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Using Prediction Intervals to Improve Information Quality of Bankruptcy Prediction Models

Marco Lam Brad S. Trinkle

USING PREDICTION INTERVALS TO IMPROVE INFORMATION QUALITY OF BANKRUPTCY PREDICTION MODELS

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ABSTRACT

The purpose of this paper is to improve the information quality of bankruptcy prediction models proposed in the literature by building prediction intervals around the point estimates generated by these models and to determine if the use of the prediction intervals in conjunction with the point estimated yields an improvement in predictive accuracy over traditional models. The authors calculated the point estimates and prediction intervals for a sample of firms from 1991 to 2008. The point estimates and prediction intervals were used in concert to classify firms as bankrupt or non-bankrupt. The accuracy of the tested technique was compared to that of a traditional bankruptcy prediction model. The results indicate that the use of upper and lower bounds in concert with the point estimates yield an improvement in the predictive ability of bankruptcy prediction models. The improvements in overall prediction accuracy and non-bankrupt firm prediction accuracy are statistically significant at the 0.01 level. The authors present a technique that (1) provides a more

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complete picture of the firm's status, (2) is derived from multiple forms of evidence, (3) uses a predictive interval technique that is easily repeated, (4) can be generated in a timely manner, (5) can be applied to other bankruptcy prediction models in the literature, and (6) is statistically significantly more accurate than traditional point estimate techniques. The current research is the first known study to use the combination of point estimates and prediction intervals to in bankruptcy prediction.

Keywords: Bankruptcy; predictive ability; prediction accuracy; prediction intervals; logistic regression

INTRODUCTION

Management decision making is only optimal when management receives accurate information. Therefore, methods that improve the information quality of prediction models are an important area of study in decision making research. The purpose of this paper is to improve the information quality of bankruptcy prediction models proposed in the literature by building prediction intervals around the point estimates generated by these models and to determine if the use of the prediction intervals in conjunction with the point estimated yields an improvement in predictive accuracy over traditional models.

There has been a long stream of research on predicting bankruptcy (Altman, Marco, & Varetto, 1994; Baixauli, Alvarez, & Módica, 2012; Beaver, 1966; Gepp, Kumar, & Bhattacharya, 2010; Li & Sun, 2013; McKee, 2003; McKee & Greenstein, 2000; Messier Jr & Hansen, 1988; Ohlson, 1980; Premachandra, Bhabra, & Sueyoshi, 2009; Tsai, Chang, Chung, & Li, 2010; Zaki, Bah, & Rao, 2011). These papers have all had as their main goal the development of accurate prediction models while using a variety of independent variables and different statistical techniques. For example, multiple discriminant analysis was used by Altman et al. (1994) and Cormier, Magnan, and Morard (1995). Logistic regression was used by Ohlson (1980) and Zaki et al. (2011). Non-parametric (Premachandra et al., 2009), semi-parametric (Hwang, Cheng, & Lee, 2007), and artificial intelligence techniques (Altman et al., 1994; Divsalar, Roodsaz, Vahdatinia, Norouzzadeh, & Behrooz, 2012; Gepp et al., 2010; Messier Jr & Hansen, 1988; Serrano-Cinca, 1997; Varetto, 1998) have also been used. Baixauli et al. (2012) combine an accounting based model with a market-based model, while (Agarwal & Taffler, 2008) compare the accounting ratio-based approach employed in this paper with market-based approaches and find no difference in their predictive ability.

A large number of articles in the extant bankruptcy literature including Zmijewski (1984), Grice and Dugan (2001), Grice and Ingram (2001), and Shumway (2001) have centered on methodological issues that arose in prior research and how to solve those issues and improve the models predictive accuracy. These issues include sample selection (Zmijewski, 1984), inappropriate application and sensitivity to periods (Grice & Dugan, 2001), the need for coefficient re-estimation (Grice & Ingram, 2001), and misspecification of the model parameters (Shumway, 2001).

