

## **Intelligent Systems**

Excersice 4- Representation / Feature Selection

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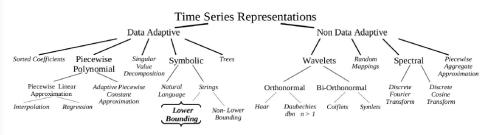


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## Data representation

# NON DATA ADAPTIVE VS. DATA ADAPTIVE APPROXIMATION





#### **FOURIER TRANSFORMATION**



The FT represents the time series in the frequency domain. The signal is constructed as a sequence of sine and cosine terms.

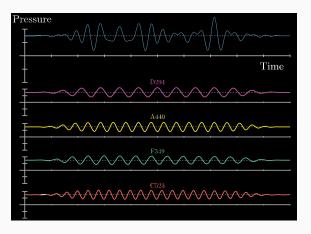


Figure 1: Fourier Transform

#### **FOURIER TRANSFORMATION**



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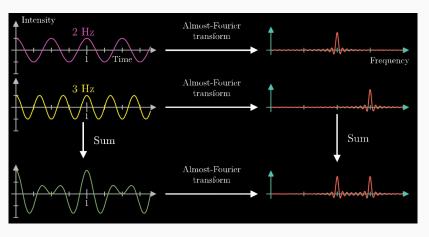


Figure 2: Fourier Transform



# A. Explain the idea of the *Shape Definition Language* and its application?

- B. Approximate the time series with the following approximations:
  - Piecewise Aggregate Approximation (PAA) with 4 segemtns.
  - Clipping to binary values (→ search the procedure on the internet).
  - Picewise Linear Approximation with 4 segments.
  - Run-Length Encoding (RLE).
- C. Aggregate the timeseries to the following statistical measures:
  - Mean
  - Standard deviation
  - Mode

nrocedure?

D. What are the advantages and disatvantages of the *clipping* 

#### 1. A SHAPE DEFINITION LANGUAGE



Given the case that the most important information of a time series can be extracted from the rough shape, limited terms of the *Shape Definition Language* (*SDL*) are enough to model it, i.e. *Up, up, stable, zero, down, Down.* These represent the different slopes of the time series. Advantageously, the representation can easily be processed by algorithms for symbolic sequences, like *Longest Common Subsequence* (*LCSS*).

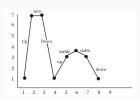


Figure 3: Shape Definition Language<sup>1</sup>

#### 1. B



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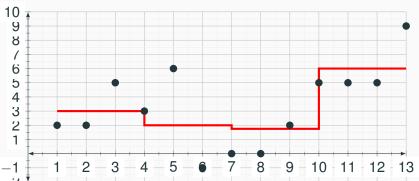
nrocedure?

D. What are the advantages and disatvantages of the *clipping* 

#### 1. B PIECEWISE AGGREGATE APPROXIMATION



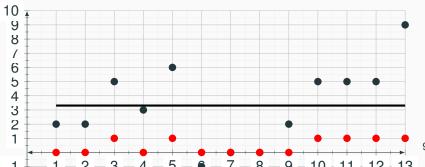
- PAA Segment Length =  $\frac{t_{\text{end}} t_{\text{start}}}{\#Segments}$
- 4 segments  $\rightarrow$  PAA Segment Length  $= \frac{13-1}{4} = 3$
- Segment 1:  $\frac{2+2+5+3}{4} = 3$
- Segment 2:  $\frac{3+6+(-1)+0}{4} = 2$
- ...



#### 1. B CLIPPING

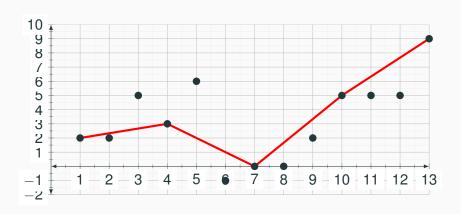


- Calculate time series' mean:  $\mu = \frac{1}{N} \sum_{i=1}^{N} x_i$
- The clipped values are given by:  $y_i^* = \begin{cases} 1 \text{ , for } x_i \geq \mu \\ 0 \text{ , otherwise} \end{cases}$
- $\mu = \frac{2+2+5+3+6+(-1)+0+2+5+5+5+9}{13} \approx 3.31$
- Clipping: 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1, 1, 1



### 1. B PIECEWISE LINEAR APPROXIMATION





#### 1. B Run-Length Encoding



- Counter {n} represents the number of repititions of the following symbol
- $2,2,5,3,6,-1,0,0,2,5,5,5,9 \rightarrow \{2\}2,5,3,6,-1,\{2\}0,2,\{3\}5,9$

#### 1. C



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nrocedure?

