K1: $1\,ly = 3{,}0 \cdot 10^{8} \cdot 365 \cdot 24 \cdot 60^{2} = 9{,}46 \cdot 10^{5}.$ (B)

K2: $65\ \text{miles/time} = 65 \cdot 1{,}609\ \text{km/time} = 105\ \text{km/time}$ (B)

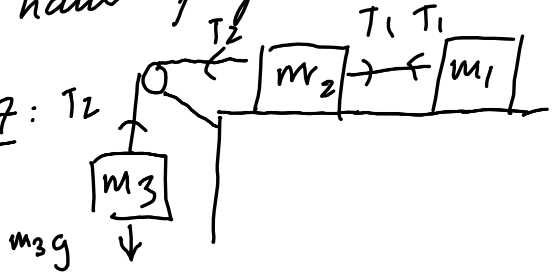
K3: $v = at \Rightarrow t = \frac{v}{a} = \frac{108}{3{,}6 \cdot 0{,}25} = 120\ s.$ (E)

K4: $2a\Delta s = v^2 - v_0^2 \Rightarrow \Delta s = \frac{v^2 - v_0^2}{2a} = 704\ m$ (A)

K5: $ma = F \Rightarrow a = \frac{F}{m} = \frac{17\,N}{68\,kg} = 0{,}25\ m/s^2.$ (C)

K6: Leshirner klesse samen etter 1 time. på lufen hans flegi'd 60 km. (D)

K7:



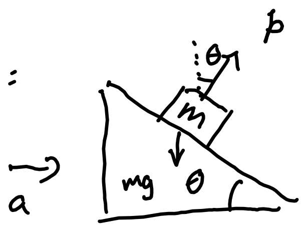
$m_1 a = T_1$ en

$(m_1 + m_2 + m_3)\, a = m_3 g$ p.a. $a = \frac{m_3 g}{m_1 + m_2 + m_3}$

og på $T_1 = \frac{m_1 m_3 g}{m_1 + m_2 + m_3} = 3{,}9\,N$

(A)

K8:



$$m\begin{pmatrix}a\\0\end{pmatrix} = \begin{pmatrix} p\sin\theta \\ p\cos\theta - mg\end{pmatrix}$$

p.a. $p = \frac{mg}{\cos\theta}$ p.a. E

K9: $a_p = \frac{r_{min}+r_{max}}{2}$ og $\frac{a_p^3}{T_p^2} = \frac{GM_S}{4\pi^2} = \frac{a_J^3}{T_J^2}$ E

$$\Rightarrow T_p = \left(\frac{a_p}{a_J}\right)^{3/2} \cdot \underbrace{T_J}_{1 \text{ ár}} = \left(\frac{49,3+29,7}{2}\right)^{3/2} = 248 \text{ ár}.$$

K10: ↓ B $F_k = \frac{kQq}{r^2}$.

K11: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{k}{m}}\cdot x = 0,47$ m/s C

K12: $\omega = \alpha t \Rightarrow t = \frac{\omega}{\alpha} = 17,5$ s en þá er

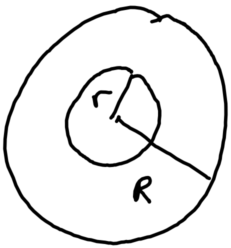
$$\frac{\Delta\theta}{2\pi} = \frac{\frac{1}{2}\alpha t^2}{2\pi} = 48,7$$ hringir svo B

$\underline{K13}$: $I = 3 \cdot \frac{1}{3} \cdot ML^2 = ML^2 = 1{,}21 \cdot 10^6$ kgm^2. (A)

$\underline{K14}$: Stutta svarið er: (C)

<u>Langa svarið er:</u> $\rho = \frac{m_J}{\frac{4\pi}{3}R^3}$ og síðan er þyngdarlögmáls-

krafturinn:



$$m a_r = -\frac{G M(r) m}{r^2}$$

og $M(r) = \rho \cdot \frac{4\pi}{3} r^3$ þ.a.

$$m a = -\frac{G \cdot \rho \cdot \frac{4\pi}{3} r^3 m}{r^2} = -\frac{G \cdot M_J m}{R^3} \cdot r$$

og $mg = \frac{G M_J m}{R_J^2}$ þ.a.