While the aforementioned papers yield improvements in model predictive accuracy and therefore improved management decision making, they all have a common issue that has room for improvement. Common among these studies is the reliance on a single derived point estimate to make the classification of bankrupt or non-bankrupt (e.g., a *z*-score in Altman, 1968 and a probability in Zmijewski, 1984). The use of a single point estimate limits decision makers to using information that, while accurate in predicting bankrupt and non-bankrupt firms with predicted values that are far from the cutoff value, is not effective in classifying firms with predicted values that are close to the cutoff value. For example, the Altman (1968) z-score model accurately predicted all firms whose z-scores were relatively small, that is smaller than 1.81, as bankrupt and all firms whose *z*-scores were relatively large, that is larger than 2.99, as non-bankrupt. However, the problem lies in what Altman referred to as the "zone of ignorance" (1968, p. 606); the *z*-scores between 1.81 and 2.99. All of his model's misclassifications took place in this area, which suggests that there is an area for improvement. Also, Zmijewski (1984) was very successful in discriminating between bankrupt and non-bankrupt firms. However, his Type 1 errors (i.e., classifying bankrupt firms as non-bankrupt) were high, with accuracy rates on bankrupt firms of less than 50%. Given that the performance on the bankrupt firms is low and that Type 1 errors are the most costly errors in bankruptcy prediction (Watts & Zimmerman, 1986), there is also room for improvement in this model.

Confidence intervals have been used in many areas of decision making, including health care (Weber et al., 2011), age replacement problems (Leger & Cleroux, 1992), and credit risk (Dunkel & Weber, 2010; Hai, Nelson, & Staum, 2010). In the same line as the current research, (Hanson & Schuermann, 2006) analyze several confidence interval generating techniques on firm credit quality and default. They find that when comparing these predictors against traditional point estimate models, that for very low rated firms, bounded models do not predict better than the point estimates. However, bounded models do predict better as the credit ratings of the firms improves. We empirically test if using confidence intervals hereafter referred to as prediction intervals, due to context when dealing with an individual observation, will also improve bankruptcy prediction models, since the majority of the firms that are missed are not in the extremes, but in the middle area or "zone of ignorance" (Altman, 1968, p. 606).

We test our primary objective of prediction interval accuracy using the variables from Zmijewski (1984). The model was a parsimonious model of three variables (return on assets, financial leverage, and liquidity) and was developed using Probit regression. This model suits our purposes because it generates a single probability point estimate for potential bankruptcy and it utilizes an often used statistical technique. Further, as Grice and Dugan (2001) show, the model is not sensitive to industry classification and is not sensitive to financial distress situations other than those used to develop the model.

We improve upon the quality of the information derived from bankruptcy models and their discriminatory accuracy by deriving a 95% prediction interval around the model's point estimates. We then used the upper and lower bounds of the prediction interval to analyze the firms in the "zone of ignorance" (Altman, 1968, p. 606) and potentially reclassify firms as bankrupt or non-bankrupt. This technique resulted in statistically significant improvements in overall accuracy rates and on classifying non-bankrupt firms over traditional point estimates. Use of the technique could be easily adjusted to achieve greater or lower Type 1 and Type 2 error rates, according to the risk tolerance of the decision maker.

This project contributes to the extant research by combining the research streams of bankruptcy prediction modeling and the development of prediction intervals. We find that by deriving prediction intervals around the point estimates of bankruptcy prediction models it is possible to improve on the model accuracy. This is an important finding for decision makers in all areas of predictive modeling, because of their need for models that provide higher quality information with improved accuracy rates.

The next section of this paper discusses the development of the confidence intervals and the hypotheses. The section "Samples and Data" contains a description of the sample and data for building the models and prediction intervals. The section "Analysis and Results" discusses the results. The paper is concluded in the final section.

PREDICTION (CONFIDENCE) INTERVALS AND HYPOTHESES

Consider a bankruptcy prediction model that calculates a bankruptcy likelihood score, Y, based on net income, X. The *target population* for this model is all the publicly traded firms *next* year. The model predicts *next* year's bankruptcy based on a sample of *last* year's firms. Assuming that the assumptions are valid, statistical inference from the sample to the population will be valid.

This model can be used for two purposes: (1) to estimate the average bankruptcy likelihood score for all firms with net income x; and (2) to calculate the bankruptcy prediction score of a randomly chosen firm that has net income x. In the latter case, we select this random firm from the subpopulation $X = x$. We denote the bankruptcy likelihood of this random firm $Y(x)$. While we do not know the value of $Y(x)$, we can estimate it based on sample data; $\hat{Y}(x) = \hat{\beta}_0 + \hat{\beta}_1 x$.

