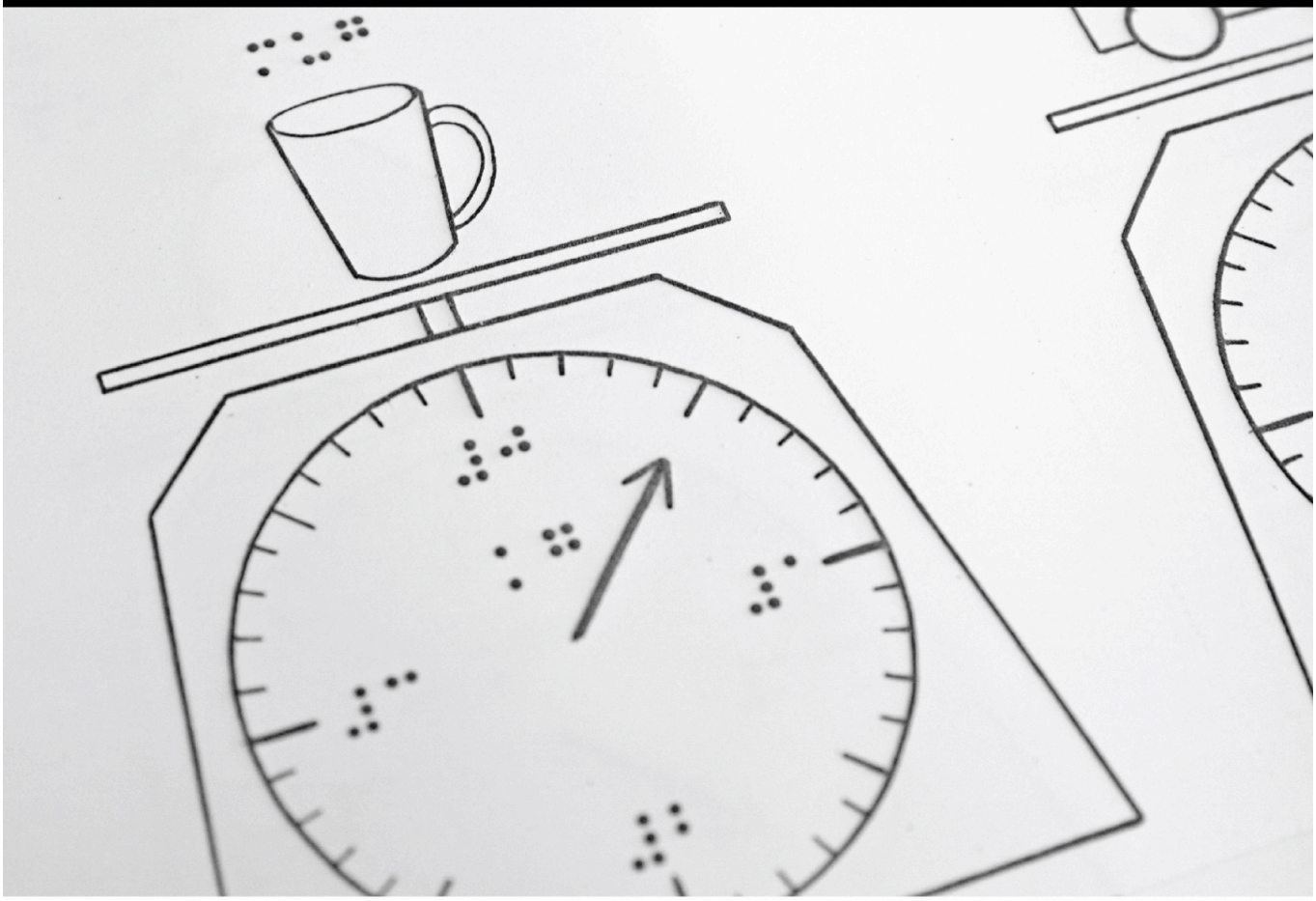
Next Sense

Advancing the RIDBC Renwick Centre experience.

Unified English Braille Training Manual

Introductory Mathematics

Josie Howse



UEBOnline

Unified English Braille Training Manual: Introductory Mathematics
Revision 7

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NextSense Institute is Australia's leading centre for research and professional studies in the field of education for children with sensory disabilities; offering webinars, short courses, and degree programs for parents, carers, educators, and health professionals. NextSense Institute is committed to providing high-quality teaching and learning opportunities. Our programs are conducted by leading national and international experts for education and health professionals who support people who are deaf, hard of hearing, blind or have low vision.

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Foreword

Mathematics is widely considered a core learning area during the formal years of education, since the study of mathematics involves reasoning, abstraction, generalisation and problem-solving, combined with use of symbols to access and express mathematical ideas and knowledge. Mathematics is, therefore, an essential component of education, employment, and everyday life in the 21st Century. The growth of accessible, inclusive digital technologies offers the opportunity for students with vision impairment or other print disabilities to participate in mathematics classrooms on an equitable basis with their sighted peers.

The **purpose** of this UEB introductory mathematics training manual and associated online training program is to clearly articulate the application of Unified English Braille (UEB) to reading and writing mathematical symbols and expressions during the primary years of education. Prior knowledge of UEB in literary contexts is considered essential before embarking on the UEB Introductory Mathematics lessons. You are referred to the UEB Online literacy training program that is available at https://uebonline.org. An understanding of UEB in literary contexts will enable professionals and parents to understand and connect language, literacy and mathematics development in children, and to plan teaching and learning experiences that are inclusive of children and young people with vision impairment.

The **target audience** for the training manual includes educators, parents and caregivers, allied health professionals, education administrators and policy makers who support or promote the use of braille as a medium for information access and communication in literacy and mathematical contexts.

The aims of UEB Introductory Mathematics are to enable professionals and parents/caregivers to:

- Develop knowledge of Unified English Braille as it is applied to the continuum of those mathematical concepts and symbols that are typically taught during the primary school years;
- Use their acquired knowledge of Unified English Braille to support braille-using students in accessing mathematics textbooks and print-based materials;
- Possess some basic skills in print-to-braille transcription; and
- Appreciate the enabling potential of knowledge of UEB in directly supporting students with vision impairment to appreciate, engage and communicate mathematical understanding in a broad range of contexts.

The Manual’s content is presented as a series of lessons that address specific topics in primary level mathematics. The lessons include practice and review exercises involving print to braille transcription. The lessons build on each other, enabling progressive consolidation and mastery of content.

NextSense acknowledges with thanks those organisations and individuals who have contributed to developing the introductory mathematics training manual and Online UEB Introductory Mathematics program – see https://uebonline.org. The UEB Online Project Team recognises the expertise of Josie Howse, our content author; online program developer Craig Cashmore of PeppaCode Pty Ltd; as well as Sonali Marathe and Tarna Cosgrove of Next Sense's Alternative Format Production department who have assured the accuracy of the braille content as well as the accessibility and layout of the training manual. The Project Team also extends its appreciation to the Duchen Family Foundation, the JLDJS Foundation, Sibley Endowment, the Skipper-Jacobs Charitable Trust and the Thomas Hare Investments Trust. Without their financial support, UEB Mathematics Online and this UEB introductory mathematics training manual would not have been possible.

We hope that this contribution to the disability field will provide professionals and parents with the required knowledge to support and encourage children and young people with vision impairment to actively and confidently engage with the exciting world of mathematics and scientific thought.

Frances Gentle
UEB Online Project Team Leader and Conjoint Lecturer,
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Contributors

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Josie Howse is an Adjunct Research Fellow with NextSense Institute and Manager of the Braille and Large Print Services, NSW Department of Education. The NSW Department of Education team provides all texts and examinations in braille, large print and e-text to students with vision impairment in the government sector and is the largest producer of alternate format student textbooks and examinations in Australia.

Josie has been working in the field of vision impairment for more than 40 years. She has held a number of executive positions at national and state levels and has extensive experience in braille code development at an international level. Josie was the editor of the 2006 Unified English Braille Primer: Australian Edition, and co-editor of the 2016 Unified English Braille: Australian Training Manual.

Josie was awarded the Public Service Medal (PSM) in the Queen's Birthday Honours list in 2007, has been listed annually in Who's Who of Australian Women since 2008, and is the recipient of a Lifetime Achievement Award from the Round Table on Information Access for People with Print Disabilities in 2012.

UEB Online Developer: Craig Cashmore

B. Eng. (Hons) - UTS

Craig holds an Engineering Degree in Telecommunications and has worked in the software development industry for over 30 years, holding senior software design, software architecture and technical management positions in companies including Jtec, Ericsson and LongReach Networks.

More recently Craig founded PeppaCode, a web and app development business focused on 'out-of-the-ordinary' strategic web and software development for small business, start-ups and educational institutions.

Some of Craig's achievements at PeppaCode include the successful launch of UEB Online for NextSense Institute and a vehicle tracking and management system for a bus operator.

Craig continues to work on new and innovative projects using modern web technologies.

Lesson 1

Introducing Number

Numeric Mode

Numeric Indicator ⠼ dots 3 and 4 5 6

The number system is formed by using the letters of the alphabet a – i for the numbers 1-9 and j for 0 preceded by the numeric indicator (⠼) dots 3456.

1	2	3	4	5	6	7	8	9	0
⠁	⠃	⠉	⠙	⠑	⠋	⠛	⠓	⠊	⠚

The numeric indicator sets numeric mode for the remainder of the symbols-sequence.

Numeric Mode Symbols

The following symbols may occur in numeric mode:

- **The Ten Digits**

1	2	3	4	5	6	7	8	9	0
⠁	⠃	⠉	⠙	⠑	⠋	⠛	⠓	⠊	⠚

Examples:

2 ⠼⠃

20 ⠼⠃⠚

21 ⠼⠃⠁

2018 ⠼⠃⠚⠁⠓

- **A Full Stop or Period as Commonly Used in Decimals**

Examples:

0.5 ⠼⠚⠲⠑

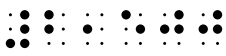
0.74 ⠼⠚⠲⠛⠙

1.67 ⠼⠁⠲⠋⠛

14.921 

- **Comma**

  Examples:

  2,500 

  9,250 

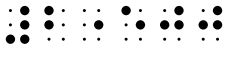
  14,921

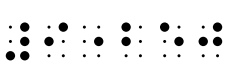
  325,961

  1,000,000 

- **Numeric Space**

  Examples:

  2 500 

  9 250 

  15 000 

  1 000 000

- **Line Continuation Indicator**

A line continuation indicator, dot 5, is placed immediately at the end of the line to indicate that the symbols-sequence does not fit and is continuing to the next line.

Since the line continuation indicator does not terminate numeric mode, a numeric indicator is not required when the sequence is continued on the next line.

When making this decision, choice of the run over position should follow mathematical structure:

- before operation signs
- before comparison signs
- before fractions
- before shapes

Further information regarding breaking of the symbols-sequence can be found in “Unified English Braille Guidelines for Technical Material” (International Council on English Braille, 2014).

