Goals

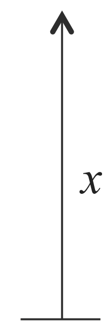
- Basic mechanics
- Control
- Design considerations
- Agility
- Component selection
- Effects of size

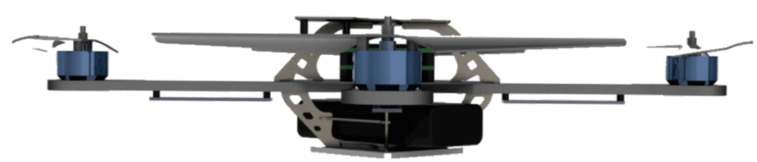
Control of height

$$\boxed{\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g}} = m\mathbf{a} \quad \rightarrow \quad a = \frac{d^2 x}{dt^2} = \ddot{x}$$

Input $$u = \frac{1}{m}\left[\sum_{i=1}^{4} k_F \omega_i^2 + m\mathbf{g}\right]$$

Second order dynamic system $u = \ddot{x}$





What input drives the robot to the desired position?

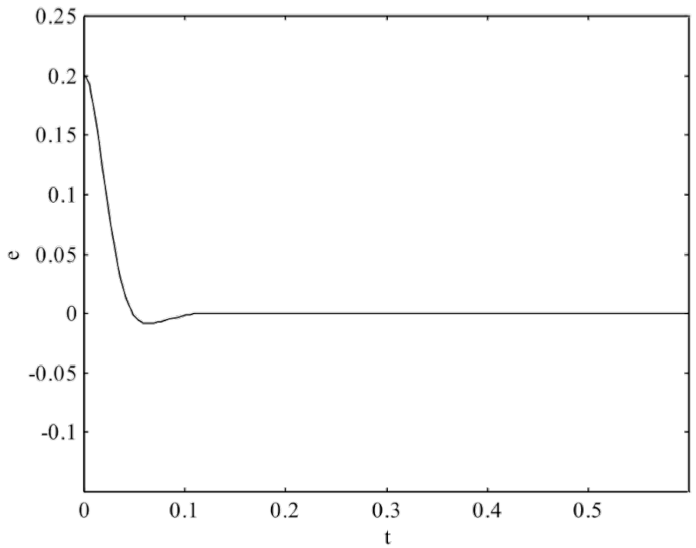
Control of a linear second-order system

Problem

State, input $x,\ u\ \in \mathbb{R}$

Plant model $\ddot{x} = u$

Want x to follow the desired trajectory $x^{des}(t)$



General Approach

Define error, $e(t)=x^{des}(t)- x(t)$

Want $e(t)$ to converge exponentially to zero

Strategy

Find u such that

$$\ddot{e} + K_v\dot{e} + K_p e = 0 \qquad K_p,\ K_v\ >\ 0$$

$$u(t) = \ddot{x}^{\mathrm{des}}(t) + K_v\dot{e}(t) + K_p e(t)$$

Feedforward Derivative Proportional

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Control for trajectory tracking in a simple second-order system

PD control

$$u(t) = \ddot{x}^{\mathrm{des}}(t) + K_v \dot{e}(t) + K_p e(t)$$

Proportional control acts like a spring (capacitance) response

Derivative control is a viscous dashpot (resistance) response

Large derivative gain makes the system overdamped and the system converges slowly

PID control

In the presence of disturbances (e.g., wind) or modeling errors (e.g. unknown mass), it is often advantageous to use PID control

$$u(t) = \ddot{x}^{\mathrm{des}}(t) + K_v \dot{e}(t) + K_p e(t) + K_i \underbrace{\int_0^t}_{\text{Integral}} e(\tau) d\tau$$

PID control generates a third-order closed-loop system

Integral control makes the steady-state error go to zero

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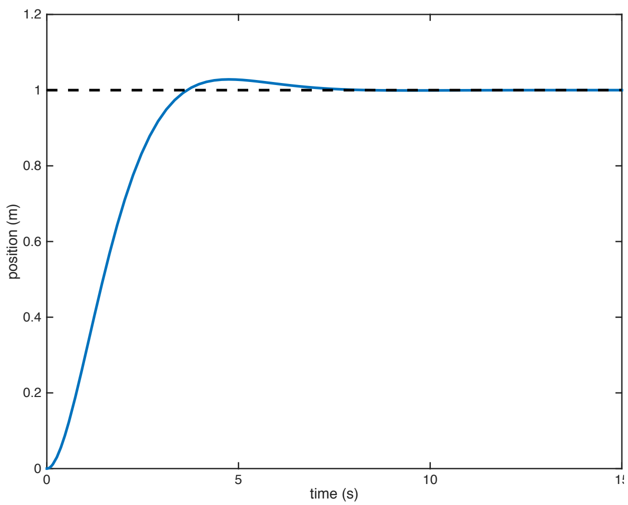
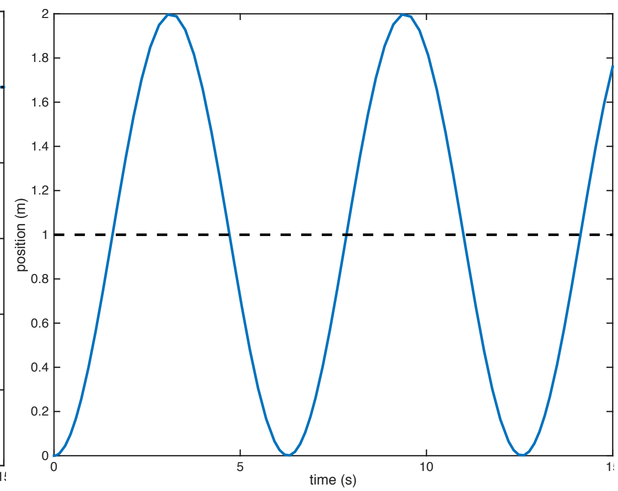
Effects of Gains for a PD Control System



