



# Benemérita Universidad Autónoma de Puebla

*Facultad de Ciencias de la Computación*

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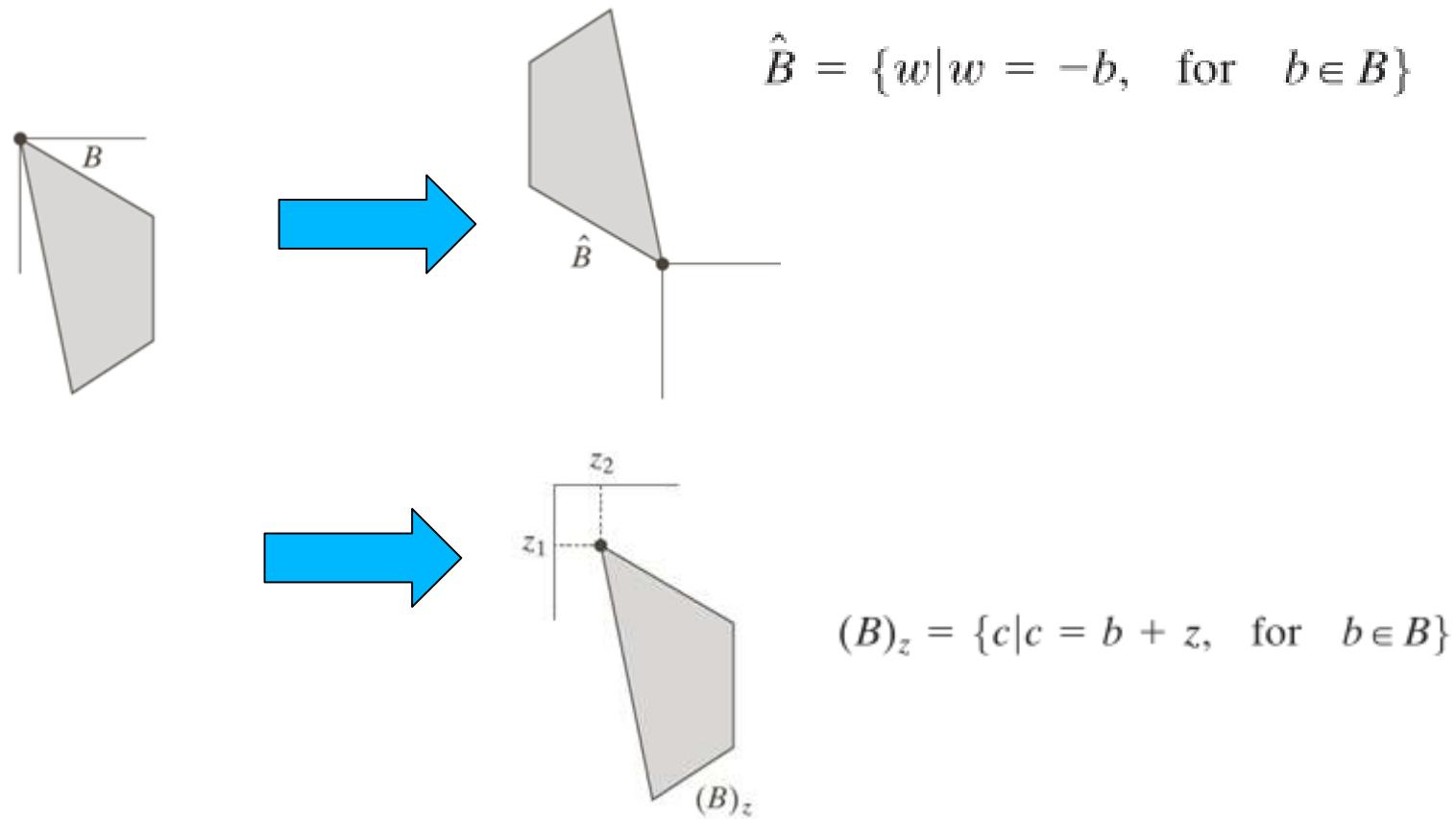


# Morfología



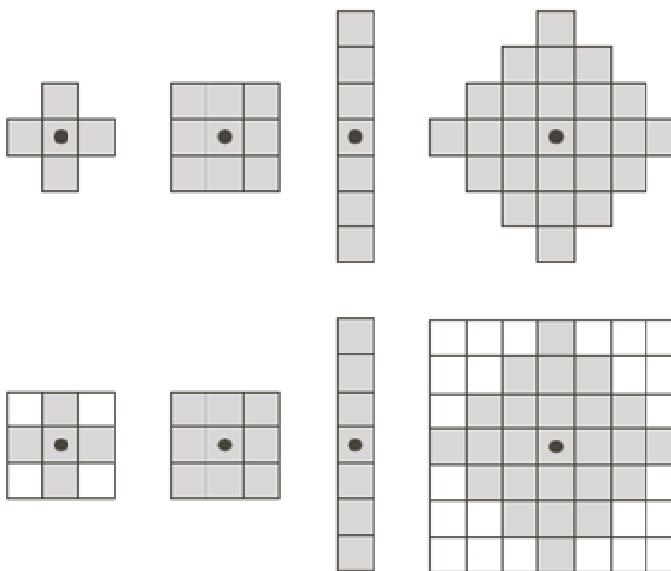


# Morfología



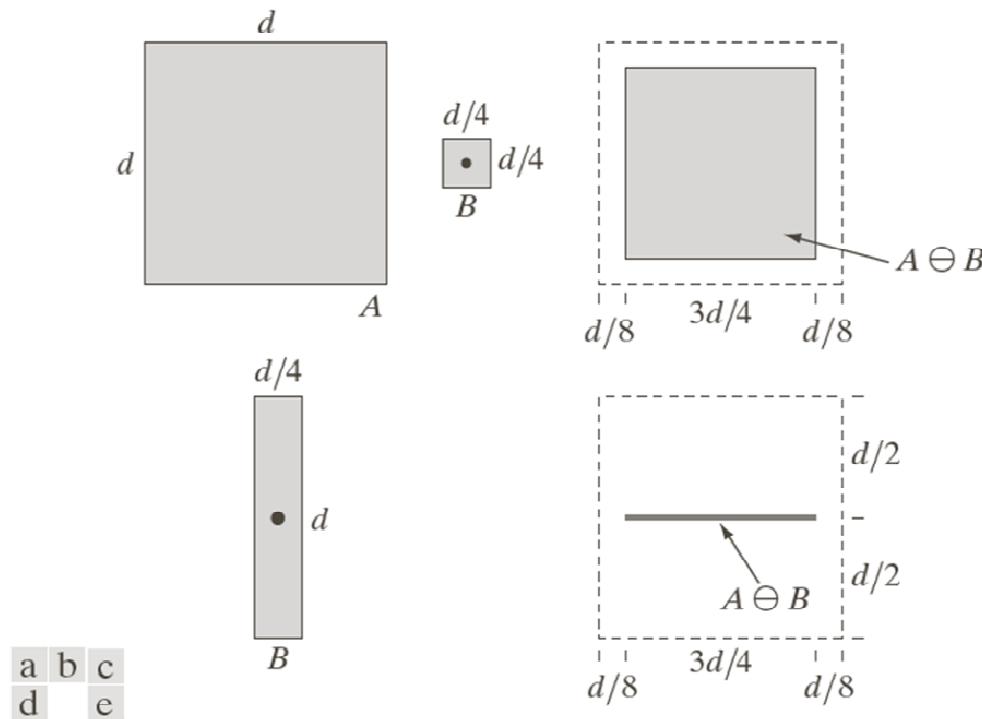


# Elemento Estructura





# Erosión



**FIGURE 9.4** (a) Set  $A$ . (b) Square structuring element,  $B$ . (c) Erosion of  $A$  by  $B$ , shown shaded. (d) Elongated structuring element. (e) Erosion of  $A$  by  $B$  using this element. The dotted border in (c) and (e) is the boundary of set  $A$ , shown only for reference.

