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Solution T-3.1

Here,

$$p(x=0, Y=0) = p(Y=0|x=0) \cdot p(x=0)$$

$$= \frac{N-M-1}{N-1} \cdot \frac{N-M}{N}$$

$$p(x=0, Y=1) = p(Y=1|x=0) \cdot p(x=0)$$

$$= \frac{M}{N-1} \cdot \frac{N-M}{N}$$

$$p(x=1, Y=0) = p(Y=0|x=1) \cdot p(x=1)$$

$$= \frac{N-M}{N-1} \cdot \frac{M}{N}$$

Now,

$$p_x(x=0) = p(x=0, Y=0) + p(x=0, Y=1)$$

$$p_y(Y=0) = p(x=0, Y=0) + p(x=1, Y=0)$$

$$= p(x=0, Y=0) + p(x=0, Y=1)$$

$$= p_x(x=0)$$

$$\therefore p_x(x=0) = \frac{N-M-1}{N-1} \cdot \frac{N-M}{N} + \frac{M}{N-1} \cdot \frac{N-M}{N}$$

$$= \frac{(N-M-1)(N-M) + M(N-M)}{N(N-1)}$$

$$= \frac{(N-M)(N-M-1+M)}{N(N-1)}$$

$$= \frac{(N-M)(N-1)}{N(N-1)}$$

$$= \frac{N-M}{N}$$

$$P_X(x=0) \cdot P_Y(y=0) = \frac{N-M}{N} \cdot \frac{N-M}{N}$$

$$\neq \frac{N-M-1}{N-1} \cdot \frac{N-M}{N} = P(x=0, y=0)$$

Therefore, the random variables x and y are dependent.

Solution T-3.2

Here,

$$P(\text{red}/\text{urn1}) = \frac{1}{2}$$

$$P(\text{red}/\text{urn2}) = \frac{3}{10}$$

$$P(\text{urn1}) = \frac{1}{2} = P(\text{urn2})$$

$$P(\text{red}) = P(\text{red}/\text{urn1}) \cdot P(\text{urn1}) + P(\text{red}/\text{urn2}) \cdot P(\text{urn2})$$

$$= \frac{1}{2} \cdot \frac{1}{2} + \frac{3}{10} \cdot \frac{1}{2}$$

$$= \frac{1}{4} + \frac{3}{20}$$

$$= \frac{5+3}{20} = \frac{8}{20} = \frac{2}{5}$$

From Bayes rule we get,

$$P(\text{urn1}/\text{Red}) = \frac{P(\text{red}/\text{urn1}) \cdot P(\text{urn1})}{P(\text{red})}$$

$$= \frac{1/2 \cdot 1/2}{2/5}$$

$$= \frac{5}{8}$$

Solution T-3.3

Let

Recession coming = A

Recession or not coming = B

Recession prediction = C.

$$P(C \mid A) = \frac{8}{10}$$

$$P(C \mid B) = \frac{1}{10}$$

$$P(A) = \frac{2}{10}$$

$$P(B) = 1 - P(A)$$

$$= \frac{8}{10}$$

And, $P(C) = P(A, C) + P(B, C)$

$$= P(C \mid A) \cdot P(A) + P(C \mid B) \cdot P(B)$$

$$= 8/10 \cdot 2/10 + 1/10 \cdot 8/10$$

$$= \frac{16+8}{100} = \frac{6}{25}$$

From Bayes Rule, we get

$$P(\text{Recession coming} \mid \text{recession pred}) = \frac{P(C \mid A)\, P(A)}{P(C)}$$

or, $P(A \mid C)$

$$= \frac{8/10 \cdot 2/10}{6/25} = 2/3$$