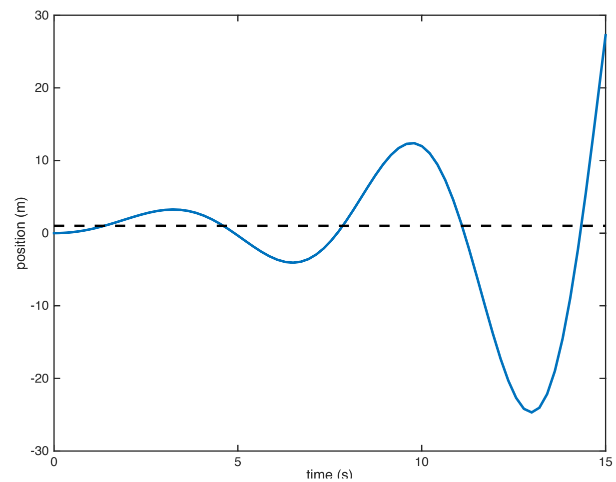
Stable

$$K_p,\ K_v\ >\ 0$$



Marginally Stable

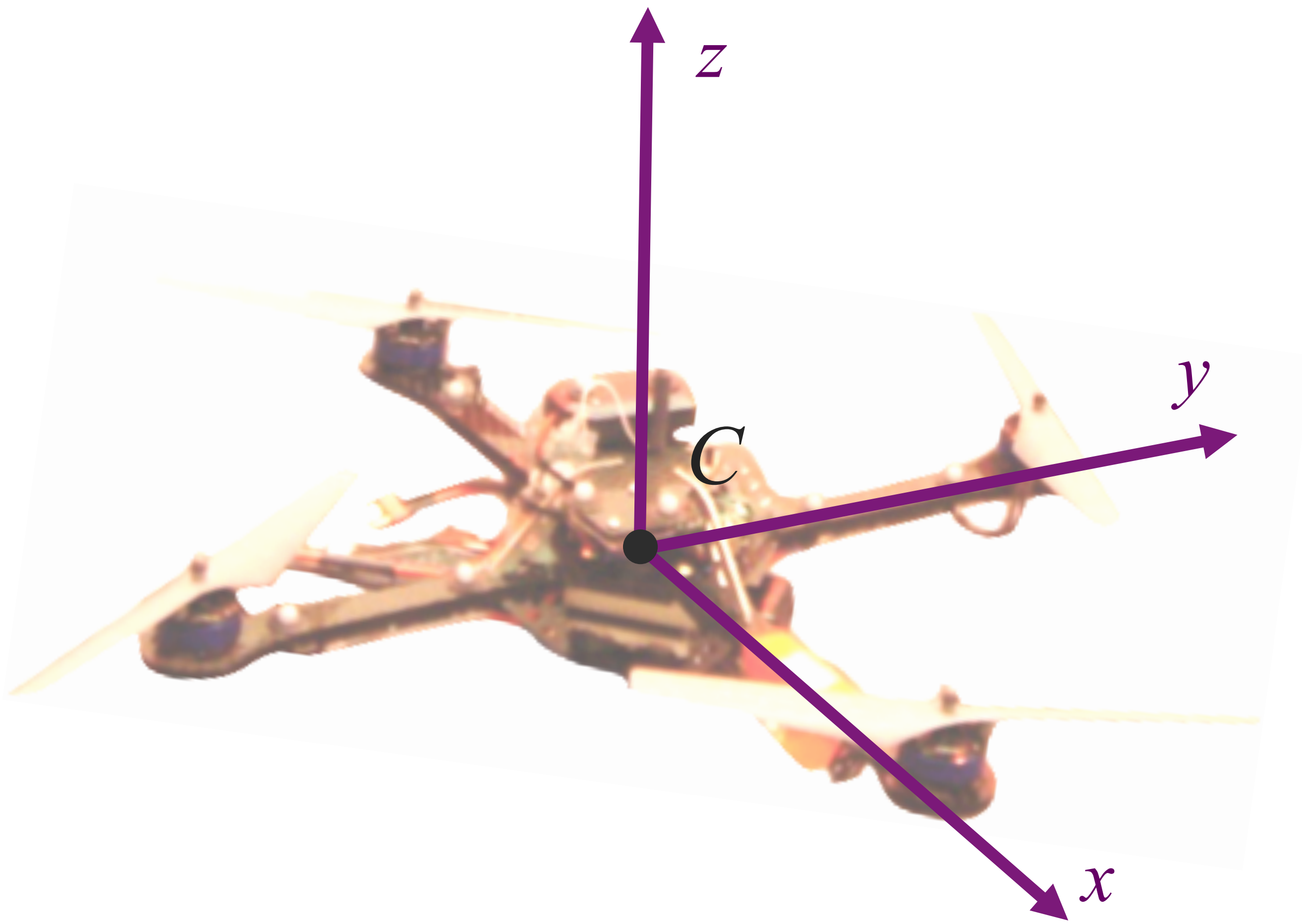
