Chapter 5

5 Integrals

5.1 Definition

Reinmann Sum

$$\int_a^b f(x)dx = \sum_{i=1}^{n} f(x_i^*)\Delta x \tag{5.1}$$

5.1.1 Testing Ground

:

$\int\int_S f(x,y)dA$

Chapter 15

15 Multiple Integrals

15.1 15.1,2,3 Double Integrals

$\sum_{i=1}^{n} f(x_i^*)\Delta x$

Double Integral of f over the rectangle R is

$$\int_R\int f(x,y)dA - \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A \tag{15.1}$$

if the limit exists.

...

15.1.1 Definition of Double Integral

$$\int_R\int f(x,y)dA = \lim_{(m,n)\to\infty}\sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^* y_{ij}^*)\Delta A \tag{15.2}$$

iltc double reinmann sum

precise definition

...

15.1.2 Over Rectangle

$$\iint_R f(x,y)dA \tag{15.3}$$

15.1.3 The Midpoint Rule

$$\iint_R f(x,y)dA \approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(xover, yover)\Delta A \tag{15.4}$$

15.1.4 Fubini's Theorem

$$\iint_R f(x,y)dA = \int_a^b \int_c^d f(x,y)dydx \int_c^d \int_a^b f(x,y)dxdy \tag{15.5}$$

$$\iint_R g(x)h(y)dA = \int_a^b g(x)dx \int_c^d h(y)dy \text{ where } R = [a,b]\times[c,d] \tag{15.6}$$

Average Value of a function $f_{ave} = \frac{1}{A(R)}\iint_R f(x,y)dA$

15.1.5 Double Integral over General Region D

$$\iint_D f(x,y)dA = \iint_R F(x,y)dA \tag{15.7}$$

Given D then

$$D = \{(x,y)|a \le x \le b, g_1(x) \le y \le g_x(x)\} \tag{15.8}$$

is

$$\iint_D f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(2)} f(x,y)dxdy \tag{15.9}$$

and the opposite for y Subregions can be fixed by solving in terms of a variable

$$\iint_D f(x,y)dA = \iint_{D_1} f(x,y)dA + \iint_{D_2} f(x,y)dA \tag{15.10}$$

Double Integral of 1 is the Area of D:

$$\iint_D 1dA = A(D) \tag{15.11}$$

15.1.6 Double Integrals in Polar Coordinates UNFIN

Polar Rectangle (disk or circle) $R = \{(r,\theta)|a \le r \le b, \alpha \le \theta \le \beta\}$ where $r^2 = x^2 + y^2, x = r\cos\theta, y = \sin\theta$

15.2 15.6,7,8 Triple Integrals

15.2.1 Definition

Triple integral of f over box B is (if the limit exists)

$$\iiint_B f(x,y,z)dV = \lim_{l,m,n\to\infty} \sum_{i=1}^{l}\sum_{j=1}^{m}\sum_{k=1}^{n} f(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*,)\Delta V \tag{15.12}$$

Fubini's Theorem for Triple Integrals on rect box $B = [a,b]\times[c,d]\times[r,s]$

$$\iiint_B f(x,y,z)dV = \int_r^s \int_c^d \int_a^b f(x,y,z)\,dx\,dy\,dz \tag{15.13}$$

General region E

$$\iiint_E f(x,y,z)dV = \iiint_B F(x,y,z)dV \tag{15.14}$$

Triple Integral bounded by regions

$$\iiint_B f(x,y,z)dV = \int_a^b \int_{g_1(x)}^{g_2(x)} \int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)\,dz\,dy\,dx \tag{15.15}$$

where terms are switched around when switching orders. Should draw two diagrams. (1) of solid region E and (2) of its projection D onto the xy plane.

3 types of regions. one depending on which dimension goes first

15.2.2 Cylindrical Coordinates

Represented by the ordered triple (r,θ,z) where r and θ are polar coords to P on x,y and z is distance from P.

$$x = r\cos\theta, y = r\sin\theta, z = z,,, r^2 = x^2+y^2, \tan\theta = \frac{y}{x}, z = z \tag{15.16}$$

Formula for triple integration in cylindrical coordiantes

$$\iiint_E f(x,y,z)dV = \int_\alpha^\beta \int_{h_1(x)}^{h_2(x)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)r\,dz\,dr\,d\theta \tag{15.17}$$

15.2.3 Spherical Coordinates

$$x = \rho \sin\phi cos\theta, y = \rho \sin\phi \sin\theta, z = \rho \cos\phi, , , \rho^2 = x^2 + y^2 + z^2 \tag{15.18}$$

the counterpart of a rectangular box is a spherical wedge.

B is the unit ball.

15.3 15.9 Change of Variables in Multiple Integrals

Transformation T from the uv-plane to xy-plane: $T(u, v) = (x, y)$ where $x = g(u, v), y = h(u, v)$ assuming that T is a C^1 transformation (g, h have continuous FOPD)

15.3.1 Vectors and Jacobian

Position vector of image

$$\vec{r}(u, v) = g(u, v)\hat{\imath} + h(u, v)\hat{\jmath} \tag{15.19}$$

The Jacobian

$$\frac{\partial(x, y)}{\partial(u, v)} = \left| \right| \tag{15.20}$$

15.3.2 Change of Variables Theorem

15.4 Applications

15.4.1

15.4.2 15.4, 15.5 Applications and Surface Area

15.4.3 15.6 applications of triple integrals

Special Case where $f(x, y, z) = 1$ for all E then the TI is the volume of E

$$V(E) = \iiint_E dV \tag{15.21}$$

For density function, the triple integral is mass. It's moments are the triple integral times a dimension...physics stuff.

There is also probability (joint desnity function)

$$P((X,Y,Z) \in E) = \iiint_E f(x,y,z)dV \tag{15.22}$$