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

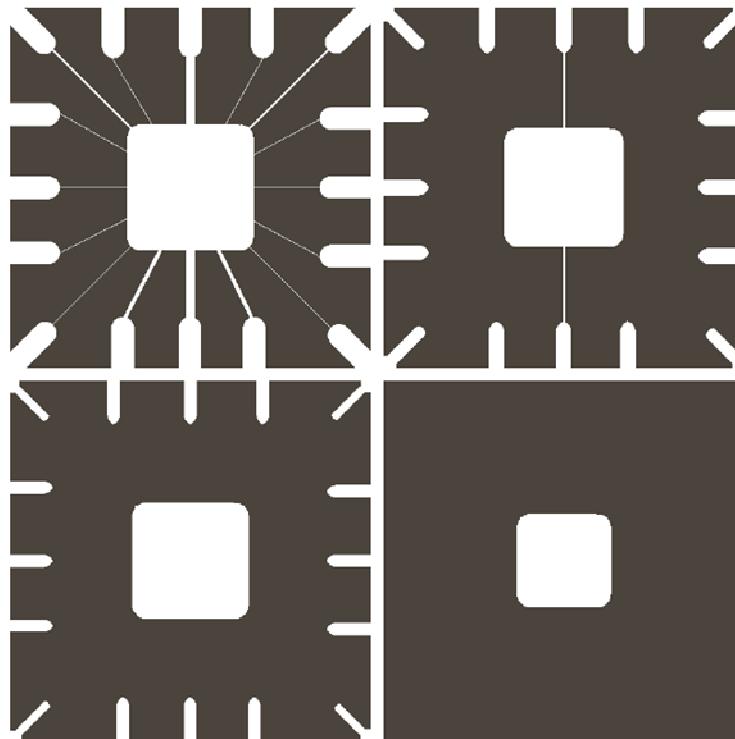
$$A \ominus B = \{z | (B)_z \cap A^c = \emptyset\}$$





# Erosión

- Aplicación:



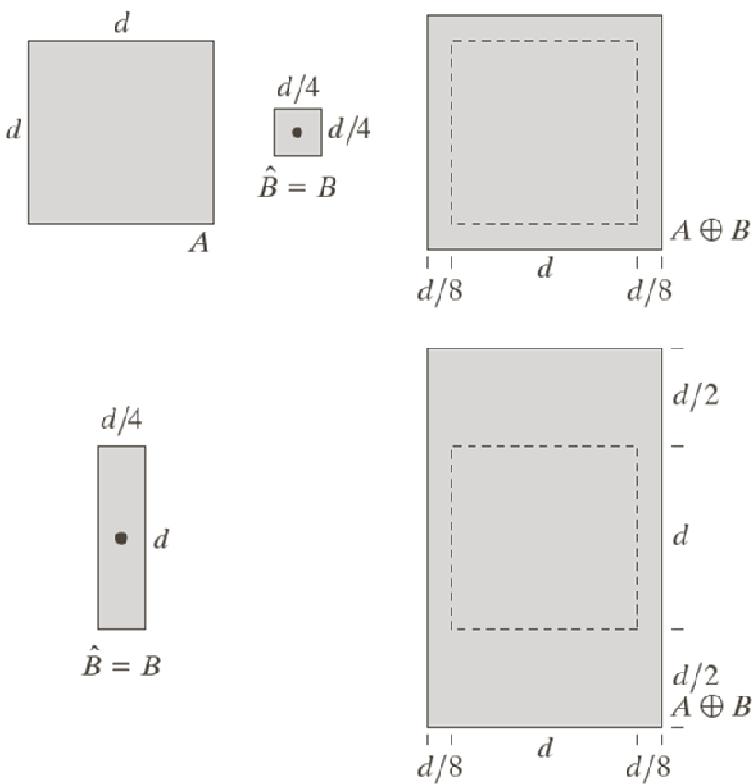
a b  
c d

**FIGURE 9.5** Using erosion to remove image components. (a) A  $486 \times 486$  binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes  $11 \times 11$ ,  $15 \times 15$ , and  $45 \times 45$ , respectively. The elements of the SEs were all 1s.





# Dilatación



a	b	c
d		e

**FIGURE 9.6**

- (a) Set  $A$ .
- (b) Square structuring element (the dot denotes the origin).
- (c) Dilation of  $A$  by  $B$ , shown shaded.
- (d) Elongated structuring element.
- (e) Dilation of  $A$  using this element. The dotted border in (c) and (e) is the boundary of set  $A$ , shown only for reference

$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \emptyset\}$$





# Dilatación

## • Aplicación

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



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0	1	0
1	1	1
0	1	0

a      b      c

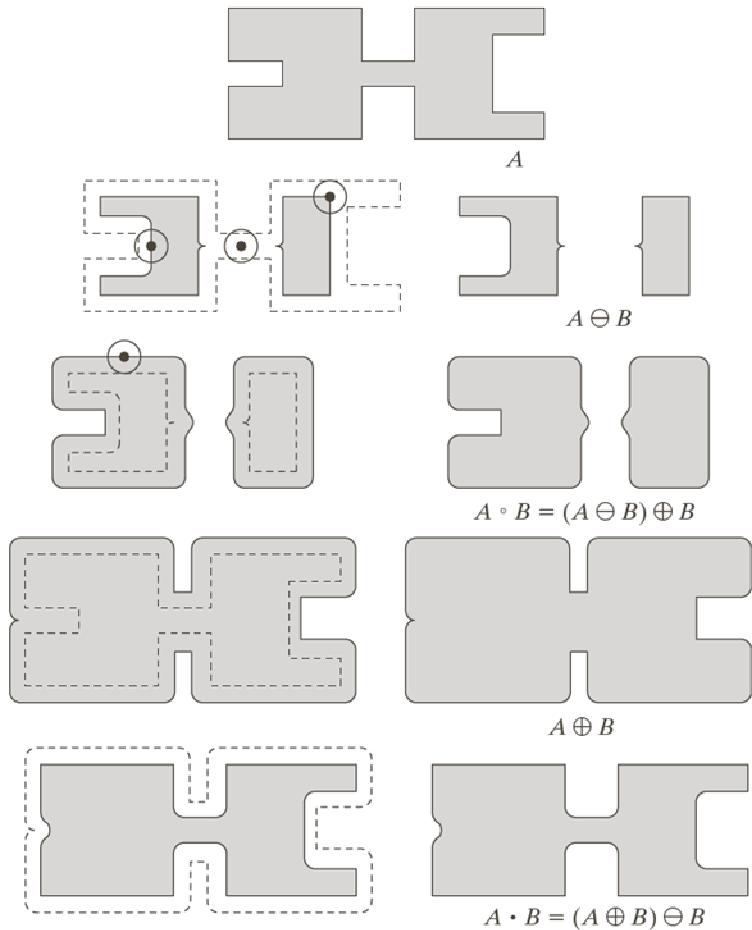
FIGURE 9.7

(a) Sample text of poor resolution with broken characters (see magnified view).  
(b) Structuring element.  
(c) Dilation of (a) by (b). Broken segments were joined.





