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Example solution for Series #11

Exercise 1 — 10 Points

You get for each correct answer 1 point, but you will lose 1 point for an incorrect answer.

a) All recursive languages are also recursively enumerable. ☒ true ◯ false

b) If there exists a $w \in \Sigma^*$ such that a DTM $\mathcal{A}$ loops on w, then $L(\mathcal{A}) = \emptyset$. ◯ true ☒ false

c) Deterministic Turing machines are more powerful than PDAs. ☒ true ◯ false

d) The tape alphabet and the input alphabet of a Turing machine are disjoint. ◯ true ☒ false

e) $\{a^n b^n c^n | n \in \mathbb{N}_0\}$ is recognisable by a 1DTM. ☒ true ◯ false

f) Each semidecidable language is also decidable. ◯ true ☒ false

g) The more tapes a DTM has the more powerful it is. ◯ true ☒ false

h) 2DPDAs are as powerful as DTMs. ☒ true ◯ false

i) 1-counter automaton are as powerful as DTMs. ◯ true ☒ false

j) $\{a^n b^n c^n | n \in \mathbb{N}_0\}$ is recognisable by an enumeration machine. ☒ true ◯ false

Exercise 2 — 7 Points

Give the following definitions:

a) 1DTM (4P)
b) 1DTM $\mathcal{A}$ loops on $x \in \Sigma^*$ (1P)
c) 1DTM $\mathcal{A}$ total (1P)
d) $L \subseteq \Sigma^*$ semidecidable (1P)

Solution:

a) $\mathcal{A} = (Q, \Sigma, \Gamma, \vdash, \sqcup, \delta, q_0, q_a, q_r)$ 1DTM with
- finite set of states Q with start state q_0
- finite input alphabet Σ
- finite tape alphabet $\Gamma \supseteq \Sigma$
- blank symbol $\sqcup \in \Gamma \backslash \Sigma$
- left endmarker $\vdash \in \Gamma \backslash \Sigma$
- transition function $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$
- accepting state q_a
- rejecting state $q_r \neq q_a$

b) $\mathcal{A}$ does not halt on x
c) $\mathcal{A}$ halts on all inputs
d) L recursively enumerable

Exercise 3 — 24 Points

a) Prove that $L' = \{ww^R w | w \in \{a, b\}\}$ is not context-free. (9P)

b) Decide whether the following claims are correct or not. Justify your answer.
   a) There exists a 1DTM $\mathcal{A}$ that accepts an infinite language without moving the head more than one position away from position 1 (notice that at position 0 we have $\vdash$). (1P)
   b) For all 1DTM $\mathcal{A}_1$ exists a 1DTM $\mathcal{A}_2$ with $L(\mathcal{A}_1) = L(\mathcal{A}_2)$ and $\mathcal{A}_2$ has only the two states q_a and q_r. (1P)
   c) For each Turing-Machine $\mathcal{A}$ on the alphabet Σ moving the head only to the right, $L(\mathcal{A}) \subseteq \Sigma^+$. (1P)

c) Construct a 1DTM for $L = \{w \in \Sigma^* \mid \exists v \in \Sigma^* : w = vv^R\}$. Describe the behaviour of your machine. (10P)

Solution:

