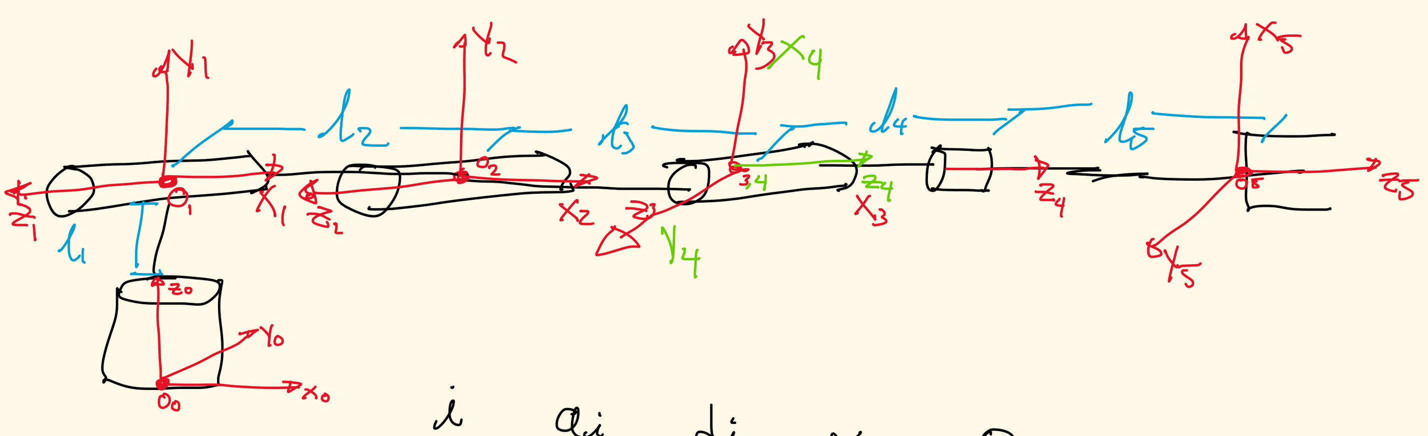
Cinemática inversa

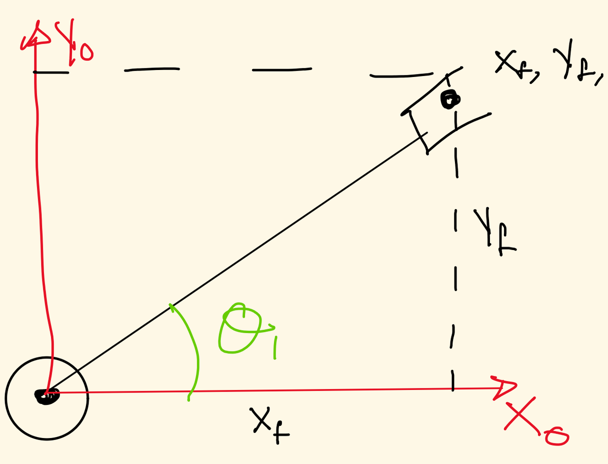
lunes, 28 de noviembre de 2022 18:07



i	a_i	d_i	α_i	θ_i
1	0	l_1	$\pi/2$	θ_1
2	l_2	0	0	θ_2
3	l_3	0	0	θ_3
4	0	0	$\pi/2$	θ_4
5	0	l_4+l_5	0	θ_5

