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Solution 4.1

We know the decision rule for a MAP classifier is $\omega_{opt} = \arg\max_{\omega_k} P(\omega_k|x)$ and discriminant function is

$$g_i(x) = P(\omega_i|x)$$
$$= P(x|\omega_i)P(\omega_i)$$

We can use log function as it is increasing monotonically

$$g_i(x) = \log P(x|\omega_i) + \log P(\omega_i)$$

For two class decision boundary is

$$g_i(x) = g_j(x)$$

$$\therefore \log P(x|\omega_i) + \log P(\omega_i) = \log P(x|\omega_j) + \log(\omega_j)$$

if $\log P(\omega_i) = \log(\omega_j)$, we get

$$\log P(x|\omega_i) = \log P(x|\omega_j)$$
$$\Rightarrow \log P(x|\omega_i) - \log P(x|\omega_j) = 0$$
$$\Rightarrow \log \frac{P(x|\omega_i)}{P(x|\omega_j)} = 0$$

So, the special required condition is the prior probability have to be equal.

Solution - 4.2

Here,

$$\omega_1 = \{(3,8), (2,6), (3,4), (4,6)\}$$

$$\omega_2 = \{(3,0), (3,-4), (1,-2), (5,-2)\}$$

$$\mu_1 = \begin{pmatrix} 3 \\ \cancel{8}6 \end{pmatrix}, \quad \Sigma_1 = \begin{pmatrix} 1/2 & 0 \\ 0 & 2 \end{pmatrix}, \quad \Sigma_1^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\mu_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \quad \Sigma_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \quad \Sigma_2^{-1} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

Here, $\Sigma = \frac{1}{n} \sum_{i}^{n} (x_i - \bar{x})(y_i - \bar{y})$

$$\det \Sigma_1 = 1, \quad \det \Sigma_2 = 4$$

Discriminant function's

$$g_1(x) = \ln p(x|\omega_1) + \ln(\omega_1)$$

$$g_2(x) = \ln p(x|\omega_2) + \ln(\omega_2)$$

and $g_1(x) = g_2(x)$.

$$\Rightarrow \ln p(x|\omega_1) = \ln p(x|\omega_2) \quad [\because \ln(\omega_1) = \ln(\omega_2)]$$

We know for multivariate normal density

$$p(x|\omega) = \frac{1}{\sqrt{(2\pi)^2 \cdot \det(\Sigma)}} e^{-\frac{1}{2}(x-\mu)^T \cdot \Sigma^{-1} \cdot (x-\mu)}$$

$$\ln p(x|\omega) = -\frac{1}{2}\ln\left[(2\pi)^2 \cdot \det(\Sigma)\right] - \frac{1}{2}(x-\mu)^t \Sigma^{-1}(x-\mu) \cdot \ln e$$

$$\therefore \ln p(x|\omega_1) = -\frac{1}{2}\left\{\ln\left[(2\pi)^2\right] + \ln(1) + (x-\mu_1)^t \Sigma_1^{-1}(x-\mu_1)\right\}$$

$$= -\frac{1}{2}\left\{\ln(2\pi)^2 + \left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \end{pmatrix}\right)^t \begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix}\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \end{pmatrix}\right)\right\}$$

$$= -\frac{1}{2}\left\{\ln(2\pi)^2 + \left(2(x_1-3) \quad \tfrac{1}{2}(x_2-6)\right)\begin{pmatrix} x_1-3 \\ x_2-6 \end{pmatrix}\right\}$$

$$= -\frac{1}{2}\left\{\ln(2\pi)^2 + 2(x_1-3)^2 + \frac{1}{2}(x_2-6)^2\right\}$$

again,

$$\ln p(x|\omega_2) = -\frac{1}{2}\left\{\ln(2\pi)^2 + \ln(4) + (x-\mu_2)^t \cdot \Sigma_2^{-1}(x-\mu_2)\right\}$$

$$= -\frac{1}{2}\left\{\ln(2\pi)^2 + \ln(4) + \left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix}\right)^t \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \cdot \left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix}\right)\right\}$$

$$= -\frac{1}{2}\left\{\ln(2\pi)^2 + \ln(4) + \left(\tfrac{1}{2}(x_1-3) \quad \tfrac{1}{2}(x_2+2)\right)\begin{pmatrix} x_1-3 \\ x_2+2 \end{pmatrix}\right\}$$

$$= -\frac{1}{2}\left\{\ln(2\pi)^2 + \ln(4) + \frac{1}{2}(x_1-3)^2 + \frac{1}{2}(x_2+2)^2\right\}$$

$$\ln p(x|\omega_1) = \ln p(x|\omega_2)$$

$$\Rightarrow -1/2\left\{\ln(2\pi)^2 + 2(x_1-3)^2 + \frac{1}{2}(x_2-6)^2\right\} = -\frac{1}{2}\left\{\ln(2\pi)^2 + \ln(4) + \frac{1}{2}(x_1-3)^2 + \frac{1}{2}(x_2+2)^2\right\}$$

$$\Rightarrow 2(x_1-3)^2 + \frac{1}{2}(x_2-6)^2 = \ln(4) + \frac{1}{2}(x_1-3)^2 + \frac{1}{2}(x_2+2)^2$$