Chapter 1

Probability

- Basic Concepts
- Properties of Probertility
- Methods of Enumeration
- Conditional Probability
- Independent Events
- Bayes’s Theorem

Section 1.1

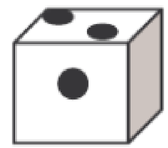
Properties of Probertility

Basic Concepts

The discipline of Statistics deals with the *collection* and *analysis* of data which is based on Probability Theory.

- Consider *Experiments* for which the outcome cannot be predicted with certainty, two definitions are given:
  - $S(\Omega)$: Sample Space (outcome space)
  - E: An Event (a subset of outcome space)

Sample Space: Example

- Ex. Roll a die 

  Outcomes: landing with a 1, 2, 3, 4, 5, or 6 face up.

  Sample Space: $S =$

Sample Space: Example (con't)

- Example 2: The gender of the next child born in certain hospital
- Example 3: The gender of the next two children born in certain hospital
- Example 4: The weight of the next child born in certain hospital

Sample Space: Example (con't)

- Example 5: Flipping a fair coin
- Example 6: Sum of rolling a pair of two dice
- Example 7: Scores (0~100) of *30* students who take MATH3332

Event: Example

- Roll a die once

  Event A: getting a “3”

  Roll a die twice

  S=

  Event B: the sum of the two numbers equal to “7”

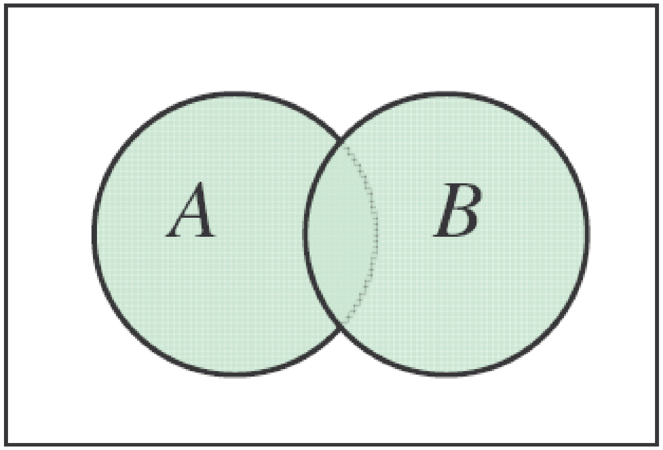
Event: Example (con't)

- Event C: the next new born child is a girl.
- Event D: the annual salary for the entry-level statistician in company EXE is \$40,000 -- \$50,000
- Event E: There are two cars stopped during one red light period at certain intersection.
- Event F: Students in this class are from Math, CS, IS, Economics, Math Edu, Theatre & Performance Studies,

Relations from the Set Theory (1)

- Union:
  - The **union** of two events A and B is the event consisting of all outcomes that are either in A or B
  - Notation: A U B
  - Read: A or B

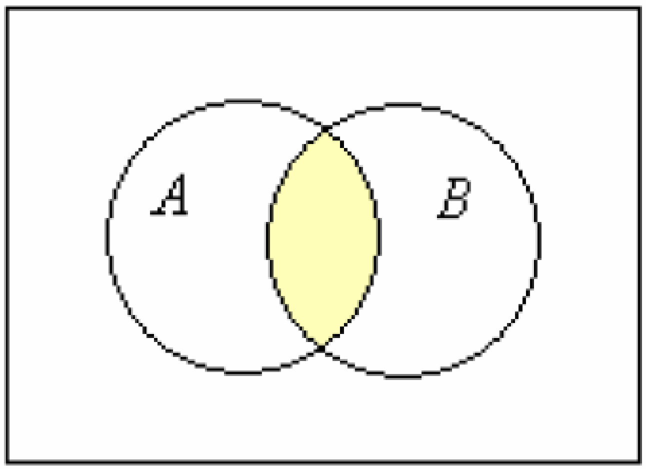
$$A \cup B$$



Relations from the Set Theory (2)

