Spring 2015
Question 6

a] $P(x_1) = f(x_1)$ -

$P(x_2|x_1) = f(x_2|x_1)$ -

$P(x_3|x_1) = f(x_3|x_1)$ -

$P(x_4|x_2,x_3) = f(x_4,x_2,x_3)$ -

$P(x_5|x_4) = f(x_4,x_5)$ -

$P(x_6|x_4) = f(x_4,x_6)$ -

$P(7|x_5,x_6) = f(x_5,x_6) = \text{constant}$

Eliminate x_4

$\Rightarrow \sum_{x_4} f(x_4,x_2,x_3)\, f(x_4,x_5)\, f(x_4,x_6)$

$\Rightarrow g(x_5,x_6,x_2,x_3)$ -

Eliminate x_1

$\Rightarrow \sum_{x_1} f(x_1)\, f(x_2|x_1)\, f(x_3|x_1)$

$\Rightarrow g'(x_2,x_3)$ -

Eliminate x_3

$\Rightarrow \sum_{x_3} g'(x_2,x_3)\, g(x_5,x_6,x_2,x_3)$

$\Rightarrow g''(x_2,x_5,x_6)$ -

Eliminate x_2

$\Rightarrow \sum_{x_2} g''(x_2,x_5,x_6) = g'''(x_5,x_6)$

Eliminate x_5

$\Rightarrow \sum_{x_5} g'''(x_5,x_6) = h(x_6)$

eliminate x_6 -> done

Complexity Time = $O(d^5)$, space = $O(4d^4)$

b) Ordering for best complexity

$x_1, x_2, x_3, x_4, x_5, x_6$

Eliminate x_1,

$\Rightarrow \sum_{x_1} f(x_1)\ f(x_2, x_1)\ f(x_3, x_1)$

$\Rightarrow g(x_2, x_3)$

Eliminate x_2,

$\Rightarrow \sum_{x_2} g(x_2, x_3)\ f(x_2, x_3, x_4)$

$\Rightarrow g'(x_3, x_4)$

Eliminate x_3,

$\Rightarrow \sum_{x_3} g'(x_3, x_4) = g''(x_4)$

Eliminate x_4

$\Rightarrow \sum_{x_4} g''(x_4)\ f(x_4, x_5)\ f(x_4, x_6)$

$\Rightarrow g'''(x_5, x_6)$

Eliminate x_5

$\Rightarrow \sum_{x_5} g'''(x_5, x_6) = h(x_6)$

Eliminate $h(x_6)$ → done

Best complexity

Time = $O(d^3)$

Space = $O(2d^2)$