ICPC Reference Sheet

Zack Ritter and Michael Hug

This document contains references for many areas of programming, and covers Dynamic programming, graph theory, data structures, and more.

Contents

Graph Theory

Dijsktra's Shortest Path Algorithm

```
// Number of vertices in the graph

#define V 9

// A utility function to find the vertex with minimum distance value, from
// the set of vertices not yet included in shortest path tree
int minDistance(int dist[], bool sptSet[])
{
   // Initialize min value
   int min = INT_MAX, min_index;

   for (int v = 0; v < V; v++)
     if (sptSet[v] == false && dist[v] <= min)
         min = dist[v], min_index = v;

   return min_index;
}

// A utility function to print the constructed distance array
int printSolution(int dist[], int n)
{
   printf("Vertex   Distance from Source\n");
   for (int i = 0; i < V; i++)
      printf("%d \t\t %d\n", i, dist[i]);
}

// Funtion that implements Dijkstra's single source shortest path algorithm
// for a graph represented using adjacency matrix representation
void dijkstra(int graph[V][V], int src)
{
    int dist[V];      // The output array.  dist[i] will hold the shortest
                       // distance from src to i

    bool sptSet[V]; // sptSet[i] will true if vertex i is included in shortest
                      // path tree or shortest distance from src to i is
finalized

    // Initialize all distances as INFINITE and stpSet[] as false
    for (int i = 0; i < V; i++)
       dist[i] = INT_MAX, sptSet[i] = false;

    // Distance of source vertex from itself is always 0
    dist[src] = 0;

    // Find shortest path for all vertices
```

```
    for (int count = 0; count < V-1; count++)
    {
       // Pick the minimum distance vertex from the set of vertices not
       // yet processed. u is always equal to src in first iteration.
       int u = minDistance(dist, sptSet);

       // Mark the picked vertex as processed
       sptSet[u] = true;

       // Update dist value of the adjacent vertices of the picked vertex.
       for (int v = 0; v < V; v++)

          // Update dist[v] only if is not in sptSet, there is an edge from
          // u to v, and total weight of path from src to  v through u is
          // smaller than current value of dist[v]
          if (!sptSet[v] && graph[u][v] && dist[u] != INT_MAX
                                        && dist[u]+graph[u][v] < dist[v])
             dist[v] = dist[u] + graph[u][v];
    }

    // print the constructed distance array
    printSolution(dist, V);
}
```

