

Question 5] BN

b) Given x_1 is missing

we can just take all possible values of x_1 i.e. $i \in d$

So time complexity = $O(nd)$

Space complexity = $O(n)$

(since we sum out d & store only the remaining values

i.e. dataset

c) x_1, x_2, x_3, x_4 is missing

$$P(x_1) = f(x_1)$$

$$P(x_2 | x_1) = f(x_1, x_2)$$

$$P(x_3 | x_2) = f(x_2, x_3)$$

$$P(x_4 | x_1) = f(x_1, x_4)$$

$$P(x_5 | x_2, x_4) = f(x_2, x_4) \quad (\text{since } x_5 \text{ is given})$$

$$P(x_6 | x_3, x_5) = f(x_3)$$

$$P(x_7 | x_4) = f(x_4)$$

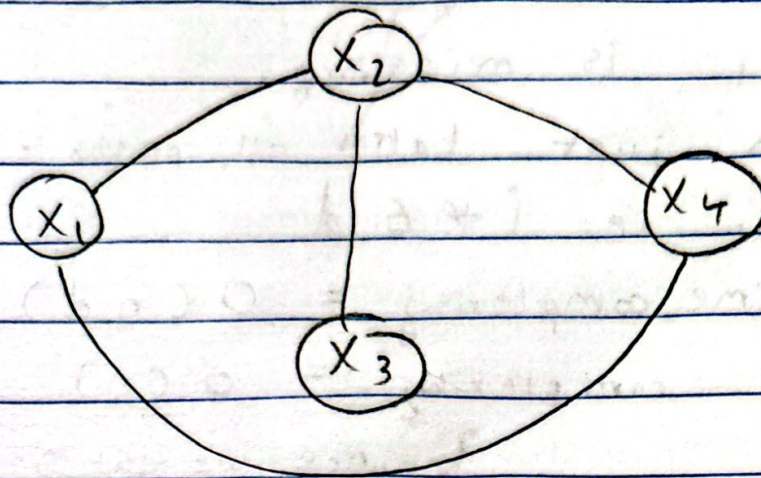
$$P(x_8) \dots \text{constant}$$

$$P(x_9) \dots \text{constant}$$

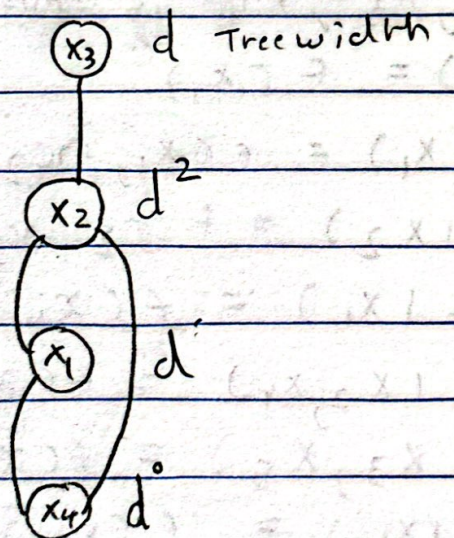
Joint probability,

$$\Rightarrow f(x_1) f(x_1, x_2) f(x_2, x_3) f(x_1, x_4) \\ f(x_2, x_4) f(x_3) f(x_4)$$

Primal Graph,



Elimination order



Tree width = 2

Time complexity $= O(nd^3)$ (n cuz n datasets points)

Space complexity $= O(nd^2)$