Because $\hat{Y}$ is only an estimate, we are interested in the quality of this estimate. Therefore, we find an interval $[L, U]$ with a specified degree of prediction. Where L and U are the lower and upper bound, respectively of the $(1-\alpha)$ prediction interval. This will let us be $(1-\alpha)$ confident, that a randomly chosen firm with $X = x$, will lie in the interval: $C[L \le Y(x) \le U]$. If we now choose a future firm observation at random and try to estimate its bankruptcy likelihood, we refer to the interval as a prediction interval. Note that the width of the prediction interval depends on the distance from the average X value in the sample. See Fig. 1 for a graphical representation for a simple linear regression model.

Our model differs from the example above in two ways. First, we use the three predictor variables identified in Zmijewski (1984). Second, when we predict bankruptcy, the dependent variable is binary; a firm will go bankrupt, 1, or will not, 0. Hence, we use logistic regression to forecast the likelihood that the firm will go bankrupt. For an in-depth overview of logistic regression models we refer the interested reader of Hosmer and Lemeshow (2000). The multiple logistic regression model in matrix notation follows in Eq. (1):

$$E\{Y_i\} = \pi_i = [1 + \exp(-\beta' X_i)]^{-1} \tag{1}$$

The fitted values are then expressed as in Eq. (2):

$$\hat{\pi}_i = [1 + \exp(-b' X_i)]^{-1} \tag{2}$$

where $b'X_i = b_0 + b_1 X_{i1} + b_2 X_{i2} + \cdots + b_{p\text{-}1} X_{i,p\text{-}1}$

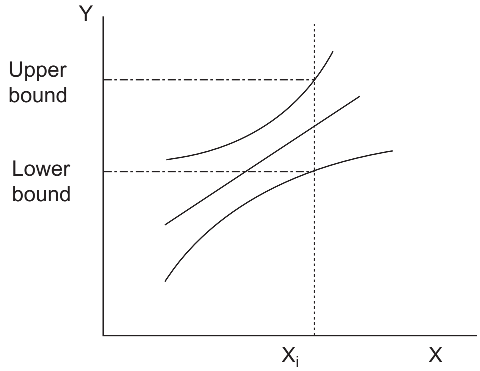

Fig. 1. Prediction Interval for an Individual *Y* Given X_i.

Next, we find the upper and lower bounds of the prediction intervals. Because we anticipate that bankrupt firms have more extreme ROA, leverage, and liquidity values, we anticipate that the width of the prediction interval for bankrupt firms is on average larger than the width of the prediction interval for non-bankrupt firms as depicted in Fig. 1. This is formally stated as:

Hypothesis 1. The average width of the prediction intervals for bankrupt firms is on average larger than the width of the prediction interval of non-bankrupt firms.

If (1) bankrupt firms differ from non-bankrupt firms in ROA, leverage, and liquidity and (2) bankrupt firms have on average larger prediction intervals than non-bankrupt firms, then using both pieces of information should improve the predictive ability of the model. We therefore hypothesize that this will lead to models that have greater predictive abilities and that will provide more decision useful information. Formally stated as:

Hypothesis 2. Overall model accuracy will be improved when the lower and upper bounds of the prediction interval are used to classify firms as going bankrupt or not going bankrupt.

In the context of this study, the model yields a probability of going bankrupt; therefore the closer the predicted value is to 1 they more likely a firm is to go bankrupt and the closer the value is to 0 the more likely the firm is to be non-bankrupt. Thus, the lower bound is more likely than the traditional point estimate to classify a firm as non-bankrupt. Using the lower bound and the point estimate will improve the decision usefulness of the model in predicting non-bankrupt firms. Formally stated as:

Hypothesis 3. Model accuracy will be improved when the lower bounds of the prediction interval are used to predict firms as non-bankrupt.

Similarly, the upper bound is more likely than the traditional point estimate to classify a firm as bankrupt. Therefore using the upper bound and the point estimate will improve the decision usefulness of the model in predicting bankrupt firms. Formally stated as:

Hypothesis 4. Model accuracy will be improved when the upper bounds of the prediction interval are used to predict firms as bankrupt.

SAMPLES AND DATA

This study utilizes data from 1991 to 2008 in order to build our models and test the hypotheses. The data was obtained from Compustat. The bankrupt firms are defined as those firms that were delisted from Compustat due to bankruptcy during the given time period. Financial firms were excluded from the sample because these firms have a unique set of reporting requirements. We use a training sample, consisting of 75% of the firms, from the 1991 to 2001 data to obtain the coefficients for our logistic regression model and the estimated variance-covariance matrix to calculate the upper and lower bounds. We use the Zmijewski (1984) model, see Eq. (3), to predict bankruptcy;

Table 1. Summary Statistics for the 2002 Holdout Sample.