Dijsktra's Algorithm for Adjacency List Representation

```
// A structure to represent a node in adjacency list
struct AdjListNode
{
    int dest;
    int weight;
    struct AdjListNode* next;
};

// A structure to represent an adjacency liat
struct AdjList
{
    struct AdjListNode *head;  // pointer to head node of list
};

// A structure to represent a graph. A graph is an array of adjacency lists.
// Size of array will be V (number of vertices in graph)
struct Graph
{
    int V;
    struct AdjList* array;
};

// A utility function to create a new adjacency list node
struct AdjListNode* newAdjListNode(int dest, int weight)
{
    struct AdjListNode* newNode =
            (struct AdjListNode*) malloc(sizeof(struct AdjListNode));
    newNode->dest = dest;
```

```c
    newNode->weight = weight;
    newNode->next = NULL;
    return newNode;
}

// A utility function that creates a graph of V vertices
struct Graph* createGraph(int V)
{
    struct Graph* graph = (struct Graph*) malloc(sizeof(struct Graph));
    graph->V = V;

    // Create an array of adjacency lists.  Size of array will be V
    graph->array = (struct AdjList*) malloc(V * sizeof(struct AdjList));

     // Initialize each adjacency list as empty by making head as NULL
    for (int i = 0; i < V; ++i)
        graph->array[i].head = NULL;

    return graph;
}

// Adds an edge to an undirected graph
void addEdge(struct Graph* graph, int src, int dest, int weight)
{
    // Add an edge from src to dest.  A new node is added to the adjacency
    // list of src.  The node is added at the begining
    struct AdjListNode* newNode = newAdjListNode(dest, weight);
    newNode->next = graph->array[src].head;
    graph->array[src].head = newNode;

    // Since graph is undirected, add an edge from dest to src also
    newNode = newAdjListNode(src, weight);
    newNode->next = graph->array[dest].head;
    graph->array[dest].head = newNode;
}

// Structure to represent a min heap node
struct MinHeapNode
{
    int v;
    int dist;
};

// Structure to represent a min heap
struct MinHeap
{
    int size;       // Number of heap nodes present currently
    int capacity;   // Capacity of min heap
    int *pos;       // This is needed for decreaseKey()
    struct MinHeapNode **array;
};

// A utility function to create a new Min Heap Node
struct MinHeapNode* newMinHeapNode(int v, int dist)
```

```c
{
    struct MinHeapNode* minHeapNode =
            (struct MinHeapNode*) malloc(sizeof(struct MinHeapNode));
    minHeapNode->v = v;
    minHeapNode->dist = dist;
    return minHeapNode;
}


// A utility function to create a Min Heap
struct MinHeap* createMinHeap(int capacity)
{
    struct MinHeap* minHeap =
        (struct MinHeap*) malloc(sizeof(struct MinHeap));
    minHeap->pos = (int *)malloc(capacity * sizeof(int));
    minHeap->size = 0;
    minHeap->capacity = capacity;
    minHeap->array =
        (struct MinHeapNode**) malloc(capacity * sizeof(struct MinHeapNode*));
    return minHeap;
}


// A utility function to swap two nodes of min heap. Needed for min heapify
void swapMinHeapNode(struct MinHeapNode** a, struct MinHeapNode** b)
{
    struct MinHeapNode* t = *a;
    *a = *b;
    *b = t;
}


// A standard function to heapify at given idx
// This function also updates position of nodes when they are swapped.
// Position is needed for decreaseKey()
void minHeapify(struct MinHeap* minHeap, int idx)
{
    int smallest, left, right;
    smallest = idx;
    left = 2 * idx + 1;
    right = 2 * idx + 2;

    if (left < minHeap->size &&
        minHeap->array[left]->dist < minHeap->array[smallest]->dist )
      smallest = left;

    if (right < minHeap->size &&
        minHeap->array[right]->dist < minHeap->array[smallest]->dist )
      smallest = right;

    if (smallest != idx)
    {
        // The nodes to be swapped in min heap
        MinHeapNode *smallestNode = minHeap->array[smallest];
        MinHeapNode *idxNode = minHeap->array[idx];

        // Swap positions
        minHeap->pos[smallestNode->v] = idx;
```

```c
        minHeap->pos[idxNode->v] = smallest;

        // Swap nodes
        swapMinHeapNode(&minHeap->array[smallest], &minHeap->array[idx]);

        minHeapify(minHeap, smallest);
    }
}

// A utility function to check if the given minHeap is ampty or not
int isEmpty(struct MinHeap* minHeap)
{
    return minHeap->size == 0;
}

// Standard function to extract minimum node from heap
struct MinHeapNode* extractMin(struct MinHeap* minHeap)
{
    if (isEmpty(minHeap))
        return NULL;

    // Store the root node
    struct MinHeapNode* root = minHeap->array[0];

    // Replace root node with last node
    struct MinHeapNode* lastNode = minHeap->array[minHeap->size - 1];
    minHeap->array[0] = lastNode;

    // Update position of last node
    minHeap->pos[root->v] = minHeap->size-1;
    minHeap->pos[lastNode->v] = 0;

    // Reduce heap size and heapify root
    --minHeap->size;
    minHeapify(minHeap, 0);

    return root;
}

// Function to decreasy dist value of a given vertex v. This function
// uses pos[] of min heap to get the current index of node in min heap
void decreaseKey(struct MinHeap* minHeap, int v, int dist)
{
    // Get the index of v in  heap array
    int i = minHeap->pos[v];

    // Get the node and update its dist value
    minHeap->array[i]->dist = dist;

    // Travel up while the complete tree is not hepified.
    // This is a O(Logn) loop
    while (i && minHeap->array[i]->dist < minHeap->array[(i - 1) / 2]->dist)
    {
        // Swap this node with its parent
```

```c
            minHeap->pos[minHeap->array[i]->v] = (i-1)/2;
            minHeap->pos[minHeap->array[(i-1)/2]->v] = i;
            swapMinHeapNode(&minHeap->array[i],  &minHeap->array[(i - 1) / 2]);

            // move to parent index
            i = (i - 1) / 2;
    }
}


// A utility function to check if a given vertex
// 'v' is in min heap or not
bool isInMinHeap(struct MinHeap *minHeap, int v)
{
    if (minHeap->pos[v] < minHeap->size)
      return true;
    return false;
}


// A utility function used to print the solution
void printArr(int dist[], int n)
{
    printf("Vertex   Distance from Source\n");
    for (int i = 0; i < n; ++i)
        printf("%d \t\t %d\n", i, dist[i]);
}


// The main function that calulates distances of shortest paths from src to
all
// vertices. It is a O(ELogV) function
void dijkstra(struct Graph* graph, int src)
{
    int V = graph->V;// Get the number of vertices in graph
    int dist[V];      // dist values used to pick minimum weight edge in cut

    // minHeap represents set E
    struct MinHeap* minHeap = createMinHeap(V);

    // Initialize min heap with all vertices. dist value of all vertices
    for (int v = 0; v < V; ++v)
    {
        dist[v] = INT_MAX;
        minHeap->array[v] = newMinHeapNode(v, dist[v]);
        minHeap->pos[v] = v;
    }

    // Make dist value of src vertex as 0 so that it is extracted first
    minHeap->array[src] = newMinHeapNode(src, dist[src]);
    minHeap->pos[src]   = src;
    dist[src] = 0;
    decreaseKey(minHeap, src, dist[src]);

    // Initially size of min heap is equal to V
    minHeap->size = V;

    // In the followin loop, min heap contains all nodes
```

```
    // whose shortest distance is not yet finalized.
    while (!isEmpty(minHeap))
    {
        // Extract the vertex with minimum distance value
        struct MinHeapNode* minHeapNode = extractMin(minHeap);
        int u = minHeapNode->v; // Store the extracted vertex number

        // Traverse through all adjacent vertices of u (the extracted
        // vertex) and update their distance values
        struct AdjListNode* pCrawl = graph->array[u].head;
        while (pCrawl != NULL)
        {
            int v = pCrawl->dest;

            // If shortest distance to v is not finalized yet, and distance to
v
            // through u is less than its previously calculated distance
            if (isInMinHeap(minHeap, v) && dist[u] != INT_MAX &&
                                    pCrawl->weight + dist[u] < dist[v])
            {
                dist[v] = dist[u] + pCrawl->weight;

                // update distance value in min heap also
                decreaseKey(minHeap, v, dist[v]);
            }
            pCrawl = pCrawl->next;
        }
    }

    // print the calculated shortest distances
    printArr(dist, V);
}
```

