

Question 3]

a) Kernel  $\Rightarrow (100 + x^T y)^2$

$x = (x_1, x_2)$ ,  $y = (y_1, y_2)$

$\Rightarrow$  Substituting  $x \rightarrow y$

$\Rightarrow (100 + (x_1, x_2)^T \cdot (y_1, y_2))^2$

$\Rightarrow (100 + x_1 y_1 + x_2 y_2)^2$

$\Rightarrow 10^4 + x_1^2 y_1^2 + x_2^2 y_2^2 + 200 x_1 y_1 + 200 x_2 y_2 + 2 x_1 y_1 x_2$

$\Rightarrow (100, x_1^2, x_2^2, \sqrt{200} x_1, \sqrt{200} x_2, \sqrt{2} x_1 x_2)^T$

$(100, y_1^2, y_2^2, \sqrt{200} y_1, \sqrt{200} y_2, \sqrt{2} y_1 y_2)^T$

Therefore,

$\phi(x) = (100, x_1^2, x_2^2, \sqrt{2} 10 x_1, \sqrt{2} 10 x_2, \sqrt{2} x_1 x_2)$

which is composed of two attributes

b) Primal formulation

$\Rightarrow \min_{w, w_0} \frac{1}{2} \|w\|^2, \text{ s.t. } \bar{y}_n (\phi(x_n)^T w) \geq 1$

with constraints

$-(\phi(0, 0)^T w + w_0) \geq 1$

$(\phi(1/3, 1)^T w + w_0) \geq 1$

$(\phi(1/3, 1)^T w + w_0) \geq 1$

where  $\phi(0, 0) = (100, 0, 0, 0, 0, 0)$

$\phi(1/3, 1) = (100, 1/9, 1, 10\sqrt{2} \cdot 1/3, 10\sqrt{2}, \sqrt{2} \cdot 1/3)$

$\phi(1/3, 1) = (100, 1/9, 1, \frac{10\sqrt{2}}{3}, 10\sqrt{2}, \frac{\sqrt{2}}{3})$



Dual formulation

$$L(d) = \sum_{i=1}^3 d_i - \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 d_i d_j \bar{y}_i \bar{y}_j K(x_i, x_j)$$

$$0 \leq d_i, i \in \{1, 3\} \quad (\text{constraint})$$

$$\Rightarrow -d_1 + d_2 + d_3 = 0 \quad \text{--- (1)}$$

$$\begin{aligned} L(d) = d_1 + d_2 + d_3 - \frac{1}{2} & \left[ d_1^2 (100 + 10, 01^T 10, 01)^2 \right. \\ & + d_2^2 (100 + 1^{-1/3}, 01^T 1^{-1/3}, 01)^2 + d_3^2 (100 + 1^{1/3}, 01^T 1^{1/3}, 01)^2 \\ & - 2d_1 d_2 (100 + 10, 01^T 1^{-1/3}, 01)^2 - 2d_1 d_3 (100 + \\ & 10, 01^T 1^{1/3}, 01)^2 + 2d_2 d_3 (100 + 1^{-1/3}, 11^T 1^{1/3}, 11)^2 \end{aligned} \quad \text{--- (2)}$$