

Fall 2019 Final  
Q.5]

Q5] a)  $P(A=1) = 0.3$

$P(B=1 | A=1) = 0.6$ ,  $P(B=1, A=0) = 0.2$   $(A) \rightarrow (B) \rightarrow (C)$

$P(C=1 | B=1) = 0.1$ ,  $P(C=1 | B=0) = 0.9$

Likelihood,

$L = P(A) P(B|A) P(C|B)$

Missing data Filled

	A	B	C	W
1 <sup>st</sup>	0	1	0	$w_1$
	0	1	1	$w_2$
3 <sup>rd</sup>	0	0	1	$w_3$
	1	0	1	$w_4$
4 <sup>th</sup>	1	1	0	$w_5$
	1	1	1	$w_6$

$P(w_1) = (1-0.3) \cdot 0.2 \cdot (1-0.1) = 0.14 \times 0.9$

$P(w_2) = (1-0.3) \cdot 0.2 \cdot 0.1 = 0.014$

$P(w_3) = (1-0.3) \cdot (1-0.2) \cdot 0.9 = 0.63 \times 0.8$

$P(w_4) = 0.3 \cdot (1-0.6) \cdot 0.9 = 0.12 \times 0.9$

$P(w_5) = 0.3 \cdot 0.6 \cdot (1-0.1) = 0.18 \times 0.9$

$P(w_6) = 0.3 \cdot 0.6 \cdot 0.1 = 0.18 \times 0.1$



Normalize weights,

$$w_1 = \frac{P(w_1)}{P(w_1) + P(w_2)} = \frac{0.14 \times 0.9}{0.14 \times 0.9 + 0.04} = 0.9 = \boxed{0.9}$$

$$w_2 = \boxed{0.1}$$

$$w_3 = \frac{0.63 \times 0.2}{0.12 \times 0.9 + 0.63 \times 0.2} = \frac{0.126}{0.126 + 0.126} = \frac{0.126}{0.252} = \frac{14}{17} = \boxed{\frac{14}{17}}$$

$$w_4 = \frac{0.108}{0.108 + 0.504} = \frac{0.108}{0.612} = \frac{27}{117} = \frac{3}{17} = \boxed{\frac{3}{17}}$$

$$w_5 = \frac{0.18 \times 0.9}{0.18 \times 0.9 + 0.18 \times 0.1} = \frac{0.162}{0.162 + 0.018} = \frac{0.162}{0.18} = 0.9 = \boxed{0.9}$$

$$w_6 = \boxed{0.1}$$

b] M-step

$$P(A=1|0) = \frac{w_4 + w_5 + w_6}{3}$$

$$P(B=1|A=1) = \frac{w_5 + w_6}{w_4 + w_5 + w_6}$$

$$P(B=1|A=0) = \frac{w_1 + w_2}{w_1 + w_2 + w_3}$$

$$P(C=1|B=1) = \frac{w_2 + w_6}{w_2 + w_6 + w_1 + w_5}$$

$$P(C=1|B=0) = \frac{w_3 + w_4}{w_3 + w_4}$$