$$K_p >\ 0, \quad K_v\ = 0$$



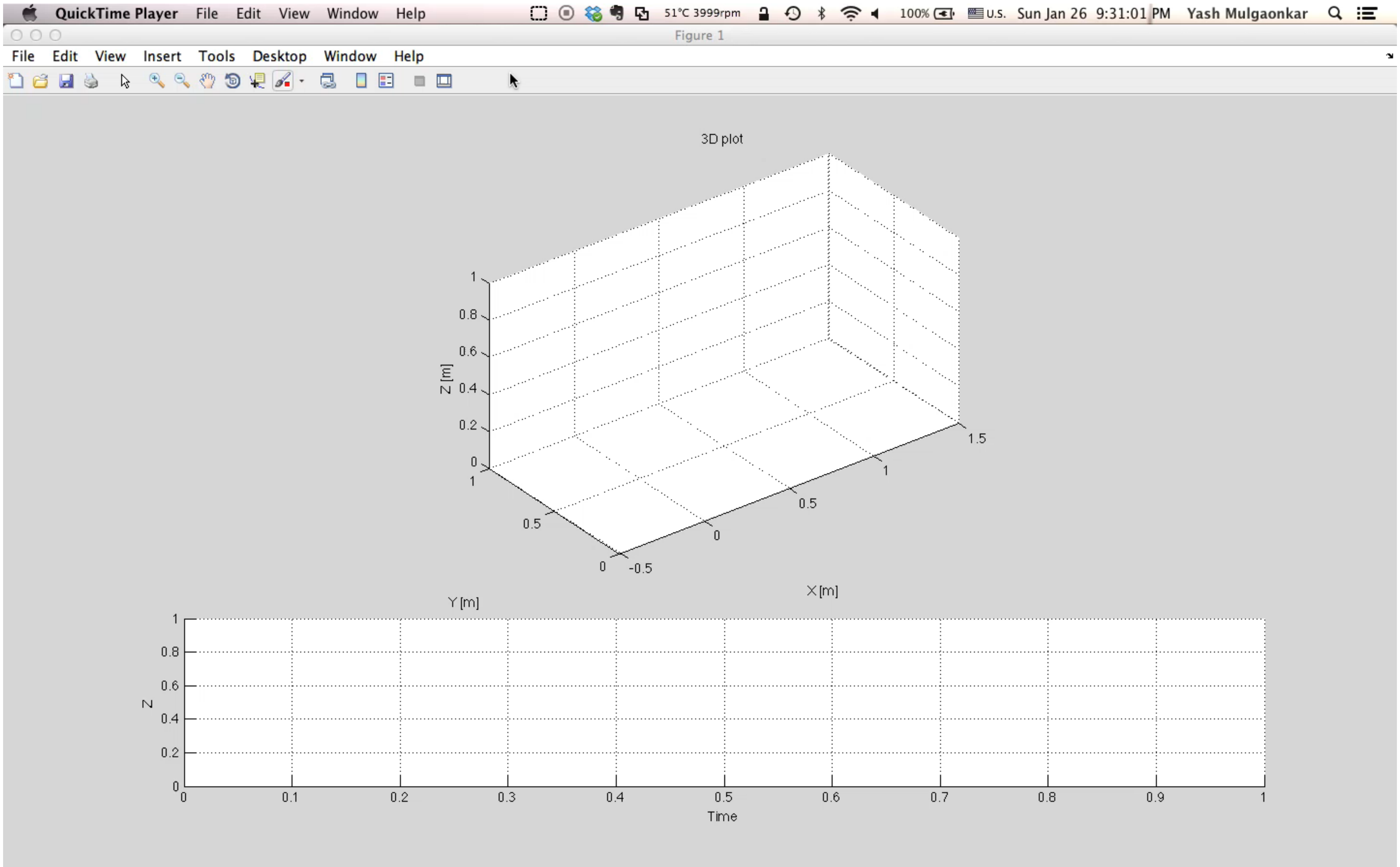
Unstable

$$K_p \text{ or } K_v\ < 0$$

