Electrical Terminal Representation of Conductor Loss in Transformers

JAMES H. SPREEN, MEMBER, IEEE

Abstract—**The formulation of a general, complete electrical terminal representation of eddy current loss in transformer windings is presented. It is shown that the effects of mutual resistances must be considered, in addition to the familiar winding resistances (self resistances), for a complete description of eddy current loss. A simple two winding structure is analyzed to display the features of the self and mutual resistance terms. This example illustrates that leakage resistance, i.e., resistance associated with opposing winding currents of equal amp turns, provides only a partial description of eddy current loss. The analysis is extended to multiple windings, with a simple three winding structure as an example.**

Introduction

DISCUSSIONS of eddy current loss in transformer windings have focused on solving eddy current problems for a known set of winding currents. Dowell [1] presented analysis results based on approximate field distributions in optimum windings carrying opposing sinusoidal currents. The opposing winding currents confine the fields to the winding region of the transformer structure. The ac resistance calculated from the eddy current power loss under these conditions was appropriately termed the leakage resistance. Venkatraman [2] used Fourier analysis to extend Dowell's results to waveforms typical of forward or buck type switched mode power converters. Carsten [3] has also extended Dowell's results using Fourier analysis, presenting graphs intended to aid design. Focusing on the synthesis problem, Jongsma [4] has re-formulated Dowell's analysis results to provide a detailed design procedure for transformers with sinusoidal currents, with extensions to nonsinusoidal waveforms via an effective frequency. Vandelac and Ziogas [5] have derived a power loss expression which, while still based on a simple approximate field distribution with sinusoidal time variation, is more general than that presented by Dowell. Further, by using a harmonic analysis of the field, they avoid the errors which readily occur if instantaneous, time domain field patterns are analyzed using an approach formulated in the frequency domain. Goldberg *et al.* [6] recognized that the leakage resistance does not provide a complete picture of the eddy current loss because of unopposed currents which may flow in the windings during operation. They used numerical analysis to improve the structure of a transformer for an application in which significant magnetizing current flows in the primary winding.

Manuscript received March 2, 1989; revised April 12, 1990.
The author is with IBM Corporation, Department 47JF/514, Neighborhood Road, Kingston, NY 12401.
IEEE Log Number 9037432.

In the references cited previously, the operational winding currents are assumed to be known, and the power losses expected in the application are calculated from the magnetic fields generated by these currents. The result is a single power loss or resistance parameter which describes the winding loss for the assumed set of currents. These analyses, then, do not provide general descriptions of the complete transformer winding losses. In order to obtain such a description, it is necessary to extend the interpretation and variety of these loss calculations. Specifically, it is necessary to rigorously relate power loss calculations to resistance parameters of an electrical terminal description of the transformer. These parameters, then, are characteristics of the transformer, not the application, and can be used to calculate winding loss and circuit behavior under any excitation conditions. This paper presents such resistance parameters, relates them to power loss calculations, and illustrates their features and behavior with examples based on simple winding structures.

Derivation of Expressions

General Basic Equations

In the frequency domain, the terminal voltages, v_1 and v_2, and terminal currents, i_1 and i_2, for a two winding transformer with winding losses are related by the expressions

$$v_1 = (j\omega L_1 + R_1)i_1 + (j\omega M + R_{12})i_2 \tag{1}$$

$$v_2 = (j\omega M + R_{12})i_1 + (j\omega L_2 + R_2)i_2. \tag{2}$$

Here, ω is the radian frequency; L_1, L_2, and M are the usual self and mutual inductances of electrically isolated but magnetically coupled windings; R_1, R_2 are the usual winding resistances; and R_{12} is a mutual resistance, usually neglected when considering winding losses [7], but which must be included to account for general eddy current losses. All six of these parameters are real; however, because of the frequency dependence of the current distribution in the winding conductors, all of them may be frequency dependent. For example, since there is no mutual resistance between electrically isolated windings at dc, the value of R_{12} must approach zero as the frequency approaches zero. In particular applications, certain combinations of these resistances, such as the leakage resis-

0885-8993/90/1000-0424$01.00 © 1990 IEEE

tance, may prove useful for modeling or circuit concepts, just as leakage inductance is often a more useful measure of magnetic coupling than is mutual inductance. However, in general, three independent parameters are needed, whether they are the resistances in (1)–(3), or parameters which can be defined in terms of these resistances.