- **Simple Fraction (as outlined in more detail in Lesson 8)**

  Example:

  $\frac{1}{2}$ 

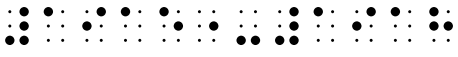
Termination of Numeric Mode

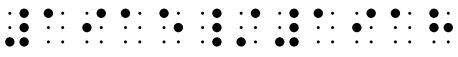
Numeric mode is terminated by a space or any symbol not listed above such as a hyphen, dash, slash/oblique stroke and colon.

In such cases, the numeric indicator will need to be repeated after these signs for the remainder of the symbols-sequence.

Examples:

1915-1918 

1915–1918 

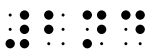
1915/1918 

5:30 am 

Ordinal Numbers

Occasionally ordinal numbers may be written in a smaller font size or in a superscript position. When print is shown in a smaller font size or in a superscript position it is ignored in braille and brailled as standard print.

Examples:

2nd 

3rd 

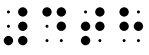
Numeric Indicators Set Grade 1 Mode

The numeric mode indicator also sets Grade 1 mode. When Grade 1 mode is in effect, contractions may not follow a number.

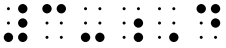
Termination of Grade 1 Mode

When Grade 1 mode is set by a numeric indicator, it is terminated by a space, a hyphen, a dash or a Grade 1 terminator. More information on Grade 1 mode will be included in Lesson 3.

Example:

4th 

1st 

3-D 

Exercise 1

1. 3 plus 7 equals 10
2. 4 minus 2 equals 2
3. 5 times 5 equals 25
4. 8 divided by 4 equals 2
5. John enjoys the music of the 60s.
6. Lara has a greater preference for the music of the 70's.
7. What is the answer if I add 21.6, 14.8 and 255.97 together?
8. 0.19 is the same way as writing .19
9. My telephone number is 02 9886 7313.
10. Class begins at 9:00 am sharp.

Extra Exercise 1

1. School finishes at 3:15 pm.
2. I am wondering whether midnight is 12 a.m. or 12 p.m.
3. Australia Day celebrations are on January 26th each year.
4. Today is 12.10.18.
5. Tomorrow is 13-10-18.
6. I plan to go overseas on 20/9/18.
7. The date can also be written 2019/08/04
8. Although more uncommon, the date can be written as 7:12:2018
9. Her birthday falls on the 1st of December.
10. We have a public holiday on the 4th April this year.

Lesson 2

Operation and Comparison Signs

Operation Signs

 + plus (dot 5, dots 2 3 and 5)

 − minus (dot 5, dots 3 and 6)

 × times, a multiplication sign as shown in print as a cross (dot 5, dots 2 3 and 6)

 · running product (dot 5, dots 2 and 5 6)

 ÷ divided by (dot 5, dots 3 and 4)

 : ratio sign as shown in print as a colon (dots 2 and 5)

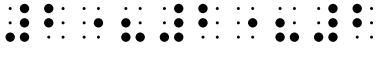
Note: Although the ratio sign shown above is used to compare two numbers, it is best treated as an operation sign for the purposes of spacing.

Operation signs are generally unspaced from numbers both preceding and following it.

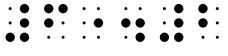
Examples:

3 + 5 

5 − 3 

2 × 2 × 2 

4 ÷ 2 

6·2 

2:4 

For a younger student or a developing braille reader the operation signs may be spaced.

It is however recommended as a teaching strategy that spaces between the operation signs be omitted as soon as the learner has mastered an understanding of mathematics.

Note: For the purposes of the exercises in UEB Mathematics Online, always unspace the operation sign from the sequence on either side.

Comparison Signs

 = equals

 ≠ not equal to

 < is less than

 > is greater than

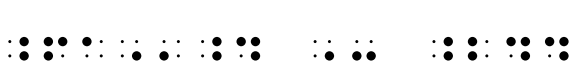
Note: Additional comparison signs will be introduced in UEB Training Manual: Advanced Mathematics.

Comparison signs are spaced from the sequences on either side.

Examples:

25 + 13 = 38 

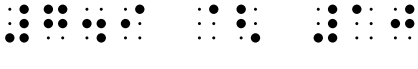
57 − 24 = 33 

61 × 4 = 244 

100 ÷ 10 = 10

56 ≠ 30 + 16

7.9 < 10 

10 > 7.9 

Exercise 2

1. 9 + 5 + 3 = 8
2. 3 500 ÷ 70 =
3. 0.7 < 0.69. True/False?
4. Area = bh = 10·3 = 30
5. 4:8 = 12:24
6. 0.69 > 0.7. True or false?
7. 50 × 100 ÷ 25 + 1 =
8. 51 × 9 = 459
9. 48 ÷ 8 = 6
10. 300 + 59 + 1 = 360

Review Exercise 2

1. Write the number that is one thousand greater than 3 499 000.
2. 0.645 012
3. 4.37 + 0.96 + 2.15 + 3.07 =
4. A stopwatch display reads 1:10:43. How many seconds is this?
5. 5.25 − 1.80 =
6. The date today can be written as 01/01/2019
7. 201 × 50 = 10,050
8. 77 ÷ 11 = 7
9. 100 ÷ 10 + 21 − 16 = 15
10. The date tomorrow can be written as 27:04:18

Lesson 3

Grade 1 Mode

Grade 1 Symbol Indicator

 Grade 1 symbol indicator (dots 5 6)

Additional Grade 1 mode indicators will be introduced in Lesson 2 of the UEB Training Manual: Advanced Mathematics.

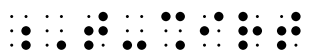
Use of Grade 1 Indicators

A braille symbol may have a Grade 1 (uncontracted) meaning and a Grade 2 (contracted) meaning.

A Grade 1 indicator is used to set Grade 1 mode when the meaning of a symbol could be misread as a contraction.

Example:

T-shirt



A Grade 1 indicator is required for any of the lower case letters a-j that immediately follow a digit, as they themselves will be read as part of the number.

Examples:

5b (not 52)

10a (not 101)

A Grade 1 indicator is required for any of the letter combinations that form a short form so they are not misread.

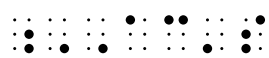
Note: In accordance with the rules for capitalisation, a capitalised word indicator is terminated by a space, a single capital letter, a nonalphabetic symbol, or a capitals terminator (dot 6 and dot 3).

Examples:

Class AB

CD-ROM

AC’s



ACs

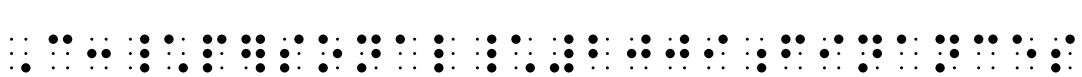
Letters of the alphabet may also have a numeric meaning as shown in Lesson 1 where the letters of the alphabet have been used as numbers with the numeric indicator preceding.

Notes:

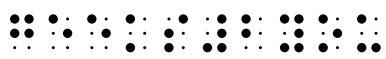
1. The numeric indicator () also sets Grade 1 mode for the remaining symbols-sequence. When Grade 1 mode is set by the numeric indicator it is terminated by a space, a hyphen, a dash or a Grade 1 indicator.
2. In the example below a Grade 1 indicator is placed in front of the word “finances” so the “f” is not read as a number 6 in the symbols-sequence.

Examples:

C:\personal\2009finances



geeks2you



1939-45

01/01/19

Exercise 3

1. 5 cm + 3 cm = 8 cm
2. 5cm + 3cm = 8cm
3. 500 g × 10 = 5000 g = 5 kg
4. 500g × 10 = 5000g = 5kg
5. School starts at 9 am each day.
6. School starts at 9am each day.
7. School finishes at 3 pm every day.
8. School finishes at 3pm every day.
9. Is a prism 2-D or 3-D?
10. All the family are going to sit in Row 15F for the concert.

Review Exercise 3

1. How many CD’s do you have?
2. We are approaching an “S” bend in the road.
3. After that next corner we will reach a “T” intersection.
4. Class 6A won the school debating competition.
5. Class 6C came second in the competition.
6. Today is 24-9-18.
7. I received 55c in my change from buying the drink.
8. How many c in 5 dollars?
9. A scale of 1:250
10. An AC adapter is required for the plug.

Lesson 4

Special Print Symbols

 ¢ cent, dot 4, dots 1 and 4 (dot 4, c)

 $ dollar, dot 4, dots 2 3 and 4 (dot 4, s)

 € euro, dot 4, dots 1 and 5 (dot 4, e)

 £ pound sterling, dot 4, dots 1 2 3 (dot 4, l)

Refer to “Unified English Braille Guidelines for Technical Material” (International Council on English Braille, 2014) for additional special print symbols for different currencies (see reference list).

 % percent, dots 4 and 6, dots 3 and 5 6 (dots 4 6, lower j)

 ° degree, dots 4 5, dots 2 and 4 5 (dots 4 5, j)

 ′ foot or minute, dots 2 3 and 5 6 (lower g)

 ″ inch or second, dots 2 3 and 5 6, dots 2 3 and 5 6 (lower g, lower g)

 (open round bracket, dot 5, dots 1 2 and 6

) close round bracket, dot 5, dots 3 and 4 5

 | tally mark (vertical line segment), dots 4 5 6

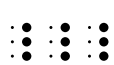
Follow the order shown in the print version for spacing, capitalisation and punctuation of abbreviations when using the signs indicated above.

Examples:

$US25 

£25 

€25 

25¢		
25%		
25°		
2′ 5″		
|||		3 tally marks
𝍸		4 tally marks with a strike through (representing 5 items)

Exercise 4

1. How many A$ in 500c?
2. How many ¢ in $5?
3. How many c in $5?
4. $10.00 − $5.50 =
5. $1.00 − 55c?
6. What is 10% of 800?
7. What is the minimum temperature if the range is 13°C and the maximum is 38°C?
8. Water boils at 100°C and freezes at 0°C.
9. 1′6″ is about half a yard and is about 45 cm or 0.45 m.
10. 50 cents can be written as $0.50, 50c or 50¢.

Review Exercise 4

1. Do I show the dollar sign as US$ or $US?
2. 9 L of water weighs 9000 g.
3. The class achieved a 70% pass rate in the exam.
4. The temperature today is 31°C.
5. She will be starting school in Kindergarten on 31/05/2019.
6. There is an “S” bend in the road ahead.
7. I will catch the train on 06.05.19.
8. April Fools’ Day is on the 1st April each year.
9. 70 ÷ 7 = 10
10. 5 − 5 = 0

Lesson 5

Omission Marks

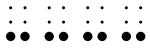
A variety of different ways are used in print to indicate that something is missing in the sequence. While decisions may be made at the time of print to braille transcription in the best interest of the student's learning, please follow the print for the purposes of these exercises.

