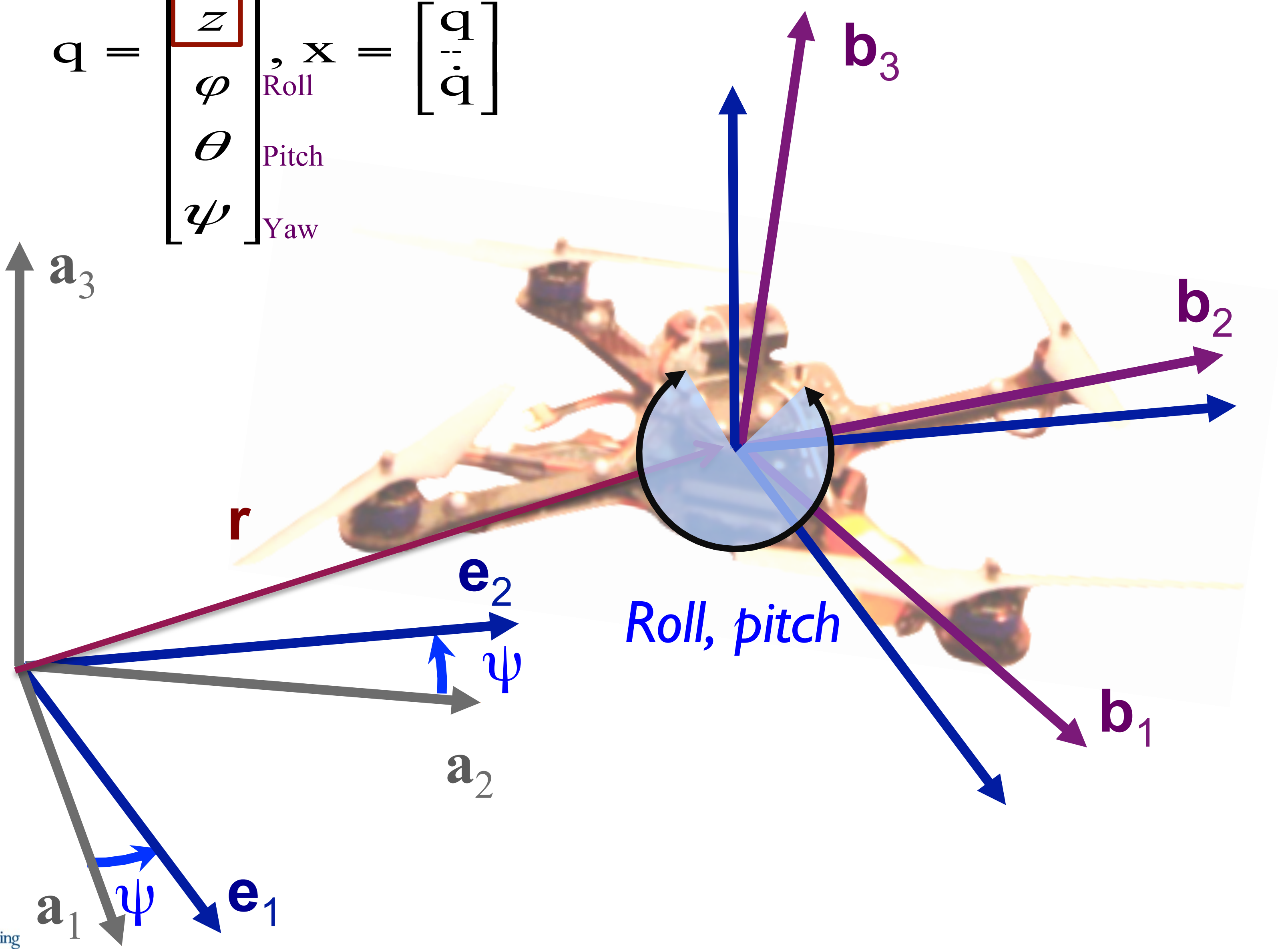
3-D Quadrotor

$$\mathbf{q} = \begin{bmatrix} \boxed{\begin{matrix} x \\ y \\ z \end{matrix}} \\ \varphi \\ \theta \\ \psi \end{bmatrix} \begin{matrix} \mathbf{r} \\ \\ \\ \text{Roll} \\ \text{Pitch} \\ \text{Yaw} \end{matrix}, \quad \mathbf{x} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}$$

