The

JAPANESE ABACUS

Its Use and Theory

BY

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FOREWORD

It gives me great pleasure that this book in English on the abacus is ready for publication. Japanese abacus operators have long cherished the desire, here finally realised, of introducing the Japanese abacus to other countries in view of the remarkable advances and developments which have been made in the instrument and its use during the past quarter of a century.

The Japanese abacus, simple and primitive looking though it be, can be operated with greater speed and efficiency than even the electric calculating machine -a fact proven in numerous tests and well documented by Mr. Kojima in his first chapter. This is particulary so in addition and subtraction, where the abacus can handle figures of any number of digits twice as fast as the electric machine. To explain the instrument's incredible speed and mystifying efficiency it is essential not only to introduce the newest improved methods of operation but also to elucidate the most advanced theories of rational bases and of bead manipulation.

In writing this practical book Mr. Kojima has kept these two requirements well in mind. The Abacus Research Institute of the Japan Chamber of Commerce and Industry has been most pleased to cooperate with him by making available its research data and correcting his manuscript in the light of all the latest information.

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Professor of Economics, Nippon University
Vice-President, All-Japan Federation of Abacus Operators

AUTHOR'S FOREWORD

This book has been written as a guide for those who, though interested in knowing more about the use and theory of the Japanese abacus, have until now been unable to find any full explanation in the English language. Chapter I presents the most important facts about the speed and efficiency of abacus calculation, with special reference to a comparison of the abacus and the electric calculating machine. Chapter II gives a brief survey of the history and development of the abacus, and Chapter III introduces the basic principles of abacus calculation.

The next three chapters explain in detail, with numerous examples, how the four processes of arithmetic are worked out on the abacus. Particular attention should be given to Chapter IV, on addition and subtraction, as it embodies the essential rules of bead manipulation. Many notes have been included to give a theoretical and scientific explanation of the rules and fundamental principles as such knowledge is not only of interest but will prove of great aid in the actual operation of the instrument. The book concludes with short chapters on decimals and mental calculation, and a selection of exercises.

Among many who kindly gave me information and suggestions, I am particularly grateful to Mr. Yoemon Yamazaki, who kindly wrote the foreword and provided me with many valuable suggestions and a large part of the information in Chapter I. He is the Vice-Chairman of the Abacus Research Institute and Advisor to the Central Committee of the Federation of Abacus Workers (hereafter referred to simply as the Abacus Committee), both organizations being under the sponsorship of the Japan Chamber of Commerce and Industry.

I also wish to express my special gratitude to Mr. Shinji Ishikawa, President of the Japan Association of Abacus Calculation, who spared himself no trouble in reading the whole of the manuscript and furnishing much important up-to-date information.

I also extend my grateful acknowledgements to Mr. Zenji Arai, Chairman of the Abacus Research Committee of the Japan Federation of Abacus Education, and Mr. Miyokichi Ban, of the above-mentioned Abacus Committee. They kindly read the whole of the manuscript and provided me with many necessary and valuable suggestions.

My grateful acknowledgements are also due to Mr. Takeo Uno on the Abacus Committee and Mr. Tadao Yamamoto, who conducts his own abacus school. They kindly read the manuscript in parts and gave me valuable suggestions.

I also wish to thank Mr. Kiyoshi Matsuzaki, of the Savings Bureau of the Ministry of Postal Administration, who kindly furnished the table on page 5.

Finally I must express my sincere thanks for many invaluable suggestions on English style from Mr. C. G. Wells, Chief Writer for the Far East Network; Mr. Harold Gosling, of the British Commonwealth Public Relations; Mr. Richard D. Lane, formerly of the Far East Network; and above all from Mr. Meredith Weatherby, of the Charles E. Tuttle Company, without whose painstaking efforts this book could not have become what it is now.

T. K.

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I - ABACUS VERSUS ELECTRIC CALCULATOR

The abacus, or *soroban* as it is called in Japan, is one of the first objects that strongly attract the attention of the foreigner in Japan. When he buys a few trifling articles at some store, he soon notices that the tradesman does not perplex himself with mental arithmetic, but instead seizes his *soroban,* prepares it by a tilt and a rattling sweep of his hand, and after a deft manipulation of rapid clicks, reads off the price. It is true that the Japanese tradesman often uses his board and beads even when the problem is simple enough to be done in one's head, but this is only because the use of the abacus has become a habit with him. If he tried, he could no doubt easily add 37 and 48 in his head. But such is the force of habit that he does not try to recognize the simplicity of any problem; instead, following the line of least resistance, he adjusts his *soroban* for manipulation, and begins clicking the beads, thus escaping any need of mental effort.

Doubtlessly the Westerner, with his belief in the powers of mental arithmetic and the modern calculating machine, often mistrusts the efficiency of such a primitive looking instrument. However, his mistrust of the *soroban* is likely to be transformed into admiration when he gains some knowledge concerning it. For the *soroban,* which can perform in a fraction of time a difficult arithmetic calculation that the Westerner could do laboriously only by means of pencil and paper, possesses distinct advantages over mental and written arithmetic. in a competition in arithmetic problems, an ordinary Japanese tradesman with his *soroban* would easily outstrip a rapid and accurate Western accountant even with his adding machine.

An exciting contest between the Japanese abacus and the electric calculating machine was held in Tokyo on November 12, 1946, under the sponsorship of the U. S. Army newspaper, the *Stars and Stripes.* In reporting the contest, the *Stars and Stripes* remarked: “The machine age took a step backward yesterday at the Ernie Pyle Theater as the abacus, centuries old, dealt defeat to the most up-to-date electric machine now being used by the United States Government. The abacus victory was decisive.”

The *Nippon Times* reported the contest as follows: “Civilization, on the threshold of the atomic age, tottered Monday afternoon as the 2,000-year-old abacus beat the electric calculating machine in adding, subtracting, dividing and a problem including all three with multiplication thrown in, according to UP. Only in multiplication alone did the machine triumph...”

The American representative of the calculating machine was Pvt. Thomas Nathan Wood of the 240th Finance Disbursing Section of General MacArthur's headquarters, who had been selected in an arithmetic contest as the most expert operator of the electric calculator in Japan. The Japanese representative was Mr. Kiyoshi Matsuzaki, a champion operator of the abacus in the Savings Bureau of the Ministry of Postal Administration.

As may be seen from the results tabulated on the following page, the abacus scored a total of 4 points as against 1 point for the electric calculator. Such results should convince even the most skeptical that, at least so far as addition and subtraction are concerned, the abacus possesses an indisputable advantage over the calculating machine. Its advantages in the fields of multiplication and division, however, were not so decisively demonstrated:

RESULTS OF CONTEST

MATSUZAKI (Abacus) vs. WOOD (Electric Calculator).

Type of Problem	Name	1st Heat	2nd Heat	3rd Heat	Score
Addition: 50 number each consisting of from 3 to 6 digits	Matsuzaki	1m 14.8s Victor	1m 16s Victor	-	1
	Wood	2m 0.2s Defeated	1m 53s Defeated	-	0
Subtraction: 5 problems, with minuends and subtrahends of from 6 to 8 digits each	Matsuzaki	1m 0.4s All correct. Victor	1m 0.8s 4 correct. No decision	1m 0s All correct. Victor	1
	Wood	1m 30s All correct. Defeated	1m 36s 4 correct. No decision	1m 22s 4 correct. Defeated	0
Multiplication: problems, each containing 5to 12 digits in multiplier and multiplicand	Matsuzaki	1m 44.6s 4 correct. Defeated	1m 19s All correct. Victor	2m 14.4s 3 correct. Defeated	0
	Wood	2m 22s 4 correct. Victor	1m 20s All correct. Defeated	1m 53.6s 4 correct. Victor	1
Division: 5 problems, each containing 5 to 12 digits in divisor and dividend	Matsuzaki	1m 36.6s All correct. Victor	1m 23.4s 4 correct. Defeated	1m 21s All correct. Victor	1
	Wood	1m 48s All correct. Defeated	1m 19s All correct. Victor	1m 26.6s 4 correct. Defeated	0
Composite problem: 1 problem in addition of 30 6-digit numbers; 3 problems in subtraction, each with two 6-digit numbers; 3 problems in multiplication, each with two figures containing a total of 5 to 12 digits; 3 problems in division, each with two figures containing a total of 5 to 12 digits	Matsuzaki	1m 21s All correct. Victor	-	-	1
	Wood	1m 26.6s 4 correct. Defeated	-	-	0
Total Score	Matsuzaki	-	-	-	4
	Wood	-	-	-	1

For reliable information on the comparative merits of the abacus and the calculating machine, we can do nothing better then turn to the Abacus Committee of the Japan Chamber of Commerce and Industry, which has made minute investigations concerning the potentialities of the Japanese abacus. The Committee has acted as judge of the semi-annual examination for abacus operators' licenses since the examinations were initiated in 1931, such licenses being divided into three classes, according to the manipulators' efficiency.

The Committee says: "In a contest in addition and subtraction, a first-grade abacus operator can easily defeat the best operator of an electric machine, solving problems twice as fast as the latter, no matter how many digits the numbers contain. If the numbers do not contain over six digits, the abacus manipulator can halve the time of the operation by relying in part upon mental calculation (a system peculiar to the abacus, to be described hereafter). In multiplication and division the first-grade abacus operator can maintain some margin of advantage over the electric calculator so long as the problem does not contain more than a total of about ten digits in multiplicand and multiplier or in divisor and quotient. The abacus and the electric machine are on a par in a problem which contains a total of ten to twelve digits. With each additional digit in a problem, the advantage of the electric calculating machine increases."

A similar view is held by Mr. Kiyoshi Matsuzaki, who made the following remark concerning the contest described in preceding pages: "In addition and subtraction even the third-grade abacus worker can hold his own against the electric calculating machine. In multiplication and division the first-grade abacus worker may have a good chance to win over the calculating machine, provided the problem does not have more than a total of ten digits in multiplicand and multiplier or in divisor and quotient. I felt nervous at the contest and made more mistakes than I might have done otherwise. My opponent may have felt the same, though. A good first-grade abacus worker ought to be able to make a better showing when he is at ease."

As examples of the proficiency required of the abacus operator, it will be of interest to cite a few problems used in the examination for abacus operators' licences.

A. ADDITION AND SUBTRACTION

No.	1	2	3	4	5
1	¥ 6 393 082.74	¥40 693 718.52	¥ 160 384.72	¥ 730.49	¥ 352 719.48
2	269.31	52 687.09	83 479 051.26	6 089 547.31	84 936.20
3	541 793.60	7 180 592.43	-21 479.50	463 195.28	92 460 385.71
4	82 706 314.95	1 715.38	9 058 627.13	97 820.56	-718 024.36
5	72 940.18	63 847 529.10	-3 780.29	3 985 271.04	45 178.62
6	3 014 725.86	26 073.94	27 915.64	10 476 825.93	8 327 605.94
7	98 156 .02	309 861.75	40 715 368.92	54 613.78	-19 062.53
8	15 726 408.39	8 714 905.26	86 203.41	218 769.45	-4 085 237.61
9	970 285.13	346.17	-504 189.76	3 428.01	25 963 180.47
10	45 963.78	295 130.86	-6 037 512.89	82 605 917.34	70 941.28
11	6 831 750.24	94 038 726.51	924.35	61 853.20	-6 798.05
12	64 371.59	69 052.74	763 815.04	250 376.19	-50 824 361.79
13	249 168.07	150 938.42	-20 849 136.57	3 576 904.82	953.16
14	70 593 826.41	43 281.65	4 102 653.98	49 021.67	3 107 425.89
15	4 352.80	7 916 403.28	95 467.83	57 316 482.90	639 507.14
計					

B. MULTIPLICATION

No.	
1	759.843 x 57.941 =
2	302.162 x 83.602 =
3	967.408 x 70.589 =
4	20.359 x 628.134 =
5	84.2697 x 9.4076 =
6	135.941 x 46.295 =
7	0.4271805 x 0.2513 =
8	669.378 x 0.31908 =
9	0.914053 x 68.087 =
10	587.216 x 17.452 =

C. DIVISION

No.	
1	4 768 788 098 / 14 593 =
2	971 837 849 / 51.682 =
3	47 408 509 168 / 49.201 =
5	0.3481095257 / 0.06457 =
5	66 014 150 202 / 92 378 =
6	3 657.6146092 / 80 914 =
7	166.4719833 / 0.6702 =
8	0. 4537275087 / 7.38609 =
9	328 399.09042 / 35.746 =
10	24 484 596 290 / 28.135 =

D. MENTAL CALCULATION

No.	1	2	3	4	5
1	¥ 74.68	¥ 3.46	¥ 52.31	¥ 8.09	¥ 90.47
2	2.98	97.98	30.64	2.41	3.51
3	50.41	6.05	—9.28	56.37	—76.29
4	83.72	2.13	14.75	1.52	—1.83
5	1.35	50.79	8.39	70.86	54.02
6	6.84	8.21	—45.05	3.94	8.35
7	95.01	19.64	6.17	96.70	29.14
8	3.27	78.30	—2.83	6.28	—6.50
9	65.10	4.56	7.14	37.19	47.26
10	4.92	82.07	91.26	48.05	1.83
計					

To receive a first-grade license an applicant must be able to work problems similar to the foregoing with 80 per cent accuracy within a time limit of five minutes each for the first three groups—A, B and C—and one minute for the fourth.

The problem in mental calculation requires a few words of explanation as the method of solving it depends directly upon a knowledge of the use of the abacus, and being an integral part of abacus technique, it is entirely different from any Western method of calculation. This abacus method of mental arithmetic is described in some detail in Chapter VIII. Suffice is to say here that the method consists in mentally visualizing an abacus and working the problem out by standard methods on the imaginary instrument. The process is easier than it sounds and accounts for the incredible and almost mystifying peaks of efficiency attained by masters of abacus operation.

To cite but one example of proficiency in this type of mental arithmetic, on May 28, 1952, during the Sixth All-Japan Abacus Contest, held in Tokyo, a master abacus operator, Mr. Yoshio Kojima, gave a demonstration of his skill in mental arithmetic.

In one minute and 18.4 seconds he gave correct answer to 50 division problems, each of which contained five to seven digits in its dividend and divisor. Next, in a twinkling of 13.6 seconds he added 10 numbers of ten digits each. Thus he set two remarkable records -and all with no aid other than the mentally visualized abacus! This means that he could have added mentally the fifteen numbers given in one of the columns of Problem A, page 6, in one-fourth of the given time limit of one minute, and in one-eighth of the time required by the best operator of an electric calculating machine.

A. ADDITION AND SUBTRACTION

No.	1	2	3	4	5
1	¥ 71 896	¥ 93 502	¥ 130 745	¥ 60 374	¥ 9 180
2	306 425	8 164	59 280	875 126	25 634
3	839	802 635	4 968	23 601	418 275
4	50 178	378	102	7 284	54 361
5	2 941	25 910	701 539	932	7 903
6	567 308	-8 756	48 075	-506 849	86 215
7	762	650 481	6 714	-39 256	903 587
8	82 037	362	429	1 023	643
9	3 694	71 049	602 893	485	71 852
10	470 589	2 913	27 564	187 683	408
11	24 310	134 795	4 931	-96 178	640 729
12	165	-19 247	93 270	-4 517	210
13	5 742	-804	315 687	-760	92 674
14	904 213	-65 798	90 312	248 951	5 096
15	68 951	746 083	856	19 074	837 921
計					

B. MULTIPLICATION

No.	
1	6 742 x 358 =
2	2 681 x 609 =
3	5 093 x 176 =
4	0.825 x 94.12 =
5	3 310 x 803 =
6	9 478 x 0.645 =
7	76 506 x 5.2 =
8	193.4 x 4.18 =
9	0.4052 x 0.267 =
10	9 718 x 703 =

C. DIVISION

No.	
1	435 633 / 921 =
2	315 56 / 805 =
3	18.998 / 236 =
4	63.162 / 0.087 =
5	223.792 / 394 =
6	400.026 / 418 =
7	180 096 / 64 =
8	0.105118 / 0.753 =
9	104 249 / 1.709 =
10	0.21918 / 5.62 =

To receive a third-grade license an applicant must be able to work problems similar to those on page 8 with 70 per cent accuracy within a time limit of five minutes for each group.

The primary advantage of the abacus is its incredible speed resulting from the mechanization or simplification of calculation, by means of which the answer to a given problem forms itself naturally or mechanically on the board, thus reducing mental labor to a minimum. The theoretical explanation of this mechanization of calculation is given in Chapter IV (see Note 3 to Example 9, Note 3 to Example 10, and Notes 3, 4 and 5 to Example 20).

Another big advantage of the abacus is its extremely moderate price, ranging generally between 25¢ and $2.50 or $3.50 to quote prices in U.S. dollar equivalents. How many times more does the ordinary calculating machine cost, to say nothing of the gleaming electric machines which abound in Western business houses?

Among many other merits of the abacus one should not overlook its handy construction, its portability, and the ease of its operational methods, which are nothing more than simplifications of the four processes of arithmetic.

The most peculiar advantage of the abacus is that a problem in addition and subtraction is worked out from left to right instead of from right to left as is the case with written arithmetic, and thus harmonizes perfectly with the normal way of reading and writing numbers. In this way a number can be added or subtracted while it is being given. For example, if the first number in the problem is 753, the operator can enter 7 on the abacus the instant he hears or sees "seven hundred," and then proceed on next to the 5 and finally the 3, whereas in written calculation he would usually have to wait until all figures were given and then start calculating backward from the 3 of 753.

The one admitted disadvantage of the abacus is that the instrument produces only a final result without preserving any record of intermediate steps. If any error is made, the whole calculation must be carried through again from beginning to end. But this seeming disadvantage is more than offset by the rapidity and accuracy which the abacus makes possible. And it is the result that counts.

The chief factor which discredits the abacus in Western eyes is the length of time and practice required to become a skilled operator. Certainly the abacus requires much more practice than the calculating machine. But this apparent disadvantage is not so great an obstacle as it is generally thought to be. Some experience and practice with this simple but highly scientific instrument will convince the reader that this Western idea is largely a prejudice. A few weeks of practice for an hour each day with proper procedures will give anyone sufficient skill to turn to the abacus instead of pencil and paper for arithmetical computation.

According to the Abacus Committee, average students, who begin their practice while in their teens, should be able to pass the examination for third-grade licenses after half a year of daily practice of one hour, and bright students or students with a mathematical bent after only three months. Generally speaking, another half year of practice will enable a third-grade abacus operator to obtain a second-grade license; and one more full year should make him a first-grade operator. As is generally the case with any other art or accomplishment, it is best to start practicing under right guidance when young. Those who take up their study of the abacus after they are out of their teens are never able to pass the first-grade examination, but it is definitely possible for them to attain to the third rank, and occasionally even to the second.

