CH 16 Counting and Choosing

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1 Multiplication Principle

THEOREM 16.1 (Multiplication Principle): Let P be a process which consists of n stages, and suppose that for each r, the rth stage can be carried out in a_r ways. Then P can be carried out in $a1 - n$ ways.
PROPOSITION 16.1: Let S be a set consisting of n ele. Then the num of different arrangements of the elements of S in order is n!

$$S = a, b, c$$
$$abc, acb, bac, bca, cab, cba$$

2 Binomial Coefficients

DEFINITION: Let n be a positive int and r an int such that $0 \leq r \leq n$ Define

$$\binom{n}{r}$$

("n choose r") to be the num of r-element subsets of $1, , , n$
PROPOSITION 16.2:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

from proof:

$$n! = \binom{n}{r} \times r! \times (n-r)!$$

nC0 and nCn = 1 and nC1 = n.
nCr are binomial coefficients from:

3 Binomial Thoerem

THEOREM 16.2 (Binomial Theorem): Let n be a pos int, and let a, b be real nums. Then:

$$\begin{aligned}(a+b)^n &= \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r \\ &= a^n + na^{n-1}b + ...seeexpansion\end{aligned}$$

1. Each expression is symmetrical about the centre:

$$\binom{n}{r} = \binom{n}{n-r}$$

and from pascal's triangle:

$$\binom{n+1}{r} = \binom{n}{r} + \binom{n}{r-1}$$

PROPOSITION 16.3: For any pos int n

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$$

see 138.

4 Ordered Selections

PROPOSITION 16.4: Let S be a set of ele
(1) The num of ordered selections of r ele of S, allowing repetitions is equal to n^r
(2) The number of ordered selections of r distinct ele of S is equal to $n(n-1)\cdots(n-r+1)$
Written as $P(n,r) = \frac{n!}{(n-r)!} = r!\binom{n}{r}$

5 Multinomial Coefficients

DEFINITION (Ordered Partition): Let n be a pos int, and let $S = 1, , , n$. A partition of S is a collection of subsets S1-k such that each ele of S lies in exactly one of these

subsets. The partition is ordered if we take account of the order in which the subsets are written.
see 140, 141
PROPOSITION 16.5:

$$\binom{n}{r1-k} = \frac{n!}{r_1!r_2!...r_k!}$$

Mult princ: $n! = \binom{n}{r1-k} r_1!r_2!...r_k!$
These are called multinomial coefficients
THEOREM 16.3 (Multinomial Theorem): Let n be a pos int, and let x1-k be real numbers. Then the expansion of $(x_1 + \cdots + x_k)^n$ is the sum of all terms of:

$$\binom{n}{r1-k} x_1^{r_1}...x_k^{r_k}$$

where r1-k are nn int such that their sum = n.