Kiel University
Christian-Albrechts-Universität zu Kiel

Faculty of Engineering

Intelligent Systems

Excersice 6 – Segmentation

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Top down segmentation

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Given two time series in Figure 1, apply a top down segmentation with a maximum approximation error of 1 by using the error function:

$$\sum_{i=1}^{n} |x_{t_i} - \tilde{x}(t_i)|,$$

where $S = \{t_i\}_{i=1}^{n}$ is a segment of length n, x_t are the measurements at time t, and $\tilde{x} : \mathbb{R} \to \mathbb{R}$ is the approximation.
For the approximation function $\tilde{x}$ use:

A. A constant function.
B. A polynom of degree 1.

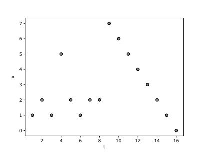

Figure 1: Some timeseries

Offline techniques

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Top-down approach

- Given: Time series, modelling or approximation method (often approximation with polynomial of degree 1) and (abort) criterion (often threshold value for approximation error)
- Wanted: Segmentation of the time series into as few sections as possible so that the specified criterion is met for all segments (e.g. approximation error smaller than specified threshold value).
- Basic idea: starting from only one segment (entire time series), each segment is further subdivided until the termination criterion for all sub-segments is fulfilled.

Offline techniques (2)

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Process (using the example of an approximation and consideration of the error as a termination criterion):

- Calculate the approximation error for the currently considered segment (initially: entire time series).
- For each possible position at which the segment can be segmented into two parts, calculate the sum of the approximation errors of the two sub-segments.
- Split the segment at the point where the greatest reduction in error is achieved.
- If the error of one of the two partial segments is greater than the given threshold value, subdivide this further

1. A Top down segmentation with constant function

Approximate n points with the help of a constant function means:

$$\tilde{x}(t) = c,$$

and choose $c \in \mathbb{R}$ that the minimum approximation error is minimised.

$$c^* = \min_{c \in \mathbb{R}} \sum_i |c - x_i|$$

(Remark: Simplified notation: $x_i := x_{t_i}$)
Through differentiation of above equation and setting it to 0:

$$c^* = \frac{1}{n} \sum_{i=1}^{n} x_i = \overline{x_i}$$

1. B

A. A constant function.

B. A polynom of degree 1.

1. B A Top down segmentation with polynom of degree 1

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Approximate n points with the help of a polynom of degree 1 means:

$$\tilde{x}(t) = at + b,$$

and choose $a, b \in \mathbb{R}$ that the minimum approximation error is minimised.

$$(a^*, b^*) = \min_{a,b \in \mathbb{R}} \sum_i |at_i + b - y_i|$$

Through differentiation of above equation* and setting it to 0:

$$a^* = \frac{\sum_{i=1}^n t_i x_i - n\bar{t}\bar{x}}{\sum_{i=1}^n t_i^2 - n\bar{t}^2}$$

$$b^* = \bar{x} - a\bar{t}$$

1. B A Top down segmentation with polynom of degree 1 - Derivation I

$$g = \sum_{i=1}^{n} (at_i + b - x_i)^2 = \sum_{i=1}^{n} a^2 t_i^2 + 2abt_i - 2at_i x_i - 2bx_i + b^2 + x_i^2$$

$$\frac{\partial g(a,b)}{\partial b} = \sum_{i=1}^{n} 2at_i - 2x_i + 2b = 0$$

$$\Leftrightarrow \sum_{i=1}^{n} at_i - \sum_{i=1}^{n} x_i + nb = 0$$

$$\Leftrightarrow b = -a\frac{\sum_{i=1}^{n} t_i}{n} + \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\Leftrightarrow b = \overline{x} - a\overline{t} \quad (*)$$

1. B A Top down segmentation with polynom of degree 1 - Derivation II

$$\frac{\partial g(a,b)}{\partial a} = \sum_{i=1}^{n} 2at_i^2 + 2bt_i - 2t_i x_i = 2\sum_{i=1}^{n} t_i(at_i + b - x_i) = 0$$

Important: Here (*) substitute b, because b depents on a (b = b(a)) and has to be taken into account for the partial derivative!

$$\Leftrightarrow 2\sum_{i=1}^{n} t_i(at_i + (\overline{x} - a\overline{t}) - x_i) = 0$$

1. B A Top down segmentation with polynom of degree 1 - Derivation III

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$$\Leftrightarrow \sum_{i=1}^{n} a t_i^2 - a\bar{t} t_i + t_i \bar{x} - t_i x_i = 0$$

$$\Leftrightarrow a \sum_{i=1}^{n} t_i^2 - \bar{t} t_i = \sum_{i=1}^{n} t_i x_i - \sum_{i=1}^{n} t_i \bar{x}$$

$$a = \frac{\sum_{i=1}^{n} t_i x_i - \sum_{i=1}^{n} t_i \bar{x}}{\sum_{i=1}^{n} t_i^2 - t_i \bar{t}} = \frac{\sum_{i=1}^{n} t_i x_i - n\bar{t}\bar{x}}{\sum_{i=1}^{n} t_i^2 - n\bar{t}^2}$$

Bottom up segmentation

2. A

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A. a constant function.

B. a polynom of degree 1.

Offline techniques (5)

Bottom-up approach:

- Requirements as for top-down procedures: Time series, error function and threshold value
- A segmentation of the time series into as few segments as possible is also required so that the threshold value is not exceeded for all segments.
- Basic idea: Starting from a segmentation as fine as possible, segments are grouped together step by step until it is no longer possible to group them together without violating the targets.

Offline techniques (6)

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Process (using the example of an approximation and consideration of the error as a termination criterion):

- Segment the entire time series into as many sub-segments as possible (e.g. for an approximation of each segment with a polynomial of degree K in segments of length K + 1).
- Calculate the increase of the approximation error for the combination of two adjacent segments.
- Combine adjacent segments with the minimum increase of the error as long as the error of the resulting segment does not exceed the threshold.

Python: Segmentation

4. A

A. **Download the jupyter notebook *07_Segmentation.ipynb* from Open Olat. Try to recap the handwritten solution by following the Jupyter Notbeook Script.**