

Spring 2018 Final

Q3] SVM

Primal form

Objective minimize  $\frac{1}{2} \|w\|^2$   
 $w, b$

$$\text{s.t. } y_i (w^T x_i + b) \geq 1$$

$$\Rightarrow -1 (|w_1, w_2|^T \cdot |0, 0| + b) \geq 1$$

$$1 (|w_1, w_2|^T \cdot |1, -1/3| + b) \geq 1$$

$$1 (|w_1, w_2|^T \cdot |1, 1/3| + b) \geq 1$$

$$\Rightarrow -b \geq 1$$

$$-w_1/3 + w_2 + b \geq 1$$

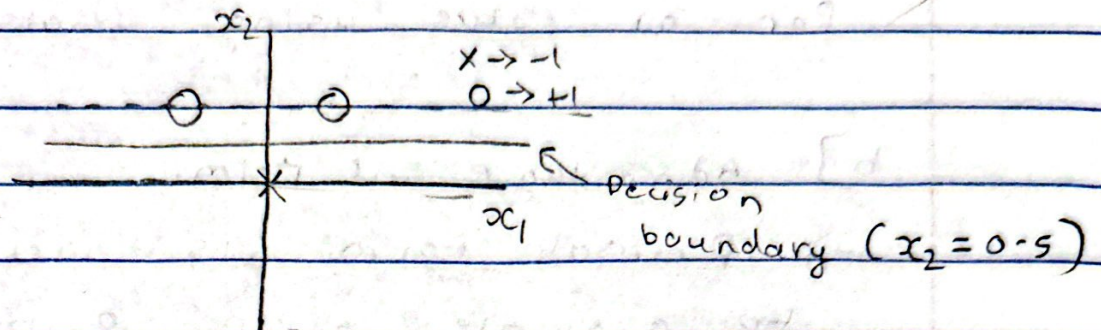
$$w_1/3 + w_2 + b \geq 1, \text{ while minimizing}$$



\* Q3] Hack, caution: don't use this method on exam

To provide precise  $w, b$

Hack the question, I mean use cheat code = plotgraph



We know decision boundary  $\Rightarrow x_2 = 0.5$

Support vectors are  $\Rightarrow x_2 = 1, x_2 = 0$

$\hookrightarrow$  these are eq<sup>n</sup> of lines

$\Rightarrow$  At support vector

$$w_1 x_1 + w_2 x_2 + b = +1 \quad (\text{positive margin})$$

$$w_1 x_1 + w_2 x_2 + b = -1 \quad (\text{negative margin})$$

Also, we know that  $w_1 = 0$  since our margins are all horizontal.

$$\Rightarrow w_2 x_2 + b = 1, \quad w_2 x_2 + b = -1$$

At ~~positive~~ <sup>negative</sup> margin,  $x_1 = 0, x_2 = 0$

$$\Rightarrow w_2 \times 0 + b = -1, \quad b = -1$$

At positive margin

$$\Rightarrow w_2 \times 1 + b = +1$$

$$\Rightarrow \underline{w_2 = 2}$$

Hm, we cheated & got right ans obviously

$$\text{width of margin} = \frac{2}{\|w\|} = \frac{2}{\sqrt{0^2 + 2^2}} = \frac{2}{2} = 1$$