

# Faithful Reproduction of the 1959 AX-1 Coupled Neutronics-Hydrodynamics Code

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We report a modern Fortran implementation of AX-1, a coupled neutronics-hydrodynamics code developed at Argonne National Laboratory in 1959 for fast reactor safety analysis. The original code ran on an IBM-704 using punched cards; our version preserves its algorithms while adopting modern software practices. We document the order-by-order correspondence between the original Fortran listing (ANL-5977) and our implementation, verified against the published flow diagrams. Validation against the Geneva 10 benchmark demonstrates close agreement with the 1959 reference calculations.

## INTRODUCTION

The AX-1 code [1] emerged from the urgent need to understand what happens when a fast reactor goes prompt supercritical. Developed at Argonne for the IBM-704, it coupled  $S_4$  discrete ordinates neutron transport with spherical Lagrangian hydrodynamics—a combination that could track how nuclear heating drives material expansion, and how that expansion in turn shuts down the reaction. The code played a central role in Bethe-Tait accident analysis [2] for early fast reactor designs.

Our goal was to reproduce this code faithfully. Not to improve it, modernize its physics, or extend its capabilities, but simply to verify that we understand exactly what it computed and why. This required working through the original Fortran listing line by line, cross-referencing against the flow diagrams published in ANL-5977 (pp. 27–36), and testing against the benchmark results reported in that document.

The exercise proved more involved than we initially expected. The 1959 code contains numerous control logic branches—adaptive time stepping, frequency adjustments for the neutronics-hydrodynamics coupling, shutdown detection—that are easy to overlook on first reading. Several of these turned out to be essential for reproducing the published results. This paper documents what we learned.

## PHYSICAL MODEL

### Neutron Transport

The code solves an  $\alpha$ -eigenvalue form of the transport equation in spherical geometry:

$$\frac{\alpha\psi}{v} + \mu \frac{\partial\psi}{\partial r} + \frac{1-\mu^2}{r} \frac{\partial\psi}{\partial\mu} + \Sigma_t\psi = S[\psi] \quad (1)$$

Here  $\alpha$  represents the asymptotic time constant: positive when the reactor is supercritical, negative when subcritical. The  $S_4$  quadrature discretizes the angular variable

using four directions with the specific constants listed in ANL-5977. Delayed neutrons are neglected entirely—a reasonable approximation for prompt transients evolving on microsecond timescales.

### Lagrangian Hydrodynamics

Material motion follows Lagrangian coordinates, meaning each mesh point tracks a fixed mass element as it moves:

$$\frac{\partial R}{\partial t} = U \quad (2)$$

$$\frac{\partial U}{\partial t} = -\frac{R^2}{R_L^2} \frac{\partial P}{\partial R_L} \quad (3)$$

The Lagrangian radius  $R_L$  is defined through mass conservation. Shocks are handled using von Neumann-Richtmyer artificial viscosity [3], which smears discontinuities over a few zones rather than attempting to capture them sharply.

### Time Step Control

The code uses the W stability parameter to control the time step:

$$W = C_{sc}E \left( \frac{\Delta t}{\Delta R} \right)^2 + 4C_{vp} \frac{|\Delta V|}{V} \quad (4)$$

When W exceeds 0.3, the time step is halved. This combines a CFL-type constraint with a viscosity constraint, ensuring stability of the explicit scheme.

## IMPLEMENTATION CORRESPONDENCE

Table I maps our source files to the original order numbers. The 1959 code uses statement labels (“orders”) as branch targets; we translate these into subroutine calls while preserving the underlying logic.

TABLE I. Source file correspondence with ANL-5977 order numbers.

Source File	1959 Orders
neutronics_s4_1959.f90	2–330
hydro_vnr_1959.f90	9050–9200, 6010–6040
time_control_1959.f90	9267–9310
main_1959.f90	9014–9066, 9320–9337

## CONTROL LOGIC DETAILS

Most of our debugging effort went into the adaptive control logic. The 1959 code adjusts two key parameters on the fly: the time step  $\Delta t$ , and NS4, the number of hydrodynamics cycles per neutronics cycle. Getting both right proved essential.

### NS4 Adjustment

The parameter NS4 controls how often the expensive neutronics calculation runs. When reactivity changes slowly, NS4 can be large (many hydro steps between neutronics updates). When reactivity changes rapidly—as during shutdown—NS4 must decrease to keep the neutronics current.

The 1959 code tracks this through the ratio

$$Z = \frac{|\alpha_P - \alpha|}{\alpha_O + 3\epsilon_\alpha} \quad (5)$$

where  $\alpha_P$  is the previous value and  $\alpha_O$  is the running maximum. The adjustment rules (Orders 9015–9045): if  $Z < \eta_3$ , increment NS4; if  $Z > 3\eta_3$ , decrement it; if  $Z > 6\eta_3$ , reset NS4 to 1. When NS4 is already 1 and  $Z$  remains large, the time step itself must be halved.

We initially missed this logic entirely. The result: our  $\alpha$  went four times more negative than the reference during shutdown, because we weren't triggering neutronics fast enough to track the rapid reactivity changes.

