Laugardagsæfing 7: 2019–2020

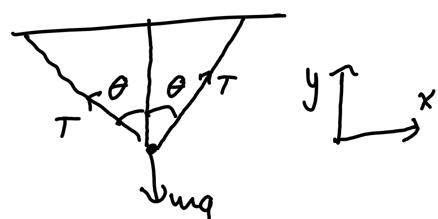
K1: $p = mv, \quad [p] = [m]\cdot[v] = kg\cdot\frac{m}{s}.$ D

K2: $$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{\left(\frac{100}{3,6}\right)}{2,5} = 11,1 \ m/s^2$$ D

K3: D

K4: E $\quad \mu = \tan\theta_{max}$ óháð þyngdarhröðuninni.

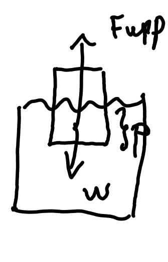
K5: Jafnvægi þ.a.

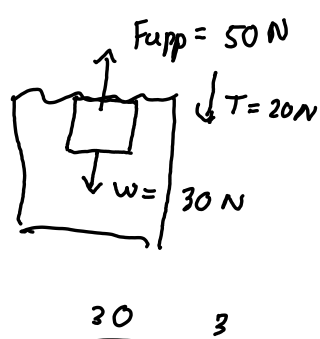


$$\underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{m\vec{a}} = \begin{pmatrix} T\sin\theta - T\sin\theta \\ 2T\cos\theta - mg \end{pmatrix}$$

þ.a. $\quad T = \dfrac{mg}{2\cos\theta}$ E

K6: $F_{upp} = w$





þ.e. $\rho_{vatn}\cdot V_{undir}\cdot g = w$

en $V_{undir} = p V_{kassi}$ svo

$p\cdot\rho_{vatn}\cdot V_{kassi}\, g = w$ þ.a. $p = \dfrac{w}{\rho_{vatn}\cdot V_{kassi}\, g} = \dfrac{30}{50} = \dfrac{3}{5}$

svo $1 - p = \underline{\underline{\dfrac{2}{5}}}$ D

K7: $v = 22$ km/klst ; $v_0 = 17{,}5$ km/klst

20 mín = 1/3 klst

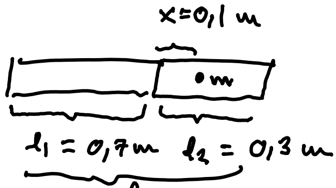
$v_0 = \frac{s}{t}$ og $v = \frac{s}{t - 1/3}$ (jöfnuhneppi með 2 óþekktum).
s og t

þ.e. $t = \frac{s}{v_0}$ og $s = vt - \frac{v}{3}$

en þá er $s = \frac{v}{v_0}s - \frac{v}{3}$ þ.a. $s = \frac{-\frac{v}{3}}{1 - \frac{v}{v_0}} = 28{,}5$ km.

Á

K8:



Heildarvægið er núll þ.a. við fáum:

$$\left(M\frac{l_1}{l}\right)g\frac{l_1}{2} = mgx + \left(M\frac{l_2}{l}\right)g\frac{l_2}{2}$$

þ.a. $M\left(\frac{l_1^2 - l_2^2}{2l}\right) = mx$ þ.a. $M = \frac{2lmx}{l_1^2 - l_2^2}$

sem gefur að $M = 25{,}0$ g. C

K9: Látum hæð borðsins vera h þá er:

$$\begin{pmatrix} D \\ h \end{pmatrix} = \begin{pmatrix} vt \\ \frac{1}{2}gt^2 \end{pmatrix}$$ þ.a. $t = \sqrt{\frac{2h}{g}}$ og því

