Robotics

Estimation and Learning
with Dan Lee

Week 4.
Localization

4.1 Odometry Modeling
4.2 Sensor Registration
4.3 Particle Filter

Penn Engineering

Week 4.

Localization

4.1 Odometry Modeling

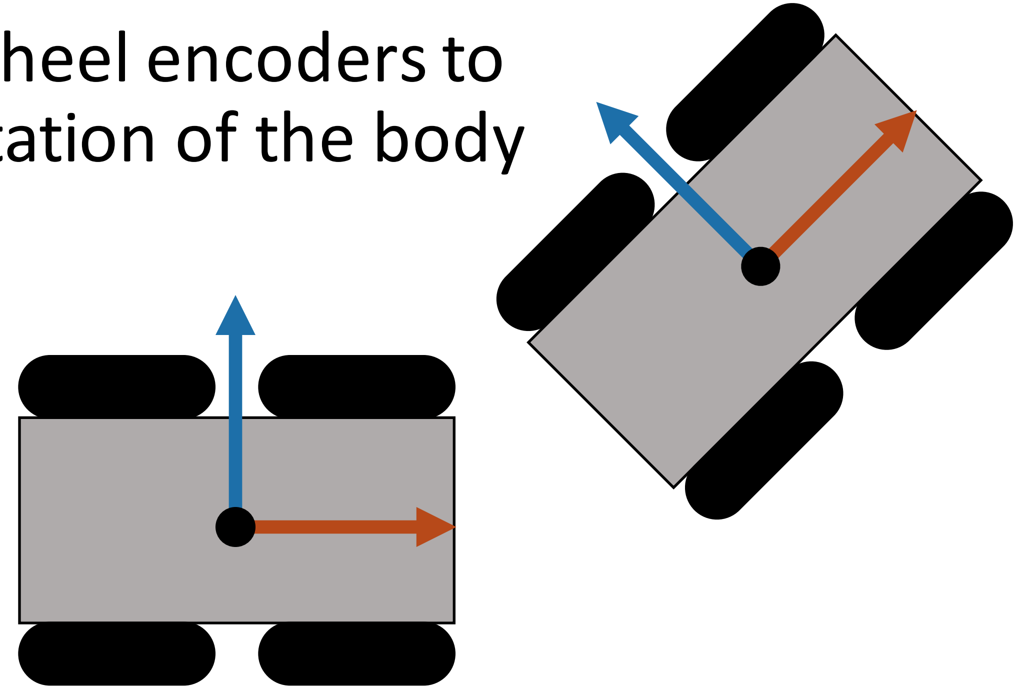
Localization

- Encoder and local sources of information can be very precise ~200,000 pulse/m [1]
- Laser range finding: 3-5 cm obstacle detection [2,3]
- RGB(-D) Vision: 1-10cm
- Gyroscope (turning)
  - Accelerometer integration provides poor accuracy
- Higher precision, but *local* information

[1] http://www.clearpathrobotics.com/husky-unmanned-ground-vehicle-robot/
[2] http://www.hokuyo-aut.jp/02sensor/07scanner/utm_30lx_ew.html
[3] http://velodynelidar.com/vlp-16-lite.html
[4] Robust Real-Time Visual Odometry for Dense RGB-D Mapping

Simple Approach

- Integrate odometry information
- Form a model of the vehicle
  - Skid steer, in this case
- Map ticks of the wheel encoders to translation and rotation of the body



Tracking Angular Movement

- Encoder ticks (e) are observed at the inner and outer radii

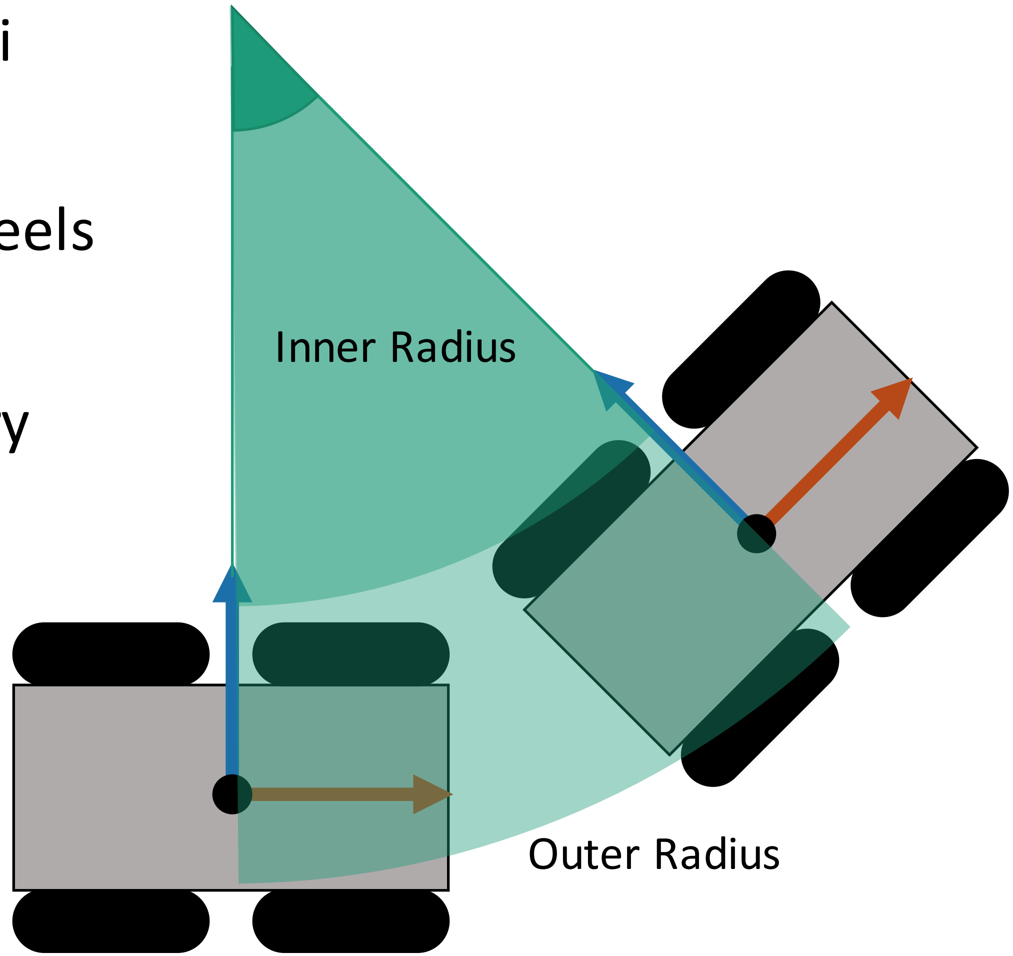
$$e_i = \theta r_i \quad e_o = \theta r_o$$

- Known width between wheels

$$\mathrm{w} = r_o - r_i$$

- Calculate angular odometry

$$\boxed{\theta = \frac{e_o - e_i}{w}}$$

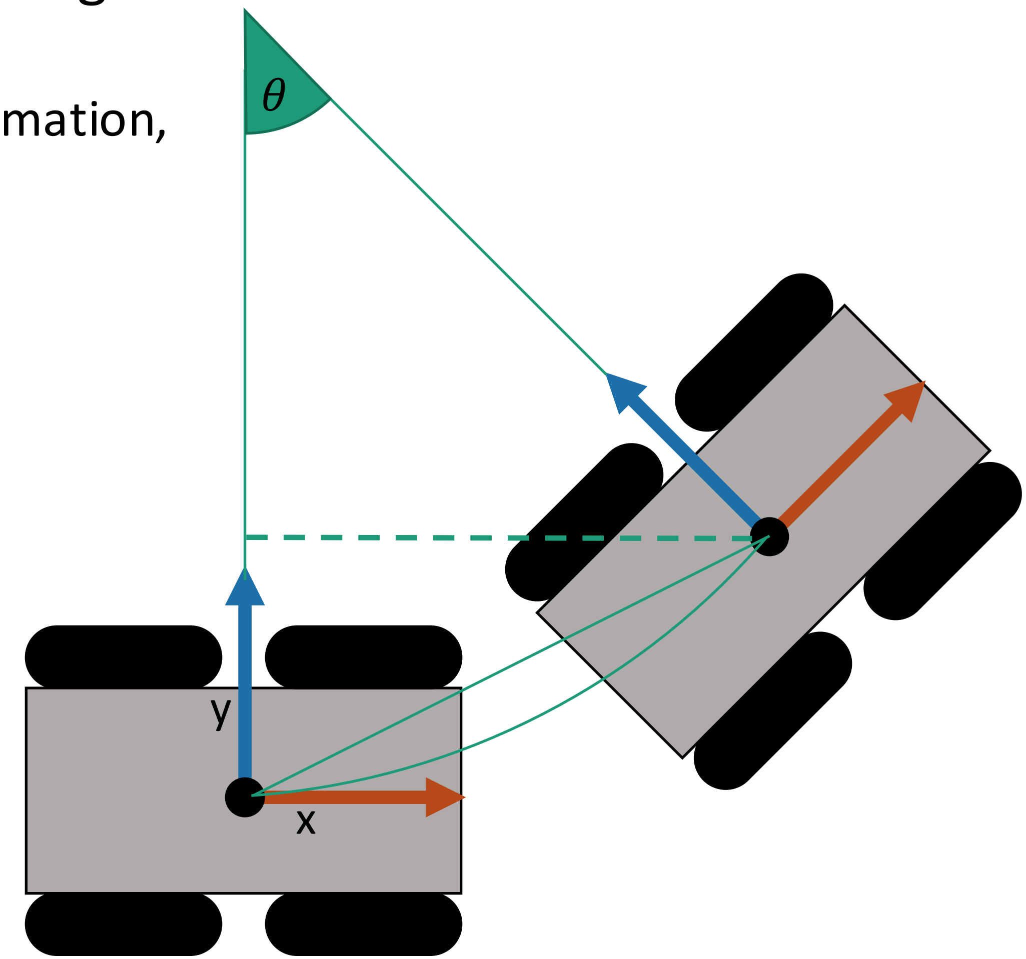


Tracking Translational Motion

- Translation requires knowledge of the angular movement
  - Use circular sector approximation, valid for small movements
- *Quiz: Spinning in Place*