Angular velocity components in B

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

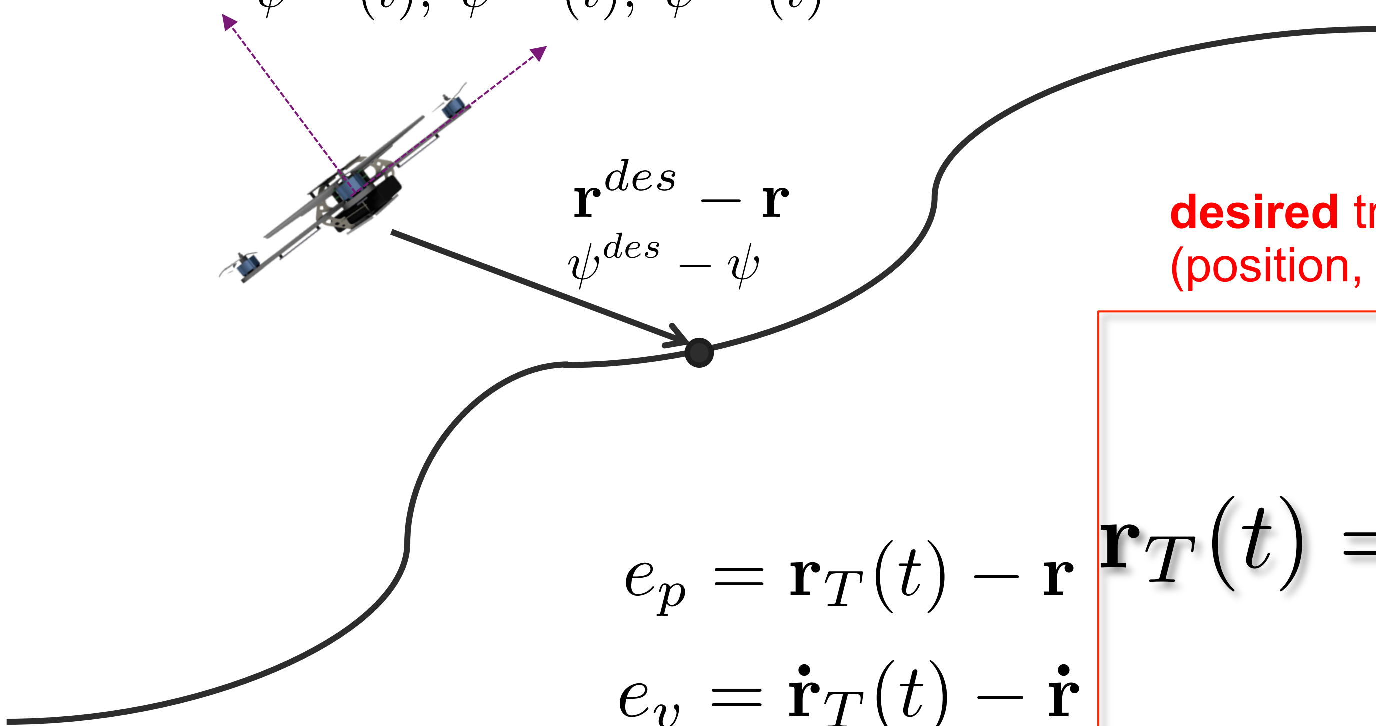


Trajectory Tracking in 3 Dimensions

Given $\mathbf{r}_T(t), \dot{\mathbf{r}}_T(t), \ddot{\mathbf{r}}_T(t)$

$$\mathbf{r}^{des}(t),\ \dot{\mathbf{r}}^{des}(t),\ \ddot{\mathbf{r}}^{des}(t)$$

$$\psi^{des}(t),\ \dot{\psi}^{des}(t),\ \ddot{\psi}^{des}(t)$$



$$\mathbf{r}_T(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \\ \psi(t) \end{bmatrix}$$

$$e_p = \mathbf{r}_T(t) - \mathbf{r}$$

$$e_v = \dot{\mathbf{r}}_T(t) - \dot{\mathbf{r}}$$

Want $(\ddot{\mathbf{r}}_T(t) - \ddot{\mathbf{r}}_c) + k_{d,x}e_v + k_{p,x}e_p = 0$

