Spring 2016
Question 7

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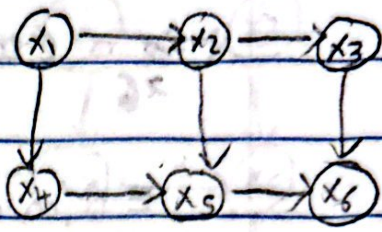

a) $P(x_1) = f(x_1)$ -

$P(x_2 | x_1) = f(x_1, x_2)$ -

$P(x_3 | x_2) = f(x_2, x_3)$ -

$P(x_4 | x_1) = f(x_4, x_1)$ -

$P(x_5 | x_4, x_2) = f(x_4, x_2, x_5)$ -

$P(x_6)$ = constant

Eliminate x_1

$\Rightarrow \sum_{x_1} f(x_1)\, f(x_1, x_2)\, f(x_1, x_4)$

$\Rightarrow g(x_2, x_4)$ - ①

Eliminate x_2

$\Rightarrow \sum_{x_2} g(x_2, x_4)\, f(x_2, x_3)\, f(x_4, x_2, x_5)$

$\Rightarrow g'(x_3, x_4, x_5)$ - ②

Eliminate x_3

$\Rightarrow \sum_{x_3} g'(x_3, x_4, x_5) = g''(x_4, x_5)$ - ③

Eliminate x_4

$\Rightarrow \sum_{x_4} g''(x_4, x_5) = h(h_5)$

Now eliminate h_5 & done

Time complexity $= O(d^4)$ (step ②)

Space complexity $= O(3d^3)$

b) $x_1 \to f(x_1)$

$x_2 \to f(x_1, x_2)$

$x_{n+1} \to f(x_1, x_{n+1})$

$x_{n+2} \to f(x_2, x_{n+1}, x_{n+2})$

Suppose we eliminate x_1 / $f(x_1)$ $f(x_1, x_2)$ $f(x_1, x_{n+1})$

we get $g(x_2, x_{n+1}) \Rightarrow O(d^3)$

Then we eliminate x_{n+1}.

we get $\underbrace{g'(x_2, x_{n+2})}_{\text{this function already exist}} \Rightarrow O(d^3)$

So, we are not adding any extra variable.

Elimination order

$(x_1, x_{n+1}, x_2, x_{n+2}, \ldots x_i, x_{n+i})$

should give us a time complexity of $O(d^3)$ &

space complexity of $O(2d^2)$

This is beacause x_i has extra edge ~~from~~ x_i to x_{n+i}, so that is why eliminating x_{n+i-1} will make sure everything on the left of x_{n+i} is eliminated & there is no extra variable