# Apertura / Cierre



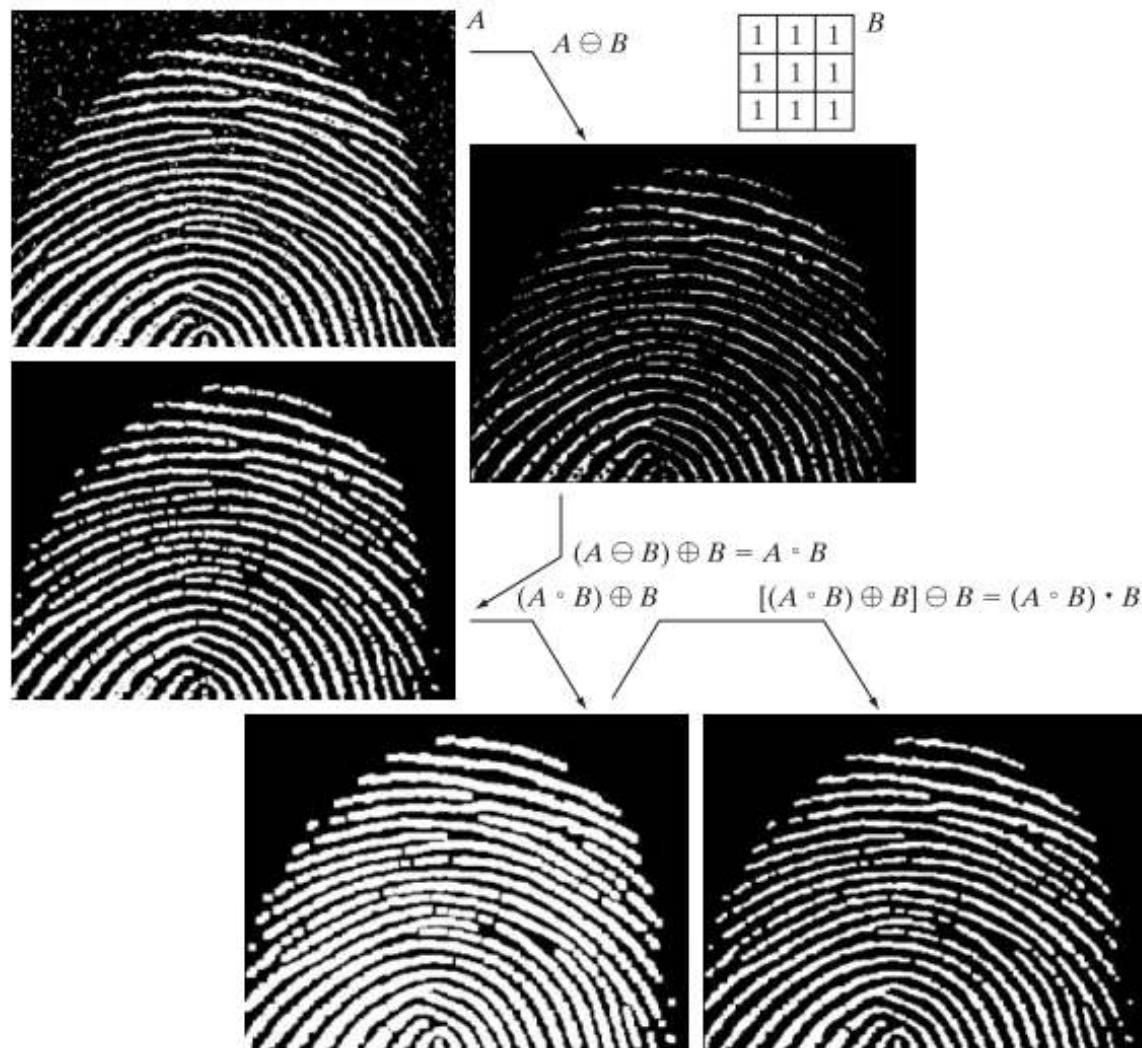
a  
b c  
d e  
f g  
h i

**FIGURE 9.10**  
Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The SE was not shaded here for clarity. The dark dot is the center of the structuring element.





# Aplicación Apertura/Cierre



**FIGURE 9.11**  
(a) Noisy image.  
(b) Structuring element.  
(c) Eroded image.  
(d) Opening of  $A$ .  
(e) Dilation of the opening.  
(f) Closing of the opening.  
(Original image courtesy of the National Institute of Standards and Technology.)





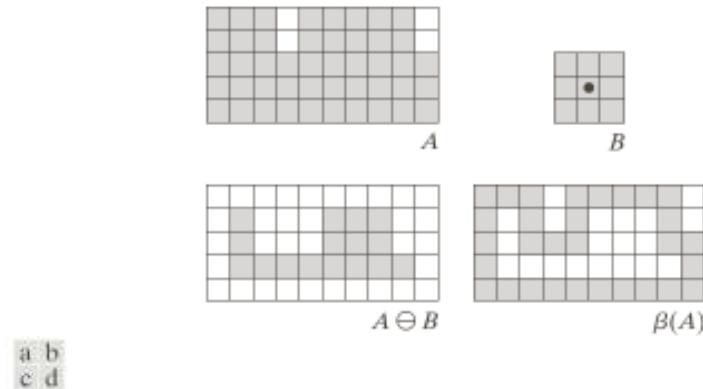
# Extracción de bordes

Con morfología ???





# Extracción de bordes



**FIGURE 9.13** (a) Set  $A$ . (b) Structuring element  $B$ . (c)  $A$  eroded by  $B$ . (d) Boundary, given by the set difference between  $A$  and its erosion.





