Principal Axes and Principal Moments

Principal axis of inertia

u is a unit vector along a principal axis if **I . u** is parallel to **u**

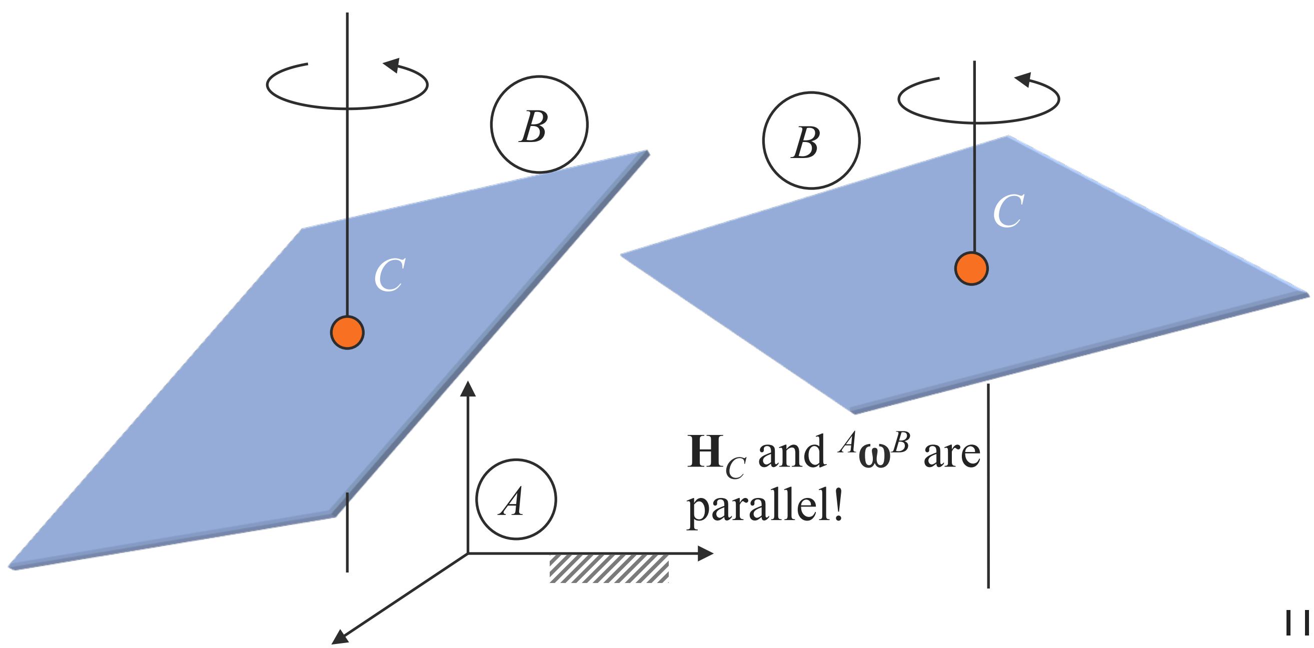
There are 3 independent principal axes!

Principal moment of inertia

The moment of inertia with respect to a principal axis, **u . I . u**, is called a principal moment of inertia.

Physical interpretation

$\mathbf{H}_C$ and ${}^A\boldsymbol{\omega}^B$ are not parallel!



$\mathbf{H}_C$ and ${}^A\boldsymbol{\omega}^B$ are parallel!

Euler's Equations

1

$$\frac{{}^{A}d\mathbf{H}_C}{dt} = \mathbf{M}_C$$

$$\begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} \begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \begin{array}{l} p \\ q \\ r \end{array} = \begin{bmatrix} M_{C,1} \\ M_{C,2} \\ M_{C,3} \end{bmatrix}$$

2

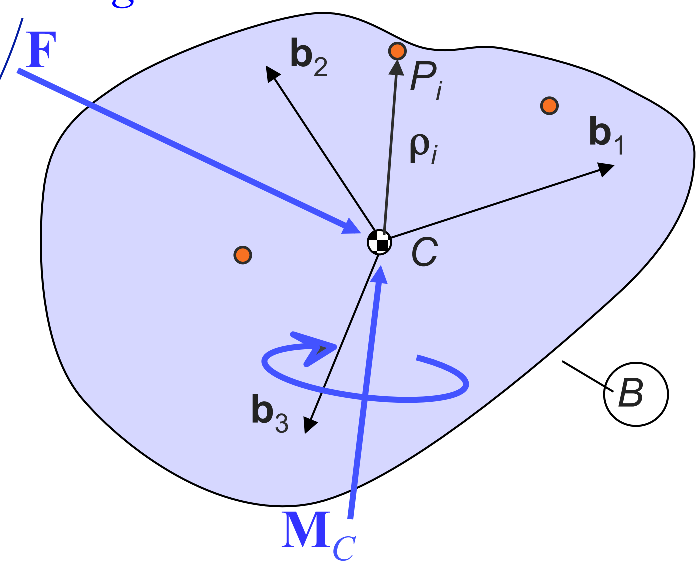
differentiating

Let $\mathbf{b}_1$, $\mathbf{b}_2$, $\mathbf{b}_3$, be along principal axes and

$${}^{A}\boldsymbol{\omega}^{B} = \omega_1\, \mathbf{b}_1 + \omega_2\, \mathbf{b}_2 + \omega_3 \mathbf{b}_3$$

$$\frac{{}^{B}d\mathbf{H}_C}{dt} + {}^{A}\omega^{B} \times \mathbf{H}_C = \mathbf{M}_C$$

$$\frac{{}^{B}d\mathbf{H}_C}{dt} = I_{11}\dot{\omega}_1\mathbf{b}_1 + I_{22}\dot{\omega}_2\mathbf{b}_2 + I_{33}\dot{\omega}_3\mathbf{b}_3$$



net external moment