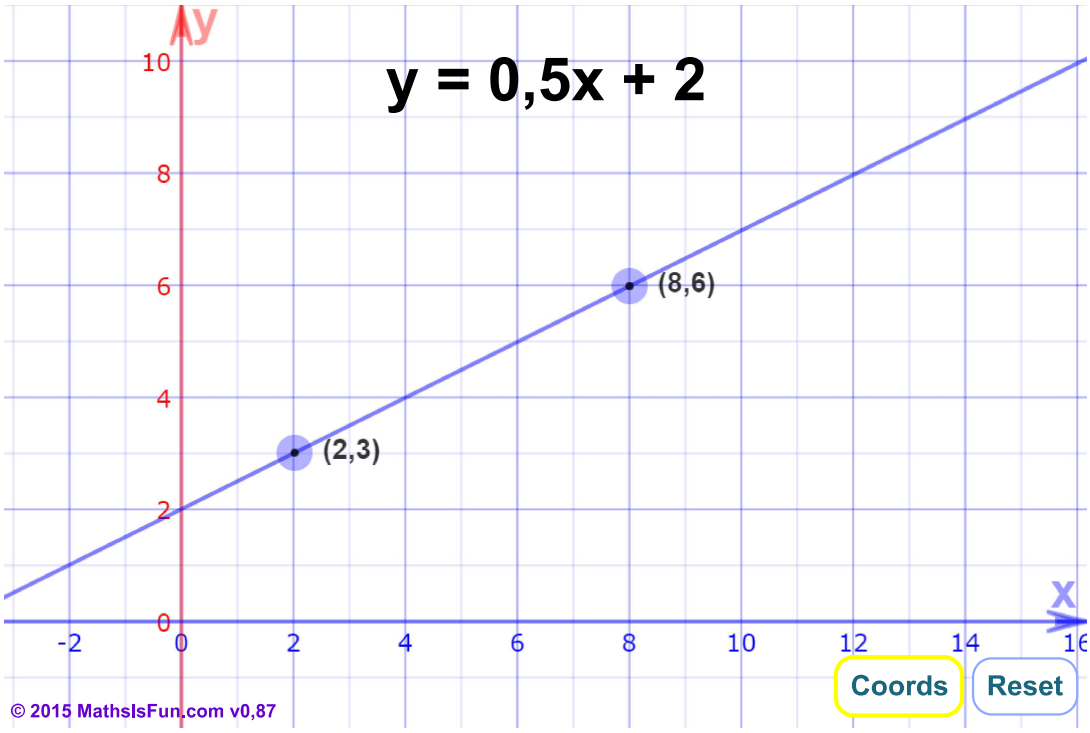
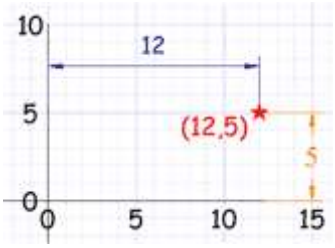
Equation of a Line from 2 Points

First, let's see it in action. Here are two points (you can drag them) and the equation of the line through them. Explanations follow.



The Points

We use Cartesian Coordinates to mark a point on a graph by **how far along** and **how far up** it is:



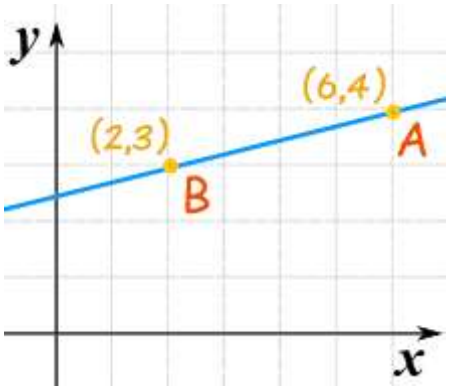
Example: The point **(12,5)** is 12 units along, and 5 units up

Steps

There are 3 steps to find the Equation of the Straight Line :

- 1. Find the slope of the line
- 2. Put the slope and one point into the "Point-Slope Formula"
- 3. Simplify

Step 1: Find the Slope (or Gradient) from 2 Points



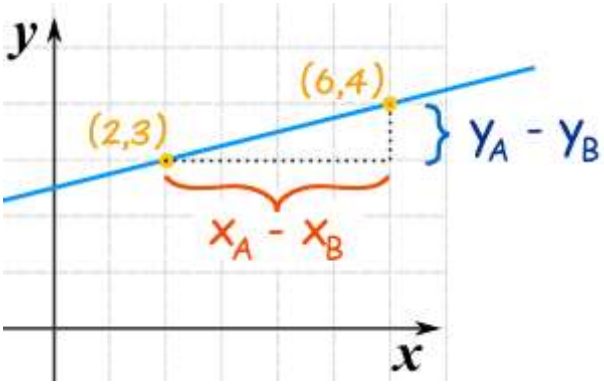
What is the slope (or gradient) of this line?