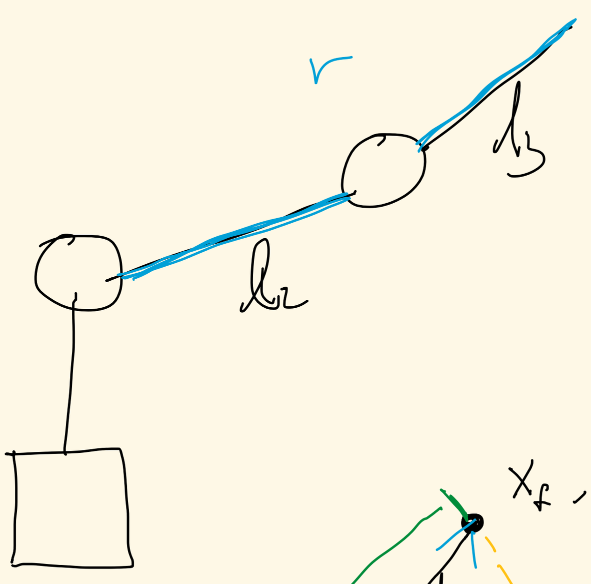
Cinematica inversa

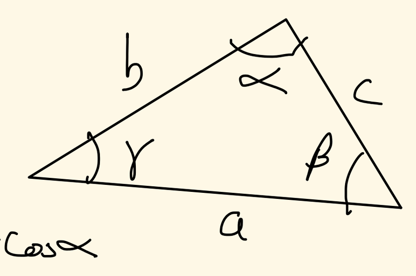
Posicionandonos en el plano X_0, Y_0



$$\boxed{\theta_1 = \operatorname{atan2}(Y_f, X_f)}$$

Posicionandonos en el plano X_1, Y_1 y haciendo el desacople de la muñeca

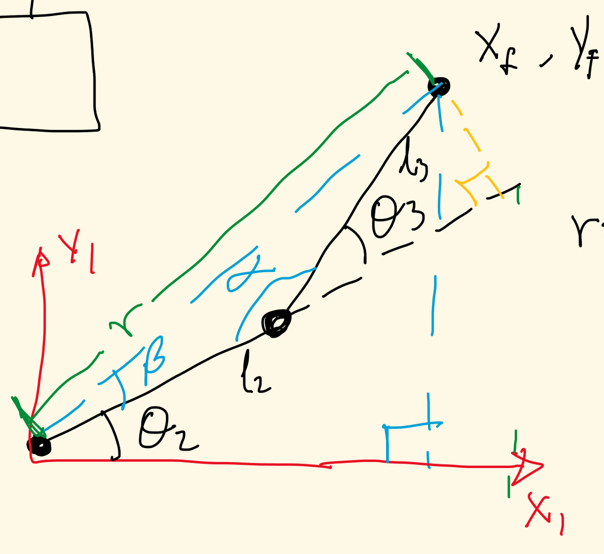




Ley de cosenos $\rightarrow \dfrac{b^2+c^2-a^2}{2bc} = \cos\alpha$

Suma de cosenos $\rightarrow \cos(x \pm y) = \cos x \cos y \mp \operatorname{sen} x \operatorname{sen} y$

$C\theta^2 + S\theta^2 = 1$



$r = \sqrt{X_f^2 + Y_f^2}$

$$\tan(\beta + \theta_2) = \frac{Y_f}{X_f}$$

$$\boxed{\operatorname{atan2}(z_f - l_1, r) - \beta = \theta_2} \quad ①$$

$$\tan\beta = \frac{Y}{X}$$

$(l_2 + l_3)\cos\theta_3 \rightarrow X$

$l_3 \sin\theta_3 \rightarrow Y$

$$\tan\beta = \frac{l_3 \sin\theta_3}{(l_2+l_3)\cos\theta_3}$$

$$\boxed{\beta = \operatorname{atan2}(l_3 \sin\theta_3, (l_2+l_3)\cos\theta_3)} \quad ②$$

$\theta_3 + \alpha = 180^0$

$\theta_3 = 180 - \alpha$

$\cos\theta_3 = \cos(180 - \alpha)$

$\cos\theta_3 = \cos(180)\cos\alpha + \sin 180 \sin\alpha$

$\cos\theta_3 = -\cos\alpha$

Por Ley de cosenos:

$$\boxed{\cos\alpha = \frac{l_2^2 + l_3^2 - r^2 - (z_f - l_1)^2}{2 l_2 l_3} \approx D}$$

$\cos\theta_3 = -D$

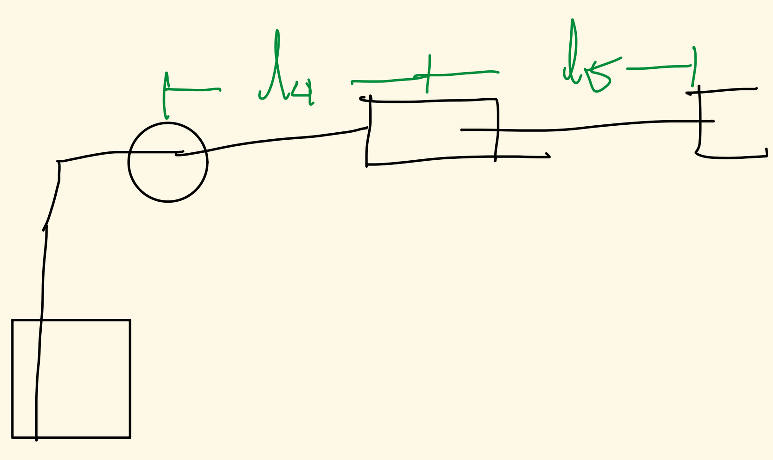
$\sin^2\theta_3 + \cos^2\theta_3 = 1 \longrightarrow \sin^2\theta_3{}^3 - D^2 = 1$