Note: In the list of signs of omission and example below a Grade 1 indicator is needed in front of the question mark as it has a contracted meaning "his" and is standing alone.

Signs of Omission

 — Long dash (shown as a centred horizontal line in print)

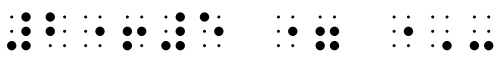
 _ Underscore

 ---- 4 hyphens

 ? Question mark

Examples:

2 + 5 = —



5 + 2 = ----

7 + 5 = ?

Exercise 5

1. 5 + 9 = _
2. 25 + 5 = ?
3. 15 × ? = 45
4. 197 + — = 215
5. 196 ÷ 4 = —
6. James has ---- caps.
7. 72 — 9 = 8
8. 72 ÷ 9 = 8
9. 4, 7, 10, —, —.
10. 2, 4, 6, 8, ?, ?.

Review Exercise 5

1. 15 – 7 = ?
2. 25 + 19 = —
3. 25/10/2018
4. $19 + $15 = $—
5. 15c – 12c =
6. 31 + ---- = 77
7. Class 5A and 6B went to the oval to play football.
8. I only received 13c as my change.
9. My initials are CD and my friend’s initials are AB
10. 16.7 + 13.3 = 30.0.

Lesson 6

Shape Indicators

There are a number of different shapes that occur in print. These shapes are often indicated by the sign for the shape indicator, that is, dots 1246 (⠩), and followed by the number of sides of the shape. The most common shape indicators are presented below.

Note: As the shape indicator also has a Grade 2 meaning, unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be needed before the shape indicator.

Examples:

⠩⠫ □ Square Box

⠩⠬ Δ Triangle

⠩⠪ O Circle

Note: In the examples below, the shape indicator needs a Grade 1 symbol indicator in front of it as the shape indicator has a contracted meaning.

Examples:

2 + 5 = □ ⠼⠃⠐⠖⠼⠑⠐⠶ ⠼⠃ ⠼⠑ ⠩⠫

5 Δ 2 = 7 ⠼⠑ ⠩⠬ ⠼⠃ ⠼⠛

7 O 5 = 2 ⠼⠛ ⠩⠪ ⠼⠑ ⠼⠃ ⠼⠑

Use of Shape Termination Indicator

⠸ Shape termination indicator, dots 1 5 6

If a shape is followed by a space, then no termination to the symbol is necessary.

If a shape symbol is followed by punctuation or is unspaced from a following symbol then the shape termination indicator must be used.

Note: While the shape termination indicator has a Grade 2 meaning, in the example below the shape termination indicator will not require the Grade 1 symbol indicator as the numeric indicator has set Grade 1 mode.

Example:

□, □ ⠩⠫⠂⠀⠩⠫

Exercise 6

1. 5 + 9 = □
2. 25 + □ = 30
3. 16 Δ 3 = 13
4. 197 + 10 = Δ
5. 196 ÷ O = 49
6. James has □ caps.
7. 72 Δ 9 = 8
8. 72 O 9 = 8
9. 4, 7, 10, □, □.
10. 2, 4, 6, 8, —, —.

Review Exercise 6

1. 159 + 77 – 21 = ?
2. 55 + □ + 48 = 120
3. Δ – 59 = 100
4. 25, 30, 35, □, 45 = 175
5. I was given 5 CDs for my birthday.
6. 3 + □ = 7
7. I sat in row 9C and my son sat in row 9D.
8. I received 55c change from $5.
9. My initials are AC and my friend's initials are AD.
10. 16.7 + 13.3 is 30.0.

Lesson 7

Roman Numerals

Roman Numerals should be brailled as if they were normal letters of the alphabet using the rules for capitalisation and Grade 1 mode.

Note: In the examples below:

1. The letters “V” and “x” have a contracted meaning when standing alone and so will require Grade 1 indicators. The letter “I” and “i” do not have a contracted meaning and so will not require Grade 1 indicators.
2. The letters CD may be read as a short form could so will also require a Grade 1 indicator.

Examples:

Read parts I, II and V.

Answer only questions i, vi and x.

CD

Exercise 7

1. X – VI =
2. C + L – XV =
3. Answer questions i, ii and iii only.
4. A lower case Roman nine is ix.
5. An upper case Roman nine is IX.
6. v + i = vi
7. M + LX =
8. iv + v + x =
9. LX is another way of writing 60.
10. What is XC + CCC in Roman Numerals?

Review Exercise 7

1. C + L = —
2. 175 ÷ 5 = O
3. 221 – 73 + 54 =
4. What colour is a $100 note?
5. What are the next 2 squared numbers:

   16, 36, 64, □, □?
6. Write 21:36 in digital time.
7. $7.93 = ---- cents.
8. Is 0.6 smaller than 0.57?
9. 67 × 20 = □
10. What number is 400 000 less than 9 640 400?

Lesson 8

Fractions

Simple Fractions, Mixed Numbers and Linear Fractions

There are a number of methods of indicating fractions in print.

Simple Numeric Fraction

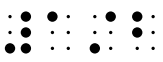
A simple numeric fraction is one whose numerator (top of the fraction line) and denominator (bottom of the fraction line) contain only:

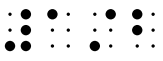
- digits,
- decimal points,
- commas, or
- separator spaces,

and, if the fraction line in print (often referred to as the vinculum) is drawn between the two vertically (or near vertically) arranged numbers.

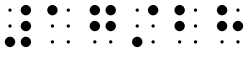
If the fraction complies fully with the definition above for a simple fraction, then a numeric fraction line symbol (⠌) is used between the numerator and the denominator and continues the numeric mode as previously mentioned in Lesson 1.

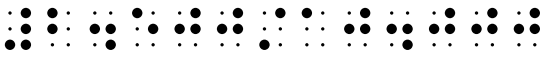
Examples:

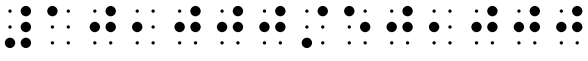

$\frac{1}{2}$ (vertically) or

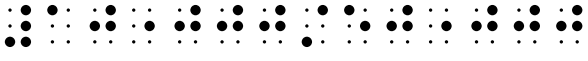

$1/_{2}$ (near vertically)


$\frac{17}{28}$ (vertically) or


$17/_{28}$ (near vertically)


$\frac{2.500}{10.000}$ (decimals)


$\frac{10{,}000}{50{,}000}$ (commas)

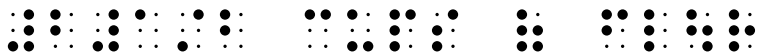

$\frac{10\,000}{50\,000}$ (separator spaces)

Mixed Numbers

Mixed numbers (shown with a whole number) immediately followed by a simple fraction should be treated as two unspaced numeric items, with the numeric indicator being repeated after the whole number and before the simple fraction.

Example:

$2\frac{1}{2}$ cups of flour

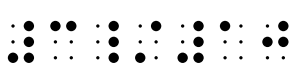


Linear Fractions

The simple numeric fraction line is not used when the print is expressed linearly (that is, where all the numbers in the print are positioned sitting horizontally on the line), using an ordinary forward slash symbol (). In such instances, the same forward slash symbol is used as a print slash and the numeric indicator will need to be repeated as the slash terminates numeric mode.

Example:

3/10



Exercise 8

1. $\frac{5}{8}$ of the class are girls
2. $5/8$ of the class are girls
3. $1^{3}/_{4}$ m = 1750 cm.
4. 0.08 + $\frac{3}{5}$
5. $^{2}/_{3}$ teaspoon of vanilla is needed in the recipe.
6. ¼ + ½ + ¾ =
7. $\frac{5.5}{4{,}000}$
8. Write 9.9/10.0 as a decimal.
9. Calculate $\frac{7.7}{2\ 000}$
10. Write $^{77}/_{100}$ as a percentage.

Review Exercise 8

1. 3 cm + 5 cm = 8 cm
2. 75 ≠ 40 + 25
3. Add (5 × 50) and (8 × 40)
4. $5.30 − $5.00 = □
5. 0.75 < 0.7. True or False?
6. How many $ in 400c?
7. 55cm + 22cm + 23cm = 1m
8. 3.9 × 4.1 = O
9. VII + XXX =
10. Anzac Day in Australia is 25th April each year.

Lesson 9

Revision Lessons 1 to 8

Review Exercise 9

1. g in 5 kilograms
2. 5 m 97 cm = —
3. 34 + □ = 41
4. How many 20c coins in $5.60?
5. L in 2,900 millilitres
6. How many minutes from 10:37 a.m. to 12:29 p.m.?
7. cm in 346 millimetres
8. Write $1\frac{56}{100}$ as a decimal.
9. Hindu Arabic for XCVIII.
10. 8 × 7 + 7 + 9 = —
11. $^1/_2 + {}^1/_3 + {}^1/_4 + {}^1/_5$ =
12. The temperature rose from 15°C to 29°C.
13. 72 Δ 8 = ___
14. 60% of $150
15. 6 litres of paint at $19.99/litre.
16. ¾ of 800 mL
17. 1/2 of $10,000 is
18. 13.2 + 15.9 =
19. 50 000 + 7 000 + 500 + 40 + 3
20. How many millimetres in 9 cm?

Lesson 10

UEB Introductory Mathematics Test

Test Exercise 10

1. (55 + 99) + (21 + 12) =
2. Convert 6101 cm to metres and centimetres.
3. 510 mm + 33 mm =
4. What is $\frac{4}{5}$ as a decimal?
5. □ ÷ 100 = 10
6. 1, 3, □, 7
7. The recipe requires $^{2}/_{3}$ cup sugar
8. My friend's initials are GD
9. $^{5.5}/_{3{,}000}$
10. The school holidays will begin on the 1st of next month.
11. 77 Δ 7 = 11
12. The ISBN is 0-14-300414-X
13. Tomorrow will be 29/10/2018
14. Write the decimal .717 as a fraction.
15. Calculate 75% as a fraction.
16. What is 30·10 as a running product?
17. 9.625 + 3.127 =
18. 5000 mm is □ centimetres?
19. 50% = $\frac{50}{100}$= 0.5 = ½
20. The temperature today is expected to reach 90°F.

