

$$t \ u_1 \dots u_n =! r$$

$$1) \Gamma, A. \Gamma \hat{\vdash}_\eta r : A \wedge \forall r'. \Gamma \hat{\vdash}_\eta r' : A \rightarrow r' = r$$

subject exp.

$$2) \Gamma \hat{\vdash}_\eta t \ u_1 \dots u_n : A$$

$$\Gamma \leq \Phi \quad A \leq \tau$$

$$\frac{\Phi \vdash r : \tau}{(\Phi, \tau) \text{ princ. pair.}}$$

$$\Gamma \hat{\vdash}_\eta t : \varphi_1 \rightarrow \dots \rightarrow \varphi_n \rightarrow A$$

$$3) \Gamma \hat{\vdash}_\eta t' : \varphi_1 \rightarrow \dots \rightarrow \varphi_n \rightarrow A$$

$$t' \ u_1 \dots u_n \rightsquigarrow r' \rightarrow r = r'$$

$$\Gamma \leq \Phi, \varphi_1 \rightarrow \dots \rightarrow \varphi_n \rightarrow A \leq \sigma_1 \rightarrow \dots \rightarrow \sigma_n \rightarrow \tau \rightarrow \Phi \vdash t' : \sigma_1 \rightarrow \dots \rightarrow \sigma_n \rightarrow \tau$$

$$t' = \dots \lambda x. \overset{w}{\dots} \quad u_n = \dots \lambda x. \overset{w}{\dots}$$

$$\Gamma = f(f(a))$$

$$A = \delta$$

$$\Phi = \{ f : d \rightarrow d, a : d \}$$

$$\Gamma = \{ f : (d \rightarrow \beta) \rightarrow (\beta \rightarrow \gamma) \mid a : d \}$$

why?

t' is in nf \leadsto subformula property \leadsto upper bound on size (t')

- what about multiple constraints?
- what about inequalities / solution size?

\hookrightarrow given a solution t we have rhs for inequalities
 \leadsto transform t to t' , preserving those rhs

$$\text{multiple constraints: } \begin{array}{l} \Gamma^1 \hat{\vdash}_\eta t : \varphi_1^1 \rightarrow \dots \rightarrow \varphi_n^1 \rightarrow A^1 \\ \vdots \\ \Gamma^m \hat{\vdash}_\eta t : \varphi_1^m \rightarrow \dots \rightarrow \varphi_n^m \rightarrow A^m \end{array}$$