Commanded acceleration, calculated by the controller

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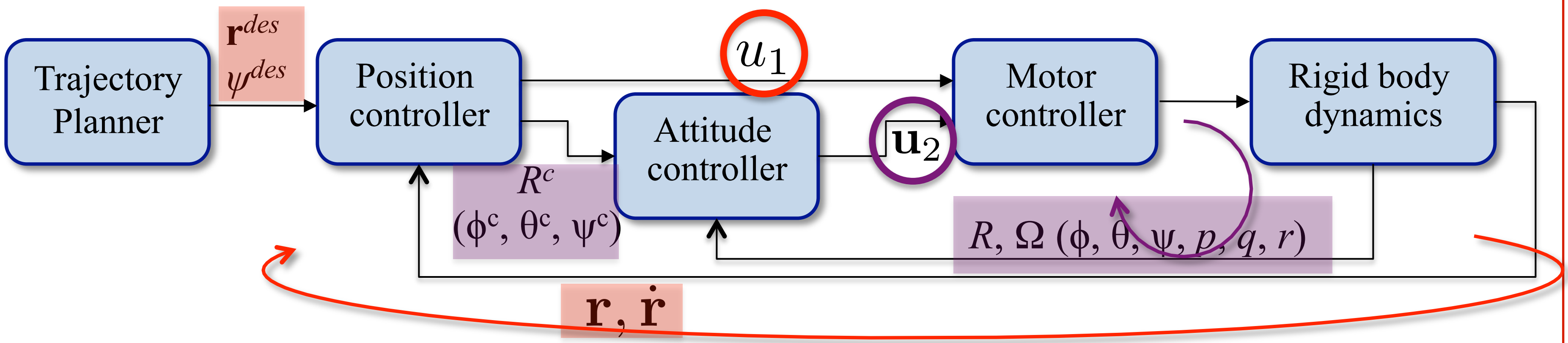
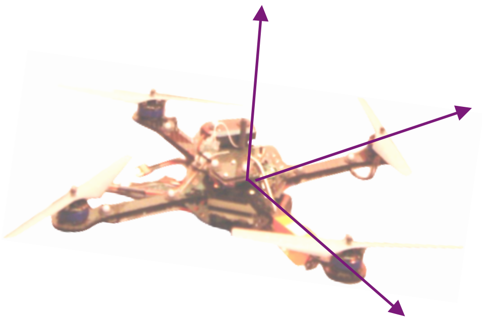


$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \underbrace{\boxed{F_1 + F_2 + F_3 + F_4}}_{u_1} \end{bmatrix}$$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\boxed{\begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}}}_{\mathbf{u}_2} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Control for Hovering



Linearize the dynamics at the hover configuration

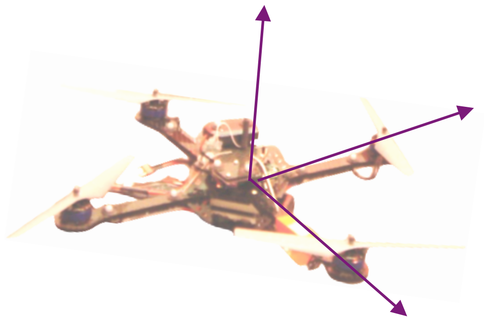
$$(u_1 \sim mg, \theta \sim 0, \phi \sim 0, \psi \sim \psi_0)$$

$$(u_2 \sim 0, p \sim 0, q \sim 0, r \sim 0)$$

$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \underbrace{F_1 + F_2 + F_3 + F_4}_{u_1} \end{bmatrix}$$

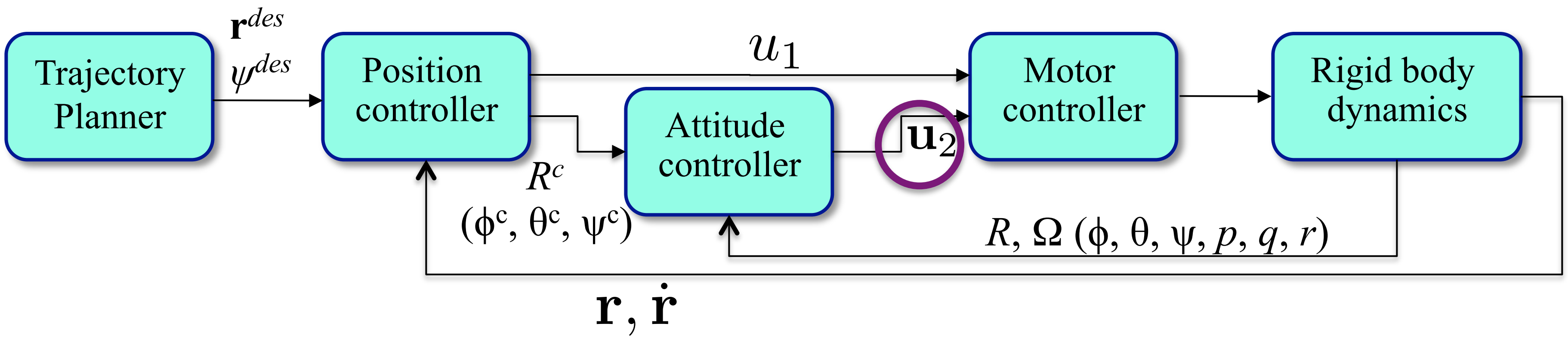
$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}}_{\mathbf{u}_2} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Control for Hovering