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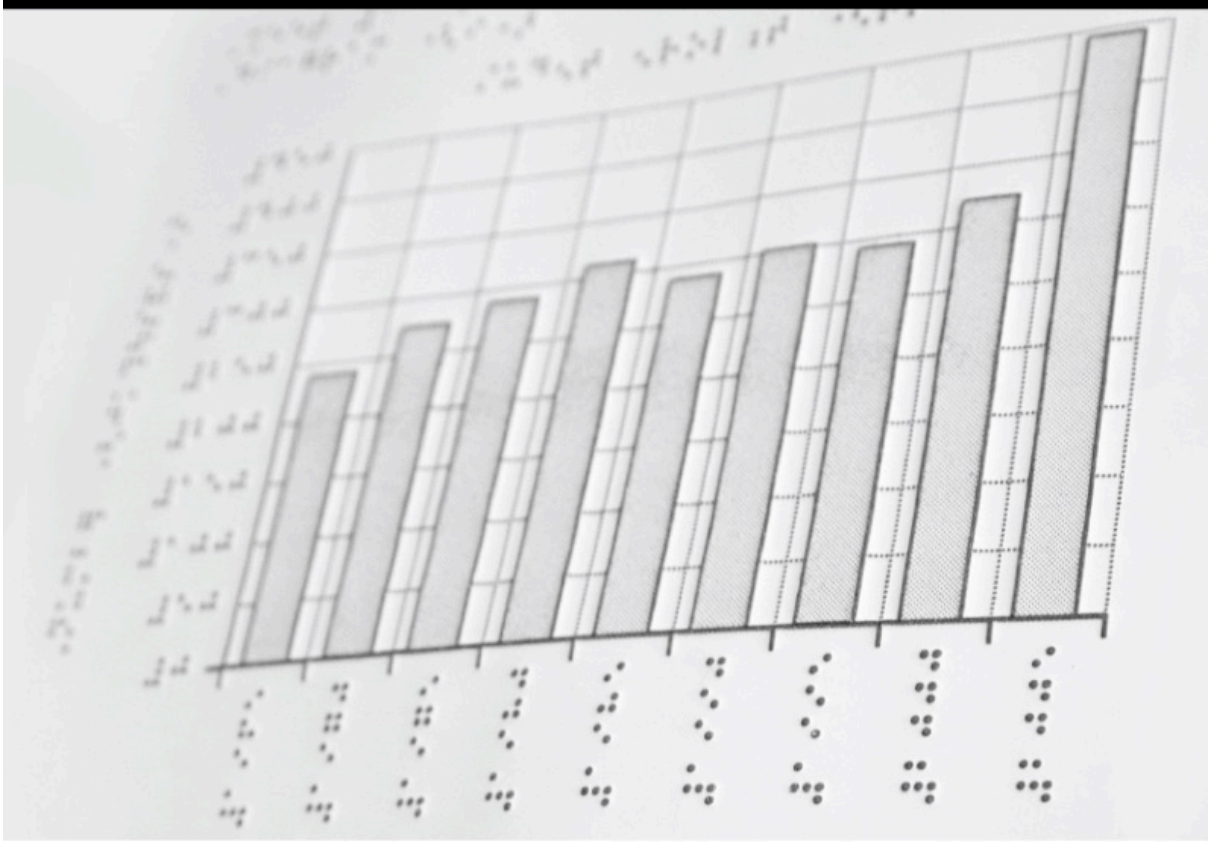
Next Sense

Advancing the RIDBC Renwick Centre experience.

Unified English Braille Training Manual

Advanced Mathematics

Josie Howse



UEBOnline

Unified English Braille Training Manual: Advanced Mathematics
Revision 6

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NextSense Institute is Australia's leading centre for research and professional studies in the field of education for children with sensory disabilities; offering webinars, short courses, and degree programs for parents, carers, educators, and health professionals. NextSense Institute is committed to providing high-quality teaching and learning opportunities. Our programs are conducted by leading national and international experts for education and health professionals who support people who are deaf, hard of hearing, blind or have low vision.

Table of Contents

Foreword

If you are reading this Foreword, it is most likely that you have successfully completed the online Unified English Braille (UEB) Introductory Mathematics training program. Please accept my congratulations for your success to date!

The **purpose** of this UEB Advanced Mathematics training program is to provide instruction in the reading and writing of secondary-level mathematical symbols in Unified English Braille. Mathematics is widely considered a core learning area and an essential requirement for the study of the STEM subjects of Science, Technology and Engineering. Mathematics teachers generally possess specialised mathematical knowledge and skills that enable them to effectively teach secondary students with a diverse range of abilities and attributes. However, students who use braille to access and communicate information require instruction from teachers who understand the braille code and are able to effectively modify print-based information into tactile form.

The **target audience** for UEB Advanced Mathematics includes mathematics teachers, teachers of braille, parents and caregivers, allied health professionals, education administrators and policymakers.

The **aims** of the UEB Advanced Mathematics training program are:

- To promote the acquisition of knowledge of Unified English Braille as it is applied to the continuum of those mathematical symbols that are typically taught during the secondary years of schooling; and
- To raise awareness of the enormous potential of braille knowledge and skills in enabling students with vision impairment to effectively access and engage with mathematics content and to communicate their mathematical understanding in a broad range of contexts.

The **instructional content** is presented as a series of lessons that address specific topics in secondary education. The lessons include practice and review exercises involving print to braille transcription. The content of each lesson builds on prior content, enabling the progressive development and consolidation of braille knowledge.

Please **note** that an additional UEB Online training program called UEB Extension Mathematics addresses mathematics content that is encountered during the senior years of secondary mathematics – see https://uebonline.org.

The **recommended UEB Online study sequence** is: (i) completion of UEB Literacy modules 1 and 2; followed by (ii) UEB Introductory Mathematics; and then (iii) UEB Advanced Mathematics and (iv) UEB Extension Mathematics. This step-by-step sequence is recommended because Unified English Braille is a single code system that encompasses the braille symbols for literary and technical information. It is

important, therefore, to be knowledgeable of the UEB symbols used in literary contexts, as these symbols are used for literary content of mathematical information.

On behalf of NextSense Institute, I extend my sincere thanks to those organisations and individuals who have contributed to developing the UEB Online mathematics training programs and supporting materials – see https://uebonline.org. This includes thanks to my Project Team colleagues, Josie Howse (content author), Craig Cashmore of PeppaCode Pty Ltd (online program developer); and NextSense's Lena Karam, Sonali Marathe and Tarna Cosgrove who have assured the accuracy and accessibility of the information presented. Sincere appreciation is also extended to the Duchen Family Foundation, the JLDJS Foundation, Sibley Endowment, the Skipper-Jacobs Charitable Trust and the Thomas Hare Investments Trust. Without their financial support, the UEB Online Mathematics training programs would not have been possible.

We hope that this contribution to the disability field will provide professionals and parents with the required knowledge to support and encourage secondary school students with vision impairment to actively and confidently engage with the exciting world of mathematics.

Frances Gentle, AO PhD
UEB Online Project Team Leader and Conjoint Lecturer,
NextSense Institute

Contributors

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BA (Ed.) – Macquarie University

Josie Howse is an Adjunct Research Fellow with the NextSense Institute

and former Manager of the Braille and Large Print Services, NSW Department of Education. The NSW Department of Education team provides all texts and examinations in braille, large print and e-text to students with vision impairment in the government sector and is the largest producer of alternative format student textbooks and examinations in Australia.

Josie has been working in the field of vision impairment for more than 40 years. She has held a number of executive positions at national and state levels and has extensive experience in braille code development at an international level. Josie was the editor of the 2006 Unified English Braille Primer: Australian Edition, and co-editor of the 2016 Unified English Braille: Australian Training Manual. Josie was awarded the Public Service Medal (PSM) in the Queen's Birthday Honours list in 2007, has been listed annually in Who's Who of Australian Women since 2008, and is the recipient of a Lifetime Achievement Award from the Round Table on Information Access for People with Print Disabilities in 2012.

UEB Online Developer: Craig Cashmore
B. Eng. (Hons) – UTS

Craig holds an Engineering Degree in Telecommunications and has worked in the software development industry for over 30 years, holding senior software design, software architecture and technical management positions in companies including Jtec, Ericsson and LongReach Networks.

More recently Craig founded PeppaCode, a web and app development business focused on 'out-of-the-ordinary' strategic web and software development for small business, start-ups and educational institutions.

Some of Craig's achievements at PeppaCode include the successful launch of UEB Online for NextSense and a vehicle tracking and management system for a bus operator. Craig continues to work on new and innovative projects using modern web technologies.

Advanced Mathematics

Lesson 1: Grade 1 Mode (Symbol Indicator) and Algebra

Grade 1 Mode

The rules for the use of Grade 1 mode in a literary context will also apply in a mathematical context. Please revise the information presented in Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics*.

A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning. The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

⠰ Grade 1 Symbol indicator (dots 56)

The following Grade 1 mode indicators will be introduced more fully in Lesson 2 of this Training Manual.

⠰⠰ Grade 1 Word Indicator

⠰⠰⠰ Grade 1 Passage Indicator

⠰⠄ Grade 1 Passage Terminator

⠐⠐⠿⠰⠰⠰ Grade 1 Passage Indicator on a line of its own

⠐⠐⠿⠰⠄ Grade 1 Passage Terminator on a line of its own

Note:

Grade 1 indicators will not be needed for simple arithmetic problems involving number, operation signs, numerical fractions and mixed numbers.

Grade 1 Symbol Indicator

- A Grade 1 symbol indicator (⠰) sets Grade 1 mode for only the next symbol.

Example:

Seat 10a

⠰⠎⠑⠁⠞ ⠁⠚⠰⠁

- A Grade 1 symbol indicator is NOT required before the letters a, i and o, because they do not have a contraction meaning when they “stand alone”.

Example:

The letters a, i and o do not need a Grade 1 indicator.

- A Grade 1 symbol indicator precedes a capitalisation indicator.

Example:

The letter C.

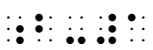
- A Grade 1 symbol indicator is required to prevent a letter from being misread as a number.

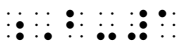
Example:

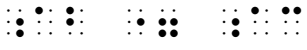
John@take2.com

- A Grade 1 symbol indicator may be required for a sequence of letters in braille that could represent an alphabetic wordsign, or a shortform such as ab (about) or ac (according), if the letter or letter sequence is “standing alone”.

Examples:

b-1 

B-1 

ab = ac 

Numeric Indicators Set Grade 1 Mode

The numeric mode indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

Examples:

4th 

1st 

geeks2you

4starhotel@webnet.com

When Grade 1 mode is set by a numeric indicator, it is terminated by a space, a hyphen, a dash or a Grade 1 terminator. More information on Grade 1 mode will be included in Lesson 2.

Examples:

3-D 

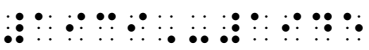
Remember, the Numeric mode is terminated by a space or any symbol **not** listed below:

- The ten digits
- Full stop (period)
- Comma;
- The numeric space
- Simple numeric fraction line; and
- The continuation indicator

Therefore, the numeric mode is terminated by symbols such as the hyphen, dash, slash/oblique stroke and colon.

Examples:

1939-45


1939–1945


24/7


8:30 am


Standing Alone Rule

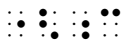
Refer to *The Rules of Unified English Braille* (Simpson, 2013) for a comprehensive definition and explanation of usage of “standing alone”.