$D = \sqrt{\frac{2h}{g}}\,v$ þ.e. $v = \underbrace{\sqrt{\frac{g}{2h}}}_{\text{fasti}}\,D$ svo

v sem fall af D gefur beina línu! Á

K10: $\frac{GM_A m}{r_A^2} = mg_A$ p.a. $g_A = \frac{GM_A}{r_A^2}$

og pá $\frac{g_A}{g_B} = \frac{\frac{GM_A}{r_A^2}}{\frac{GM_B}{r_B^2}} = \frac{M_A}{M_B}\cdot\frac{r_B^2}{r_A^2}$

en $M_A = \rho\frac{4}{3}\cdot\pi r_A^3$ so

$r_A = 2r_B$

$$\frac{g_A}{g_B} = \frac{\rho\frac{4\pi}{3}r_A^3}{\rho\frac{4\pi}{3}r_B^3}\cdot\frac{r_B^2}{r_A^2} = \frac{r_A}{r_B} = \frac{2r_B}{r_B} = 2.$$ D

K11: $\frac{Gm_e^2}{r^2} = \frac{ke^2}{r^2}$ p.a. $m_e = \sqrt{\frac{k}{G}}\cdot e$

so $m_e = 1{,}86\cdot 10^{-9}$ C

K12: B

K13: $h_1 = \frac{1}{2}at^2$; $2gh_2 = v_1^2$ p.a. $h_2 = \frac{v_1^2}{2g}$ D

en $v_1 = at$ p.a. $h = h_1 + h_2 = \frac{1}{2}at^2\left(1+\frac{a}{g}\right) = 1100$ m

K14: A $E = K + U$

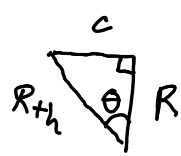
$-\frac{GM_J m}{2r} = \frac{GM_J m}{2r} - \frac{GM_J m}{r}$ so $\Delta E = -\Delta K$ so K eykst um 4 J.

fjarlægd frá jørðini

K15: Jörðin snýst um horn $\omega_{jörð} \cdot \tau = \theta$

þar sem $\omega_{jörð} = \frac{2\pi}{T} = \frac{2\pi}{24 \cdot 60^2} = 7{,}27 \cdot 10^{-5}$

er hornhraði jarðarinnar í snúning um sjálfa sig.



Pýþagóras gefur $c = \sqrt{(R+h)^2 - R^2}$

Kósínusreglan gefur svo að:

$$\cos\theta = \frac{(R+h)^2 + R^2 - \left((R+h)^2 - R^2\right)}{2R(R+h)} = \frac{R}{R+h}$$

svo

$$\omega_{jörð}\,\tau = \theta = \arccos\left(\frac{R}{R+h}\right) \quad C$$

þ.a. $\tau = \dfrac{\arccos\left(\frac{R}{R+h}\right)}{\omega_{jörð}} = 222$ sek $= 3$ min; 42 sek

Dæmi 1: Árekstur

(a)

Skriðþungavarðveizla gefur,

$$mv_0 = Mu + mv_1$$

Höfum síðan:

$$\begin{pmatrix} s \\ h \end{pmatrix} = \begin{pmatrix} ut \\ \frac{1}{2}gt^2 \end{pmatrix} \quad \text{þ.a.} \quad t = \sqrt{\frac{2h}{g}} \quad \text{og þá}$$

$$u = \frac{s}{t} = \sqrt{\frac{g}{2h}} \cdot s = 19{,}8 \text{ m/s}$$

en þá er $v_1 = \dfrac{mv_0 - Mu}{m} = v_0 - \dfrac{M}{m} \cdot u = 104 \text{ m/s}.$

Höfum síðan

$$\begin{pmatrix} D \\ h \end{pmatrix} = \begin{pmatrix} v_1 t \\ \frac{1}{2}gt^2 \end{pmatrix} \quad \text{þ.a.} \quad D = v_1 t = v_1 \cdot \sqrt{\frac{2h}{g}} = \underline{\underline{105 \text{ m}}}$$

(b) $\Delta K = \frac{1}{2}mv_0^2 - \frac{1}{2}mv_1^2 - \frac{1}{2}Mu^2 = 1157 \text{ J}$

og þá $\eta = \dfrac{\Delta K}{K_0} = \dfrac{1157}{1250} = 0{,}93$

þ.e. 93 % af orkunni tapaðist í varma.

