120AB Study Guide

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1 Notation

Note: finite or countably infinite is just countable (see infinities in proofsbook)
Notation: uppercase letter such as Y to denote an rv and a lowercase letter such as y to denote a particular value of that rv.
$(Y = y)$ is the set of all points in S assigned to the value y by rv Y.
$P(Y = y)$ is the probability that Y takes on the value y, defined as the sum of th proba of all sp in S that are assigned to value y. sometimes denoted by $p(y)$
...

2 Probability

DeMorgan's Laws: $\overline{A \cap B} = \overline{A} \cup \overline{B}$ and $\overline{A \cup B} = \overline{A} \cap \overline{B}$
Distributive laws:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Experiment: process by which observation is made
Event: E (simplest possible outcome)
SS (Sample Space): set of all possible sample points (sp) S of an experiment
dss (Discrete Sample Space): contains either a countable number of distinct sp
Mutually exclusive (ME) sets are ME events.
Compound Events: unions of sets of sp of simple events
A simple event E_i is included in event A iff A occurs whenever E_i occurs.
Event (dss): collection of sample points (any subset of S)

Relative Frequency Definition 2.6:
S is an SS with an experiment. To every event A in S a number P(A) the probability of A such that:

Axiom 1: $P(A) \geq 0$
Axiom 2: $P(S) = 1$
Axiom 3: If $A_{1-\ldots}$ form a sequence of pairwise ME events in S

$(A_i \cap A_j = \emptyset, if i \neq j) then$

$$P(A_1 \cap A_2 \cap A_3 \cup ...) j \sum_{i=1}^{\infty} P(A_i)$$

Sample-Point Method (2.5)
1.
2.
3.
4.
5.

Multiplication Principle (Fundamental Rule of Counting) (mn rule)
Permutations:

$$P_r^n = P(n,r) = seeproofsbook = \frac{n!}{(n-r)!}$$

Partitions: n objects into k groups containing n1-k objects where each object appears exactly in one group $\sum_{i=1}^{k} n_i = n$ is

$$N = \binom{n}{n1-k} = \frac{n!}{n1-k!}$$

see multinomials (2.6)
Combinations: n choose r

$$C_r^n = \binom{n}{r} = \frac{P_r^n}{r!} = \frac{n!}{r!(n-r)!}$$

Bayes' Theorem ()Conditional Probability (2.7)): probability of A given B

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Independent Events: iff

$$P(A|B) = P(A)$$
$$P(B|A) = P(B)$$
$$P(A \cap B) = P(A)P(B)$$

Multiplicative Law of Probability: intersection of events (and)

$$\begin{aligned} P(A \cup B) &= P(A)P(B|A) \\ &= P(B)P(A|B) \\ \text{if independent } P(A \cap B) &= P(A)P(B) \end{aligned}$$

Additive Law of Probability: The probability of the union of two events A and B is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

ME events $P(A \cup B) = P(A) + P(B)$

$P(A) = 1 - P(\overline{A})$
Event Composition:
1.
2.
3.
4.
(HELP) Law of Total Probability (2.10): For some pos int k, let the sets B_{1-k} be such that
1. $S = B_1 \cup .... \cup B_k$
2. $B_i \cap B_j = \emptyset$, for $i \neq j$.
Then the collection B_{1-k} is a partition of S.
such that $P(B_i) > 0$, for $i = 1 - k$ Then for any event A:

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

rv (Random Variable): real-valued function for which the domain is a SS.
Population: larger body of data where samples are taken from
(HELP) Replacement:
srs (simple random sampling): N and n is population and sample. A ssrs is each of the $\binom{N}{n}$ samples has an equal probability of being selected, the sampling is said to be random.

3 Discrete

Discrete - rv Y is discrete if can assume a countable number of distinct values
pd (Probability Distribution) - collection of probabilities
Probability Function for Y - $p(y)$
pd for drv Y that shows $p(y)$ for all y. Must be:
1. $0 \leq p(y) \leq 1$ for all y
2. $\sum_y p(y) = 1$, where summation is over all y with nonzero $p(y)$.

params (Parameters) - numerical descriptive measures for $p(y)$.
ev (Expected Value) for drv:

$$E(Y) = \sum_y yp(y)$$

if $p(y)$ is accurate of population frequency distribution then $E(Y) = \mu$ the population mean.
ev of g(Y) a real-valued funciton of Y:

$$E[g(Y)] = \sum_{\text{all } y} g(y)p(y)$$

variance of a drv Y with mean $E(Y) = \mu$: the ev of $(Y - \mu)^2$:

$$V(Y) = E[(Y - \mu)^2]$$

sd of drv Y: positive square root of V(Y)
if p(y) is accurate for population then $V(Y) = \sigma^2$, sd is σ.
Theorems (closed under addition and scalar multiplication):

$$E(c) = c$$
$$E[cg(Y)] = cE[g(Y)]$$
$$E[g_1(Y) + ... + g_k(Y)] = E[g_1(Y)] + ... + E(g_k(Y)])$$
$$V(Y) = \sigma^2 = E[(Y - \mu)^2] = E(Y^2) - \mu^2$$

3.1 binomial pd

...

3.2 geometric pd

...

4 Continuous

5 Multivariate

6 Functions

7 Sampling Distributions

8 Estimation

9 Estimators

10 Hypothesis Testing

11 2.3

11.1