D. What are the advantages and disatvantages of the *clipping* 

#### 1. C STATISTICAL MEASURES:



Given *N D*-dimensional points  $x_i \in \mathbb{R}^d$ , i = 1, ..., N, the following statistical measures can be defined:

- *Mean*:  $\mu = \frac{1}{N} \sum_{i=1}^{N} x_i \rightarrow 3.31$
- Standard deviation:  $\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i \mu)^2} \rightarrow 2.84$
- Mode:  $m = argmax_x |\{y|y \in \{x_1, \dots, x_N\}, x = y\}| \rightarrow 5$

#### 1. D



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  - Picewise Linear Approximation with 4 segments.
  - Run-Length Encoding (RLE).
- C. Aggregate the timeseries to the following statistical measures:
  - Mean
  - Standard deviation
  - Mode
- D. What are the advantages and disatvantages of the

#### 1. D Pro & Cons of Clipping



What are the advantages and disatvantages of the *clipping* procedure?

#### **Advantages:**

- Extreme compression rate (float → bool)
- Simple representation (only binary values)
- Rough patterns can be found easily

#### Disatvantage:

Very unprecise representation of the signal

### 1. E



- A. Explain the idea of the *Shape Definition Language* and its application?
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D. What are the advantages and disatvantages of the *clipping* 

#### 1. E AAPA vs. PAA I



What is the main difference between the *Adaptive Picewise Aggregate Approximation (APAA)* and the *PAA?* 

#### Main difference:

- Variable length of the (temporal) sections
- Adaptive according to local details of a time series
  - Sections of frequent movement will be parted in smaller intervals
  - A Section without significant information will be represented as a large interval

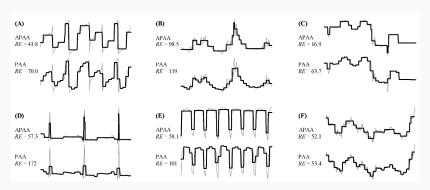
#### 1. E AAPA vs. PAA II



What is the main difference between the *Adaptive Picewise Aggregate Approximation (APAA)* and the *PAA?* 

#### Advantage:

 The error (e.g. Least-Squares) between the raw data and the representation is reduced.<sup>2</sup>



Principal component analysis

#### 2. A & B



- A. What is the goal of the *Principal Component Analysis* (*PCA*) and what is its basic assumption.
- B. What is the benefit of the PCA?
- C. Describe the following items:
  - Zero-mean feature
  - Variance
  - Standard deviation
  - Covariance matrix
  - Arithmetic mean
  - Eigenvector
  - Eigenvalue
  - Projection onto new feature space
- D. How can we get a dimensionality reduction with the means of Eigenvalues?

#### 2.A& B PCA GOALS AND BENEFITS



What is the goal of the *Principal Component Analysis* (*PCA*) and what is its basic assumption. **Basic assumption**:

 The larger the variance, the higher is the level of information.

#### Goal:

- Dimensionality reduction:
  - Remove dimensions with poor information gain
  - Generate new dimensions that better fit to the structure of the data, i.e. at which the data has the largest variance.

#### Benefits:

- Reduce computing time: Through the usage of data mining algorithms applied on dimensionality-reduced data sets.
- Feature selection: Very easy; just select the PCA meta features

#### 2. C & D



- A. What is the goal of the *Principal Component Analysis* (*PCA*) and what is its basic assumption.
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- D. How can we get a dimensionality reduction with the means of Eigenvalues?