$$\boxed{y = \frac{e_o + e_i}{2}\cos\theta}$$

$$\boxed{x = \frac{e_o + e_i}{2}\sin\theta}$$



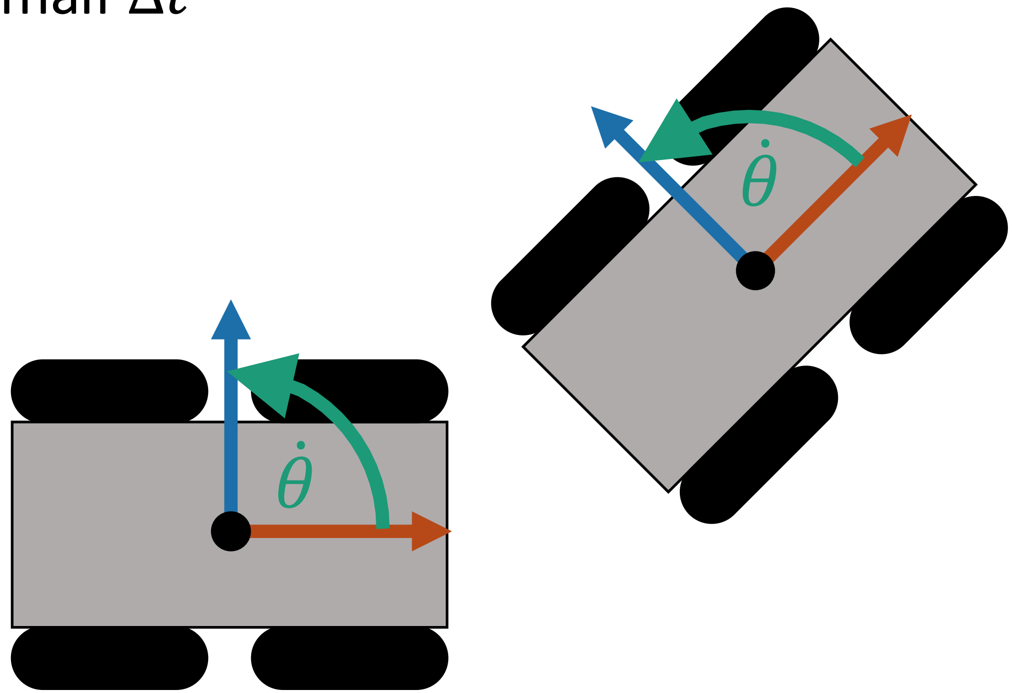
Aiding Pure Encoder Odometry

- Encoder measurements can be noisy
- Angular estimate feeds into translation, propagating error
- *Solution:* Utilize more precise gyroscope for angular change
- Gyroscope is accurate for small Δt

$$\boxed{\theta = \dot{\theta}\Delta t}$$

$$\boxed{y = \frac{e_o + e_i}{2}\cos\theta}$$

$$\boxed{x = \frac{e_o + e_i}{2}\sin\theta}$$



Simple Approach Characteristics

- Local frame of reference of the robot starting point
- *Issue:* Encoders suffer from slippage, missing counts
- *Issue:* Gyroscope integration suffers from drift
- Utilizing maps of the world can correct localization errors

Robotics

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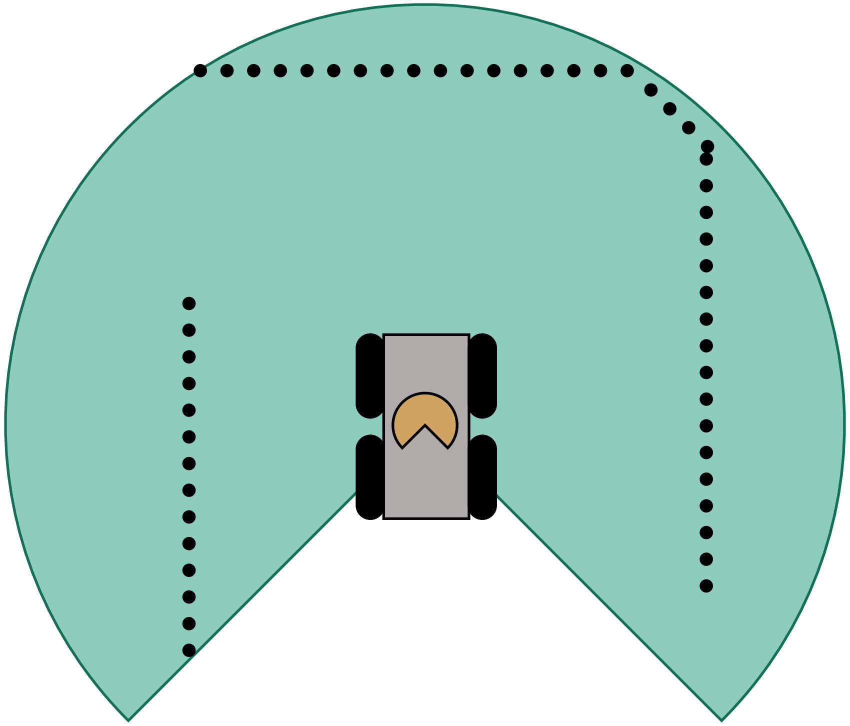
Week 4.
Localization

4.2 Map Registration

Penn Engineering

LIDAR Depth Sensor

- Depth measurements made in polar coordinates
- Continuous readings, r, at discrete angles, θ



LIDAR Depth Sensor

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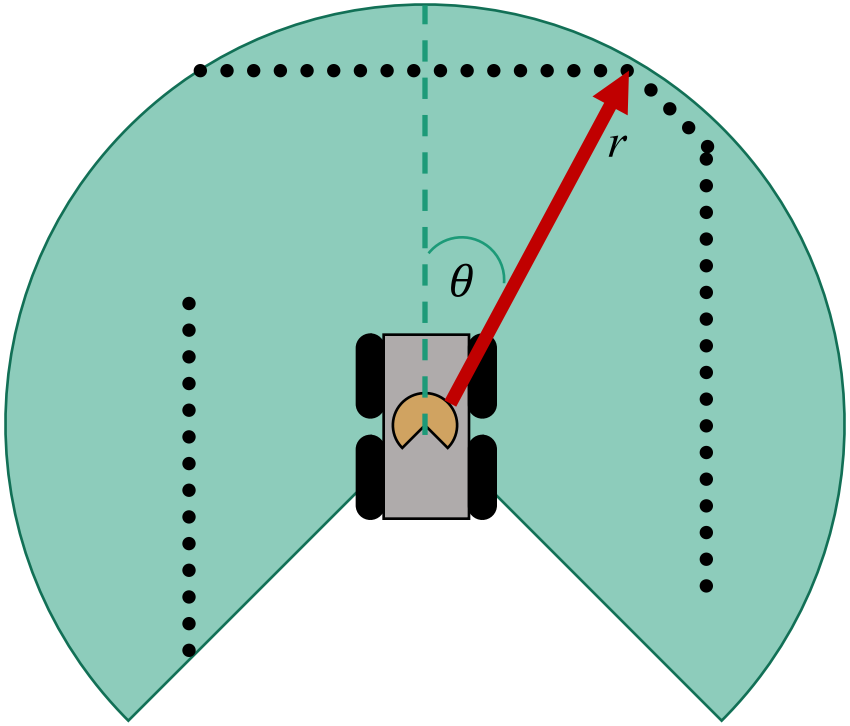