Prim's Minimum Spanning Tree for Adjacency List Representationi

```
// A structure to represent a node in adjacency list
struct AdjListNode
{
    int dest;
    int weight;
    struct AdjListNode* next;
};

// A structure to represent an adjacency liat
struct AdjList
{
    struct AdjListNode *head;  // pointer to head node of list
};

// A structure to represent a graph. A graph is an array of adjacency lists.
// Size of array will be V (number of vertices in graph)
struct Graph
```

```
{
    int V;
    struct AdjList* array;
};

// A utility function to create a new adjacency list node
struct AdjListNode* newAdjListNode(int dest, int weight)
{
    struct AdjListNode* newNode =
            (struct AdjListNode*) malloc(sizeof(struct AdjListNode));
    newNode->dest = dest;
    newNode->weight = weight;
    newNode->next = NULL;
    return newNode;
}

// A utility function that creates a graph of V vertices
struct Graph* createGraph(int V)
{
    struct Graph* graph = (struct Graph*) malloc(sizeof(struct Graph));
    graph->V = V;

    // Create an array of adjacency lists.  Size of array will be V
    graph->array = (struct AdjList*) malloc(V * sizeof(struct AdjList));

     // Initialize each adjacency list as empty by making head as NULL
    for (int i = 0; i < V; ++i)
        graph->array[i].head = NULL;

    return graph;
}

// Adds an edge to an undirected graph
void addEdge(struct Graph* graph, int src, int dest, int weight)
{
    // Add an edge from src to dest.  A new node is added to the adjacency
    // list of src.  The node is added at the begining
    struct AdjListNode* newNode = newAdjListNode(dest, weight);
    newNode->next = graph->array[src].head;
    graph->array[src].head = newNode;

    // Since graph is undirected, add an edge from dest to src also
    newNode = newAdjListNode(src, weight);
    newNode->next = graph->array[dest].head;
    graph->array[dest].head = newNode;
}

// Structure to represent a min heap node
struct MinHeapNode
{
    int v;
    int key;
};
```

```c
// Structure to represent a min heap
struct MinHeap
{
    int size;       // Number of heap nodes present currently
    int capacity;   // Capacity of min heap
    int *pos;       // This is needed for decreaseKey()
    struct MinHeapNode **array;
};

// A utility function to create a new Min Heap Node
struct MinHeapNode* newMinHeapNode(int v, int key)
{
    struct MinHeapNode* minHeapNode =
            (struct MinHeapNode*) malloc(sizeof(struct MinHeapNode));
    minHeapNode->v = v;
    minHeapNode->key = key;
    return minHeapNode;
}

// A utilit function to create a Min Heap
struct MinHeap* createMinHeap(int capacity)
{
    struct MinHeap* minHeap =
        (struct MinHeap*) malloc(sizeof(struct MinHeap));
    minHeap->pos = (int *)malloc(capacity * sizeof(int));
    minHeap->size = 0;
    minHeap->capacity = capacity;
    minHeap->array =
        (struct MinHeapNode**) malloc(capacity * sizeof(struct MinHeapNode*));
    return minHeap;
}

// A utility function to swap two nodes of min heap. Needed for min heapify
void swapMinHeapNode(struct MinHeapNode** a, struct MinHeapNode** b)
{
    struct MinHeapNode* t = *a;
    *a = *b;
    *b = t;
}

// A standard function to heapify at given idx
// This function also updates position of nodes when they are swapped.
// Position is needed for decreaseKey()
void minHeapify(struct MinHeap* minHeap, int idx)
{
    int smallest, left, right;
    smallest = idx;
    left = 2 * idx + 1;
    right = 2 * idx + 2;

    if (left < minHeap->size &&
        minHeap->array[left]->key < minHeap->array[smallest]->key )
      smallest = left;

    if (right < minHeap->size &&
```

```c
          minHeap->array[right]->key < minHeap->array[smallest]->key )
        smallest = right;

    if (smallest != idx)
    {
        // The nodes to be swapped in min heap
        MinHeapNode *smallestNode = minHeap->array[smallest];
        MinHeapNode *idxNode = minHeap->array[idx];

        // Swap positions
        minHeap->pos[smallestNode->v] = idx;
        minHeap->pos[idxNode->v] = smallest;

        // Swap nodes
        swapMinHeapNode(&minHeap->array[smallest], &minHeap->array[idx]);

        minHeapify(minHeap, smallest);
    }
}


// A utility function to check if the given minHeap is ampty or not
int isEmpty(struct MinHeap* minHeap)
{
    return minHeap->size == 0;
}


// Standard function to extract minimum node from heap
struct MinHeapNode* extractMin(struct MinHeap* minHeap)
{
    if (isEmpty(minHeap))
        return NULL;

    // Store the root node
    struct MinHeapNode* root = minHeap->array[0];

    // Replace root node with last node
    struct MinHeapNode* lastNode = minHeap->array[minHeap->size - 1];
    minHeap->array[0] = lastNode;

    // Update position of last node
    minHeap->pos[root->v] = minHeap->size-1;
    minHeap->pos[lastNode->v] = 0;

    // Reduce heap size and heapify root
    --minHeap->size;
    minHeapify(minHeap, 0);

    return root;
}

// Function to decreasy key value of a given vertex v. This function
// uses pos[] of min heap to get the current index of node in min heap
void decreaseKey(struct MinHeap* minHeap, int v, int key)
{
```

```c
    // Get the index of v in  heap array
    int i = minHeap->pos[v];

    // Get the node and update its key value
    minHeap->array[i]->key = key;

    // Travel up while the complete tree is not hepified.
    // This is a O(Logn) loop
    while (i && minHeap->array[i]->key < minHeap->array[(i - 1) / 2]->key)
    {
        // Swap this node with its parent
        minHeap->pos[minHeap->array[i]->v] = (i-1)/2;
        minHeap->pos[minHeap->array[(i-1)/2]->v] = i;
        swapMinHeapNode(&minHeap->array[i],  &minHeap->array[(i - 1) / 2]);

        // move to parent index
        i = (i - 1) / 2;
    }
}


// A utility function to check if a given vertex
// 'v' is in min heap or not
bool isInMinHeap(struct MinHeap *minHeap, int v)
{
   if (minHeap->pos[v] < minHeap->size)
     return true;
   return false;
}


// A utility function used to print the constructed MST
void printArr(int arr[], int n)
{
    for (int i = 1; i < n; ++i)
        printf("%d - %d\n", arr[i], i);
}


// The main function that constructs Minimum Spanning Tree (MST)
// using Prim's algorithm
void PrimMST(struct Graph* graph)
{
    int V = graph->V;// Get the number of vertices in graph
    int parent[V];   // Array to store constructed MST
    int key[V];      // Key values used to pick minimum weight edge in cut

    // minHeap represents set E
    struct MinHeap* minHeap = createMinHeap(V);

    // Initialize min heap with all vertices. Key value of
    // all vertices (except 0th vertex) is initially infinite
    for (int v = 1; v < V; ++v)
    {
        parent[v] = -1;
        key[v] = INT_MAX;
        minHeap->array[v] = newMinHeapNode(v, key[v]);
        minHeap->pos[v] = v;
```

```
    }

    // Make key value of 0th vertex as 0 so that it
    // is extracted first
    key[0] = 0;
    minHeap->array[0] = newMinHeapNode(0, key[0]);
    minHeap->pos[0]   = 0;

    // Initially size of min heap is equal to V
    minHeap->size = V;

    // In the followin loop, min heap contains all nodes
    // not yet added to MST.
    while (!isEmpty(minHeap))
    {
        // Extract the vertex with minimum key value
        struct MinHeapNode* minHeapNode = extractMin(minHeap);
        int u = minHeapNode->v; // Store the extracted vertex number

        // Traverse through all adjacent vertices of u (the extracted
        // vertex) and update their key values
        struct AdjListNode* pCrawl = graph->array[u].head;
        while (pCrawl != NULL)
        {
            int v = pCrawl->dest;

            // If v is not yet included in MST and weight of u-v is
            // less than key value of v, then update key value and
            // parent of v
            if (isInMinHeap(minHeap, v) && pCrawl->weight < key[v])
            {
                key[v] = pCrawl->weight;
                parent[v] = u;
                decreaseKey(minHeap, v, key[v]);
            }
            pCrawl = pCrawl->next;
        }
    }

    // print edges of MST
    printArr(parent, V);
}
```