	Non-Bankrupt ($n = 1{,}303$)			Bankrupt ($n = 26$)		
	ROA	Leverage	Liquidity	ROA	Leverage	Liquidity
Average	−0.02325	0.416002	1.617683	−0.29768	0.579397	0.812024
Min	−3.0745	0.000164	0.059501	−1.68469	0.007403	0.077253
Median	0.023121	0.365082	1.310105	−0.15222	0.482959	0.683273
Max	0.772618	2.359241	41.96923	0.113505	2.282125	3.04187

$N = 1{,}329$.
ROA = Return on Assets = net income divided by total assets.
Leverage = total debt divided by total assets.
Liquidity = current assets divided by current liabilities.

$$y = -\alpha - \beta \times \text{Return on assets} + \beta \times \text{Financial leverage} - \beta \times \text{liquidity} \tag{3}$$

The variables are defined as in Zmijewski (1984): Return on Assets = net income divided by total assets; financial leverage = total debt divided by total assets; and liquidity = current assets divided by current liabilities. Summary statistics for the variables of the holdout sample data set are shown in Table 1.

ANALYSIS AND RESULTS

We began with re-estimating the coefficients of the Zmijewski (1984) model on the training data sets. This was done in keeping with Grice and Ingram (2001) who show that coefficients of bankruptcy models need to be re-estimated as the time period changes. The re-estimation resulted in models with coefficients of the same signs as Zmijewski (1984). The re-estimated models are given in Table 2.

Then, we use the traditionally developed models to get a baseline for their predictive abilities against the holdout samples. Similar to Altman (1968), we selected the decision value to be used to classify firms such that we predicted 50% of the bankrupt firms correctly in our training sample. Our results for the holdout sample show that the model is accurate 65.9% of the time, with 57.7% accuracy on bankrupt firms and 66.1% accuracy on non-bankrupt firms. See Table 3 for details on the predictive abilities of the traditionally developed models.

We note that the accuracy achieved by the model suggests that the model might be mis-specified, as suggested by Shumway (2001). However, the purpose of this research is not to develop a new bankruptcy prediction

Table 2. Re-estimated Coefficients for the Zmijewski (1984) Model.

Variables	Coefficients	Standard Error	Wald Chi-Square	*P*-Value
Constant	−1.90684	0.0298	4083.3956	<.0001
ROA	−1.05383	0.1037	103.2298	<.0001
Leverage	0.815013	0.0553	217.1267	<.0001
Liquidity	0.004899	0.00297	2.7270	0.0987

$N = 13{,}227$.

ROA = Return on Assets = net income divided by total assets.

Leverage = total debt divided by total assets.

Liquidity = current assets divided by current liabilities.

Table 3. Predictive Abilities of the Model for 2002 Data.

		Predicted		Percentage Correct
		Bankrupt	Non-Bankrupt	
Actual	Bankrupt	15	11	57.7%
	Non-bankrupt	442	861	66.1%

Overall accuracy rate = 65.9%.
N = 1,329.

Table 4. Classification Matrices for Lower and Upper Bound 2002 Data.

		Predicted					
		Lower Bound			Upper Bound		
		Bankrupt	Non-bankrupt	Percentage correct	Bankrupt	Non-bankrupt	Percentage correct
Actual	Bankrupt	14	12	53.8%	20	6	76.9%
	Non-bankrupt	368	935	71.8%	521	782	60.0%
	Overall accuracy			71.4%			60.3%

N = 1,329.

model but rather to suggest an approach to address firms that are in the "zone of ignorance."

Next, we used the coefficients from the traditionally developed model and the respective variance-covariance matrix to derive prediction intervals around the point estimates from the traditional model. The average width of the prediction interval for bankrupt firms and non-bankrupt firms are 0.0188 and 0.0095, respectively. The difference is statistically significant at the 0.01 level, $t = 2.92$, $p = 0.0037$, supporting hypothesis 1.

We then used the upper and lower bound to further investigate the firms in the "zone of ignorance." See Table 4 for details on the predictive abilities of lower and upper bounds.