#### 2. C & D PCA STEPS I



**Given:** Data set with *N* samples and *D* dimensions (features).

#### 1. Standardisation:

• Calculate the arithmetic mean for each feature *j*:

$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{j,i}$$

• Zero-mean sample:

$$\mathbf{X}_{\mathbf{j},\mathbf{i}}' = \mathbf{X}_{\mathbf{j},\mathbf{i}} - \mu_{\mathbf{j}}$$

• Calculate the standard deviation for each feature *j*:

$$\sigma_j = \sigma_j = \sqrt{\frac{1}{N-1} \sum_{i=1}^N X_{j,i}^{\prime 2}}$$

• Standardisation for every feature *j*:

$$X_{j,i} = \frac{X'_{j,i}}{\sigma_j}$$

### 2. C & D PCA STEPS II



#### 2. Calculate covariance matrix:

• 
$$s_{j,j'} = \frac{1}{N-1} \sum_{i=1}^{N} x'_{j,i} \cdot x'_{j',i}$$
  
•  $S = \begin{pmatrix} s_{1,1} & \cdots & s_{1,D} \\ \vdots & \ddots & \vdots \\ s_{D,1} & \cdots & s_{D,D} \end{pmatrix}$ 

#### 2. C & D PCA STEPS III



#### 3. Calculate eigenvectors and eigenvalues:

- Eigenvalues represent the ratio of the variance along their corresponding eigenvectors
- The eigenvector with the largest eigenvalue represents the direction, in which the data has the largest variance
- The eigenvector with the second largest eigenvalue represents a orthogonal direction w.r.t. the first eigenvector, in which the data has the second largest variance, etc.
- Eigenvectors are the principal components (PC), which are more suitable to model the structure of the data than the original features (assuming that high information gain corresonds to high variance).

#### 2. C & D PCA STEPS IV



- **4. Dimensionality reduction:** Choose the most important eigenvalues according to their eigenvalues:
  - Method 1: The sum of the remaining eigenvalues shall be larger than a predefined ratio (e.g. 0.75) of the sum of all eigenvalues.
  - Method 2: Dimensions shall be removed if the eigenvalue of the corresponding eigenvector is lower than the mean of all eigenvalues.

#### 4. Projection onto the new feature space:

• Spanned by the new selected eigenvectors (*PC*s).

# **Python PCA**



- A. Download the file *04\_Representation.ipyn* from *OpenOlat*.
- B. In order to solve the tasks, you can use the library *numpy*.
- C. Compare your results afterwards with the help of *sklearn*.

**Feature Selection Basics** 



- A. What are the tasks and goals of feature selection?
- B. What are benefits of feature selection?
- C. Describe the term "weakly relevant but non-redundant features"?
- D. Creating feature subsets, what is the advantage of Random Generation (RG) over Sequential Forward Generation (SFG), Sequential Backward Generation (SBG), and Bidirectional Generation (BG)?
- E. Enumerate and describe the three different search strategies for finding an adequate subset of features.Additionally, mention their advantages and disadvantages.

#### 1. A



What are the tasks and goals of feature selection?

#### 1. A Tasks & goals of feature selection



#### Task:

- Find and select the optimal subset of the feature set.
- The subset should meet the following conditions:
  - Provide the highest accuracy for the chosen task (classification, regresseion).
  - Minimal number of features simultaneously guaranteeing at least a required minimum accuracy. (no redundant features)
  - Minimum costs (in computing time and monetary).
  - · Improved comprehensability.

#### Goal:

 Distinguish important (good) features from unimportant (bad) features.



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#### 1. B BENEFITS OF FEATURE SELECTION



#### Benefits:

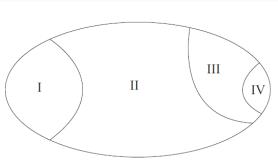
- Saving of time through less amount of data (fewer feautures, possibly fewer patterns)
- Better data quality by elimination of unimportant (e.g. unecessary or high-noisy) information.
- Better comprehensability of the results of the pattern recognition process in smaller dimensional spaces; facilitated evaluation; interpretability can be impossibly by given a high number of features
- Reduced costs, for example by only buying the relevant features (e.g. direct marketing) or by reduced computing resources (e.g. in technical applications like signal processing)



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## 1. C WAEKLY RELEVANT, BUT NON-REDUNDANT FEATURES





I: irrelevante Merkmale

II: schwach relevante Merkmale

III: schwach relevante, aber nicht-redundante Merkmale

IV: stark relevante Merkmale

III+IV: optimale Teilmenge



- A. What are the tasks and goals of feature selection?
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#### 1. DRG, SFG, SBG, BG



Creating feature subsets, what is the advantage of Random Generation (RG) over Sequential Forward Generation (SFG), Sequential Backward Generation (SBG), and Bidirectional Generation (BG)?