Prim's Minimum Spanning Tree Algorithm

```
// Number of vertices in the graph
#define V 5

// A utility function to find the vertex with minimum key value, from
// the set of vertices not yet included in MST
int minKey(int key[], bool mstSet[])
{
    // Initialize min value
    int min = INT_MAX, min_index;
```

```
    for (int v = 0; v < V; v++)
      if (mstSet[v] == false && key[v] < min)
          min = key[v], min_index = v;


    return min_index;
}


// A utility function to print the constructed MST stored in parent[]
int printMST(int parent[], int n, int graph[V][V])
{
    printf("Edge   Weight\n");
    for (int i = 1; i < V; i++)
        printf("%d - %d    %d \n", parent[i], i, graph[i][parent[i]]);
}


// Function to construct and print MST for a graph represented using adjacency
// matrix representation
void primMST(int graph[V][V])
{
      int parent[V]; // Array to store constructed MST
      int key[V];    // Key values used to pick minimum weight edge in cut
      bool mstSet[V];   // To represent set of vertices not yet included in MST

      // Initialize all keys as INFINITE
      for (int i = 0; i < V; i++)
          key[i] = INT_MAX, mstSet[i] = false;

      // Always include first 1st vertex in MST.
      key[0] = 0;      // Make key 0 so that this vertex is picked as first
vertex
      parent[0] = -1; // First node is always root of MST

      // The MST will have V vertices
      for (int count = 0; count < V-1; count++)
      {
         // Pick thd minimum key vertex from the set of vertices
         // not yet included in MST
         int u = minKey(key, mstSet);

         // Add the picked vertex to the MST Set
         mstSet[u] = true;

         // Update key value and parent index of the adjacent vertices of
         // the picked vertex. Consider only those vertices which are not yet
         // included in MST
         for (int v = 0; v < V; v++)

            // graph[u][v] is non zero only for adjacent vertices of m
            // mstSet[v] is false for vertices not yet included in MST
            // Update the key only if graph[u][v] is smaller than key[v]
           if (graph[u][v] && mstSet[v] == false && graph[u][v] <  key[v])
             parent[v]  = u, key[v] = graph[u][v];
      }
```

```
    // print the constructed MST
    printMST(parent, V, graph);
}
```

