Lausnir

K1: $[K] = [\frac{1}{2}mv^2] = kg\frac{m^2}{s^2}$ (C)

K2: Hraðinn efst er núll þ.a.

$$2a\Delta s = v^2 - v_0^2 \Rightarrow v_0 = \sqrt{2gh}$$

og þá er $v = v_0 - gt \Rightarrow t = \frac{v_0}{g}$

svo heildartíminn er $T = 2t = \frac{2v_0}{g} = \sqrt{\frac{8h}{g}}$

þ.e. $T = \sqrt{\frac{8 \cdot 40}{9{,}82}} = \underline{\underline{5{,}71\ s}}$ (C)

K3: $mgh = \mu mgd \Rightarrow d = \frac{h}{\mu} = \frac{5}{0{,}2} = 25\ m.$

(E).

K4: $m_{Fe}v_{Fe} = (m_{Fe} + m_{Fi})v_e$

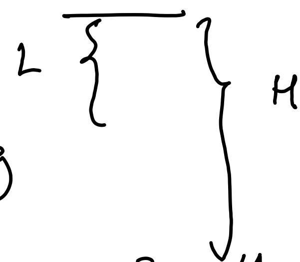
$$\Delta K = \frac{1}{2}m_{Fe}v_{Fe}^2 - \frac{1}{2}(m_{Fe} + m_{Fi})v_e^2 = \frac{1}{2}m_{Fe}v_{Fe}^2 - \frac{1}{2}\frac{m_{Fe}^2}{m_{Fe} + m_{Fi}}v_{Fe}^2$$

$$= \frac{1}{2}m_{Fe}v_{Fe}^2\left(1 - \frac{m_{Fe}}{m_{Fe} + m_{Fi}}\right) = \frac{1}{2}m_{Fe}v_{Fe}^2\left(\frac{m_{Fi}}{m_{Fe} + m_{Fi}}\right) = 266\ kJ.$$

(A)

K5: Mesta hröðunin er þegar

hann snertir vatnsborðið:



$$m a_{max} = kx - mg = k(H-L) - mg$$

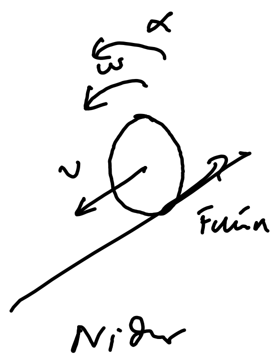
en $mgH = \frac{1}{2} k(H-L)^2 \Rightarrow k(H-L) = \frac{2mgH}{H-L}$

svo:

$$a_{max} = \left(\frac{2H}{H-L} - 1\right) g = \left(\frac{H+L}{H-L}\right) g = 15{,}1 \ m/s^2.$$

(B)

K6: Einföld mynd:



svo svarið er (D). Alltaf upp.

K7: Bilað dæmi: Svarið átti að vera (C)

K8: $s = s_0 + v_0 t + \frac{1}{2} a_0 t^2 + \frac{1}{6} j t^3$

og $s_0 = v_0 = a_0 = 0$ svo $s = \frac{1}{6} \cdot 2 \cdot 4^3 = \frac{2^6}{3} = 21{,}3\,m.$

K9: (C)

K10:
$$\frac{L_B}{L_A} = \frac{m_B v_B r_B}{m_A v_A r_A} = \frac{v_B r_B}{v_A r_A}$$

síðan er $\frac{v^2}{r} = \frac{GM}{r^2} \Rightarrow v = \sqrt{\frac{GM}{r}}$

svo
$$\frac{L_B}{L_A} = \frac{\sqrt{\frac{GM}{r_B}}\, r_B}{\sqrt{\frac{GM}{r_A}}\, r_A} = \sqrt{\frac{r_B}{r_A}} = \sqrt{\frac{2 r_A}{r_A}} = \sqrt{2}.$$

