

# ★ Spring 2016 Midterm

Q1] Entropy  $X_1 = \frac{4}{10} \left[ -\frac{1}{4} \log \frac{1}{4} - \frac{3}{4} \log \frac{3}{4} \right]$

$+ \frac{6}{10} \left[ -\frac{1}{3} \log \frac{1}{3} - \frac{2}{3} \log \frac{2}{3} \right]$

Entropy  $X_2 = \frac{3}{10} [0] + \frac{7}{10} \left[ -\frac{2}{7} \log \frac{2}{7} - \frac{5}{7} \log \frac{5}{7} \right]$

Entropy  $X_3 = \frac{6}{10} [1] + \frac{4}{10} [1]$

Entropy  $X_4 = \frac{4}{10} [1] + \frac{6}{10} [1]$

$x_1$	$x_3$	$x_4$	$y$
1	1	0	1
0	0	1	1
0	1	1	-1
0	0	0	-1
0	0	0	-1
1	0	0	-1
0	1	1	-1
0	1	1	-1

Choose  $X_2$ ,

Level 2,

Entropy  $X_1$  is least

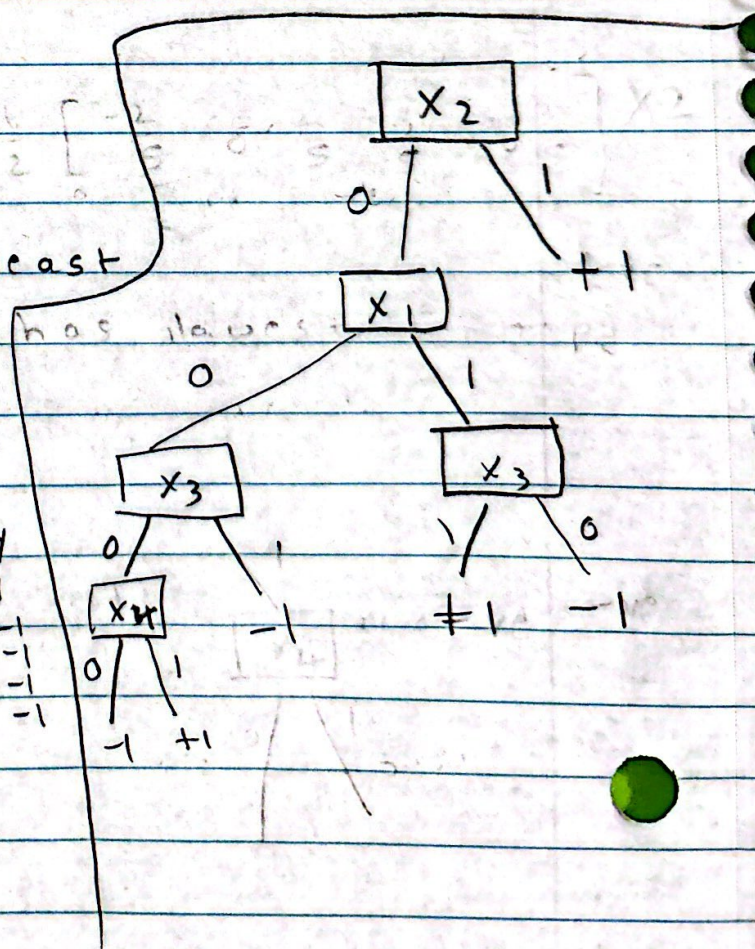
Choose  $X_1$  it has lowest

Level 3,

Let  $X_1 = 1, X_1 = 0$

$x_3$	$x_4$	$y$
1	0	1
0	0	-1
0	0	-1
1	1	-1

Choose  $X_3$

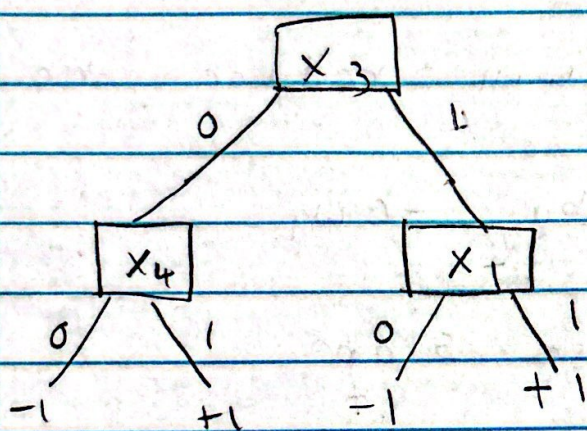




b] How to find solution?