In recent years the abacus has enjoyed an amazing increase in popularity in Japan. For example, in the last seven years since the war's end, the successful examinees for the first-grade license have totaled approximately 5 700, those for the secondgrade 28 000, and those for the third-grade 217 000, making a total of 250 700. This figure for the last seven years is almost five times that of seventeen years immediately preceding the war.

The abacus has even found its way into the curriculum of all grade schools as one of the elements of arithmetic, and there are now numerous abacus schools to meet the needs of those preparing to go into business. In short, the abacus has become such a popular favorite that it is to be found in practically every household.

How are we to account for the sudden spurt in the popularity of the old-fashioned abacus, here in the middle of the mechanized twentieth century? Undoubtedly the principal explanation lies in the fact that its operational methods have recently been markedly simplified and improved. As will be explained in Chapter VI, the old method of division required the memorization of a difficult division table, and was the chief factor which alienated the average, non-commercial Japanese from the abacus. Once this difficulty was overcome by the introduction of the newer method of division -so much simpler and, in a sense, so much more accurate that it marked a milestone in the improvement of abacus technique- the instrument rapidly attained the universal popularity which it now enjoys.

But how account for the almost exclusive use of the abacus in offices and firms which could well afford electric calculating machines? Let statistics give the answer. According to figures compiled by the Abacus Committee, in the conduct of an average business the four types of arithmetical calculation occur in about the following proportions: addition seventy percent, subtraction five percent, multiplication twenty percent, and division five percent. As previously mentioned, the abacus can add and subtract faster than the electric calculating machine. As for problems in multiplication and division, those which contain more than a total of ten digits in their multiplicand and multiplier or in their divisor and quotient are exceptional. This means that a good operator can work out most multiplication and division problems as fast as or even faster on an abacus than on an electric calculation machine, to say nothing of the much slower non-electric machine. Hence, it is not surprising that the abacus is used almost exclusively in all Japanese commercial establishments, from the tiny store to the giant corporation. To give exact figures, in Japanese business ninety-two percent of all calculation is done on the abacus, five percent on calculating machines (mostly of the manually operated variety), and the remaining three percent by means of calculating tables, slide rules or written and mental arithmetic.

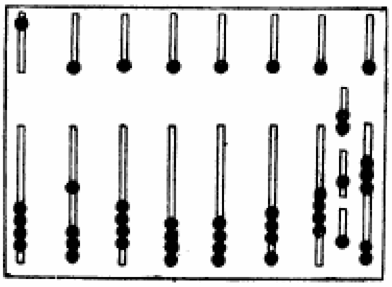
At the present time various experiments are being undertaken to improve still further the operational technique of the abacus. But this little handbook will introduce only the best of the established methods and verified theories, essential for learning to operate the abacus with good understanding and rapidity. Once the basic rules have been mastered, the secret of acquiring skill in abacus operation lies in constant daily practice.

Problems involving the extraction of roots can also be solved on the abacus with great rapidity. But the extraction of roots, which is rarely used in everyday and business calculation, is outside the scope of this book.

In concluding our comparison of the abacus and the calculating machine, we shall not go so far as to make the rash assertion that the abacus is worthy of immediate adoption by Western countries, where calculating machines are readily available. But we do feel justified in saying that the *soroban* is at least worthy of study and consideration by Westerners and that by introducing to the West this unique and practical example of Eastern science we will be repaying a modicum of our heavy debt of gratitude for the great amount of Western civilization which we have adopted here in Japan.

II – BRIEF HISTORY OF THE ABACUS

The imperfect numerical notation and the scarcity of suitable writing materials in ancient times are presumed to have given rise to the need for devices of mechanical calculation. While the definite origin of the abacus is obscure, there is some reason for believing that its earliest form was a reckoning table covered with sand or fine dust, in which figures were drawn with a stylus, to be erased with the finger when necessary. The English word *abacus* is etymologically derived from the Greek *abax,* meaning a reckoning table covered with dust, which in turn comes from a Semitic word meaning dust or a reckoning table covered with dust or sand. In time this sanddust abacus gave place to a ruled table upon which counters or disks were arranged on lines to indicate numbers. Various forms of this line abacus were in common use in Europe until the opening of the seventeenth century. In rather remote times, a third form of abacus appeared in certain parts of the world. Instead of lines on which loose counters were laid, the table had movable counters sliding up and down grooves.



ROMAN GROOVED ABACUS

All three types of abacuses were found at some time or other in ancient Rome —the dust abacus, the line abacus, and the grooved abacus. Out of this last type yet a fourth form of the abacus was developed—one with beads sliding on rods fixed in a frame. This form, the bead or rod abacus, with which calculations can be made much more quickly than on paper, is still used in China, Japan, and other parts of the world. In Europe, after the introduction of Arabic numerals, instrumental arithmetic ceased to make much progress and finally gave way altogether to the graphical as the supply of writing materials became gradually abundant.

As for the Orient, a form of the counting-rod abacus, called *ch'eou* in China and *sangi* in Japan, had been used since ancient times as a means of calculation. The Chinese abacus itself seems, according to the best evidence, to have originated in Central or Western Asia. There is a sixth-century Chinese reference to an abacus on which counters were rolled in grooves. The description of this ancient Chinese abacus and the known intercourse between East and West give us good reason to believe that the Chinese abacus was suggested by the Roman. The Chinese write in vertical columns from above downwards. If they ever are compelled to write in a horizontal line, they write from right to left. But the abacus is worked from left to right. This is another indication that the abacus was not indigenous to China. The present Chinese bead abacus, which is generally called *suan-pan* (arithmetic board) in Mandarin and *soo-pan* in the southern dialect, was a later development, probably appearing in the twelfth century, and did not come into common use till the fourteenth century. It is only natural that the people of the Orient, having retained a system of numerical notation unsuited for calculation, should have developed the abacus to a high degree, and its continuous universal use even after the introduction of Arabic numerals is eloquent testimony to the great efficiency achieved in its development.

The Japanese word for abacus, *soroban,* is probably the Japanese rendering of the Chinese *suan-pan.* Although the *soroban* did not come into popular use in Japan until the seventeenth century, there is no doubt that it must have been known to Japanese merchants at least a couple of centuries earlier. In any case, once this convenient instrument of calculation became widely known in Japan, it was studied extensively and intensively by many mathematicians, including Seki Kowa (1640 — 1708), who discovered a native calculus independent of the Newtonian theory. As a result of all this study, the form and operational methods of the abacus have undergone one improvement after another. Like the present-day Chinese *suan-pan,* the *soroban* long had two beads above the beam and five below. But toward the close of the nineteenth century it was simplified by reducing the two beads above the beam to one, and finally around 1920 it acquired its present shape by omitting yet another bead, reducing those below the beam from five to four.

Thus the present form of the *soroban* is a crystallization of labor and ingenuity in the field of Oriental mathematics and science. We feel sure that the *soroban,* enjoying widespread use in this mechanical age on account of its distinct advantages over the lightning calculating machine, will continue to be used in the coming atomic age as well.

III – BASIC PRINCIPLES OF CALCULATION

The abacus is a simple instrument for performing rapid arithmetical calculation. It consists of an oblong wooden frame or board holding a number of vertically arranged rods, on which wooden beads, balls, or counters slide up and down. A beam running across the board divides the rods into two sections: upper and lower. The most common type of abacus in Japan has twenty-one bamboo rods, and is about twelve inches long by two inches wide. But larger types with twenty-seven or thirty-one rods, and smaller ones with seventeen or thirteen rods, are also used. As described at the conclusion of the preceding chapter, and as may be seen in the accompanying illustration, the number of beads per rod has been progressively reduced, in the interests of simplicity and ease of operation, from seven to six, and finally to five. Until recently an abacus with five beads in the lower section of each rod was in general use. But this type of abacus has now been largely replaced by a one with four beads on each rod below the beam.

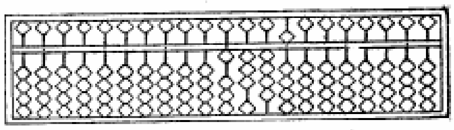


FIG. 1. MODERN JAPANESE ABACUS

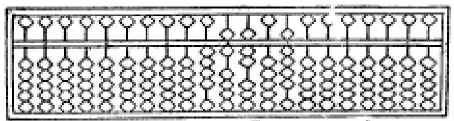


FIG. 2. OLDER-TYPE JAPANESE ABACUS

The abacus is based on the decimal system. For convenience in calculation the beam is marked with a unit point at every third rod. These unit points serve to indicate the decimal point and other units of decimal measure. For example, select any rod near the center of the board which is marked with a unit point, and call this the unit rod of the problem. Then the first rod to its left is the tens' rod, the second is the hundreds' rod, the third rod (marked with another unit point) is the thousands' rod, etc. On the other hand, the first rod to the right of the unit rod is the tenths', the second is the hundredths', the third (likewise marked with another point) is the thousandths' rod, etc.

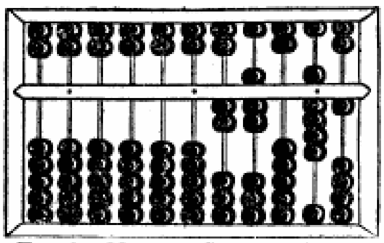


FIG. 3. MODERN CHINESE ABACUS

Each of the four beads on the lower section of a rod has the value of 1, while the bead on the upper section of a rod has the value of 5. Each of the 1-unit beads below the beam obtains its value when it is moved up toward the beam, and loses its value when it is moved back down to its former position. On the other hand, each of the 5-unit beads above the beam obtains its value when it is moved down to the beam and loses its value when it is moved back up.

The beads in Figure 1, using the third unit point from the right to designate the unit rod, represent the number 1 345, while Figure 2 shows 46 709.

Before using the abacus, make sure that all the beads are in the neutral position representing zero. This is done by moving up all 5-unit beads and moving down all 1-unit beads. In clearing the abacus for use, hold the left end with your left middle finger on its upper edge and your left thumb on its lower edge, and move all beads down by slanting the upper edge toward your body. After leveling the abacus again, raise all 5-unit beads by moving the right index finger from left to right along the upper edge of the beam.

When calculating on the abacus, use two fingers: the right index finger and thumb. Some operators use only the index finger, but experiments show that it is more efficient to use the thumb as well. Nearly all experts use two fingers. Use the index linger to move

5-units beads up and down and to move 1-unit beads down, while using the thumb only to move 1-unit beads up. For instance, to place the figure 7 on the abacus with only the index finger requires two successive motions— first move down a 5-unit bead, and then move up two 1-unit beads—whereas these motions can be performed simultaneously with two fingers, with a corresponding increase in efficiency.

Moreover, in our everyday actions we commonly employ two or more fingers, say in picking up something or in holding a pen, and the hand is so made that the index finger almost always requires the assistance of the thumb. This accounts for the proven fact that, in the long run, it is much less tiring to operate the abacus with two fingers than with but one.

Experiments also show that the index linger can move beads down more quickly and accurately than the thumb, while on the other hand the thumb can move beads up with greater speed, force, and accuracy than the index finger.

The best and quickest way to acquire skill in abacus manipulation is to use the index finger and thumb in strict accord with the prescribed rules for bead manipulation. The correct finger movements will be indicated in detail for a number of problems in the next chapter. They should be carefully heeded and practiced many times until you can flick your two fingers as nimbly and effortlessly as the fingers of a pianist glide over the keys in executing a sonata.

Another important secret for acquiring rapid skill in abacus calculation is always to keep your fingers close to the beads. Never raise your fingers high from the beads nor put them deep between the beads. Glide the beads up and down by touching their ridges just slightly with the tips of your fingers.

The guiding principles for the movement of beads, as followed hereafter, may be summarized thus:

General Rules for Moving Beads

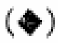
1. Move down a 5-unit bead and move up one or more 1-unit beads as the same time. (See Example 5, next chapter.)
2. First move down one or more 1-unit beads, and then move up a 5-unit bead. (See Example 6.)
3. In quick succession first move down a 5-unit bead, and then one or more 1-unit beads. (See Examples 7 and 9.)
4. In quick succession first move up one or more 1-unit beads, and then a 5-unit head. (See Examples 8 and 9.)
5. In addition, after finishing operation on the unit rod, move up a 1-unit bead on the tens' rod. (See Examples 11, 13, 15, 17 and 19.)
6. In subtraction, after subtracting a 1-unit bead from the tens' rod, operate on the unit rod. (See Examples 12, 14, 16, 18 and 20.)

FIG. 4

When working with the abacus, sit up straight at a desk. A good posture will have much to do with the speed and accuracy of your calculations.

Finally, in studying the illustrations which accompany the examples given throughout the rest of the book, the following key should be kept in mind:

Key to Illustrations

1. A white  bead is one which is in its original position and has no numerical value.
2. A striped  bead is one which has just been moved, thereby having either obtained or lost its numerical value.
3. A black  bead is one which obtained numerical value in a previous step.
4. ¯ indicates that beads are to be moved down with the index finger.
5. - indicates that beads are lo be moved up with the index finger.
6.  indicates that beads are to be moved up with the thumb.
7. Figures in parentheses accompanying the foregoing signs indicate the order in which beads are to be moved.

IV. ADDITION AND SUBTRACTION

There are four principal arithmetical calculations on the abacus: addition, subtraction, multiplication, and division. Of these, addition and subtraction are basic processes, for unless you know how to add and subtract on the abacus, you cannot multiply or divide. In our daily life and business accounts, addition is used far more frequently than the other processes and is most important of all.

The central part of the abacus is generally used for addition and subtraction. However, when many large numbers are to be added, the first number is set at the right side of the abacus, because the working extends to the left. In any case a one digit number and the last digit of a larger number should always be set on a unit rod, that is, on a rod marked with a unit point.

1. Adding and Subtracting One-Digit Numbers

Example 1. 1 + 2 = 3

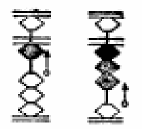


FIG. 5 FIG. 6

Step 1: Set the number 1 by moving up one 1-unit bead with the thumb (Fig. 5). See that you set 1 on a unit rod marked with a unit point.

Step 2: Add 2 to 1 by moving up, on the same rod, two more 1-unit beads, using the thumb (Fig. 6).

Note 1 : This example illustrates the procedure used in adding one or more 1-unit beads. The problems to which this procedure applies are:

1+1	2+1	3+1	5+1	6+1	7+1	8+1
1+2	2+2		5+2	6+2	7+2	
1+3			5+3	6+3		
			5+4			

Note 2 : Hereafter, such phrases as “move up one 1-unit bead,” “move down three 1-unit beads,” etc., will be shortened to “set 1,” “move down 3,” etc.

Example 2. 3 - 2 = 1

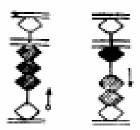


FIG. 7 FIG. 8

Step 1: Set 3 with the thumb (Fig. 7).

Step 2: Subtract 2 by moving down two 1-unit beads with the index finger (Fig. 8).

Note: This example illustrates the procedure used in subtracting one or more 1-unit beads. The problems to which this procedure applies are:

2—1	3—1	4—1	6—1	7—1	8—1	9—1
	3—2	4—2		7—2	8—2	9—2
		4—3			8—3	9—3
						9—4

Example 3. 2 + 5 = 7

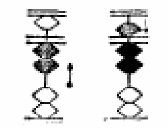


FIG. 9 FIG. 10

Step 1: Set 2 (Fig. 9).

Step 2: Move down 5 with the index finger (Fig. 10).

Note: This example illustrates the procedure used in adding a 5-unit bead. The problems to which this procedure applies are:

1+5 2+5 3+5 4+5

Example 4. 7 - 5 = 2

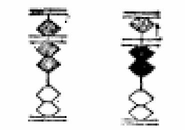


FIG. 11 FIG. 12

Step 1: Set 7 (Fig. 11).

Step 2: Move up 5 with the index finger (Fig. 12).

Note: This example shows the procedure used in subtracting a 5-unit bead. The problems to which this procedure are:

5—5 6—5 8—5 9—5

Example 5. 2 + 6 = 8

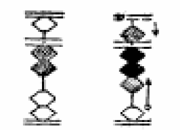


FIG. 13 FIG. 14

Step 1: Set 2 (Fig. 13).

Step 2: Move down 5 with the index finger and move up 1 with the thumb at the same time (Fig. 14).

Note : This example illustrates the procedure used in adding both a 5-unit bead and one or more 1-unit beads. The problems to which this procedure applies are:

1+6	2+6	3+6
1+7	2+7	
1+8		

Example 6. 8 — 6 = 2

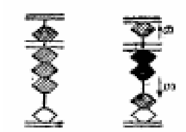


FIG. 15 FIG. 16

Step 1: Set 8 (Fig. 15).

Step 2: After moving down 1 with the index finger, move up 5 with the same finger (Fig. 16).

Note 1: This example shows how to subtract both one or more 1-unit beads and a 5-unit bead. The problems to which this procedure applies are:

9—9	9—8	9—7	9—6
	8—8	8—7	8—6
		7—7	7—6
			6—6

Note 2: The above-mentioned procedure is preferable to moving up 5 first and 1 next with the index finger. As is explained in Note 2 of Example 15, in some cases the latter procedure makes it difficult to move the fingers nimbly, e. g., the addition of 4 to 9.

Example 7. 4 + 1 = 5

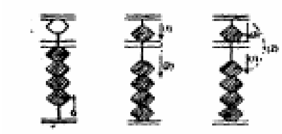


FIG. 17 FIG. 18 FIG. 19

Step 1: Set 4 (Fig. 17).

Step 2: Move down 5 first and 4 next in close succession with the index finger (Fig. 18).

Note 1: This example illustrates the procedure used in setting 5 when the addition of two numbers makes 5. The problems to which this procedure applies are:

4+1 3+2 2+3 1+4

Note 2: Fig. 19 illustrates the incorrect way to perform Step 2. Note that the correct way (Fig. 18) requires but a single continuing down stroke of the finger whereas the incorrect way requires three separate movements: (1) move down 4, (2) move the finger back up, and (3) move down 5, resulting in a loss of time and effort.

Example 8. 5 —1 = 4

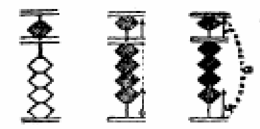


FIG. 20 FIG. 21 FIG. 22

Step 1: Set 5 (Fig. 20).

Step 2: First move up 4 with the thumb, and then move up 5 with the index finger in close succession (Fig. 21). Flick the thumb and the index finger with the idea of performing the two motions at the same time.

