Definition (position)

1

$t|_p$ = get_subterm $t\ p$ (as in lambdaw)
$t[p \leftarrow t']$ = replace_subtree $t'\ p\ t$
We say that p is a position of a variable z (bound at some $p_b \preccurlyeq p$) when $t|_p = \lambda \bar{y}.\, z\, \bar{u}$.
prefix order

Definition (accessible)

We say that p is accessible wrt. (in)equation $t\, u_1 \ldots u_m = r$ when the normal form of $t[p \leftarrow \underline{nil}]\, u_1 \ldots u_m$ has an occurrence of nil. (nil is a distinguished constant that does not occur in rhs).
We say that p is accessible when it is accessible wrt. some (in)equation in the instance at hand.

Definition (descendant)

We say that p_1 is a descendant under binding of p_0 in t when

1) $p_1 = p_0$ or
2) there is $p_2 : p_0 \preccurlyeq p_2 \preccurlyeq p_1$ such that
   a) p_2 is a descendant under binding of p_0 and
   b) $t|_{p_2} = \lambda \bar{y}.\, z\, \bar{u}$, $(\bar{u} = [u_1 \ldots u_m])$
   c) $p_1 = p_2 \cdot i \cdot p_1'$,
   d) $u_i = \lambda y_1' \ldots y_m'.\, z'\, \bar{u}'$,
   e) $t|_{p_1} = \lambda \bar{y}''.\, y_j'\, \bar{u}''$ for some $j: 1 \leq j \leq m$.

Definition (observed)

We say that a position p of a variable z bound at a position $p_b \preceq p$ is observed by a position $p_o : p_b \preceq p_o \prec p$ when p_o is a position of some variable z' that is a descendant under binding of some position $p_b' \prec p_b$.

The position p is observed in t when it is observed by some position p_o.

Definition (measure simplistic)

We define measure $\mu(t) = \langle \mu_1(t), |t| \rangle$

size of t

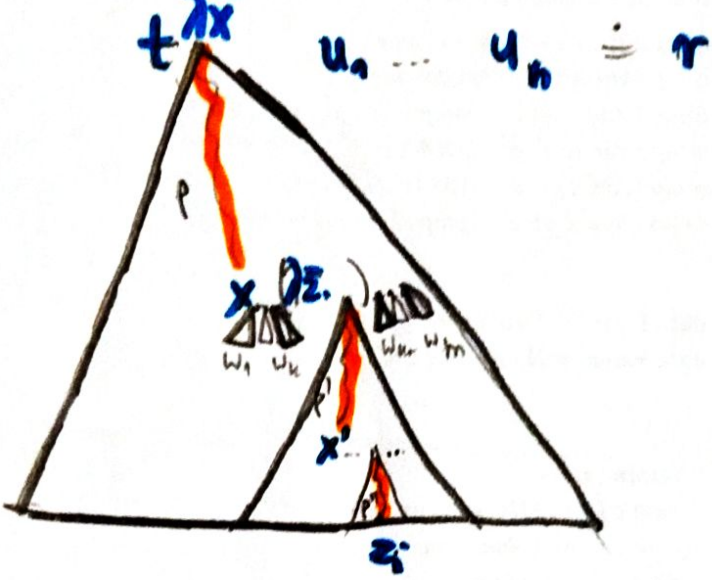
$\mu_1(t)$ = the number of positions of variables in t s.t.

(1) they are observed in t,

(2) they are accessible.

Reflections for 4th order (simplistic)

[1] Typical situation when unfolding is applied:



This occurrence of z_i is obscured by the (visible) position of x'.

[2] Condition for (simplistic) unfolding (we assume that the game tree incorporates plays for all (in)equations):

[2a] For each node in the game tree of the form (at position σ)

$$\text{node } (0, \rho, \Theta)_e \ [\text{child}]$$

(e says that we deal in the position of the game tree relevant for (in)equation e)

(i.e. each node that elaborates the variable x from the picture above)