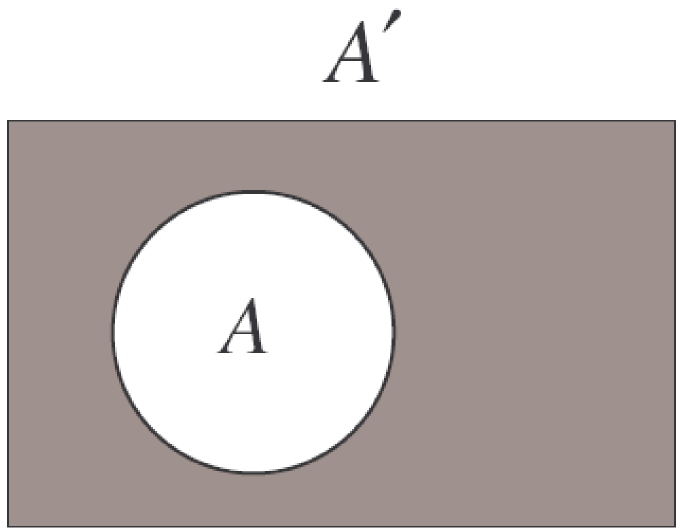
- Intersection:
  - The **intersection** of two events A and B is the event consisting of all outcomes that are in both A and B
  - Notation: $A \cap B$
  - Read: A and B

$$A \bigcap B$$



Relations from the Set Theory (3)

- Complement:
  - The **complement** of an events A is the set of all outcomes in S that are not contained in A
  - Notation: A’



Example: Rolling a die

- $S=\{1, 2, 3, 4, 5, 6\}$

  $A=\{1, 2, 3\}$; $B=\{1, 3, 5\}$

  $A \cup B =$

  $A \cap B =$

  $A' =$

  $A' \cap B =$

  $A' \cup B' =$

  $(A \cap B)' =$

Example: Basketball Tournament

- Four universities – 1, 2, 3, and 4 – are participating in a holiday basketball tournament. In the first round, 1 will play 2 and 3 will play 4. Then the two winners will play for the championship, and the two losers will also play. One possible outcome can be denoted by 1324 (1 beats 2 and 3 beats 4 in first-round games, and then 1 beats 3 and 2 beats 4).

  (1) list all outcomes in S.

  (2) Let A denote the event that 1 wins the tournament. List outcomes in A.

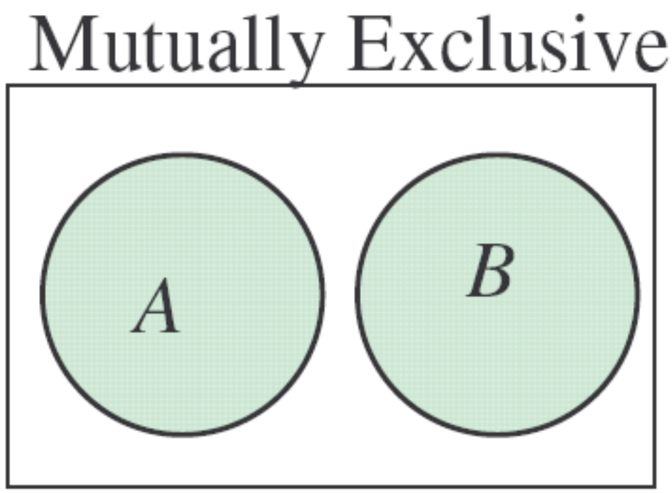
  (3) Let B denote the event that 2 gets into the championship game. List outcomes in B.

  (4) What are the outcomes in $A \cup B$ and in $A \cap B$? What are the outcomes in A'?

Relations from the Set Theory (4)

- Mutually exclusive /disjoint event
  - When A and B have no outcomes in common, they are **mutually exclusive** or **disjoint** event.

    I.e. $A \cap B = \Phi$



Mutually Exclusive: Example

- EX1: when rolling a die, if event A={2, 4, 6} (even) and event B={1, 3, 5} (odd), then A and B are mutually exclusive

- EX2: When drawing a single card from a standard deck of cards, if event A={heart, diamond} (red) and event B={spade, club} (black), then

  ____

Mutually Exclusive: More Than Two Events

- Given events A1, A2, A3, …, these events are said to be mutually exclusive (or pairwise disjoint if

Relations from the Set Theory (5)

- Exhaustive Events
  - A1, A2, …, Ak are *exhaustive events* if

$$\cup_{i=1}^{k} A_i = A_1 \cup A_2 \cup \ldots \cup A_k = S$$

- Mutually exclusive and exhaustive events
  - A1, A2, …, Ak are *mutually exclusive and exhaustive events* if