Note 1 : This example illustrates the procedure of subtracting a number from 5. The problems to which this procedure applies are:

5—1	5—2	5—3	5—4
	6—2	6—3	6—4
		7—3	7—4
			8—4

Note 2: As explained in Step 2 of this example, when 1 is subtracted from 5, four 1-unit beads and one 5-unit bead should be pushed up at the same time. But if the beginner finds it hard to make the two motions at the same time, he may perform each separately by first pushing up four 1-unit beads, and then a 5-unit bead (Fig. 21).

Fig. 22 illustrates the incorrect way to perform Step 2. Note that the correct way requires but a single continuous up stroke of the thumb and the index finger, whereas the incorrect way requires three separate movements. This means that your finger or at least your attention has to travel further, resulting in a loss of time and effort.

Example 9. 4 + 3 = 7

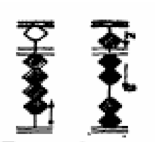


FIG. 23 FIG. 24

Step 1: Set 4 (Fig. 23).

Step 2: First move down 5, and then 2 in close succession with the index finger (Fig. 24).

Note 1 : This example illustrates the procedure used in adding a 5-unit bead and subtracting one or more 1-unit beads. The problems to which this procedure applies:

4+1	4+2	4+3	4+4
	3+2	3+3	3+4
		2+3	2+4
			1+4

Note 2: Since three 1-unit beads cannot be added to the four 1-unit beads, 5 is added and 2 is subtracted to offset the excess. This operation may be represented in the form of the equation: 4 + 3 = 4 + (5 — 2) = 7

Note 3: When working the foregoing example, do not think: Since 3 plus 4 equals 7, I must form 7 on the board. Instead, simply remember that 2 is the complementary digit with which 3 makes 5, and by flicking down 5 and 2 in rapid succession, allow the sum 7 to form itself naturally on the board. There are only two groups of complementary digits for 5: 3 and 2, and 4 and 1. Operation by means of complementary digits is much simpler and less liable to error than the ordinary mode of calculation. For further explanation of calculation by means of complementary digits, see Note 3 to Example 20.

Example 10. 7 - 3 = 4

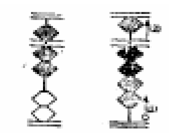


FIG. 25 FIG. 26

Step 1: Set 7 (Fig. 25).

Step 2: In close succession, first move up 2 with the thumb, and then move up the 5 with the index finger with the idea of performing the two motions at the same time (Fig. 26).

Note 1: This example illustrates the procedure for adding one or more 1-unit beads and subtracting one 5-unit bead. Problems to which applicable:

5—1	5—2	5—3	5—4
	6—2	6—3	6—4
		7—3	7—4
			8—4

Note 2: Since three 1-unit beads cannot be subtracted from the two 1-unit beads, 2 is added and 5 is subtracted. This operation may be represented by the equation:

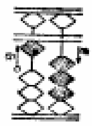
7 - 3 = 7 + 2 - 5 = 4

Note 3: When working this example, do not think: 3 from 7 leaves 4, so 4 must be formed on the board. Instead, simply remember that 2 is the complementary digit with which *3* makes 5, and allow the result to form itself naturally on the board. (See Note 3 to Example

20.)

Example 11. 3 + 7 = 10


B
FIG. 27


A B
FIG. 28

Step 1: Set 3 on a unit rod, which we shall call B (Fig. 27).

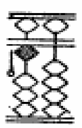
Step 2: In close succession, first move down then 3 on B with the index finger, and then move up 1 on the tens' rod, here called A, with the thumb (Fig. 28). Flick the index finger and the thumb in a twisting manner so that you may perform the two motions at the same time.

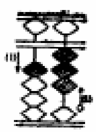
Note 1: This example shows how to set 10 when it is the sum of two digits. This procedure requires the subtraction of one or more 1-unit beads. Problems to which applicable:

1+9 2+8 3+7 4+6

Note 2: In working out this example, do not move up 1 on the tens' rod until you have moved down 3 on the unit rod B. If you follow this incorrect procedure, you will never improve in bead calculation. For the theoretical reasons for the advantages of the correct procedure, see Note 5 (The Order of Operation) to Example 20.

Example 12. 10 — 7 = 3


A B
FIG. 29


A B
FIG. 30

Step 1: Set 10. This is done by simply moving up one bead on the tens' rod A (Fig. 29).

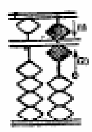
Step 2: First remove the 10 by moving down the 1 on the tens' rod A with the index finger, and then move up 3 on the unit rod B with the thumb (Fig. 30).

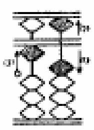
Note 1: This example shows how to subtract 10 and add one or more 1-unit beads. Problems to which such procedure applies:

10—9 10—8 10—7 10—6

Note 2: In working out this example, be sure to move down 1 on the tens' rod A before moving up 3 on the unit rod B. For the theoretical reasons for the advantages of the correct procedure, see Note 5 (The Order of Operation) to Example 20.

Example 13. 6 + 4 = 10


A B
FIG. 31


A B
FIG. 32

Step 1: Set 6 on the unit rod B.

Step 2: Move down 1 on B with the index finger, then move up 5 on B with the same finger, and finally move up 1 on the tens' rod A with the thumb. Work the index finger and the thumb with the idea of performing the last two motions at the same time.

Note: This example shows how to form the sum 10 when it is made by the addition of two digits. This procedure, requiring the subtraction of both one or more 1-unit beads and a 5-unit bead, applies to the problems:

6+4 7+3 8+2 9+1

Example 14. 10 — 4 = 6

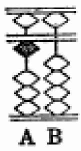


FIG. 33

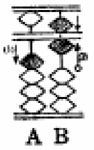


FIG. 34

Step 1: Set 1 on the tens' rod A.

Step 2: First move down the 1 on A with the index finger, then move down 5 and move up 1 on B at the same time.

Note: This example shows how to subtract 10 and add both a 5-unit bead and one or more 1-unit beads. Applicable problems:

10—4 10—3 10—2 10—1

Example 15. 9 + 4 = 13

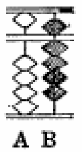


FIG. 35

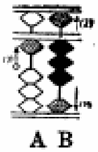


FIG. 36

Step 1: Set 9 on the unit rod B.

Step 2: Move down 1 on B with the index finger, then move up 5 on B with the same finger and finally move up 1 on A with the thumb. Work the index finger and the thumb with the idea of performing the last two motions at the same time.

Note 1 : This example shows how to add 10 after subtracting one or more 1-unit beads and a 5-unit bead. Applicable problems:

9+1	9+2	9+3	9+4
	8+2	8+3	8+4
		7+3	7+4
			6+4

Note 2: Do not reverse motions 1 and 2 of Step 2. If you do, your operation will slow down. Because after moving up 5, you will find it hard to move down 1 on B and move up 1 on A at the same time in the manner of twisting your fingers, although this latter procedure is workable in some cases, for example, in adding 4 to 6 or 7. This is the main reason why experts, in working out Example 6 (8 — 6 = 2), disfavor the procedure of moving up 5 first, and moving down 1 next.

Example 16. 13 — 4 = 9

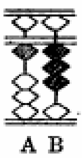


FIG. 37

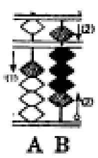


FIG. 38

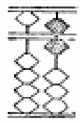
Step 1: Set 13 on AB.

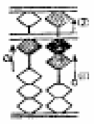
Step 2: After moving down the 1 on A, move down 5 and move up 1 on B at the same time.

Note: This example shows how to add both a 5-unit bead and one or more 1-unit bead after subtracting 10. Applicable problems.

11—2	11—3	11—4
	12—3	12—4
		13—4

Example 17. 6 + 6 = 12


A B
FIG. 39

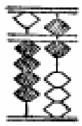

A B
FIG. 40

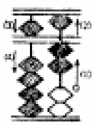
Step 1: Set 6 on the unit rod B.

Step 2: Move up 1 on B (Motion 1), move up 5 on B (Motion 2), and move up 1 on A (Motion 3). Work the two fingers with the idea of performing the first two motions at the same time.

Note 1: This example shows how to add 1 to the tens' rod after adding one or more 1-unit beads and subtracting a 5-unit bead on the unit rod. Applicable problems:

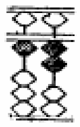
5+6	6+6	7+6	8+6
5+7	6+7	7+7	
5+8	6+8		
5+9			

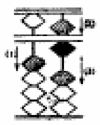

A B
FIG. 41


A B
FIG. 42

Note 2: It is possible to perform the last two motions of Step 2 above at the same time after completing the first motion. But this procedure should not be followed, as it does not work in some cases. For example, the problem, "46 + 6 = 52" (Figs. 41 and 42) or "96 + 6 = 102," can be worked in no other way than that indicated.

Example 18. 12 — 6 = 6


A B
FIG. 43


A B
FIG. 44

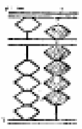
Step 1: Set 12 on AB.

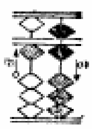
Step 2: After moving down the 1 on A, move down 5 and 1 on B in succession.

Note: This example shows how to add a 5-unit bead and subtract one or more 1-unit beads after subtracting 10. Applicable problems:

11—6			
12—6	12—7		
13—6	13—7	13—8	
14—6	14—7	14—8	14—9

Example 19. 9 + 7 = 16


A B
FIG. 45


A B
FIG. 46

Step 1: Set 9 on the unit rod B.

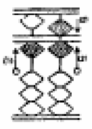
Step 2: Move down 3 on B, and move up 1 on A. Perform the two motions mechanically at the same time as if twisting the thumb and the index finger.

Note 1: This example shows how to subtract one or more 1-unit beads from the unit rod and add Applicable problems:

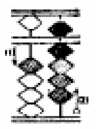
2+9	3+9	4+9	6+9	7+9	8+9	9+9
	3+8	4+8		7+8	8+8	9+8
		4+7			8+7	9+7
						9+6

Note 2: Since 7 cannot be added to the 9 on the unit rod B, 3, the complementary digit of 7 for 10, is subtracted and 10 is added. This operation may be represented by the equation: 9 + 7 = 9 - 3 + 10 = 16

Example 20. 16 - 7 = 9



A B
Fig. 47



A B
Fig. 48

Step 1: Set 16 on AB.

Step 2: Move down the 1 on A, and move up 3 on B.

Note 1: When setting a two-digit number on the board, as in Step 1, always set the tens' digit first.

Note 2: This example shows how to subtract 10 and add one or more 1-unit beads. Applicable problems:

15—6	15—7	15—8	15—9
	16—7	16—8	16—9
		17—8	17—9
			18—9

Note 3: Since 7 cannot be subtracted from the 6 on the unit rod, 10 is subtracted from rod A, and 3, the complementary digit of 7 for 10, is added. The basis for this operation may be represented by the equation: 16 - 7 = 16 - 10 + 3 = 9

Note 4: Mechanization of Operation.

The fundamental principle which makes abacus operation simple and speedy is mechanization. To give a theoretical explanation, the mechanical operation of the abacus is designed to minimize your mental labor and limit it to the unit rod, without carrying it to the tens' rod, by means of the complementary digits for 10 and 5, and to let the result form itself mechanically and naturally on the board.

To give an example, in adding 7 to 9, the student accustomed to the Western mode of calculation will probably form 16 on the board as a result of mental calculation to the effect that 9 and 7 is 16. But such procedure is in every way inferior to the above-mentioned mechanical one. Not only does this Western method require mental exertion and time but it is liable to cause perplexity and errors.

When a problem of addition and subtraction is worked on the board, the procedure is very simple. Addition and subtraction, which involve two rods, are simplified by means of a complementary digit, that is, the digit necessary to give the sum 10 when added to a given digit. For instance, suppose we have to add 7 on a rod where there is 9; then we think or say, "7 and 3 is 10," and subtract 3 from the rod in question, and add 1 to next rod on the left. When we have to subtract 7 from 16, we think or say, "7 from 10 leaves 3," and subtract 1 from the next rod on the left, and add 3 to the rod in question. This means that

10 is always reduced to 1, and added or subtracted on the tens' rod. Therefore, after recalling the complementary digit, the operator has simply to perform either of the two mechanical operations: subtracting the complementary digit and adding 1 on the tens' rod (in addition) or subtracting 1 on the tens' rod and adding the complementary digit (in subtraction). The result then will naturally form on the board. No matter how many digits may be contained in the numbers to be added or subtracted, the entire operation is performed by applying this mechanical method to each digit in turn.

The same mechanical method applies to the operations which require the analysis of 5 (see Examples 7 to 10). Suppose we have to add 3 to 4; then we merely think of the complementary digit of 3 for 5 (that is, the digit necessary to give the sum 5 when added to 3), and we move down the 5-unit digit and two 1-unit digits on the rod in question. Then the result will naturally appear on the board. Any attempt to calculate the answer mentally will retard the operation.

10 has only five groups of complementary digits: 9 and 1, 8 and 2, 7 and 3, 6 and 4, and 5 and 5, while 5 has only two: 4 and 1, and 3 and 2. Accordingly, the use of the mechanized method requires no more mental effort than that of remembering one of the elements of each of these very few pairs of complementary digits. This is the fundamental reason which makes calculation by means of the complementary digit much simpler and speedier and less liable to error than the ordinary way of mental or written calculation.

The following examples will show how much more laborious the ordinary calculation is. In written calculation we proceed from right to left. For instance in the problem 99 + 88 + 77 + 66, we first add the unit digits, thinking "9 + 8 = 17, 17 + 7 = 24 and 24 + 6 = 30." Next we add the 30 to the 90 of 99 and work on. In the problem 567— 89, we cannot subtract the 9 from 7, so borrowing 10 from the 6 in the tens' place, we get 8. Next proceeding to the tens' place we again find that we cannot subtract the 8 from the remaining 5 of the minuend, so we borrow 1 from the remaining 5 in the hundreds' place, and we get 7 in the tens' place and the answer 478. These processes involve laborious mental exertion.

On the other hand, all calculations on the abacus proceed from left to right, that is, from the highest to the lowest digit. This accords with our natural customary practice of naming or remembering all numbers from the highest to the lowest digit. Therefore, to set numbers on the board is to calculate numbers.

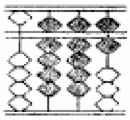
In conclusion, incredibly speedy abacus operation is mainly attributable to three reasons: mechanical operation by means of the complementary digit, left to right operation, and the previously explained dozen rules of rational or scientific bead manipulation. These are the reasons why, no matter how rapidly numbers may be mentioned, as long as they are given distinctly, the skilled abacus operator can add and subtract without any error, irrespective of how many digits the numbers may contain.

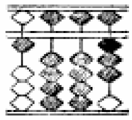
Note 5: The Order of Operation.

When addition involves two rods, as in the example 9 + 7=16, he sure to subtract 3 from the unit rod, and next add 10 in the form of 1 to the tens' rod. Thus 9 + 7 = 9 — 3 +10 = 16.

The idea of 7 in the terms of the complementary digit is " 7 = 10—3." So you would be tempted to add 10 first and subtract 3 next. But as already pointed out, such a procedure which involves unnecessary shifts of attention between the unit rod and the tens' rod, should not be followed. Because in adding 7 to 9, you will naturally first observe the unit rod to add 7. Now subtract 3 from the unit rod, and then, proceeding to the tens' rod, add 1. This procedure requires only two shifts of attention and operation, On the other hand, if you did not subtract 3 first, you would have to come back to the unit rod to subtract 3 after adding 1 to the tens' rod. This inferior procedure would delay your operation.

The advantage of the correct procedure becomes even clearer in some problems containing more than one digit. For instance, in adding 7 to 996, note the decided advantage of forming 3 and proceeding mechanically straight to the left to clear the tens' and hundreds' rods of their 9 and to set 1 on the thousands', as shown in Figs. 49 and 50.


A B C D
FIG. 49


A B C D
FIG. 50

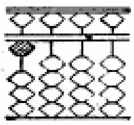
When subtraction involves two rods, as in the example 15—7 = 8, be sure to subtract 10 in the form of 1 from the tens' rod, and next add 3 to the unit rod. Thus:

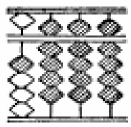
$$15 - 7 = 15 - 10 + 3 = 8.$$

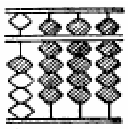
In subtracting 7, naturally you will first look at the unit rod; then you will see that it is impossible to subtract 7 from 6 and that you must borrow 10 from the tens' rod. At this instant subtract 10 in the form of 1, and then add 3, i. e., the complementary digit of 7 for 10, to the unit rod. So following this natural order of attention, first subtract the 10 and then add the 3. If you were to reverse this order, you would have to shift your attention back again to the tens' rod to subtract 10 after adding 3. Thus this wrong procedure would cause needless shifting of attention and delay operation. Note that failure to use the complementary digit would necessitate the less efficient method of mental calculation.

The correct procedure is especially advantageous in some problems containing more than one digit, e.g., the problem 1 000—1 = 999.

Following the correct procedure, in this problem we can proceed mechanically straight from left to right (Fig. 52), while the incorrect procedure (Fig. 53) involves the loss of time and labor.


A B C D
FIG. 51


A B C D
FIG. 52


A B C D
FIG. 53

When setting numbers of two or more digits, set the tens' first. Also, when adding or subtracting numbers of two or more digits, add or subtract beginning with the highest-place digit. This is another fundamental rule which will produce efficiency. As previously explained, when a number is named or given, beginning with the highest digit, it can be mentally remembered or set and calculated much more naturally and easily on the board than beginning with the lowest digit. This method is opposite to that of written calculation, which is started back. ward with the last digit after a number has been given.

2. Adding and Subtracting Two-Digit Numbers

When setting two-digit numbers, set the tens' first. Also when adding and subtracting two-digit numbers, add and subtract the tens' first, On the abacus always operate from left to right. This is a fundamental rule based on efficiency. The efficiency of this rule is especially true in the calculation of large numbers, as in Examples 26 and 27. As explained in the introduction, since a number is named or given, beginning with the highest digit, it can be mentally remembered or set and calculated much more naturally and easily on the board, beginning with the highest-place digit than with the lowest place. Written calculation is started backward with the last digit after a number has been given, whereas on the abacus a number is calculated while it is being given, in other words, to set a number is to calculate it.

Example 21. 14 + 25 = 39

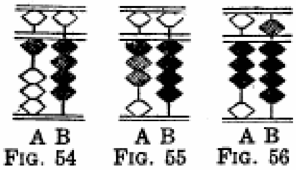


FIG. 54 FIG. 55 FIG. 56

AB	
1 4	Step 1
+ 2	Step 2
3 4	Result of Step 2
+ 5	Step 3
3 9	Result of Step 3

Step 1: Set 14 on AB, with the 4 appearing on the unit rod B (Fig. 54).

Step 2: Add the 2 of 25 to the 1 on A with the thumb. This gives 3 on A and 34 on AB (Fig. 55).

Step 3: Add the remaining 5 of 25 to the 4 on B with the forefinger. This gives 9 on B. The answer is 39 (Fig. 56).