↳ Find 2 or 3 attribute that do not have contradictory  $Y$  output

$X_1, X_3$  &  $X_4$  satisfy this condition



c] we will prefer b) tree because it has smaller height so it is likely to be optimal. Also tree with more nodes representing the same concept are more likely to overfit the training examples

Question 2]

a] Refer to Spring 2015 midterm  
Q2] a]



b]  $w_1 x_1 + w_2 x_2 = 0$

$$\text{slope} = \frac{-w_1}{w_2} = - \left( \frac{w_1}{w_2} \right)$$

\* slope increases when  $w_1$  decreases

$-\frac{c}{2}(w_1)^2$  will reduce the value

of  $w_1$  by  $-cw_1$

\* will increase slope

- L2  $\rightarrow$  Yes, is possible if we select medium large value of  $c$  slope will increase from L1. So it is possible to have L2
- L3  $\rightarrow$  No, for L3 to be possible ~~stop~~  $c$  has to be negative
- L4  $\rightarrow$  Yes, but the value of  $c$  has to be very large to increase slope by that much.  
~~It is~~ It is not completely vertical, so it is theoretically possible



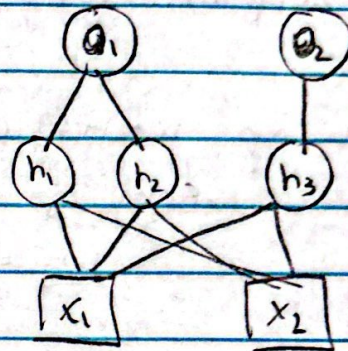
Q3]  $y_1 = \text{XOR} = (x_1 \wedge x_2) \vee (-x_1 \wedge -x_2)$   
 $y_2 = (x_1 \wedge -x_2)$

$$h_1 = x_1 \wedge x_2$$

$$w_0 = -1.5, w_1 = 1, w_2 = 1$$

$$h_2 = -x_1 \wedge -x_2$$

$$w_0 = 0.5, w_1 = -1, w_2 = -1$$



$$o_1 = h_1 \vee h_2$$

$$w_0 = 1.5, w_1 = 1, w_2 = 1$$

$$h_3 = x_1 \wedge -x_2$$

$$w_0 = -0.5, w_1 = 1, w_2 = -1$$

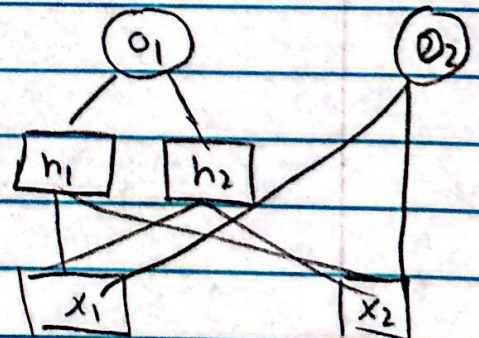
$$o_2 = h_3$$

$$w_0 = 0, w_1 = 1$$

All units ~~are~~  $h_1, h_2, h_3, o_1, o_2$   
 are signed units i.e. if  $\text{bias} > 0, 1$   
 else 0

Alternate  
 solution,

skip  $h_3$   
 if allowed





$$b] \quad \frac{\partial E}{\partial w_j} = - \sum_{i=1}^m (\hat{y}_i - o_i)^2 (x_j + x_j^{1-s})$$

Algorithm

Loop until convergence

for  $j$  in features  $n$

$$w_j = w_j - \sum_{d=1}^m (y_d - o_d)^2 (x_{jd} + x_{jd}^{1-s})$$

end for

end loop

↓  
equivalent

to  $x_{jd}$

if  $i$  represents  
 $i^{\text{th}}$  feature from 1  
to  $n$