CONCEPTUAL QUESTIONS

1. Can the magnitude of the displacement vector be more than the distance traveled? Less than the distance traveled? Explain.
2. If $\vec{C} = \vec{A} + \vec{B}$, can $C = A + B$? Can $C > A + B$? For each, show how or explain why not.
3. If $\vec{C} = \vec{A} + \vec{B}$, can $C = 0$? Can $C < 0$? For each, show how or explain why not.
4. Is it possible to produce a vector quantity by adding, subtracting, multiplying, or dividing two scalar quantities?
5. How would you define the *zero vector* $\vec{0}$?
6. Two vectors have lengths of 4 units each. What is the range of possible lengths obtainable for the vector representing the sum of the two?
7. Can a vector have zero magnitude if one of its components is nonzero? Explain.
8. Two vectors of unequal magnitudes can never add up to a zero vector. Does this hold true for three unequal vectors? Explain with an example.
9. Are the following statements true or false? Explain your answer.
   a. The magnitude of a vector can be different in different coordinate systems.
   b. The direction of a vector can be different in different coordinate systems.
   c. The components of a vector can be different in different coordinate systems.

EXERCISES AND PROBLEMS

Exercises

Section 3.1 Scalars and Vectors

Section 3.2 Using Vectors

1. | Trace the vectors in FIGURE EX3.1 onto your paper. Then find (a) $\vec{A} + \vec{B}$ and (b) $\vec{A} - \vec{B}$.

FIGURE EX3.1

FIGURE EX3.2

2. | Trace the vectors in FIGURE EX3.2 onto your paper. Then find (a) $\vec{A} + \vec{B}$ and (b) $\vec{A} - \vec{B}$.

Section 3.3 Coordinate Systems and Vector Components

3. | a. What are the x- and y-components of vector $\vec{E}$ shown in FIGURE EX3.3 in terms of the angle θ and the magnitude E?
   b. For the same vector, what are the x- and y-components in terms of the angle ϕ and the magnitude E?

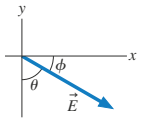

FIGURE EX3.3

4. || A velocity vector 40° below the positive x-axis has a y-component of −10 m/s. What is the value of its x-component?
5. | A position vector in the first quadrant has an x-component of 10 m and a magnitude of 12 m. What is the value of its y-component?
6. | Draw each of the following vectors. Then find its x- and y-components.
   a. $\vec{a}$ = (3.5 m/s^2, negative x-direction)
   b. $\vec{v}$ = (440 m/s, 30° below the positive x-axis)
   c. $\vec{r}$ = (12 m, 40° above the positive x-axis)
7. || Draw each of the following vectors. Then find its x- and y-components.
   a. $\vec{v}$ = (7.5 m/s, 30° clockwise from the positive y-axis)
   b. $\vec{a}$ = (1.5 m/s^2, 30° above the negative x-axis)
   c. $\vec{F}$ = (50.0 N, 36.9° counterclockwise from the positive y-axis)
8. | Let $\vec{C}$ = (3.15 m, 15° above the negative x-axis) and $\vec{D}$ = (25.6 m, 30° to the right of the negative y-axis). Find the x- and y-components of each vector.
9. | A runner is training for an upcoming marathon by sprinting around a circular track 80 m in diameter at a constant speed. Let a coordinate system have its origin at the center of the circle, with the x-axis pointing east and the y-axis, north. The runner starts at (x, y) = (40 m, 0 m) and runs 1.5 times around the track in a clockwise direction. What is his displacement vector? Specify magnitude and direction in your answer.