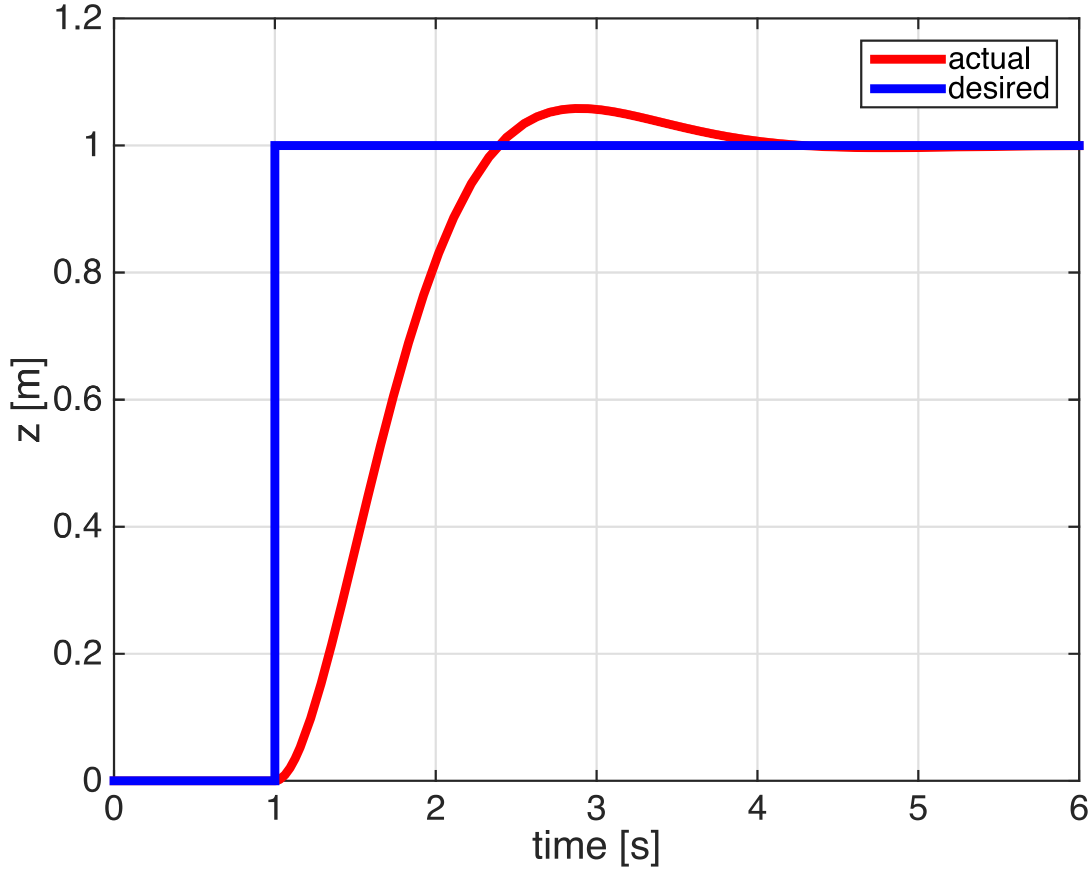
Control of quadrotor height



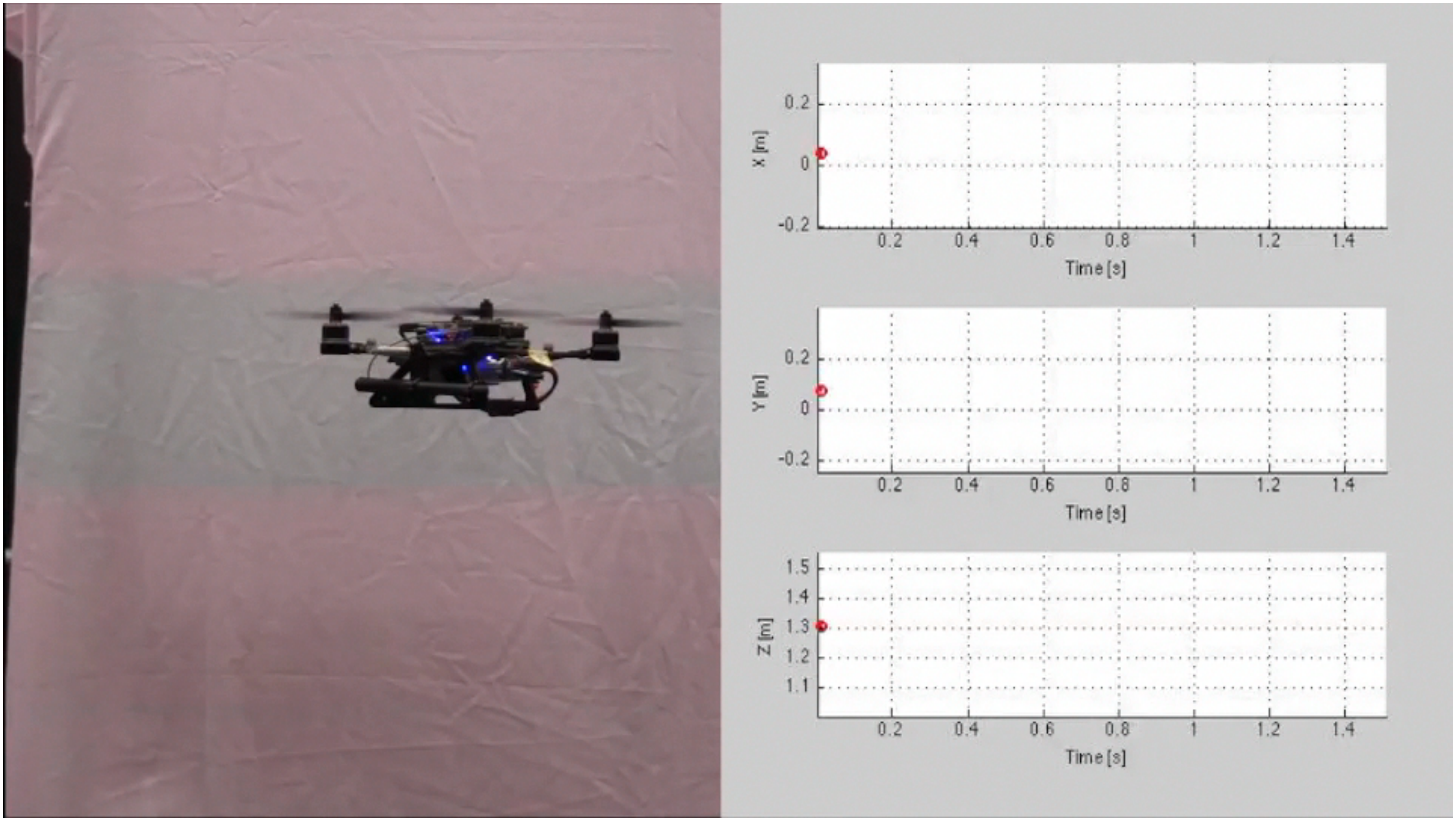