The same procedure can be expressed diagrammatically as seen above.

Example 22. 45 + 27 = 72

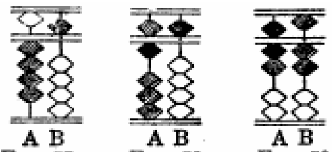


FIG. 57 FIG. 58 FIG. 59

AB	
4 5	Step 1
+ 2	Step 2
6 5	
+ 7	Step 3
7 2	

Step 1: Set 45 on AB (Fig. 57).

Step 2: Add the 2 of 27 to the 4 on A. This gives 6 on A and 65 on AB (Fig. 58).

Step 3: Add the remaining 7 of 27 to the 5 on B. This gives 2 on B and 7 on A, as 1 is carried to the 6 on A. The answer is 72 (Fig. 59).

Example 23. 79 — 23 = 56

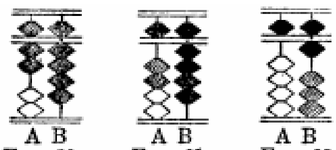


FIG. 60 FIG. 61 FIG. 62

AB	
7 9	Step 1
− 2	Step 2
5 9	
− 3	Step 3
5 6	

Step 1: Set 79 on AB (Fig. 60).

Step 2: Subtract the 2 of 23 from the 7 on A. This leaves 5 on A and 59 on AB (Fig. 61).

Step 3: Subtract the remaining 3 of 23 from the 9 on B. This leaves 6 on B. The answer is 56 (Fig. 62).

Example 24. 83 — 49 = 34

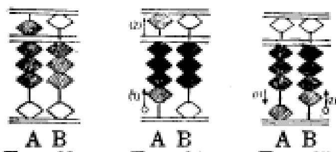


FIG. 63 FIG. 64 FIG. 65

AB	
8 3	Step 1
− 4	Step 2
4 3	
− 9	Step 3
3 4	

Step 1: Set 83 on AB (Fig. 63).

Step 2: Subtract the 4 of 49 from the 8 on A. This leaves 4 on A and 43 on AB (Fig. 64).

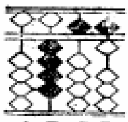
Step 3: Subtract the remaining 9 of 49 from the 3 on B. As you cannot subtract 9 from 3, borrow 1 from A This leaves 3 on A and enables you to subtract 9 from 10 on B. Add, to the 3 on B, the remainder 1 of 9 from 10, and you get 4 on B. The answer is 34 (Fig. 65).

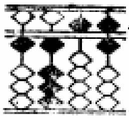
3. Adding and Subtracting Numbers of Over Two Digits

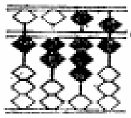
The methods used in adding or subtracting numbers containing three or more digits are the same as those just described in the case of two-digit numbers. Two problems each in addition and subtraction will suffice to make this clear.

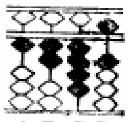
Example 25. 456 + 789 = 1 245

	ABCD	
	4 5 6	Step 1
+	7	Step 2
	1,1 5 6	
+	8	Step 3
	1,2 3 6	
+	9	Step 4
	1,2 4 5	


A B C D
FIG. 66


A B C D
FIG. 67


A B C D
FIG. 68


A B C D
FIG. 69

Step 1: Set 456 on BCD (Fig. 66).

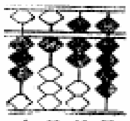
Step 2: Add the 7 of 789 to the 4 on B. This gives you 11 on AB and 1 156 on ABCD (Fig. 67).

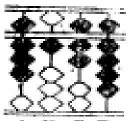
Step 3: Add the 8 of the remaining 89 to the 5 on C. This gives you 23 on BC and 1 236 on ABCD (Fig. 68).

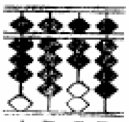
Step 4: Add the remaining 9 to the 6 on D. This gives you 45 on CD. The answer is 1 245. (Fig. 69).

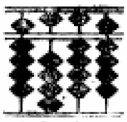
Example 26. 3 179 + 5 876 = 9 055

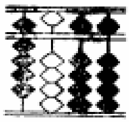
	ABCD	
	3,1 7 9	Step 1
+	5	Step 2
	8,1 7 9	
+	8	Step 3
	8,9 7 9	
+	7	Step 4
	9,0 4 9	
+	6	Step 5
	9,0 5 5	


A B C D
FIG. 70


A B C D
FIG. 71


A B C D
FIG. 72


A B C D
FIG. 73


A B C D
FIG. 74

Step 1: Set 3 179 on ABCD (Fig. 70).

Step 2: Add the 5 of 5 876 to the 3 on A. This gives you 8 on A and 8 179 on ABCD.

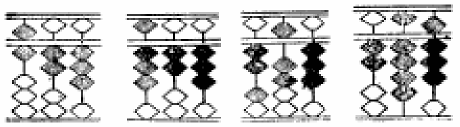
Step 3: Add the 8 of the remaining 876 to the 1 on B. This gives you 9 on B and 8 979 on ABCD (Fig. 72).

Step 4: Add the 7 of the remaining 76 to the 7 on C. This gives you 904 on ABC and 9 049 on ABCD (Fig. 73).

Step 5: Add the remaining 6 to the 9 on D. This gives you 55 on CD. The answer is 9 055 (Fig. 74).

Example 27. 623 — 375 = 248

Step 1: Set 623 on ABC (Fig. 75).



A B C A B C A B C A B C

Fig. 75 Fig. 76 Fig. 77 Fig. 78

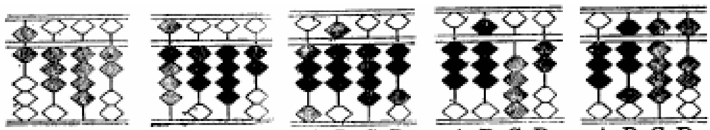
```
   A B C
   6 2 3   Step 1
 - 3       Step 2
   3 2 3
 -   7     Step 3
   2 5 3
 -     5   Step 4
   2 4 8
```

Step 2: Subtract the 3 of 375 from the 6 on A. This leaves 3 on A and 323 on ABC (Fig. 76).

Step 3: Subtract the 7 of the remaining 75 from the 2 on B by borrowing 1 from the 3 on A. This leaves 25 on AB and 253 on ABC (Fig. 77).

Step 4: Subtract the remaining 5 from the 3 on C by borrowing 1 from the 5 on B. This leaves 48 on BC. The answer is 248 (Fig. 78).

Example 28. 6 342 — 2 547 = 3 795



A B C D A B C D A B C D A B C D A B C D

Fig. 79 Fig. 80 Fig. 81 Fig. 82 Fig. 83

```
   A B C D
   6,3 4 2   Step 1
 - 2         Step 2
   4,3 4 2
 -   5       Step 3
   3,8 4 2
 -     4     Step 4
   3,8 0 2
 -       7   Step 5
   3,7 9 5
```

Step 1: Set 6 342 on ABCD, with 6 on the thousands' rod and 2 on the ones' (Fig. 79).

Step 2: Subtract the 2 of 2 547 from the 6 on A. This leaves 4 on A and 4 342 on ABCD (Fig. 80).

Step 3: Subtract the 5 of the remaining 547 from the 3 on B by borrowing 1 from the 4 on A. This leaves 38 on AB and 3 842 on ABCD (Fig. 81).

Step 4: Subtract the 4 of the remaining 47 from the 4 on C. This leaves 0 on C on 3 802 on ABCD (Fig. 82).

Step 5: Subtract the remaining 7 from the 2 on D by borrowing 1 from the 8 on B. This leaves 795 on BCD. The answer is 3 795 (Fig. 83).

4. Exercises

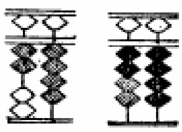


Fig. 84 Fig. 85

Probably the most convenient way of dealing with problems containing a long file of numbers, such as the following, is to use the top edge of the abacus as a marker. For example, in problem 1 on page 52, first place the top edge of the abacus immediately under the first number, 24, and form it on the board (Fig. 84); then move the abacus down until the next number, 20, appears directly under the beads in use, and add that number on the beads (Fig. 85); and continue in this fashion to the end of the problem.

Abacus calculation is also greatly facilitated by having someone call out the successive numbers. Numbers should be read distinctly and quickly. For example, the number 123 456 789 should be given as "one, two, three million; four, five, six thousand; seven, eight, nine."

The best exercise for attaining skill is to add 123 456 789 fine times. If your sum is correct, it will be 1 111 111 101. Again, add 789 fine times on the rods GHI, with I as the unit rod; next add 456 on DEF fine times; finally add 123 on ABC nine times, and you will get the same sum. Subtract 123 456 789 from 1 111 111 101 nine times, and you will end with zero. These three exercises involve every procedure used in problems of addition and subtraction. A third-grade abacus operator can work each of the three exercises in one minute, and a first-grade operator in thirty seconds.

I. Adding and Subtracting Two-Digit Numbers

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
24	55	22	11	33	55	66	55
20	44	66	44	77	55	44	77
–33	–55	–88	–33	–88	–*55*	–22	–80
–11	–22	77	44	99	54	33	69
22	55	–66	–22	–88	35	–44	–76
12	–11	55	33	77	–55	55	88
–23	–55	–66	–44	–99	75	–33	–66
11	***11***	***0***	***33***	***11***	***164***	***99***	***67***

II. Adding and Subtracting Three-Digit Numbers

(1)	(2)	(3)	(4)	(5)
222	345	561	621	158
665	762	259	946	782
778	473	846	–255	–345
555	528	667	428	566
335	981	445	564	*444*
778	811	778	–392	–657
222	176	289	734	216
889	634	265	855	774
443	367	778	–628	–889
223	189	665	–476	–677
5 110	***5 266***	***5 553***	***2 397***	***322***

III. Adding and Subtracting Four-Digit Numbers

(1)	(2)	(3)	(4)	(5)
3,627	9,105	2,456	7,081	6,924
1,508	2,746	8,193	5,469	8,570
9,472	1,809	5,647	–2,505	1,439
6,345	5,321	7,038	3,748	–3,268
8,160	4,684	9,825	4,917	–7,015
2,079	3,263	3,741	–6,803	9,847
4,384	5,162	6,580	–6,294	5,192
7,819	7,038	1,269	1,372	2,603
5,623	8,574	4,001	9,620	–3,786
1,950	9,970	9,372	8,135	4,051
50, 967	***57,672***	***58, 122***	***24,740***	***24, 557***

V. MULTIPLICATION

There are several methods of multiplication on the abacus. The one introduced in the following pages is a recent method which is generally considered the best and is now the standard method taught in grade schools. In describing the method, standard terminology will be used. Thus, for example, in the problem 5 x 2 = 10, 5 will be called the multiplicand, 2 the multiplier, and 10 the product.

It is customary to set the multiplicand at the central part of the abacus and the multiplier to the left, leaving two or three rods unused between the two numbers, just enough to separate them clearly but not too widely. The decision of the Abacus Committee in favor of two unused rods will be followed in our problems here.

The method of multiplication used here gives the product immediately to the right of the multiplicand. There is a less favored method which gives the first figure of the product immediately to the left.

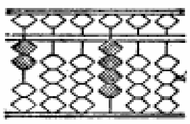
Two main reasons can be given for setting the multiplicand on the right and the multiplier on the left. Since the abacus is operated with the right hand, a reverse order of setting the two figures would cause the multiplier to be hidden by the hand much of the time. Moreover, in case the multiplier is a large number, too much space would have to be left between it and the multiplicand. Otherwise the product, which is produced at the right of the multiplicand would extend right into the multiplier.

Although the use of a unit rod marked with a unit point does not have as much bearing in problems of multiplication and division as in addition and subtraction, it does facilitate calculation in many ways. Therefore, in the following problems the unit figure of the multiplicand is set on a unit rod. In the case of the multiplier, however, so long as it is not a fractional number, the unit rod is disregarded and the unit figure is set on the third rod to the left of the multiplicand.

As for the order of setting the multiplicand and the multiplier, since the unit figure of the multiplicand must be set on a unit rod, it is advisable for the beginner to set the multiplicand ahead of the multiplier. It should be noted, however, that experts can locate both the multiplicand and the multiplier at a glance. So they very often set the multiplier ahead of the multiplicand, thus saving the time required in shifting the hand back to the left after setting the multiplicand. Even more frequently experts set only the multiplicand on the board not even troubling to set the multiplier.

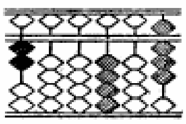
1. Multiplying by One-Digit Numbers

Example 1. 4 x 2 = 8



A B C D E F

FIG. 86



A B C D E F

FIG. 87

A	B	C	D	E	F	
2	0	0	4	0	0	Step 1
					8	Step 2
2	0	0	0	0	8	Result

Step 1: Set the multiplicand 4 on the unit rod D and the multiplier 2 on rod A, thus leaving two vacant rods between the numbers as in Fig. 86

Step 2: Multiply the multiplicand 4 by the mu1tiplier 2. Set the product 8 on F, the second rod to the right of the multiplicand, and clear rod D of its 4. Fig. 87 shows the result of Step 2.

Note: The accompanying diagram shows another way to illustrate the same problem. Here the two figures in the row designated Step 1 indicate that the multiplier 2 and the multiplicand 4 have been set on A and D respectively. The two figures in the row designated Result show the multiplier 2 remaining on A and the product 8 which has been set on F as the result of the multiplication.

All the following examples will be illustrated in the two ways shown above.

The reasons for clearing off the multiplicand after its multiplication will be given at the end the section on multiplication by two-digit numbers.

Example 2. 8 x 6 = 48

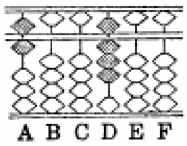


FIG. 88

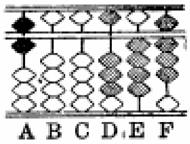


FIG. 89

```
A B C D E F
      ·     ·
6 0 0 8 0 0   Step 1
        4 8   Step 2
6 0 0 0 4 8
```

Step 1: Set 8 on the unit rod D and 6 on A, leaving two vacant rods (Fig. 88).

Step 2: Multiplying 8 by 6, set the product 48 on EF, and clear D of its 8. In this step, the first rod to the right of the multiplicand, designated E, is the tens' rod of the product 48 (Fig. 89).

Note: Some experts say it is desirable to clear away the multiplicand before setting the product. For instance, in the above example, they say that the product 48 should be set on EF after clearing D of its 8. This method has the merit of saving the time of shifting the hand back to the left to clear off the multiplicand after setting the product. But the Abacus Committee frowns upon this procedure, saying that, especially for beginners, it is apt to cause confusion in that the multiplicand must be carried in the memory after it has been cleared away from the board.

Example 3. 24 x 7 = 168

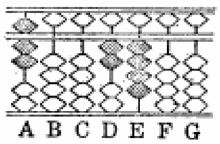


FIG. 90

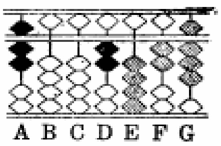


FIG. 91

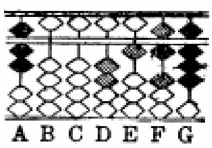


FIG. 92

```
A B C D E F G
  ·     ·
7 0 0 2 4 0 0   Step 1
          2 8   Step 2
7 0 0 2 0 2 8
        1 4     Step 3
7 0 0 0 1 6 8
```

Step 1: Set 24 on DE, with E as the unit rod, and set 7 on A (Fig. 90).

Step 2: Multiplying the 4 in 24 by 7, set the product 28 on FG, and clear E of its 4 (Fig. 91).

Step 3: Multiplying the remaining 2 in 24 by 7, set the product 14 on EF, thereby adding this new product to the 28 on FG, and clear E of its 2. This makes a total of 168 on EFG, which is the answer (Fig. 92).

Note 1: The reason for setting the product 14 on the rods EF, which are one place higher than FG, is obvious. When adding 14, do not take the trouble of thinking that this product is 140 in actual value and that therefore this must be set on EF. Instead just mechanically set the 1 in 14 on E and add the 4 in 14 to the previous 2 on F, and let the result form itself automatically.

Note 2: In Step 2, F is the tens' rod of the product 28, while in Step 3, E is the tens' rod of the product 14. In each step of multiplication, the first rod to the right of that figure in the multiplicand which is multiplied is the tens' rod of the product.

Note 3: When there are two digits in the multiplicand, first multiply the last digit by the multiplier and then the first digit.

2. Multiplying by Two-Digit Numbers

Example 4. 8 x 17 = 136

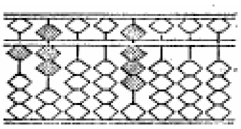


FIG. 93

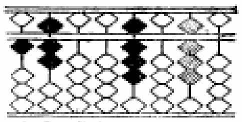


FIG. 94

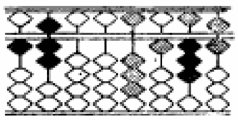


FIG. 95

A	B	C	D	E	F	G	H	
1	7	0	0	8	0	0	0	Step 1
						8		Step 2
						+5	6	Step 3
1	7	0	0	0	1	3	6	

Step 1: Set 8 on the unit rod E and 17 on AB (Fig. 93).

Step 2: Multiplying the 8 by the 1 in 17, set the product 8 on G (Fig. 94).

Step 3: Multiplying the 8 by the 7 in 17, set the product 56 on GH, and clear E of its 8. Since you already have 8 on G, you get, on FGH, a total of 136, which is the answer (Fig. 95).

Note: When there are two digits in the multiplier, first multiply the multiplicand by the first digit of the multiplier and next by the last digit of the multiplier.

Example 5. 46 x 23 = 1,058

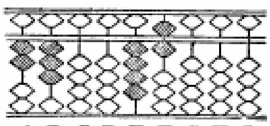


FIG. 96

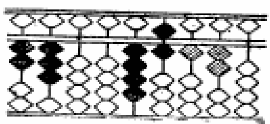


FIG. 97

A	B	C	D	E	F	G	H	I	
2	3	0	0	4	6	0	0	0	Step 1
						1	2		Step 2
							+1	8	Step 3
2	3	0	0	4	0	1	3	8	
						+8			Step 4
						+1	2		Step 5
2	3	0	0	0	1	0	5	8	

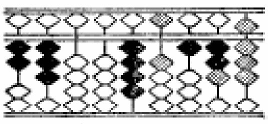


FIG. 98

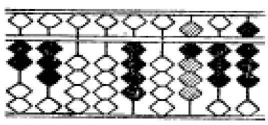


FIG. 99

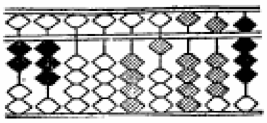


FIG. 100

Step 1: Set 46 on EF, with E as the unit rod, and set 23 on AB (Fig. 96).

