CH 18 Equivalence Relations

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1 Relation

RELATION: Let S be a set, a relation on S is:
Choose a subset R of the Cartesian product $S \times S$, or R consist of some of the ordered pairs $(s,t), s,t \in S$. For those ordered pairs $(s,t) \in R$, we write $s\ t$ and say s is related to t. $(s,t) \notin R, s \nsim t$, is a relation on S
EQUIVALENCE RELATION: Let S be a set, and let be a relation on S. Then is an equivalence relation if for all $a, b, c \in S$:

$$
\begin{aligned}
(i) a\ a & \text{ this says } \quad \text{is reflexive} \\
(ii) & \text{ if } a\ b \text{ then } b\ a (thissays\ issymmetric) \\
(iii) & \text{ if } a\ b \text{ and } b\ c \text{ then } a\ c \text{ says } \quad \text{is transitive}
\end{aligned}
$$

2 Equivalence Classes

Let S be a set and an equivalence relation on S. For $a \in S$, define

$$cl(a) = \{s | s \in S, s\ a\}$$

Thus, $cl(a)$ is the set of things that are related to a. The subset $cl(a)$ is called an equivalence class of . The EC of are the subsets $cl(a)$ as a ranges over the ele of S.

3 Prop 18.1

Let S be a set and let be an ER on S. Then the EC of form a partition of S.