

Faiz Ahmed  
user: stu225473

### Exercise 1:

- a) Equivalence relations are reflexive, antisymmetric, and transitive. == **False**  
Reason: In mathematics, an equivalence relation is a binary relation that is reflexive, symmetric and transitive.
- b) In a group  $(G, \cdot, e)$  where  $a^{-1}$  denotes the inverse element of  $a \in G$ , the inverse element of  $a \cdot b$  is  $b^{-1} \cdot a^{-1}$ . == **False**  
Reason:  $b$  is an element of  $G$
- c) Define the complement as  $A^c = U - A$  for a set  $U$  and  $A \subseteq U$ . The complement operation is idempotent. == **True**  
In computing, an idempotent operation is one that has no additional effect if it is called more than once with the same input parameters. For example, removing an item from a set can be considered an idempotent operation on the set.  
In mathematics, an idempotent operation is one where  $f(f(x)) = f(x)$ . For example, the  $\text{abs}()$  function is idempotent because  $\text{abs}(\text{abs}(x)) = \text{abs}(x)$  for all  $x$ .
- d) The intersection on sets is commutative. == **True**  
Example: Let  $A = \{x : x \text{ is a whole number between 4 and 8}\}$  and  $B = \{x : x \text{ is an even natural number less than 10}\}$ .  
 $A \cap B = \{5, 6, 7\} \cap \{2, 4, 6, 8\} = \{6\} = \{2, 4, 6, 8\} \cap \{5, 6, 7\} = B \cap A$ .
- e) The usual subtraction  $:- Z \times Z \rightarrow Z$  is associative. == **False**
- f) The natural numbers  $N$  are not closed under division. == **True**  
A set is closed (under an operation) if and only if the operation on any two elements of the set produces another element of the same set. If the operation produces even one element outside of the set, the operation is not closed.
- g) Functions are left-total and right-unique relations. = **False**  
A function  $f : A \rightarrow B$  is a binary relation over  $A$  and  $B$  that is right-unique.  
(For this reason, a binary relation that is right-unique is often called functional).
- h) Each group is abelian w.r.t. its operation. == **False**  
Ans: Group has four (closure, associativity, identity, Inverse ) axioms but Abelian Group has five ( $+$ , commutativity ) axioms
- i)  $(Z, +, \cdot, 0, 1)$  is a field with the usual addition and multiplication. == **True**
- j) Two sets  $A$  and  $B$  are equal iff  $A \subseteq B$  and  $B \subseteq A$ . = **True**