Step 2: Multiplying the 6 in 46 by the 2 in 23, set the product 12 on GH (Fig. 97).

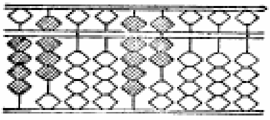
Step 3: Multiplying the same 6 in 46 by the 3 in 23, set the product 18 on HI and clear F of its 6. Since you have 12 on GH, you get a total of 138 on GH (Fig. 98). Remember that each time the same digit in the multiplicand is multiplied by one digit after another in the multiplier, the value of the product is reduced by one rod or place.

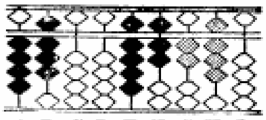
Step 4: Multiplying the 4 in 46 by the 2 in 23, set the product 8 on G. This makes a total of 938 on GH (Fig. 99).

Step 5: Multiplying the same 4 in 46 by the 3 in 23, set the product 12 on GH and clear E of its 4. This leaves the answer 1 058 on FGHI (Fig. 100).

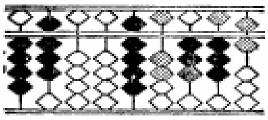
Note: In case both the multiplier and the multiplicand have two digits, (1) multiply the last digit of the multiplicand by the first digit of the multiplier; (2) multiply the same digit of the multiplicand by the last digit of the multiplier; (3) multiply the first digit of the multiplicand by the first digit of the multiplier; and (4) multiply the same first digit of the multiplicand by the last digit of the multiplier. This is the fundamental rule of multiplication.

Example 6. 97 x 48 = 4 656


A B C D E F G H I
Fig. 101


A B C D E F G H I
Fig. 102

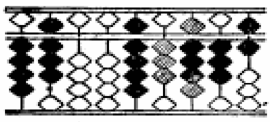
ABCDEFGHI	
4 8 0 0 9 7 0 0 0	Step 1
2 8	Step 2
+ 5 6	Step 3
4 8 0 0 9 0 3 3 6	
+ 3 6	Step 4
+ 7 2	Step 5
4 8 0 0 0 4 6 5 6	

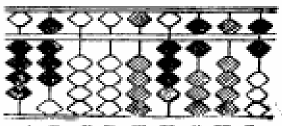

A B C D E F G H I
Fig. 103

Step 1: Set 97 on EF, with F as the unit rod, and set 48 on AB (Fig. 101).

Step 2: Multiplying the 7 in 97 by the 4 in 48, set the product 28 on GH (Fig. 102).

Step 3: Multiplying the same 7 in 97 by the 8 in 48, set the product 56 on HI, and clear F of its 7. Since you have 28 on GH, you get a total of 336 on GHI (Fig. 103).


A B C D E F G H I
Fig. 104


A B C D E F G H I
Fig. 105

Step 4: Multiplying the remaining 9 in 97 by the 4 in 48, set the product 36 on FG. This makes a total of 3 936 on FGHI (Fig. 104).

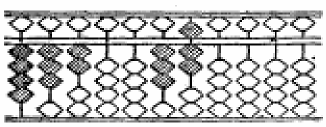
Step 5: Multiplying the same 9 in 97 by the 8 in 48, set the product 72 on GH, and clear E of its 9. This gives you, on FGHI, a total of 4 656, which is the answer (Fig. 105).

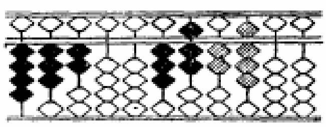
Note: The preceding examples will have indicated the desirability of clearing off each digit in the multiplicand after its multiplication by all the digits in the multiplier. If you did not do so, you would be greatly inconvenienced in operation. This is especially the case when the multiplicand is a large number. First, you would often find it hard to tell which of the digits in the multiplicand you had multiplied by all the digits in the multiplier. Second, this incorrect procedure would necessitate the removal of the multiplier further to the right beyond the product of the correct procedure by as many digits as there are in the multiplicand.

3. Multiplying by Numbers of Over Two-Digits

No matter how many digits the multiplier may have, the principle of multiplication is the same as that of multiplying by two-digit numbers. You have only to see that you do not mistake the order of multiplication and the rods on which to set products.

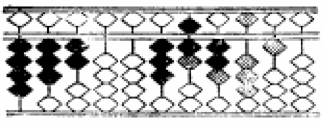
Example 7. 37 x 432 = 15 984


A B C D E F G H I J K
Fig. 106

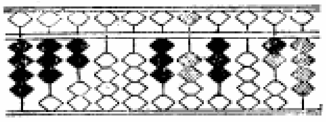

A B C D E F G H I J K
Fig. 107

Step 1: Set 37 on FG, with G as the unit rod, and set 432 on ABC (Fig. 106).

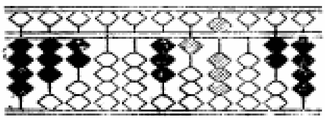
Step 2: Multiplying the 7 in 37 by



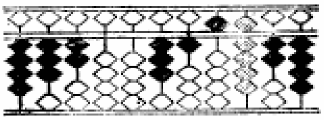
A B C D E F G H I J K
FIG. 108



A B C D E F G H I J K
FIG. 109

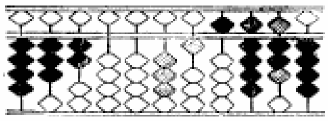


A B C D E F G H I J K
FIG. 110



A B C D E F G H I J K
FIG. 111

```
A B C D E F G H I J K
4 3 2 0 0 3 7 0 0 0 0   Step 1
              2 8       Step 2
                2 1     Step 3
                  1 4   Step 4
4 3 2 0 0 3 0 3 0 2 4
            1 2         Step 5
                9       Step 6
                  6     Step 7
4 3 2 0 0 0 1 5 9 8 4
```



A B C D E F G H I J K
FIG. 112

the 4 in 432, set the product 28 on HI (Fig. 107).

Step 3: Multiplying the same 7 in 37 by the 3 in 432, set the product 21 on IJ. This makes a total of 301 on HIJ (Fig. 108).

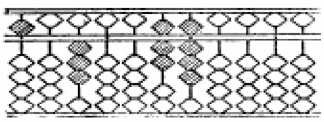
Step 4: Multiplying the same 7 in 37 by the 2 in 432, set the product 14 on JK, and clear G of its 7. This makes a total of 3,024 on HIJK (Fig. 109).

Step 5: Multiplying the 3 in 37 by the 4 in 432, set the product 12 on GH. This makes a total of 15,024 on GHIJK (Fig. 110).

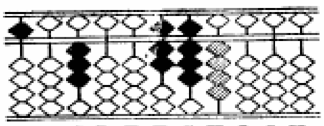
Step 6: Multiplying the 3 in 37 by the 3 in 432, set the product 9 on 1. This makes a total of 15,924 on GHIJK (Fig. 111).

Step 7: Multiplying the 3 in 37 by the 2 in 432, set the product 6 on J, and clear F of its 3. This makes, on GHIJK, a total of 15,984, which is the answer (Fig. 112).

Example 8. 78 x 503 = 39 234

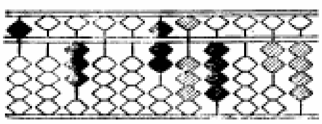


A B C D E F G H I J K
FIG. 113

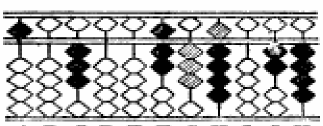


A B C D E F G H I J K
FIG. 114

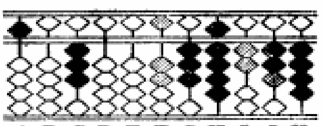
```
A B C D E F G H I J K
5 0 3 0 0 7 8 0 0 0 0   Step 1
              4 0       Step 2
                 +2 4   Step 3
5 0 3 0 0 7 0 4 0 2 4
           +3 5         Step 4
               +2 1     Step 5
5 0 3 0 0 0 3 9 2 3 4
```



A B C D E F G H I J K
FIG. 115



A B C D E F G H I J K
FIG. 116



A B C D E F G H I J K
FIG. 117

Step 1: Set 78 on FG, with G as the unit rod, and set 503 on ABC (Fig. 113).

Step 2: Multiplying the 8 in 78 by the 5 in 508, set the 4 of the product 40 on H (Fig. 114).

Step 3: Multiplying the same 8 in 78 by the 3 in 503, set the product 24 on JK and clear G of the 8. This makes a total of 4,024 on HIJK. In setting this product skip rod I as the second figure of the multiplier 503 is zero. In other words, the product must be set on JK instead of on IJ (Fig. 115).

Step 4: Multiplying the 7 in 78 by the 5 in 503, set the product 35 on GH. This makes a total of 39 024 on GHIJK (Fig. 116).

Step 5: Multiplying the same 7 in 78 by the 3 in 503, set the product 21 on IJ instead of HI, as the second figure of 503 is zero, and clear F of its 7. This leaves, on GHIJK, a total of 39 234, which is the answer (Fig. 117).

4. Exercises

Group I

1.	34 x 4 = 136	11.	21 x 23 = 483
2.	23 x 5 =115	12.	12 x 32 = 384
3.	12 x 4 = 48	13.	21 x 43 = 903
4.	82 x 3 = 96	14.	12 x 56 = 672
5.	21 x 5 = 105	15.	31 x 64 = 1 984
6.	33 x 45 = 1 485	16.	43 x 56 = 2 408
7.	52 x 56 = 2 912	17.	32 x 64 = 2 048
8.	23 x 65 = 1 495	18.	53 x 76 = 4 028
9.	53 x 75 = 3 975	19.	23 x 83 = 1 909
10.	25 x 85 = 2 125	20.	35 x 96 = 8 860

Group II

1.	112 x 23 = 2 576	11.	1 023 x 34 = 34 782
2.	123 x 35 = 4 305	12.	3 243 x 45 = 145 935
3.	212 x 46 = 9 752	13.	4 352 x 58 = 252 416
4.	845 x 57 = 19 665	14.	5 624 x 67 = 376 808
5.	423 x 64 = 27 072	15.	6 712 x 78 = 523 536
6.	513 x 76 = 38 9S8	16.	132 x 334 = 44 088
7.	607 x 87 = 52 809	17.	234 x 456 = 106 704
8.	452 x 85 = 38 420	18.	431 x 467 = 201 277
9.	631 x 95 = 59 945	19.	546 x 686 = 374 556
10.	608 x 97 = 58 976	20.	756 x 879 = 664 524

VI. DIVISION

There are two fundamental methods of division on the abacus. The older method, though still favored by some, has fallen out of general use since about 1930 because it requires the memorization of a special division table. The newer method, which is the easier to learn because it uses the multiplication instead of the division table, is the standard one now taught in grade schools, and will be introduced in the following pages. Strictly speaking, it is not new, as it has long been used, but only in a very limited use until around 1930 when it was improved and publicized. Standard terminology will be used in describing the method. For example, in the problem 50 / 5 = 10, 50 is the dividend, 5 the divisor, and 10 the quotient.

It is customary to set the dividend a little to the right of the central part of the abacus and the divisor at the left. The two numbers are generally separated by three or four unused rods. As the Abacus Committee favors leaving four unused rods between the two numbers, the following examples will adhere to that practice.

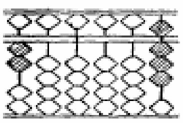
The method of division used here gives the first digit of the quotient between the dividend and divisor. Two main reasons can be given for setting the dividend on the right and the divisor on the left. One is that since the abacus is operated with the right hand, the reverse order of setting the two numbers would cause the multiplier to be hidden by the hand much of the time, as in the case of multiplication. The other is that in case the dividend is indivisible by the divisor, the reverse order would cause the quotient to extend right into the divisor.

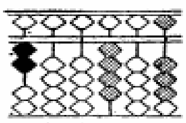
In division, as in multiplication, the use of the unit rod is not too essential, but does facilitate calculation in many ways. Therefore, the unit figure of the dividend is always set on a unit rod. When the divisor is a whole number, however, we shall disregard the unit rod, and simply set the divisor in such a way that its last digit is located on the fifth rod to the left of the dividend.

As for the order of setting the dividend and divisor, since the last digit of the dividend must be set on a unit rod, it is advisable for the beginner to set the dividend before setting the divisor. As is the case with multiplication, however, experts often reverse the procedure, setting the divisor first or not at all.

1. Dividing by One-Digit Numbers

Example 1. 8/ 2 = 4


A B C D E F
Fig. 118


A B C D E F
Fig. 119

A	B	C	D	E	F	
2	0	0	0	0	8	Step 1
			4			Step 2
					−8	
2	0	0	4	0	0	Result

Step 1: Set the dividend 8 on rod F and the divisor 2 on rod A, with four vacant rods between the two numbers. Make sure that F is a unit rod marked with a unit point (Fig. 118).

Step 2: Mentally divide 8 by 2 (8 / 2 = 4); set the quotient 4 on D, the second rod to the left of the dividend; and clear F of its 8. Fig. 119 and the row of figures designated Result in the diagram show the result of this step.

Example 2. 837+ 3 = 279

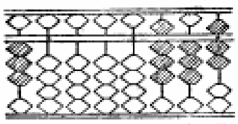


Fig. 120

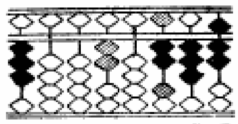


Fig. 121

```
ABCDEFGH
300008 3 7   Step 1
   2         Step 2
    -6
30020237
    7        Step 3
    -21
30027027
      9      Step 4
      -27
30027900
```

Step 1: Set 837 on the rods FGH, with H as the unit rod, and set 3 on A (Fig. 120).

Step 2: Compare the 3 with the 8 in 837. 3 goes into 8 twice with 2 left over. Set the quotient figure 2 on D, the second rod to the left of 8 in 837. Next multiply the divisor 3 by this quotient figure 2, and subtract the product 6 from the 8 on F. This leaves 2 on F (Fig. 121).

Step 3: Compare the 3 with 23 on FG. The 2 on F is the remainder left over as a result of the previous step. 3 goes into 23 seven times with 2 left over. Set 7 as the quotient figure on E. Next multiply the divisor 3 by this 7, and subtract the product 21 from the 23 on FG. This leaves 2 on G (Fig. 122).

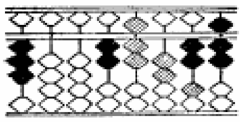


Fig. 122

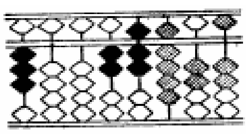


Fig. 123

Step 4: Compare the 3 with 27 on GH. The 2 on G is the remainder left over as a result of the second step. 3 goes into 27 nine times. Set the quotient figure 9 on F. Next multiply the 3 by this 9, and subtract the product 27 from the 27 on GH. This clears GH and leaves the answer 279 on DEF (Fig. 123).

Note: Answers to problems in division can be easily checked by multiplication. Thus, to check the foregoing answer, simply multiply the quotient 279 on DEF by the divisor 3, that is, the number you originally divided by, and you will get the product 837 on FGH, i. e., the same rods on which you had 837 as the dividend. By this checking the student will see that the position of the quotient in division is that of the multiplicand in multiplication, and that the position of the dividend in division is that of the product in multiplication. Therefore, we may say that the methods of multiplication and division introduced in this book form the counterpart of each other.

Example 3. 6 013 / 7 = 859

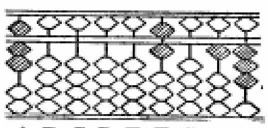


Fig. 124

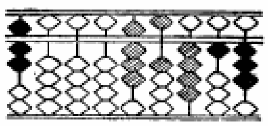


Fig. 125

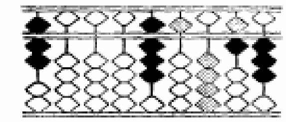
```
ABCDEFGHI
700006013   Step 1
    8       Step 2
    -56
700080413
     5      Step 3
     -35
700085063
      9     Step 4
      -63
700085900
```

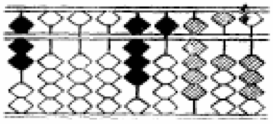
Step 1: Set 6,013 on FGHL with 1 as the unit rod, and set 7 on A (Fig. 124).

Step 2: Compare the divisor 7 with the 6 in 6,013. 7 will not go into 6. So compare the 7 with the 60 in 6,013. 7 goes into 60 eight times. In this case set the quotient figure 8 on E. the first rod to the left of the first digit of the dividend. Next multiply the divisor 7 by this 8, and subtract the product 56 from the 60 on FG. This leaves 4 on G (Fig. 125).

Step 3: Compare the 7 with 41 on GH 7 goes into 41 five times. Set the quotient figure 5 on

F. Next multiply the 7 by this 5, and subtract the product 35 from the 41 on GH. This leaves 6 on H (Fig. 126).


A B C D E F G H I
FIG. 126


A B C D E F G H I
FIG. 127

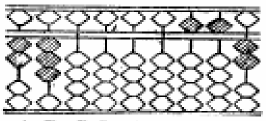
Step 4: Compare the 7 with 63 remaining on HI. 7 goes into 63 nine times. Set the quotient figure 9 on G. Next multiply the 7 by this 9, and subtract the product 63 from the 63 remaining on HI. This clears HI, and leaves the answer 859 on EFG (Fig. 127).

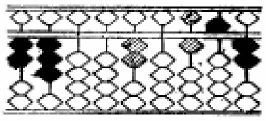
Note: When the divisor is larger than the first digit of the dividend, compare it with the first two digits of the dividend. In this case set the quotient figure on the first rod to the left of the first digit of the dividend. The chief merit of this procedure is that, in checking, the quotient multiplied by the divisor gives the product on the very rods on which the dividend was located previous to its division.

This procedure is the same as the principle of graphic division. In dividing 36 by 2, you write the quotient figure 1 above the 3 in 36. But in dividing 36 by 4, you write the quotient figure 9 above the 6 in 36. On the abacus board the quotient figure cannot be put above the dividend. So in dividing 36 by 2, the first quotient figure 1 is set on the second rod to the left of 36, while in dividing 36 by 4, the quotient figure 9 is set on the first rod to the left of 36.

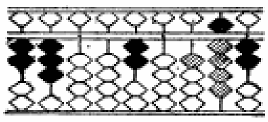
2. Dividing by Two-Digit Numbers

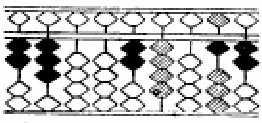
Example 4. 552 / 23 = 24

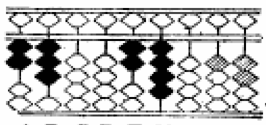

A B C D E F G H I
FIG. 128


A B C D E F G H I
FIG. 129

A	B	C	D	E	F	G	H	I	
2	3	0	0	0	0	5	5	2	Step 1
				2					Step 2
						−4			
							−6		Step 3
2	3	0	0	2	0	0	9	2	
					4				Step 4
							−8		
							−1	2	Step 5
2	3	0	0	2	4	0	0	0	


A B C D E F G H I
FIG. 130


A B C D E F G H I
FIG. 131


A B C D E F G H I
FIG. 132

Step 1: Set 552 on GHI, with 1 as the unit rod, and set 23 on AB (Fig. 128).