A letter or letter sequence is considered “standing alone” when:

- it is preceded and followed by a space, a hyphen or a dash (of any length); or
- common punctuation and indicator symbols intervene between the letter or letters-sequence and the **preceding** or **following** space, hyphen or dash or
- a word with an interior apostrophe is considered to be “standing alone” under the specific provisions referring to contractions, alphabetic wordsigns, strong wordsigns and shortforms.

Examples:

(c



(C

“can

“p’s and q’s”

General Note

Decisions often need to be made in a mathematical context about whether to use:

- a Grade 1 symbol indicator or
- a Grade 1 word indicator or
- a Grade 1 passage indicator with a Grade 1 terminator or
- a Grade 1 passage indicator on a line of its own with a Grade 1 passage terminator on a line of its own.

Often there is a choice about Grade 1 indicators in mathematical contexts, with any of the options (above) being equally correct. Decisions about option selection are generally associated with user and transcriber preferences, including consideration for simplicity or functionality.

Algebra

- Algebra is the part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations.
- Simple algebraic equations which include letters may need Grade 1 symbol indicators where the letters stand alone, or the letters a-j immediately follow the numbers and the letters themselves may be misread as numbers.
- The ratio sign () has a Grade 2 (contracted) meaning, so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the ratio sign.
- Remember that the numeric indicator also sets Grade 1 mode for the next symbols-sequence. When Grade 1 mode is set by a numeric indicator, it is terminated by a space, a hyphen, a dash or a Grade 1 terminator.

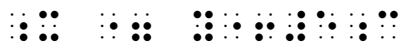
Notes:

1. The presentation of algebraic expressions in print is often shown in italics. This is generally ignored in braille.
2. If the braille representation for a print sequence in the following exercises does not fit on the line, then the first preference would be to break:
   - before comparison signs,
   - before operation signs, or
   - before a mathematical unit such as
     - fractions
     - functions
     - radicals
     - items with modifiers such as superscripts or bars
     - shapes or arrows
     - anything enclosed in print or braille grouping symbols.

Usually the best place to break is before a comparison sign or an operation sign.

Examples:

$x = y + 5c$



Seat 6a

The ratio of $x : y$

The ratio $5 : 10$

Exercise 1

1. $a + b = c$
2. $x + y = z$
3. $2x = y$
4. $am - an$
5. x-axis
6. X-axis
7. The product of a and b, (ab), gives the area.
8. The ratio $p: q$
9. $Ab = Xb + Yb$
10. $4r \div 2r =$

Extra Exercise 1

1. Expand $(a + 2)(a + 4)$
2. Expand and simplify $(3a + 5)(a + 1)$
3. $(q + 4)(q - 2)$
4. $2x(3x - 1) + 3(3x - 1)$
5. $ab + cd = gd$
6. $(m + 3)(m + 3) = m(m + 3) + 3(m + 3)$
7. $21a \div 7a = 3$
8. $mn + yz =$
9. $7b + 5b =$
10. $(t - 4)(t - 7)$

Lesson 2: Grade 1 Mode (Word and Passage) and Fractions (continued)

Grade 1 Mode

The rules for the use of Grade 1 mode in a literary context will also apply in a mathematical context. Please revise the information presented in Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics* and Lesson 1 of this Training Manual.

A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning. The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

⠰ Grade 1 Symbol Indicator (dots 56)

⠰⠰ Grade 1 Word Indicator

⠰⠰⠰ Grade 1 Passage Indicator

⠰⠄ Grade 1 Passage Terminator

⠠⠠⠿⠰⠰⠰ Grade 1 Passage Indicator on a line of its own

⠠⠠⠿⠰⠄ Grade 1 Passage Terminator on a line of its own

Note:

Grade 1 indicators will not be needed for simple arithmetic problems involving number, operation signs, numerical fractions and mixed numbers.

Grade 1 Word Indicator

- The Grade 1 word indicator (⠰⠰) sets Grade 1 mode for the following sequence of symbols or the remainder of the current symbol sequence.
- The effect of a Grade 1 word indicator is terminated by a space or a Grade 1 terminator.
- Remember that the numeric indicator (⠼) also sets Grade 1 mode for the next symbols-sequence.
- Complex algebraic expressions that do **not** include a comparison sign are best shown using Grade 1 word mode.
- More examples of the use of Grade 1 word mode in a mathematical context will be presented later in this manual.

Note:

The example below is a General Fraction and will be dealt with in more detail in Lesson 2.

Example:

Replace I with $\frac{E}{R}$

Grade 1 Passage Indicator and Grade 1 Passage Terminator

- A Grade 1 passage indicator (⠰⠰⠰) sets Grade 1 mode for the next passage.
- A Grade 1 passage is terminated by the Grade 1 terminator (⠰⠄).
- Complex algebraic equations that include a comparison sign are best enclosed in Grade 1 passage indicators. This will ensure that isolated letters and indicators such as superscripts, subscripts, fractions, radicals, arrows and shapes are well defined without the need for grade symbol indicators.
- More examples of the use of Grade 1 passage mode in a mathematical context will be presented later in this manual.

Grade 1 Passage Terminator

The Grade 1 passage terminator (⠰⠄) follows immediately after the last affected symbols-sequence of a Grade 1 passage.

Example:

$x = y - z, \quad y = x + z, \quad z = y - x$

Grade 1 Passage Indicator and Terminator on a line of their own

Where it is better to preserve the natural alignment of a text, for example, in a computer program or a set of equations in mathematics, place the Grade 1 passage indicator (⠰⠰⠰) on a separate line above the equations (consider the margin as a suitable location), and the Grade 1 terminator (⠰⠄) on a separate line below the equations. (Refer to the example below which shows this use with a set of quadratic equations using superscripts. Superscripts will be explained further in Lesson 4 of this Training Manual).

When the Grade 1 passage indicator and terminator is placed on a line of its own, each indicator should be preceded by the dot locator for “use” (⠐⠐⠸). A braille symbol which has only lower dots in the cell and which is isolated from other text may be misread.

Examples:

Solve the following quadratic equations:

1. $x^2 - x - 2 = 0$
2. $x^2 - 4x - 3 = 0$
3. $2x^2 - x = 1$

⠸⠭⠘⠼⠃⠀⠤⠀⠭⠀⠤⠀⠼⠃⠀⠨⠅⠀⠼⠚

⠸⠭⠘⠼⠃⠀⠤⠀⠼⠙⠭⠀⠤⠀⠼⠉⠀⠨⠅⠀⠼⠚

⠸⠭⠘⠼⠃⠀⠤⠀⠭⠀⠨⠅⠀⠼⠁

⠸⠭⠘⠼⠃⠀⠤⠀⠭⠀⠤⠀⠼⠁⠀⠨⠅⠀⠼⠚

⠸⠭⠘⠼⠃⠀⠤⠀⠭⠀⠤⠀⠼⠁⠀⠨⠅⠀⠼⠚

⠸⠭⠘⠼⠃⠀⠤⠀⠭⠀⠨⠅⠀⠼⠚

General Note

Decisions often need to be made in a mathematical context about whether to use:

- a Grade 1 symbol indicator or
- a Grade 1 word indicator or
- a Grade 1 passage indicator with Grade 1 terminator or
- a Grade 1 passage indicator on a line of its own with a Grade 1 passage terminator on a line of its own.

Often there is a choice about Grade 1 indicators in mathematical contexts, with any of the decision options (above) being equally correct. Decisions about option selection are generally associated with user and transcriber preferences, including consideration for simplicity or functionality.

Fractions

Lesson 8 of the *Unified English Braille Training Manual: Introductory Mathematics* introduced how to braille simple fractions, linear fractions and mixed numbers (that is, a whole number followed immediately by a simple fraction).

It is important to understand the definition of a **simple fraction**, that is, what elements and only those elements that can be considered to be a simple fraction, and therefore require use of the simple fraction line, (⠳ dots 34) between the numerator and the denominator.

If a fraction does **not** comply with the **simple numeric fraction** definition (below) for whatever reason, then it will be a **general fraction** and will require a different approach, using different signs.

Simple numeric fraction definition (Revision)

Simple numeric fractions were introduced in Lesson 8 of the *Unified English Braille Training Manual: Introductory Mathematics*.

Definition: A simple numeric fraction is one whose numerator (top of the fraction line) and denominator (bottom of the fraction line) contain only:

- digits,
- decimal points,
- commas, or
- separator spaces,
- **and**, if the fraction line in print (often referred to as the vinculum) is drawn between the two vertically (or near vertically) arranged numbers as shown in the print.

If the fraction complies fully with the definition above for a simple fraction, then a numeric fraction line symbol (⠌) should be used between the numerator and the denominator and the numeric fraction line symbol continues the numeric mode and the numeric indicator will not need to be repeated after the fraction line.

Examples:

⠼⠁⠌⠼⠃	$\frac{1}{2}$	(vertically) or
⠼⠁⠌⠼⠃	$1/_{2}$	(near vertically)
⠼⠁⠋⠌⠼⠃⠊	$\frac{196}{28}$	(vertically) or
⠼⠁⠋⠌⠼⠃⠊	$196/_{28}$	(near vertically)
⠼⠃⠄⠑⠚⠚ ⠌ ⠼⠁⠚⠄⠚⠚⠚	$\frac{2.500}{10.000}$	(decimals)
⠼⠁⠚⠂⠚⠚⠚ ⠌ ⠼⠑⠚⠂⠚⠚⠚	$\frac{10{,}000}{50{,}000}$	(commas)
⠼⠁⠚⠀⠚⠚⠚ ⠌ ⠼⠑⠚⠀⠚⠚⠚	$\frac{10\ 000}{50\ 000}$	(separator spaces)

General Fractions

General Fraction Indicators

The braille symbols shown below for the opening and closing general fraction indicators have a contracted (Grade 2) meaning and so have been shown with a Grade 1 indicator in front of them.

A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning. The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

⠰⠆ Opening general fraction indicator

⠰⠰ Closing general fraction indicator

⠰⠤ General fraction line

- If the numerator or denominator of a fraction is not entirely numeric, as defined above for a simple fraction, then the general fraction indicators should be used. Write the opening general fraction indicator (⠰⠆), then the numerator (top) expression, then the general fraction line symbol (⠰⠤), then the denominator (bottom) expression and finally, the closing general fraction indicator (⠰⠰) (see examples below).
- The numerator and denominator may be any kind of expression, including fractions of either simple numeric or general type.
- Remember Grade 1 mode has been set by the numeric indicator (⠼) and is terminated by the space, hyphen, dash or Grade 1 terminator.
- The following examples of General Fractions show the choice that can be made when using the Grade 1 indicator. There is usually a preferred option.