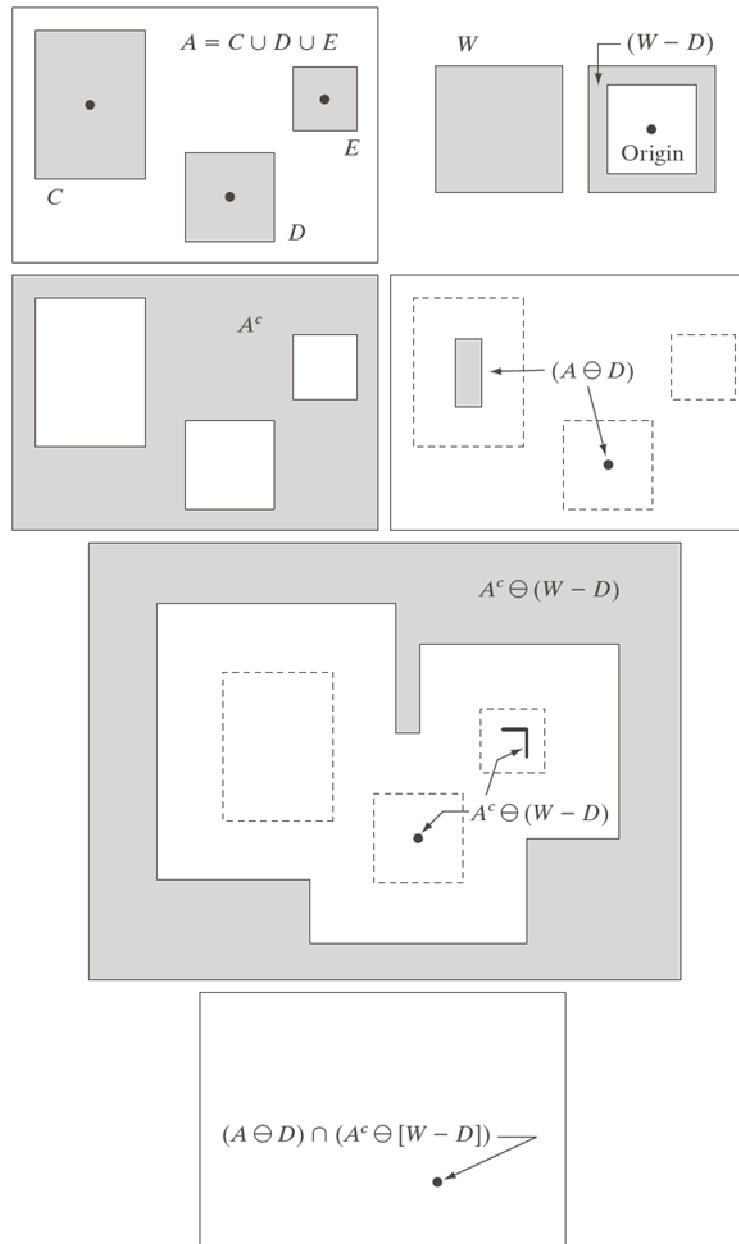
# Extracción de bordes





# Transformación Hit-or-Miss

$$A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$$



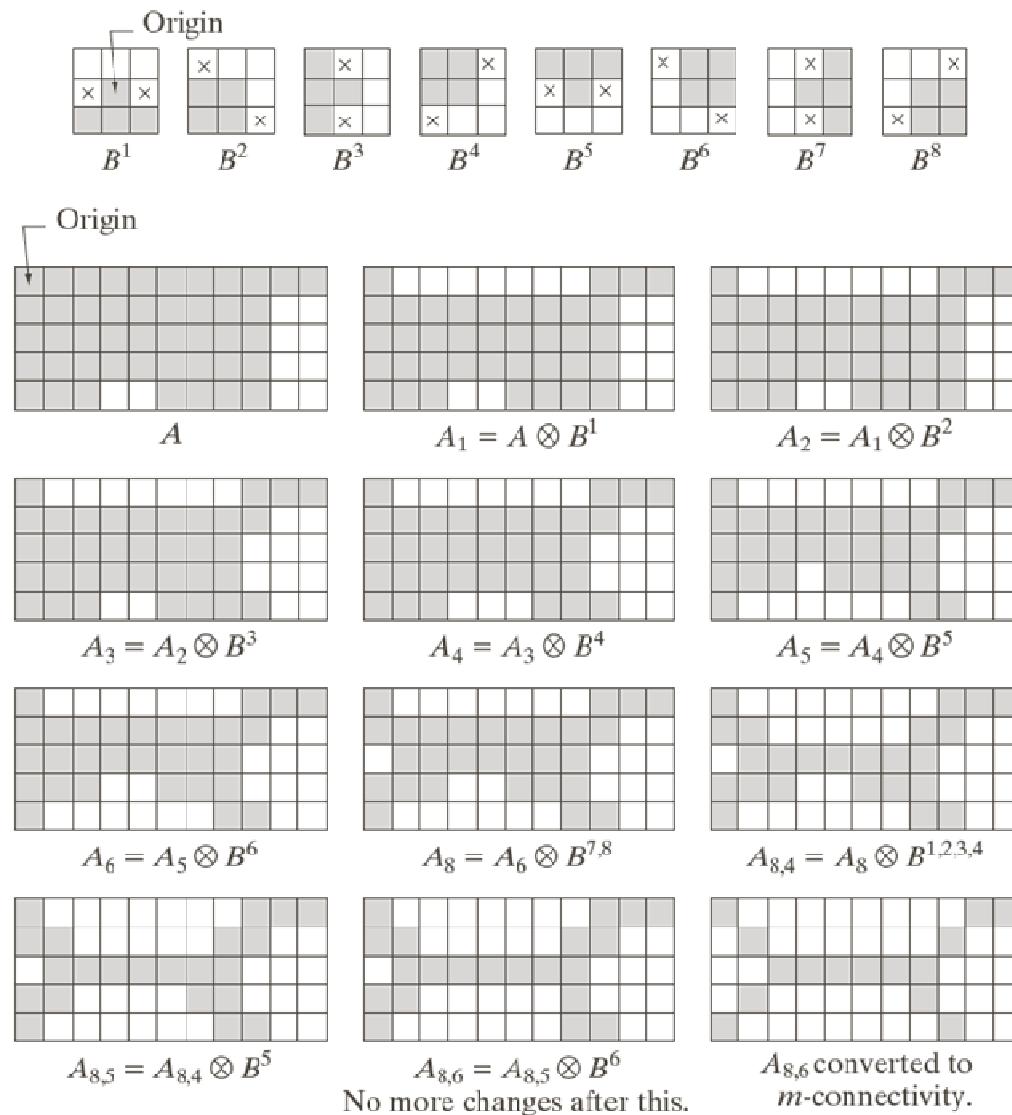
**FIGURE 9.12**  
 (a) Set  $A$ . (b) A window,  $W$ , and the local background of  $D$  with respect to  $W$ ,  $(W - D)$ .  
 (c) Complement of  $A$ . (d) Erosion of  $A$  by  $D$ .  
 (e) Erosion of  $A^c$  by  $(W - D)$ .  
 (f) Intersection of (d) and (e), showing the location of the origin of  $D$ , as desired. The dots indicate the origins of  $C$ ,  $D$ , and  $E$ .



# Adelgazamiento



$$\begin{aligned} A \otimes B &= A - (A \oplus B) \\ &= A \cap (A \oplus B)^c \end{aligned}$$



**FIGURE 9.21** (a) Sequence of rotated structuring elements used for thinning. (b) Set A. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first four elements again. (l) Result after convergence. (m) Conversion to  $m$ -connectivity.





# Adelgazamiento

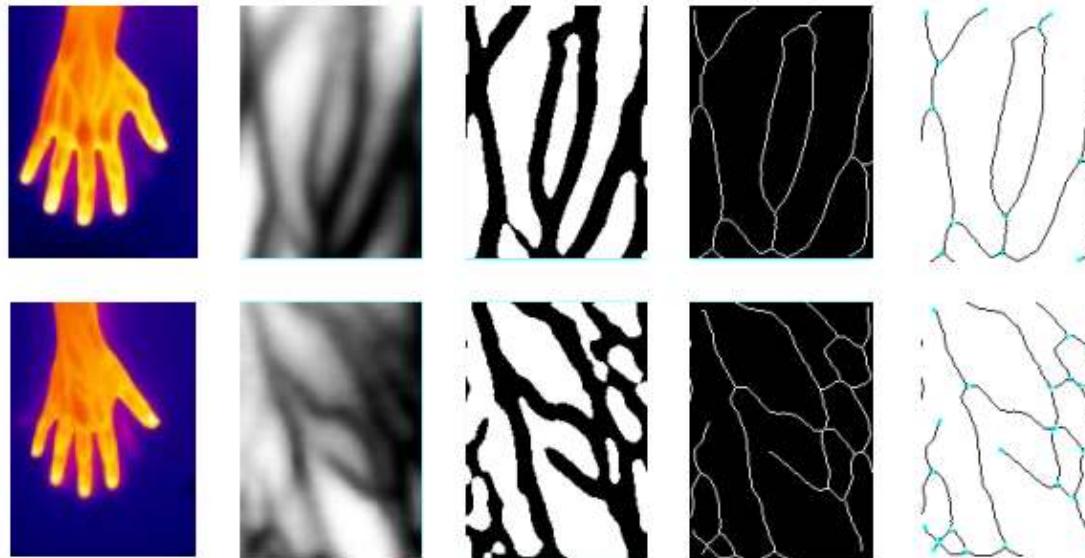
## Hand Vein Infrared Image Segmentation for Biometric Recognition

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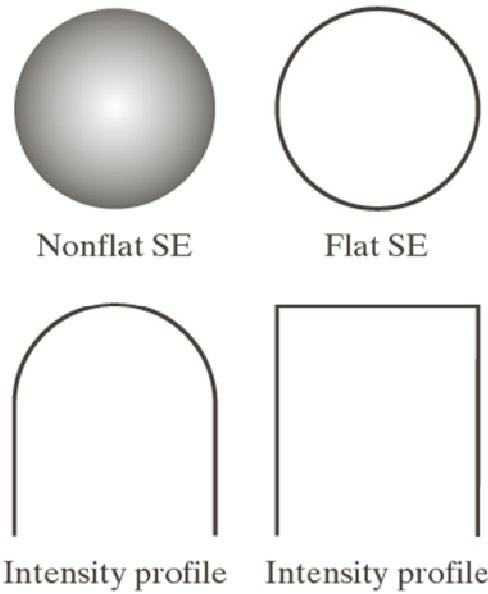