The power loss, averaged over one period of the sinusoidal excitation, expressed in terms of the resistances and general winding currents i_1 and i_2 is given by

$$P = \tfrac{1}{2}R_1 i_1 i_1^* + \tfrac{1}{2}R_2 i_2 i_2^* + \tfrac{1}{2}R_{12}(i_1 i_2^* + i_1^* i_2). \quad (3)$$

(The * indicates complex conjugate.)

Specific Conditions

The object, then, is to start with a model of the physical structure of a transformer and derive expressions for the three resistances in (1) and (2). A set of terminal currents is assumed, the magnetic fields and current density distributions calculated from the assumed currents, and the winding losses calculated from the current densities and the fields. In principle, any three distinct and (not related by a simple scaling multiplier) sets of currents could be used to obtain three equations in the three unknown resistances. The following conditions are used here:

condition 1: i_1 real, $i_2 = 0$ (4a)

$$P_1 = \tfrac{1}{2}R_1 (i_1)^2 \quad (4b)$$

condition 2: $i_1 = 0$, i_2 real (5a)

$$P_2 = \tfrac{1}{2}R_2 (i_2)^2 \quad (5b)$$

condition 3: i_1 real, $i_2 = -i_1\sqrt{\frac{L_1}{L_2}}$ (6a)

$$P_3 = \frac{1}{2}\left(R_1 + \frac{L_1}{L_2}R_2 - 2\sqrt{\frac{L_1}{L_2}}R_{12}\right)(i_1)^2. \quad (6b)$$

Here, the power loss P_m, calculated from (3), is the average power loss over one sinusoidal period for terminal currents specified by condition m. These specific currents need not duplicate or approximate winding currents which a transformer might experience in an application. The purpose of setting these conditions is to extract expressions for the resistances, which can then be used to calculate the winding power loss under any set of winding currents.

Current condition 3 may be regarded as a generalized specification of the winding currents often assumed in leakage impedance calculations. Typically, the inductance of a winding increases as the square of the number of turns in the winding. Thus, for two windings of turns N_1 and N_2 sharing a common flux path, the ratio of the self inductances is related to the physical turns ratios by

$$\sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2}. \quad (7)$$

For structures in which (7) is satisfied, current condition 3, as previously noted, becomes the usual equal, opposite amp-turns condition often assumed for leakage inductance calculations [8]. The resistance factor in the parentheses in (6b) is then the leakage resistance, as seen at winding 1. It contains the first winding resistance R_1, and a transformed or reflected second winding resistance $(N_1/N_2)^2 R_2$, with the often neglected mutual resistance appearing as if partially reflected as $2(N_1/N_2)R_{12}$.

Determination of Resistances

In principle, any method, either analytical or numerical, of obtaining power loss values for specified excitation currents can be used with (4)–(6) to obtain values for the resistances. In this paper, in order to illustrate the general nature of the resistances, enough simplifying assumptions are invoked to allow analytic expressions to be obtained for these parameters. The assumptions are itemized briefly, next. More detailed discussion of such calculations can be found in the text by Perry [9], for example. He discusses such aspects as corrections for coil curvature, and provides some comparison of calculations with measurements. It should be noted that direct experimental determination of these resistances from actual measurements is difficult. Measurements can include the effects of such factors as core loss and distributed capacitances. These effects, not discussed in this paper, can add significant complications to the interpretation of measured resistance values.