Examples:

$$\frac{2+3}{4-1}$$

⠰⠆⠼⠃⠐⠼⠉⠰⠤⠼⠙⠐⠼⠁⠰⠰

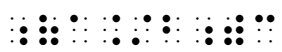
$$\frac{x+y}{x-y}$$

⠰⠆⠭⠐⠽⠰⠤⠭⠤⠽⠰⠰

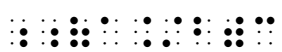
OR

⠼⠆⠐⠼⠉⠰⠤⠼⠙⠐⠼⠁⠰⠰

$\frac{a}{b}c$

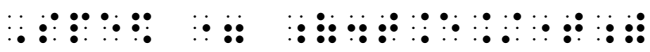


OR



$Speed = \frac{distance}{time}$

(the first example shown in braille below is the preferred option so as to retain more contracted words in the literary equation)



OR

OR

$\frac{x \times y}{p \times q} = \frac{xy}{pq}$

OR

OR

the preferred option below, due to the use of a comparison sign used in the sequences and Grade 1 indicators that are required on both sides of the equation.

Notes:

1. It is important to remember that the opening and closing fraction indicators each have a Grade 2 (contracted) meaning and so if Grade 1 mode has not been established for some reason, then the opening and closing fraction indicators will require Grade 1 indicators. For more information about Grade 1 mode, refer to Lesson 1 of this Training Manual and also Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics*.
2. As referred to in the General Note in Lesson 1, there is often a choice when selecting Grade 1 indicators in mathematical contexts, with any of the Grade 1 mode options (symbol, word or passage) being equally correct. The choice of

which Grade 1 indicator is associated with user and transcriber preferences, including consideration for simplicity or functionality.

3. For the purpose of the UEB Online exercises that are associated with this UEB Training Manual: Advanced Mathematics, use the following criteria for implementing Grade 1 mode:
   - Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
   - Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode, except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
   - Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hints:

- Questions 1-5 in Exercise 2 below are general fractions because they contain letters.
- Questions 6-10 in Exercise 2 below are general fractions because they contain something more than digits, commas, decimal points or numeric spaces.
- Remember the numeric indicator (⠼) also sets Grade 1 mode for the remainder of the symbols-sequence.
- When Grade 1 mode is set by the numeric indicator (⠼) it is terminated by a space, hyphen, dash or Grade 1 terminator.

Exercise 2

1. Evaluate $\frac{mn}{3} \div \frac{m}{n}$
2. $\frac{PQR}{XZ}$
3. $\frac{PqR}{xz}$
4. $y = \frac{x}{2}$
5. $\frac{3a}{5} + \frac{a}{5}$
6. $\frac{2^1/_2}{x+y}$
7. $\frac{8x-24y}{8}$
8. $\frac{4m+12}{3} \times \frac{6m}{m+3}$
9. $\frac{6(h+5)}{h+9}$
10. Simplify $\frac{5w+10}{5}$

Review Exercise 2

1. $\frac{4+3+2}{6-3+8}$
2. $\frac{2/3}{5}$
3. $A = \frac{h}{2}(a+b)$
4. $\frac{3(h+5)}{h+9}$
5. $\frac{5.3}{4{,}200}$
6. $\frac{\$55}{5}$
7. $\frac{24m}{3cm}$
8. $\frac{3}{10\,000}$
9. $\frac{4{,}000}{10}$
10. $\frac{\frac{x}{2}+\frac{y}{3}}{x+y}$

Lesson 3: Operation and Comparison Signs (continued)

Operation Signs

⠖ + plus (dot 5, dots 2 3 and 5)

⠤ − minus (dot 5, dots 3 and 6)

⠦ × times, a multiplication sign as shown in print as a cross (dot 5, dots 2 3 and 6)

⠲ . a running product sign, shown as a dot, is another means of showing multiplication in print (dot 5, dots 2 and 5 6)

⠌ ÷ divided by (dot 5, dots 3 and 4)

⠒ : ratio sign as shown in print as a colon (dots 2 and 5)

⠬ ± plus or minus (plus over minus)

⠭ ∓ minus or plus (minus over plus)

Notes:

1. For the purposes of the UEB Online exercises that are associated with this UEB Training Manual: Advanced Mathematics, always **unspace** the operation sign from the sequence on either side.
2. Although the ratio sign shown above is used to compare two numbers, it is best treated as an operation sign for the purposes of spacing.
3. The ratio sign terminates the effect of the numeric indicator and will therefore need to be repeated before the number that follows.

Examples:

6:12

⠼⠋⠒⠼⠁⠃

2·4·7

⠼⠃⠲⠼⠙⠲⠼⠛

Comparison Signs

⠶ = is equal to

⠶⠡ ≠ not equal to (i.e. a line cutting through an equals sign)

⠣ < is less than

 > is greater than

 ≤ is less than or equal to

 ≥ is greater than or equal to

 ≈ is approximately equal to (i.e. a tilde over a tilde)

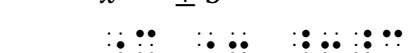
Notes:

1. For the purposes of the UEB Online exercises associated with this UEB Training Manual: Advanced Mathematics, always **space** the comparison sign from the sequence on either side.
2. There are a number of print representations to show “is approximately equal to”. Refer to “*Unified English Braille Guidelines for Technical Material*” (International Council on English Braille, 2014) for equivalent braille representations. However, only the “is approximately equal to” sign shown above may be used in the UEB Online exercises associated with the mathematics training manuals.
3. A Grade 1 indicator is needed if the algebraic letter stands alone.
4. The triangle sign used in geometry is the same sign in braille as the shape indicator (shown as a triangle shape in Lesson 6 of the *Unified English Braille Training Manual: Introductory Mathematics*).
5. Refer to the rules for the use of the shape termination indicator as outlined in Lesson 6 of the *Unified English Braille Training Manual: Introductory Mathematics*.
6. The letters a-j will require a Grade 1 indicator if immediately following a number as the letters themselves will be read as part of the number.

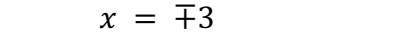
Examples:

$a \times b = c$

$x = \pm 3$

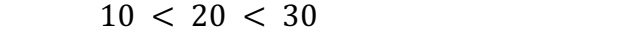


$x = \mp 3$



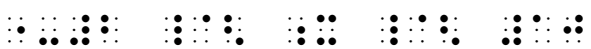
$4x + 5x \neq 8x$

$10 < 20 < 30$



$30 > 20 > 10$

$-2 \leq x \leq 10$



$10 \geq x \geq -2$

$3.9 \times 4.1 \approx 16$

Remember:

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice when selecting Grade 1 indicators in mathematical contexts, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. The choice is associated more with user and transcriber preferences, including consideration for simplicity or functionality.

For the purpose of the exercises in UEB Online that are associated with this UEB Mathematics training program, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 3

1. $16 - 5 \geq 10$
2. $3 + 3 \leq 3 \times 3$
3. $15 < 17$
4. $29 > 23$
5. $a + b \neq c$
6. $15xy \div 3x = \frac{15xy}{3x}$
7. $3(4 - 2x) \geq 18$
8. $2.5 \times 7.7 \approx 19$
9. $x = \pm 15$
10. $3a + 2b = 5c$

Review Exercise 3

1. $5 + -2 =$
2. $9 + -6 = +3$
3. $.672$
4. $\frac{1\,745\,711}{6\,527}$
5. $\frac{\$55.50}{5}$
6. $\frac{20m}{4cm}$
7. $\frac{a}{b+c}$
8. $\frac{x}{4{,}000}$
9. Expand $(2x - 5)(3x + 7)$
10. $\frac{\frac{p}{5}+\frac{q}{10}}{p+q}$

Lesson 4: Indices

Superscripts and Subscripts

A **superscript** is a distinguishing symbol (such as a numeral or letter) that is written immediately above, OR above and to the right or left of another character.

A **subscript** is a distinguishing symbol (such as a numeral or letter) that is written immediately below, OR below and to the right or left of another character.

Level Change Indicator

The superscript and subscript level change indicators and the braille grouping indicators shown below also have a contracted (Grade 2) meaning and so have been shown with a Grade 1 indicator in front of them. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning.

⠰⠔ Level change up

⠰⠢ Level change down

⠰⠔ Expression directly above (this symbol will be introduced in more detail in the UEB Training Manual: Extension Mathematics).

⠰⠢ Expression directly below (this symbol will be introduced in more detail in UEB Training Manual: Extension Mathematics).

Braille Grouping Indicator

⠰⠣ Open braille grouping indicator (there is no print representation for this symbol in braille)

⠰⠜ Closing braille grouping indicator (there is no print representation for this symbol in braille)

Note:

When both a left-hand subscript and superscript are shown together in print (such as in atomic mass numbers), they are brailled with the subscript first followed by the superscript.

Examples:

x^n

⠭⠰⠔⠝

x_n

⠭⠰⠢⠝

$^{-}2 + {}^{-}3 = {}^{-}5$

OR

$^{238}_{92}U$

Definition of an Item

An “item” is defined as any single symbol(s) that follows immediately after the level change indicator. It is therefore important to make clear to the reader exactly what symbol(s) will be included as a consequence of the level change indicator.

There is a defined list, shown below, of the specific symbols considered to be the “next item”:

1. An entire number, i.e. the initiating numeric indicator and all succeeding symbols within the numeric mode (which would include any decimal points, commas, numeric separator spaces, or simple numeric fraction lines).
2. An entire general fraction, enclosed in general fraction indicators (⠰) and (⠰) as shown in Lesson 2 of this module. Note that Grade 1 indicators will be required if the sequence is not already in Grade 1 mode.
3. An entire radical expression, enclosed in radical indicators.
4. An arrow (shown in UEB Training Manual: Extension Mathematics).
5. An arbitrary shape.
6. Any expression enclosed in matching pairs of round parentheses, square brackets or curly braces.

If none of the above conditions apply, then the “item” is only the next symbol and may require braille grouping indicators (⠐) opening and (⠰) closing to ensure the whole of the superscript or subscript has been captured. Note that Grade 1 indicators will be required if the sequence is not already in Grade 1 mode.

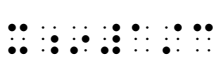
Any expression enclosed in the braille grouping indicators shown above will subsequently make it clear to the reader exactly what symbols are included as part of the level change and considered to be the “next item”.

Examples:

2^2

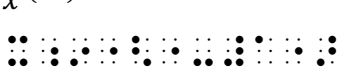
$x^{2.2}$

$x^{1/3}$

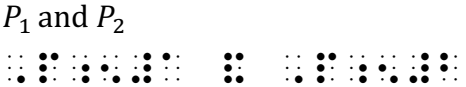


Footnote^{56}

$x^{(-1)}$



P_1 and P_2



H_2O

x^{2y}

OR

Notes:

1. The rule for the definition of an item outlined above is critical to transcribing mathematics into braille correctly.
2. If none of the groupings listed above under “Definition of an Item” apply to the expression, then the item is **only** the next individual symbol.
3. Use the opening (⠸⠣) and closing (⠸⠜) grouping indicator (also shown above) to capture the whole of the print sequence if the definition of an item is not met.
4. Remember the numeric indicator (⠼) also sets Grade 1 mode for the remainder of the symbols-sequence.
5. When Grade 1 mode is set by the numeric indicator (⠼), it is terminated by a space, hyphen, dash or Grade 1 terminator.