# Morfología (Grises)





# Elemento estructura



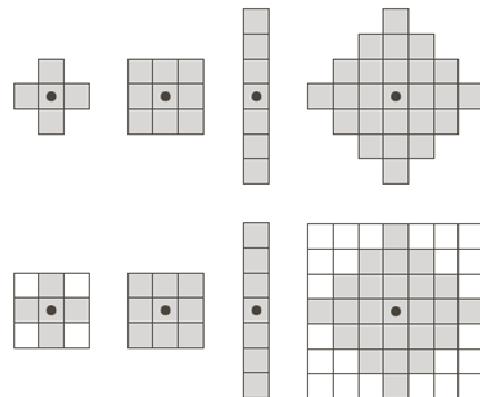
$$\hat{b}(x, y) = b(-x - y).$$



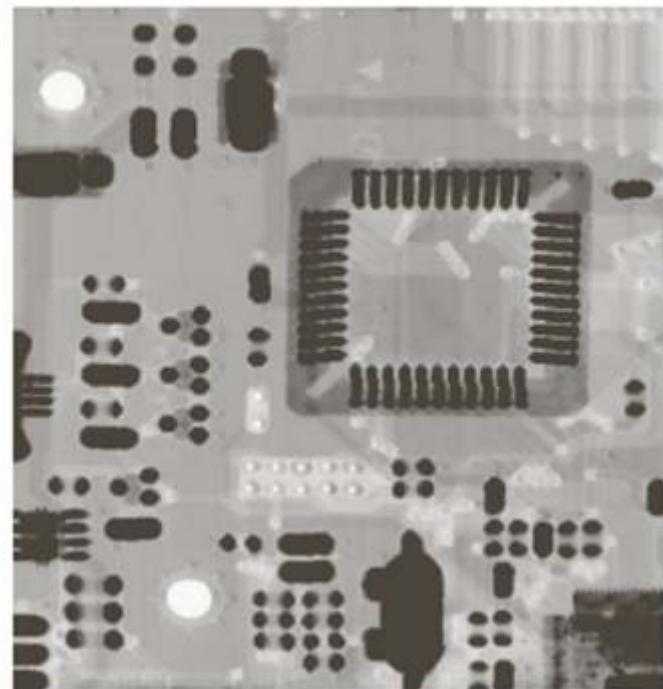
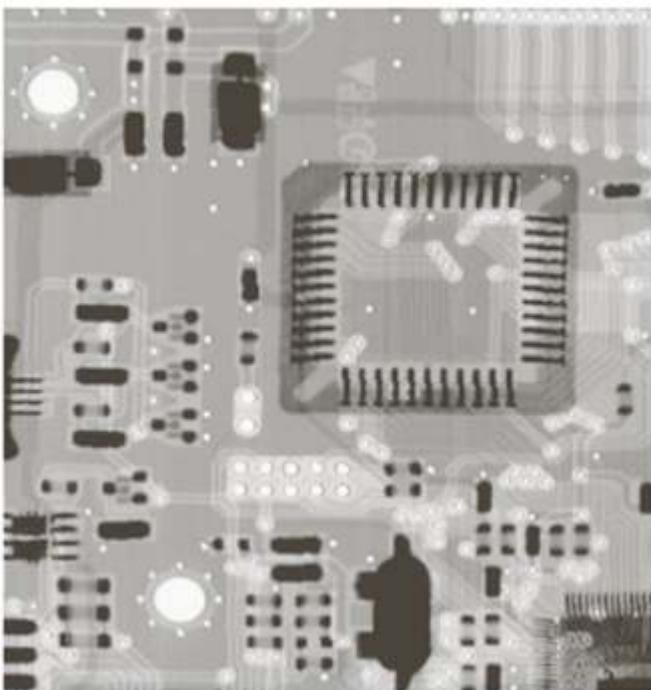


# Elemento estructura

- Plano, no continuo



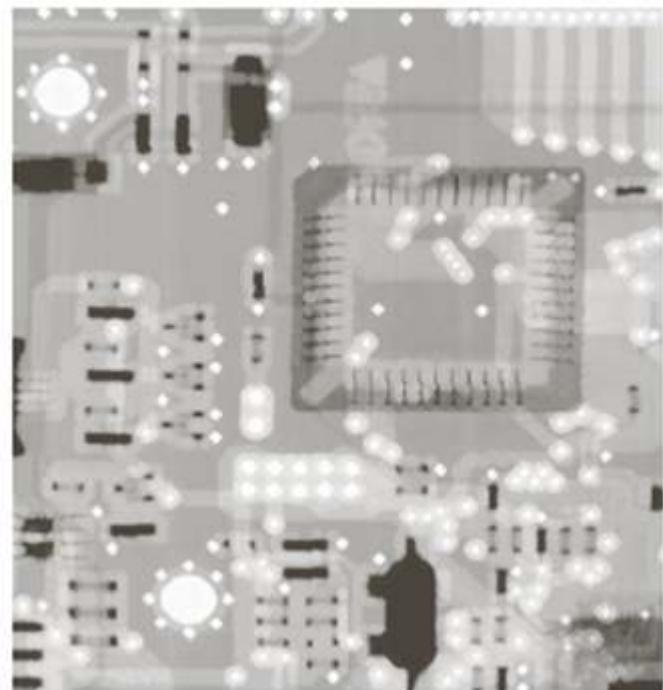
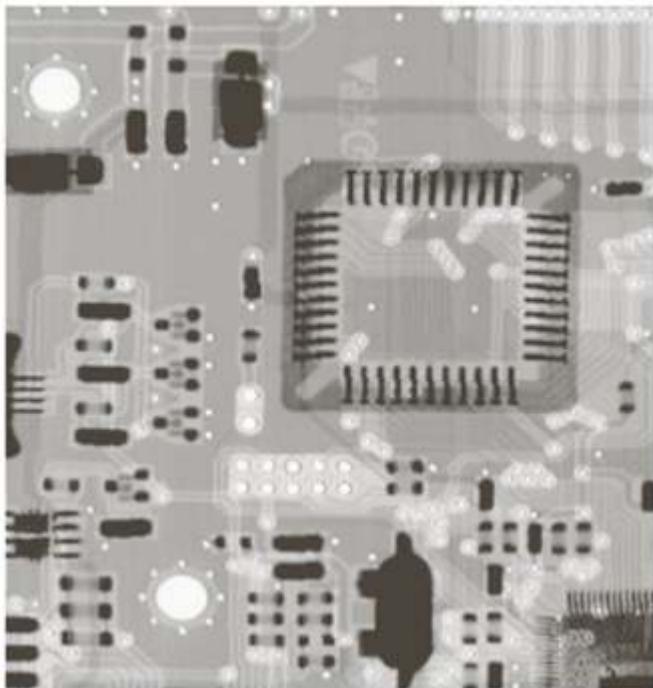
# Erosión



$$[f \ominus b](x, y) = \min_{(s, t) \in b} \{f(x + s, y + t)\}$$



# Dilatación



$$[f \oplus b](x, y) = \max_{(s, t) \in b} \{f(x - s, y - t)\}$$





# Elem. Est. no plano



Nonflat SE

$$[f \ominus b_N](x, y) = \min_{(s, t) \in b_N} \{f(x + s, y + t) - b_N(s, t)\}$$

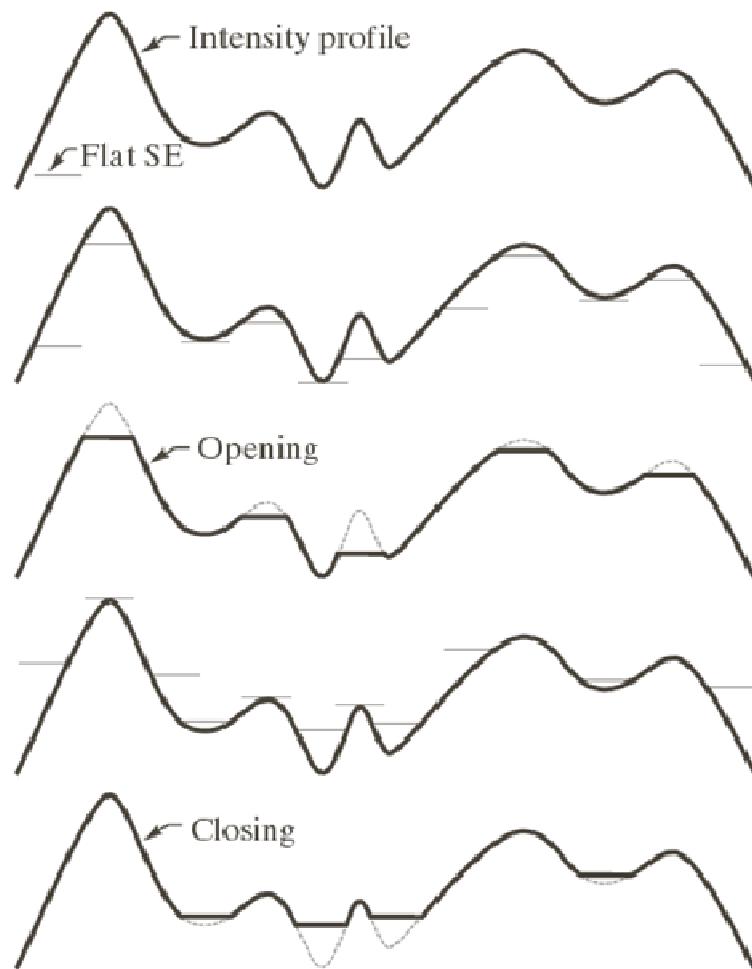
$$[f \oplus b_N](x, y) = \max_{(s, t) \in b_N} \{f(x - s, y - t) + b_N(s, t)\}$$