Simulation - PD Control of height



PD Controller



PD Controller



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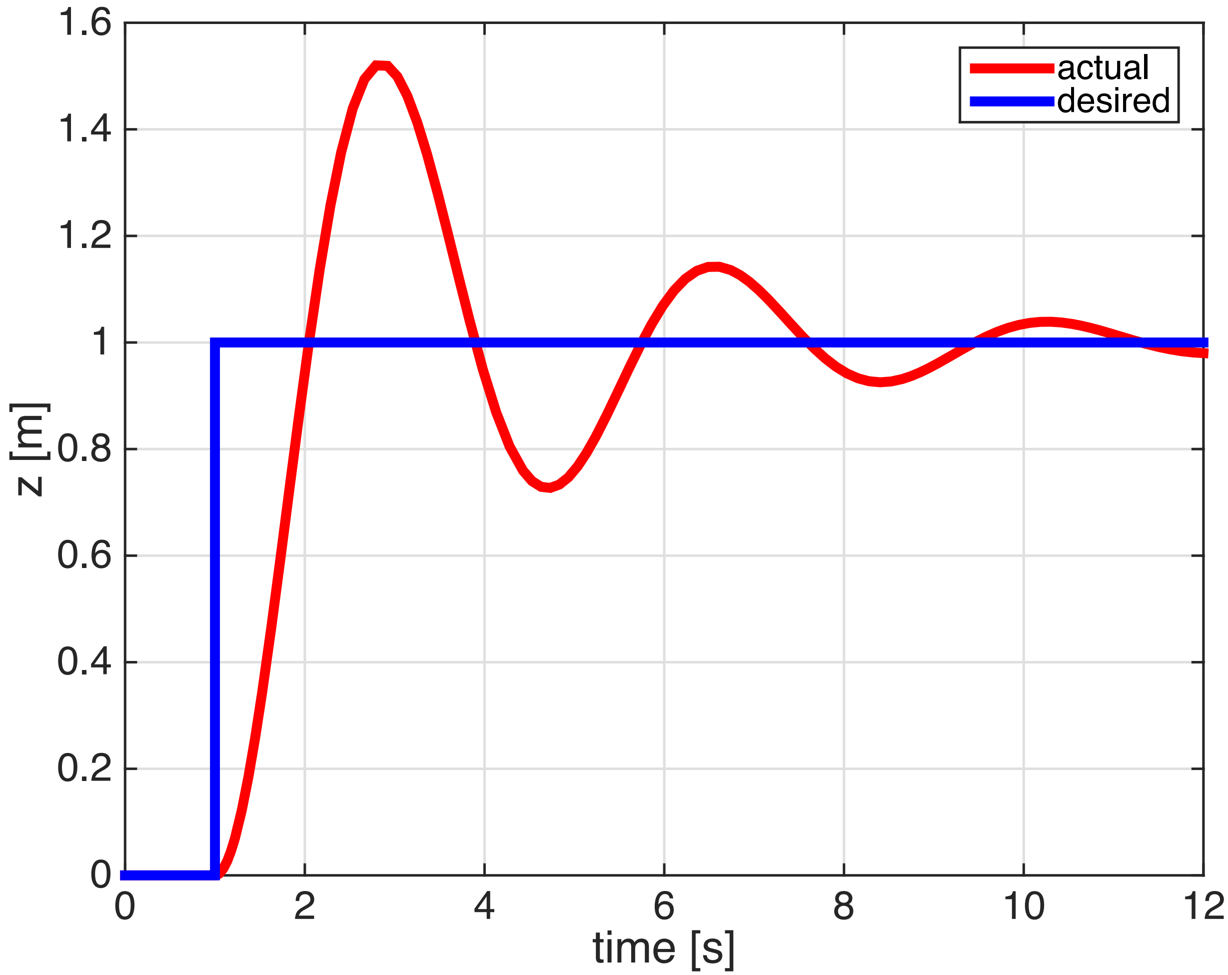
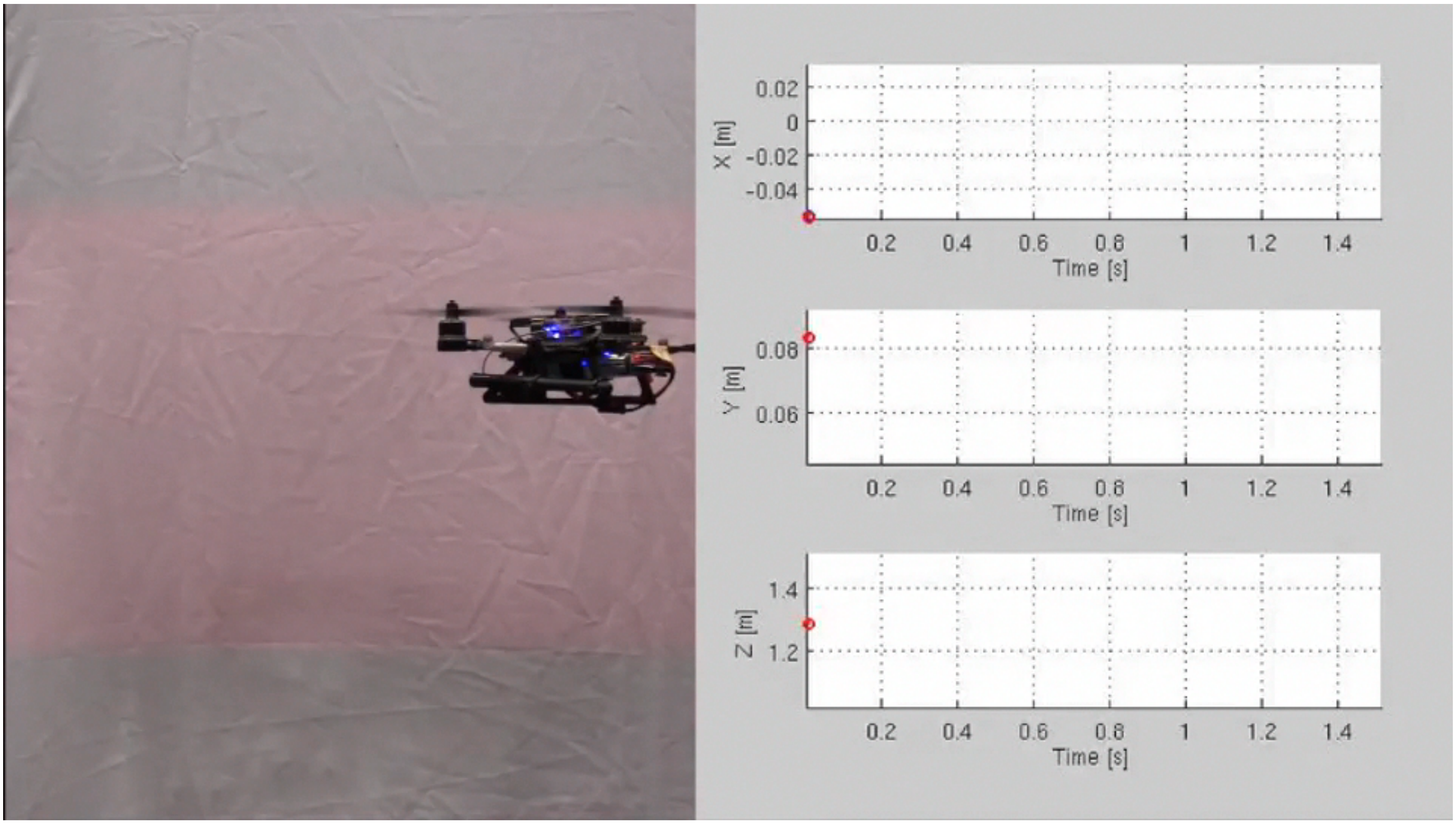
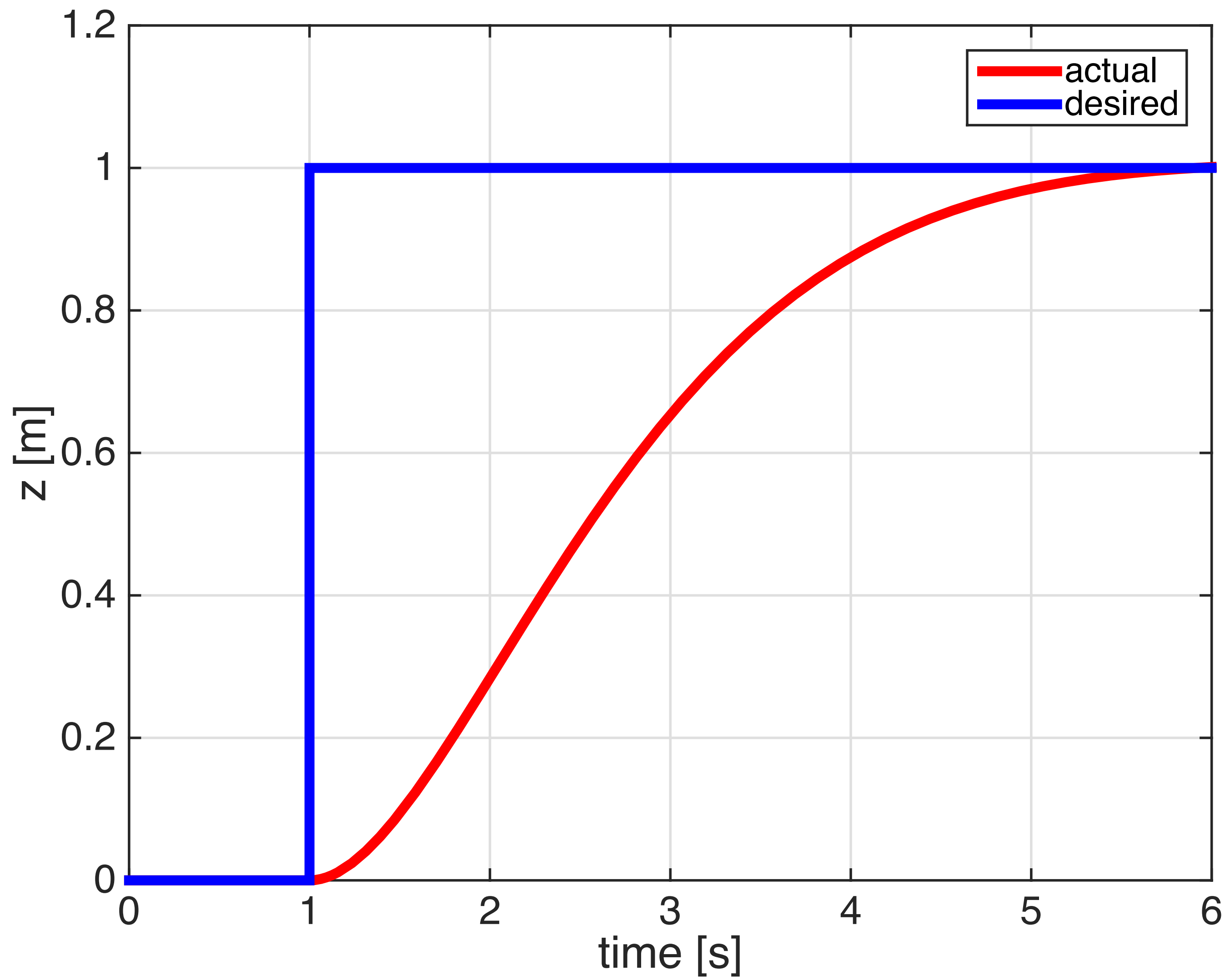
High K_p