Kruskal's Minimum Spanning Tree Algorithm

```c
// a structure to represent a weighted edge in graph
struct Edge
{
    int src, dest, weight;
};

// a structure to represent a connected, undirected and weighted graph
struct Graph
{
    // V-> Number of vertices, E-> Number of edges
    int V, E;

    // graph is represented as an array of edges. Since the graph is
    // undirected, the edge from src to dest is also edge from dest
    // to src. Both are counted as 1 edge here.
    struct Edge* edge;
};

// Creates a graph with V vertices and E edges
struct Graph* createGraph(int V, int E)
{
    struct Graph* graph = (struct Graph*) malloc( sizeof(struct Graph) );
    graph->V = V;
    graph->E = E;

    graph->edge = (struct Edge*) malloc( graph->E * sizeof( struct Edge ) );

    return graph;
}

// A structure to represent a subset for union-find
struct subset
{
    int parent;
    int rank;
};

// A utility function to find set of an element i
// (uses path compression technique)
int find(struct subset subsets[], int i)
{
    // find root and make root as parent of i (path compression)
    if (subsets[i].parent != i)
        subsets[i].parent = find(subsets, subsets[i].parent);
```

```c
        return subsets[i].parent;
}

// A function that does union of two sets of x and y
// (uses union by rank)
void Union(struct subset subsets[], int x, int y)
{
    int xroot = find(subsets, x);
    int yroot = find(subsets, y);

    // Attach smaller rank tree under root of high rank tree
    // (Union by Rank)
    if (subsets[xroot].rank < subsets[yroot].rank)
        subsets[xroot].parent = yroot;
    else if (subsets[xroot].rank > subsets[yroot].rank)
        subsets[yroot].parent = xroot;

    // If ranks are same, then make one as root and increment
    // its rank by one
    else
    {
        subsets[yroot].parent = xroot;
        subsets[xroot].rank++;
    }
}

// Compare two edges according to their weights.
// Used in qsort() for sorting an array of edges
int myComp(const void* a, const void* b)
{
    struct Edge* a1 = (struct Edge*)a;
    struct Edge* b1 = (struct Edge*)b;
    return a1->weight > b1->weight;
}

// The main function to construct MST using Kruskal's algorithm
void KruskalMST(struct Graph* graph)
{
    int V = graph->V;
    struct Edge result[V];  // Tnis will store the resultant MST
    int e = 0;  // An index variable, used for result[]
    int i = 0;  // An index variable, used for sorted edges

    // Step 1:  Sort all the edges in non-decreasing order of their weight
    // If we are not allowed to change the given graph, we can create a copy of
    // array of edges
    qsort(graph->edge, graph->E, sizeof(graph->edge[0]), myComp);

    // Allocate memory for creating V ssubsets
    struct subset *subsets =
        (struct subset*) malloc( V * sizeof(struct subset) );

    // Create V subsets with single elements
    for (int v = 0; v < V; ++v)
```

```
    {
        subsets[v].parent = v;
        subsets[v].rank = 0;
    }

    // Number of edges to be taken is equal to V-1
    while (e < V - 1)
    {
        // Step 2: Pick the smallest edge. And increment the index
        // for next iteration
        struct Edge next_edge = graph->edge[i++];

        int x = find(subsets, next_edge.src);
        int y = find(subsets, next_edge.dest);

        // If including this edge does't cause cycle, include it
        // in result and increment the index of result for next edge
        if (x != y)
        {
            result[e++] = next_edge;
            Union(subsets, x, y);
        }
        // Else discard the next_edge
    }

    // print the contents of result[] to display the built MST
    printf("Following are the edges in the constructed MST\n");
    for (i = 0; i < e; ++i)
        printf("%d -- %d == %d\n", result[i].src, result[i].dest,
                                            result[i].weight);

    return;
}
```