$$(f \ominus b)^c = (f^c \oplus \hat{b})$$

$$(f \oplus b)^c = (f^c \ominus \hat{b})$$



# Apertura-Cierre



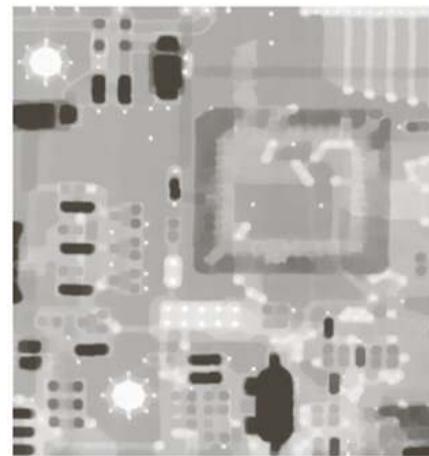
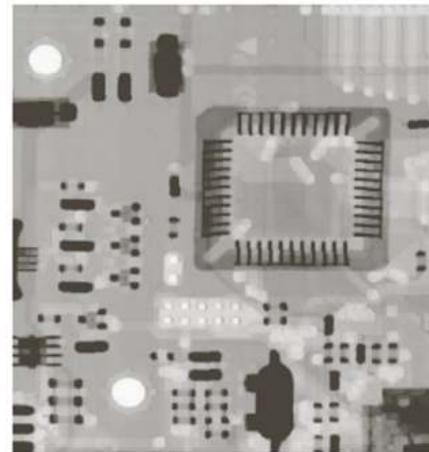
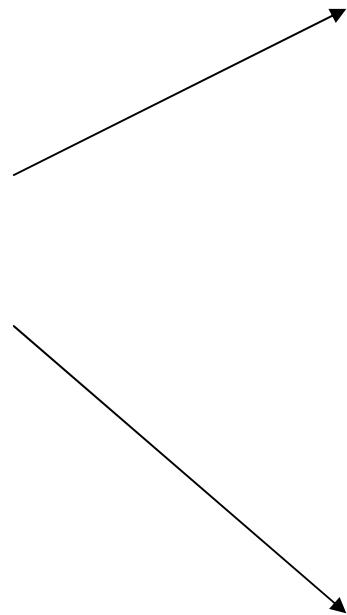
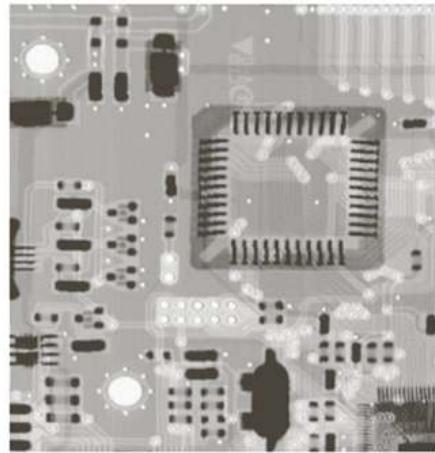
a  
b  
c  
d  
e

**FIGURE 9.36**  
Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal.  
(c) Opening.  
(d) Flat structuring element pushed down along the top of the signal.  
(e) Closing.



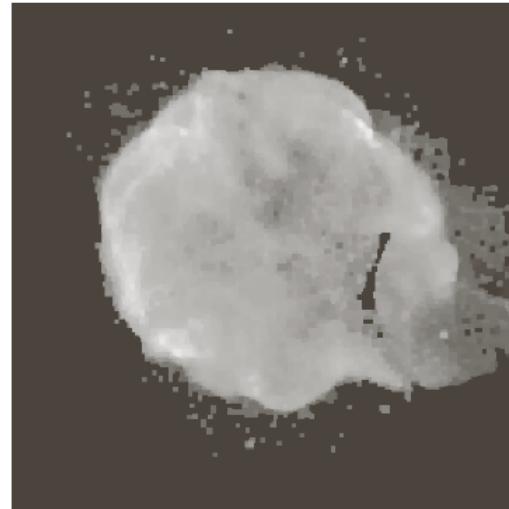
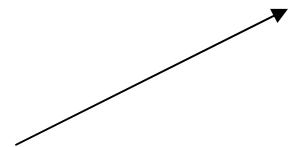
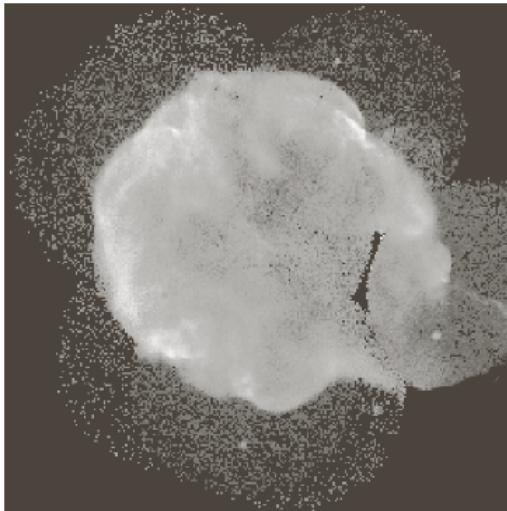


# Apertura-Cierre

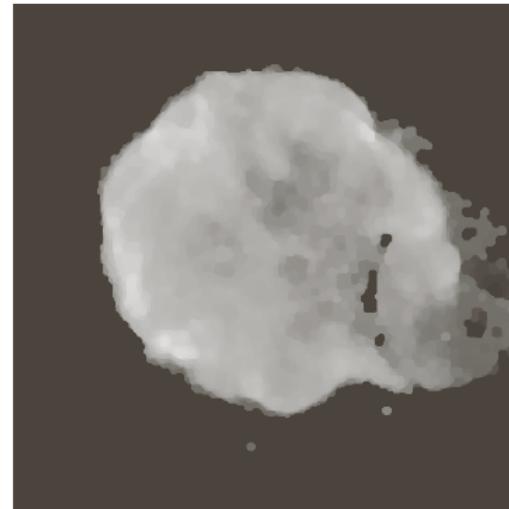
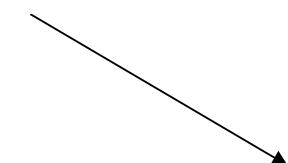




# Suavizado



Radio=3



Radio=5

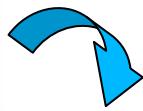




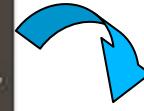
# Gradiente



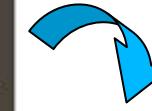
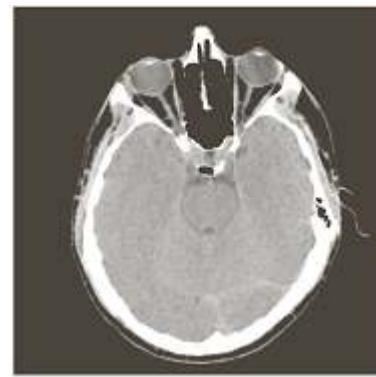
# Gradiente