there is a further node (at some position $\sigma' > \sigma$) of the form

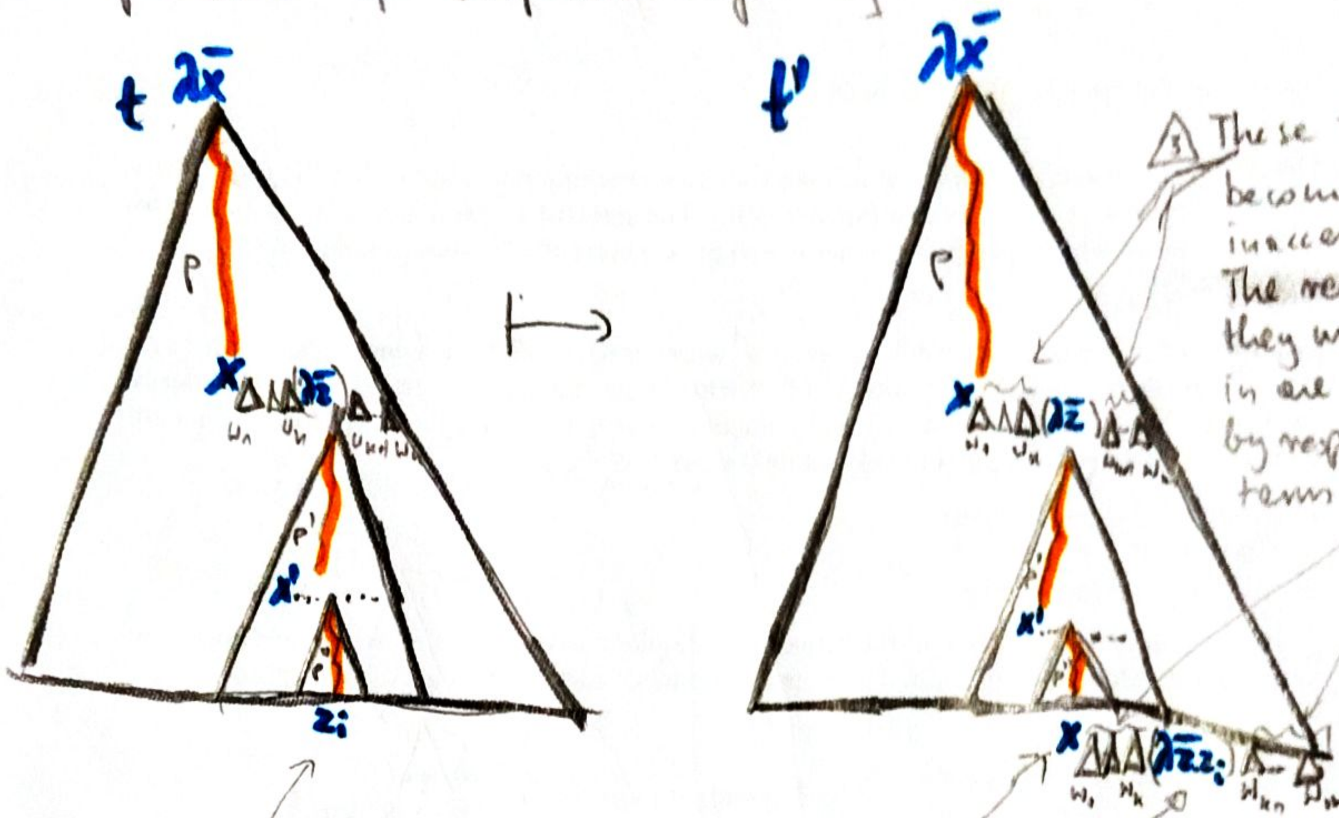
$$\text{node } (0, \rho \cdot k \cdot \rho' \cdot k' \cdot \rho'', \Theta')_e \ \text{childs}'$$

without intermittent reference to positions in $w_1 \ldots w_m$ ($\neq w_k$).

[2b] The node child is node $(0, \rho \cdot k, \Theta'')$ childs''

[2c] The term resulting from σ is the same as one resulting from σ'

[3] Operation of simplistic unfolding



⓪ By [2] this position is accessible in all (in)equations. So are all preceding positions.

① By the virtue of the game this position is accessible in all (in)equations too. This position is not covered now.

② As x is interpreted in game in the same way (at least in nth order) the game proceeds to this node, which is interpreted as in t.

③ These terms become inaccessible. The reductions they were involved in are taken up by respective terms here

Follow comments in the sequence ⓪, ①, ...

[4] This kind of unfolding is properly defined, but is not enough – different (in)equations may lead to different:

- occurrences of zi (that's not a problem),
- occurrences of z_j different than z_i,
- arguments of x ($w_{k'}$ instead of w_k),

Therefore the general requirement in condition [2] that all nodes of a particular shape have a later node that refers to the same occurrence of a variable is too restrictive.

Definition (measure)

We define measure $\mu(t) = \langle \mu_1(gt), |t| \rangle$

gt — gene tree of t; $|t|$ — size of t

$\mu_1(gt)$ = the sum of sizes of all segments $[\sigma, \tau]$ in gt s.t.

- $gt|_\sigma = \text{node}(0, p, \Theta)_e\ [\text{child}]$

  $gt|_{\sigma\cdot\tau} = \text{node}(0, p\cdot\gamma, \Theta')_e\ [\text{child}']$

- $t|_p = \lambda\bar{z}''. x\ [W_1 \ldots W_n\ W_{n+1} \ldots W_k]$ $\qquad W_k = \lambda\bar{z}. W_k'$

  $t|_{p\cdot\gamma} = \lambda\bar{z}''; z_i\ []$

  $\gamma = k\cdot p'\cdot k'\cdot p''$ using the notation from page 3

- Θ' on the domain of Θ equals Θ. $(\Theta'|_{dom(\Theta)} = \Theta)$

- p is observed by some $p\cdot k\cdot p'$ in t

The notion of observed position can be defined in terms of genes only

5 Condition for unfolding. Let t be a solution, gt the gene tree of the e.

There are segments $[\sigma, \tau]$ in gt and $[p, \gamma]$ in t s.t.

$\text{COND}(\sigma, \tau, p, \gamma)$:

2a $gt|_\sigma = \text{node}(0, p, \Theta)_e\ [\text{child}]$

  $gt|_{\sigma\tau} = \text{node}(0, p\cdot\gamma, \Theta')_e\ [\text{child}']$

2b Θ' on the domain of Θ equals Θ.

2c p is observed by some $p\cdot k\cdot p'$ in t

2d For each segment $[\sigma', \tau']$ s.t. $\text{COND}(\sigma', \tau', p, \gamma)$ holds the result of reduction of the gene tree of σ' is the same as the result of $\sigma'\cdot\tau'$ (i.e. τ' does not contribute).

6 Operation of unfolding

Exactly the same as the simplistic unfolding.