Step 2: Compare the 2 in 23 with the 5 in 552. 2 goes into 5 two times. Set the quotient figure 2 on E, the second rod to the left of the 5 in 552. Next multiply the 2 in 23 by this quotient figure 2, and subtract the product 4 from the 5 on G. This leaves 1 on G (Fig. 129).

Step 3: Now multiply the 3 in 23 by the same quotient figure 2, and subtract the product 6 from 15 on GH. This leaves 9 on H (Fig. 130).

Step 4: Compare the 2 in 23 with the 9 on H. 2 goes into 9 four times. Set the quotient figure 4 on F. Next multiply the 2 in 23 by this quotient figure 4, and subtract the product 8 from the 9 on H. This leaves 1 on H (Fig. 131).

Step 5: Multiply the 3 in 23 by the same 4, and subtract the product 12 from the 12 remaining on HI. This clears HI and leaves the answer 24 on EF (Fig. 132).

Example 5. 6 308 / 83 = 76

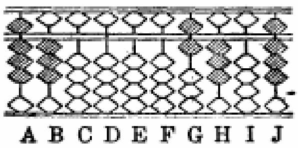


FIG. 133

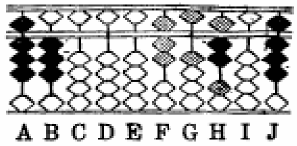


FIG. 134

A	B	C	D	E	F	G	H	I	J	
8	3	0	0	0	0	6	3	0	8	Step 1
					7					Step 2
					−	5	6			
						−	2	1		Step 3
8	3	0	0	0	7	0	4	9	8	
						6				Step 4
						−	4	8		
							−	1	8	Step 5
8	3	0	0	0	7	6	0	0	0	

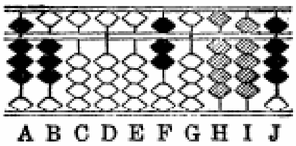


FIG. 135

Step 1: Set 6 308 on GHIJ, with J as the unit rod, and set 83 on AB (Fig. 133).

Step 2: Compare the 8 in 83 with the 6 in 6,308. 8 will not go into 6. So compare the 8 with the 63 in 6,308. 8 goes into 63 seven times. Set the quotient figure 7 on F, the first rod to the left of the 6 in 6,308. Next multiply the 8 in 83 by this 7 in the quotient, and subtract the product 56 from the 63 on GH. This leaves 7 on H (Fig. 134).

Step 3: Multiply the 3 in 83 by the same quotient figure 7, and subtract the product 21 from 70 on HI. This leaves 49 on HI (Fig. 135).

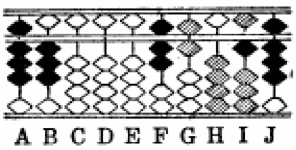


FIG. 136

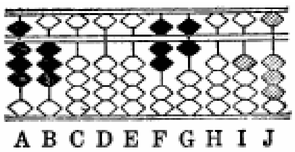


FIG. 137

Step 4: Compare the 8 in 83 with the 49 on HI. 8 goes into 49 six times. Set the quotient figure 6 on G. Next multiply the 8 in 83 by this 6, and subtract the product 48 from the 49 on HI. This leaves 1 on I (Fig. 136).

Step 5: Multiply the 3 in 83 by the same quotient figure 6, and subtract the product 18 from 18 on IJ. This clears IJ. and leaves the answer 76 on FG (Fig. 137).

Note: In case the divisor is a two-digit number, do not take the trouble of comparing its two digits with the first two or three digits of the dividend to work out the correct quotient figure mentally. Simply compare the first digit of the divisor with that of the dividend. When the first digit of the divisor is larger than that of the dividend, compare it with the first two digits of the dividend. In case quotient figures tried are incorrect, correct them by the methods shown in Examples 6, 7, and 8 instead of perplexing yourself with mental arithmetic. Thus make the most of the chief advantage of the abacus, the complete mechanical process which minimizes mental labor, and experience will enable you to find correct quotient figures at a glance.

Example 6. 4,698 / 54 = 87

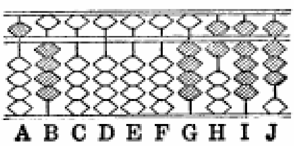


FIG. 138

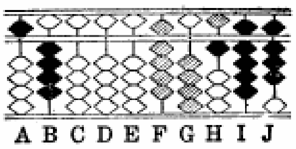


FIG. 139

This example shows how the process of division must be revised when too large a quotient figure has been used.

Step 1: Set 4 698 on GHIJ, with J as the unit rod, and set 54 on AB (Fig. 138).

Step 2: The 5 in 54 will not go into the 4 in 4 698. So compare the 5 with the 46 in 4 698. 5 goes into 46 nine times. Now suppose you have tried 9 as the quotient figure instead of the correct 8 and have set it on F. Then you will multiply the 5 in 54 by 9, and subtract the product 45 from the 46 on GH. This leaves 1 on H. Next multiplying the 4 in 54 by the same 9, you will find that the product 36 is larger than the 19 remaining on HI and that you ought to have tried a quotient figure one less than 9 (Fig. 139).

```
A B C D E F G H I J
5 4 0 0 0 0 4 6 9 8  Step 1
          9          Step 2
          - 4 5
          -(3 6)
5 4 0 0 0 9 0 1 9 8
          - 1        Step 3 (revision)
              + 5
5 4 0 0 0 8 0 6 9 8
              - 3 2  Step 4 (revision)
5 4 0 0 0 8 0 3 7 8
            7        Step 5
            - 3 5
              - 2 8  Step 6
5 4 0 0 0 8 7 0 0 0
```

Step 3: To revise the incorrect quotient figure 9 to 8, subtract 1 from the 9 on F, and you get the new quotient figure 8 on F. Next multiply the 5 in 54 by 1, i. e., the difference between the quotient figures 9 and 8, and add the product 5 to the 1 on H. Now you have 6 on H (Fig. 140).

Step 4: Multiply the 4 in 54 by the new quotient figure 8, and subtract the product 32 from 69 on HI. This leaves 37 on HI (Fig. 141).

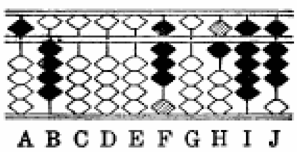


FIG. 140

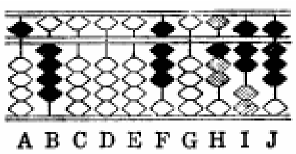


FIG. 141

Step 5: Compare the 5 in 54 with the 37 on HI. 5 goes into 37 seven times. So set the quotient figure 7 on G. Next multiply the 5 in 54 by this 7, and subtract the product 35 from the 37 on HI. This leaves 2 on I (Fig. 142).

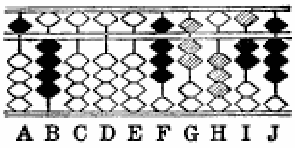


FIG. 142

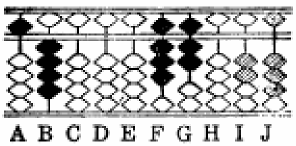


FIG. 143

Step 6: Multiply the 4 in 54 by the same quotient figure 7, and subtract the product 28 from 28 remaining on IJ. This clears IJ and leaves the answer 87 on FG (Fig. 143).

Example 7. 1 666 / 17 = 98

This example shows how a problem of division is worked when the first digit of both divisor and dividend are the same.

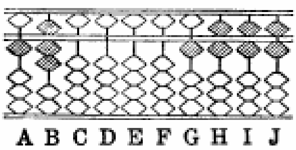


FIG. 144

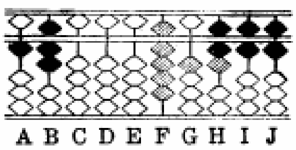


FIG. 145

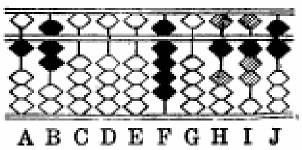


FIG. 146

```
A B C D E F G H I J
1 7 0 0 0 0 1 6 6 6  Step 1
          9          Step 2
            - 9
            - 6 3    Step 3
1 7 0 0 0 9 0 1 3 6
            9        Step 4
            - 9
            -(6 3)
1 7 0 0 0 9 9 0 4 6
          - 1        Step 5
              + 1
              - 5 6  Step 6
1 7 0 0 0 9 8 0 0 0
```

Step 1: Set 1 666 on GHIJ, with J as the unit rod, and set 17 on AB (Fig. 144).

Step 2: When the first digit of the divisor and the dividend are the same, as in this example, compare the second digits of the two numbers. In such a situation, if the second digit of the dividend is smaller than that of the divisor, try 9 as the quotient figure. If 9 is too large, try 8 as in Step 5 of this example. If 8 is still too large, go on trying a quotient figure one less till the correct one is found. In such a case 9 is the figure likeliest to be correct.

Now try 9 as the quotient figure and set it on F, the first rod to the left of the first digit of the dividend. Next multiply the 1 in 17 by this 9 and subtract the product 9 from 16 on GH. This leaves 7 on H (Fig. 145).

Step 3: Multiply the 7 in 17 by this same 9, and subtract the product 63 from 76 on HI. This leaves 13 on HI (Fig. 146).

Step 4: The 1 in 17 and the 1 remaining on H are the same. So compare the 7 in 17 and the 3 remaining on I. 3 is smaller than 7. So try 9 as the quotient figure and set it on G. Now multiply the 1 in 17 by this 9 and subtract the product 9 from the 13 on HI. This leaves 4 on 1. Next, multiplying the 7 in17 by this same 9, you will see that the product 63 is larger than 46 remaining on IJ. So you will find that you ought to have tried 8 as the quotient figure (Fig. 147).

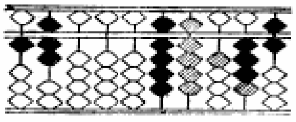


FIG. 147

Step 5: To revise the incorrect quotient figure 9 to 8, subtract 1 from the 9 on G. Next you must revise the division in Step 4. So multiply the 1 in 17 by 1, the difference between the 9 and 8, and add the product 1 to the 4 remaining on I. Then you get 5 on I (Fig. 148).

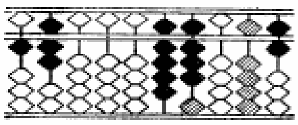


FIG. 148

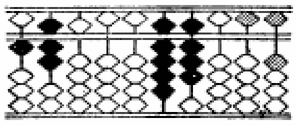


FIG. 149

Step 6: Multiply the 7 in 17 by the new quotient figure 8 and subtract the product 56 from 56 on IJ. This clears IJ and leaves the answer 98 on FG (Fig. 149).

Note: In cases where the first digits of both the divisor and the dividend are the same, if the second digit of the dividend is larger than that of the divisor, set 1 as the quotient figure on the second rod to the left of the first digit of the dividend. An instance is given in Example 9.

Example 8. 7 644 / 84 = 91

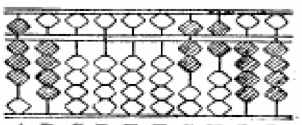


FIG. 150

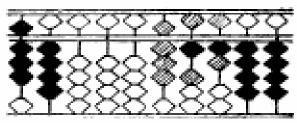


FIG. 151

This example is to show how division is to be revised when the quotient figure tried is too small.

Step 1: Set 7 644 on GHIJ, with J as the unit rod, and set 84 on AB (Fig. 150).

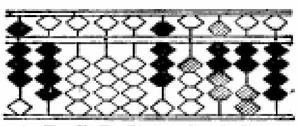


FIG. 152

Step 2: The 8 in 84 will not go into the 7 in 7 644. So compare the 8 with the 76 in 7 644. 8 goes into 76 nine times. So you ought to try 9 as the quotient figure. But suppose by mistake you have tried 8 as the quotient figure instead of the correct 9 and have set it on F. Then you will multiply the 8 in 84 by 8 and subtract the product 64 from the 76 on GH. This will leave 12 on GH (Fig. 151).

A	B	C	D	E	F	G	H	I	J	
8	4	0	0	0	0	7	6	4	4	Step 1
					8					Step 2
						−6	4			
							−3	2		Step 3
8	4	0	0	0	8	0	9	2	4	
					+1					Step 4
							−8	4		(Revision)
8	4	0	0	0	9	0	0	8	4	
						1				Step 5
								−8		
									−4	Step 6
8	4	0	0	0	9	1	0	0	0	

Step 3: Multiplying the 4 in 84 by the same quotient figure 8, you will subtract the product 32 from 124 on GHI. Then you will find that the remainder 92 is larger than 84 and that you ought to have tried 9, i.e., a quotient figure one more than 8 (Fig. 152).

Step 4: To revise the incorrect quotient figure 8 to 9, add 1 to the quotient figure 8 on F. Next multiply the divisor 84 by 1, i. e., the difference between the two quotient figures, 8 and 9, and subtract the product 84 from the 92 on HI. This leaves 8 on I (Fig. 153).

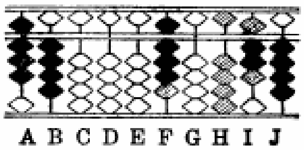


FIG. 153

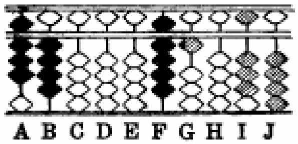


FIG. 154

Step 5: The 8 in 84 and the 8 remaining on I are the same. So compare the 4 in 84 with the 4 remaining on and you can see that they are also the same. Therefore, set the quotient figure 1 on G. Now, multiplying the 8 in 84 by 1, subtract the product 8 from the 8 on I. Next multiplying the 4 in 84 by the same 1, subtract the product 4 from the 4 on J. This clears IJ and leaves the quotient 91 on FG (Fig. 154).

Example 9. 3 978 / 234 = 17

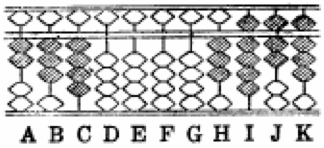


FIG. 155

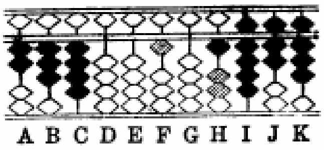


FIG. 156

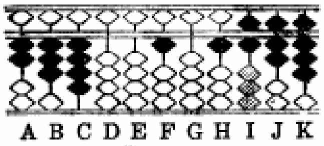


FIG. 157

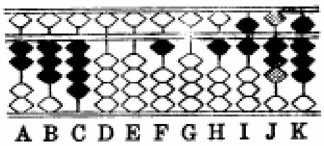


FIG. 158

```
ABCDEFGHIJK
23400003978  Step 1
     1       Step 2
    -2
     -3      Step 3
      -4     Step 4
23400101638
      8      Step 5
    -16
     -(24)
23400180038
     -1      Step 6
      +2
23400100238
      7
     -21     Step 7
      -28    Step 8
23400170000
```

Step 1: Set 3,978 on HIJK, with K as the unit rod, and set 234 on ABC (Fig. 155).

Step 2: Compare the 2 in 234 with the 3 in 3 978. 2 goes into 3 one time. Set the quotient figure 1 on F, the second rod to the left of the 3 in 3978. Now multiply the 2 in 234 by this quotient figure 1, and subtract the product 2 from the 3 on H. This leaves 1 on H (Fig. 156).

Step 3: Multiply the 3 in 234 by the same quotient figure 1, and subtract the product 3 from 9 on 1. This leaves 6 on 1 and 167 on HIJ (Fig. 157).

Step 4: Multiply the 4 in 234 by the same quotient figure 1, and subtract the product 4 from 7 on J. This leaves 3 on J and 1638 on HIJK (Fig. 158).

Step 5: Compare the 2 in 234 with the 16 remaining on HI. 2 goes into 16 eight times. Suppose you have tried 8 as the quotient figure instead of the correct 7 and have set it on G. Then you will multiply the 2 in 234 by 8, and subtract the product 16 from the 16 on HI. This clears HI. Next, multiplying the 3 in 234 by the same 8, you will find that the product 24 is larger than 3 remaining on J, and that you ought to have tried a quotient figure one less than 8 (Fig. 159).

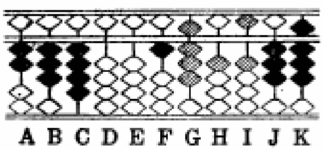


FIG. 159

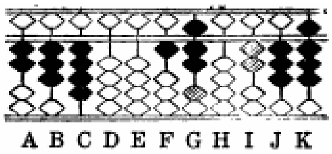


FIG. 160

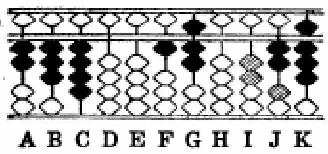


FIG. 161

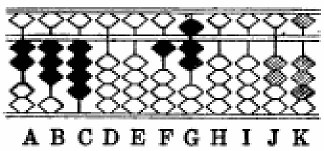


FIG. 162

Step 6: To revise the incorrect quotient figure 8 to 7, subtract 1 from the 8 on G, and you get the new quotient figure 7 on G. Next multiply the 2 in 234 by 1, i. e., the difference between the quotient figures 8 and 7, and set the product 2 on I. Now you have 2 on I and 234 on IJK (Fig. 160).

Step 7: Multiply the 3 in 234 by the new quotient figure 7, and subtract the product 21 from 23 on IJ. This leaves 2 on 3 and 28 on JK (Fig. 161).

Step 8: Next multiply the 4 in 234 by the same new quotient figure 7, and subtract the product 28 from 28 on JK. This clears JK and leaves the answer 17 on FG (Fig. 162).

Example 10. 7,061 / 307 = 23

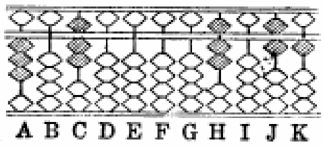


FIG. 163

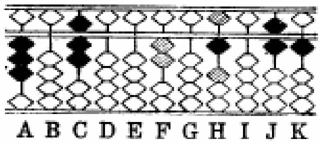


FIG. 164

Step 1: Set 7 061 on HIJK, with K as the unit rod, and set 307 on ABC, leaving as always four vacant rods between the two numbers (Fig. 163).

