CH 19 Functions

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1 Function

DEFINITION (Function): Let S and T be sets. A function from S to T is a rule that assigns to each $s \in S$ a single ele of T, denoted by $f(s)$. We write

$$f : S \to T$$

to mean that f is a funtion from S to T. If $f(s) = t$, we often say f sends $s \to t$.
DEFINITION (Image): iIf $f : S \to T$ is a function, the image of f is the set of all ele of T that are equal to $f(s)$ for some $s \in S$. We write $f(S)$ for the image of f. Thus

$$f(S) = f(s)|s \in S$$

example

2 Important Functions

(I) We say f is onto if the image $f(S) = T$; if for every $t \in T$ there exists $s \in S$ such that $f(s) = t$
(II) We say f is one-to-one if whenever $s_1, s_2 \in S$ with $s_1 \neq s_2$, then $f(s_1) \neq f(s_2)$; f is 1-1 if f sends different elements of S to different elements of T. Or for all $s_1, s_2 \in S$

$$f(s_1) = f(s_2) \Rightarrow s_1 = s_2$$

(III) We say that f is a bijection if f is both onto and 1-1
onto or surjective functions or surjections
1-1 or injective functions or injections
PROPOSITION 19.1: Let $f : S \to T$ be a function, where S and T are finite sets.
(i) If f is onto, then $|S| \geq |T|$.
(ii) If f is 1-1, then $|S| \leq |T|$.
(iii) If f is a bijection, then $|S| = |T|$.

3 Pigeonhole Principle

If we put n+1 or more pigeons into n pigeonholes, then there must be a pigeonhole containing more than one pigeon.

4 Inverse Functions

DEFINITION: Let $f : S \to T$ be a bijection. The inverse funciton of f is the function from $T \to S$ that sends each $t \in T$ to the unique $s \in S$ such that $f(s) = t$. We denote the inverse function by $f^{-1} : T \to S$. Thus, for $s \in S, t \in T$,

$$\begin{aligned} f^{-1}(t) = s \Leftrightarrow f(s) &= t \\ f^{-1}(f(s)) = s \text{ and } f(f^{-1}(t)) &= t \end{aligned}$$

5 Composition of Functions

DEFINITION: Let S, T, U be sets, and let $f : S \to T$ and $g : T \to U$ be functions. The composition of f and g is the function $g \circ f : S \to U$, which is defined:

$$(g \circ f)(s) = g(f(s)) \text{ for all } s \in S$$

Identity functions:

$$f^{-1} \circ f = \iota_S, f \circ f^{-1} = \iota_T$$

PROPOSITION 19.2: Let S, T, U be setse, and let $f : S \to T$ and $g : T \to U$ be functions. Then:
(i) if f and g are both 1-1, so is $g \circ f$
(ii) if f and g are both onto, so is $g \circ f$
(iii) if f ang g are both bijections, so is $g \circ f$.

6 Counting Functions

PROPOSITION 19.3: Let S, T be finite sets, with $|S| = m, |T| = n$. Then the num of functions from S to T is equal to n^m.