Section 3.4 Unit Vectors and Vector Algebra

10. || Draw each of the following vectors, label an angle that specifies the vector's direction, then find its magnitude and direction.
    a. $\vec{B} = -4.0\hat{\imath} + 4.0\hat{\jmath}$
    b. $\vec{r} = (-2.0\hat{\imath} - 1.0\hat{\jmath})$ cm
    c. $\vec{v} = (-10\hat{\imath} - 100\hat{\jmath})$ m/s
    d. $\vec{a} = (20\hat{\imath} + 10\hat{\jmath})$ m/s^2
11. || Draw each of the following vectors, label an angle that specifies the vector's direction, and then find the vector's magnitude and direction.
    a. $\vec{A} = 3.0\hat{\imath} + 7.0\hat{\jmath}$
    b. $\vec{a} = (-2.0\hat{\imath} + 4.5\hat{\jmath})$ m/s^2
    c. $\vec{v} = (14\hat{\imath} - 11\hat{\jmath})$ m/s
    d. $\vec{r} = (-2.2\hat{\imath} - 3.3\hat{\jmath})$ m
12. || Let $\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$, $\vec{B} = 2\hat{\imath} - 4\hat{\jmath}$, and $\vec{C} = \vec{A} + \vec{B}$.
    a. Write vector $\vec{C}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.
    c. What are the magnitude and direction of vector $\vec{C}$?
13. | Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{C} = \vec{A} + \vec{B}$.
    a. Write vector $\vec{C}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$.
    c. What are the magnitude and direction of vector $\vec{C}$?
14. | Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{D} = \vec{A} - \vec{B}$.
    a. Write vector $\vec{D}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{D}$.
    c. What are the magnitude and direction of vector $\vec{D}$?
15. | Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{E} = 2\vec{A} + 3\vec{B}$.
    a. Write vector $\vec{E}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{E}$.
    c. What are the magnitude and direction of vector $\vec{E}$?

16. | Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{F} = \vec{A} - 4\vec{B}$.
    a. Write vector $\vec{F}$ in component form.
    b. Draw a coordinate system and on it show vectors $\vec{A}$, $\vec{B}$, and $\vec{F}$.
    c. What are the magnitude and direction of vector $\vec{F}$?
17. | Let $\vec{E} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{F} = 2\hat{\imath} - 2\hat{\jmath}$. Find the magnitude of
    a. $\vec{E}$ and $\vec{F}$ b. $\vec{E} + \vec{F}$ c. $-\vec{E} - 2\vec{F}$
18. | Let $\vec{B} = (5.0\text{ m}, 30°\text{ counterclockwise from vertical})$. Find the x- and y-components of $\vec{B}$ in each of the two coordinate systems shown in FIGURE EX3.18.

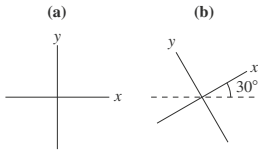

FIGURE EX3.18

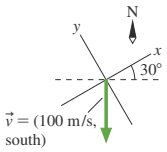

FIGURE EX3.19

19. | What are the x- and y-components of the velocity vector shown in FIGURE EX3.19?
20. || For the three vectors shown in FIGURE EX3.20, $\vec{A} + \vec{B} + \vec{C} = 1\hat{\jmath}$. What is vector $\vec{B}$?
    a. Write $\vec{B}$ in component form.
    b. Write $\vec{B}$ as a magnitude and a direction.

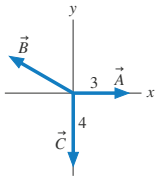

FIGURE EX3.20

21. | The *magnetic field* inside an instrument is $\vec{B} = (2.0\hat{\imath} - 1.0\hat{\jmath})$ T where $\vec{B}$ represents the magnetic field vector and T stands for *tesla,* the unit of the magnetic field. What are the magnitude and direction of the magnetic field?

Problems

22. || Let $\vec{A} = (3.0\text{ m}, 20°\text{ south of east})$, $\vec{B} = (2.0\text{ m, north})$, and $\vec{C} = (5.0\text{ m}, 70°\text{ south of west})$.
    a. Draw and label $\vec{A}$, $\vec{B}$, and $\vec{C}$ with their tails at the origin. Use a coordinate system with the x-axis to the east.
    b. Write $\vec{A}$, $\vec{B}$, and $\vec{C}$ in component form, using unit vectors.
    c. Find the magnitude and the direction of $\vec{D} = \vec{A} + \vec{B} + \vec{C}$.
23. || CALC The position of a particle as a function of time is given by $\vec{r} = (5.0\hat{\imath} + 4.0\hat{\jmath})t^2$ m, where t is in seconds.
    a. What is the particle's distance from the origin at $t = 0$, 2, and 5 s?
    b. Find an expression for the particle's velocity $\vec{v}$ as a function of time.
    c. What is the particle's speed at $t = 0$, 2, and 5 s?
24. || a. What is the angle ϕ between vectors $\vec{E}$ and $\vec{F}$ in FIGURE P3.24?
    b. Use geometry and trigonometry to determine the magnitude and direction of $\vec{G} = \vec{E} + \vec{F}$.
    c. Use components to determine the magnitude and direction of $\vec{G} = \vec{E} + \vec{F}$.