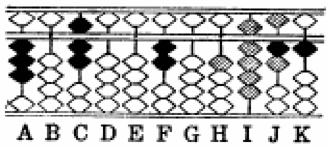


FIG. 165

```
A B C D E F G H I J K
3 0 7 0 0 0 0 7 0 6 1   Step 1
          2             Step 2
            − 6
                − 1 4   Step 3
3 0 7 0 0 2 0 0 9 2 1
            3           Step 4
              − 9
                − 2 1   Step 5
3 0 7 0 0 2 3 0 0 0 0
```

Step 2: Comparing the 3 in 307 and the 7 in 7,061 you can see that 3 goes into 7 two times. Set the quotient figure 2 on F. Next multiply the 3 in 307 by this 2, and subtract the product 6 from the 7 on H. This leaves 1 on H (Fig. 164).

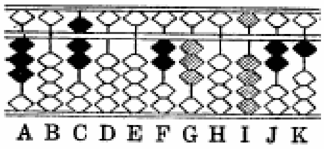


FIG. 166

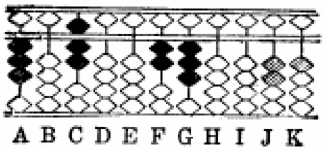


FIG. 167

Step 3: Multiply the 7 in 307 by the same quotient figure 2, and setting the product 14 on IJ, subtract it from 106 on HIJ. This leaves 92 on IJ. Since the second digit in 307 is zero, see that you set the product 14 on IJ instead of HI (Fig. 165).

Step 4: The 3 in 307 goes into the 9 on I three times. So set the quotient figure 3 on G. Next multiply the 3 in 307 by this quotient figure 3 and subtract the product 9 from the 9 on 1. This leaves 21 on JK (Fig. 166).

Step 5: Multiply the 7 in 307 by the same quotient figure 3 and subtract the product 21 from the 21 on JK. This clears JK and leaves the answer 23 on FG (Fig. 167).

4. Exercises

Group I

1.	24 / 2 = 12	11.	132 / 12 = 11
2.	36 / 3 = 12	12.	441 / 21 = 21
3.	115 / 5 = 23	18.	1 495 / 65 = 23
4.	204 / 6 = 34	14.	2 451 / 57 = 43
5.	357 / 7 = 51	15.	4 293 / 81 = 53
6.	424 / 4 = 106	16.	9 384 / 92 = 102
7.	4 008 / 8 = 501	17.	5 134 / 34 =151
8.	7 470 / 9 = 830	18.	4 635 / 45 = 103
9.	5 202 / 2 = 2 601	19.	15 990 / 78 = 205
10.	2 804 / 4 = 701	20.	84 056 / 14= 6 004

Group II

1.	8 296 / 68 = 122	11.	6 342 / 453 = 14
2.	4 270 / 14 = 305	12.	9 728 / 304 = 32
3.	11 100 / 75 = 148	13.	38 920 / 695 = 56
4.	7 560 / 28 = 270	14.	46 113 / 809 = 57
5.	24 957 / 47 = 531	15.	26 460 / 147 =180
6.	42 024 / 51 = 824	16.	215 940 / 236 = 915
7.	48 052 / 82 = 586	17.	178 712 / 502 = 356
8.	87 608 / 94 = 932	18.	459 780 / 970 = 474
9.	21 245 / 35 = 607	19.	874 038 / 418 = 2 091
10.	73 264 / 76 = 964	20.	690 988 / 761 = 908

VII. DECIMALS

In addition and subtraction the unit point serves as the mark of a decimal point, and the calculation of decimal problems is quite the same as that of whole numbers.

However, in multiplication and division, you cannot easily find the unit rod of the product and that of the quotient unless you know two rules covering the position of the decimal point of the product and two others covering the position of the decimal point of the quotient. These four rules may be best explained and illustrated in paired counterparts. The first pair of rules applies to whole or mixed-decimal numbers, and the second to decimal fractions.

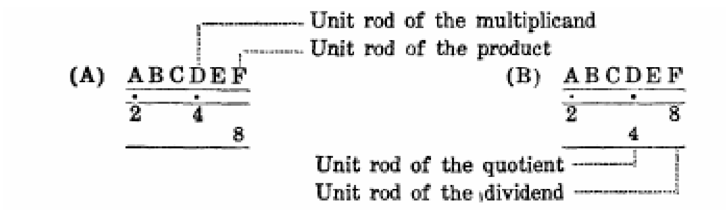
Rule A. When the multiplier is a whole or a mixed decimal, the unit rod of the product moves to the right of that of the multiplicand by as many rods plus one as there are whole digits in the multiplier.

Rule B. When the divisor is a whole or a mixed decimal number, the unit rod of the quotient moves to the left of the unit rod of the dividend by as many rods plus one as there are whole digits in the divisor.

Rule C. When the multiplier is a decimal fraction whose first significant figure is in the tens place, the last digit of the product is formed on the first rod to the right of the last digit of the multiplicand. Call this the basic rod. Then, each time the value of this multiplier is reduced by one place, the last digit of the product shifts by one rod to the left of this basic rod.

Rule D. When the divisor is a decimal fraction whose first significant figure is in the tens place, the last digit of the quotient is formed on the first rod to the left of the last digit of the dividend. Call this the basic rod. Then, each time the value of this divisor is reduced by one place, the last digit of the quotient shifts by one rod to the right of this basic rod.

Example 1. (A) 4 x 2 = 8 (B) 8 / 2 = 4



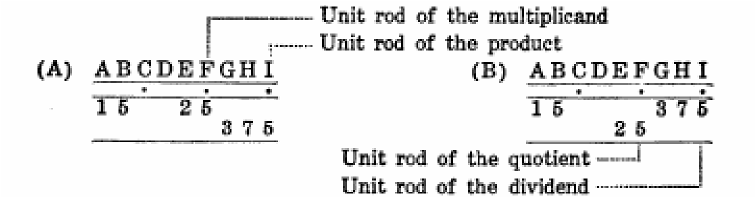
As seen in the first diagram above, showing the position of the multiplier (Rod A), the multiplicand (Rod D), and the product (Rod F), when the multiplier is a one-digit number, the unit rod of the product moves by two rods to the right of that of the multiplicand. In other words, the last digit of the product is formed on the second rod to the right of that of the multiplicand.

As seen in the second diagram above, showing the position of the divisor (Rod A), the dividend (Rod F), and the quotient (Rod D), when the divisor is a one-digit number, the unit rod of the quotient moves by two rods to the left of that of the dividend. In other words, the last digit of the quotient is formed on the second rod to the left of that of the dividend.

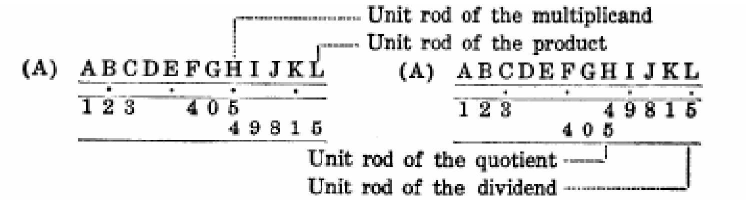
Example 2. (A) 25 x 15 = 375 (B) 375 / 15 = 25

The first diagram below shows that when the multiplier is a two-digit number, the last digit of the product is formed on the third rod to the right of that of the multiplicand.

The second diagram below shows that when the divisor is a two-digit number, the last digit of the quotient is formed on the third rod to the left of that of the dividend.



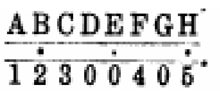
Example 3. (A) 405 X 123 = 49 815 (B) 49 815 / 123 = 405



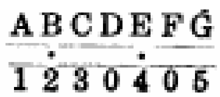
The first diagram above shows that when the multiplier is a three-digit number, the last digit of the product is formed on the fourth rod to the right of that of the multiplicand.

The second diagram shows that when the divisor is a three. digit number the last digit of the quotient is formed on the fourth rod to the left of that of the dividend.

Note on Example 3 (B): In case the dividend is separated from the divisor with four vacant rods, as in this example, the quotient product is clearly distinguishable from the divisor, since two vacant rods are left between them, thus:

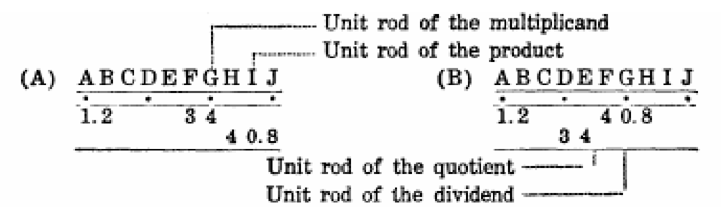


But if the dividend were separated from the divisor with only three vacant rods, the quotient produced would be hardly distinguishable from the quotient, since only one vacant rod would be left between them thus:



From this example the reader will see that in case the second figure of the quotient is a cipher, the quotient is hardly distinguishable from the divisor. Thus it is always preferable to set the divisor on the fifth instead of the fourth rod to the left of the dividend.

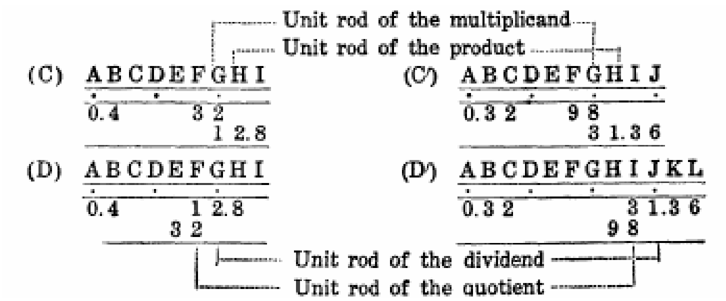
Example 4. (A) 34 x 1.2 = 40.8 (B) 40.8 / 1.2 = 34



Observe the first diagram above, and you will find that when the multiplier is a mixed number, the last whole digit of the product moves to the right of that of the multiplicand by as many rods plus one as there are whole digits in the multiplier.

Observe the second diagram, and you will find that when the divisor is a mixed number, the last whole digit of the quotient moves to the left of that of the dividend by as many rods plus one as there are whole digits in the divisor.

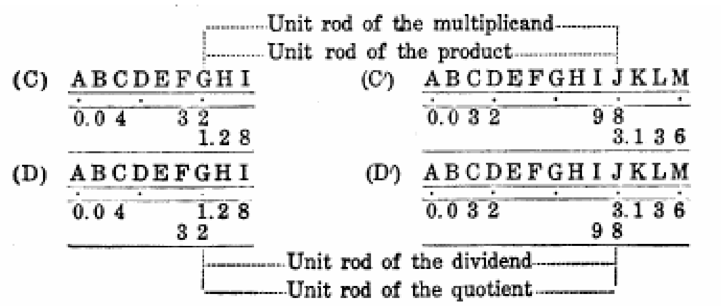
Example 5. (C) 32 x 0.4 =12.8 (C') 98 x 0.32 = 31.36
(D) 12.8 / 0.4 = 32 (D') 31.36 / 0.32 = 98



Diagrams C and C' above show that when the divisor is a decimal fraction whose first significant figure is in the tenth place, the last whole digit of the product is formed on the first rod to the right of that of the multiplicand.

Diagrams D and D', above show that when the divisor is a decimal fraction whose first significant figure is in the tenth place, the last whole digit of the quotient is formed on the first rod to the left of that of the dividend.

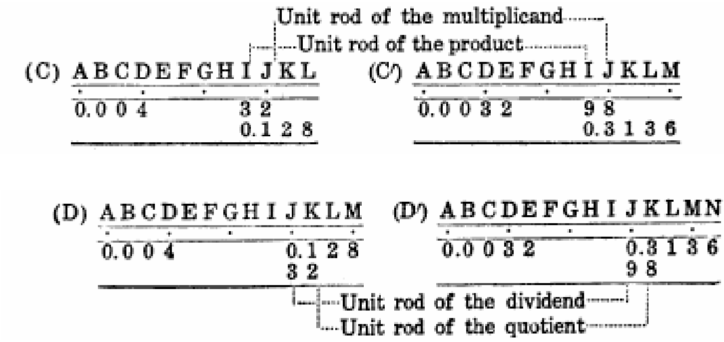
Example 6. (C) 32 x 0.04 = 1.28 (C') 98 x 0.032 = 3.136
(D) 1.28 / 0.04 = 32 (D') 3.136 / 0.032 = 98



Diagrams C and C' above show that when the multiplier is a decimal fraction whose first significant figure is in the hundredth place, the last whole digit of the product is formed on the very rod on which that of the multiplicand is located.

Diagrams D and D' above show that when the divisor is a decimal fraction whose first significant figure is in the hundredth place, the last whole digit of the quotient is formed on the very rod on which that of the dividend is located.

Example7. (C) 32 x 0.004 = 0.128 (C') 98 x 0.0032 = 0.3136
(D) 0.128 / 0.004 = 32 (D') 0.3136 / 0.0032 = 98



Diagrams C and C' above show that when the multiplier is a decimal fraction whose first significant figure is in the thousandth place, the last whole digit of the product is formed on the first rod to the left of the last whole digit of the multiplicand.

Diagrams D and D' show that when the divisor is a decimal fraction whose first significant figure is in the thousandth place, the last whole digit of the quotient is formed on the first rod to the right of the last whole digit of the dividend.

VIII. MENTAL CALCULATION

All abacus experts can calculate mentally with miraculous rapidity. On an average they are twice as quick in mental calculation as on the abacus. It is possible for anyone to attain astonishing rapidity in such mental calculation by proper practice. The secret lies in applying abacus calculation to mental arithmetic by visualizing abacus manipulation.

Here are the vital points:

1. For example, in adding 24 to 76, close your eyes and visualize the beads of an abacus set to 76. Then mentally add 24 onto the beads, aiding your visualization of the abacus by flicking the index finger and thumb of your right hand as if really calculating on an abacus.

2. When adding a series of numbers, say, 24 + 76 + 62 + 50, aid your memory by folding one of your left fingers each time the sum has come up to 100.

3. At first, practice the addition of numbers of two or more digits which come up to a round sum, for example, 76 + 24, or 222 + 555 + 223, and the like.

4. Remember, practicing a few minutes at a time for many days is worth more than practicing hours on a single day.

EXERCISES

Constant daily practice is essential if one is to become proficient in the use of the abacus. The following exercises, prepared and arranged in accordance with the most up-to-date methods, have been kindly furnished by Professor Miyokichi Ban, an outstanding abacus authority. They will provide a good beginning for the serious student, who can then find more problems in any ordinary arithmetic book. Also note that problems in multiplication and division may be used as problems in addition and subtraction respectively.

The exercises are arranged so that a student can measure his progress against the yardstick of the Japanese licensing system, the required standard of proficiency for the particular grade being given at the beginning of each group. The possessor of a first, second or third grade license, as awarded by the Abacus Committee, is officially qualified for employment in a public corporation or business house. Licenses for the lower grades are given on the basis of unofficial examinations conducted by numerous private abacus schools.

The exercises are also chosen to give the maximum of variety to the problems, with each digit from zero to nine receiving equal attention - an essential requirement for improvement in abacus operation. The system followed is that just initiated by the Central Abacus Committee after long and careful research.

I - Sixth Grade Operator

Group A
(1 set per minute, or entire group with 70% accuracy in 10 minutes.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
528	967	482	106	815	360	769	241	654	185
160	239	251	543	302	829	420	952	516	730
427	650	147	928	491	213	195	309	740	698
951	108	598	710	852	308	–513	756	495	246
719	243	120	954	–169	497	–854	487	536	809
452	758	–643	329	–401	932	508	360	–785	953
106	491	–839	267	–958	589	274	617	–320	721
843	536	304	695	576	690	421	508	–197	370
385	702	987	514	740	147	963	873	201	617
690	871	439	870	183	674	–307	420	873	164
724	460	-671	308	–235	850	–631	196	124	902
381	629	–305	796	–673	201	–175	689	–482	596
203	984	–526	632	340	765	286	204	–968	480
579	315	760	807	927	148	840	795	319	342
634	870	215	481	264	756	392	138	203	875
7 782	8 823	1 319	8 940	3 054	7 959	2 588	7 545	1 909	8 688

Group B
(1 set per minute or entire group with 70% accuracy in 10 minutes.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
728	36	627	52	271	13	806	4 105	936	3 094
631	87	75	718	6 104	624	54	78	21	65
4 089	52	238	60	42	51	–716	29	6 084	413
50	705	–94	42	35	379	–81	712	–18	817
92	68	–426	8 096	487	785	439	67	–745	23
175	96	–81	621	93	41	5 021	834	–50	71
47	349	7 513	97	312	2 095	48	203	62	938
904	2 138	59	481	–78	36	27	40	329	26
72	510	60	305	–5 083	984	–78	526	–1 203	7 041
593	74	–3 041	574	20	63	–3 605	81	–459	86
61	421	-52	85	961	542	–953	9 036	48	259
86	907	139	19	–854	807	130	17	76	95
8 260	53	807	73	–69	1 068	69	395	597	508
354	619	40	9 038	–705	70	92	58	30	672
13	8 042	968	264	96	92	247	649	817	40
16 155	14 157	6 832	20 520	1 632	7 650	1 500	16 830	6 525	14 148

Group C
(70% accuracy, 5 minutes.)

(1) 187 x 53 = 9 911
(2) 245 x 21 = 5 145
(3) 309 x 19 = 5 871
(4) 408 x 38 = 15 504
(5) 561 x 60 = 33 660
(6) 620 x 42 = 26 040
(7) 716 x 90 = 64 440
(8) 832 x 57 = 47 424
(9) 954 x 74 = 70 596
(10) 973 x 86 = 83 678

Group D
(70% accuracy, 5 minutes.)

(1) 1 725 x 51 = 87 975
(2) 2 698 x 24 = 64 752
(3) 3 980 x 30 = 119 400
(4) 4 509 x 65 = 293 085
(5) 5 062 x 73 = 369 526
(6) 6 874 x 68 = 467 432
(7) 7 431 x 80 = 594 480
(8) 8 146 x 12 = 97 752
(9) 9 357 x 49 = 458 493
(10) 8 230 x 97 = 798 310

Group E
(70% accuracy, 5 minutes.)

(1) 960 / 24 = 40
(2) 810 / 45 = 18
(3) 7 505 / 79 = 95
(4) 6 640 / 80 = 83
(5) 5 920 / 16 = 37
(6) 4 080 / 68 = 60
(7) 3 127 / 53 = 59
(8) 2 160 / 30 = 72
(9) 1 152 / 72 = 16
(10) 2 184 / 91 = 24

Group F
(70% accuracy, 5 minutes.)