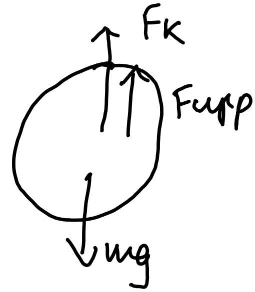
(B)

K11: $a = \frac{r_{min} + r_{max}}{2}$ og $\frac{a^3}{T^2} = \frac{a_{jörð}^3}{T_{jörð}^2}$

$$\Rightarrow a = \left(\frac{T}{T_{jörð}}\right)^{2/3} \cdot 1\,AU = (75)^{2/3}\,AU = 17{,}8\,AU$$

svo $r_{max} = 2 \cdot 17{,}8 - r_{min} = 35\,AU.$

K12:



þurfum $F_{upp} \geq mg - F_k = 589$ N

er $F_{upp} = \rho \frac{4\pi}{3} R^3 g$ so

$$R_{min} = \left(\frac{3(mg - F_k)}{4\pi \rho g}\right)^{1/3} = 2.27 \text{ m}$$

K13: Stálkúlan (A).

K14: $x'(t) = -x_0 w \sin\left(wt + \frac{\pi}{6}\right)$

sum er mest þegar $\sin\left(wt + \frac{\pi}{6}\right) = 1$ so

$V_{max} = x_0 w = 6 \cdot 2 = 12$ m/s.

K15: þurfum h = massamiðjan. Tá er

$$h = \frac{m \cdot 0 + 2m \cdot \frac{L}{2}}{3m} = \frac{1}{3} L. \qquad (C)$$

Dæmi 1:

(a) $E_0 = \frac{1}{2} k_1 d^2$

Hraði fyrir árekstur: $\frac{1}{2} m_1 v_0^2 = \frac{1}{2} k_1 d^2$

þ.a. $v_0 = \sqrt{\frac{k_1}{m_1}}\, d$ og skriðþungavarðveisla gefur að:

$$m_1 v_0 = (m_1+m_2)u \quad \text{og þá} \quad u = \frac{m_1 v_0}{m_1+m_2}$$

en $\frac{1}{2} k_2 x^2 = \frac{1}{2}(m_1+m_2)u^2$ gefur að

$$x = \sqrt{\frac{(m_1+m_2)}{k_2}}\, u = \sqrt{\frac{m_1+m_2}{k_2}} \cdot \frac{m_1 v_0}{m_1+m_2}$$

$$= \sqrt{\frac{m_1+m_2}{k_2}} \cdot \frac{m_1}{m_1+m_2} \cdot \sqrt{\frac{k_1}{m_1}}\, d$$

$$= \left(\sqrt{\frac{k_1}{k_2}} \cdot \sqrt{\frac{m_1}{m_1+m_2}}\right) d$$

(b) $\Delta K = \frac{1}{2} m_1 v_0^2 - \frac{1}{2}(m_1+m_2) u^2$

$$= \frac{1}{2} m_1 v_0^2 - \frac{1}{2}(m_1+m_2) \frac{m_1^2 v_0^2}{(m_1+m_2)^2}$$

$$= \frac{1}{2} m_1 v_0^2 \left(1 - \frac{m_1}{m_1+m_2}\right) = \frac{1}{2} m_1 v_0^2 \left(\frac{m_2}{m_1+m_2}\right)$$

$$= \frac{1}{2} m_1 \cdot \frac{k_1}{m_1} d^2 \cdot \left(\frac{m_2}{m_1+m_2}\right)$$

$$= \frac{1}{2} k_1 d^2 \left(\frac{m_2}{m_1+m_2}\right),$$

Bohr-líkanið:

(a) Samkvæmt hringhreyfingu er

$$m_e \frac{v_n^2}{r_n} = \frac{ke^2}{r_n^2} \Rightarrow v_n = \sqrt{\frac{ke^2}{m_e r_n}} \quad \text{svo}$$

$$L_n = m_e v_n r_n = m_e \sqrt{\frac{ke^2}{m_e r_n}}\, r_n = n\hbar$$

$$\Rightarrow \sqrt{r_n} = \frac{n\hbar}{\sqrt{m_e k e^2}} \quad \text{svo} \quad r_n = \left(\frac{\hbar^2}{m_e k e^2}\right) n^2$$