Dilata



Erosiona



$$g = (f \oplus b) - (f \ominus b)$$

Resta





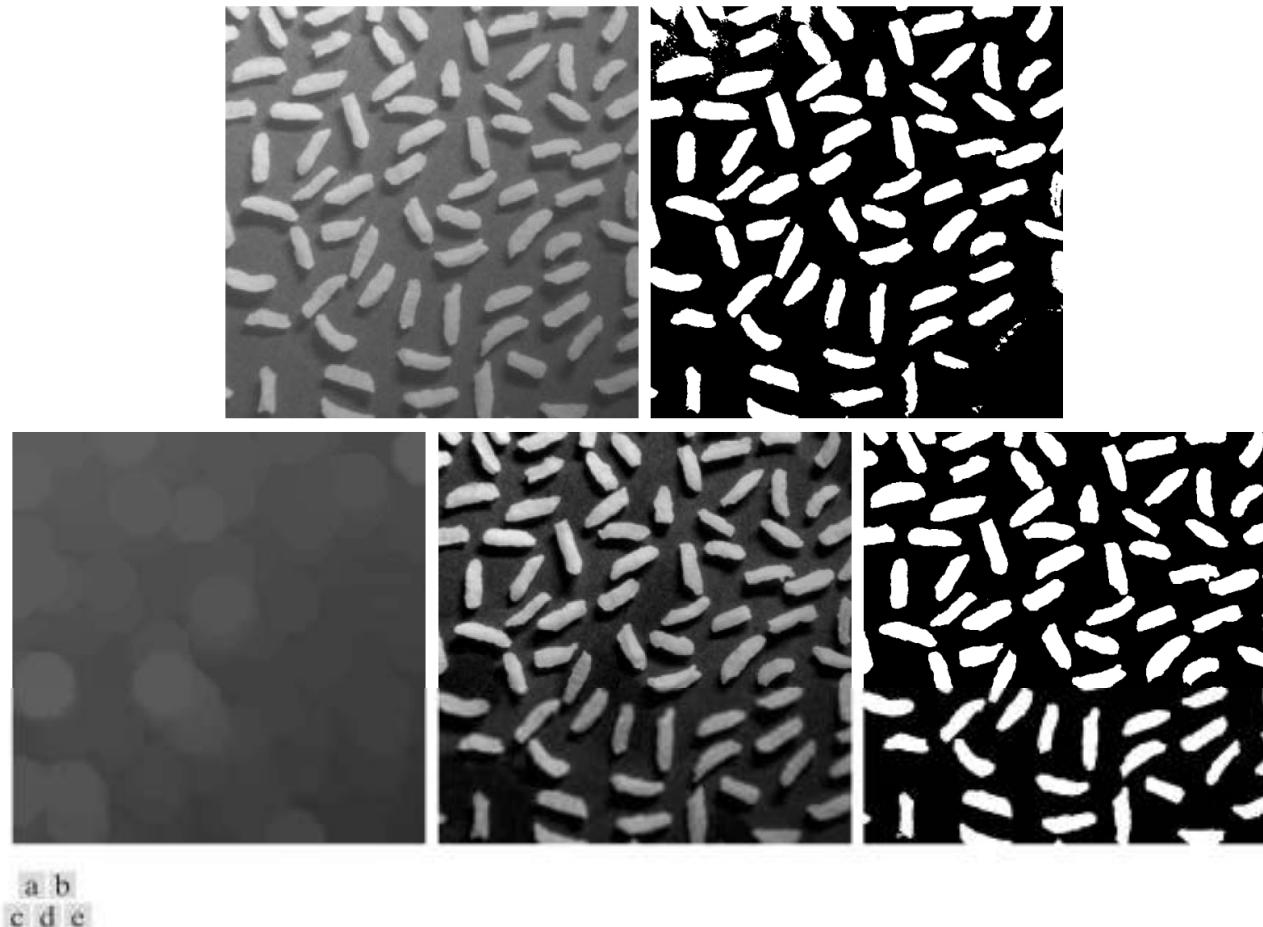
# Top hat- Bottom hat

$$T_{\text{hat}}(f) = f - (f \circ b)$$

$$B_{\text{hat}}(f) = (f \bullet b) - f$$



# Top hat- Bottom hat



**FIGURE 9.40** Using the top-hat transformation for *shading correction*. (a) Original image of size  $600 \times 600$  pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.



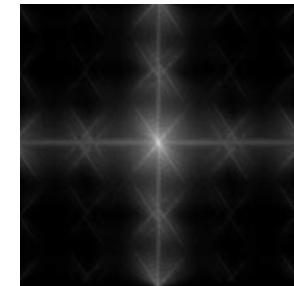
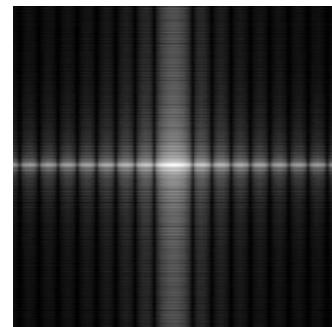
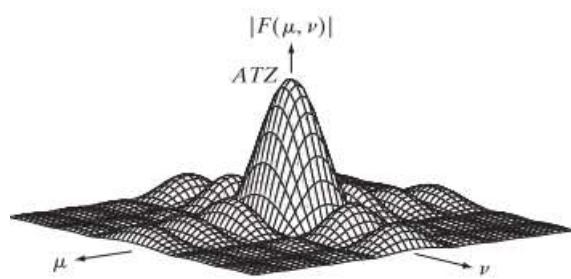
# PDI

# Dominio de la Frecuencia





# Transformada de Fourier

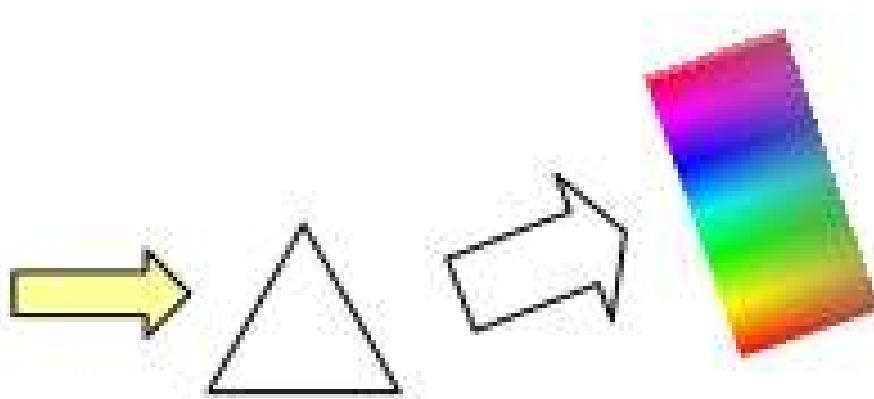
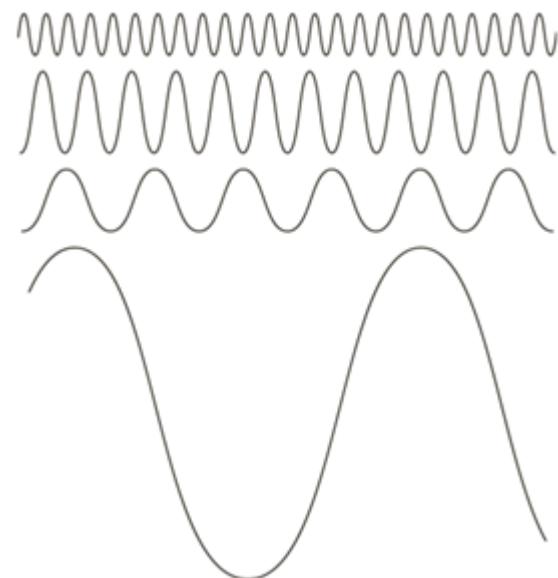




# Transformada de Fourier



- Jean Baptiste Joseph Fourier  
– 1822





# Transformada de Fourier

$$F(U,V) = \langle f(x,y) | K(x,y) \rangle$$

- En una dimensión:

$$F(u) = \langle f(x) | e^{-i2\pi ux} \rangle = \int_{-\infty}^{\infty} f(x)e^{-i2\pi ux} dx$$

- Espectro de Fourier

$$\sqrt{R(u)^2 - I(u)^2} = \sqrt{R(u)^2 + I(u)^2}$$





# T.F. Discreta

- Una dimensión:

$$F(u) = \sum_{x=x_1}^{x_2} f(x) e^{-i2\pi ux} \Delta x = \sum_{x=x_1}^{x_2} f(x) e^{-i2\pi ux}$$

- Dos dimensiones:

$$F(u, v) = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x, y) e^{-2\pi i \left( \frac{ux}{N} + \frac{vy}{M} \right)}$$

$$= \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} f(x, y) \left[ \cos\left(2\pi\left(\frac{ux}{N} + \frac{vy}{M}\right)\right) - i \sin\left(2\pi\left(\frac{ux}{N} + \frac{vy}{M}\right)\right) \right]$$





