Wasserstein Motifs: Optimal Transport for Food Web Alignment

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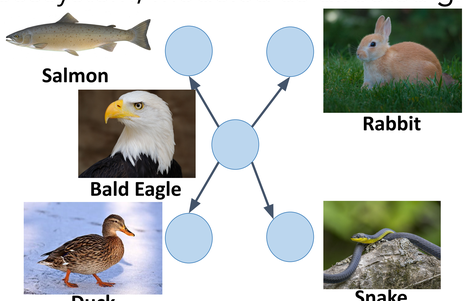
Why are Food Webs Important?

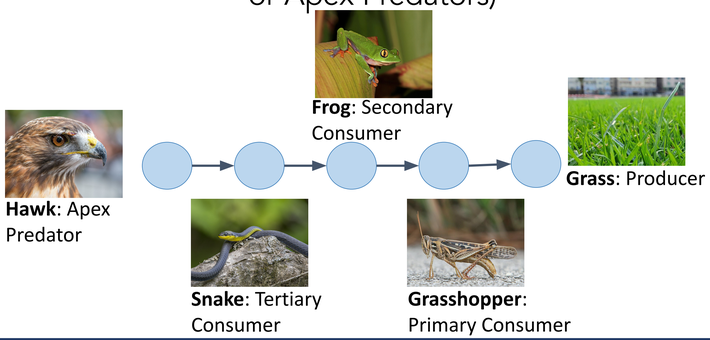
- Understanding Ecosystem Stability and Resilience
- Biodiversity and Conservation Planning
- Nutrient and Energy Flow Dynamics

What is a Food Web?

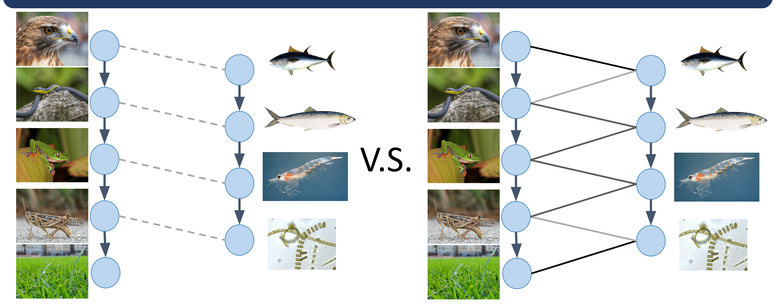
- Food webs are ecological networks that represent species interactions (e.g. predator→prey) within an ecosystem, modeled as directed graphs



- Species in food webs occupy distinct roles (Producers, Primary/Secondary/Tertiary consumers, or Apex Predators)



Food Web Alignment: Hard vs Soft



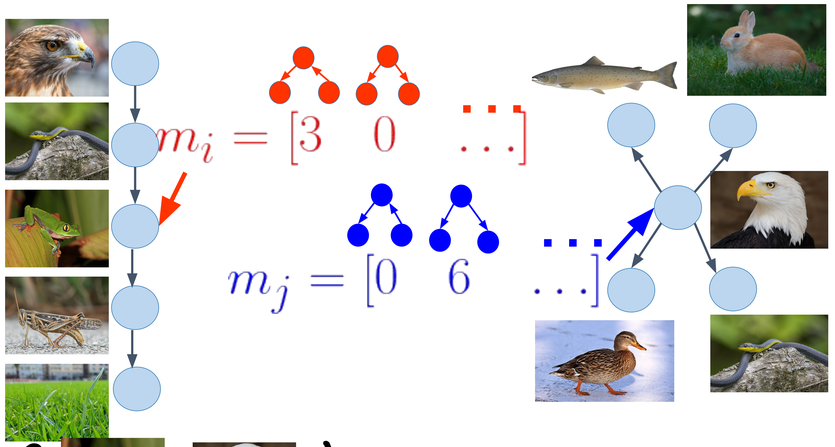
How do we quantify alignment quality?

Only if we could assign a "Cost" for each possible alignment…

$C($  $,$ $) = ?$

Introduction to Network Motifs

How do we characterize differences in network structure?



$C($  $,$  $) = d(m_i, m_j) = \|m_i - m_j\|_2$

Optimal Transport Formulations

Minimization over all possible alignments | Sum across all alignments made | **Difference in functional traits** between node i and node j | **Decision Variable** indicating alignment | Penalize non-alignments

$$\min_{T \in \mathbb{R}^{p \times q}} \sum_{i,j} d(m_i, m_j) T_{ij} - \epsilon \sum_{i,j} T_{ij}$$

Sum of **incident weights** must be consistent

$s.t.$

$$T\mathbf{1}_q \preceq \mathbf{1}_p \quad T^\top \mathbf{1}_p \preceq \mathbf{1}_q \quad T_{ij} \in \{0,1\} \quad \forall i,j$$

