CH 2 Probability

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0.1 Intro

Probability: measure of one's belief in the occurence of a future event
Random events with a stable relative frequency are called RANDOM or STOCHASTIC.

0.2 Probability and Inference

see FIG 2.1 (Frequency Distribution)
probability of observed probabilities is important

0.3 Set Notation

captial letters for sets, $A = \{a_1, a_2, a_3\}$
S is a universal set if it denotes the set of all ele under consideration
A is a subset of B if every ele in A is also in B $A \subset B$
null or empty set is ENTER NULL SYMBOL HERE
union of A and B is the set of all points in A or B or both denoted by $AunionB$
intersection of A and B is the set of all points in A and B denoted by $AintersectionB$ or AB
if A is a subset of S, then the complement of A, $\overline{A}$ is the set of points that are in S but not in A.
$Aunion\overline{A} = S$
Two sets A,B are disjoint or mutually exclusive if $AintersectionB = null$ (no points in common)
A and $\overline{A}$ are mutually exclusive
See Diagrams

0.3.1 Most important set algebra

distributive laws

DeMorgan's laws:

0.4 The Discrete Case

0.4.1 Definition: Expiriment

Experiment: the proces by which an observation is made
Events: outcomes of an experiment (denoted by capital letters here)
Compound Event: an event that can be decomposed into other events
Simple Events: events that cannot be decomposed
Each point in a set is a sample point that points to an experiment

0.4.2 Simple Event

Simple event cannot be decomposed and corresponds to one and only one sample point. (denoted by E_x)

0.4.3 Sample Space

Sample Space: set consisting of all possible sample points for an experiement

0.4.4 Discrete Sample Space

DSS: contains either a finite or a countable number of ditinct sample points

0.4.5 Event

Event in a DSS S is a collection of sample points - any subset of S

0.4.6 Definition of Probability

S is a SS with an experiment. to every event A in S $P(A)$ is the probability of A so that the 3 axioms: 1. RF of occurence of any event must be $P(A) \geq 0$. negative doesn't make sense
2. RF of the whole SS S must be unity. $P(S) = 1$
3. if two events are ME(disjoint), the RF of their union is the sum of their RF.
or

if $A_1, A_2, A_3...$ form a sequence of pairwise ME events in S ($A_i \cap A_j = \emptyset$ if $i \neq j$), then

$$P(A_1 \cup A_2 \cup ...) = \sum_{i=1}^{\infty} P(A_i)$$

0.5 The Sample-Point Method

1. Define experiment and determine how to describe one simple event
2. List simple events in experiment and test that they can't be decomposed. This is SS S.
3. Assign resonable probabilities to SP in S, making certain of the AoP.
4. Define event of interest, A, as a specific collection of sample points. (all points where A occurs)
5. Find $P(A)$ by summing probabilities of SP in A.

0.6 Tools for Counting SP

$P(A) = n_a/N$if N equiprobable SP and A contains n_a SP

0.6.1 mn Rule Theorem

with m ele a1tom and n ele b1ton, it is possible to form $mn = m \times n$ pairs containing one ele from each group.
See FIG2.9, example 2.7
Follows with any number of sets (mnp)

0.6.2 Definition of a Permutation

an ordered arrangement of r distinct objects is a permutation. the num of ways of ordering n distinct object taken r at a time will be P_r^n

0.6.3 Permutation Theorem

made with ex2.7 and mn rule then divided by N SP

$$P_r^n = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

returns the num of SP

0.6.4 Partition Theorem

num of ways partitioning n distinct objects into k distinct groups containing n1tok objects, respectively, where each obj appearse in exactly one group and $\sum_{i=1}^{k} n_i = n$ is

$$N = \binom{n}{n1tok} = \frac{n}{n_1!n_2!\cdots n_k!}$$

the terms in the pmatrix are the multinomial coefficients bc they occur in the expansion of the multinomial y1tok to the nth power.

0.6.5 Definition of Combination

(Special case of Partitioning)
num of combinations of n objects taken r at a time is the num of subsets, each of size r, that can be formed from the n objects defined as C_r^n or $\binom{n}{r}$

0.6.6 Combination Theorem

num of unordered subsets of size r chosen (WITHOUT REPLACEMENT) from n available objects is

$$\binom{n}{r} = C_r^n = \frac{P_r^n}{r!} = \frac{n!}{r!(n-r)!}$$

$\binom{n}{r}$ is referred to as binomial coefficients because they occur in the binomial expansion of $(x+y)^n$

0.7 Conditional Probability and Independence of Events

0.7.1 Definition of Conditional Probability

conditional probability of an event A, given that an event B has occured is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

provided $P(B) > 0$
SEE FIG2.1 and Explanation of 2.9

0.7.2 Independence

A, B are independent if any one are true:
$P(A|B) = P(A)$
$P(B|A) = P(B)$
$P(A \cap B) = P(A)P(B)$ if none are true events are dependent

0.8 Two Laws of Probability

0.8.1 The Multiplicative Law of Probability

(derived from conditional probability) probability of intersection of A and B is

$$\begin{aligned} P(A \cap B) &= P(A)P(B|A) \\ &= P(B)P(A|B) \end{aligned}$$

if a and b are independent:

$$p(A \cap B) = P(A)P(B)$$

can be extended to intersection of any number of events

0.8.2 The Additive Law of Probability

the P of the union of two events A and B is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

if ME ($P(A \cap B) = 0$), then

$$P(A \cup B) = P(A) + P(B)$$

can be extended to more unions
"A or B"

0.8.3 Not Theorem

if A is an event:

$$P(A) = 1 - P(\overline{A})$$

0.9 The Event-Composition Method

1. Define experiment
2. Visualize nature of SP
3. Write equation expressing the event of interest as a composition of events (using unions, intersections, complements)
4. Apply two laws of probability to compositions to find $P(A)$
see sum of a geometric series

0.10 The Law of Total Probability and Bayes' Rule

give S as a union o ME subsets

0.10.1 Definition of Partition

for some positive int k, let the sets B1tok be: 1. $S = B_1 \cup B_2 \cdot B_k$
2. $B_i \cap B_j = \emptyset$), for $i \neq j$
then the collection of sets B1tok is a partition of S

0.10.2 The Law of Total Probability

assume B1tok is a partition of S, such that $P(B_i) > 0$ for i1tok. then for any A:

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

0.10.3 Baye's Rule

assume B1tok is a partition of S, such that $P(B_i) > 0$ for i1tok. then

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$$

from definition of conditional probability and law of total probability
see EX2.23 and FIG2.13

0.11 Numerical Events and Random Variables

events of major interest identified by num is numerical events (NE?)

0.11.1 Definition of a Random Variable

RV: a real-valued function for which the domain is a SS.
random variables have random values

0.12 Random Sampling

statistical experiment involves observation of a sample selected from a larger body of data called a population

0.12.1 Two Methods of Sample Selection

with replacement
without replacement
method of sampling: design of an experiment affects the information

0.12.2 Random Sampling

Let N and n represet num of ele in population and sample. If sampling is conducted so that each of $\binom{N}{n}$ samples has an equal P of being selected, the sampling is said to be random, and the result is a random sample.