LIDAR Depth Sensor

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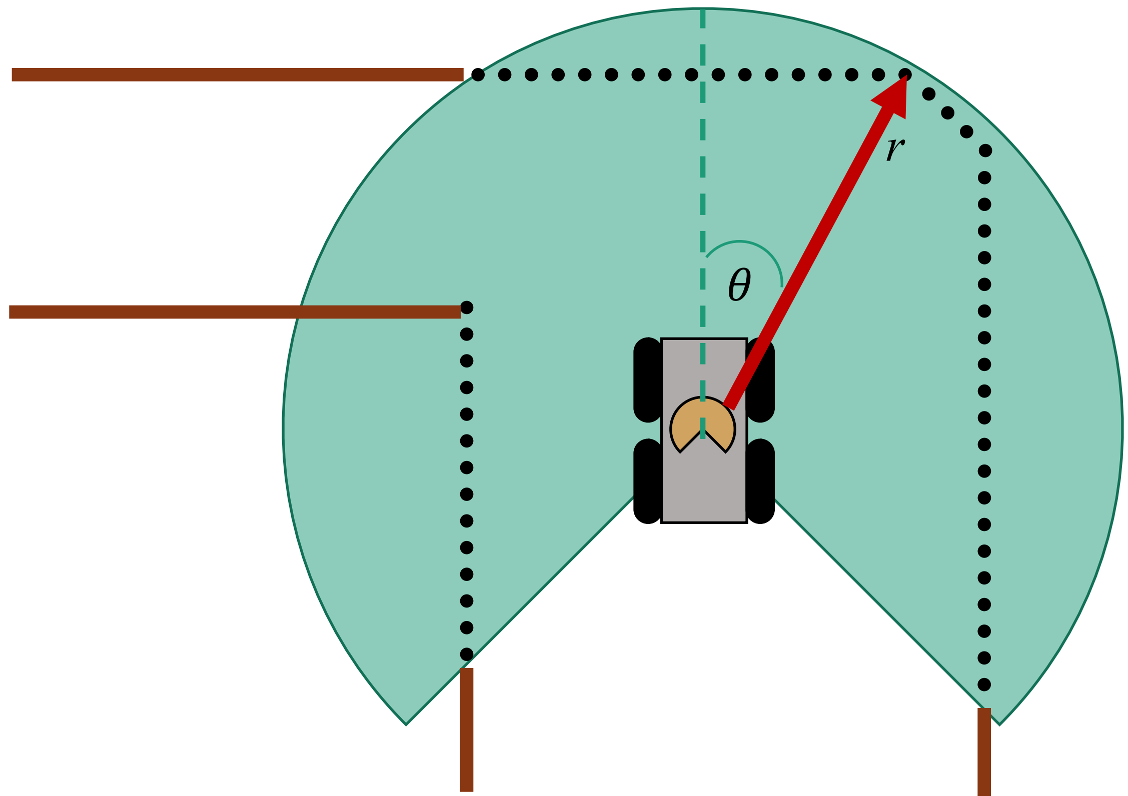
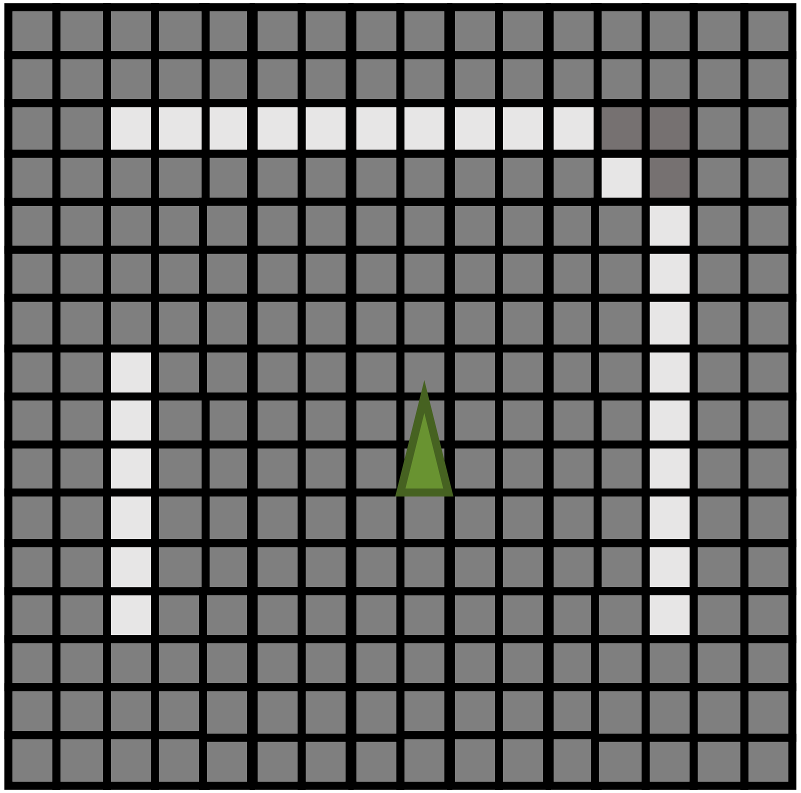


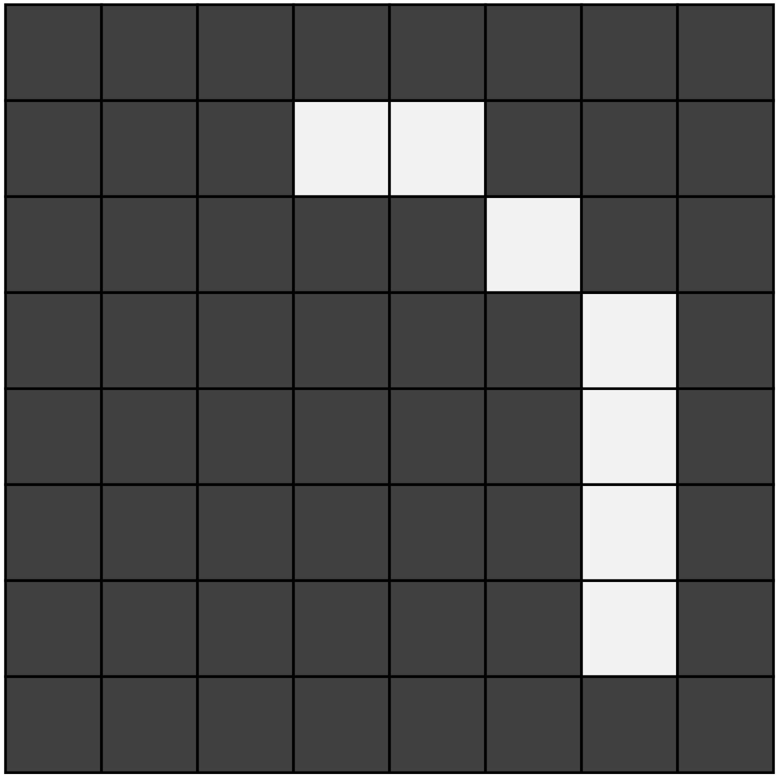
Map Representation

- Discrete Grid representing 2D space *(see Week 3)*
- White cells represent the presence of an obstacle