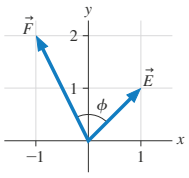

FIGURE P3.24

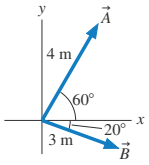

FIGURE P3.25

25. || FIGURE P3.25 shows vectors $\vec{A}$ and $\vec{B}$. Find vector $\vec{C}$ such that $\vec{A} + \vec{B} + \vec{C} = \vec{0}$. Write your answer in component form.
26. ||| FIGURE P3.26 shows vectors $\vec{A}$ and $\vec{B}$. Find $\vec{D} = 2\vec{A} + \vec{B}$. Write your answer in component form.

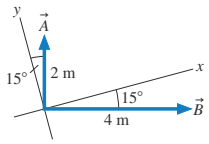

FIGURE P3.26

27. || Find a vector that points in the same direction as the vector $(\hat{\imath} + \hat{\jmath})$ and whose magnitude is 1.
28. || While vacationing in the mountains you do some hiking. In the morning, your displacement is $\vec{S}_{\text{morning}} = (2000\text{ m, east}) + (3000\text{ m, north}) + (200\text{ m, vertical})$. After lunch, your displacement is $\vec{S}_{\text{afternoon}} = (1500\text{ m, west}) + (2000\text{ m, north}) - (300\text{ m, vertical})$.
    a. At the end of the hike, how much higher or lower are you compared to your starting point?
    b. What is the magnitude of your net displacement for the day?
29. || The minute hand on a watch is 1.5 cm in length. What is the displacement vector of the tip of the hand
    a. from 8:00 a.m. to 8:20 a.m.?
    b. from 8:00 a.m. to 9:00 a.m.?
30. || You go to an amusement park with your friend Betty, who wants to ride the 30-m-diameter Ferris wheel. She starts the ride at the lowest point of a wheel that, as you face it, rotates counterclockwise. What is her displacement vector when the wheel has rotated by an angle of 60°? Give your answer as a magnitude and direction.
31. || Ruth sets out to visit her friend Ward, who lives 50 mi north and 100 mi east of her. She starts by driving east, but after 30 mi she comes to a detour that takes her 15 mi south before going east again. She then drives east for 8 mi and runs out of gas, so Ward flies there in his small plane to get her. What is Ward's displacement vector? Give your answer (a) in component form, using a coordinate system in which the y-axis points north, and (b) as a magnitude and direction.
32. | A cannon tilted upward at 30° fires a cannonball with a speed of 100 m/s. What is the component of the cannonball's velocity parallel to the ground?
33. | You are fixing the roof of your house when the head of your hammer breaks loose and slides down. The roof makes an angle of 30° with the horizontal, and the head is moving at 3.5 m/s when it reaches the edge. What are the horizontal and vertical components of the head's velocity just as it leaves the roof?
34. | Jack and Jill ran up the hill at 4 m/s. The horizontal component of Jill's velocity vector was 3.5 m/s.
    a. What was the angle of the hill?
    b. What was the vertical component of Jill's velocity?

