Fall
2020
midterm-1

Q3] SVM

a) $K(x,y) = (225 + x^T y)^2$

$x = (x_1, x_2)$, $y = (y_1, y_2)$

$\Rightarrow (225 + |x_1, x_2| \cdot |y_1, y_2|)^2$

$\Rightarrow (225 + x_1 y_1 + x_2 y_2)^2$

$\Rightarrow 225^2 + x_1^2 y_1^2 + x_2^2 y_2^2 + 450 x_1 y_1 + 450 x_2 y_2$
$+ 2 x_1 y_1 x_2 y_2$

$\Rightarrow |225, x_1^2, x_2^2, \sqrt{450} x_1, \sqrt{450} x_2, \sqrt{2} x_1 x_2|$
$|225, y_1^2, y_2^2, \sqrt{450} y_1, \sqrt{450} y_2, \sqrt{2} y_1 y_2|$

$\therefore \phi(x) = [225, x_1^2, x_2^2, \sqrt{450} x_1, \sqrt{450} x_2, \sqrt{2} x_1 x_2]$
$x = (x_1, x_2)$

$\therefore$ This is a valid kernal since we can gerate a valid feature space.

b] Primal

Objective $\underset{w, b}{\text{minimize}} \ \frac{1}{2} \|w\|^2$

s.t. $-(|225, 0, 0, 0, 0, 0|^T w + b) \geq 1$
$(|225, 1/9, 1, -\sqrt{450}/3, \sqrt{450}, -\sqrt{2}/3|^T w + b) \geq 1$
$(|225, 1/9, 1, \sqrt{450}/3, \sqrt{450}, \sqrt{2}/3|^T w + b) \geq 1$
$-(|225, 0, 1, 0, -\sqrt{450}, 0|^T w + b) \geq 1$

★ Dual formulation

$\Rightarrow \sum_{i=1}^{4} \alpha_i y_i = 0, \quad -\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4 = 0, \quad s.t.\ \alpha_i \geq 0$

$\Rightarrow L(\alpha) = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \frac{1}{2} \Big[\alpha_1^2 (225 + 0)^2 +$

$\alpha_2^2 (225 + |-1/3, 1| \cdot |-1/3, 1|)^2 + \alpha_3^2 (225 + |1/3, 1| \cdot |1/3, 1|)^2 +$
$\alpha_4^2 (225 + |0, -1| \cdot |0, -1|)^2 - 2\alpha_1\alpha_2 (225 + 0)^2 - 2\alpha_1\alpha_3 (225)^2$
$+ 2\alpha_1\alpha_4 (225 + 0)^2 + 2\alpha_2\alpha_3 (225 + |-1/3, 1| \cdot |1/3, 1|)^2$
$- 2\alpha_2\alpha_4 (225 + |-1/3, 1| \cdot |0, -1|)^2 - 2\alpha_3\alpha_4 (225 + |1/3, 1| \cdot |0, -1|$

★ Support vectors

Therefore Support vectors should be $(0, 0), (-1/3, 1), (1/3, 1)$