Dynamic Programming

Longest Increasing Subsequence

```
/* lis() returns the length of the longest increasing subsequence in
    arr[] of size n */
int lis( int arr[], int n )
{
    int *lis, i, j, max = 0;
    lis = (int*) malloc ( sizeof( int ) * n );

    /* Initialize LIS values for all indexes */
    for ( i = 0; i < n; i++ )
        lis[i] = 1;

    /* Compute optimized LIS values in bottom up manner */
```

```
   for ( i = 1; i < n; i++ )
      for ( j = 0; j < i; j++ )
         if ( arr[i] > arr[j] && lis[i] < lis[j] + 1)
            lis[i] = lis[j] + 1;

   /* Pick maximum of all LIS values */
   for ( i = 0; i < n; i++ )
      if ( max < lis[i] )
         max = lis[i];

   /* Free memory to avoid memory leak */
   free( lis );

   return max;
}
```

Longest Common Subsequence

```
int lcs( char *X, char *Y, int m, int n )
{
   if (m == 0 || n == 0)
      return 0;
   if (X[m-1] == Y[n-1])
      return 1 + lcs(X, Y, m-1, n-1);
   else
      return max(lcs(X, Y, m, n-1), lcs(X, Y, m-1, n));
}
```

Minimum Cost Path

```
int minCost(int cost[R][C], int m, int n)
{
    int i, j;

    // Instead of following line, we can use int tc[m+1][n+1] or
    // dynamically allocate memoery to save space. The following line is
    // used to keep te program simple and make it working on all compilers.
    int tc[R][C];

    tc[0][0] = cost[0][0];

    /* Initialize first column of total cost(tc) array */
    for (i = 1; i <= m; i++)
       tc[i][0] = tc[i-1][0] + cost[i][0];

    /* Initialize first row of tc array */
    for (j = 1; j <= n; j++)
       tc[0][j] = tc[0][j-1] + cost[0][j];
```

```
        /* Construct rest of the tc array */
        for (i = 1; i <= m; i++)
            for (j = 1; j <= n; j++)
                tc[i][j] = min(tc[i-1][j-1], tc[i-1][j], tc[i][j-1]) + cost[i][j];


        return tc[m][n];
}
```

Matrix Chain Multiplication

```
// Matrix Ai has dimension p[i-1] x p[i] for i = 1..n
int MatrixChainOrder(int p[], int n)
{

    /* For simplicity of the program, one extra row and one extra column are
       allocated in m[][].  0th row and 0th column of m[][] are not used */
    int m[n][n];

    int i, j, k, L, q;

    /* m[i,j] = Minimum number of scalar multiplications needed to compute
       the matrix A[i]A[i+1]...A[j] = A[i..j] where dimention of A[i] is
       p[i-1] x p[i] */

    // cost is zero when multiplying one matrix.
    for (i = 1; i < n; i++)
        m[i][i] = 0;

    // L is chain length.
    for (L=2; L<n; L++)
    {
        for (i=1; i<=n-L+1; i++)
        {
            j = i+L-1;
            m[i][j] = INT_MAX;
            for (k=i; k<=j-1; k++)
            {
                // q = cost/scalar multiplications
                q = m[i][k] + m[k+1][j] + p[i-1]*p[k]*p[j];
                if (q < m[i][j])
                    m[i][j] = q;
            }
        }
    }


    return m[1][n-1];
}
```