35. | A pine cone falls straight down from a pine tree growing on a 20° slope. The pine cone hits the ground with a speed of 10 m/s. What is the component of the pine cone's impact velocity (a) parallel to the ground and (b) perpendicular to the ground?
36. | Kami is walking through the airport with her two-wheeled suitcase. The suitcase handle is tilted 40° from vertical, and Kami pulls parallel to the handle with a force of 120 N. (Force is measured in *newtons,* abbreviated N.) What are the horizontal and vertical components of her applied force?
37. || Dee is on a swing in the playground. The chains are 2.5 m long, and the tension in each chain is 450 N when Dee is 55 cm above the lowest point of her swing. Tension is a vector directed along the chain, measured in *newtons,* abbreviated N. What are the horizontal and vertical components of the tension at this point in the swing?
38. || Your neighbor Paul has rented a truck with a loading ramp. The ramp is tilted upward at 25°, and Paul is pulling a large crate up the ramp with a rope that angles 10° above the ramp. If Paul pulls with a force of 550 N, what are the horizontal and vertical components of his force? (Force is measured in *newtons,* abbreviated N.)
39. || Tom is climbing a 3.0-m-long ladder that leans against a vertical wall, contacting the wall 2.5 m above the ground. His weight of 680 N is a vector pointing vertically downward. (Weight is measured in *newtons,* abbreviated N.) What are the components of Tom's weight parallel and perpendicular to the ladder?
40. || The treasure map in **FIGURE P3.40** gives the following directions to the buried treasure: "Start at the old oak tree, walk due north for 500 paces, then due east for 100 paces. Dig." But when you arrive, you find an angry dragon just north of the tree. To avoid the dragon, you set off along the yellow brick road at an angle 60° east of north. After walking 300 paces you see an opening through the woods. Which direction should you go, and how far, to reach the treasure?

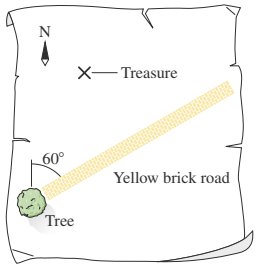

FIGURE P3.40

41. ||| The bacterium *E. coli* is a single-cell organism that lives in the BIO gut of healthy animals, including humans. When grown in a uniform medium in the laboratory, these bacteria swim along zig-zag paths at a constant speed of 20 μm/s. **FIGURE P3.41** shows the trajectory of an *E. coli* as it moves from point A to point E. What are the magnitude and direction of the bacterium's average velocity for the entire trip?

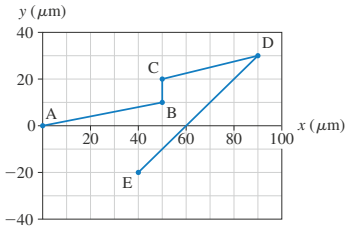

FIGURE P3.41

42. || A flock of ducks is trying to migrate south for the winter, but they keep being blown off course by a wind blowing from the west at 6.0 m/s. A wise elder duck finally realizes that the solution is to fly at an angle to the wind. If the ducks can fly at 8.0 m/s relative to the air, what direction should they head in order to move directly south?
43. || **FIGURE P3.43** shows three ropes tied together in a knot. One of your friends pulls on a rope with 3.0 units of force and another pulls on a second rope with 5.0 units of force. How hard and in what direction must you pull on the third rope to keep the knot from moving?

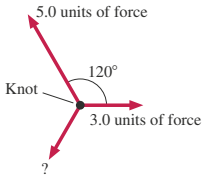

FIGURE P3.43

44. || Four forces are exerted on the object shown in **FIGURE P3.44**. (Forces are measured in *newtons*, abbreviated N.) The *net force* on the object is $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 4.0\hat{\imath}$ N. What are (a) $\vec{F}_3$ and (b) $\vec{F}_4$? Give your answers in component form.

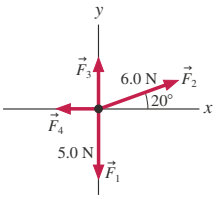

FIGURE P3.44

45. || **FIGURE P3.45** shows four electric charges located at the corners of a rectangle. Like charges, you will recall, repel each other while opposite charges attract. Charge B exerts a repulsive force (directly *away from* B) on charge A of 3.0 N. Charge C exerts an attractive force (directly *toward* C) on charge A of 6.0 N. Finally, charge D exerts an attractive force of 2.0 N on charge A. Assuming that forces are vectors, what are the magnitude and direction of the net force $\vec{F}_{net}$ exerted on charge A?

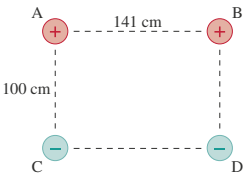

FIGURE P3.45