* Fall 2023

Q4] SVM

a) Primal Formulation

=> Primal form with a kernal & slack

$$\Rightarrow \min_{w, w_0, \xi} \frac{1}{2}\|w\|^2 + C \sum_{n=1}^{3} \xi_n$$

s.t. $\xi_i \geq 0$

$$\bar{y}_n (\phi(x_n)^T w + w_0) \geq 1 - \xi_n$$

Given Kernal $(1 + x_i^T x_j)^2$

let $x_i^T = [x_1, x_2]$; $x_j = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix}$

$$\phi(x) = (1 + x_1 x_1' + x_2 x_2')^2$$

$$= 1 + x_1^2 x_1'^2 + x_2^2 x_2'^2 + 2 x_1 x_1' + 2 x_2 x_2' + 2 x_1 x_1' x_2 x_2'$$

trying to seperate x & x'

$$\Rightarrow [1, x_1^2, x_2^2, \sqrt{2} x_1, \sqrt{2} x_2, \sqrt{2} x_1 x_2]^T \cdot [1, x_1'^2, x_2'^2, \sqrt{2} x_1', \sqrt{2} x_2', \sqrt{2} x_1' x_2']$$

$$\phi([x_1, x_2]) = [1, x_1^2, x_2^2, \sqrt{2} x_1, \sqrt{2} x_2, \sqrt{2} x_1 x_2)$$

Given, values of x_1, x_2 we can write

$$\Rightarrow \quad -(\phi([0.2, 0.2])^T w + w_0) \geq 1 - \xi_1,$$
$$(\phi([-0.6, 1.2])^T w + w_0) \geq 1 - \xi_2,$$
$$(\phi([-0.3, 1.5])^T w + w_0) \geq 1 - \xi_3$$

where,

$$\phi([0.2, 0.2]) = [1, 0.4, 0.4, \sqrt{2}\times 0.2, \sqrt{2}\times 0.2, \sqrt{2}\times 0.4]$$
$$\phi([-0.6, 1.2]) = [1, 0.36, 1.44, -\sqrt{2}\times 0.6, \sqrt{2}\times 1.2, -\sqrt{2}\times 0.72]$$
$$\phi([-0.3, 1.5]) = [1, 0.9, 2.25, -\sqrt{2}\times 0.3, \sqrt{2}\times 1.5, -\sqrt{2}\times 0.45]$$

$$\xi_1, \xi_2, \xi_3 \geq 0$$

b] Dual formation

$\Rightarrow$ dual form with slack penalty

$$L(\alpha) = \sum_{i=1}^{3} \alpha_i - \frac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \alpha_i \alpha_j \bar{y}_i \bar{y}_j K(x_i, x_j)$$

$$\text{s.t} \quad 0 \leq \alpha_i \leq C, \quad \sum_{i=1}^{3} \alpha_i \bar{y}_i = 0$$

$\hookrightarrow \quad -\alpha_1 + \alpha_2 + \alpha_3 = 0$

$$\Rightarrow L(\alpha) = \alpha_1 + \alpha_2 + \alpha_3 - \frac{1}{2}\Big[\alpha_1^2 (1 + [0.2, 0.2]^T [0.2, 0.2])^2$$
$$- 2\alpha_1\alpha_2 (1 + [0.2, 0.2]^T [-0.6, 1.2])^2 - 2\alpha_1\alpha_3 (1 + [0.2, 0.2]^T [-0.3, 1.5])^2$$
$$+ 2\alpha_2\alpha_3 (1 + [-0.6, 1.2]^T [-0.3, 1.5])^2 + \alpha_2^2 (1 + [-0.6, 1.2]^T [-0.6, 1.2])^2$$
$$+ \alpha_3^2 (1 + [-0.3, 1.5]^T [-0.3, 1.5])^2\Big]$$

At last if we need to simplify it further