Note that the lower bound results in greater overall accuracy because of the relative large number of non-bankrupt numbers firms predicted correctly at the expense of predicting fewer bankrupt firms correctly. In our data sets, bankrupt firms are only about 0.8% of the population. Hence by setting a relatively large decision value, the overall prediction accuracy will increase at the expense of the prediction accuracy of bankrupt firms. Similarly, the upper bound predicted a larger percentage of bankrupt firms correctly at the expense of misclassifying more non-bankrupt firms.

We used the evidence about the width of prediction intervals for bankrupt and non-bankrupt firms, that is, H1, and the upper and lower bound to potentially reclassify firms in the "zone of ignorance" (Altman, 1968, p. 606). In this context, we classify a prediction interval as small when the range of the interval is closer to that of the average non-bankrupt firm (e.g., 0.0095) and we classify a prediction interval as large when the range of the interval is closer to that of the average bankrupt firm (e.g., 0.0188). In particular, when the lower bound and point estimate are in disagreement, we reclassify a firm as "non-bankrupt" when the width of the prediction interval is small and keep the classification of "bankrupt" if the width of the prediction interval is large. Similarly, when the point estimate and upper bound are in disagreement, we reclassify a firm from "non-bankrupt" to "bankrupt" if the width of the prediction interval is relatively large and keep the classification if the width is relatively small. To illustrate, let's consider a firm with a point estimate of 0.1507 and a 95% prediction interval of (0.1437, 0.1578). When using 0.151 as the decision value, the point estimate would classify this firm as non-bankrupt while the upper bound classifies this observation as bankrupt. Because the width of the prediction

Table 5. Predictive Abilities of the Model Using Lower Bounds, Point Estimates, and Upper Bounds for 2002 Data.

Panel A: Classification accuracy for point estimate and combined models

	Predicted					
	Point estimate			Combined		
Actual	Bankrupt	Non-bankrupt	Percentage correct	Bankrupt	Non-bankrupt	Percentage correct
Bankrupt	15	11	57.7%	17	9	65.4%
Non-bankrupt	442	861	66.1%	386	917	70.4%
Overall accuracy			65.9%		70.3%	

Panel B: Reconciliation point estimate and combined model

	Predicted					
	Bankrupt			Non-Bankrupt		
Actual	Point estimate	Adjustment lower bound	Adjustment upper bound	Point estimate	Adjustment lower bound	Adjustment upper bound
Bankrupt	15	−0	+2	11	+0	−2
Non-bankrupt	442	−64	+8	861	+64	−8

Table 6. Predictive Abilities of the Model Using Lower Bounds, Point Estimates, and Upper Bounds for 2003–2008 Data.

Panel A: Classification accuracy for point estimate and combined models

		Point Estimate			Combined		
	Actual	Bankrupt	Non-bankrupt	Percentage correct	Bankrupt	Non-bankrupt	Percentage correct
2003	Bankrupt	23	21	52.3%	23	21	52.3%
	Non-bankrupt	1787	2459	57.9%	1720	2526	59.5%
	Overall accuracy			57.9%			59.4%
2004	Bankrupt	21	13	61.8%	22	12	64.7%
	Non-bankrupt	1886	2513	57.4%	1764	2635	59.9%
	Overall accuracy			57.4%			59.9%
2005	Bankrupt	17	9	65.4%	17	9	65.4%
	Non-bankrupt	1715	2807	62.1%	1572	2950	65.2%
	Overall accuracy			62.1%			65.2%
2006	Bankrupt	21	12	63.6%	21	12	63.6%
	Non-bankrupt	1646	3028	64.8%	1541	3133	67.0%
	Overall accuracy			64.8%			67.0%
2007	Bankrupt	16	28	36.4%	16	28	36.4%
	Non-bankrupt	1735	3142	64.4%	1549	3328	68.2%
	Overall accuracy			64.2%			68.0%
2008	Bankrupt	13	21	38.2%	13	21	38.2%
	Non-bankrupt	1621	3149	66.0%	1532	3238	67.9%
	Overall accuracy			65.8%			67.7%

