

$$\hat{H} = \sum_{i,j} h_{ij} \hat{E}_{IJ} + \hat{U} + \sum_i v'_i \hat{n}_i = \sum_{i,j} h_{ij} \hat{E}_{IJ} + \hat{U} - \sum_i \mu_i \hat{n}_i = \sum_{i \neq j} h_{ij} \hat{E}_{IJ} + \hat{U} + \sum_i v_i^{ext} \hat{n}_i \quad (1)$$

Note that we split one-electron operator on 2 contributions: where $i \neq j$ and where $i = j$. This is also used in python code (`Molecule.t` and `Molecule.v_ext` respectively)

x	Description
<code>Molecule.Ne</code>	Number of electrons in the lattice
<code>Molecule.Ns</code>	Number of atoms in the lattice
<code>Molecule.u</code>	1D array that represents on site repulsion: $x_i = U_{i,i,i,i} = \langle ii ii \rangle$
<code>Molecule.t</code>	2D array that represents one-electron Hamiltonian where $i \neq j$: $x_{ij} = \begin{cases} 0 & \text{i and j are not connected} \\ t_{ij} & \text{otherwise} \end{cases}$ it is analog to the $\langle \chi_i \hat{h} \chi_j \rangle$
<code>Molecule.v_ext</code>	1D array; sum of $h_{ii} - \mu_i = h_{ii} + v'_i = v_i^{ext}$. It is predefined by our system and it doesn't change during our calculations
<code>Molecule.equiv_atom_groups</code>	dictionary of different sites that are equivalent (and because of that recalculation would be just repetitive. It is a dict with keys $[0, N_{sites}]$ and for each entry value is tuple of all sites (according to Hamiltonian matrix) e.g., $\{0: (0, 3), 1: (1, 2, 4, 5)\}$
<code>Molecule.embedded_mol</code>	It is a representation of a cluster in <code>class_Quant_NBody.QuantNBody</code> class which is class created by me on top of Saad's library <code>Quant_NBody</code>

1 Kohn-Sham

x	Description
Molecule.mu_s	1D vector that ensures that KS system will have correct density with only 1-electron interactions + additional potential called Hartree-Exchange correlation potential: $x_i = \mu^s = \mu^{ext} - v^{Hxc}$
Molecule.h_ks	2D vector. It combines kinetic energy part of one electron Hamiltonian ($i \neq j$) and Kohn-Sham potential: $x_{ij} : h_{ij}^{KS} = h_{i \neq j} - \mu_i^s \delta_{ij}$
Molecule.wf_ks	2D array that represents all wave functions of non-interacting system ordered by energy. Coefficients for the j -th wave function on the site i on the lattice is represented by x_{ij} . This means that wave function is presented by vector $\vec{v}_k = x_{ik}$
Molecule.epsilon_s	1D vector of Kohn-Sham energies
Molecule.y_a	2D array of 1RDM. y due to similarity to γ and a to remind that this is only density per spin (α). It is generated from KS equation.
Molecule.n_ks	1D vector that is made from diagonal elements of γ_α . It represents occupation numbers for each site that are important for CASCI part of algorithm.
Molecule.mu_hxc	1D vector that is the connection between KS and CASCI. In KS part it is represent Hxc chemical potential ($x_i = \mu_i^{Hxc} = -v_i^{Hxc}$) and in CASCI it represents impurity potential: $x_i = \tilde{\mu}_i^{imp}$

2 CAsCI

Here are variables that are nested into CAsCI method

x	Description
site_group	Key of a dictionary that goes over all equivalent atom groups.
site_id	index of 1 (first) site in the group where we actually calculate CAsCI
y_a_correct_imp	2D matrix that is a copy of γ_α with permuted indices in such way that our our impurity gets index 0 (this is important since Householder doesn't transform site with index 0). This is done by simply switching columns (<code>site_id</code> , 0) and then also same rows.
v	1D Householder vector that creates bath and environment orbitals. $x_i = v_i = \begin{cases} 0 & i = 0 \\ \frac{\gamma_{10}-\xi}{\sqrt{2\xi(\xi-\gamma_{10})}} & i = 1 \\ \frac{\gamma_{i0}}{\sqrt{2\xi(\xi-\gamma_{10})}} & i \geq 2 \end{cases}$ where $\xi = -\text{sign}(\gamma_{10})\sqrt{\sum_{j>0} \gamma_{j0}^2}$
P	2D Transformation matrix that transforms 1RDM: $x_{ij} = P_{ij} = \delta_{ij} - 2v_i v_j$
y_a_tilde	This is also 1RDM but in Householder transformed space: $X = P \cdot \gamma_\alpha \cdot P$. Note that this is not Molecule class object because now when we have embedding different impurities will have different Householder transformations.
h_tilde	In save way we transformed 1RDM we also transform one-electron Hamiltonian. This is then $P \cdot (t + v^{ext}) \cdot P$
h_tilde_dimer	just cropped transformed one-electron Hamiltonian that will be input for CAsCI in <code>Quant_NBody</code>
u_0_dimer	4D tensor that represents $\langle ij kl \rangle$. In our case this is for dimer with all values 0 ("Non-interacting bath") but impurity ($x_{0000} = U$)

<code>mu_imp</code>	This is number that corresponds to chemical potential of the impurity. At each cycle it is read from Hxc potential at site that corresponds to <code>site_id</code> and in the end written there.
<code>optimized_v_imp_obj</code>	object that is returned by <code>scipy.optimize.minimize</code> optimizer. It has information about how close we were able to come to KS result and optimized impurity chemical potential.
<code>cost_function_CASCI</code>	function that calculates square difference of occupation in KS system and CASCI at given impurity potential.