fall 2022 midterm-1

Question2]

a) $K(x_1, x_2) = \phi(x_1)^T \phi(x_2)$

$x_1 = (a_1, b_1)$, $x_2 = (a_2, b_2)$

$\Rightarrow (25 + |a_1, b_1| \cdot |a_2, b_2|)^2$

$\Rightarrow (25 + a_1 a_2 + b_1 b_2)^2$

$\Rightarrow 625 + a_1^2 a_2^2 + b_1^2 \times b_2^2 + 50 a_1 a_2 + 50 b_1 b_2 + 2 a_1 b_1 a_2 b_2$

$\Rightarrow |25, a_1^2, b_1^2, \sqrt{50} a_1, \sqrt{50} b_1, \sqrt{2} a_1 b_1|$

$|25, a_2^2, b_2^2, \sqrt{50} a_2, \sqrt{50} b_2, \sqrt{2} a_2 b_2|$

$\therefore \phi(x) = |25, a^2, b^2, \sqrt{50} a, \sqrt{50} b, \sqrt{2} ab|$

b] primal Form

Objective, $\min_{w,b} \frac{1}{2} \|w\|^2$

s.t. $-(|25, a/4, 1/4, \sqrt{50}\, 3/4, \sqrt{50}/4, \sqrt{2}\, 3/4|^T w + b) \geq 1$

$(|25, 16, 1, \sqrt{50}/-4, \sqrt{50}, -\sqrt{2}\, 4|^T w + b) \geq 1$

$(|25, 16, 1, \sqrt{50}/4, \sqrt{50}, \sqrt{2}\, 4|^T w + b) \geq 1$

$(|25, 9, 4, -\sqrt{50}\, 3, -\sqrt{50}\, 2, \sqrt{2}\, 6|^T w + b) \geq 1$

* Dual Formation

$$\Rightarrow \sum_{i=1}^{4} \alpha_i y_i = 0, \quad -\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4 = 0$$

$$\text{s.t. } \alpha_i \geq 0, \quad i \in \{1, 2, 3, 4\}$$

$$L(\alpha) = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \frac{1}{2}\Big[\alpha_1^2 (1 + \langle 3/2, 1/2 \rangle \cdot \langle 3/2, 1/2 \rangle$$

$$+ \alpha_2^2 (25 + \langle -4, 1 \rangle \cdot \langle -4, 1 \rangle)^2 + \alpha_3^2 (25 + \langle 4, 1 \rangle \cdot \langle 4, 1 \rangle)^2 +$$

$$\alpha_4^2 (25 + \langle -3, -2 \rangle \cdot \langle -3, -2 \rangle)^2 + 2\alpha_1\alpha_2 (25 + \langle 3/2, 1/2 \rangle \cdot \langle -4, 1 \rangle)^2$$

$$- 2\alpha_1\alpha_3 (25 + \langle 3/2, 1/2 \rangle \cdot \langle 4, 1 \rangle)^2 + 2\alpha_1\alpha_4 (25 + \langle 3/2, 1/2 \rangle \cdot \langle -3, -2 \rangle)^2$$

$$+ 2\alpha_2\alpha_3 (25 + \langle -4, 1 \rangle \cdot \langle 4, 1 \rangle)^2 - 2\alpha_2\alpha_4 (25 + \langle -4, 1 \rangle \cdot \langle -3, -2 \rangle)^2$$

$$- 2\alpha_3\alpha_4 (25 + \langle 4, 1 \rangle \cdot \langle -3, -2 \rangle)^2$$