More Notations and Definitions

- A -- event A

- P(A) – Probability of event A

  -- Chance that event A will happen

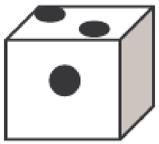
- *Probability* is a set function *P* that assigns to each event A, $A \subseteq S$, a number P(A), called the probability of the event A, such that the following properties are satisfied:

Axioms of Probability

- Axiom 1
- Axiom 2
- Axiom 3 If all A_i's are mutually exclusive, then

Axiom: Example

- Ex. Roll a die



  Sample space is S={1, 2, 3, 4, 5, 6}

  Axiom 1 implies:

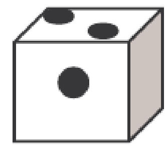
  Axiom 3 implies:

- Ex: a single coin is tossed, the sample space is S={H,T}

  Let p represent a fix number between __ and __, and P(H)=p, then P(T)=________.

Axiom: Example

- Ex. Roll a die



  Sample space is S={1, 2, 3, 4, 5, 6}

  Axiom 1 implies:

  Axiom 3 implies:

- Ex: a single coin is tossed, the sample space is S={H,T}

  Let *p* represent a fix number between __ and __, and P(H)=*p*, then P(T)=________.

Interpreting Probability

- The axioms serve only to rule out assignments inconsistent with our intuitive notions of probability.
- The axioms do not completely determine an assignment of probabilities to events.
- The appropriate assignment depends on the manner in which the experiment is carried out and also on the interpretation of probability
- The interpretation that is most frequently is based on the notion of relative frequencies.

Probability: Example

- A disk 2 inches in diameter is thrown at random on a tiled floor, where each tile is a square with sides 4 inches in length. Let C be the event that the disk will land entirely on one tile. How much is P(C)?

Properties of Probability

For any event A, $P(A) = 1 - P(A')$.

$P(\varnothing) = 0$

If A and B are mutually exclusive, then $P(A \cap B) = 0$.

Example

- A fair coin is flipped successively until the same face is observed on successive flips. Let A = {x: x=3, 4, 5, …}; that is, A is the event that it will take three or more flips of the coin to observe the same face on two consecutive flips. Find P(A).

Properties of Probability

$$P(A) \leq P(B) \text{ if } A \subset B.$$

$$P(A) \leq 1 \text{ for each event } A.$$

For any two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Example

- A faculty leader was meeting two students in Paris, one arriving by train from Amsterdam and the other arriving by train from Brussels at approximately the same time. Let A and B be the events that the respective trains are on time. Supple we know from past experience that $P(A) = 0.93$, $P(B) = 0.89$, and $P(A \cap B) = 0.87$. Find the probability that at least one train is on time.

Additional Examples

- In a certain residential suburb, 60% of all households subscribe to the metropolitan newspaper published in a nearby city, 80% subscribe to the local paper, and 50% of all households subscribe to both papers. If a household is selected at random, what is the probability that it subscribes to

  (1) at least one of the two newspapers?

  (2) exactly one of the two newspapers?

Equally Likely Outcomes

- If the sample space contains N outcomes,
  S={E1, E2, …, EN}
  and each outcomes has the same chance to happen, then equal probability will be assigned to all N simple events

- Now consider an event A, with N(A) denoting the number of outcomes contained in A, then

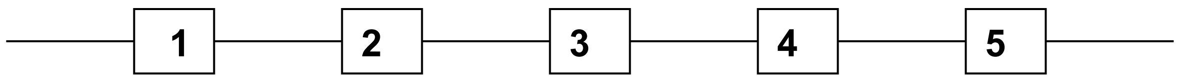
  P(A) =

Example

- A card is drawn from a well-shuffled deck of 52 playing cards. What is the probability that it is a queen or heart?

Additional Examples

- Consider a system of five identical components connected in series, as illustrated in the following:



Denote a component that fails by *F* and one that doesn't fail by S (for success}

S={FFFFF, FFFFS, FFFSF, FFSFF, FSFFF, SFFFF,
FFFSS, FFSFS, FSFFS, ……..
…….., SSSSS}

Let A denote the event that the system fails, then

A={

P(A) =

More Properties

- For three events A, B, and C

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$