Logic and Theoretical Foundation of Computer Science

LaTFoCS

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Kiel University

Dependable Systems Group

SEIT 350 JAHREN GANZ WEIT OBEN

CAU

Satisfiability, Validity, and Consequence in Propositional Logic

Motivation

Consider four chairs and Peter, Anne, Mary, and Paul. We have the following constraints:

- Peter wants to have two neighbours.
- Mary does not want to sit next to Paul.
- Paul wants to sit only with neighbouring women.
- Anne wants to sit on chair 3.

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Do the all get to sit?

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Be careful with the negation!

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- iff φ is satisfiable $\square$

How to decide whether a formula has a property?

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Definition

Let $\Psi \subseteq \Phi$. An algorithm A is a decision procedure for Ψ if for all $\varphi \in \Phi$ it returns `true` iff $\phi \in \Psi$.

Remarks on Decision Procedures

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Definition

A decision procedure is called a refutation procedure if the property is proven by refuting the negation of the formula.

Easy decision procedure for satisfiability

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What is the problem with this decision procedure?

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Example

$\Psi_1 = \{p, \neg p \lor q, q \land r\}$ is satisfiable, $\Psi_2 = \{p, \neg p \land q\}$ not.

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The proof is left to the reader.

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Notice: φ needs to be `true` only under all interpretations satisfying Ψ; we do not care for the remaining interpretations.

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Set $\varphi = q \vee r \vee p$. Since all β set at least one atom to `true`, φ is satisfied in all of them. Notice: we don't care for the remaining to interpretations in which φ is also true and neither for the one, φ is not satisfied for.

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- our language for talking **about** formulae contains now $\equiv$ and $\models$

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- $\models$ is the counterpart of $\rightarrow$

Properties of $\models$

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- everything that is modelled by Ψ holds under these circumstances

Theories

Definition

$T \subseteq \Phi$ closed under logical consequence iff for all $\varphi \in \Phi$, $T \models \varphi$ implies $\varphi \in T$ - in this case T is called a theory and all elements of T are called theorems

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  - we claim something
  - we proof that it holds within this setting

Axioms

Firstly the formal definition:

Definition

A theory T is axiomatisable iff there exists $A \subseteq \Phi$ with $T = \{\varphi | A \models \varphi\}$ - in this case A contains the axioms of T.

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You may always assume that

- Peano-Axioms hold
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What else can we assume to be true? What else are we allowed to use in a proof?

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Math(etimatical proof) describes the way from A to B but nobody tells you how to come to A or especially what you want in B.

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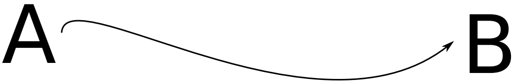

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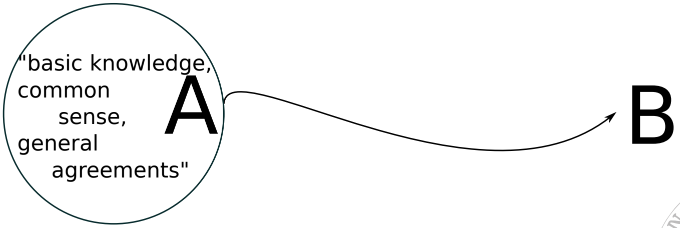

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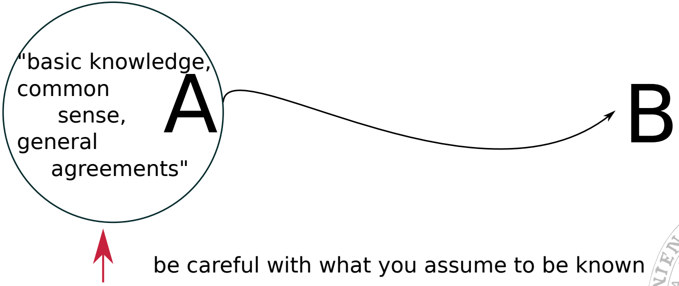

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in general two scenarios: learning purposes and *real* life

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- common sense if it is not part of the new stuff
    - if you are said to prove that the difference between two even numbers is even, you are not allowed to write "obvious by common sense"
- since neither we nor you can rely on the common bachelor studies, ask if you are unsure what you are allowed to use

Dangerous Words

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what is the problems with these words?

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- in a p2p relation you can only guess what is known and what not

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Similar

- in the word meaning: similis (Latin)
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- take care of different parts but with decent work and a scrap paper you will get from one part to the other

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  - Can I assume w.l.o.g. each human being to be either male or female?

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Be careful and think more than twice if you have perhaps *with loss of generality*.

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- Try to prove some of the *easy* stuff on your own!