Examples:

x^2

x^{-2}

OR

Hint:

- The superscript and subscript signs each have a Grade 2 (contracted) meaning so will always require Grade 1 indicators if the sequence is not already shown in Grade 1 mode. For more information about Grade 1 mode, refer to Lesson 1 and Lesson 2 of this Training Manual and also Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics*.

Examples:

y^n

y_n

x^{-n}

OR

x_{-n}

OR

a^{2n}

OR

x_{2n}

OR

Remember:

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice when selecting Grade 1 indicators in mathematical contexts, with any of the options for Grade 1 mode (symbol, word or passage) being equally correct.

However, for the purpose of the UEB Online exercises that are associated with this UEB Training Manual: Advanced Mathematics, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 4

1. 10^2
2. x^{b+1}
3. $x^{\frac{1}{2}}$
4. $x^{1/2}$
5. x_4
6. x_{2+}
7. x^2
8. P_x
9. $a^{-n} = \frac{1}{a^n}$
10. $a^n\, a^m = a^{n+m}$

Review Exercise 4

1. $0.0025 = 2.5 \times 10^{-3}$
2. x_{a+b}
3. $a_2 + b_3$
4. $x^y z$
5. $\frac{x^{-9}}{4}$
6. $\frac{x^y}{z}$
7. $\frac{x^{1.2}}{4}$
8. $\frac{x^{-\frac{3}{4}}}{4}$
9. $7e_{2x}$
10. $e^{x^3}y$

Lesson 5: Roots and other Radicals

The opening and closing radical signs shown below also have a contracted (Grade 2) meaning and so have been shown with a Grade 1 indicator in front of them. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a Grade 2 (contraction) meaning and Grade 1 mode has not already been established in the sequence.

 $\sqrt{}$ Open radical (root) sign, with vinculum

 Close radical (root) sign (there is no print representation for this braille symbol)

 $\surd$ Square root sign without a vinculum

Square Roots

The expression inside the square root sign in print (known as the **radicand**) should be preceded by the open radical (root) sign and followed by the close radical (root) sign. The radicand sign may be any expression whatsoever and may therefore contain radicals as well as other mathematical structures.

A **vinculum** is a horizontal line used in mathematical notation for a specific purpose.

In the examples below, the vinculum is used as part of the notation of the radical to highlight the radicand whose root is being indicated.

Examples:

$\sqrt{16} = 4$

$\sqrt{x+y}$ (square root of x + y)

OR

Radical Index (Cube Roots etc)

The radical index, if present, is printed above and to the left of the radical sign. The index is shown in braille as a superscript expression immediately following the opening radical symbol.

Notes:

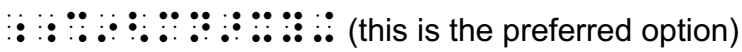
1. Grade 1 mode may be necessary, using a Grade 1 symbol indicator, a Grade 1 word indicator, or a Grade 1 passage indicator with Grade 1 passage terminator. Refer to Lesson 1 and Lesson 2 of this Training Manual and also Lesson 3 of the *Unified English Braille Training Manual: Introductory Mathematics* for further reference if required.
2. Ensure that whatever is showing under the root sign has been closed after the last item by using the close radical sign (⠸⠜).
3. When Grade 1 mode is set by the numeric indicator (⠼), it is terminated by a space, hyphen, dash or Grade 1 terminator.
4. Remember the rules relating to the “next item” (as explained in Lesson 4 of this Training Manual) when using superscripts and subscripts.

Examples:

$\sqrt[3]{8} = 2$ (cubed root of 8 equals 2)

OR

$\sqrt[mn]{xy}$

 (this is the preferred option)

OR

Square Root sign on its own with no vinculum

Sometimes print omits the horizontal line (vinculum) above the radicand. A root sign showing the vinculum means exactly the same, and the use (or otherwise) of the vinculum is only a preference by publishers. A closing indicator is not required when this root sign with no vinculum is used in braille.

Notes:

1. When introducing the topic of simple roots to younger children, it is better to introduce the concept of roots with a vinculum requiring the opening and closing indicators from the beginning of their learning.
2. If, however, the square root sign is being used as an isolated graphic symbol √ then the root sign without a vinculum (⠸⠩) can be used.

3. Remember the numeric indicator (⠼) also sets Grade 1 mode for the remainder of the symbols-sequence.

Examples:

The √ sign is often shown in the text.

⠼⠁⠣ ⠩⠼⠁⠋ ⠷ ⠼⠙ ⠨⠅ ⠼ ⠼⠁⠋⠐⠶⠼⠙

$\sqrt{16} = 4$

⠩⠼⠁⠋⠜ ⠐⠶ ⠼⠙

Remember:

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice when selecting Grade 1 indicators, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. However, for the purpose of the UEB Online exercises that are associated with this UEB Mathematics training manual, use the following criteria for implementing Grade 1 mode when completing the online exercises:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 5

1. $\sqrt{25} = 5$
2. $\sqrt{3^2} + 4$
3. $3\sqrt{28} - 2\sqrt{7}$
4. $\sqrt{2t} \times \sqrt{8} = 4\sqrt{3}$
5. $\sqrt{16t} = 4\sqrt{3}$
6. $\frac{6}{2\sqrt{3}}$
7. $5(2\sqrt{3} + 3)$
8. $\sqrt{a^2 + b^2})$
9. $E = \sqrt{1 - \frac{b^2}{a^2}}$
10. $V = \sqrt{2gR}$

Review Exercise 5

1. $x = \sqrt{x^2}$
2. $x = \frac{5 + \sqrt{45}}{2}$
3. $(a - 4)^2$
4. $(b - 4) = \pm\sqrt{3}$
5. $-(5x - 3) \geq 2$
6. $F = \frac{mv^2}{r}$
7. $m = \frac{y_2 - y_1}{x_2 - x_1}$
8. 10^{-4}
9. $(2r)^{\frac{1}{2}}$
10. $T_n = a + (n - 1)d$

Lesson 6: Shape Indicators (continued) and Miscellaneous Symbols

Shape Indicators

The shape indicators shown below also have a contracted (Grade 2) meaning and so have been shown with a Grade 1 indicator in front of them. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning and Grade 1 mode has not already been established in the sequence. For more information about Grade 1 mode, refer to Lesson 1 of this Training Manual and also Lesson 3 of *Unified English Braille Training Manual: Introductory Mathematics*.

⠸⠩ shape indicator

⠸⠹ shape termination indicator (there is no print representation for this braille symbol)

⠸⠩⠫⠹  square (as introduced in Lesson 6, *Unified English Braille Training Manual: Introductory Mathematics*)

⠸⠩⠫⠩  triangle, equilateral (as introduced in Lesson 6, *Unified English Braille Training Manual: Introductory Mathematics*)

⠸⠩⠫⠱ pentagon

⠸⠩⠫⠩  hexagon

⠸⠩⠫⠳  octagon

⠸⠩⠻⠩ parallelogram

⠸⠩⠿  circle (as introduced in Lesson 6, *Unified English Braille Training Manual: Introductory Mathematics*)

Transcriber defined shapes

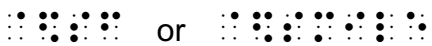
⠐⠩ transcriber-assigned shape indicator

Note:

A transcriber-assigned shape indicator should not be used if the print symbol has already an assigned braille sign. The indicator should precede a short series of initials or a single Grade 1 word. The definitions of all the transcriber-assigned shape indicators used in the transcription should be available to the reader in either a transcriber's note or on a special page.

Example:

A smiling face icon could be shown in either of the following ways:

 or

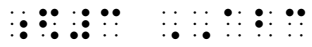
Use of the shape termination indicator

 shape termination indicator

- If a shape is followed by a space, then no termination sign is required.
- If the shape is followed by punctuation or is unspaced from a following symbol, then the shape termination indicator must be used.
- The shape termination indicator has a Grade 2 (contracted) meaning so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the shape termination indicator.
- Remember the numeric indicator () also sets Grade 1 mode for the remainder of the symbols-sequence.

Examples:

∆ ABC



∆ABC

Miscellaneous Symbols (continued)

Not all miscellaneous symbols are included in this course. For a more extensive list of miscellaneous symbols and their equivalent braille representations, please refer to the “Unified English Braille Guidelines for Technical Material” (International Council on English Braille, 2014).

 ′ foot or minute (shown as a prime sign)

 ″ inch or second (shown as a double prime sign)

 ! factorial sign

 ∴ therefore sign

 ∠ angle sign

 ∥ is parallel to

 ⊥ is perpendicular to

⠶⠶ ≡ is congruent/equivalent to (three horizontal lines)

⠸⠸⠸ ||| is similar to (three vertical lines)

⠷⠷ [open square bracket

⠾⠾] close square bracket

⠷⠷ { open curly bracket

⠾⠾ } close curly bracket

⠠ dot 5 continuation indicator (used when the braille sequence is too long for the line and needs to be broken)

⠀ visible blank space (an omission, which often occurs in fractions)

⠸⠌ / cancelling sign

⠐⠐ . recurring decimal (dot over the top of a number)

⠸⠩ ✓ tick sign (not to be confused with the root sign)

Notes:

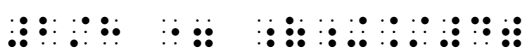
1. In general, the spacing of symbols follows the print.
2. Some of the signs listed above have a Grade 2 (contracted) meaning. A Grade 1 indicator will therefore be necessary if the sequence is not already in Grade 1 mode.
3. If the braille representation for a print sequence does not fit on the line, then a dot 5 continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing for the remainder of the print sequence. Usually the preferred place to break is before a comparison sign or an operation sign.
4. Braille grouping signs are needed for the recurring decimal to explicitly show the character that has the dot above.
5. Remember when both a subscript and superscript are shown together in print, they are brailled with the subscript first followed by the superscript.

