CH 6 Complex Numbers

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October 21, 2020

1 Intro

i is $i^2 = -1$.
A complex number is $z = a + bi$, where the real part is $a = Re(z)$ and the imaginary part is $b = Im(z)$
Addition and multiplication is defined for complex nums (39)

$$\frac{1-i}{1+i} = -i$$

$\mathbb{C}$ is the set of all complex numbers
$\mathbb{R} \subseteq C$
for quadratics:
if $b^2 \geq 4ac$ then $\in \mathbb{R}$,
if $b^2 \leq 4ac$ then $\in \mathbb{C}$

2 Geometry

Complex conjugate of $z = a + bi$ is defined as:
$\overline{z} = a - bi$ (reflected over real axis)
Modulus of z is distance from the origin to z (—z—) $z\overline{z} = |z|^2$
Argument of z is the angle θ
Polar form of z: $z = r(\cos\theta + i\sin\theta)$
where $a = r\cos\theta$ and $b = r\sin\theta$ and $|z| = r$
Principal argument is in $-\pi < \theta \leq \pi$ written as $arg(z)$, because multiples of 2pi are same angle
Ex $arg(1-i) = -\frac{\pi}{4}$

3 De Moivre's Theorem

Complex plane or argand diagram
If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and If $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ Then $z_1 z_2$ has mmodulus $r_1 r_2$ and arugment $\theta_1\theta_2$
Says: mult a C z by $\cos\theta + i\sin\theta$ ROTATES z counterclockwise through the angle θ
Ex: multipliciation by i rotates $\frac{pi}{2}$

3.1 Prop 6.1

Let $z = r(\cos\theta + i\sin\theta)$ and let n be a pos int:

$$(i) z^n = r^n(\cos n\theta + i\sin n\theta)$$
$$(ii) z^{-n} = r^{-n}(\cos n\theta - i\sin n\theta)$$

4 The $e^{i\theta}$ Notation

$e^{i\theta} = \cos\theta + i\sin\theta$
Common:

$$\begin{aligned} e^{2\pi i} &= 1 \\ e^{\pi i} &= -1 \\ e^{\frac{pi}{2} i} &= i \\ e^{\frac{pi}{4} i} &= \frac{1}{\sqrt{2}}(1+i)e^{i\theta} && = e^{i(\theta+2k\pi)} \end{aligned}$$

all $e^{i\theta}$ has modulus 1 (unit circle)
$z = re^{i\theta}$ where $r = |z|$ and $\theta = arg(z)$

$$e^{i\theta}e^{i\phi} = e^{i(\theta+\phi)}$$
$$(e^{i\theta})n = e^{in\theta}$$

5 Roots of Unity

cube roots of unity are each $\frac{2\pi}{3}$ away from eachother
nth Roots of unity: if n is a pos int, then the complex nums that satisfy the equation are:

$$z^n = 1$$

5.1 Prop 6.3

Let n be a pos int and $w := e^{\frac{2\pi i}{n}}$ Then the nth roots of unity are the n complex nums:

$$1, w, w^2, ..., w^{n-1}$$

and are evenly spaced around the unit circle