$E[g(Y1tok)] = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(y1, y2, \ldots, y_k) \times f(y1tok)dy1t0k$$

0.6 Special Theorem

0.6.1 EV of a Constant Theorem

$$E(c) = c$$

0.6.2 Scalar Multiplication EV Theorem

Let g(Y1, Y2) be a fxn of the RV Y1 and Y2. then

$$E[cg(Y_1, Y_2)] = cE[g(Y_1, Y_2)]$$

0.6.3 Addition EV Theorem

Let Y1 and Y2 be RV and g1tok(Y1,Y2) be fxns of Y1 and Y2

$$E[g_{1tok}(Y_1, Y_2)] = E[g_{1tok}(Y_1, Y_2)]$$

0.6.4 Independent RV EV Fxn Theorem

Let Y1 and Y2 be independent RV and g(Y1) and h(Y2) be fxns

$$E[g(Y_1)h(Y_2)] = E[g(Y_1)]E[h(Y_2)]$$

provided expectations exist

0.7 The Covariance of Two Random Variables

Dependence has two measures: covariance and correlation
Covariance is the avg val of the EV fo the product of deviances

0.7.1 Definition of Covariance

If Y1 and Y2 are RV w/ means μ_1, μ_2, the covariance of Y1 and Y2

$$\text{Cov}(Y_1, Y_2) = E[(Y_1 - \mu_1)(Y_2 - \mu_2)]$$

larger the abs val of Cov, the greater linear dependence between Y1 and Y2.
Positive: Y1 increases as Y2 increases
Negative: Y1 decreases as Y2 increases
Zero: uncorrelated

0.7.2 Correlation Coefficient

$$\rho = (\text{Cov}\frac{Y_1, Y_2}{\sigma_1 \sigma_2})$$

Sign of the correlation coefficient is the same as the sign of the Cov

0.7.3 Computation of Cov Theorem

If Y1 and Y2 are RV w/ means μ_1, μ_2

$$\mathrm{Cov}(Y_1, Y_2) = E[(Y_1 - \sigma_1)(Y_2 - \sigma_2)] = E(Y_1Y_2) - E(Y_1)E(Y_2)$$

0.7.4 Independence Cov Theorem

IF Y1 and Y2 and independent RV

$$\mathrm{Cov}(Y_1, Y_2) = 0$$

Independent RV must be uncorrelated
CONVERSE: if the Cov is zero, the variables need not be independent

0.8 The EV and Variance of Linear Fxns of RV