Problem with SEG, SRG, and RG:

Problem with SFG, SBG, and BG:

Incrementally removing the most unimportant or adding the most important feature may not end in an optimal solution!

Goal of *RG*:

Prevent local optima

#### 1. E



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#### 1. E SEARCH STRATEGIES



- Exhaustive Search:
  - + Searches through all combinations of features, finds optimum
  - Runtime
- Heuristic Search:
  - Heuristics are applied (e.g. based on expert knowledge)
  - + Certain paths of the search space → Reduced runtime
  - Not guaranteed to finde the optimum
- Non-deterministic Search:
  - Random selection of the next feature subset
  - + Fixed number of iterations → Reduced runtime
  - Global optimum not very likely

## Inconsistency Rate

- A. Look at the Table. This data set represents relevant data for the decision wheter to play tennis or not. The column "Play" represents the class of the sample. First apply binning of the temperature values, in order to reduce the continuous temperature value range to three ordinal values. Also, take care of an equal interval size.
- B. Calculate the inconsistency rate (IR) of the new data set. Why does the IR plays a role for the feature selection?

#### 2. A BINNING I



| Outlook  | Temperature | Humidity | Windy | Play |
|----------|-------------|----------|-------|------|
| overcast | 24          | high     | false | no   |
| rainy    | 12          | normal   | false | no   |
| sunny    | 18          | low      | true  | yes  |
| overcast | 13          | low      | true  | no   |
| sunny    | 23          | high     | true  | yes  |
| rainy    | 24          | normal   | false | yes  |
| rainy    | 19          | high     | true  | no   |
| overcast | 17          | normal   | false | yes  |
| sunny    | 14          | high     | false | yes  |
| overcast | 21          | high     | false | no   |
| sunny    | 17          | low      | true  | yes  |
| rainy    | 18          | high     | true  | no   |
| rainy    | 22          | normal   | false | yes  |
| sunny    | 12          | high     | false | yes  |
| overcast | 10          | low      | true  | no   |
| sunny    | 11          | high     | false | no   |
| overcast | 12          | low      | true  | yes  |
| overcast | 20          | high     | false | yes  |
| sunny    | 16          | low      | true  | no   |
| rainy    | 15          | high     | true  | yes  |
| rainy    | 21          | normal   | false | no   |

#### 2. A BINNING II



#### Values:

10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24

#### Bins:

- 10, 11, 12, 13, 14 Cold
- 15, 16, 17, 18, 19 Mild
- 20, 21, 22, 23 ,24 Warm

#### 2. A BINNING III



| Outlook  | Temperature | Humidity | Windy | Play |
|----------|-------------|----------|-------|------|
| overcast | warm        | high     | false | no   |
| rainy    | cold        | normal   | false | no   |
| sunny    | mild        | low      | true  | yes  |
| overcast | cold        | low      | true  | no   |
| sunny    | warm        | high     | true  | yes  |
| rainy    | warm        | normal   | false | yes  |
| rainy    | mild        | high     | true  | no   |
| overcast | mild        | normal   | false | yes  |
| sunny    | cold        | high     | false | yes  |
| overcast | warm        | high     | false | no   |
| sunny    | mild        | low      | true  | yes  |
| rainy    | mild        | high     | true  | no   |
| rainy    | warm        | normal   | false | yes  |
| sunny    | cold        | high     | false | yes  |
| overcast | cold        | low      | true  | no   |
| sunny    | cold        | high     | false | no   |
| overcast | cold        | low      | true  | yes  |
| overcast | warm        | high     | false | yes  |
| sunny    | mild        | low      | true  | no   |
| rainy    | mild        | high     | true  | yes  |
| rainy    | warm        | normal   | false | no   |



- A. Look at the Table. This data set represents relevant data for the decision wheter to play tennis or not. The column "Play" represents the class of the sample. First apply binning of the temperature values, in order to reduce the continuous temperature value range to three ordinal values. Also, take care of an equal interval size.
- B. Calculate the inconsistency rate (IR) of the new data set. Why does the IR plays a role for the feature selection?

