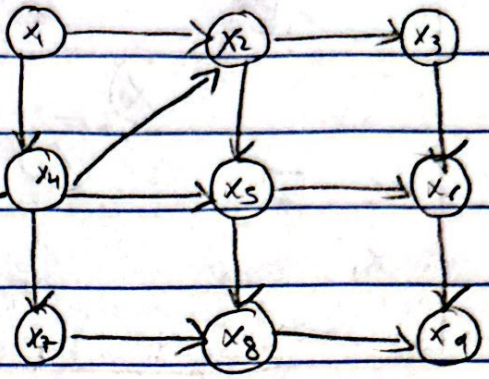
Fall 2023 midterm-2

Question 1

Bayesian networks



$P(x_1) = f(x_1)$ -

$P(x_2 | x_4, x_1) = f(x_2, x_4, x_1)$

$P(x_3 | x_2) = f(x_3, x_2)$ -

$P(x_4 | x_1) = f(x_4, x_1)$ -

$P(x_5 | x_2, x_4) = f(x_5, x_2, x_4)$

$P(x_6 | x_5, x_3) = f(x_6, x_5, x_3)$ -

$P(x_7 | x_4) = f(x_7, x_4)$ -

$P(x_8 | x_5, x_7) = f(x_8, x_7, x_5)$ -

$P(x_9)$ -> constant x_9 given

Eliminate x_7

$$\Rightarrow \sum_{x_7} f(x_7, x_4)\, f(x_8, x_7, x_5)$$

$$\Rightarrow g(x_4, x_8, x_5) \rightarrow (1)$$

Eliminate x_3

$$\Rightarrow \sum_{x_3} f(x_3, x_2)\, f(x_6, x_5, x_3)$$

$$\Rightarrow g'(x_2, x_5, x_6) \rightarrow (2)$$

Eliminate x_1

$$\Rightarrow \sum_{x_1} f(x_1)\, f(x_4, x_1)\, f(x_2, x_4, x_1)$$

$$\Rightarrow g''(x_4, x_2) \rightarrow (3)$$

Eliminate x_2

$$\Rightarrow \sum_{x_2} f(x_5, x_2, x_4)\, g'(x_2, x_5, x_6)\, g''(x_4, x_2)$$

$$\Rightarrow g'''(x_5, x_4, x_6) \rightarrow (4)$$

Eliminate x_4

$$\Rightarrow \sum_{x_4} g(x_4, x_8, x_5)\ g'''(x_5, x_4, x_6)$$

$$\Rightarrow g''''(x_8, x_5, x_6) \rightarrow (5)$$

Eliminate x_5

$$\Rightarrow \sum_{x_5} g''''(x_8, x_5, x_6)$$

$$\Rightarrow h(x_8, x_6) \rightarrow (6)$$

Eliminate x_6

$$\Rightarrow \sum_{x_6} h(x_8, x_6) = h'(x_8) \hookrightarrow (7)$$

Eliminate x_8 from $h'(x_8) \rightarrow (8)$

Step	Time complexity	Space complexity
(1)	d^4	d^3
(2)	d^4	d^3
(3)	d^3	d^2
(4)	d^4	d^3
(5)	d^4	d^3
(6)	d^3	d^2
(7)	d^2	d
(8)	d	0

Space complexity $= O(3d^3)$

Time complexity $= O(d^4)$