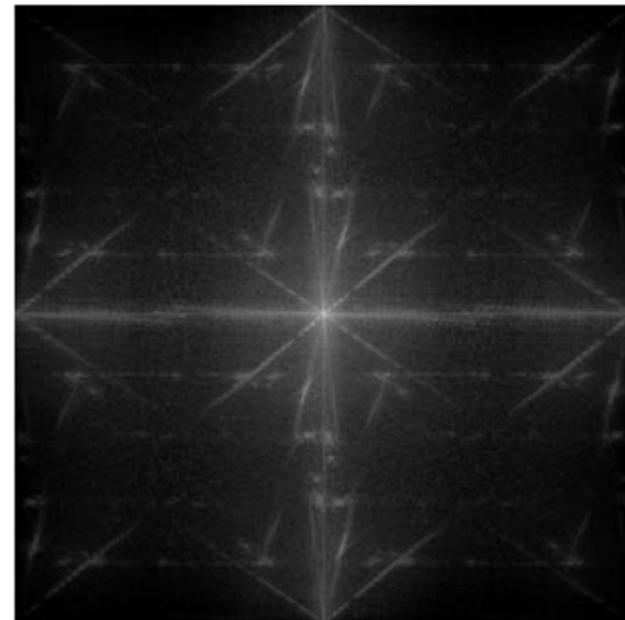
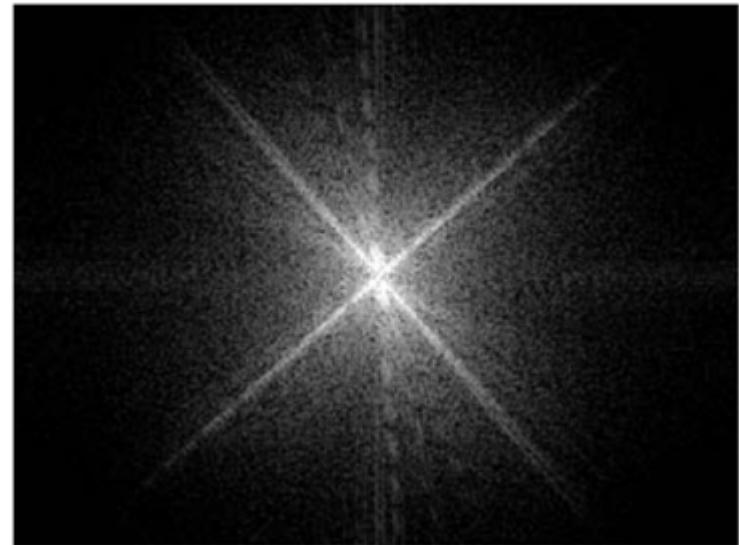
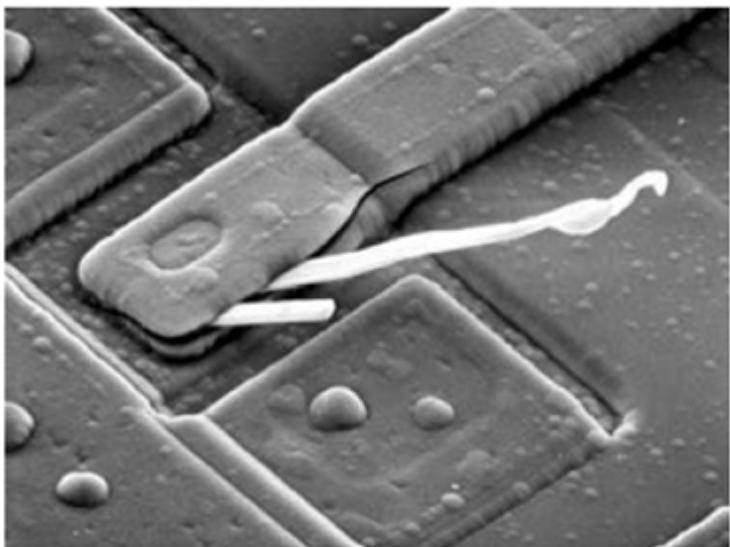
# T.F. Inversa

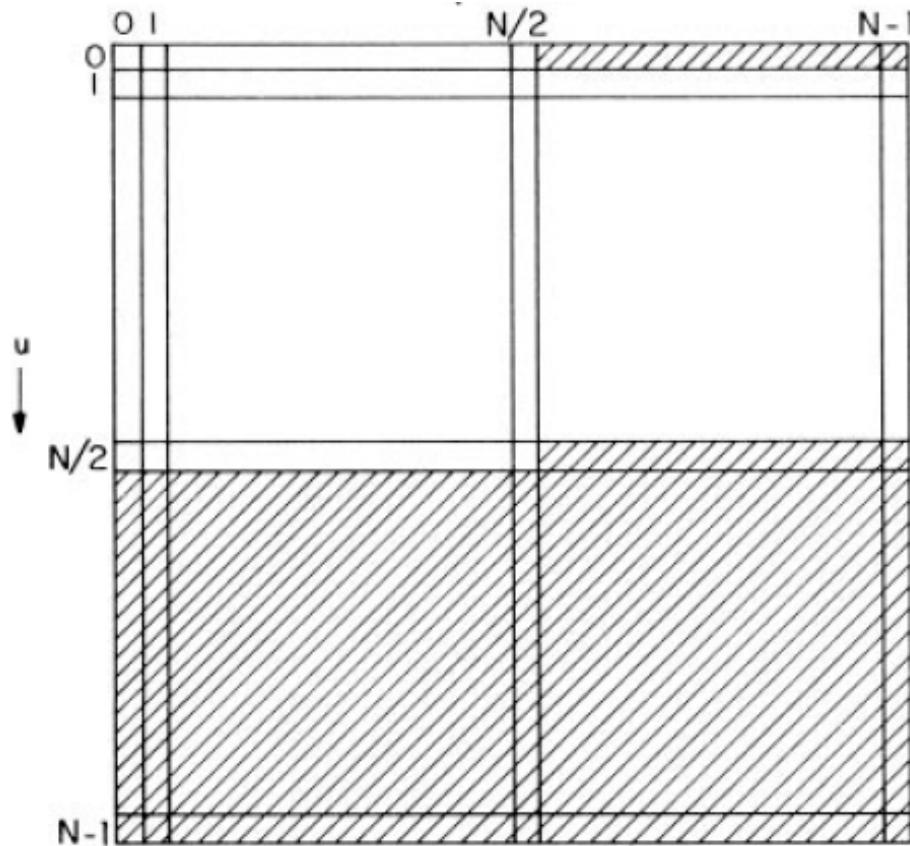
$$F^{-1}(u, v) = \frac{1}{NM} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} F(u, v) e^{-2\pi i (\frac{ux}{N} + \frac{vy}{M})}$$





# Transformada de Fourier



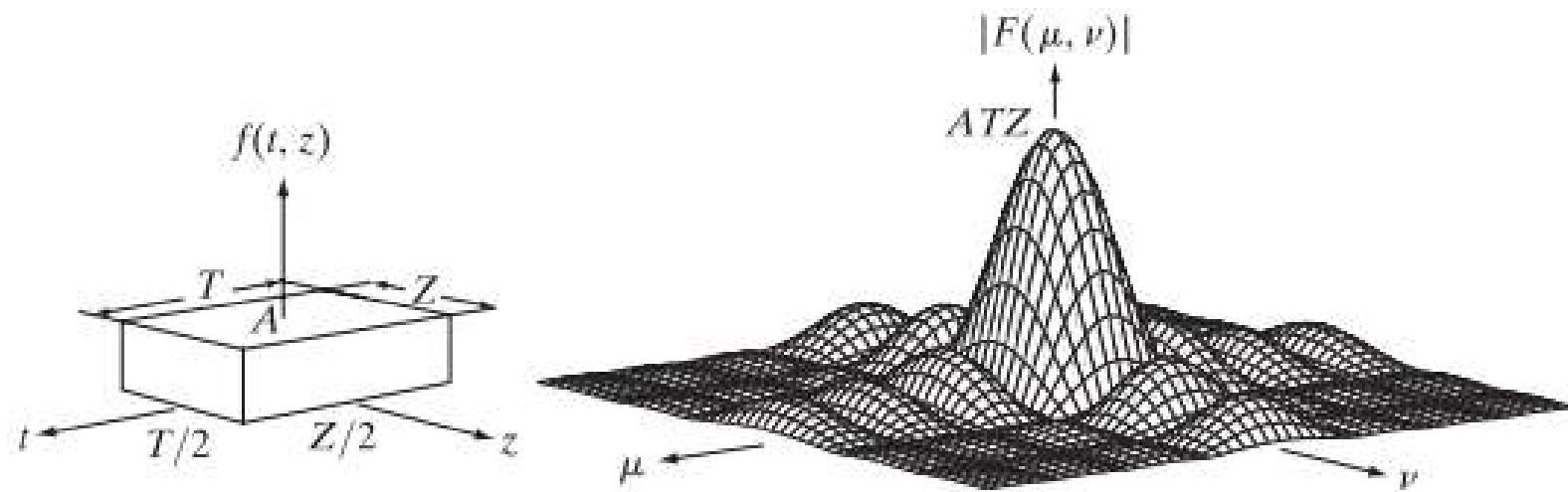


Dominio de las frecuencias de la TF





# Transformada de Fourier





# Implementación

- Calcular U,V a partir de cada x,y
- Aclarar espectro:  $\ln(1+\alpha|F|)$
- Mapear a rango válido:

$$v'_i = \frac{v_i - min_A}{max_A - min_A} (new\_max_A - new\_min_A) + new\_min_A.$$

- $F^{-1}$  ??

