Fall 2023

Quesin 2] BN, EM Algorithm

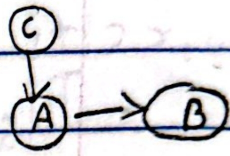
a] $P(C=1) = \theta$, $P(C=0) = 1-\theta$

$P(A=1 | C=1) = \alpha$, $P(A=0 | C=1) = 1-\alpha$

$P(A=1 | C=0) = \beta$, $P(A=0 | C=0) = 1-\beta$

$P(B=1 | A=1) = \gamma$, $P(B=0 | A=1) = 1-\gamma$

$P(B=1 | A=0) = \lambda$, $P(B=0 | A=0) = 1-\lambda$

Bayesian Network $\longrightarrow$ 

Joint Probability,

$\Rightarrow$ $P(C)\ P(C|A)\ P(B|A)$

$P(1a) = P(C=0)\ P(C=0 | A=0)\ P(B=0 | A=0)$

$= (1-\theta)(1-\beta)(1-\lambda)$

Similarly,

$P(1b) = (1-\theta)(1-\beta)\lambda$

$P(2a) = \theta(1-\alpha)(1-\lambda)$

$P(2b) = \theta(1-\alpha)\lambda$

$P(3a) = \theta(1-\alpha)(1-\lambda)$

$P(3b) = \theta\alpha(1-\gamma)$

$P(4a) = (1-\theta)\beta(1-\gamma)$

$P(4b) = \theta\alpha(1-\gamma)$

Normalized weights,

$W = \dfrac{\text{Prior}}{\text{Prior for both cases}}$ $\left(\text{example } w_1 = \dfrac{P(1a)}{P(1a) + P(1b)}\right)$

i.e. 0 & 1

$$w_1 = \frac{P(1a)}{P(1a)+P(1b)} = \frac{(1-\theta)(1-\beta)(1-\lambda)}{[(1-\theta)(1-\beta)(1-\lambda) + (1-\theta)(1-\beta)(\lambda)]}$$

Similarly,

$$w_2 = \frac{(1-\theta)(1-\beta)\lambda}{[(1-\theta)(1-\beta)(1-\lambda) + (1-\theta)(1-\beta)\lambda]}$$

$$w_3 = \frac{\theta(1-\alpha)(1-\lambda)}{[\theta(1-\alpha)(1-\lambda) + \theta\lambda(1-\alpha)]}$$

$$w_4 = \frac{\theta\lambda(1-\alpha)}{[\theta(1-\alpha)(1-\lambda) + \theta\lambda(1-\alpha)]}$$

$$w_5 = \frac{\theta(1-\alpha)(1-\lambda)}{[\theta(1-\alpha)(1-\lambda) + \theta\alpha(1-\gamma)]}$$

$$w_6 = \frac{\theta\alpha(1-\gamma)}{[\theta(1-\alpha)(1-\lambda) + \theta\alpha(1-\gamma)]}$$

$$w_7 = \frac{(1-\theta)\beta(1-\gamma)}{[(1-\theta)\beta(1-\gamma) + \theta\alpha(1-\gamma)]}$$

$$w_8 = \frac{\theta\alpha(1-\gamma)}{[(1-\theta)\beta(1-\gamma) + \theta\alpha(1-\gamma)]}$$

b] M - Step

-> This step is basically recalculating probabilities $\theta, \alpha, \beta, \gamma, \lambda$ based on new weights

$$\theta_{new} = \frac{\text{Sum of weights where } C=1}{\text{Sum of all weights}}$$

$$\theta_{new} = \frac{w_3 + w_4 + w_5 + w_6 + w_8}{w_1 + w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8}.$$

$$\alpha_{new} = \frac{\text{Sum of } w, \text{ given } A=1 \ \& \ C=1}{\text{sum of weights } (A=0, C=1) + (A=1, C=0)}$$

$$\alpha_{new} = \frac{w_6 + w_8}{w_3 + w_4 + w_5 + w_6 + w_8}$$

$$\beta_{new} = \frac{\text{Sum of } w, \text{ given } A=1 \ \& \ C=0}{\text{Sum of } w, \text{ given } (A=0, C=0) + (A=1, C=0)}$$

$$= \frac{w_7}{w_1 + w_2 + w_7}$$

$$\gamma_{new} = \frac{\text{Sum of } w, \text{ given } B=1 \ \& \ A=1}{\text{Sum of } w, \text{ given } (B=0, A=1) + (B=1, A=1)}$$

$$\gamma_{new} = \frac{0}{w_6 + w_7 + w_8} = 0$$

$$\lambda_{new} = \frac{\text{Sum of } w, \text{ given } B=1 \ \& \ A=0}{\text{Sum of } w, \text{ given } (B=0, A=0) + (B=1, A=0)}$$

$$= \frac{w_2 + w_4}{w_1 + w_2 + w_3 + w_4 + w_5}$$