Mathematical Interest Theory Study Guide

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Note many symbols here have a $ after them but is omitted for simplicity

1 Growth

Interest: if an investment amount K grows to S then the difference is $S - K$ interest (non negative)
investment opportunities theory: (economic productivity of capital) borrows then shares profit
Time preference theory: money now, rather than later
Excuse for interest: lender should be compensated for potential loss
principal: amount of money investor loans / is borrowed
Amount function: $A_K(t), \{t|t \geq 0\}$ for principal K. Where $A_K(0) = K$
Accumulation function: for principal of $1 $a(t), A_1(t)$ are standard to write. $a(0) = 1$
Typically $A_K(t) = Ka(t)$
amount of interest: if $t_2 > t_1 \geq 0$, then $A_K(t_2) - A_K(t_1)$ is the amount of interest if K between t12
effective interest rate: for t1 to t2

$$i_{[t_1,t_2]} = \frac{a(t_2) - a(t_1)}{a(t_1)}$$
$$i_{[t_1,t_2]} = \frac{A_K(t_2) - A_K(t_1)}{A_K(t_1)}$$

n-th time period: if n is pos int the interval $[n-1,n]$ and i_n is $i_{[n-1,n]}$

$$i_n = \frac{a(n)-a(n-1)}{a(n-1)}$$
$$a(n) = a(n-1)(1+i_n)$$

A fxn by simple interest rate s:

$$A_K(t) = K(1+st)$$
$$a(t) = 1+st$$

Simple interest rarely used for long duration loans since $\{i_n\}$ converges to 0 (book 16).
Exact Simple Interest (actual/actual): days of loan / days in year
Ordinary Simple Interest (30m/360):

$$d = 360(y_2-y_1)+30(m_2-m_1)+(d_2-d_1)$$

Compound Interest (usual): $i = i_1 = a(1)-1$
if $a(t)$ has period interest rates $i_n = i$, then for all non negative integers k (by induction)

$$a(k) = (1+i)^k$$
$$a(t) = (1+i)^t,\ t \geq 0$$

$a(s+t) = a(s)a(t)$
floor function: math.floor()
Fiscal policy: government's decisions with spending and taxation. spending decreases rates, taxation increases rates
Monetary policy: regulation of money supply and interest rates by banks (Federal Reserve: Federal Funds Rate)
prime rate: rate that bank charges to it's best customers
Amount of discount: KD where D is the discount rate. The borrower will have to pay KD to receive use of K.

$K - KD = (1 - D)K$
effective discount rate on interval:

$$d_{t_1,t_2} = \frac{a(t_2) - a(t_1)}{a(t_2)}$$
$$d_{t_1,t_2} = \frac{A_K(t_2) - A_K(t_1)}{A_K(t_2)}$$

d for nth time period:

$$d_n = \frac{a(n) - a(n-1)}{a(n)}$$
$$a(n-1) = a(n)(1 - d_n)$$

i and D are equivalent for $[t_1, t_2]$ if for each \$1 invested at t_1, wo rates produce the same accumulated value at t_2 or have the same accumulation function.
i and D equivalent iff: if L is loan amount and i interest rate:

$$L = L(1 + i_{[t_1,t_2]})(1 - d_{[t_1,t_2]})$$
$$1 = (1 + i_{[t_1,t_2]})(1 - d_{[t_1,t_2]})$$

so:

$$i_{[t_1,t_2]} = \frac{d_{[t_1,t_2]}}{1 - d_{[t_1,t_2]}}$$
$$d_{[t_1,t_2]} = \frac{i_{[t_1,t_2]}}{1 + i_{[t_1,t_2]}}$$
$$(1 + i_n)(1 - d_n) = 1$$
$$i_n = \frac{d_n}{1 - d_n}$$

Discount Function: $v(t) = \frac{1}{a(t)}$ where $v(t_0)$ is the money you must invest at t=0 to have 1 after t_0 years
invest at t_1 to have S at t_2 then $Sv(t_2)a(t_1) = S\frac{a(t_1)}{a(t_2)} = S\frac{v(t_2)}{v(t_1)}$
discount factor: $v = \frac{1}{1+i}$
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