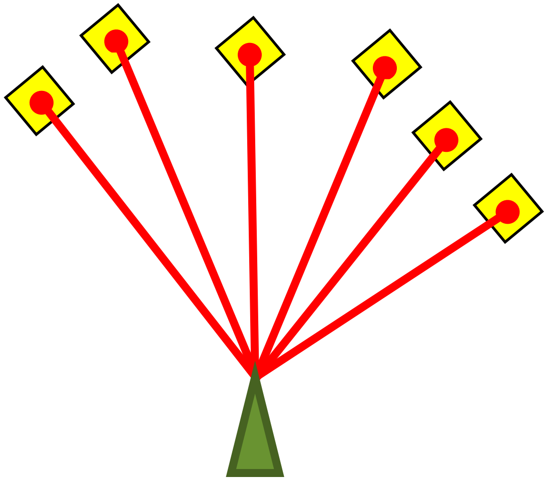


Map Measurements

Map

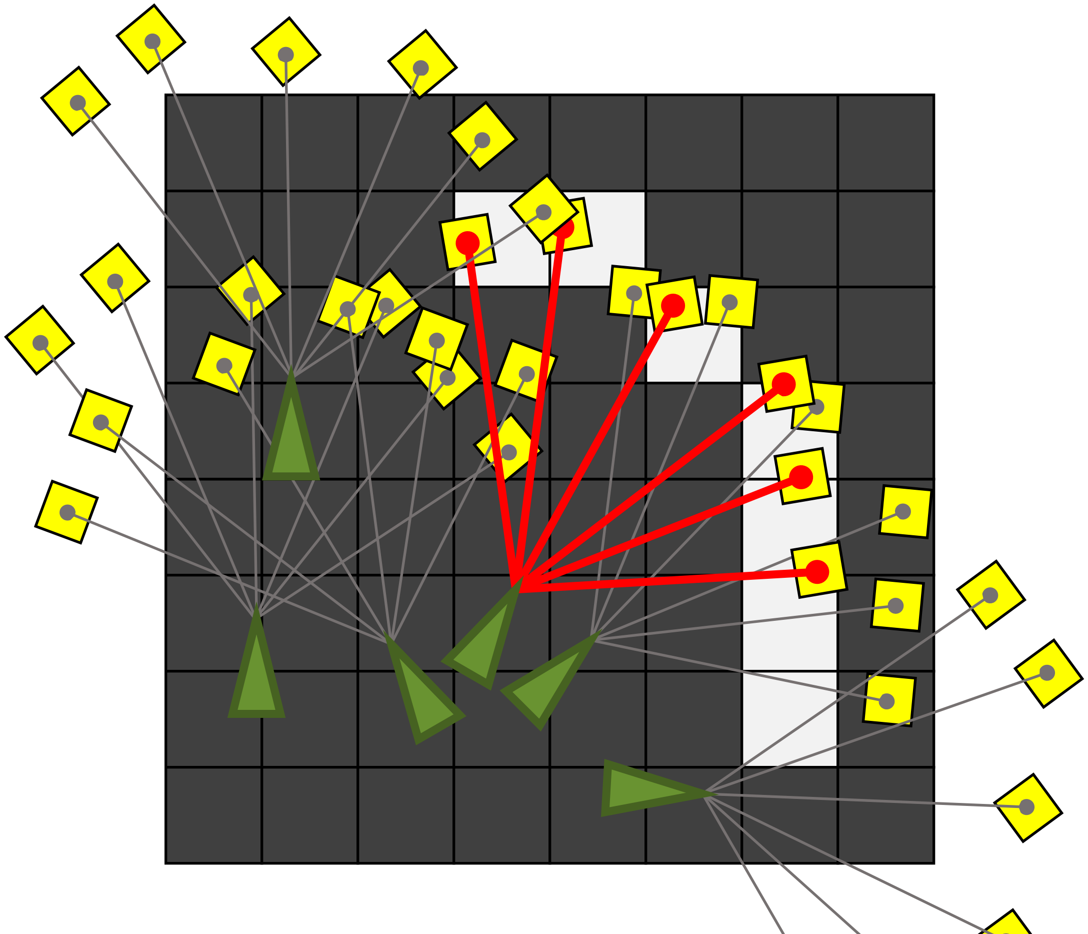


Measurements



Map Registration

- Find robot pose that best explains the measurements



Map Registration

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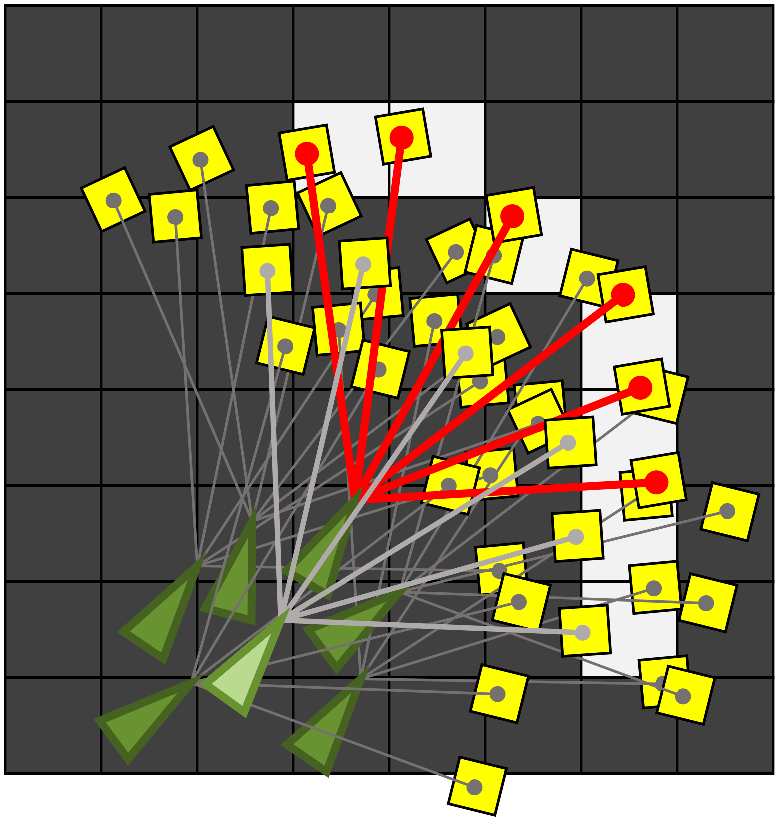
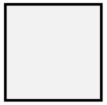
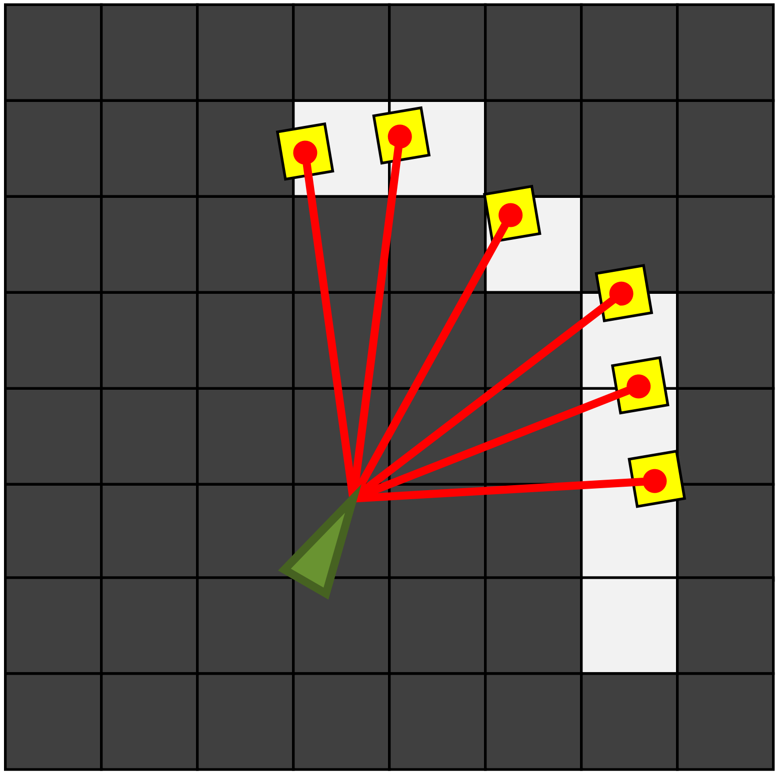


Map Registration

- Correlate laser obstacles with map obstacles 
- Correlate laser free space with map free space 

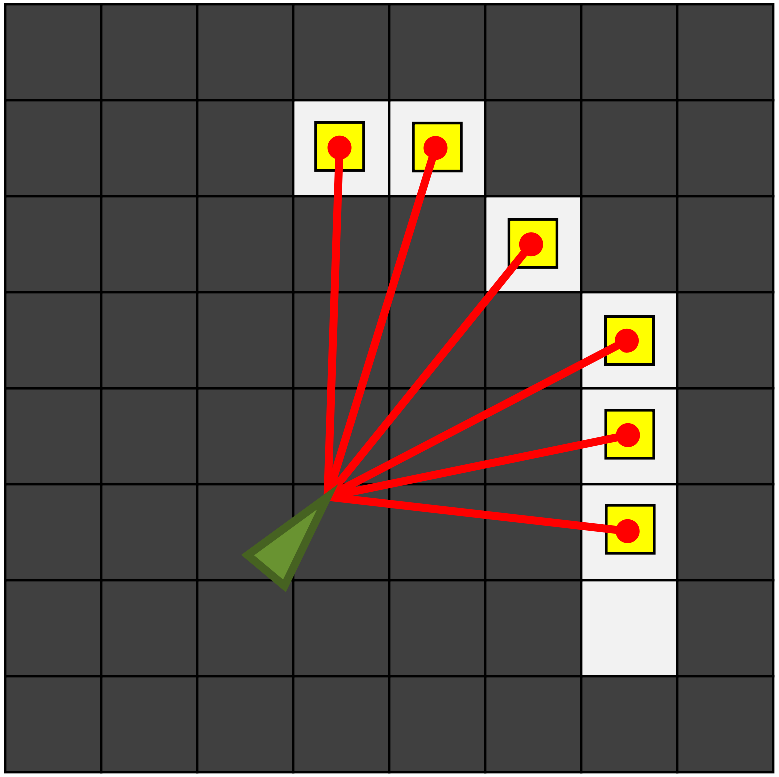


$$\sum_r \delta\left(p_x + r\cos(p_\theta + r_\theta)\,, p_y + r\sin(p_\theta + r_\theta)\right) \cdot m(x, y)$$

Map Registration

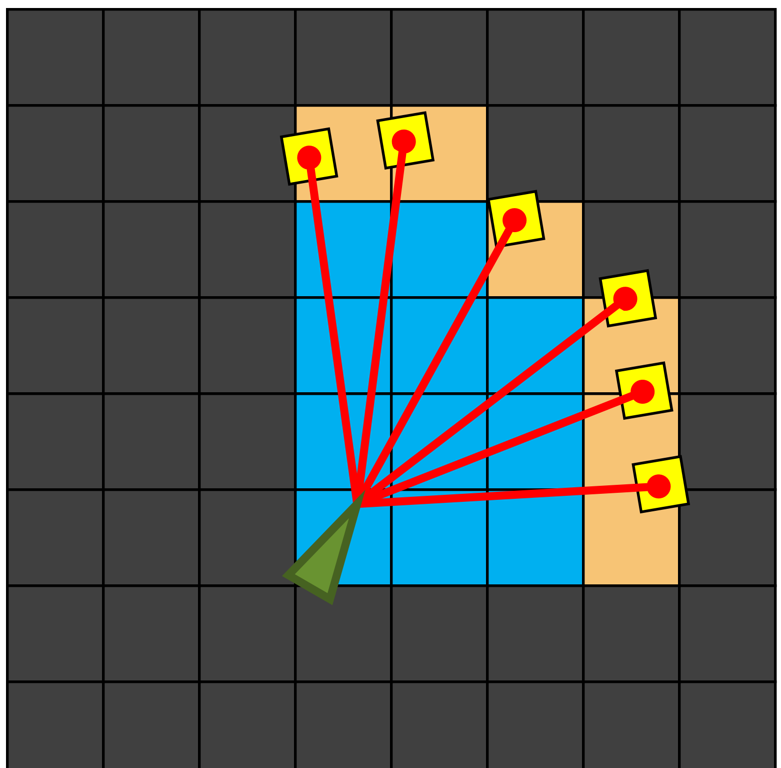
Find the best pose

$$\max_{p} \sum_{r} \delta\big(p_x + r\cos(p_\theta + r_\theta)\,, p_y + r\sin(p_\theta + r_\theta)\big) \cdot m(x, y)$$



Map Registration

- Laser scans penetrate free space
- Laser scans return distance to obstacle



Map Registration

- Adding pose uncertainty

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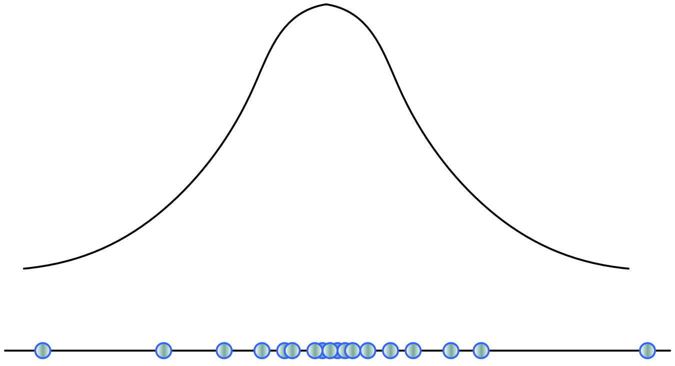
Localization