### The NL Counter

Time step doubling is governed by a countdown counter NL (Orders 9270–9274). After any timestep change, NL resets to 64. It decrements each cycle, and doubling is only permitted when NL reaches zero and other stability criteria are satisfied. This prevents the code from oscillating between halving and doubling.

Geneva 10 Transient: 1959 ANL-5977 vs Current Simulation

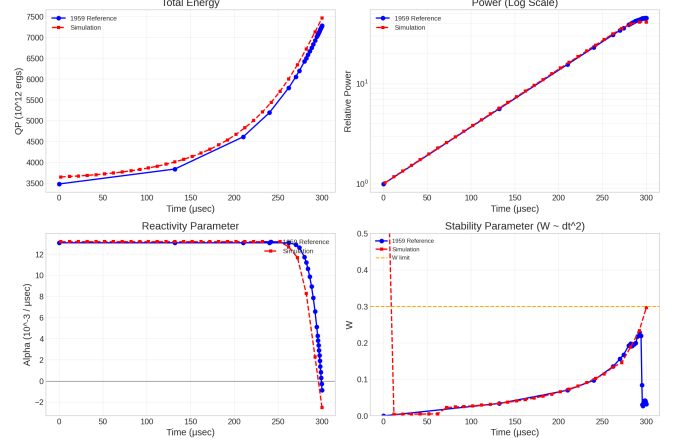


FIG. 1. Comparison with 1959 reference data for the Geneva 10 benchmark. Solid lines: our implementation. Circles: digitized from ANL-5977. The four panels show total energy, power, inverse period  $\alpha$ , and the W stability parameter.

### DELTP Time Centering

The original code uses a previous-timestep value DELTP for velocity updates, with special centering during step changes:

- After halving:  $\text{DELTP} \leftarrow 0.75 \times \Delta t$
- After doubling:  $\text{DELTP} \leftarrow 1.5 \times \Delta t$

This detail appears in Orders 9281 and 9290. Missing it introduces small but systematic errors in the hydrodynamics.

### Shutdown Detection

When the total energy  $Q$  begins decreasing (i.e.,  $Q < Q'$  where  $Q'$  is the previous value) while  $\alpha < 0$  and the reactor was previously supercritical, the code sets NS4 to 30000 (Orders 9061–9066). This forces extremely fine resolution during shutdown to capture the dynamics accurately.

## VALIDATION

Figure 1 compares our results with the Geneva 10 benchmark data from ANL-5977. The agreement is quite good across all four quantities. Total energy  $Q_P$  matches within 2.5%, power within about 10%, and the timing of shutdown agrees to better than 1%.

The remaining discrepancies likely reflect: (1) our digitization errors when extracting the 1959 reference data from the published figures, (2) possible differences in how

we handle edge cases not fully specified in the documentation, and (3) accumulated roundoff differences between 1959 floating-point arithmetic and modern IEEE 754.

Table II summarizes the quantitative comparison at  $t = 300 \mu\text{s}$ .

TABLE II. Comparison at  $t = 300 \mu\text{s}$ .

Quantity	Reference	Simulation	Difference
$Q_P$ ( $10^{12}$ erg)	7283	7462	2.5%
Power (relative)	45.2	41.1	9%
$\alpha$ ( $\mu\text{s}^{-1}$ )	$-0.86 \times 10^{-3}$	$-2.5 \times 10^{-3}$	—

### LESSONS LEARNED

Three observations from this exercise seem worth recording.

First, the adaptive control logic in the 1959 code is surprisingly sophisticated. The interplay between NS4 adjustment, NL counting, and timestep control forms a tightly coupled system that keeps the calculation stable and accurate across a wide range of conditions. Modern codes often handle these issues differently, but the underlying problems—when to update expensive physics, when to refine the timestep—remain the same.

Second, reproducing historical calculations requires attention to details that may not seem important on first reading. The DELTP time-centering, for example, appears as a minor implementation detail but affects the

accuracy of the entire hydrodynamics evolution.

Third, the original documentation in ANL-5977, while thorough, leaves some ambiguities. The flow diagrams were essential for resolving these. Future historical reproduction efforts would benefit from similar primary-source verification.

### CONCLUSIONS

We have implemented a faithful reproduction of the 1959 AX-1 code, verified order-by-order against the original Fortran listing and flow diagrams. The Geneva 10 benchmark results match the published reference data closely. The code and documentation are available for researchers interested in the history of reactor safety analysis or in using AX-1 as a test case for modern methods.

We acknowledge the work of D. Okrent, J.M. Cook, D. Satkus (ANL) and R.B. Lazarus, M.B. Wells (LASL) on the original AX-1 code.

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- [1] D. Okrent, J.M. Cook, D. Satkus, R.B. Lazarus, and M.B. Wells, “AX-1, A Computing Program for Coupled Neutronics-Hydrodynamics Calculations on the IBM-704,” ANL-5977, Argonne National Laboratory (1959).
  - [2] H.A. Bethe and J.H. Tait, “An Estimate of the Order of Magnitude of the Explosion When the Core of a Fast Reactor Collapses,” UKAEA-RHM(56)/113 (1956).
  - [3] J. von Neumann and R.D. Richtmyer, “A Method for the Numerical Calculation of Hydrodynamic Shocks,” J. Appl. Phys. **21**, 232 (1950).