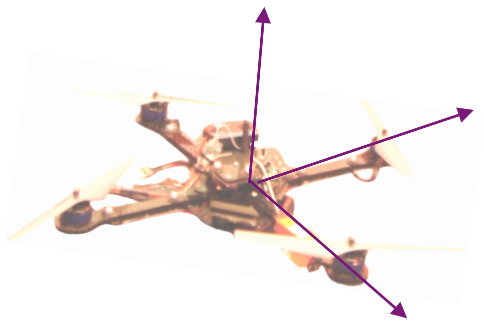
$$\left(u_2 \sim 0, p \sim 0, q \sim 0, r \sim 0\right)$$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\boxed{\begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}}}_{\mathbf{u}_2} - \cancelto{0}{\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}}$$



$$\mathbf{u}_2 = \begin{bmatrix} k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_c - p) \\ k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_c - q) \\ k_{p,\psi}(\psi_c - \psi) + k_{d,\psi}(r_c - r) \end{bmatrix}$$

Control for Hovering



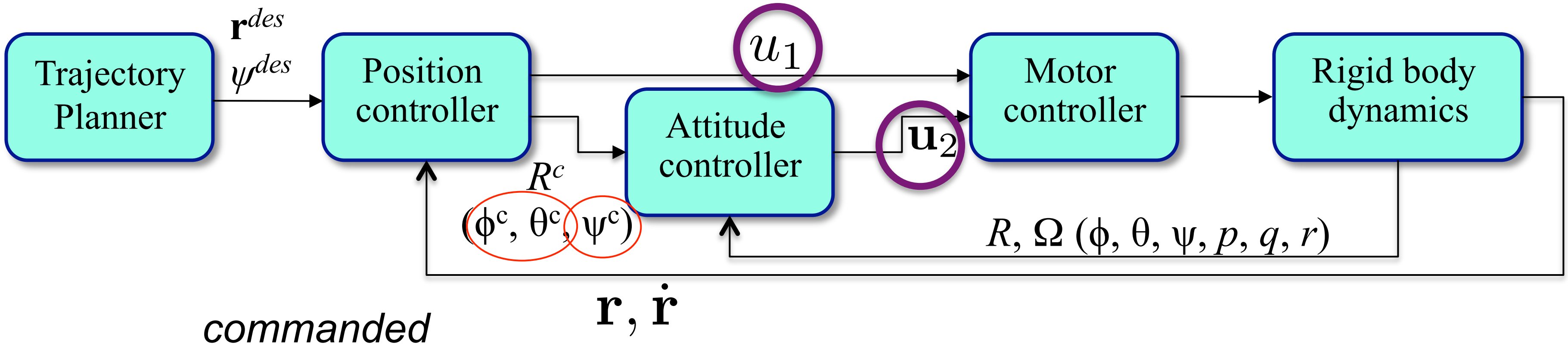
$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \underbrace{\boxed{F_1 + F_2 + F_3 + F_4}}_{u_1} \end{bmatrix}$$

Linearization

$$(u_1 \sim mg, \theta \sim 0, \phi \sim 0, \psi \sim \psi_0)$$

$$\ddot{r}_1 = \ddot{x} = g(\theta \cos\psi + \phi \sin\psi)$$

$$\ddot{r}_2 = \ddot{y} = g(\theta \sin\psi - \phi \cos\psi)$$



commanded

actual (feedback)

$$(\ddot{r}_{i,des} - \ddot{r}_{i,c}) + k_{d,i}(\dot{r}_{i,des} - \dot{r}_i) + k_{p,i}(r_{i,des} - r_i) = 0$$

specified

$$\boxed{u_1 = m(g + \ddot{r}_{3,c})}$$

$$\phi_c = \frac{1}{g}(\ddot{r}_{1,c}\sin\psi_{des} - \ddot{r}_{2,c}\cos\psi_{des})$$

$$\theta_c = \frac{1}{g}(\ddot{r}_{1,c}\cos\psi_{des} + \ddot{r}_{2,c}\sin\psi_{des})$$

$$\psi_c = \psi^{des}$$

$$\mathbf{u}_2 = \begin{bmatrix} k_{p,\phi}(\phi_c - \phi) + k_{d,\phi}(p_c - p) \\ k_{p,\theta}(\theta_c - \theta) + k_{d,\theta}(q_c - q) \\ k_{p,\psi}(\psi_c - \psi) + k_{d,\psi}(r_c - r) \end{bmatrix}$$

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