Knapsack Problem

```c
// Returns the maximum value that can be put in a knapsack of capacity W
int knapSack(int W, int wt[], int val[], int n)
{
   int i, w;
   int K[n+1][W+1];

   // Build table K[][] in bottom up manner
   for (i = 0; i <= n; i++)
   {
       for (w = 0; w <= W; w++)
       {
           if (i==0 || w==0)
               K[i][w] = 0;
           else if (wt[i-1] <= w)
                   K[i][w] = max(val[i-1] + K[i-1][w-wt[i-1]],  K[i-1][w]);
           else
                   K[i][w] = K[i-1][w];
       }
   }

   return K[n][W];
}
```

Longest Palindromic Subsequence

```c
// Returns the length of the longest palindromic subsequence in seq

int lps(char *str)

{

   int n = strlen(str);

   int i, j, cl;

   int L[n][n];  // Create a table to store results of subproblems



   // Strings of length 1 are palindrome of lentgh 1

   for (i = 0; i < n; i++)

      L[i][i] = 1;
```

```
    // Build the table. Note that the lower diagonal values of table are
    // useless and not filled in the process. The values are filled in a
    // manner similar to Matrix Chain Multiplication DP solution (See
    // http://www.geeksforgeeks.org/archives/15553). cl is length of
    // substring
    for (cl=2; cl<=n; cl++)
    {
        for (i=0; i<n-cl+1; i++)
        {
            j = i+cl-1;
            if (str[i] == str[j] && cl == 2)
               L[i][j] = 2;
            else if (str[i] == str[j])
               L[i][j] = L[i+1][j-1] + 2;
            else
               L[i][j] = max(L[i][j-1], L[i+1][j]);
        }
    }


    return L[0][n-1];
}
```

Maximum Sum Increasing Subsequence

```
/* maxSumIS() returns the maximum sum of increasing subsequence in arr[] of
   size n */
int maxSumIS( int arr[], int n )
{
   int *msis, i, j, max = 0;
   msis = (int*) malloc ( sizeof( int ) * n );

   /* Initialize msis values for all indexes */
```

```
   for ( i = 0; i < n; i++ )
      msis[i] = arr[i];

   /* Compute maximum sum values in bottom up manner */
   for ( i = 1; i < n; i++ )
      for ( j = 0; j < i; j++ )
         if ( arr[i] > arr[j] && msis[i] < msis[j] + arr[i])
            msis[i] = msis[j] + arr[i];

   /* Pick maximum of all msis values */
   for ( i = 0; i < n; i++ )
      if ( max < msis[i] )
         max = msis[i];

   /* Free memory to avoid memory leak */
   free( msis );

   return max;
}
```

Ugly Numbers

```
/* Function to get the nth ugly number*/
unsigned getNthUglyNo(unsigned n)
{
    unsigned *ugly =
            (unsigned *)(malloc (sizeof(unsigned)*n));
    unsigned i2 = 0, i3 = 0, i5 = 0;
    unsigned i;
    unsigned next_multiple_of_2 = 2;
    unsigned next_multiple_of_3 = 3;
    unsigned next_multiple_of_5 = 5;
    unsigned next_ugly_no = 1;
    *(ugly+0) = 1;

    for(i=1; i<n; i++)
    {
       next_ugly_no = min(next_multiple_of_2,
                          next_multiple_of_3,
                          next_multiple_of_5);
       *(ugly+i) = next_ugly_no;
       if(next_ugly_no == next_multiple_of_2)
       {
           i2 = i2+1;
           next_multiple_of_2 = *(ugly+i2)*2;
       }
       if(next_ugly_no == next_multiple_of_3)
       {
           i3 = i3+1;
           next_multiple_of_3 = *(ugly+i3)*3;
       }
       if(next_ugly_no == next_multiple_of_5)
       {
```

```
            i5 = i5+1;
            next_multiple_of_5 = *(ugly+i5)*5;
        }
    } /*End of for loop (i=1; i<n; i++) */
    return next_ugly_no;
}
```

