Group member

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Solution:

Given,

$$f_x(x|\theta) = f_x(x|\lambda) = \lambda e^{-\lambda x}$$

Likelihood function is

$$L(\lambda) = \prod_{i=1}^{n} f_x(x_i|\lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i}$$

log-likelihood function

$$\ln L(\lambda) = \sum_{i=1}^{n} \ln(f_x(x_i|\lambda)) = \sum_{i=1}^{n} \ln(\lambda e^{-\lambda x_i})$$

$$= \sum_{i=1}^{n} (\ln \lambda - \lambda x_i \ln e)$$

$$= \sum_{i=1}^{n} \ln \lambda - \sum_{i=1}^{n} \lambda x_i$$

$$= n \ln \lambda - \lambda \sum_{i=1}^{n} x_i$$

Necessary condition for optimization

$$\frac{d}{d\lambda} \ln L(\lambda) = 0$$

$$\Rightarrow \frac{d}{d\lambda}\left(n \ln \lambda - \lambda \sum_{i=1}^{n} x_i\right) = 0$$

$$\Rightarrow \frac{n}{\lambda} = \sum_{i=1}^{n} x_i$$

$$\Rightarrow \lambda = \frac{1}{1/n \sum_{i=1}^{n} x_i} = \frac{1}{\mu_x}$$