For analysis, the structure of many transformer windings can be approximated by a simple planar conducting sheet model, following the approach of Dowell [1]. The curvature of the conductors is neglected; edge and end effects are neglected. The physical model is thus simplified to that illustrated in Fig. 1: a sheet conductor in the y-z plane, carrying current i in the y direction, assumed to be infinite in the $\pm y$ direction, with magnetic fields of amplitudes H_a and H_b present at the $x = 0$ and $x = h$ surfaces. H_a and H_b are the fields from all sources, including current in the conductor sheet itself. The power loss per unit length, $P/1$ in such a sheet can be expressed in terms of the amplitudes H_a and H_b of these sinusoidally varying magnetic fields. The derivation of the power loss is treated in the appendix of [5]. The result, simplified and expressed in a form convenient for this work, is

$$P/1 = \frac{1}{2}\frac{b\rho}{\delta}\left[(H_a - H_b)^2 F(h/\delta) + 2H_a H_b G(h/\delta)\right], \quad (8)$$

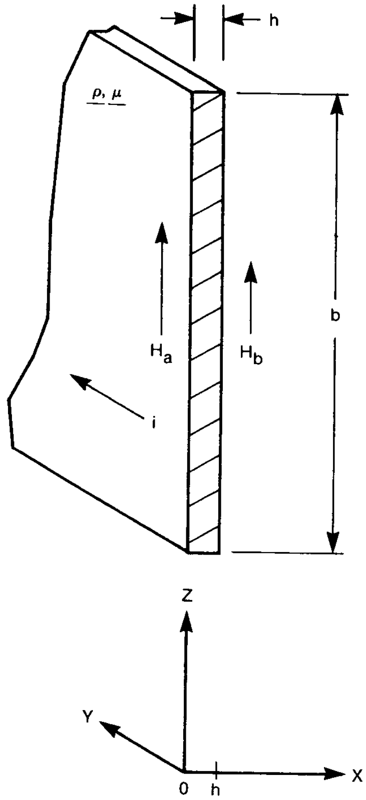


Fig. 1. Model for eddy current loss calculations: Sheet conductor with magnetic fields.

where

- b width of conductor, as illustrated in Fig. 1
- h thickness of the conductor
- ρ resistivity of the conductor
- δ usual skin depth $= \sqrt{2\rho/\omega\mu}$ (9)
- ω radian frequency
- μ permeability of the conductor.

$$F(x) = \frac{\sinh(2x) + \sin(2x)}{\cosh(2x) - \cos(2x)} \tag{10}$$

$$G(x) = \frac{\sinh(x) - \sin(x)}{\cosh(x) + \cos(x)}. \tag{11}$$

Here, for simplicity, only foil windings are considered. However, this result can be extended to other types of windings, such as round wire, by appropriately adjusting ρ and h with a layer fill factor and a round-to-square conversion, as described by Snelling [10], for example. The functions $F(x)$ and $G(x)$ defined in (10) and (11) are related to the parameters M' and D' used by Dowell [1]:

$$M' = (h/\delta)F(h/\delta) \quad \text{and } D' = 2(h/\delta)G(h/\delta). \tag{12}$$

Further, the function $F_1(x)$ used in [5] and [9] is equal to $F(x)$ in (10), and the function $G(x)$ in [9] is equal to $G(x)$ in (11). Since in the analysis here, the fields H_a and H_b are generated by the winding currents specified as real in (4a), (5a), and (6a), it is not necessary to include the possibility of phase shifts between these two fields, as is done in [5].

For a conductor sheet carrying no net current, but immersed in a uniform field generated by other sources, the loss expression contains only the second term in the brackets in (8), since when $H_a = H_b$, the first term vanishes. Similarly, for a winding at the edge of a winding region within which the field is confined by suitably chosen winding currents (such as equal, opposite amp turns), the second term vanishes because either H_a or H_b is zero.

Summary of Calculational Approach

The approach, then, is to: 1) assume winding currents corresponding to one of the conditions specified in (4a), (5a), or (6a); 2) calculate the magnetic fields present in the windings using simple sheet current models for the field sources (so the field calculation is nearly by inspection); 3) use (8) to calculate the power loss in each conductor; and 4) equate the total power loss in all conductors to the appropriate power expression in (4b), (5b), or (6b). Performing these steps for each condition provides expressions for the three resistances.