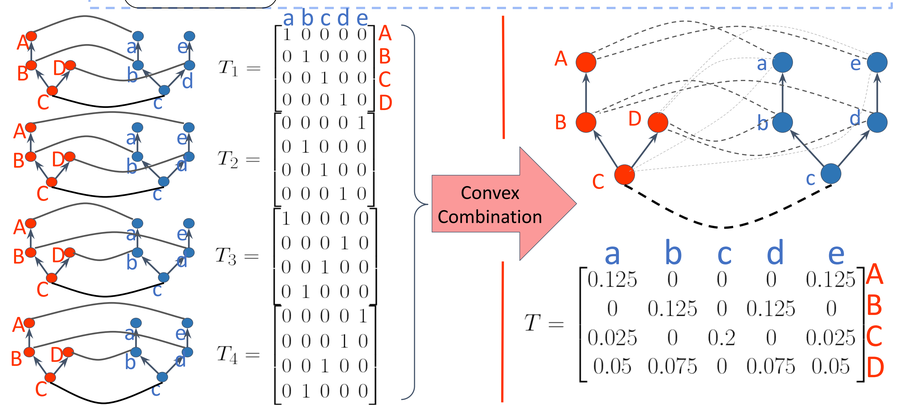
Each node is aligned to at most 1 node

Enforcing Binary(Hard) Alignments

OR

$$T\mathbf{1}_q \preceq \frac{1}{p}\mathbf{1}_p \quad T^\top \mathbf{1}_p \preceq \frac{1}{q}\mathbf{1}_q \quad T_{ij} \in [0,1] \quad \forall i,j$$

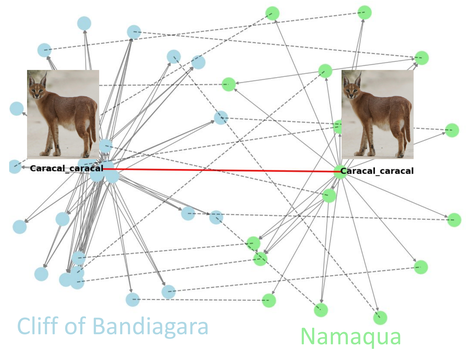
Enforcing Probabilistic (Soft) Alignments



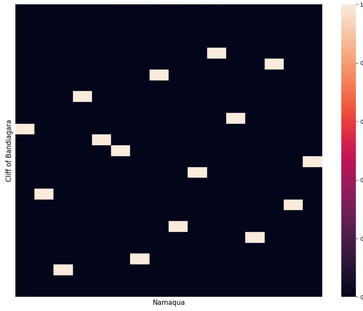
Convex Combination

$$T = \begin{bmatrix} 0.125 & 0 & 0 & 0 & 0.125 \\ 0 & 0.125 & 0 & 0.125 & 0 \\ 0.025 & 0 & 0.2 & 0 & 0.025 \\ 0.05 & 0.075 & 0 & 0.075 & 0.05 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

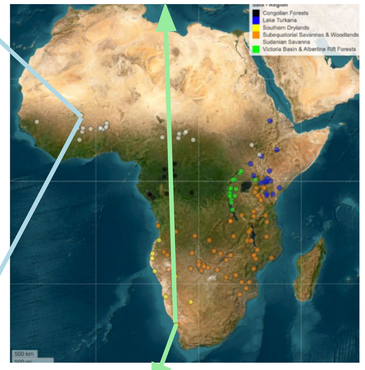
Results: Sub-Saharan African Food Webs



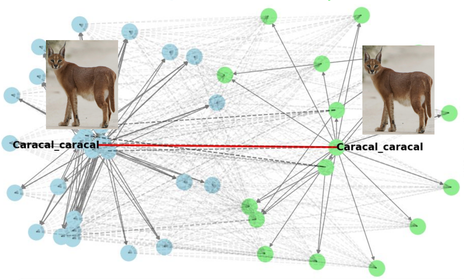
Cliff of Bandiagara

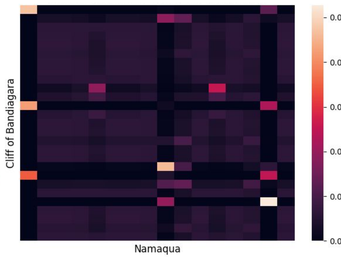


Namaqua



Cliff of Bandiagara | Namaqua





Future Works

- Incorporate **Sparsity Constraints** to the soft alignment problem to obtain sparser solutions
- Implement a **momentum-accelerated** optimization algorithm to achieve faster convergence for the soft alignment problem
- Explore **Alternative cost functions** which takes dissimilarity between **neighboring** nodes into consideration

References

1. Bramon Mora, B., Gravel, D., Gilarranz, L.J. *et al.* Identifying a common backbone of interactions underlying food webs from different ecosystems. *Nat Commun* 9, 2603 (2018). https://doi.org/10.1038/s41467-018-05056-0

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