$$m a = -\frac{mg}{R} r$$

svo við ályktum að $\boxed{\ddot{r} = -\frac{g}{R_J} r}$

svo við höfum fengið einfalda sveifluhreyfingu

með horntíðni $\omega = \sqrt{\frac{g}{R_J}}$ en þá er

$$r(t) = R_J \cos\left(\sqrt{\frac{g}{R_J}}\, t\right)$$ og tíminn sem það

tekur að fara frá Íslands til Ástralíu og aftur til baka

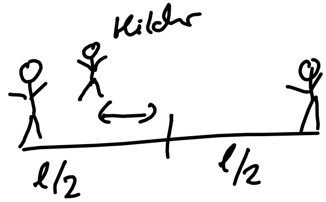
í gegnum miðju jarðarinnar er

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R_J}{g}} = 5061 \text{ s} \approx 1 \text{ klst og } 24 \text{ mín.}$$

K15:

Þar sem skriðþunginn er varðveittur byrjar sleðinn að fara til hægri en þegar Vésteinn grípur boltann þá stöðvast sleðinn aftur. (C)

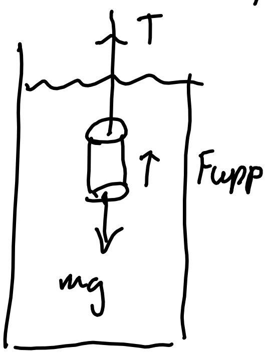
K16:



$$mrg\frac{l}{2} + mrgx = Mg\frac{l}{2}$$ (E)

þ.a. $x = \left(\frac{M}{mr} - 1\right)\cdot\frac{l}{2} = 1{,}125\,m = 1{,}1\,m$

K17:



$T + F_{upp} = mg$ $\qquad \rho_{gull} = \frac{m_{gull}}{V_{gull}} \Rightarrow V_{klump} = \frac{m_{gull}}{\rho_{gull}}$

þ.a. $T = mg - F_{upp} = \rho_{gull}\cdot V_{klump}\cdot g - \rho_{vatn}\cdot V_{klump}\, g$

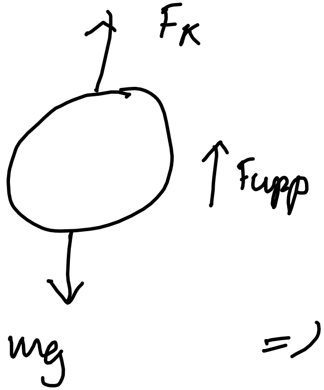
$= \left(\rho_{gull} - \rho_{vatn}\right)\cdot\frac{m_{gull}}{\rho_{gull}}\cdot g$

$= \left(1 - \frac{\rho_{vatn}}{\rho_{gull}}\right) m_{gull}\, g = 230\ N.$ (C)

K18: Boltinn hefur hraða: $\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} V_{bíll} \\ -V_{Albert} \end{pmatrix}$ en það er engin breyting í xy-planinu (bara z) þ.a. þessir hraðar haldast fastir allan tímann. Staða boltans er þá í xy-planinu $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} v_B t \\ -v_A t \end{pmatrix}$ þ.a. $t = \frac{x}{v_B}$ og þá sést

$$y = -v_A \cdot \frac{x}{v_x} = -\frac{v_A}{v_B}x$$ (D)

K19:



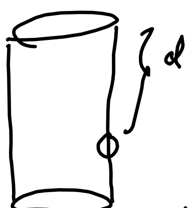
$$F_K + F_{upp} = mg$$

$$F_K + \frac{4\pi}{3}R^3\rho_{luft}\cdot g = mg$$

$$\Rightarrow R = \left(\frac{mg - F_K}{\frac{4\pi}{3}\rho_{luft}\cdot g}\right)^{1/3} = 2,27\ m.$$

B

K20:



Flatemål gatsins er $A = r^2\pi$.