Examples

Example 1: Two Foil Windings with Shield

As an example, consider a very simple transformer with two foil windings and a shield, shown in cross section in Fig. 2. Each winding is one turn. The foil for winding 1 is designated conductor 1, the foil for winding number 2 is designated conductor 2, and the foil forming the shield is designated conductor 3. Any capacitive current from the windings, through the shield to ground, is assumed to be negligible: the shield is assumed to carry no net current.

Resistance Evaluation

For condition 1, only winding 1 carries current, producing a uniform field at the positions of the other two conductors. The fields at the surfaces of the conductors are

$$H_{a1} = -H_{b1} = \frac{1}{2}\frac{i_1}{b}, \tag{13a}$$

$$H_{a2} = H_{b2} = H_{a3} = H_{b3} = \frac{1}{2}\frac{i_1}{b}. \tag{13b}$$

Here, the magnetic fields H_{an} and H_{bn} are the amplitudes of the fields present at the two surfaces of conductor n.

Clearly, from (8), the eddy current losses in conductors 2 and 3, under this current condition, are described by only the second term: the first term vanishes for conductors in uniform fields. Equation (8) is used to calculate

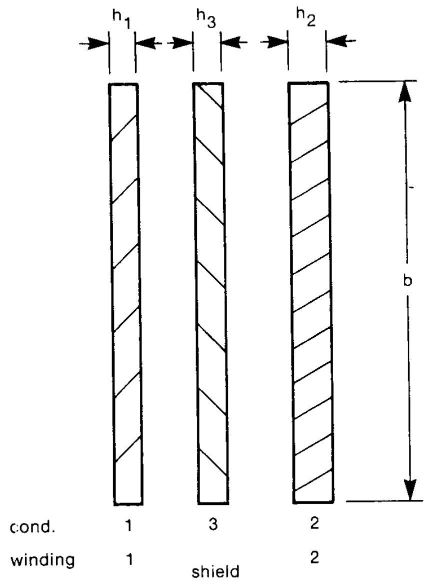


Fig. 2. Geometry for example 1: Two foil windings and a shield.

the power loss in each conductor, with the fields expressed in (13). The power losses are then summed and set equal to the power loss in (4b) to obtain an expression for the resistance R_1. Assuming that all three conductors have the same length of turn λ, and are formed of the same conducting material, the results of these calculations are

$$R_1 = \frac{\rho\lambda}{b\delta}\left[F(h_1/\delta) - \frac{1}{2}G(h_1/\delta) + \frac{1}{2}G(h_2/\delta) + \frac{1}{2}G(h_3/\delta)\right]. \tag{14}$$

Here, h_n is the thickness of conductor n, as illustrated in Fig. 2. Note that eddy current losses in all the conductors contribute to R_1. Thus, at frequencies for which significant eddy current loss occurs, R_1 cannot be given its usual low frequency interpretation as representing dissipation in the conductor forming winding 1.

A similar calculation can be used to obtain an expression for R_2:

$$R_2 = \frac{\rho\lambda}{b\delta}\left[F(h_2/\delta) - \frac{1}{2}G(h_2/\delta) + \frac{1}{2}G(h_1/\delta) + \frac{1}{2}G(h_3/\delta)\right]. \tag{15}$$

For this simple structure, each winding is assumed to have one turn, and the two self inductances are assumed to be equal. Therefore, current condition 3, (6a), reduces to equal, opposite amps flowing in windings 1 and 2. For this current pattern, the field is a uniform field contained between windings 1 and 2, with no field outside the windings:

$$H_{a1} = 0, \quad H_{b1} = \frac{i_1}{b} \tag{16a}$$

$$H_{a2} = \frac{i_1}{b}, \quad H_{b2} = 0 \tag{16b}$$

$$H_{a3} = H_{b3} = \frac{i_1}{b}. \tag{16c}$$

Again, (8) is used to calculate the power loss in each conductor, with the conductor surface fields specified in (16). Summing these power losses and equating them to the expression in (6b), simplified for equal self inductances, yields the relation

$$R_1 + R_2 - 2R_{12} = \frac{\rho\lambda}{b\delta}\left[F(h_1/\delta) + F(h_2/\delta) + 2G(h_3/\delta)\right]. \tag{17}$$

