MATH3332 **Homework 2, Due Oct. 02 before class.**

Please do the following exercises in the textbook. Please show your calculation. Otherwise, points will be deducted.

Chapter 2

Exercise 2.1-1

2.1-1. Let the p.m.f. of X be defined by $f(x) = x/9$, $x = 2, 3, 4$.

(a) Draw a bar graph for this p.m.f.
(b) Draw a probability histogram for this p.m.f.

Exercise 2.1-3

2.1-3. For each of the following, determine the constant c so that $f(x)$ satisfies the conditions of being a p.m.f. for a random variable X, and then depict each p.m.f. as a bar graph:

(a) $f(x) = x/c, \quad x = 1, 2, 3, 4.$
(b) $f(x) = cx, \quad x = 1, 2, 3, \ldots, 10.$
(c) $f(x) = c(1/4)^x, \quad x = 1, 2, 3, \ldots.$
(d) $f(x) = c(x + 1)^2, \quad x = 0, 1, 2, 3.$
(e) $f(x) = x/c, \quad x = 1, 2, 3, \ldots, n.$
(f) $f(x) = \dfrac{c}{(x + 1)(x + 2)}, \quad x = 0, 1, 2, 3, \ldots.$

Exercise 2.1-7(a)(c)

2.1-7. Let a random experiment be the casting of a pair of unbiased six-sided dice, and let X equal the smaller of the outcomes if they are different and the common value if they are equal.

(a) With reasonable assumptions, find the p.m.f. of X.
(c) Let Y equal the range of the two outcomes (i.e., the absolute value of the difference of the largest and the smallest outcomes). Determine the p.m.f. $g(y)$ of Y for $y = 0, 1, 2, 3, 4, 5$.

Exercise 2.1-11

2.1-11. In a lot (collection) of 100 light bulbs, there are 5 bad bulbs. An inspector inspects 10 bulbs selected at random. Find the probability of finding at least one defective bulb. HINT: First compute the probability of finding no defectives in the sample.

Exercise 2.2-1

2.2-1. Find $E(X)$ for each of the distributions given in Exercise 2.1-3.

HINT: Part (c). Note that the difference $E(X) - (1/4)E(X)$ is equal to the sum of a geometric series.

Exercise 2.2-3

2.2-3. In a particular lottery, 3 million tickets are sold each week for 50¢ apiece. Out of the 3 million tickets, 12,006 are drawn at random and without replacement and awarded prizes: twelve thousand \$25 prizes, four \$10,000 prizes, one \$50,000 prize, and one \$200,000 prize. If you purchased a single ticket each week, what is the expected value of this game to you?

Exercise 2.2-5

2.2-5. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the p.m.f.

$$f(x) = \frac{5 - x}{10}, \qquad x = 1, 2, 3, 4.$$

If the patient is to receive \$200 from an insurance company for each of the first two days in the hospital and \$100 for each day after the first two days, what is the expected payment for the hospitalization?

Exercise 2.2-7

2.2-7. In the gambling game chuck-a-luck, for a \$1 bet it is possible to win \$1, \$2, or \$3 with respective probabilities 75/216, 15/216, and 1/216. One dollar is lost with probability 125/216. Let X equal the payoff for this game and find $E(X)$. Note that when a bet is won, the \$1 that was bet, in addition to the \$1, \$2, or \$3 that is won, is returned to the bettor.

Exercise 2.3-1

2.3-1. Find the mean and variance for the following discrete distributions:

(a) $f(x) = \dfrac{1}{5}, \quad x = 5, 10, 15, 20, 25.$
(b) $f(x) = 1, \quad x = 5.$
(c) $f(x) = \dfrac{4 - x}{6}, \quad x = 1, 2, 3.$

Exercise 2.3-5

2.3-5. Consider an experiment that consists of selecting a card at random from an ordinary deck of cards. Let the random variable X equal the value of the selected card, where Ace = 1, Jack = 11, Queen = 12, and King = 13. Thus, the space of X is $S = \{1, 2, 3, \cdots, 13\}$. If the experiment is performed in an unbiased manner, assign probabilities to these 13 outcomes and compute the mean μ of this probability distribution.

Exercise 2.3-15 (a)(b) For part b, just do the first 10 observations in the first row.

2.3-15. Let X equal the larger outcome when a pair of four-sided dice is rolled. The p.m.f. of X is

$$f(x) = \frac{2x - 1}{16}, \qquad x = 1, 2, 3, 4.$$

(a) Find the mean, variance, and standard deviation of X.

