Logical and Theoretical Foundations of Computer Science

LaTFoCS

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Dependable Systems Group

Why logic?

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The Art of Thinking

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- Confirmatio: set out arguments as well as evidence
- Refutatio: refute opposing arguments
- Conclusio: summary of the argument

Syllogisms

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- Some doctors are fools.
  All fools are rich.
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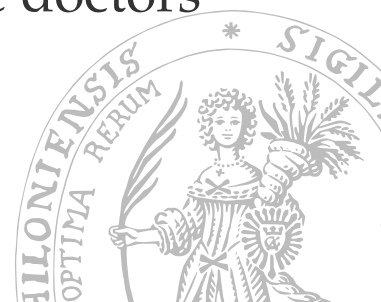

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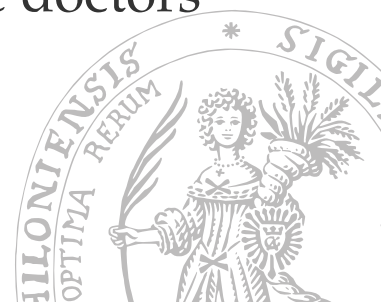

... and why in Computer Science?

automated conclusions in

- autonomous driving
- meteorology
- washing machines
- medicine

How to transform our knowledge into Computers?

1. we need a formalism to describe it (formulae)
2. we need a formalism how to interprete it (interpretation, models)
3. we need some ensurance that everything works as intended
   - either formula or negation are true (Consistency)
   - everything provable is really true (Soundness)
   - everything true is provable (Completeness)
   - the axioms are independent of each other (Indepence)

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- But what are computers?
- What is computation?
- As decently as we look into logic, we have to look into the definition of computation!

What is computation?

Definition (Computation (Oxford Dictionary))

1.
   - (mass noun) The action of mathematical calculation.
   - (count noun) ‘statistical computations’
2. The use of computers, especially as a subject of research or study.

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while TRUE
   i++;
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- Turing: Halt-Problem is undecidable!

Theory of Computation

We have to investigate what computation means in detail!

- Which kinds of problems do we have?
- How to encode problems?
- What is the definition of computable/decidable?

Problems and Encoding

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Here only Decision-Problems!

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informal:

- Output: **yes**, **no** (resp. 0/1 or true/false)

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- set of all possible inputs $A \subseteq \Sigma^*$
- set of all *yes*-instances $B \subseteq A$

Course Outline

- Propositional Logic
- Application to *real world* proofs.
- Theory of Computation: Chomsky Hierarchy
- Predicate Logic
- Application to *real world* proofs.
- Complexity Theory: P, NP, NPC