#### 2. B Inconsistency Rate



| Muster     | Klasse: Yes | Klasse: No | IZ           |
|------------|-------------|------------|--------------|
| overcast 1 | 1           | 2          | IZ = 3-2 = 1 |
| overcast 2 | 1           | 2          | IZ = 3-2 = 1 |
| overcast 3 | 1           | 0          | IZ = 1-1 = 0 |
| rainy 1    | 0           | 1          | IZ = 1-1 = 0 |
| rainy 2    | 2           | 1          | IZ = 3-2 = 1 |
| rainy 3    | 1           | 2          | IZ = 3-2 = 1 |
| sunny 1    | 2           | 1          | IZ = 3-2 = 1 |
| sunny 2    | 1           | 0          | IZ = 1-1 = 0 |
| sunny 3    | 2           | 1          | IZ = 3-2 = 1 |

• 
$$IR = \frac{1+1+0+0+1+1+1+0+1}{21} = \frac{6}{21} = \frac{2}{7}$$

#### Why should you check the dataset of inconsistencies after the feature selection?

 Feature selection can produce inconsistencies by removing features.

#### FEATURE SELECTION WITH IR I



Let  $D_i$  and  $D_j$  be two (sub)sets of features with inconsistency rates  $IR(D_i)$  and  $IR(D_i)$ , then:

- $D_i$  and  $D_j$  are indistinguishable, if  $IR(D_i) = IR(D_j)$  and  $|D_i| = |D_i|$
- $D_i$  is preferred to  $D_j$ , if:
  - $IR(D_i) = IR(D_j)$  and  $|D_i| < |D_j|$
  - $IR(D_i) < IR(D_j)$  and  $|D_i| <= |D_j|$

#### FEATURE SELECTION WITH IR II



|       | Haar- |       |         | Sonnen- |          |
|-------|-------|-------|---------|---------|----------|
|       | farbe | Größe | Gewicht | creme   | Ergebnis |
| $i_1$ | 1     | 2     | 1       | 0       | 1        |
| $i_2$ | 1     | 3     | 2       | 1       | 0        |
| $i_3$ | 2     | 1     | 2       | 1       | 0        |
| $i_4$ | 1     | 1     | 3       | 0       | 1        |
| $i_5$ | 3     | 2     | 3       | 0       | 1        |
| $i_6$ | 2     | 3     | 3       | 0       | 0        |
| $i_7$ | 2     | 2     | 3       | 0       | 0        |
| $i_8$ | 1     | 1     | 1       | 1       | 0        |

Which features would you select in this data set?

#### **Minimal Inconsistency Rate** 0 for:

- Hair color and sun creame
- Hair color, size, weight
- Any super set of the two sets above

Result: Hair color and sun creame

**Feature Selection Techniques** 



- A. What is the difference between the wrapper and the filter?
- B. Calculate the Information Gain of every feature in Table 1. Sort your results and begin with the most important one.
- C. What is known by "Automated Branch and Bound Algorithmus" and what are its properties? Create an ABB search tree from the data of Table 1.

#### 3. A WRAPPER VS. FILTER



There are two basic methodologies for the feature selection:

- Filter analyse the quality of features in the feature space.
- Wrapper analyse the quality of features according to a suitable Machine Learning algorithm (e.g. Classification)

In principle, Wrapper achieve better results, as the actual Machine Learning task is taken into account. Though, the computing time is much higher, sometimes not computable in a reasonable amount of time. Hybrid approaches are possible.