This expression is the leakage resistance for this structure. It corresponds to the real part of the total leakage impedance calculated by Dowell [1]. The various terms which can be identified with individual conductors, e.g., $F(h_1/\delta)$ with conductor 1 and $F(h_2/\delta)$ with conductor 2, correspond to the resistances of the winding portions considered in Dowell's derivation and its extensions and reformulations [2]–[4]. Comparison of these terms with the expressions in (14) and (15) forces an important conclusion: the leakage resistance contribution of an individual winding is not equivalent to the self resistance of the winding. The total leakage resistance cannot be apportioned between the two windings to obtain R_1 and R_2. Thus, in general, the leakage resistance does not provide a complete description of the winding losses.

Substituting for R_1 and R_2 from (14) and (15) yields an expression for the mutual resistance R_{12} of this example:

$$R_{12} = -\frac{1}{2}\frac{\rho\lambda}{b\delta}G(h_3/\delta). \tag{18}$$

This expression for R_{12} is negative. However, the total power dissipated in the transformer is always positive, since $R_1 \times R_2 > R_{12}^2$, which guarantees that the power loss given in (9) is positive for any set of winding currents.

Behavior at Frequency Limits

The low frequency behavior of R_{12} is easy to examine if (16) is rewritten as

$$R_{12} = -\frac{1}{2}\frac{\rho\lambda}{bh_3}(h_3/\delta)G(h_3/\delta). \tag{19}$$

At low frequencies, the skin depth δ becomes very large, so the ratio h_3/δ becomes very small. For small x, the product $xG(x)$ behaves as $x^4/6$. Since the skin depth in (9) is related to the square root of the inverse of the frequency, this behavior translates to

$$|R_{12}|(h_3/\delta \text{ small}) \sim \omega^2. \tag{20}$$

As was anticipated, the mutual resistance vanishes in the limit of ω approaching zero, i.e., excitation by direct current.

For large x, $G(x)$ approaches 1. Therefore, the high frequency limiting value of R_{12} is given by

$$R_{12}(h_3/\delta \text{ large}) = -\frac{1}{2}\frac{\rho\lambda}{b\delta}. \tag{21}$$

The limiting high frequency value is independent of conductor thickness, since the current flows within a skin depth of the surface.

The low frequency behavior of R_1 is determined by the first term in the brackets in (14), since this term approaches a finite limit, in contrast to all the $G(h/\delta)$ type terms, which approach zero. The low frequency limit is easily obtained by noting that the product $xF(x)$ approaches 1 for x small. Therefore,

$$R_1(\delta \text{ large}) = \frac{\rho\lambda}{bh_1}. \tag{22}$$

The limit is specified as δ large to indicate that the skin depth δ must be much larger than any of the conductor thickness h_1, h_2, or h_3, for this result to be valid. This expression is simply the dc resistance of winding 1. Here, since the loss has been related to resistances, the dc value can be obtained by taking the limit directly, without a factor of 2 adjustment used in the power loss calculations of [5].

Both $F(x)$ and $G(x)$ approach 1 for x large. Therefore, the high frequency limiting value for R_1 is given by

$$R_1(\delta \text{ small}) = \frac{3}{2}\frac{\rho\lambda}{b\delta}. \tag{23}$$

The limit is specified as δ small to indicate that the skin depth δ must be much smaller than any of the conductor thicknesses h_1, h_2, or h_3 for this result to be valid. Again, the result is independent of the conductor thicknesses, since the current flow is effectively within a skin depth of the surface. Because of the presence of the other conductors, the value of R_1 in (23) is three times the limiting value associated with the loss in conductor 1. This result emphasizes that only in the low frequency limit can R_1 be interpreted as representing power loss in the conductor which forms winding 1.

