Question a] Bayesian Networks

$$P(x_1 | x_5, x_7) = f(x_1, x_5, x_7) †$$

$$P(x_2 | x_1) = f(x_2, x_1) †$$

$$P(x_3 | x_2) = f(x_2, x_3) †$$

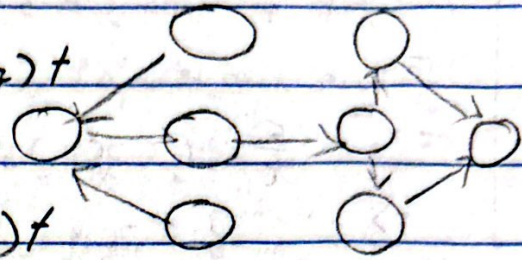$$P(x_4 | x_6, x_8) = f(x_4, x_6, x_8) = \text{constant}$$

$$P(x_5) = f(x_5) †$$

$$P(x_6 | x_3) = f(x_3, x_6) †$$

$$P(x_8 7) = f(x_7) †$$

$$P(x_3 | x_8) = f(x_3, x_8) †$$

a) Eliminate X_1

$$\Rightarrow \sum_{x_1} f(x_1, x_5, x_7) f(x_1, x_2) = g(x_2, x_5, x_7)$$

Eliminate X_2

$$\Rightarrow \sum_{x_2} g(x_2, x_5, x_7) f(x_2, x_3) = g'(x_3, x_5, x_7)$$

Eliminate X_3

$$\Rightarrow \sum_{x_3} g'(x_3, x_5, x_7) f(x_3, x_6) f(x_3, x_8) = g''(x_5, x_6, x_7, x_8)$$

Eliminate X_5

$$\Rightarrow \sum_{x_5} f(x_5) g''(x_5, x_6, x_7, x_8) = g'''(x_6, x_7, x_8)$$

Eliminate X_6

$$\Rightarrow \sum_{x_6} g'''(x_6, x_7, x_8) = h(x_7, x_8)$$

Eliminate X_7

$$\Rightarrow \sum_{x_7} h(x_7, x_8) f(x_7) = h'(x_8)$$

Eliminate $x_8 \rightarrow$ done

complixity,

Time $= O(d^5)$ (for $g''(x_5, x_6, x_7, x_6$

Space $= O(4d^4)$

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b] Eliminate x_5

$\sum_{x_5} f(x_5) f(x_1, x_5, x_7) = g(x_1, x_7) -$

Eliminate x_7

$\sum_{x_7} f(x_7) g(x_1, x_7) = g'(x_1) -$

Eliminate x_1

$\sum_{x_1} g'(x_1) f(x_2, x_1) = g''(x_2) -$

Eliminate x_2

$\sum_{x_2} g''(x_2) f(x_2, x_3) = g'''(x_3)$

Eliminate x_3

$\sum_{x_3} f(x_3, x_6) g'''(x_3) f(x_3, x_8) = h(x_6, x_8$

Eliminate x_6

$\sum_{x_6} h(x_6, x_8) = h'(x_8)$

Eliminate x_8 & done

complexity,

Time $= O(d^3)$

Space $= O(2d^2)$