CH 5 Eigenvectors and Eigenvalues

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0.1 Eiegenvectors and Eigenvalues

recheck EV and Evec. ****************

0.1.1 Definition

Eigenvector (Evec) of nxn A is NZ vec such that $A\boldsymbol{x} = \lambda\boldsymbol{x}$ for some λ.
Eigenvalue (EV) is that scalar λ of A if there is a nontrivial solution $\boldsymbol{x}$ of the equation.
or the equation

$$(A - \lambda I)\boldsymbol{x} = \mathbf{0}$$

$\boldsymbol{x}$ is the Evec corresponding to λ

0.1.2 Eigenspace

set of all solutions of that equation is the nullspace of $A - \lambda I$. The set is a subS of $\mathbb{R}^n$ called the eigenspace (ES) of A corresponding to λ.
has ZV and all EV of λ
See FIG3 (A acts on ES as dilation)

0.1.3 EV of Triangular Theorem

EV of triMX are the entries on its main diag.
0 is an EV of A IFF A is not invertible.

0.1.4 Linearly Independent EV Theorem

if v are EV corresponding to λ of nxn A, then the set v is LI.

0.1.5 EV and Difference Equations

SINCE APPLICATION later

0.2 The Characteristic Equation

0.2.1 Determinants SEE CH3

0.2.2 The Characteristic Equation

a scalar is an EV of nxn A IFF $\boldsymbol{\lambda}$ satisfies $\det(A - \lambda I) = 0$
solving gives a characteristic polynomial of A.
Some EV have multiplicities.
Complex roots have complex EV (CEV).

0.2.3 Similarity Theorem

if nxn A,B are similar, then they have same characteristic polynomial and the same EV with same multiplicites.
1. see warning
2. Similarity $\neq$ RE, RO usually changes EV.

0.2.4 APPLICATIONS to Dynamical Systems

later

0.3 Diagonalization

EV-Evec info can be displayed as $A = PDP^{-1}$ where D is DMX. SMX is diagonalizable if A is similar to a DMX.

0.3.1 The Diagonalization Theorem

nxn A is diagonalizable IFF A has n LD Evec. $A = PDP^{-1}$ where D is DMX, IFF the cols of P are n LI Evec of A. diag entries of D are Evec of A that correspond to the Evec in P.
MEANING: A is diagonalizable IFF enough Evec to form basis of $\mathbb{R}^n$ which is called the eigen vector basis of $\mathbb{R}^n$. (EvecB)

0.3.2 Diagonalizing Matricies

Find an IMX P, and DMX D to satisfy equation in Diagonalization Theorem.
1. Find the EV of A (characteristic equation)
2. Find n linearly independent Evec of A
3. Construct P from the vec in step 2
4. Construct D from the corresponding EV.

0.3.3 Diagonalizable Theorem

nxn MX w/ n distinct EV is diagonalizable.

0.3.4 Not Distinct EV Diagonalizable Theorem

a. for $1 \leq k \leq p$ the dim of ES for λ_k is less than or equal the multiplicity of the EV λ_k
b. A is diagonalizable IFF the sum of dim of ES equals n, that happens IFF: (i) characteristic polynomial factors into linear factors, (ii) the dim of ES for each λ_k equals the multiplicity of λ_k
c. if A is diagonalizable and $\mathcal{B}_{\|}$ is a basis for the ES wrt λ_k for each k, then total collection of vec in sets B 1-p forms an Evec basis for $\mathbb{R}^n$
SEE EX6

0.4 Eigenvectors and Linear Transformations

0.4.1 MX of LT

$$[T(\boldsymbol{x})]_C = M[\boldsymbol{x}]_B$$

Mis called the MX for T relative to the bases B and C
See FIG1, FIG2, EX 1

0.4.2 Linear Transformations from V into V

HELP, SEE EX2

0.4.3 Diagonal Matrix Representation Theorem

Suppose APDP (from above) where D is a diagonal nxx. If B is basis for $\mathbb{R}^n$ formed from col of P, then D is the B-MX for the transformation

$$\boldsymbol{x} \mapsto A\boldsymbol{x}$$

MEANING: describing same LT with different bases

0.4.4 Similarity of MX Representations

the set of all MX similar to A coincides with the set of all MX representations of the transformation

$$\boldsymbol{x} \mapsto A\boldsymbol{x}$$

0.5 Complex Eigenvalues

0.5.1 See Examples

complex Evec describe rotations.
See FIG1

0.5.2 Real and Imaginary Parts of Vectors

Re$\boldsymbol{x}$ is real part of complex conjugate,
Im$\boldsymbol{x}$ is imaginary part of complex conjugate,
Properties of complex conjugates carry over to complex matrix algebra. see 300.

0.5.3 Evec and EV of Real MX that acts on $\mathbb{C}^n$

$$A\overline{\boldsymbol{x}} = \overline{A\boldsymbol{x}} = \overline{\lambda\boldsymbol{x}} = \overline{\lambda}\overline{\boldsymbol{x}}$$

when A is real, its CEV occur in conjugate pairs.
See EX 6/FIG2/FIG3:
angle ϕ is the argument of $\lambda = a + bi$.
transformation $\boldsymbol{x} \mapsto C\boldsymbol{x}$ can be viewed as the composition of a rotation through ϕ and scaling by $|\lambda|$

0.5.4 APCP Theorem

let A be real 2x2 w/ CEvec $\lambda = a - bi(b \neq 0)$ and an associated Evec $\boldsymbol{v}$ in $\mathbb{C}^2$, then:

$$A = PCP^{-1}\,\text{where}P = [\text{Re}\boldsymbol{v}\;\text{Im}\boldsymbol{v}]\text{and}C = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

a plane is invariant under A if it is a rotated plane.

0.6 APPLICATIONS Discrete Dynamical Systems

later in round 2

0.7 APPLICATIONS to Differential Equations

later in differential equations

0.8 Iterative Estimated Eigenvalues

0.8.1 The Power Method

0.8.2 The Inverse Power Method