Similar expressions can be obtained for the limiting values of R_2.

Example 2: Three Foil Windings

The approach can be readily extended to transformers with multiple windings. In general, for N windings, there are then $N(N - 1)/2$ mutual resistances, in addition to the familiar N winding self resistances.

As an example, consider the simple three winding transformer arrangement shown in cross section in Fig. 3.

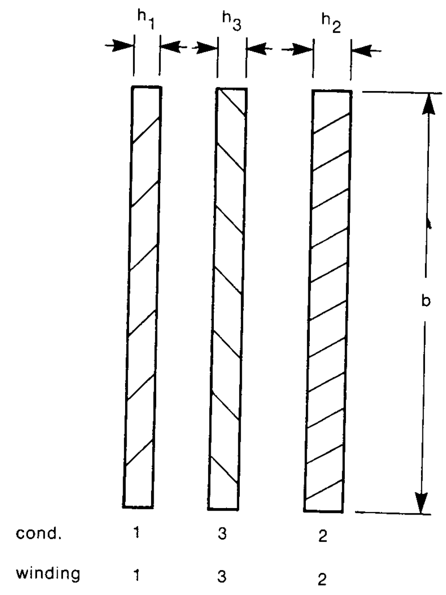


Fig. 3. Geometry for example 2: Three foil windings.

Windings 1 and 2, designated conductors 1 and 2, are secondary windings with one turn each. They might represent +5-V and −5-V outputs, designed for unequal load currents. The third winding, representing a single turn primary winding, conductor 3, is sandwiched between the secondaries, an arrangement which might be implemented for tight magnetic coupling. Three self and three mutual resistances are required to characterize the winding loss of this structure. Any six independent sets of currents, with associated power loss expressions, could be used to derive expressions for the six resistances. Here, conditions which are extensions of two winding conditions are used: three conditions with only one winding carrying current and three conditions with two windings carrying current, the third open, corresponding to pair by pair application of condition 3.

Note that Fig. 3 is identical to Fig. 2, except that conductor 3 is regarded as a winding. The self resistances are determined with all other windings open. Therefore, for determining self resistances, the arrangement in Fig. 3 is indistinguishable from the arrangement in Fig. 2. Expressions for R_1 and R_2 are thus given in (14) and (15). Similarly, an expression for R_3 is readily obtained by permuting subscripts in (14):

$$R_3 = \frac{\rho\lambda}{b\delta}\left[F(h_3/\delta) - \frac{1}{2}G(h_3/\delta) + \frac{1}{2}G(h_1/\delta) + \frac{1}{2}G(h_2/\delta)\right]. \tag{24}$$

The mutual resistance R_{12} between the two secondary windings must be equal to the expression in (18), since to determine R_{12}, winding 3 is left open, and therefore is equivalent to the shield in Fig. 2. However, an expression of the form of that in (18) cannot be assumed to describe the remaining mutual resistances. Equation (18) is appropriate for a structure in which the open circuited conductor is between the conductors carrying current. In contrast, current condition 3, (6a), applied to windings 1 and 3, with 2 open, corresponds to an arrangement in which the open circuited conductor is outside the field contained between windings 1 and 3. Following the calculational approach outlined in the first example yields

$$R_{13} = \frac{1}{2} \frac{\rho\lambda}{b\delta} G(h_2/\delta). \tag{25}$$

Similarly, derivation of the third mutual resistance yields

$$R_{23} = \frac{1}{2} \frac{\rho\lambda}{b\delta} G(h_1/\delta). \tag{26}$$

Comments on Examples

These examples were chosen to illustrate the nature of a general representation of winding loss. To that end, the structures avoided complexities such as multiple layers, which are discussed in the various references [1]–[5], and [9]. Further, the field patterns were assumed to be very simple: spatially uniform fields with all end, edge, and curvature effects of the sheet currents neglected. Such a simple field model may be adequate for windings excited as in condition 3, (6a), in which the field is approximately confined within the windings. However, the analysis results for the structure in [6], for example, show that the eddy current loss of the magnetizing current is sensitive to the core geometry and the winding placement within the window. In the terms used here, R_1, R_2, and R_{12} then may depend upon core and window parameters, not just winding geometry. While the examples in this paper serve the illustrative purpose, clearly, models of practical transformers may require more refined field and loss calculations.

