CH 7 Polynomial Equations

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1 Polynomial Equation

is in the form: $p(x) = 0$ where p(x) is a polynomial
Degree is the highest power of x with a NZ coefficient
1st degree root: $-\frac{b}{a}$
2nd degree root: quadratic formula
3rd degree root:

2 Solution of Cubic Equations

Consider $x^3 + ax^2 + bx + c = 0$
Get rid of x^2. By $y = x + \frac{a}{3}$.
Then $y^3 = (x + \frac{a}{3)})^3 = x^3 + ax^2 + \frac{a^2}{3}x + \frac{a^3}{27}$.
So it becomes $y^3 + b'y + c' = 0$ from some b' c' written as:
$y^3 + 3hy + k = 0$
Write $y = u + v$ Then:

$$\begin{aligned} y^3 = (u+v)^3 &= u^3 + v^3 + 3u^2v + 3uv^2 \\ &= u^3 + v^3 + 3uv(u+v) \\ &= u^3 + v^3 + 3uvyy^3 - 3uvy - (u^3 + v^3) && = 0 \end{aligned}$$

Which has $u + v$ as a root
Find u, v that:
$h = -uv, k = -(u^3 + v^3)$
$v^3 = -\frac{h^3}{u^3}, u^3 - \frac{h^3}{u^3} = -k$, so

$$u^6 + ku^3 - h^3 = 0$$

This is the quadratic equation

$$u^3 = \frac{1}{2}(-k + \sqrt{^2 + 4h^3})$$

$$v^3 = -k - u^3 = \frac{1}{2}(-k - \sqrt{kk^2 + 4h^3})$$

Plug back into $y = u + v$

$$\sqrt[3]{\frac{1}{2}(-k + \sqrt{k^2 + 4h^3})} + \sqrt[3]{\frac{1}{2}(-k - \sqrt{k^2 + 4h^3})}$$

3 Higher Degrees

There are no formulas for roots of ¿4th degree equations

4 The Fundamental Theorem of Algebra

Every polynomial equation of a degree at least 1 has a root $\in \mathbb{C}$
see proof CH 24

5 Theorem 7.2

Every polynomial of degree n factorizes as a product of linear polynomials and has exactly n roots $\in \mathbb{C}$ (counting repeats).

6 Theorem 7.3 (Complex Conjugates)

Every real polynomial factorizes as a product of real linear and real quadratic polynomials and has its non-real roots appearing in complex conjugate pairs
see 55/56 for proof

7 Relationships between Roots, Prop 7.1

Let the roots of:

$$x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$$

be $\alpha_1, \alpha_2, \ldots, \alpha_n$. If s_1 denotes the sum of the roots, s_2 denotes the sum of all the products of pairs of roots,...

$$\begin{aligned} s_n &= \alpha_1 \alpha_2 \ldots \alpha_n \\ &= (-1)^n a_0 \end{aligned}$$

see 56/57