Spring 2023

Question 2] SVM

Dual formation

$\Rightarrow \sum_{i=1}^{4}\sum_{j=1}^{4} \alpha_i \alpha_j \cdot K(x_i, x_j)\, y_i y_j$

$\Rightarrow \sum_{i=1}^{4} \alpha_i y_i = 0, \quad -\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4 = 0, \quad$ s.t $\alpha_i \geq 0$ $i \in \{1, 2$

$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 +$

$\alpha_1^2 (1 + \langle -1, 1\rangle \cdot \langle -1, 1\rangle)^2 + \alpha_2^2 (1 + \langle -2, 1\rangle \cdot \langle -2, 1\rangle)^2 +$

$\alpha_3^2 (1 + \langle 2, 1\rangle \cdot \langle 2, 1\rangle)^2 + \alpha_4^2 (1 + \langle -3, -2\rangle \cdot \langle -3, -2\rangle)^2$

$-2\alpha_1\alpha_2 (1 + \langle -1, 1\rangle \cdot \langle -2, 1\rangle)^2 - 2\alpha_1\alpha_3 (1 + \langle -1, 1\rangle \cdot \langle 2, 1\rangle)^2$

$+ 2\alpha_1\alpha_4 (1 + \langle -1, 1\rangle \cdot \langle -3, -2\rangle)^2 + 2\alpha_2\alpha_3 (1 + \langle -2, 1\rangle \cdot \langle 2, 1\rangle)^2$

$-2\alpha_2\alpha_4 (1 + \langle -2, 1\rangle \cdot \langle -3, -2\rangle)^2 - 2\alpha_3\alpha_4 (1 + \langle 2, 1\rangle \cdot \langle -3, -2\rangle)^2$

$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 +$

$\alpha_1^2 (1 + 1 + 1)^2 + \alpha_2^2 (1 + 4 + 1)^2 + \alpha_3^2 (1 + 4 + 1) + \alpha_4^2 (1 + 9 + 4)$

$-2\alpha_1\alpha_2 (1 + 2 + 1)^2 - 2\alpha_1\alpha_3 (1 - 2 + 1)^2 + 2\alpha_1\alpha_4 (1 + 3 - 2)^2$

$+ 2\alpha_2\alpha_3 (1 - 2 + 1)^2 - 2\alpha_2\alpha_4 (1 + 6 - 2)^2 - 2\alpha_3\alpha_4$

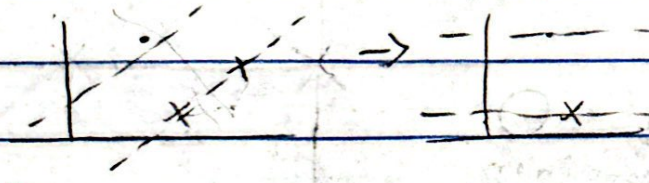
$(1 + -6 - 2)^2$

$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \frac{1}{2}[9\alpha_1^2 + 6^2\alpha_2^2 + 6^2\alpha_3^2 + 14^2\alpha_4^2$

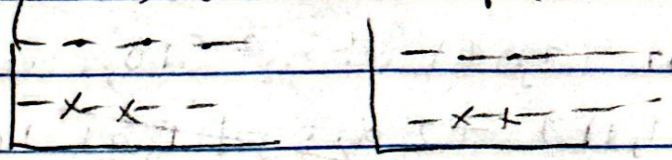
$-2\alpha_1\alpha_2 \times 4^2 + 2\alpha_1\alpha_4 \times 2^2 - 2\alpha_2\alpha_4 \times 5^2 - 2\alpha_3\alpha_4 \times 7^2]$

b] Removing support vectors from training set, will either increase or remains same the margin
This is because removing a support vector will only relax the constrains or remain same.
Relaxed constrains have potential to increase the margin, but may never decrease it.

Increases example



Remains same example



c] Given a linear SVM, the examples that are not support vectors could be removed and that will still produce exact same margin.
This is because SVM classification only depends on which side of boundary a point lies ~~which~~. The margin & boundary are only made up of support vectors.
We can also remove support vectors if there are muliple of them (since a