Table 6. *(Continued)*

Panel B: Reconciliation point estimate and combined model

		Predicted					
		Bankrupt			Non-bankrupt		
	Actual	Point estimate	Adjustment lower bound	Adjustment upper bound	Point estimate	Adjustment lower bound	Adjustment upper bound
2003	Bankrupt	23	−1	+1	21	+1	−1
	Non-bankrupt	1787	−142	+75	2459	+142	−75
2004	Bankrupt	21	−0	+1	13	+0	−1
	Non-bankrupt	1886	−181	+59	2513	+181	−59
2005	Bankrupt	17	−0	+0	9	+0	−0
	Non-bankrupt	1715	−206	+63	2807	+206	−63
2006	Bankrupt	21	−0	+0	12	−0	+0
	Non-bankrupt	1646	−167	+62	3028	+167	−62
2007	Bankrupt	16	−0	+0	28	+0	−0
	Non-bankrupt	1735	−232	−46	3142	+232	+46
2008	Bankrupt	13	−0	+0	21	+0	−0
	Non-bankrupt	1621	−130	+41	3149	+130	−41

interval is relatively large (0.0141), we reclassify this firm from "non-bankrupt" to "bankrupt." We report the results for the predictive ability of the lower bound and upper bound in Table 5 Panel A. We report the combined model, that is, the model using the point estimate and the upper and lower bound, and the reconciliation in Table 5 Panel B.

This resulted in models with predictive accuracies that were statistically significantly better, $\chi^2 = 11.4$; $p < 0.001$, than the traditionally developed models in overall accuracy. This supports our second hypothesis (H2). We also find that the percentage of non-bankrupt firms that was predicted accurately was statistically significant higher, $Z = 2.36$; $p < 0.01$, resulting in a lower Type 2 error rate. This supports our third hypothesis (H3). However, we did not find sufficient evidence to conclude that the percentage of bankrupt firms predicted accurately increased. Hence, we did not find support for our fourth hypothesis (H4). Note that dependent upon the purpose of the assessment, the user could take the more conservative approach and only consider the point estimate and the adjustments based on the upper bound. This could be appropriate when an auditor considers issuing a going concern opinion or a clean opinion in an audit where a more conservative approach is warranted due to the high detrimental impact of issuing a going concern opinion on a firm that deserves a clean opinion (Type 1 error).

In Table 6, we show the number of firms that would be re-classified based on the upper and lower bound, panel A, as well as the predictive accuracy of the combined model, panel B, for years 2003–2008.

The results for 2003–2008 confirm the findings reported for the 2002 data. Our second hypothesis (H2) that the model with the lower and upper bounds provides better overall accuracy is supported, $P = 0.037$ for 2003, and $P < 0.01$ for 2004–2008, respectively. Our third hypothesis (H3) that the model with the lower bound is statistically significantly better in classifying non-bankrupt firms is supported in 2004–2008, $p < 0.01$, and marginally supported in 2003, $P = 0.07$. Our fourth hypothesis that the upper and lower bound improve the classification of bankrupt firms is not supported.

CONCLUSION

In this project, we improved the predictive ability of a traditional bankruptcy prediction model by building prediction intervals around the point estimates of the model. The upper and lower bounds of the prediction intervals, the width of the prediction intervals, and the point estimates were used in concert to classify firms as bankrupt or non-bankrupt. As with prior research, when classifying firms that had very high or very low point estimates the point estimates yielded reasonably accurate predictions. However, when classifying firms that were in Altman's (1968, p. 606) "zone of ignorance" (i.e., those that are closer to the cutoff point) the use of the prediction interval yielded classifications that were statistically significantly more accurate than the point estimates alone.

It is imperative to management decision making that the information used be decision useful. The technique discussed in this paper yields a more decision useful classification, in that it (1) provides a more complete picture of the firm's status, (2) is derived from multiple forms of evidence, uses a predictive interval technique that is easily repeated, (3) can be generated in a timely manner, (4) can be applied to other bankruptcy prediction models in the literature (e.g., Altman, 1968; Shumway, 2001), and (5) is statistically significantly more accurate than traditional point estimate techniques. This technique can also be used in many forms of predictive analytics beyond bankruptcy prediction.

An additional advantage of our approach is that by varying the cutoff value one can adjust the model's performance to accommodate for different levels of riskiness on Type 1 and Type 2 error rates. One can also choose to use only the upper or lower bounds along with the point estimate to adjust for the level of conservatism in their decision making. In summary, we conclude that when building bankruptcy prediction models, it is necessary to use both point estimates and the prediction intervals around them when making the classifications; this will result in more decision useful information.

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