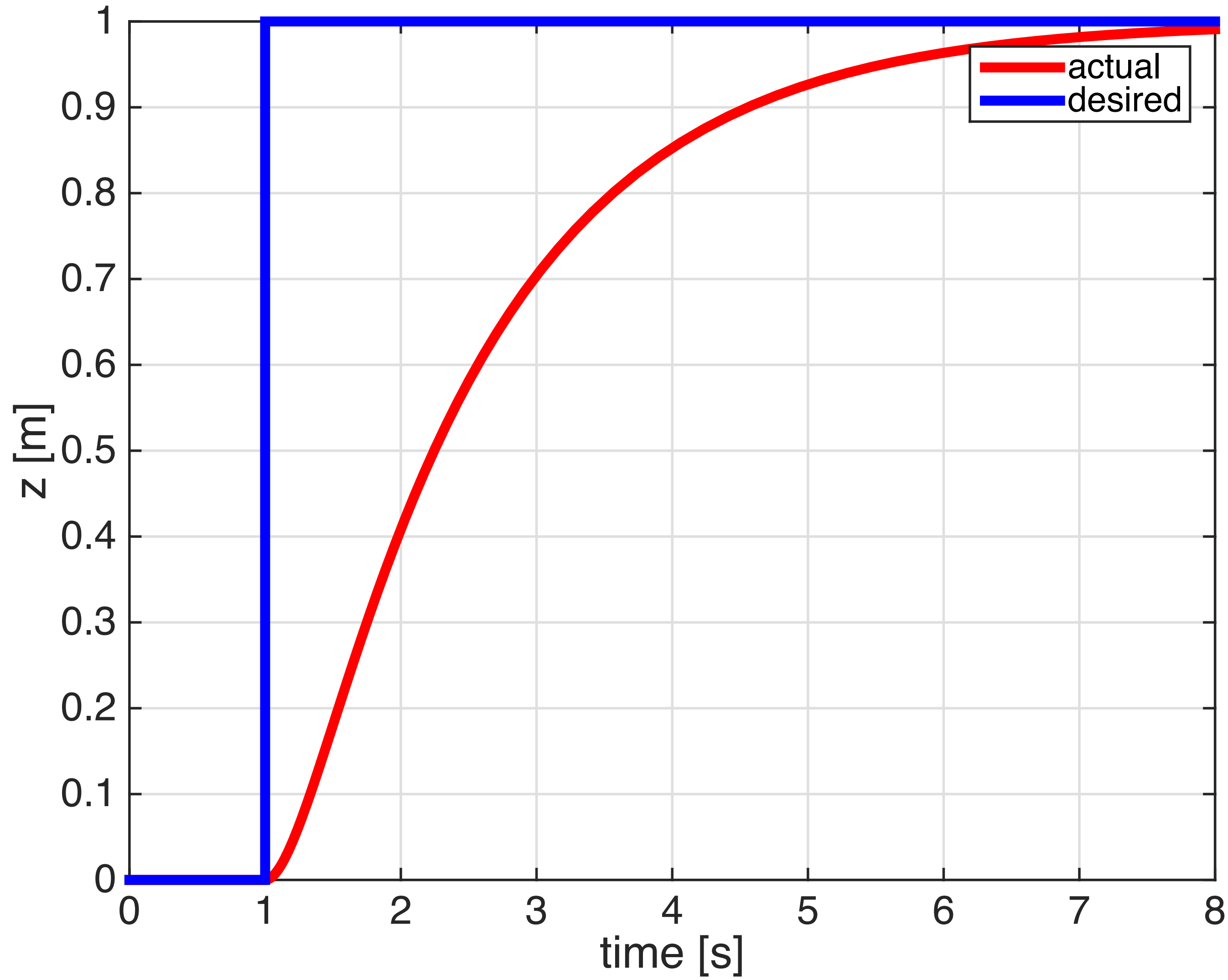



0.02
0
-0.02
-0.04
X [m]
0.2 0.4 0.6 0.8 1 1.2 1.4
Time [s]
0.08
0.06
Y [m]
0.2 0.4 0.6 0.8 1 1.2 1.4
Time [s]
1.4
1.2
Z [m]
0.2 0.4 0.6 0.8 1 1.2 1.4
Time [s]

Low K_p (soft response)



High K_v (overdamped)