We know two points:

- point "A" is (6,4) (x is 6 when y is 4)
- point "B" is (2,3) (x is 2 when y is 3).

The slope is the **change in height** divided by the **change in horizontal distance**.

Looking at this diagram ...



... the formula is:

$$\text{Slope } m = \frac{\text{change in y}}{\text{change in x}} = \frac{y_A - y_B}{x_A - x_B}$$

So we:

- subtract the Y values,
- subtract the X values
- then divide

Like this:

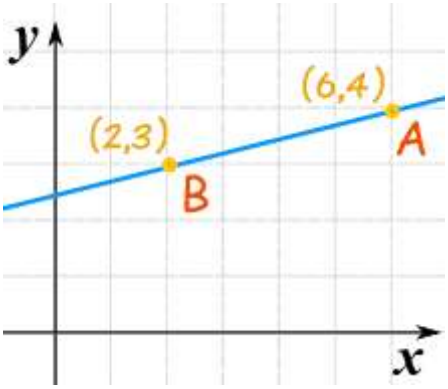
$$m = \frac{\text{change in y}}{\text{change in x}} = \frac{4-3}{6-2} = \frac{1}{4} = 0{,}25$$

It doesn't matter which point comes first, it still works out the same. Try swapping the points:

$$m = \frac{\text{change in y}}{\text{change in x}} = \frac{3-4}{2-6} = \frac{-1}{-4} = 0{,}25$$

Step 2: The "Point-Slope Formula"

Now put the slope and one point into the "Point-Slope Formula"



Start with the "point-slope" formula ($\mathbf{x_1}$ and $\mathbf{y_1}$ are the coordinates of a point on the line):

$$y - y_1 = m(x - x_1)$$

We can choose **any point** on the line for $\mathbf{x_1}$ and $\mathbf{y_1}$, so let's just use point $(2,3)$:

$$y - 3 = m(x - 2)$$

We already calculated the slope "m":

$$\mathbf{m} = \frac{\text{change in y}}{\text{change in x}} = \frac{4-3}{6-2} = \frac{1}{4}$$

And we have:

$$y - 3 = (1/4)(x - 2)$$

That is an answer, but we can simplify it further

Step 3: Simplify

Start with:	$y - 3 = (1/4)(x - 2)$
Put the 1/4 on to x and −2:	$y - 3 = x/4 - 2/4$
Add 3 to both sides:	$y = x/4 - ½ + 3$
Calculate $-½ + 3 = 5/2$:	$y = x/4 + 5/2$

And we get

$$y = x/4 + 5/2$$

Which is now in the Slope-Intercept ($\mathbf{y = mx + b}$) form.

Check It!

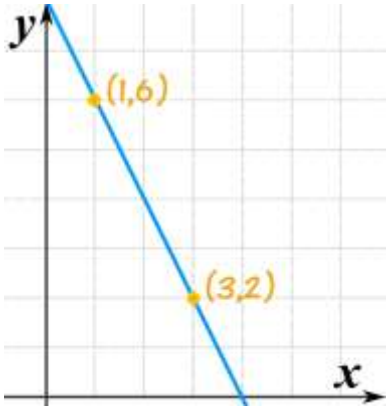
Let us confirm by testing with the second point (6,4):

$$\mathbf{y} = \mathbf{x}/4 + 5/2 = \mathbf{6}/4 + 2{,}5 = 1{,}5 + 2{,}5 = \mathbf{4}$$

Yes, when x=6 then y=4, so it works!

Another Example

Example: What is the equation of this line?



Start with the "point-slope" formula:

$$y - y_1 = m(x - x_1)$$

Put in these values:

- $x_1 = 1$
- $y_1 = 6$
- $m = (2-6)/(3-1) = -4/2 = -2$

And we get:

$$y - 6 = -2(x - 1)$$

Simplify to Slope-Intercept (**y = mx + b**) form:

$$y - 6 = -2x + 2$$

$$y = -2x + 8$$

DONE!

The Big Exception

The previous method works nicely except for one particular case: a **vertical line**:

A vertical line's gradient is undefined (because we cannot divide by 0):

$$m = \frac{y_A - y_B}{x_A - x_B} = \frac{4 - 1}{2 - 2} = \frac{3}{0} = \text{undefined}$$

But there is still a way of writing the equation: use **x=** instead of **y=**, like this:

$$x = 2$$

Question 1 Question 2 Question 3 Question 4 Question 5 Question 6 Question 7 Question 8 Question 9 Question 10