CH 3 Determinants

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0.1 Vector Spaces and Subspaces

0.1.1 Vector Space Definition

A nonempty set V of vectors, where addition and scalar mult are defined by the 10 commandments and 3 more. SEE 192,193

0.1.2 Subspaces

See CH2 for DEF
subspace is guaranteed a vector space
subspace is used when at least 2 vector spaces are in mind

0.1.3 Vector Space, Span

Linear Combination: any sum of scalar multiples of vectors, Span is set of all vec that can be written as LC
if vectors are in a VS, then Span of vectors is a subspace of V.

0.2 Null Spaces, Column Spacess, and Linear Transformations

0.2.1 Null Space of Matrix

see CH2 notes
set of all $\boldsymbol{x} \in \mathbb{R}^n$ mapped to zero vector via LT

0.2.2 Null Space is Subspace Theorem

NS is subS of $\mathbb{R}^n$. Set of all Solutions to $A\boldsymbol{x} = \mathbf{0}$ of m HLE in n unknowns is a subS of $\mathbb{R}^n$

0.2.3 Explicit Description Nul A

solving $A\boldsymbol{x} = \mathbf{0}$ produces explicit description of Nul A
SEE EXAMPLE 3

0.2.4 Col Space of MX

Col$A = $ Span$\{\boldsymbol{a_1}, \ldots, \boldsymbol{a_n}\}$
Col space is a subS of $\mathbb{R}^n$

$$\text{Col}A = \{\boldsymbol{b} : \boldsymbol{b} = A\boldsymbol{x}\text{for some}\boldsymbol{x} \in \mathbb{R}^n\}$$

Col space is range of LT
CS of mxn A is all of $\mathbb{R}^m$ IFF the equation $A\boldsymbol{x} = \boldsymbol{b}$ has a sol for each $\boldsymbol{b} \in \mathbb{R}^m$

0.2.5 Contrast between Nul A and Col A

when MX not square, Nul A and Col A are separate.
when square, Nul A and Col A share ZV, and some others in special cases
SEE 206

0.2.6 Kernel and Range of LT

a LT from VS into another VS is a rule that assigns each $\boldsymbol{x}$ to a unique number $T(\boldsymbol{x})$ such that vector addition and scalar mult is defined.
Kernel (null space of LT)
Range (col space of LT)
see 3Blue1Brown
DIFFERENTIATION IS A LINEAR TRANSFORMATION see ex7

0.3 Linear Independent Sets; Bases

0.3.1 THEOREM 4: sec 1.7

0.3.2 Definition of Basis see previous ch

0.3.3 Spanning Set Theorem

S is a set of vectors in V, H is span of that set of vectors
a. if one vec in S is a LC of the remaining, then the set formed by S removing that vec still spans H
b. if $H \neq \{\mathbf{0}\}$, some subset of S is a basis of H
basis is a spanning set small as possible to be linearly independent

0.3.4 Bases for Nul A and Col A

base for Nul A is in 4.2 LIS example.
base of Col A SEE EX 8. NPC is an LC of the PC.
FACT: PC of A form a basis for Col A. (EFM)

0.4 Coordinate Systems

0.4.1 The Unique Representation Theorem

let B be the basis, then for each $\boldsymbol{x} \in V$ there exists a unique set of scalars c that

$$\boldsymbol{x} = c_1\boldsymbol{b_1} + \cdots + c_n\boldsymbol{b_n}$$

(Just think of basis vectors spanning around space)

0.4.2 Definition of Coordinates (see CH2)

coordinate mapping by B

0.4.3 Change of Coordinates Matrix

$$P_{\mathcal{B}} = \begin{bmatrix} \boldsymbol{b_1} & \cdots & \boldsymbol{b_n} \end{bmatrix}$$

and $\boldsymbol{x} = \cdots + c_n\boldsymbol{b_n}$

$$\boldsymbol{x} = P_{\mathcal{B}}\big[\boldsymbol{x}\big]_{\mathcal{B}}$$

where $P_{\mathcal{B}}$ is a change-of-coordinates MX from B to standard basis in $\mathbb{R}^n$

0.4.4 Coordinate mapping is 1-1 LT Theorem 8

B is basis for VS V, then the coordinate mapping

$$\boldsymbol{x} \mapsto [\boldsymbol{x}]_{\mathcal{B}}$$

is a 1-1 LT from V onto $\mathbb{R}^n$

0.5 Dimension of a VS

0.5.1 ¿n Basis set is Linearly Dependent Theorem

if basis B has n vectors, then any set in V that has more than n vectors is LD.

0.5.2 All bases of a VS Theorem

If a VS V has a basis of n vec, then every basis of V must have n vec

0.5.3 Definition of Dimension

if V is spanned by a finite set, then V is finite-dimensional, and dim V is the num of vec in basis for V. dim of 0 VS is zero. If V is not spanned by finite set, V is infinite-dimensional.

0.5.4 Subspaces of Finite-Dimensional Space Theorem(FDS)

SUBSPACE THEOREM 11: let H be subS of FDVS V. any LI set in H can be expanded to a basis for H, H is FD and dim $H \leq \dim V$

0.5.5 The Basis Theorem

see CH 2

0.5.6 Dimensions of Nul A and Col A

dim Nul A is num of FV in $A\boldsymbol{x} = \mathbf{0}$
dim Col A is num of PC in A

0.6 Rank

0.6.1 Row Space

set of all LC of row vec is row space RS of A, denoted by Row A. Row A is a subS of $\mathbb{R}^n$

0.6.2 Row Space Theorem

If two MX are RE, then RS is the same. If B is in EFM, the NZ rows of B form a basis for the RS of A as well as B.

0.6.3 Rank Theorem see CH2

SEE EX5

0.6.4 Rank, Invertible MX Theorem

all true/all false:
m. The columns of A form a basis of $\mathbb{R}^n$
n. Col$A = \mathbb{R}^n$
o. dim Col$A = n$
p. rank$A = n$
q. Nul$A = \{\mathbf{0}\}$
r. dim Nul$A = 0$ pg 254 for Row space of IMX Theorem

0.7 Change of Basis

0.7.1 Change of Coordinates MX from B to C Theorem

B is b basis and C is c basis of V, then there is unique nxn MX such that

$$[\boldsymbol{x}]_{\mathcal{C}} = P_{\mathcal{C}\leftarrow\mathcal{B}}[\boldsymbol{x}]_{\mathcal{B}}$$

SEE FIG2
inverse is b larrow c

0.7.2 Change of Basis in $\mathbb{R}^n$

B to the standard basis E is still B
SEE 243 for formula and notation $P_C^{-1}P_B = P_{C\leftarrow B}$

0.8 APPLICATIONS to Difference Equations

later

0.9 APPLICATIONS to Markov Chains

learn in stats