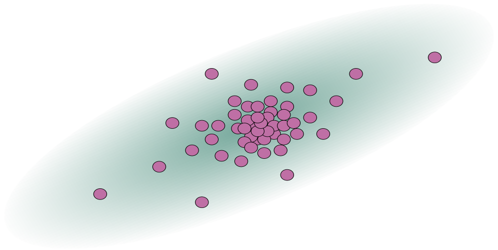
4.3 Particle Filter

Penn Engineering

Particle Filter

- Samples approximate a probability distribution
- Fast and efficient non-parametric model
- Ability to represent multimodal distributions
  - Mixtures of Gaussians, multi-hypothesis Kalman Filter



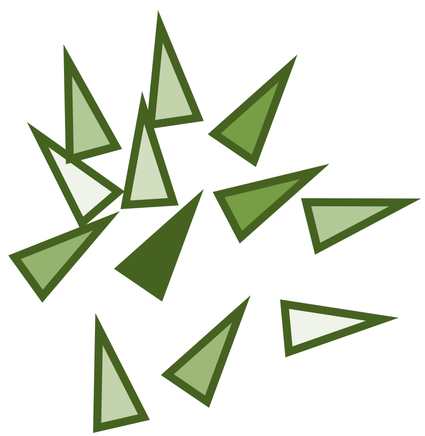


Particle Filter

- Dirac Delta function
  - Sigma is going to zero, Gaussian distribution
- Particle Filter : Limit of Gaussian mixtures when $\sigma \to 0$ (variances shrink to zero)

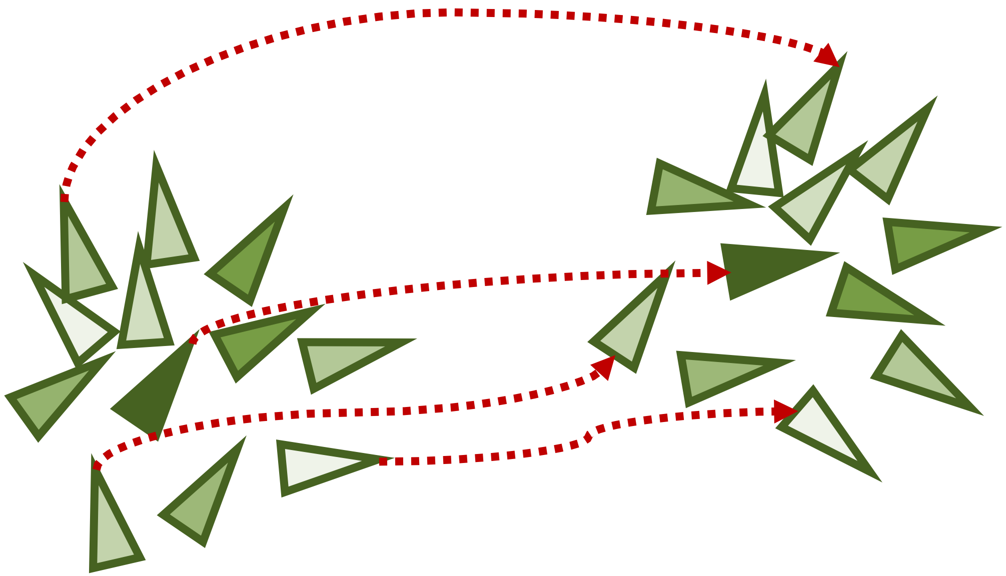
Initial Population

- Initial group of particles represents the underlying distribution of the belief state
- Particle is comprised of (pose, weight)
- Here, darker colors represents a higher *weight*
  - Represents probability, such that *weight = prob(pose)*



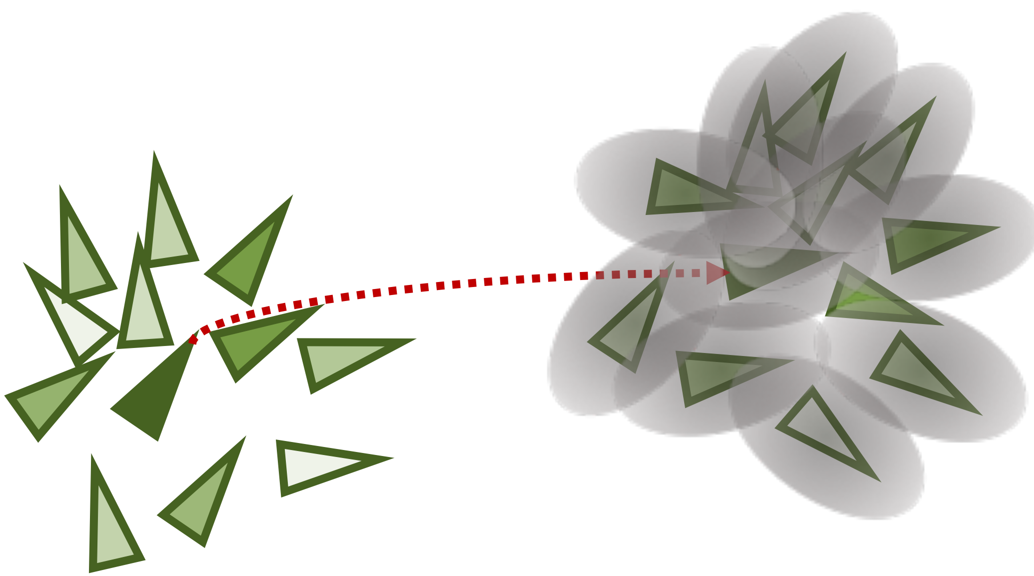
Odometry Update

- Move the particles based on odometry information
- Each particle represents a possible pose, so individually must be moved via its local frame



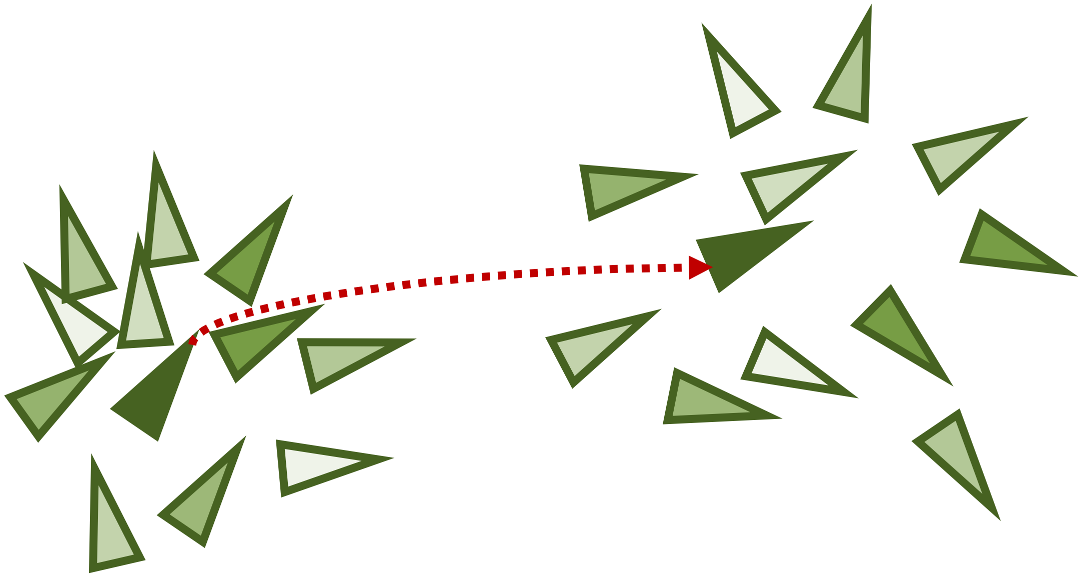
Odometry Update

- Include odometry noise model
- Sampled for each particle from the odometry noise distribution $p_i' = p_i + \mathcal{N}(0, \Sigma)$