#### 3. A FILTER



**Filter** analyse the quality of features in the feature space.

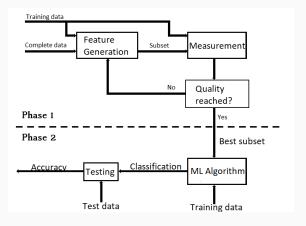


Figure 6: Filter Pipeline

#### 3. A WRAPPER



**Wrapper** analyse the quality of features according to a suitable Machine Learning algorithm (e.g. Classification)

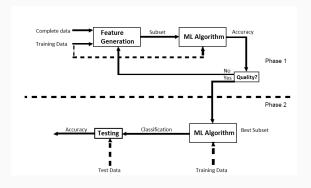


Figure 7: Wrapper Pipeline



- A. What is the difference between the wrapper and the filter?
- B. Calculate the Information Gain of every feature in Table 1. Sort your results and begin with the most important one.
- C. What is known by "Automated Branch and Bound Algorithmus" and what are its properties? Create an ABB search tree from the data of Table 1.



Calculate the Information Gain of every feature in Table 1. Sort your results and begin with the most important one.

#### 3. B Information Gain



• 
$$IG(d) = I(X) - \sum_{l=1}^{L} \frac{|X_{d_l}|}{|X|} I(X_{d_l})$$

• 
$$I(X_{d_l}) = -\sum_{c=1}^{C} p x_{d_l}(c) * log_2 p x_{d_l}(c)$$

• 
$$I(X) = -\sum_{c=1}^{C} px(c) * log_2px(c)$$

#### 3. B ENTROPY



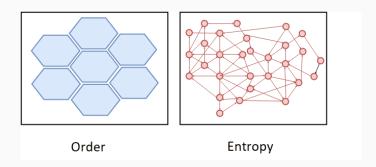


Figure 8: Order vs. Entropy

#### 3. B ENTROPY



- Entropy is a measure for for information based on the probabilty distribution of events
- Hence, it is also a quantification for disorder or chaos
- The more chaotic a system is, the more information is necessary to describe it.
- General idea:
  - Given an unlikely event, then the information we gain, if it occurs is high
  - Given a likely event, then the information we gain, if it occurs is low

#### 3. B Information Gain II



#### Shannon's entropy of events

Shannon's entropy (information content) I of an event with a probability of occurance p is defined as:

$$I = \log_2 \frac{1}{p} = -\log_2 p$$

The unit of information is bit.3

#### Examples:

- I of the event Head in a "Head or Tails" game:  $\log_2 \frac{2}{1} = 1$
- I of the event 5 rolling a dice :  $\log_2 \frac{6}{1} \approx 2,59$
- I of the event odd number rolling a dice:  $\log_2 \frac{6}{3} = 1$

<sup>&</sup>lt;sup>3</sup>If the logarithm of base 2 is used.

#### 3. B Information Gain III



#### Information of events

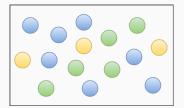
$$I = \log_2 \frac{1}{p} = -\log_2 p$$

- blue ball:  $I = -\log_2 \frac{7}{15} = 1.09$
- green ball:

$$I = -\log_2 \frac{5}{15} = 1.58$$

yellow ball:

$$I = -\log_2 \frac{3}{15} = 2.32$$



#### 3. B Information Gain IV



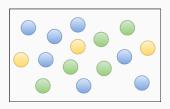
#### **Entropie**

The entropy of a series of events is defined as:

$$H = -\sum_{i=1}^n p_i \log_2 p_i,$$

where *n* is the number of possible events.

• 
$$H = -\left[\frac{7}{15} * \log_2 \frac{7}{15}\right] - \left[\frac{5}{15} * \log_2 \frac{5}{15}\right] - \left[\frac{3}{15} * \log_2 \frac{3}{15}\right] = 1.51$$