Conclusion

The general representation of eddy current loss in a two winding transformer requires three parameters: two self resistances and one mutual resistance, for example. For an N winding transformer, $N(N + 1)/2$ parameters are required, not just N winding resistances. Calculations based on simple structures illustrate the importance of mutual resistances and demonstrate that leakage resistance provides only a partial description of eddy current loss in windings. Further, the examples show that the geometry of all conductors may contribute to each resistance. For example, R_1 is not the contribution which winding 1 makes to the leakage resistance, nor is it associated with the power dissipated in only the conductors which comprise winding 1. It represents the total loss in all conductors if only winding 1 carries net current.

Thus, the presence of internal eddy current effects in the conductors at high frequencies requires the introduction of additional resistance parameters, e.g., mutual resistances, and requires an extended interpretation of the usual self resistances, in order to obtain a complete description of conductor loss in terms of general terminal voltages and currents of a transformer.

References

[1] P. L. Dowell, "Effects of eddy currents in transformer windings," in *Proc. IEE*, vol. 113, no. 8, pp. 1387–1394, Aug. 1966.

[2] P. S. Venkatraman, "Winding eddy current losses in switch mode power transformers due to rectangular wave currents," in *Proc. Powercon 11*, Ventura, CA: Power Concepts, Inc., 1984, pp. 1-11.

[3] B. Carsten, "High frequency conductor losses in switchmode magnetics," *Technical Papers First High Frequency Power Conversion Conference*, Ventura, CA, Intertec Communications, Inc., 1986, pp. 155-176.

[4] J. Jongsma, "Transformer winding design," *High-Frequency Ferrite Power Transformer and Choke Design.* Philips Technical Publication no. 207, The Netherlands: Philips, 1986, Part 3.

[5] J. P. Vandelac and P. D. Ziogas, "A novel approach for minimizing high-frequency transformer copper losses," *IEEE Trans. Power Electron.*, vol. 3, no. 3, pp. 266–277, July 1988.

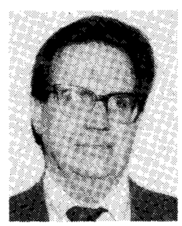
[6] A. F. Goldberg, J. G. Kassakian, and M. F. Schlecht, "Issues related to 1-10 MHz transformer design," in *18th Annual IEEE Power Electronics Specialists Conference Rec.*, Piscataway, NJ, 1987, pp. 379–386.

[7] MIT Staff, *Magnetic Circuits and Transformers.* Cambridge, MA: MIT Press, 1943, p. 625.

[8] L. F. Blume, A. Boyajian, G. Camilli, T. C. Lennox, S. Minneci, and V. M. Montsinger, *Transformer Engineering.* New York: Wiley, 1951, pp. 72–75.

[9] M. P. Perry, *Low Frequency Electromagnetic Design.* New York: Marcel Dekker, 1985, pp. 34–54, 73–83.

[10] E. C. Snelling, *Soft Ferrites.* Cleveland, OH: CRC Press, 1969, pp. 346-348.



James H. Spreen (S'69–M'77) received the B.S. degree in engineering from Case Institute of Technology, Cleveland, OH, and the Ph.D. degree in electrical engineering from Massachusetts Institute of Technology, Cambridge, MA, in 1971 and 1977, respectively.

He is presently a Senior Engineer with International Business Machines Corporation, Kingston, NY, where he works in advanced system packaging. He has also worked in various technical areas, including design and test of microwave and millimeter wave components, research on magnetic domain wall dynamics, computer simulation of integrated circuit fabrication processes, and advanced technology for power converters.

Dr. Spreen is a member of the Electrochemical Society.