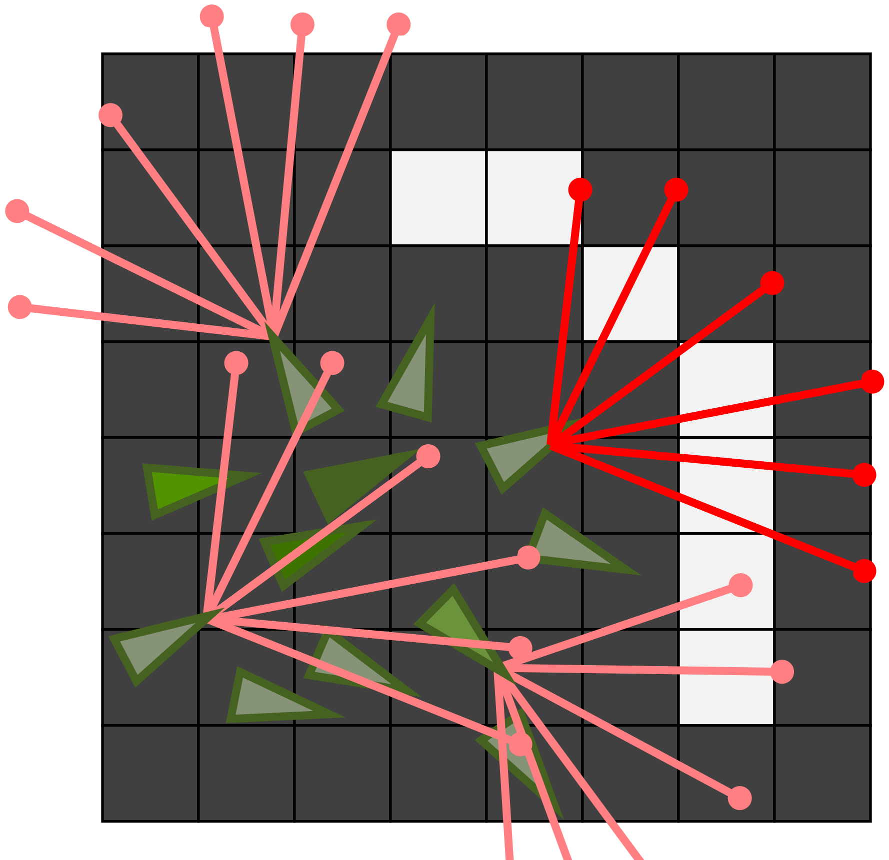
Odometry Update

- Dispersion of particles represents the added uncertainty from moving



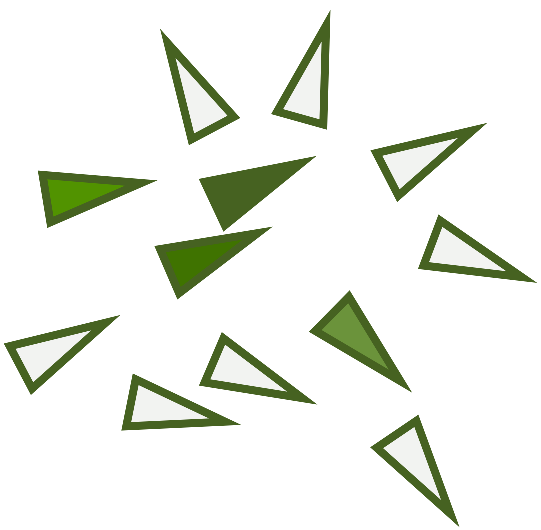
Correlation Update

- The weights of the particles can be updated based on LIDAR correlation data, $w_i' = w_i \cdot corr(p_i)$



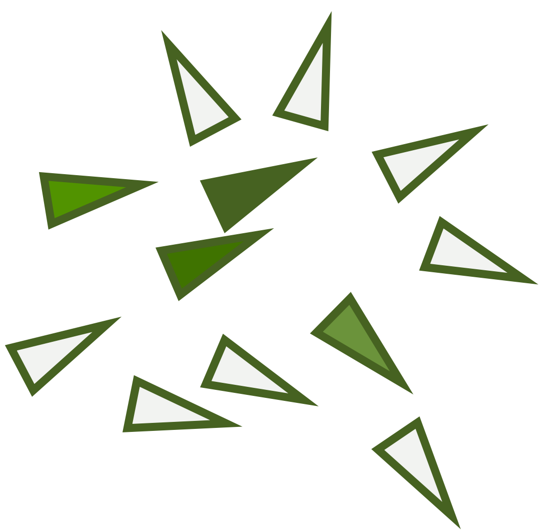
Correlation Update

- The new set of particles capture the distribution after odometry and sensor measurement
- However, this may not be the optimal set to represent the distribution



Particle Resampling

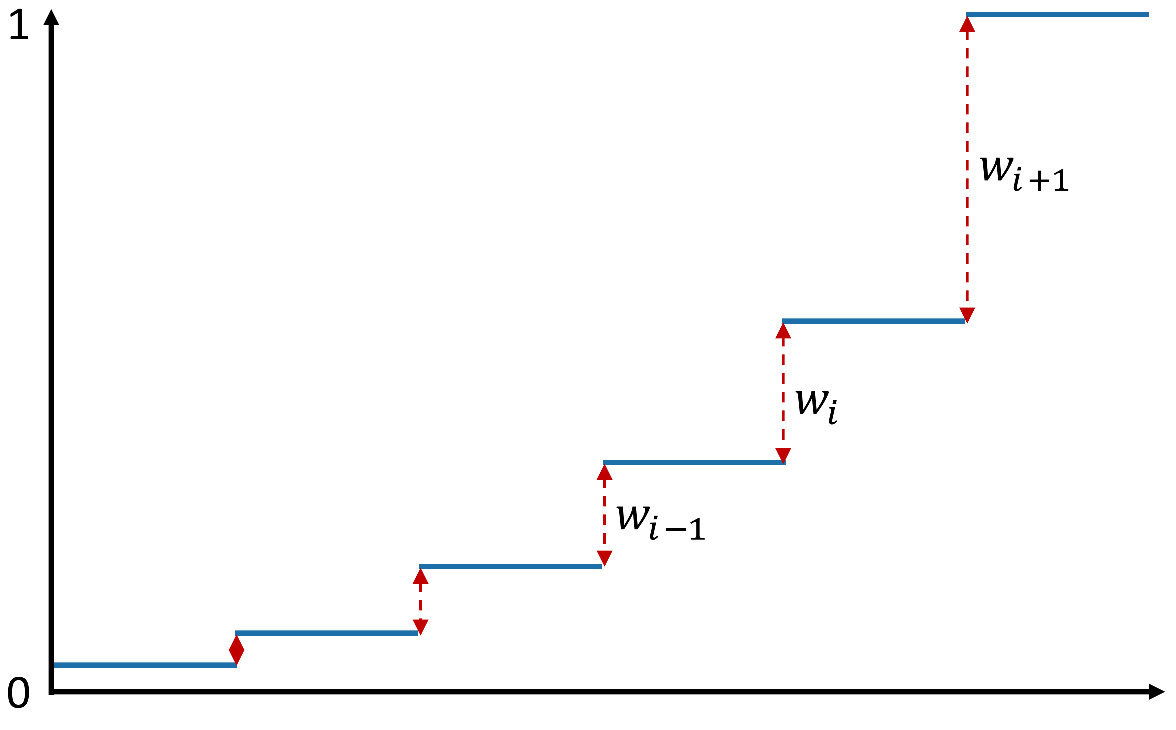
- Check a resampling criterion – the number of effective particles
- If the number of effective particles is too low, then *resample* to increase the effective number



$$n_{effective} = \frac{\left(\sum_i w_i\right)^2}{\sum_i w_i^2}$$

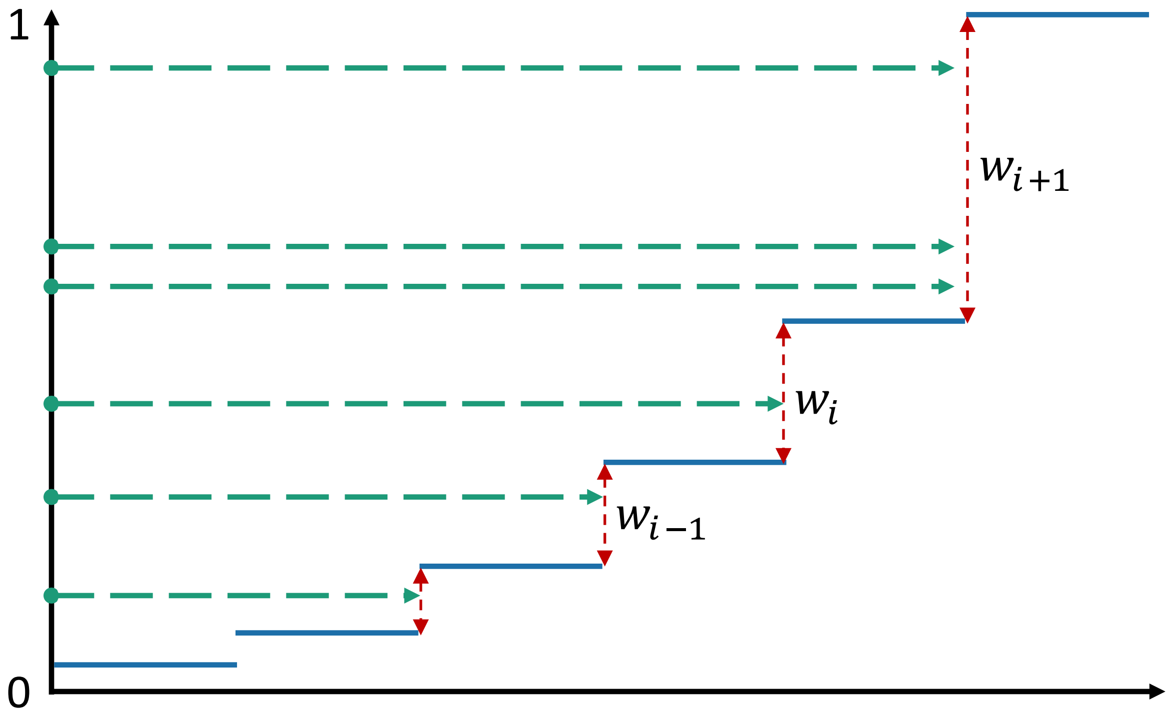
Particle Resampling

- Use the cumulative probability to aid in resampling
- Sum of normalized weights is 1



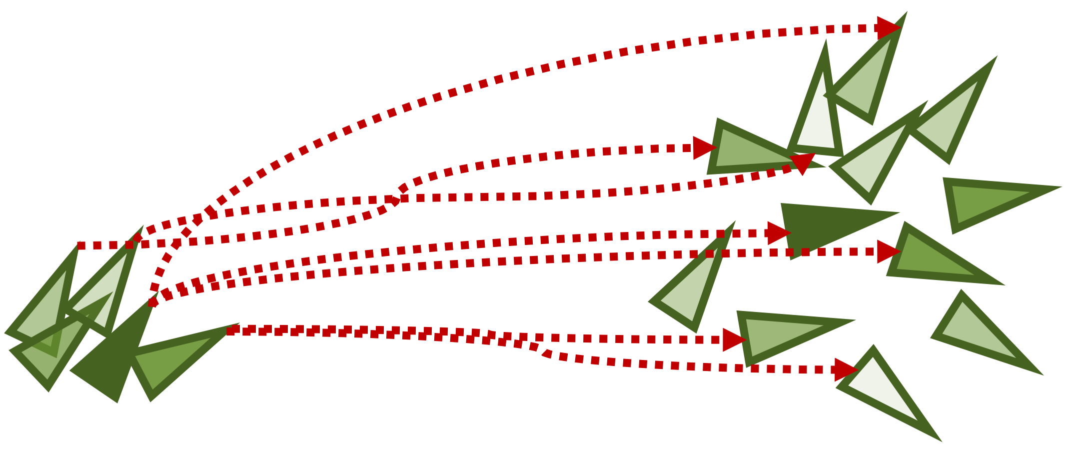
Particle Resampling

- Sample number uniformly between 0 and 1 of the cumulative range, and find which w_i includes that number



Particle Resampling

- The particles with the indices found in the resampling approach become the new set of particles to be fed into the next odometry update
- Particles may be duplicated, but the odometry noise will differentiate these particles.