Straumshastigheita ut finst med Bernoulli:

$$\rho_{vatn}\cdot gd = \frac{1}{2}\rho_{vatn}\cdot v^2 \Rightarrow v = \sqrt{2gd}$$ og

så er straumraten:

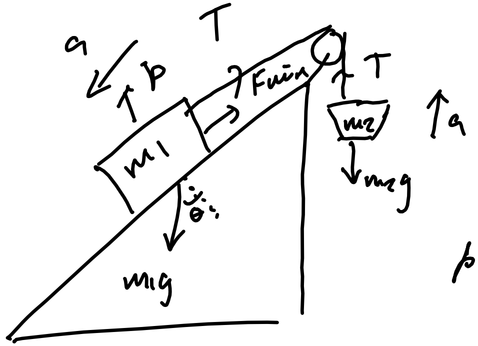
$$Av = r^2\pi\cdot\sqrt{2gd} = 0,038\ m^3/s = 38\ kg/s$$

D

Skriflegt dæmi 1:

$$m_1\begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} m_1 g\sin\theta - T - F_{nún} \\ p - m_1 g\cos\theta \end{pmatrix}$$

(a)



$$m_2 a = T - m_2 g$$

þ.a. $(m_1+m_2)a = m_1 g\sin\theta - \mu m_1 g\cos\theta - m_2 g$

þ.a. $a = \dfrac{m_1(\sin\theta - \mu\cos\theta) - m_2}{m_1+m_2}\, g$.

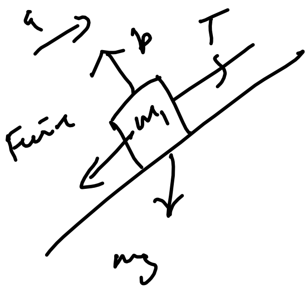
(b) Ef $a=0$ þá er $m_1(\sin\theta - \mu\cos\theta) = m_2$ þ.a. $\dfrac{m_1}{m_2} = \beta = \dfrac{1}{\sin\theta - \mu\cos\theta}$.

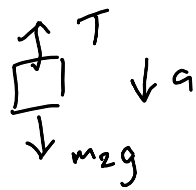
Þurfum samt að hafa að hornið θ þ.a. kubburinn geti byrjað að renna svo hér þurfum við að krefjast þess að $\theta > \arctan(\mu)$ þ.e. að $\tan\theta > \mu$ en þá er $\beta > 0$. því þá er $\mu\cos\theta < \tan\theta\cos\theta = \sin\theta$. Við ályktum því að

$$\beta_{max} = \frac{1}{\sin\theta - \mu\cos\theta}.$$

Við athugum að á einhverjum tímapunkti mun m_2 vera nógu þungur til að draga kubbinn í hina áttina.

Eru þá er núningskrafturinn í hina áttina svo við fáum:





$$m_2 a = m_2 g - T$$

$$m_1 a = T - m_1 g \sin\theta - \mu m_1 g \cos\theta$$

þ.a. $$a = \frac{m_2 - m_1(\sin\theta + \mu\cos\theta)}{m_1 + m_2} g$$

og þá $a = 0$ ef $m_2 = m_1(\sin\theta + \mu\cos\theta)$ þ.e.

$$\beta_{min} = \frac{m_1}{m_2} = \frac{1}{\sin\theta + \mu\cos\theta}$$ þ.a. ef

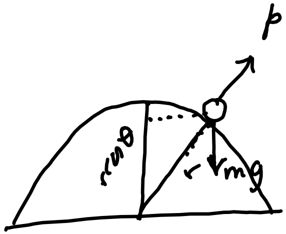
$$\beta \in [\beta_{min}, \beta_{max}] = \left[\frac{1}{\sin\theta + \mu\cos\theta}; \frac{1}{\sin\theta - \mu\cos\theta}\right]$$ eða ef

$\theta < \arctan(\mu)$ og $\beta > \beta_{min}$.

Dæmi 2:

(a) $$mgr = mgr\cos\theta + \frac{1}{2}mv^2$$

þ.a. $$v = \sqrt{2gr(1-\cos\theta)}.$$



(b) $$m\frac{v^2}{r} = p - mg\cos\theta \Rightarrow p = m\frac{v^2}{r} + mg\cos\theta$$

Athugum þegar $p=0$ þá er

$$m\frac{v^2}{r} = mg\cos\theta$$

$$m\cdot\frac{2gr(1-\cos\theta)}{r} = mg\cos\theta$$

$$\Rightarrow 2(1-\cos\theta) = \cos\theta$$

$$\Rightarrow \cos\theta = \frac{2}{3} \Rightarrow \theta = \arccos\left(\frac{2}{3}\right) = 48^\circ.$$