#### 3. B Information Gain V



• 
$$IG(d) = I(X) - \sum_{l=1}^{L} \frac{|X_{d_l}|}{|X|} I(X_{d_l})$$

• 
$$I(X_{d_l}) = -\sum_{c=1}^{C} p x_{d_l}(c) * log_2 p x_{d_l}(c)$$

• 
$$I(X) = -\sum_{c=1}^{C} px(c) * log_2px(c)$$

#### 3. B Information Gain VI



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| Outlook  | Temperature | Humidity | Windy | Play |
|----------|-------------|----------|-------|------|
| Sunny    | Hot         | High     | False | No   |
| Sunny    | Hot         | High     | True  | No   |
| Overcast | Hot         | High     | False | Yes  |
| Rainy    | Mild        | High     | False | Yes  |
| Rainy    | Cool        | Normal   | False | Yes  |
| Rainy    | Cool        | Normal   | True  | No   |
| Overcast | Cool        | Normal   | True  | Yes  |
| Sunny    | Mild        | High     | False | No   |
| Sunny    | Cool        | Normal   | False | Yes  |
| Rainy    | Mild        | Normal   | False | Yes  |
| Sunny    | Mild        | Normal   | True  | Yes  |
| Overcast | Mild        | High     | True  | Yes  |
| Overcast | Hot         | Normal   | False | Yes  |
| Rainy    | Mild        | High     | True  | No   |

#### 3. B Information Gain VII



- Calculate the information Berechne den Informationsgehalt der Klassenverteilung
  - $I(X) = -\sum_{c=1}^{C} px(c) * log_2 px(c)$
  - $I(X) = -\frac{5}{14}log_2(\frac{5}{14}) \frac{9}{14}log_2(\frac{9}{14}) = 0.94$
- 2. Calculate the information content of every feature value (= manifestation or possible instance):
  - $I(X_{d_i}) = -\sum_{c=1}^{C} px_{d_i}(c) * log_2 px_{d_i}(c)$
  - $I(X_{sunny}) = -\frac{3}{5}log_2(\frac{3}{5}) \frac{2}{5}log_2(\frac{2}{5}) = 0.97$
  - $I(X_{rainy}) = -\frac{2}{5}log_2(\frac{2}{5}) \frac{3}{5}log_2(\frac{3}{5}) = 0.97$
  - $I(X_{overcast}) = 0 log_2(\frac{4}{4}) = 0$
- 3. Calculate the Information Gain of every feature:
  - $IG(Outlook) = I(X) \sum_{l=1}^{L} \frac{|X_{d_l}|}{|X|} I(X_{d_l})$
  - $IG(Outlook) = 0.94 (\frac{5}{14} * 0.97 + \frac{5}{14} * 0.97 + \frac{4}{14} * 0) = 0.247$

#### 3. B Information Gain VIII



- 4. Compare the features by sorting them by their Information Gain values:
  - IG(Outlook) = 0.247, IG(Humidity) = 0.15, IG(Windy) = 0.05, IG(Temp) = 0.03



- A. What is the difference between the wrapper and the filter?
- B. Calculate the Information Gain of every feature in Table 1.Sort your results and begin with the most important one.
- C. What is known by "Automated Branch and Bound Algorithmus" and what are its properties? Create an ABB search tree from the data of Table.

#### 3. C ABB SEARCH TREE



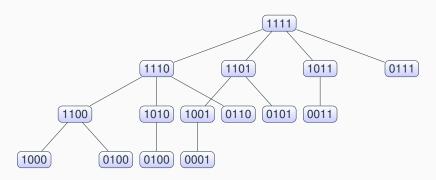


Figure 9: ABB Search Tree

### Remark: (1, 1, 1, 1) corresponds to

(Outlook, Temperature, Humidity, Windy)

#### 3. C ABB SEARCH TREE



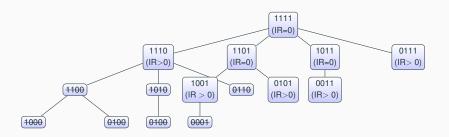


Figure 10: ABB Suchbaum Pruning

⇒ One out of the feature selections (Outlook, Temperature, Windy) and (Outlook, Humidity, Windy) will be chosen.

# SAX Algorithm with Python



**A.** Download the jupyter notebook **4\_SAX.ipynb** from Open Olat. First, calcluate the Euclidean distance of the two time series. Afterwards, apply the steps of the SAX algorithm and compare the distance of the two strings. What attracts your attention? Which paramaters can be adapted two achieve better results?

#### 4. A



Download the jupyter notebook **4\_SAX.ipynb** from Open Olat. First, calcluate the Euclidean distance of the two time series. Afterwards, apply the steps of the SAX algorithm and compare the distance of the two strings. What attracts your attention? Which paramaters can be adapted two achieve better results?

## Signature Task



**A.** Construct different feautures for the signatures and calculate the IG and IR values.