þá er $a = \dfrac{\hbar^2}{m_e k e^2} = 5{,}25 \cdot 10^{-11}$ m

(b) $$E_n = \frac{1}{2} m_e v_n^2 - \frac{ke^2}{r_n} = \frac{1}{2} m_e \cdot \left(\frac{ke^2}{m_e r_n}\right) - \frac{ke^2}{r_n}$$

$$= -\frac{ke^2}{2r_n} = -\frac{\left(\frac{ke^2}{2a}\right)}{n^2} = -\frac{E_1}{n^2}$$

svo $$E_1 = \frac{ke^2}{2a} = \frac{m_e k^2 e^4}{2\hbar^2} = 2.2 \cdot 10^{-18}\ \text{J} = 13{,}7\ \text{eV}.$$

Orkan sem losnar er:

$$\Delta E = E_n - E_m = -E_1\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = E_1\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

Höfum að ljóseindin hefur alm $\Delta E = \frac{hc}{\lambda}$

svo $\frac{1}{\lambda} = \frac{\Delta E}{hc} = \frac{E_1}{hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$

svo $R = \frac{E_1}{hc} = 1.1 \cdot 10^7\ m^{-1}$

(d) $L_E = m_E V_E r_E$ og $V_E = \sqrt{\frac{GM_S}{R_E}}$

svo $m_E\sqrt{GM_S r_E} = n\hbar$ svo

$$n = \frac{m_E\sqrt{GM_S r_E}}{\hbar} = \frac{5{,}97 \cdot 10^{24} \cdot \sqrt{6{,}67 \cdot 10^{-11} \cdot 2 \cdot 10^{30} \cdot 1{,}5 \cdot 10^{11}}}{1{,}05 \cdot 10^{-34}}$$

$$= 2{,}54 \cdot 10^{74}$$

$E_n = -\frac{GM_S M_J}{r_n}$ svo $\Delta E = \frac{GM_S M_E}{r_{n-1}} - \frac{GM_S M_E}{r_n}$

en $M_E\sqrt{GM_S} \cdot \sqrt{r_E} = n\hbar$ svo $r_n = \frac{n^2\hbar^2}{M_E^2 GM_S}$

En $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$

p.e. $\lambda = \frac{hc}{G M_S m_E \left(\frac{1}{r_{n-1}} - \frac{1}{r_n}\right)} = \frac{hc}{G M_S m_E}\left(\frac{1}{r_{n-1}} - \frac{1}{r_n}\right)^{-1}$

en $r_n = \frac{n^2 \hbar^2}{m_E^2 G M_S}$ zo

$$\lambda = \frac{hc}{G M_S m_E}\left(\frac{m_E^2 G M_S}{(n-1)^2 \hbar^2} - \frac{m_E^2 G M_S}{n^2 \hbar^2}\right)^{-1}$$

$$= \frac{2\pi \hbar^3 c}{G^2 M_S^2 m_E^3}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)^{-1}$$

n groot!

$$= \frac{2\pi \hbar^3 c}{G^2 M_S^2 m_E^3}\left(\frac{n^2 - n^2 + 2n - 1}{n^2 (n-1)^2}\right)^{-1} \approx \frac{2\pi \hbar^3 n^3 c}{G^2 M_S^2 m_E^3}$$

zo $\lambda = \frac{2\pi \hbar^3 n^3 c}{G^2 M_S^2 m_E^3} = \frac{2\pi \cdot (1{,}05)^3 \cdot (2{,}54)^3 \cdot 3}{(6{,}67)^2 \cdot (2)^2 \cdot (5{,}97)^3} \cdot 10^{18}$

$= 9{,}44 \cdot 10^{15} = \underline{\underline{1 \text{ lichtjaar!}}}$