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Localization

4.4 Iterative Closest Point (ICP) Algorithm

Penn Engineering

Review: EM Algorithm

$$\arg\max_{\theta} F(X|\theta)$$

Initialize $\hat{\theta}$

E-step

Find a lower bound $G\left(\theta|\hat{\theta}\right)$

M-step

$\hat{\theta}_{new} = \arg\max_{\theta} G(\theta|\hat{\theta})$

Stop if converged

General EM

$$\arg\max_{\boldsymbol{\mu},\Sigma} \sum_{i=1}^{N} \ln\left\{\frac{1}{K}\sum_{k=1}^{K} g_k(\mathbf{x}_i|\boldsymbol{\mu}_k,\Sigma_k)\right\}$$

Initialize $\boldsymbol{\mu}$ and Σ

E-step

Update z_k^i

M-step

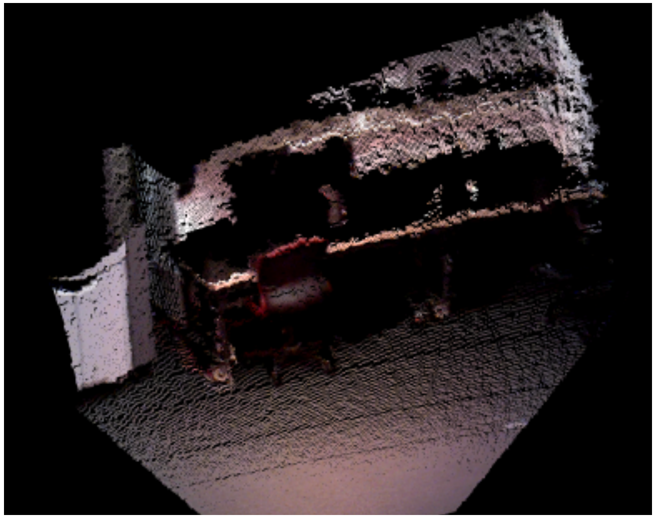
Update $\boldsymbol{\mu}$ and Σ

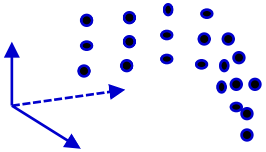
Stop if converged

EM for GMM

Review: 3D Map Representation

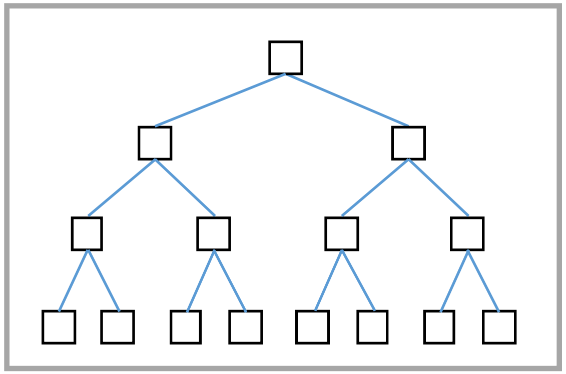
3D point cloud measurement





Map visualized in 3D

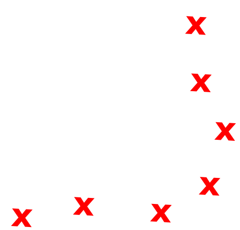
Implementation Example



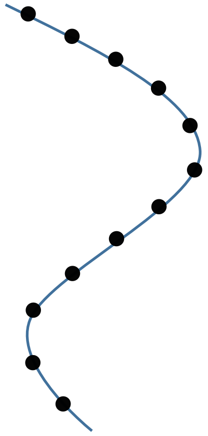
ICP Algorithm

- Problem: Register two point sets X and Y.

Measurement (X)



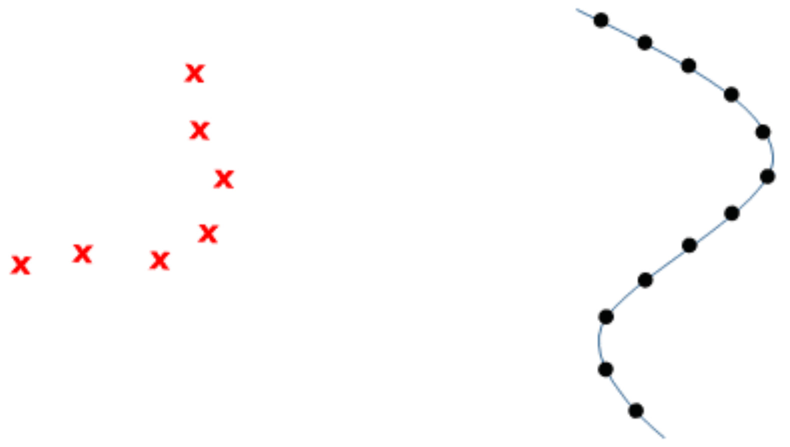
Model (Y)



ICP Algorithm

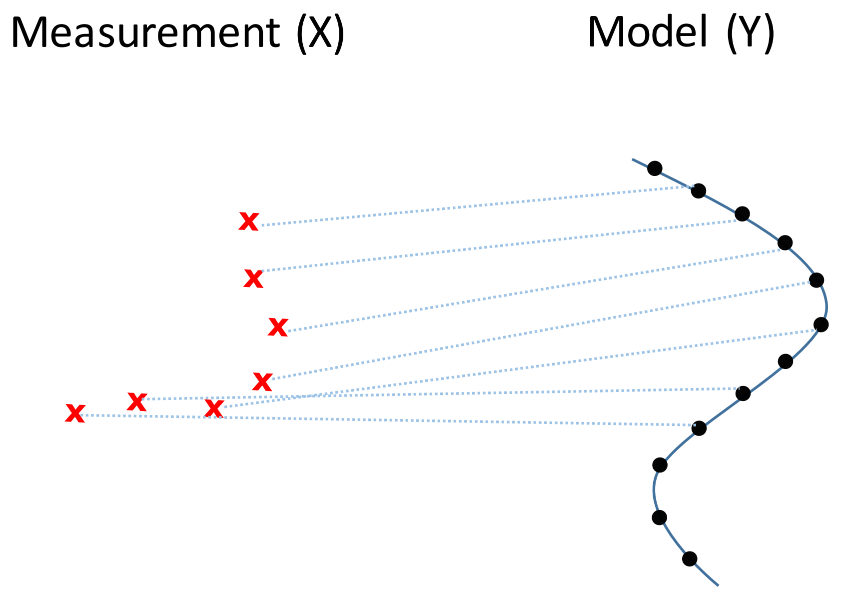
- Problem 1: Rotation and translation?

Measurement (X) Model (Y)



ICP Algorithm

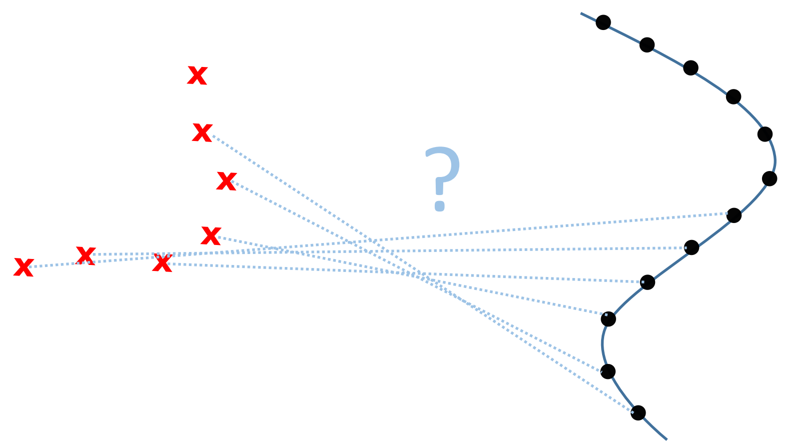
- Problem 2: Correspondences?



