Spring 2018
midterm 1

Q3] SVM

a] Dual form without slack penalty

$$L(\alpha) = \sum_{j=1}^{4}\sum_{i=1}^{4} \alpha_i \alpha_j \bar{y}_i \bar{y}_j \, K(x_i, x_j)$$

given $\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4 = 0$ — ①, $\alpha_i \geq 0$

$$\Rightarrow L(\alpha) = \alpha_1^2 (1)(1)(10,01 \cdot 10,01) + \alpha_2^2 (1)(1)(10,11 \cdot 10,11)$$

$$+ \alpha_3^2 (1)(1)(11,01 \cdot 11,01) + \alpha_4^2 (-1)(-1)(11,11 \cdot 11,11)$$

$$+ 2\alpha_1\alpha_2 (1)(1)(10,01 \cdot 10,11) + 2\alpha_1\alpha_3 (1)(1)(10,01 \cdot 11,01)$$

$$+ 2\alpha_1\alpha_4 (1)(-1)(10,01 \cdot 11,11) + 2\alpha_2\alpha_3 (1)(1)(10,11 \cdot 11,01)$$

$$+ 2\alpha_2\alpha_4 (1)(-1)(10,11 \cdot 11,11) + 2\alpha_3\alpha_4 (1)(-1)(11,01 \cdot 11,11)]$$

$$+ \sum_{i=1}^{4} \alpha_i$$

...ar ...oing ...iz ...hen $i=1, j=2$, $i=2, j=1$

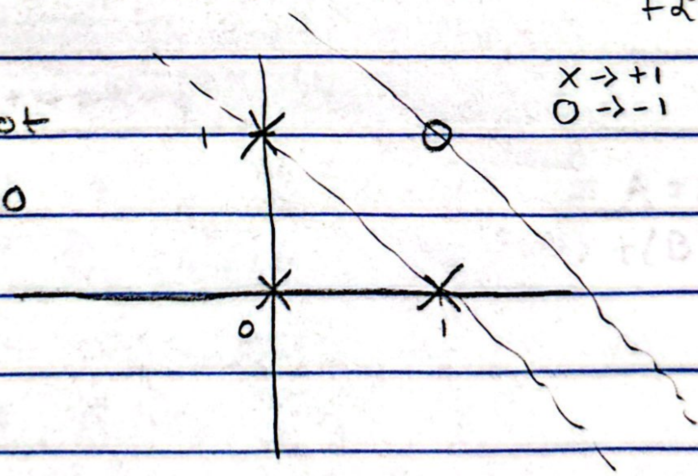
expand this

$$\Rightarrow L(\alpha) = 0 + \alpha_2^2 + \alpha_3^2 + 2\alpha_4^2 + 0 + 0 + 0 + 0$$

$$- 2\alpha_2\alpha_4 - 2\alpha_3\alpha_4 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

$$\Rightarrow L(\alpha) = \alpha_2^2 + \alpha_3^2 + 2\alpha_4^2 - 2\alpha_2\alpha_4 - 2\alpha_3\alpha_4 + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$$

b] α_1 is clearly not support vector, $\alpha_1 = 0$



$X \rightarrow +1$
$O \rightarrow -1$

solve for $\alpha_2, \alpha_3, \alpha_4$

$$\frac{\partial L(\alpha)}{\partial \alpha_2} = 0$$

$$\Rightarrow 2\alpha_2 - 2\alpha_4 + 1 = 0$$

$$\Rightarrow \alpha_2 = \alpha_4 - \frac{1}{2} \quad \text{— ②}$$

Scanned with CamScanner

similarly,

$$\alpha_3 = \alpha_4 - \frac{1}{2} \quad — (3)$$

Substitute (2), (3) in (1), $\alpha_1 = 0$

$$\Rightarrow 0 + \alpha_4 - \frac{1}{2} + \cancel{\alpha_4} - \frac{1}{2} - \cancel{\alpha_4} = 0$$

$$\Rightarrow \alpha_4 = 1$$

$$\alpha_2 = \frac{1}{2}, \alpha_3 = \frac{1}{2}, \alpha_1 = 0$$

& support vectors are points 2, 3, 4