Bellman-Ford Algorithm

```
struct Edge
{
    int src, dest, weight;
};

// a structure to represent a connected, directed and weighted graph
struct Graph
{
    // V-> Number of vertices, E-> Number of edges
    int V, E;

    // graph is represented as an array of edges.
    struct Edge* edge;
};

// Creates a graph with V vertices and E edges
struct Graph* createGraph(int V, int E)
{
    struct Graph* graph = (struct Graph*) malloc( sizeof(struct Graph) );
    graph->V = V;
    graph->E = E;

    graph->edge = (struct Edge*) malloc( graph->E * sizeof( struct Edge ) );

    return graph;
}

// A utility function used to print the solution
void printArr(int dist[], int n)
{
    printf("Vertex   Distance from Source\n");
    for (int i = 0; i < n; ++i)
        printf("%d \t\t %d\n", i, dist[i]);
}

// The main function that finds shortest distances from src to all other
// vertices using Bellman-Ford algorithm.  The function also detects negative
// weight cycle
void BellmanFord(struct Graph* graph, int src)
{
    int V = graph->V;
    int E = graph->E;
    int dist[V];
```

```c
    // Step 1: Initialize distances from src to all other vertices as INFINITE
    for (int i = 0; i < V; i++)
        dist[i]   = INT_MAX;
    dist[src] = 0;

    // Step 2: Relax all edges |V| - 1 times. A simple shortest path from src
    // to any other vertex can have at-most |V| - 1 edges
    for (int i = 1; i <= V-1; i++)
    {
        for (int j = 0; j < E; j++)
        {
            int u = graph->edge[j].src;
            int v = graph->edge[j].dest;
            int weight = graph->edge[j].weight;
            if (dist[u] + weight < dist[v])
                dist[v] = dist[u] + weight;
        }
    }

    // Step 3: check for negative-weight cycles.  The above step guarantees
    // shortest distances if graph doesn't contain negative weight cycle.
    // If we get a shorter path, then there is a cycle.
    for (int i = 0; i < E; i++)
    {
        int u = graph->edge[i].src;
        int v = graph->edge[i].dest;
        int weight = graph->edge[i].weight;
        if (dist[u] + weight < dist[v])
            printf("Graph contains negative weight cycle");
    }

    printArr(dist, V);

    return;
}
```

Floyd-Warshall Algorithm

```c
// Number of vertices in the graph
#define V 4

/* Define Infinite as a large enough value. This value will be used
   for vertices not connected to each other */
#define INF 99999

// A function to print the solution matrix
void printSolution(int dist[][V]);

// Solves the all-pairs shortest path problem using Floyd Warshall algorithm
void floydWarshell (int graph[][V])
{
```

```
    /* dist[][] will be the output matrix that will finally have the shortest
       distances between every pair of vertices */
    int dist[V][V], i, j, k;

    /* Initialize the solution matrix same as input graph matrix. Or
       we can say the initial values of shortest distances are based
       on shortest paths considering no intermediate vertex. */
    for (i = 0; i < V; i++)
        for (j = 0; j < V; j++)
            dist[i][j] = graph[i][j];

    /* Add all vertices one by one to the set of intermediate vertices.
      ---> Before start of a iteration, we have shortest distances between all
      pairs of vertices such that the shortest distances consider only the
      vertices in set {0, 1, 2, .. k-1} as intermediate vertices.
      ----> After the end of a iteration, vertex no. k is added to the set of
      intermediate vertices and the set becomes {0, 1, 2, .. k} */
    for (k = 0; k < V; k++)
    {
        // Pick all vertices as source one by one
        for (i = 0; i < V; i++)
        {
            // Pick all vertices as destination for the
            // above picked source
            for (j = 0; j < V; j++)
            {
                // If vertex k is on the shortest path from
                // i to j, then update the value of dist[i][j]
                if (dist[i][k] + dist[k][j] < dist[i][j])
                    dist[i][j] = dist[i][k] + dist[k][j];
            }
        }
    }

    // Print the shortest distance matrix
    printSolution(dist);
}
```

Binary Search Tree

```
/* A Dynamic Programming based function that calculates minimum cost of
   a Binary Search Tree. */
int optimalSearchTree(int keys[], int freq[], int n)
{
    /* Create an auxiliary 2D matrix to store results of subproblems */
    int cost[n][n];

    /* cost[i][j] = Optimal cost of binary search tree that can be
       formed from keys[i] to keys[j].
       cost[0][n-1] will store the resultant cost */

    // For a single key, cost is equal to frequency of the key
    for (int i = 0; i < n; i++)
```

```
        cost[i][i] = freq[i];

    // Now we need to consider chains of length 2, 3, ... .
    // L is chain length.
    for (int L=2; L<=n; L++)
    {
        // i is row number in cost[][]
        for (int i=0; i<=n-L+1; i++)
        {
            // Get column number j from row number i and chain length L
            int j = i+L-1;
            cost[i][j] = INT_MAX;

            // Try making all keys in interval keys[i..j] as root
            for (int r=i; r<=j; r++)
            {
                // c = cost when keys[r] becomes root of this subtree
                int c = ((r > i)? cost[i][r-1]:0) +
                         ((r < j)? cost[r+1][j]:0) +
                         sum(freq, i, j);
                if (c < cost[i][j])
                    cost[i][j] = c;
            }
        }
    }
    return cost[0][n-1];
}
```