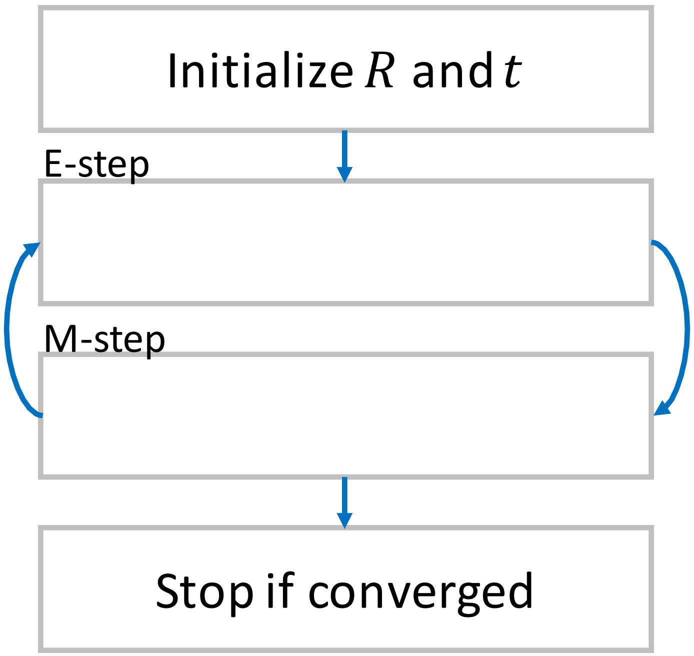
ICP Algorithm

- Problem 2: Correspondence

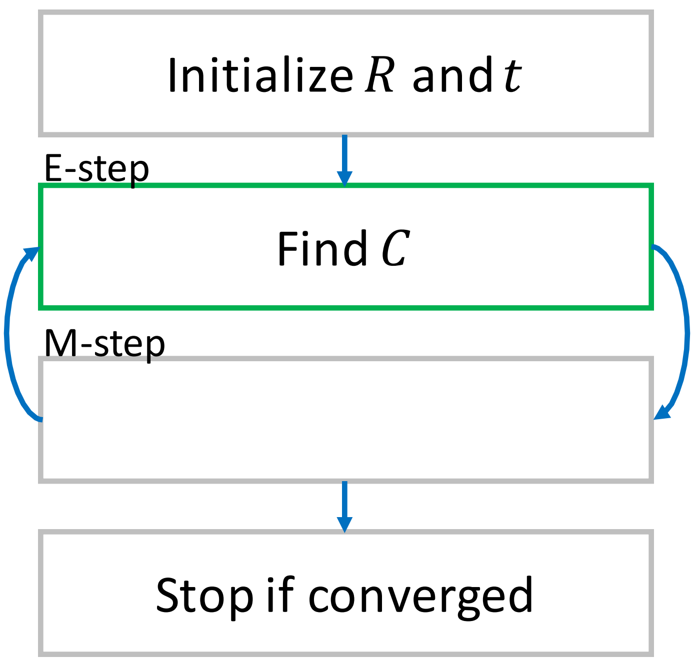
Measurement (X) Model (Y)



ICP Algorithm

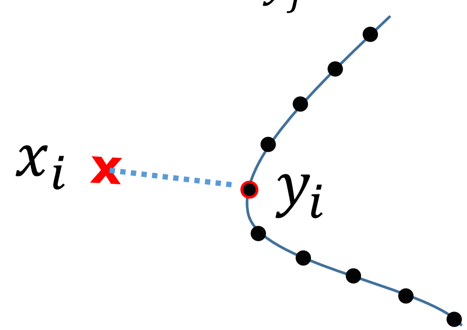


ICP Algorithm

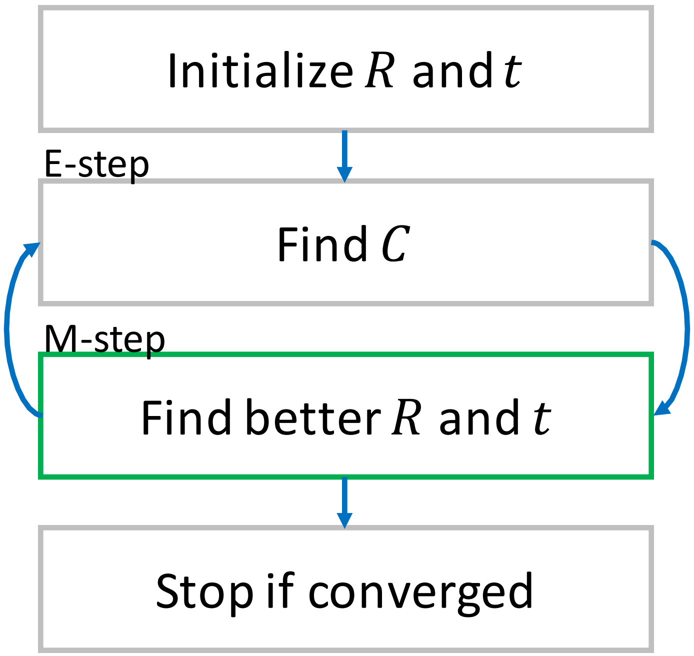


Closest point!

$$y_i = \arg\min_{y_j \in Y} \| x_i - y_j \|$$



ICP Algorithm



$$R, t = \arg\min \sum_{x_i, y_i \in C} \|d(x_i, y_i)\|^2$$

[SOLUTION] K. Arun, T. Huang, and S. Blostein, "Least-squares fitting of two 3D point set", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 9(5), pp. 698–700, 1987.

ICP Algorithm

Initialize R and t

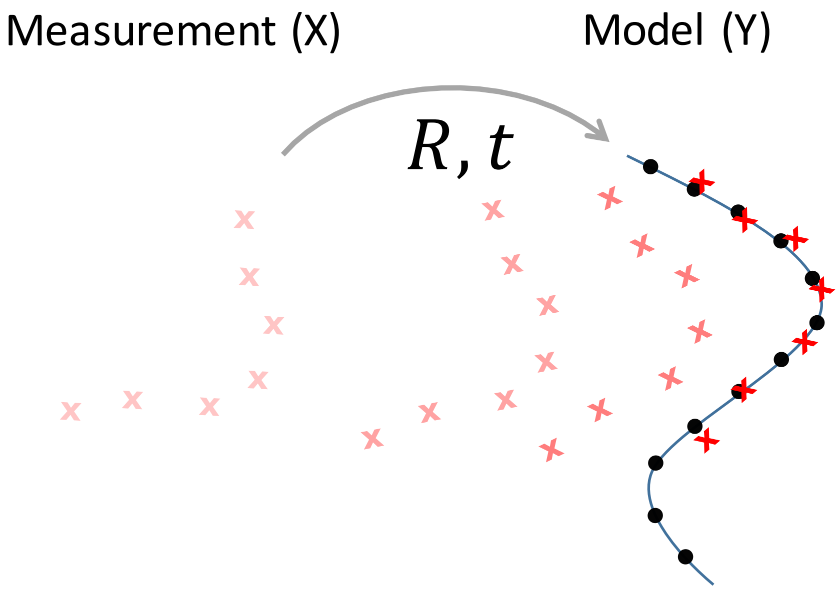
E-step

Update C

M-step

Update R and t

Stop if converged



ICP: Example

Initialize R and t

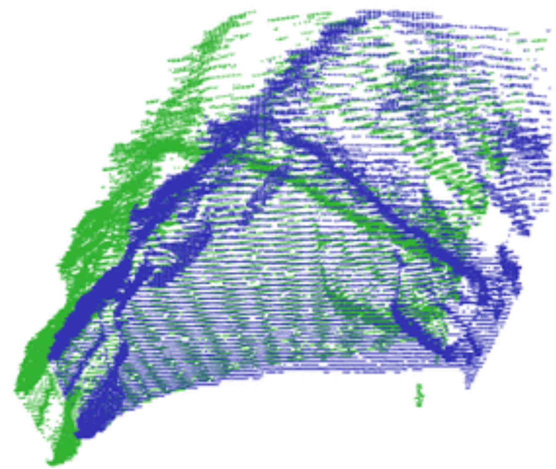
E-step

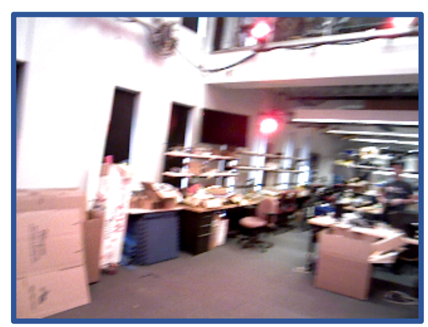
Update C

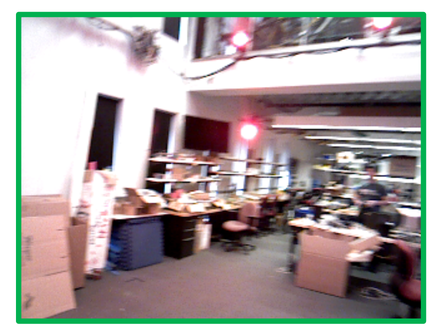
M-step

Update R and t

Stop if converged

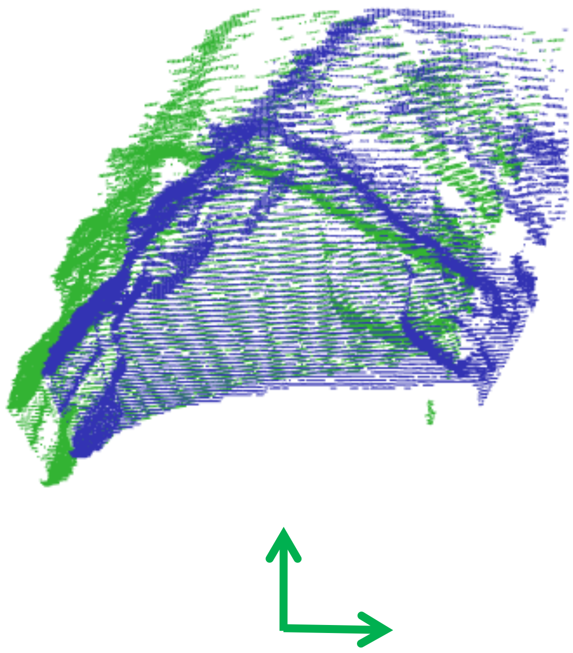






ICP: Motion Increment

Raw measurements are
in the local coordinate frame.



Registration gives the motion
increment of the body w.r.t the model