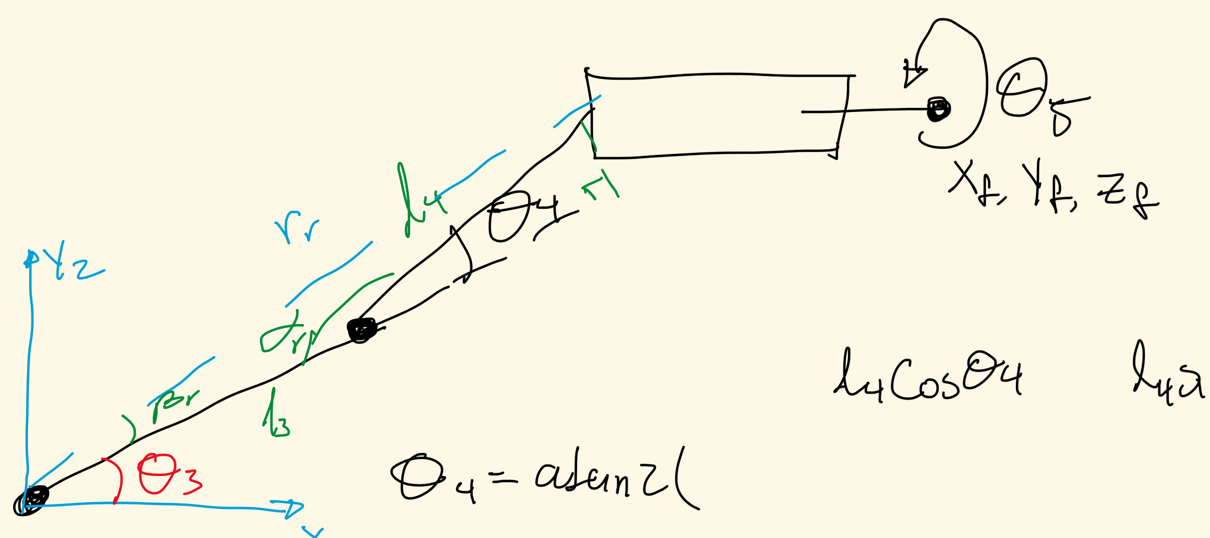
$\sin^2\theta_3 = 1 + D^2$

$\sin\theta_3 = \sqrt{1+D^2}$

$$\boxed{\theta_3 = \operatorname{atan2}(\pm\sqrt{1+D^2}, D)}$$

Viendo el desacople tenemos y situandonos en el plano X_3, Y_3





$l_4 \cos\theta_4$ $\quad l_4 \sin\theta_4$

$\theta_4 = \operatorname{atan2}($

$r_r = \sqrt{l_3^2 + l_4^2}$

$\theta_4 + \alpha_r = 180^\circ$

$\theta_4 = 180 - \alpha_r$

$\cos\theta_4 = \cos 180 \cos\alpha_r + \sin 180 \sin\alpha_r$

$\cos\theta_4 = -\cos\alpha_r$

$$\cos\alpha_r = \frac{l_3^2 + l_4^2 - r_r^2 - (z_f - l_1)^2}{2 l_3 l_4} \longrightarrow D_r$$

$\cos\theta_4 = -D_r$

$\sin\theta_4^2 + \cos\theta_4^2 = 1$

$\sin\theta_4^2 - D_r^2 = 1$

$\sin\theta_4 = \sqrt{1 + D_r^2}$

$$\boxed{\theta_4 = \operatorname{atan2}(\pm\sqrt{1+D_r^2}, D_r)}$$

$$\boxed{\theta_5 = \operatorname{atan2}(Z_f, Y_f)}$$