(1) ¥ 986 / 34 = ¥ 29
(2) ¥ 855 / 19 = ¥ 45
(3) ¥ 7 280 / 80 = ¥ 91
(4) ¥ 6 240 / 78 = ¥ 80
(5) ¥ 5 092 / 67 = ¥ 76
(6) ¥ 4 128 / 96 = ¥ 43
(7) ¥ 390 / 13 = ¥ 130
(8) ¥ 2 320 / 40 = ¥ 58
(9) ¥ 1 550 / 25 = ¥ 62
(10) ¥ 884 / 52 = ¥ 17

II - Fifth Grade Operator

Group A
(70% accuracy, 10 minutes.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
425	619	502	7 245	167	895	3 471	912	237	498
839	153	698	461	9 035	7 043	584	463	4 093	6 309
5 302	762	2 013	956	716	509	628	9 324	308	146
791	8 523	147	179	3 540	2 918	309	647	7 410	285
514	—478	9 684	317	295	451	5 672	1 509	—984	5 037
1 283	—694	5 726	420	138	734	491	854	—2 536	708
960	—7 081	409	8 096	869	1 086	—236	6 082	—841	3 572
2 048	377	971	—543	327	427	—7 018	165	965	219
683	1 049	3 056	—835	5 609	308	4 763	978	8 074	867
4 067	812	843	2 684	952	872	517	240	129	421
794	9 235	329	708	4 786	164	902	2 086	715	1 653
3 176	—504	760	1 032	128	6 217	8 059	731	—5 268	820
952	—6 380	4 215	—319	8 073	9 620	—126	3 108	—607	9 046
805	426	837	—908	409	563	—340	597	259	714
671	905	158	—6 527	241	35	—895	375	136	935
23 310	7 724	30 348	12 966	35 285	31 842	16 781	28 071	12 090	31 230

Group B
(70% accuracy, 10 minutes.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
3 128	504	196	837	6 059	463	958	7 342	217	592
940	6 142	2 501	7 590	3 240	8 594	149	3 180	1 048	403
8 437	879	385	2 601	918	3 027	4 807	453	9 731	1 780
1 056	—5 263	6 210	9 082	136	—7 380	758	9 508	5 109	6 814
9 582	—198	1 479	—649	2 847	—2 759	3 146	4 261	8 925	509
865	7 920	8 937	—3 078	9 325	—902	6 291	937	—2 346	7 921
5 297	691	7 068	—429	1 706	6 178	530	8 679	—697	2 078
7 809	4 702	927	8 563	695	4 813	2 085	518	7 163	8 637
254	9 087	4 253	930	8 014	632	5 239	6 024	3 508	981
2 375	6 815	5 406	—358	4 587	9 264	7 621	2 187	4 386	3 146
718	3 960	8 045	—1 876	273	850	1 375	9 840	—6 029	4 065
6 042	—1 536	712	—6 145	7 208	1 096	8 407	5 706	—275	8 254
163	—8 473	9 634	7 201	462	—5 709	4 362	695	—5 314	672
4 301	—348	843	5 914	3 159	—641	9 024	1 039	430	5 890
1 649	2 057	3 521	4 726	5 341	8 175	613	726	852	9 763
52 116	26 939	60 117	34 909	53 970	25 701	55 365	61 095	26708	61 005

Group C
(70% accuracy, 10 minutes.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
359	9 535	604	413	2 190	4 157	516	8 594	740	1 507
7 569	174	2 895	390	647	823	7 082	250	6 294	960
408	812	731	706	574	496	395	913	103	8 023
163	3 720	1 048	6 054	1 697	273	279	–341	857	649
914	–647	269	318	481	5 082	153	–7 082	5 019	752
792	–1 093	817	8 249	156	761	6 042	361	431	836
5 021	356	4 580	–634	3 078	–845	821	2 473	520	6 395
8 630	2 680	932	–267	423	–3 978	3 150	749	648	548
325	906	126	–9835	310	–109	264	605	3 976	125
206	185	658	142	962	634	4 987	128	837	409
127	–263	9 053	5 061	835	2 014	730	9 086	2 719	287
4 381	–8 472	316	975	5 289	361	674	–597	301	9 168
948	–598	470	–7 529	206	–580	968	–1 632	4 658	314
6 057	704	3 749	–807	4 058	–6 792	401	–805	285	7 431
874	419	527	182	739	905	5 893	467	961	270
36 774	8 418	26 775	3 418	21 645	3 202	32 355	13 169	28 349	37 674

Group D
(70 % accuracy, 5 minutes.)

(1) 942 x 495 = 466 290
(2) 839 x 457 = 383 423
(3) 723 x 980 = 708 540
(4) 680 x 134 = 91 120
(5) 508 x 268 = 136 144
(6) 417 x 873 = 364 041
(7) 396 x 629 = 249 084
(8) 204 x 316 = 64 464
(9) 165 x 501 = 82 665
(10) 751 x 702 = 527 202

Group E
(70% accuracy, 5 minutes.)

(1) 848 x 276 = 96 048
(2) 854 x 965 = 824 110
(3) 902 x 804 = 725 208
(4) 627 x 108 = 67 716
(5) 105 x 519 = 54 495
(6) 570 x 843 = 480 510
(7) 489 x 751 = 367 239
(8) 613 x 397 = 243 361
(9) 236 x 632 = 149 152
(10) 791 x 420 = 332 220

Group F
(70% accuracy 5 minutes.)

(1) 9 724 / 26 = 374
(2) 8 151 / 13 = 627
(3) 7 739 / 71 = 109
(4) 62 560 / 80 = 782
(5) 5 556 / 12 = 463
(6) 49 572 / 54 = 918
(7) 30 150 / 67 = 450
(8) 23 128 / 98 = 236
(9) 17 535 / 35 = 501
(10) 43 855 / 49 = 895

Group G
(70% accuracy 5 minutes.)

(1) 63 207 / 90 = 7 023
(2) 533 484 / 87 = 6 132
(3) 420 616 / 74 = 5 684
(4) 113 148 / 63 = 1 796
(5) 278 772 / 52 = 5 361
(6) 85 075 / 41 = 2 075
(7) 366 873 / 39 = 9 407
(8) 98 504 / 28 = 3 518
(9) 72 885 / 15 = 4 859
(10) 214 240 / 26 = 8 240

III - Fourth Grade Operator

Group A
(70% accuracy, 10 minutes.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
6 374	4 561	3 458	9 526	1 459	7 201	2 951	5 482	8 954	6 753
5 021	9 753	2 983	4 198	3 146	6 759	9 160	8 035	4 710	1 832
7 913	3 670	4 120	8 973	4 723	1 093	3 294	4 973	3 986	8 094
9 265	–1 256	6 309	7 269	2 368	2 346	8 643	5 106	5 603	2 869
4 537	–5 904	5 092	9 085	6 912	8 712	–4 712	3 683	9 215	7 150
5 084	8 329	2 148	–6 450	5 279	6 507	–1 035	1 290	5 087	3 908
8 762	2 048	5 871	–8 317	2 905	3 874	–5 368	9 624	1 693	9 201
7 190	6 827	1 397	–3 802	7 816	4 158	7 214	2 067	6 854	4 127
3 856	–7 895	7 539	1 236	3 047	8 527	6 870	6 541	2 401	9 382
4 280	–3 047	2 710	7 084	8 102	4 096	5 907	3 712	–6 049	5 240
8 152	–2 739	6 087	6 735	5 680	3 648	–7 582	6 154	–8 731	8 476
2 409	6 180	8 264	–4 391	9 834	5 930	–4 326	7 298	–3 278	6 395
6 371	5 912	4 675	–5 140	5 091	2 489	8 059	4 870	7 142	7 564
1 948	4 106	9 561	2 604	6 470	1 360	9 781	7 951	9 320	5 013
9 603	1 438	3 406	5 172	7 538	9 125	6 403	8 309	7 562	4 671
90 765	31 983	73 620	33 782	80 370	75 825	45 259	85 095	54 469	90 675

Group B
(70% accuracy, 10 minutes.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
2 453	5 906	3 629	8 431	9 504	6 712	72 934	1 846	4 165	8 679
5 192	98 710	41 568	7 846	1 029	8 950	1 653	30 782	1 594	15 032
67 941	4 825	9 205	20 753	85 934	4 167	3 817	4 935	3 872	6 985
36 029	5 174	80 453	4 501	2 318	60 843	9 051	8 071	51 943	2 803
1 683	9 631	–2 687	59 327	–7 840	6 679	1 329	6 329	92 381	41 697
4 808	3 712	–31 405	7 698	–43 126	1 703	–53 682	95 874	–6 705	9 306
8 574	20 893	8 716	3 086	–2 735	29 078	–8 306	3 105	–4 372	5 821
3 129	8 074	4 932	64 279	3 052	8 634	–2 148	5 763	–87 156	4 068
48 531	2 689	–17 043	1 058	9 607	39 581	80 597	78 491	9 034	37 459
4 067	60 243	–5 890	6 912	17 269	7 824	5 431	40 982	13 507	2 516
73 215	87 069	–9 134	2 769	8 573	2 307	14 970	2 417	5 420	70 128
2 750	1 574	4 716	90 635	–54 618	40 592	–7 026	7 650	7 263	8 749
10 896	6 458	76 352	5 804	–6 481	91 486	–25 469	63 208	–20 489	3 290
7 208	53 961	2 879	8 140	70 396	6 251	4 205	9 016	–8 016	9 347
9 645	2 307	5 021	1 972	4 125	8 095	6 748	2 569	2 698	60 175
285 621	371 226	171 312	323 211	97 007	316 902	104 104	361 038	65 139	286 055

Group C
(70% accuracy, 10 minutes.)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
2 701	58 976	378	683	8 472	4 583	75 219	839	13 597	9 218
342	4 109	917	50 746	601	13 659	327	523	763	40 137
60 179	249	4 586	–1 597	7 340	732	139	19 768	27 458	572
9 630	80 651	1 469	–265	45 981	403	–6 875	5 017	89 162	1 065
71 524	1 590	70 854	619	790	29 581	–906	240	430	56 387
8 495	–468	3 042	7 301	13 267	690	50 864	6 754	2 049	941
50 723	–5 107	793	25 478	6 154	32 071	8 542	30 482	–98 625	259
268	–67 328	2 138	9 032	24 896	947	41 038	1 937	–506	7 308
956	782	96 205	–359	974	8 765	–751	684	–4 317	32 596
814	8 245	7 621	–40 168	32 058	1 296	–60 213	93 521	5 231	483
5 209	92 361	5 839	–8 274	613	95 820	–2 496	8 075	70 318	60 725
43 167	873	19 057	63 921	50 789	7 415	9 547	42 816	–2 679	864
1 083	70 934	415	34 580	9 523	178	37 081	7 109	–985	24 139
548	–3 012	80 246	904	862	80 264	320	24 365	6 140	8 074
86 937	–654	68 120	2 817	1 035	6 304	4 693	690	804	1 690
342 576	237 201	361 180	145 418	203 355	282 708	156 524	242 820	108 840	244 458

Group D
(70% accuracy, 5 minutes.)

(1) 92 854 x 84 = 7 799 736
(2) 86 213 x 59 = 5 086 567
(3) 73 041 x 90 = 6 573 690
(4) 60 378 x 16 = 966 048
(5) 51 762 x 27 = 1 397 574
(6) 47 609 x 70 = 3 382 630
(7) 30 427 x 32 = 973 664
(8) 29 185 x 63 = 1 838 655
(9) 18 596 x 45 = 836 820
(10) 54 930 x 81 = 4 449 330

Group E
(70% accuracy, 5 minutes.)

(1) 1 375 x 562 = 772 750
(2) 2 610 x 148 = 386 280
(3) 3 784 x 625 = 2 365 000
(4) 4 208 x 201 = 845 808
(5) 5 429 x 874 = 4 744 946
(6) 6 057 x 903 = 5 469 471
(7) 7 906 x 417 = 3 296 802
(8) 8 591 x 730 = 6 271 430
(9) 9 832 x 986 = 9 694 352
(10) 4 163 x 359 = 1 494 517

Group F
(70% accuracy 5 minutes.)

(1) 9 108 x 379 = 3 451 932
(2) 8 240 x 568 = 4 680 320
(3) 7 894 x 740 = 5 841 560
(4) 6 372 x 953 = 6 072 516
(5) 5 423 x 182 = 986 986
(6) 4 617 x 194 = 895 698
(7) 3 581 x 807 = 2 889 867
(8) 2 056 x 625 = 1 285 000
(9) 1 905 x 401 = 763 905
(10) 3 769 x 236 = 889 484

Group G
(70% accuracy 5 minutes.)

(1) 2 647 x 3 740 = 9 899 780
(2) 3 068 x 2 698 = 8 277 464
(3) 9 854 x 7 219 = 71 136 026
(4) 1 370 x 4 805 = 6 582 850
(5) 8 401 x 6 457 = 54 245 257
(6) 4 936 x 9 523 = 47 005 528
(7) 6 125 x 5 184 = 31 752 000
(8) 2 519 x 8 306 = 20 922 814
(9) 7 093 x 1 962 = 13 916 466
(10) 5 782 x 3 071 = 17 756 522

Group H
(70% accuracy 5 minutes.)

(1)	379 428 / 42 =	9 034
(2)	706 860 / 85 =	8 316
(3)	235 662 / 31 =	7 602
(4)	658 145 / 97 =	6 785
(5)	406 164 / 68 =	5 973
(6)	87 362 / 19 =	4 598
(7)	115 710 / 30 =	3 857
(8)	55 637 / 23 =	2 419
(9)	94 240 / 76 =	1 240
(10)	325 134 / 54 =	6 021

Group I
(70% accuracy 5 minutes)

(1)	94 235 / 401 =	235
(2)	87 040 / 256 =	340
(3)	752 128 / 832 =	904
(4)	64 220 / 380 =	169
(5)	548 784 / 927 =	592
(6)	431 748 / 603 =	716
(7)	385 746 / 478 =	807
(8)	219 537 / 519 =	423
(9)	107 912 / 164 =	658
(10)	620 895 / 795 =	781

Group J
(70% accuracy 5 minutes.)

(1)	636 768 / 792 =	804
(2)	35 984 / 208 =	173
(3)	194 394 / 537 =	362
(4)	403 425 / 815 =	495
(5)	86 163 / 373 =	231
(6)	531 340 / 620 =	857
(7)	932 832 / 984 =	948
(8)	286 090 / 469 =	610
(9)	363 726 / 501 =	726
(10)	74 314 / 146 =	509

Group K
(70% accuracy 5 minutes.)

(1)	553 149 / 6 829 =	81
(2)	273 375 / 3 645 =	75
(3)	97 351 / 1 453 =	67
(4)	761 664 / 7 934 =	96
(5)	81 018 / 4 501 =	18
(6)	84 942 / 2 178 =	39
(7)	424 901 / 8 017 =	53
(9)	120 048 / 5 002 =	24
(9)	611 640 / 6 796 =	90
(10)	394 044 / 9 382 =	42

IV - Third Grade Operators

Group A
(70% accuracy, 5 minutes.)

(1)	(2)	(3)	(4)	(5)
8 127	526	105 942	41 306	28 640
659	4 192	835	7 962	135
17 492	60 271	94 516	95 641	86 029
961 037	358 604	62 481	529	401 286
5 208	–963	83 672	890 375	514
638 125	–71 850	1 450	–6 813	37 269
80 734	–5 397	238 107	–380 276	2 478
9 270	409 715	396	784	903 851
25 816	842	740 138	978 250	18 394
401 369	–17 438	253	3 795	549 076
756	–732 609	57 048	12 047	3 702
36 594	90 386	6 729	604 518	153
520 943	6 127	78 915	–421	695 718
481	28 459	406 329	–53 608	4 367
74 308	813 045	9 067	–29 134	70 925
2 780 919	943 910	1 885 878	2 164 955	2 802 537

(6)	(7)	(8)	(9)	(10)
350 624	79 328	60 382	952	14 538
93 041	8 653	847	38 207	751
–68 729	184 705	2 415	706 394	26 374
–134	31 894	531	2 460	652 096
791 560	936	891 270	875	8 725
45 287	507 269	63 092	410 936	479 163
2 759	421	–726	9 283	680
59 641	60 584	–308 619	65 148	13 849
–6 358	245 139	–15 974	931 054	804 975
–804 293	372	704 368	729	5 812
–15 807	4 715	–3 051	54 071	30 427
983	693 807	–57 249	13 567	209
7 410	16 042	425 698	6 182	90 412
362	2 610	9 703	207 648	587 936
201 876	80 597	56 184	89 513	3 160
658 222	1 917 072	1 828 871	2 537 019	2 719 107

Group B

(70% accuracy 10 minutes. Calculate problems 1-10 to the nearest thousandth; 11-20 to the nearest dollar.)

(1)	4 097 x 238 =	975 086	(11)	$ 2 594 x 376 =	$ 975 344
(2)	5 638 x 149 =	840 062	(12)	$ 4 608 x 0.189 =	$ 871
(3)	14 902 x 52 =	774 904	(13)	$ 7 832 x 897 =	$ 7 025 304
(4)	7 105 x 0.098 =	696.29	(14)	$ 94 120 x 6.4 =	$ 602 368
(5)	9 674 x 603 =	5 833 422	(15)	$ 8 029 x 738 =	$ 5 925 402
(6)	63.25 x 7.64 =	483.23	(16)	$ 975 x 45.12 =	$ 43 992
(7)	853 x 4.017 =	3 426 501	(17)	$ 5 176 x 0.625 =	$ 3 235
(8)	0.3081 x 0.926 =	0.285	(18)	$ 3 061 x 903 =	$ 2 764 083
(9)	2 984 x 351 =	1 047 384	(19)	$ 6 843 x 201 =	$ 1 375 443
(10)	0.2176 x 87.5 =	19.04	(20)	$ 6 549 x 643 =	$ 4 211 007

Group C

(70% accuracy 10 minutes. Calculate problems 1—10 to the nearest thousandth; 11—20 to the nearest dollar.)

(1)	937 015 / 965 =	971	(11)	$ 99 / 0.368 =	$ 269
(2)	0.08988 / 6.42 =	0.014	(12)	$ 83 619 / 27 =	$ 3 097
(3)	0.070654 / 0.136 =	0.520	(13)	$ 71 967 / 149 =	$ 483
(4)	63 366 / 708 =	89.5	(14)	$ 649 612 / 7 061 =	$ 92
(5)	55.426 / 214 =	0.259	(15)	$ 560 / 0.875 =	$ 640
(6)	415 473 / 591 =	703	(16)	$ 415 693 / 593 =	$ 701
(7)	315.333 / 45.9 =	6.87	(17)	$ 33 154 / 60.5 =	$ 548
(8)	280 932 / 82 =	3 426	(18)	$ 2 485 / 2.84 =	$ 875
(9)	17.316 / 0.037 =	468	(19)	$ 122 563 / 901 =	$ 136
(10)	241 893 / 7 803 =	31	(20)	$ 54 / 0.432 =	$ 125