(b) Calculate the sample mean, sample variance, and sample standard deviation of the following 100 simulated observations of X, and compare these answers with those found in part (a):

4 4 4 4 2 2 2 3 1 4 3 3 2 3 2 4 4 2 3 4

Exercise 2.4-1

2.4-1. An urn contains 7 red and 11 white balls. Draw one ball at random from the urn. Let $X = 1$ if a red ball is drawn, and let $X = 0$ if a white ball is drawn. Give the p.m.f., mean, and variance of X.

Exercise 2.4-3

2.4-3. On a six-question multiple-choice test there are five possible answers for each question, of which one is correct (C) and four are incorrect (I). If a student guesses randomly and independently, find the probability of

(a) Being correct only on questions 1 and 4 (i.e., scoring C, I, I, C, I, I).

(b) Being correct on two questions.

Exercise 2.4-5

2.4-5. In a lab experiment involving inorganic syntheses of molecular precursors to organometallic ceramics, the final step of a five-step reaction involves the formation of a metal–metal bond. The probability of such a bond forming is $p = 0.20$. Let X equal the number of successful reactions out of $n = 25$ such experiments.

(a) Find the probability that X is at most 4.

(b) Find the probability that X is at least 5.

(c) Find the probability that X is equal to 6.

(d) Give the mean, variance, and standard deviation of X.

Exercise 2.4-7(a)(b)(c)

2.4-7. Suppose that 2000 points are selected independently and at random from the unit square $\{(x, y) : 0 \le x < 1, 0 \le y < 1\}$. Let W equal the number of points that fall into $A = \{(x, y) : x^2 + y^2 < 1\}$.

(a) How is W distributed?

(b) Give the mean, variance, and standard deviation of W.

(c) What is the expected value of $W/500$?

Exercise 2.4-11

2.4-11. A random variable X has a binomial distribution with mean 6 and variance 3.6. Find $P(X = 4)$.

Exercise 2.4-15

2.4-15. It is claimed that for a particular lottery, 1/10 of the 50 million tickets will win a prize. What is the probability of winning at least one prize if you purchase (a) 10 tickets or (b) 15 tickets?

Exercise 2.4-19

2.4-19. Many products can operate if, out of n parts, k of them are working. Say $n = 20$ and $k = 17$, $p = 0.05$ is the probability that a part fails, and assume independence. What is the probability that the product operates?

Exercise 2.4-23

2.4-23. Your stockbroker is free to take your calls about 60% of the time; otherwise he is talking to another client or is out of the office. You call him at five random times during a given month. (Assume independence.)

(a) What is the probability that he will take every one of the five calls?

(b) What is the probability that he will accept exactly three of your five calls?

(c) What is the probability that he will accept at least one of the calls?

Exercise 2.5-5

2.5-5. For each question on a multiple-choice test, there are five possible answers, of which exactly one is correct. If a student selects answers at random, give the probability that the first question answered correctly is question 4.

Exercise 2.5-7

2.5-7. Apples are packaged automatically in 3-pound bags. Suppose that 4% of the time the bag of apples weighs less than 3 pounds. If you select bags randomly and weigh them in order to discover one underweight bag of apples, find the probability that the number of bags that must be selected is

(a) At least 20.

(b) At most 20.

(c) Exactly 20.

Exercise 2.5-9

2.5-9. An excellent free-throw shooter attempts several free throws until she misses.

(a) If $p = 0.9$ is her probability of making a free throw, what is the probability of having the first miss on the 13th attempt or later?

(b) If she continues shooting until she misses three, what is the probability that the third miss occurs on the 30th attempt?

Exercise 2.6-1

2.6-1. Let X have a Poisson distribution with a mean of 4. Find

(a) $P(2 \le X \le 5)$.

(b) $P(X \ge 3)$.

(c) $P(X \le 3)$.

Exercise 2.6-3

2.6-3. Customers arrive at a travel agency at a mean rate of 11 per hour. Assuming that the number of arrivals per hour has a Poisson distribution, give the probability that more than 10 customers arrive in a given hour.

Exercise 2.6-5

2.6-5. Flaws in a certain type of drapery material appear on the average of one in 150 square feet. If we assume a Poisson distribution, find the probability of at most one flaw appearing in 225 square feet.

Exercise 2.6-9

2.6-9. A store selling newspapers orders only $n = 4$ of a certain newspaper because the manager does not get many calls for that publication. If the number of requests per day follows a Poisson distribution with mean 3,

(a) What is the expected value of the number sold?

(b) How many should the manager order so that the chance of running out is less than 0.05?