Dæmi 2:

(a) $2gh_0 = v_0^2$

og $2gh_1 = v_1^2$ þ.a.

$$C_R = \frac{v_1}{v_0} = \frac{\sqrt{2gh_1}}{\sqrt{2gh_0}} = \sqrt{\frac{h_1}{h_0}}.$$

(b) Ef C_R er fast þá gildir að

$$C_R = \sqrt{\frac{h_1}{h_0}} = \sqrt{\frac{h_2}{h_1}} = \dots = \sqrt{\frac{h_n}{h_{n-1}}}$$

$h_{n-1} = C_R^2 h_{n-2}$

en þá er $h_n = C_R^2 h_{n-1} = \dots = C_R^{2n} h_0$

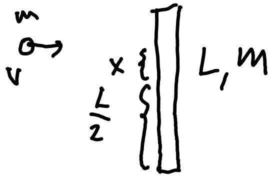
(c) $h_0 = \frac{1}{2} g t_0^2$ þ.a. $t_0 = \sqrt{\frac{2h_0}{g}}$.

(d) þá er $t_n = 2 \cdot \sqrt{\frac{2h_n}{g}} = 2 \cdot \sqrt{\frac{2 \cdot C_R^{2n} h_0}{g}} = 2C_R^n t_0$.

(e)
$$\begin{aligned}
\tau &= t_0 + t_1 + \dots \\
&= t_0\left(1 + 2C_R + 2C_R^2 + \dots\right) \\
&= t_0\left(-1 + 2\left(1 + C_R + C_R^2 + \dots\right)\right) \\
&= t_0\left(-1 + \frac{2}{1 - C_R}\right) = t_0\left(\frac{1 + C_R}{1 - C_R}\right) = \sqrt{\frac{2h_0}{g}}\left(\frac{1 + C_R}{1 - C_R}\right).
\end{aligned}$$

Dæmi 3:

(a)



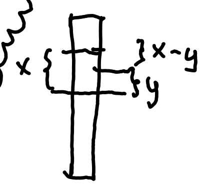
$$mv = (M+m)u$$

þ.a. $u = \dfrac{mv}{M+m}$.

(b) $y_1 = \dfrac{L}{2}$ og $y_2 = \dfrac{M\frac{L}{2} + m\left(\frac{L}{2}+x\right)}{M+m} = \dfrac{L}{2} + \dfrac{mx}{M+m}$.

$$y := y_2 - y_1 = \frac{mx}{M+m}.$$

$$x - y = \left(1 - \frac{m}{M+m}\right)x = \left(\frac{M}{M+m}\right)x$$



(c) $L_1 = mv(x-y)$

(d) $$I_y = \underbrace{\frac{1}{12}ML^2 + My^2}_{\text{Steiner}} + \underbrace{m(x-y)^2}_{\text{punktmassi}}$$

$$= \frac{1}{12}ML^2 + \frac{Mm^2}{(M+m)^2}x^2 + \frac{mM^2}{(M+m)^2}x^2$$

$$= \frac{1}{12}ML^2 + \left(\frac{Mm}{M+m}\right)x^2.$$

(e) $L_1 = I_y\,\omega$ þ.a. $\omega = \dfrac{L_1}{I_y}$

(f) Þá þarf $u = \omega \cdot \left(\frac{L}{2} + y\right)$. (Af hverju?)

Fáum því:

$$\frac{mv}{M+m} = \frac{L_1}{I_y}\left(\frac{L}{2} + \frac{mx}{M+m}\right) = \frac{mv(x-y)}{I_y}\left(\frac{L}{2} + \frac{mx}{M+m}\right)$$

þ.a. $$\frac{I_y}{M+m} = \frac{Mx}{M+m}\left(\frac{L}{2} + \frac{mx}{M+m}\right)$$

þ.a. $$\frac{1}{12}\frac{M}{M+m}L^2 + \frac{Mm}{(M+m)^2}x^2 = \frac{M}{M+m}\cdot\frac{xL}{2} + \frac{mM}{(M+m)^2}x^2$$

þ.a. $$\frac{1}{12}\left(\frac{M}{M+m}\right)L^2 = \frac{1}{2}\left(\frac{M}{M+m}\right)xL$$ þ.a. $$\underline{\underline{x = \frac{1}{6}L}}$$