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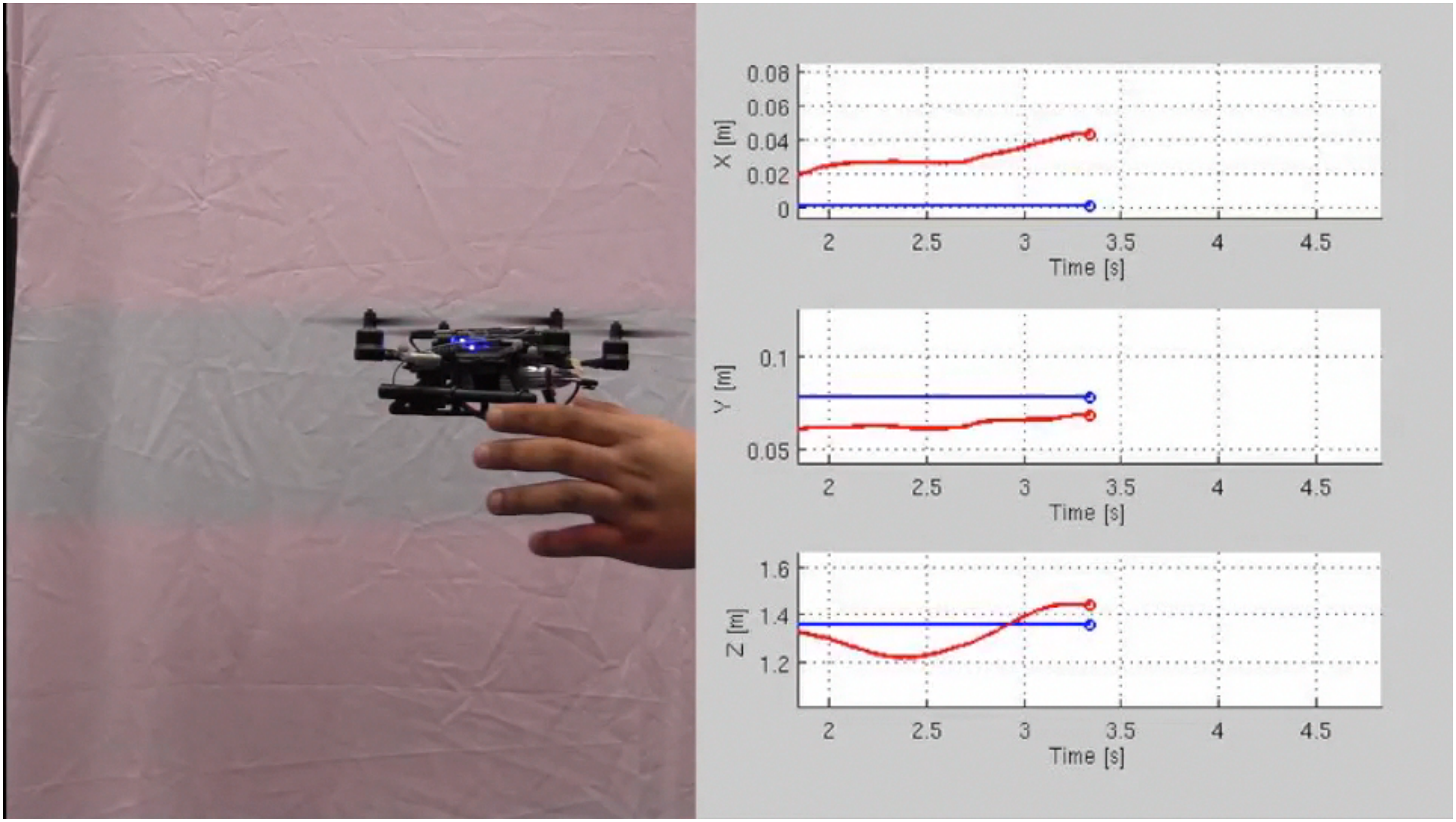
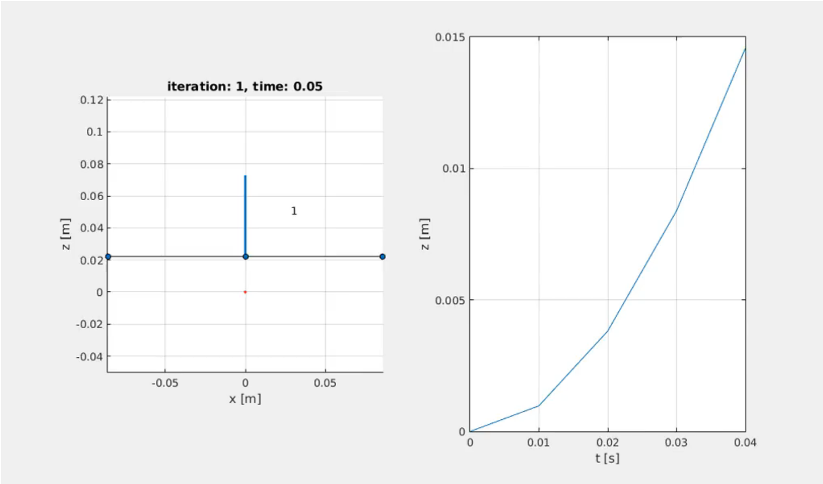

0.08
0.06
0.04
0.02
0
X [m]
2
2.5
3
3.5
4
4.5
Time [s]
0.1
0.05
Y [m]
2
2.5
3
3.5
4
4.5
Time [s]
1.6
1.4
1.2
Z [m]
2
2.5
3
3.5
4
4.5
Time [s]

Exercise

- You are given a simulator which models a PD controller for the height of a quadrotor.
- The aim of the exercise is to tune the proportional gain (Kp) of the controller in order to get a desired response from the system. The derivative gain (Kd) is kept constant.
- You should aim to get a response which has a rise time of less than 1s and a maximum overshoot of less than 5% similar to the one shown in the video below.



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