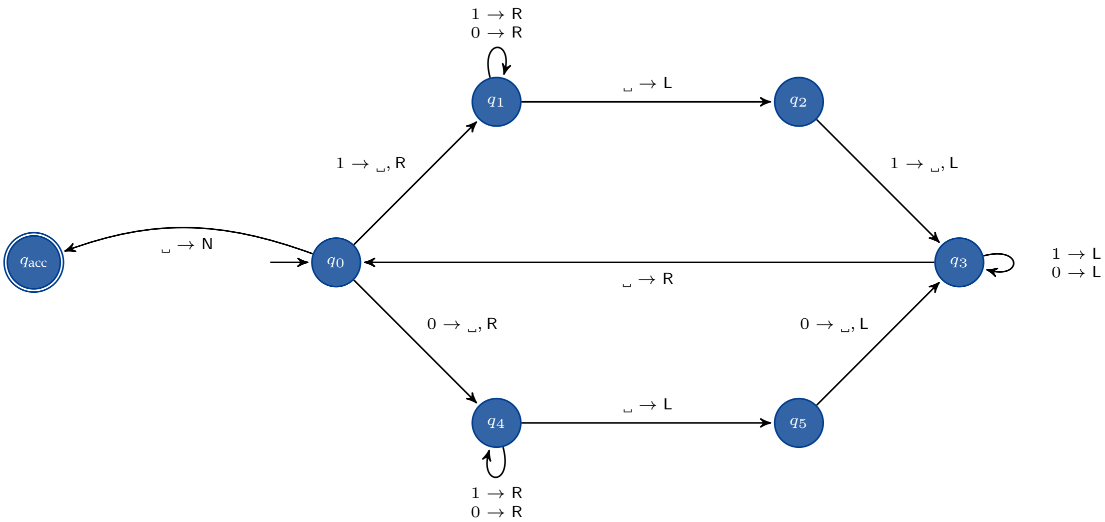
a) Suppose that L' is context-free. (0.5P)
By the Pumping Lemma for context-free language there exits $p \in \mathbb{N}$. (0.5P)
Set $z = a^p b^{2p} a^{2p} b^p$. (0.5P)
Then we have $|z| = 6p \geq p$ and by $z = (a^p b^p)(a^p b^p)^R(a^p b^p)$ we have $z \in L'$. (1P)
Consequently there exists $u, v, w, x, y \in \Sigma^*$ with (0.5P)

   (1) $z = uvwxy$ (0.5P)
   (2) $|vx| > 0$ (0.5P)
   (3) $|vwx| \leq p$ (0.5P)
   (4) $\forall i \in \mathbb{N}_0 : uv^iwx^iy \in L'$. (0.5P)

Sketch of proof for the case-analysis:
case 1: $|u| = 0$ (0.5P)
Then $vwx \in \{a\}^*$ (0.5P)
and uv^0wx^0y has strictly less a in the beginning than in the end. (0.5P)
case 2: $|u| \leq p$ (0.5P)
Then vwx is at most in the first b-block. (0.5P)
By doubling v and x the number of b in the first b-block is increased but not the number of b in the last b-block. (0.5P)
The remaining cases are analogous. (0.5P)
Thus in each case there exists an $i \in \mathbb{N}_0$ such that $uv^iwx^iy \notin L'$ (0.5P)
and consequently L' is not context-free.

b)
   a) The claim is correct. If $q_0 = q_a$ holds, $\mathcal{A}$ accepts every word and thus for the input alphabet Σ, $\mathcal{A}$ accepts Σ^*.
   b) The claim is not correct. Since $\mathcal{A}_2$ has only the states q_a and q_r, the initial state is one of them. This implies that $\mathcal{A}_2$ accepts either Σ^* or $\emptyset$.
   c) The claim is not correct. If q_0 is again q_a we accept Σ^*. The direction of the head is not important.

c)



Description of the machine: The machine compares always the first letter with the last letter. Are both the same they are replaced by ␣. If the tape is empty, we accept.

You can visualise the machine on `https://turingmachinesimulator.com/` with the following instructions

```
name: palindromes
init: q0
accept: qacc

q0,1
q1,_,>

q0,0
q4,_,>

q0,_
qacc,_,-

q1,0
q1,0,>

q1,1
q1,1,>

q1,_
q2,_,<

q2,1
q3,_,<

q3,1
q3,1,<

q3,0
q3,0,<

q3,_
q0,_,>

q4,0
q4,0,>

q4,1
q4,1,>

q4,_
q5,_,<

q5,0
q3,_,<
```