Examples:

$6'3''$

$\frac{2}{8} = \frac{}{4}$

OR



$4! = 4 \cdot 3 \cdot 2 \cdot 1$ (shown using a running product)

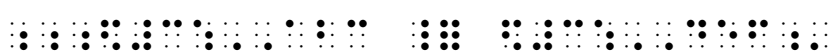
$\therefore x = 2$

$\angle A + \angle B = 90°$

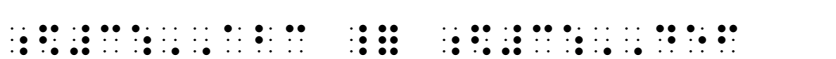
$AB \parallel CD$

$AB \perp CD$

$\Delta ABC \equiv \Delta DEF$



OR



$\Delta ABC \,|||\, \Delta DEF$

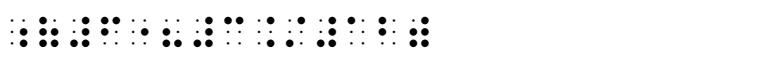
OR

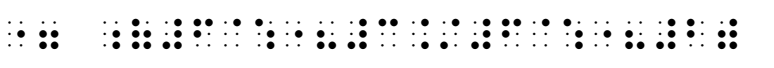
$$\frac{6 \times 3}{12} = \frac{\not{6} \times 3}{\not{6} \times 2}$$

OR

Note the use of the dot 5 continuation indicator in the following example which may not be necessary when the = sign is taking the whole of the expression to the next line.

OR

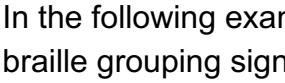




Yes ✓

In the following examples the recurring decimal has been shown using the braille grouping signs.

$0.\dot{7}$

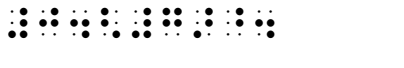


$0.48\dot{3}4\dot{5}$

OR

In the same example as above, and because the digits are implicitly grouped in print with recurring decimals, this can also be shown as:

$0.48\dot{3}4\dot{5}$



Remember:

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- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 6

1. $0.35\dot{1}2\dot{3}$
2. $\angle EFG$ is adjacent to $\angle GFH$
3. $\triangle ABC \equiv \triangle DEF$
4. $\angle OCA \neq \angle OCB$
5. $3(8+5) = 3 \times \square + 3 \times \square$
6. $EF \parallel GH$
7. $\therefore PQ \perp RS$
8. {Craig, Frances, Tarna}
9. $\angle ABC = \angle DEF$
10. $\triangle EDF \;|||\; \triangle FDG$

Review Exercise 6

1. $(x+5)^2 = x^2 \square x + 25$
2. $P(x) = 5 - 3x + x^2$
3. $(9y+1)(7y+2) = 0$
4. $v = \sqrt{\frac{g}{k}}$
5. $A = \frac{1}{2}h(a+b)$
6. $S = \frac{n}{2}(a+l)$
7. $\frac{2y}{5} = 11 - \frac{y}{3}$
8. $T_n = ar^{n-1}$
9. $\sqrt{8.41 \times 10^{-8}}$
10. $(xy)^3$

Lesson 7: Functions

Trigonometric functions

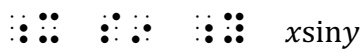
Common trigonometric functions are Sine, Cosine and Tangent. These functions are usually abbreviated in print as sin, cos and tan.

1. Sine (Sin), Cosine and Tangent may be contracted unless already in Grade 1 mode.
2. Where the function name is **preceded** or **followed** by a lowercase letter, a space may be needed between the letter and the function to remove any ambiguity as to where the function name begins and ends.

Examples:


$\sin x$


$\cos y$


$x\sin y$

If the function name is directly preceded or followed by a number, then the number should be written **unspaced** from the function name.

Remember the numeric indicator (⠼) also sets Grade 1 mode for the next symbols-sequence.

Examples:


$5\tan 45°$


$7\sin 45°$


$\sin 45$


$x\tan 60$


$x\sin 60$

A space is not needed if the function name is already separated by a bracket or by a braille indicator, such as a capitalisation indicator, a fraction indicator, or a Greek letter indicator, which is explained more fully in Lesson 8 of this Training Manual.

Remember the opening and closing fraction indicators have a Grade 2 (contracted) meaning so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the opening and closing fraction indicator.

Examples:

 $\sin(A + B)$

 $\mathrm{Cos}A$

 $\mathrm{Tan}50°$

 $\log \frac{x}{2}$

 $\sin\theta$

Logarithmic functions

The logarithmic function is usually written as log or Log and may be followed by a subscript indicating the base. A logarithm to base “e” is called a natural log and is often abbreviated to ln.

Examples:

$5\log x$

$\log_2 8 = 3$

$\ln e = 1$

Remember:

As referred to in the General Note in Lesson 1 of this Training Manual, there is often a choice of Grade 1 indicators in mathematical contexts, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. However, for the purpose of the UEB Online exercises that are associated with this training manual, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
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Exercise 7

1. $\text{Sin}\ A$
2. $\text{Log}(x+y)$
3. $6\tan 90°$
4. $\text{Sin}30$
5. $5\text{Sin}45$
6. $7\cos 5x$
7. $\log a\ +\ \log b\ =\ \log ab$
8. $\log_4 = -2$
9. $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$
10. $\dfrac{\sin Z}{35} = \dfrac{\sin 70°}{7}$

Review Exercise 7

1. $\dfrac{5}{\square} = \dfrac{30}{36}$
2. $\text{In}\ \triangle DEF, \angle E = 90°$
3. $\sqrt{9a^8}$
4. $\left(\dfrac{25}{49}\right)^{\frac{1}{2}}$
5. $\sqrt{m^2}$
6. $\dfrac{18\cos 12°}{13\tan 68°}$
7. $\sqrt[6]{101.9}$
8. $\text{In}\ \triangle KLM, \angle M = 27°51'$
9. $\sin Z = \dfrac{35\sin 70°}{45}$
10. $\log_e\left(\dfrac{2}{3}\right)$

Lesson 8: Greek Letters

Greek letters are used extensively in Mathematics. While only a small number have been used in this Training Manual, the principles for use remain the same for all. Refer to the “Unified English Braille Guidelines for Technical Material” (International Council on English Braille, 2014) for a complete list.

	α	alpha (lower case)		A	Capital
	β	beta (lower case)		B	Capital
	δ	delta (lower case)		Δ	Capital
	ε	epsilon (lower case)		E	Capital
	γ	gamma (lower case)		Γ	Capital
	θ	theta (lower case)		Θ	Capital
	λ	lambda (lower case)		Λ	Capital
	μ	mu (lower case)		M	Capital
	π	pi (lower case)		Π	Capital
	σ	sigma (lower case)		Σ	Capital

Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicator, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct. However, for the purpose of the UEB Online exercises associated with this training manual, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
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- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Exercise 8

1. $A = \frac{\theta}{360} \times 2\pi r$
2. $V = \frac{1}{3}\ \pi r^2\ h$
3. $A = 2\pi r^2 + 2\pi rh$
4. $S =\ \theta r(r + l)$
5. $\therefore\ \Sigma x = 195$
6. $V = \frac{1}{3}\ \pi$
7. $C = 2\pi r$
8. Standard deviation (σ_n)
9. Mean $= \frac{\Sigma fx}{\Sigma f}$
10. surface area $= \pi rs +\ \pi r^2$

Review Exercise 8

1. $0\ \leq\ x\ \leq\ 2\pi$
2. $\frac{25\pi}{8} m^2$
3. $(\log_e x)^4$
4. $sin\theta\ =\ \frac{14.6\ \sin 48°}{12.6}$
5. $\angle DAC =\ \angle ACB$
6. $\therefore \triangle ABC \equiv \triangle ABD$
7. $\sqrt[3]{8u^{18}}$
8. $25gh\ \div\ \square = 5g$
9. $y = -\frac{1}{2}(x + 4)(2 - x)$
10. $DE \parallel AC$ and $CE{:}EB$ = 2:3

Lesson 9: Review Test

Congratulations on reaching this Review lesson. The content of each lesson has built upon preceding lessons, with the overall structure of this Training Manual designed to reinforce several foundational principles, including the following:

1. The numeric indicator (⠼) sets numeric mode **and** Grade 1 mode for the remainder of the symbols-sequence.
2. Numeric mode is transitive over the 10 digits, the full stop, the comma, the numeric space, the simple fraction line and the continuation indicator and is terminated by symbols such as the hyphen, dash, slash/oblique stroke and colon whereby the numeric indicator will need to be repeated.
3. When Grade 1 mode is established by the numeric indicator, it is terminated by a space, hyphen, dash and Grade 1 terminator.
4. A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.
5. Understanding of when a fraction is considered a “simple fraction”. If the sequence does not satisfy the definition of a simple fraction, then it must be treated as a general fraction.
6. The rules associated with “the next item” should be considered and are particularly relevant in the use of subscripts and superscripts.

Remember:

As referred to in the General Note in Lesson 1 of the UEB Training Manual: Advanced Mathematics and throughout the preceding lessons, the choice of options for Grade 1 mode (symbol, word or passage) are equally correct in mathematical contexts. However, for the purpose of completing the following Review Test in the UEB Online course, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Review Test

1. $A = \pi r^2$
2. $\frac{4\sqrt{3}\times\sqrt{18}}{\sqrt{12}}$
3. $time = \frac{distance}{speed}$
4. $4r^4 \div 2r$
5. $x^{7.9}$
6. $A = P(1 + r)^n$
7. $3\sqrt{8}$
8. x^{-2}
9. x^3y
10. $(2x + 9)^2$
11. $\frac{x^{\frac{1}{4}}}{4}$
12. $\sqrt[3]{2x}$
13. $\{2\ ,4,\ -6\}$
14. $P(E) = \frac{n(E)}{n(S)}$
15. $\sqrt{96} \div \sqrt{12}$
16. $y = -\frac{1}{2}(x - 1)^5$
17. $AB \parallel PQ$
18. $\therefore \triangle MNO \equiv \triangle PQR$
19. $3! = 3 \cdot 2 \cdot 1$
20. $\frac{3x^2-6x}{x^2+x-6}$

Lesson 10: Advanced Test

Congratulations on reaching this Advanced Test. The content of each lesson has built upon preceding lessons, with the overall structure of this Training Manual designed to reinforce several foundational principles, including the following:

1. The numeric indicator (⠼) sets numeric mode **and** Grade 1 mode for the remainder of the symbols-sequence.
2. Numeric mode is transitive over the 10 digits, the full stop, the comma, the numeric space, the simple fraction line and the continuation indicator and is terminated by symbols such as the hyphen, dash, slash/oblique stroke and colon whereby the numeric indicator will need to be repeated.
3. When Grade 1 mode is established by the numeric indicator, it is terminated by a space, hyphen, dash and Grade 1 terminator.
4. A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.
5. Understanding of when a fraction is considered a “simple fraction”. If the sequence does not satisfy the definition of a simple fraction, then it must be treated as a general fraction.
6. The rules associated with “the next item” should be considered, and are particularly relevant in the use of subscripts and superscripts.

Remember:

As referred to in the General Note in Lesson 1 of the UEB Training Manual: Advanced Mathematics, and throughout the preceding lessons, the choice of options for Grade 1 mode (symbol, word or passage) are equally correct in mathematical contexts. However, for the purpose of completing the following Advanced Test in the UEB Online course, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Advanced Test

1. $\sqrt{a^2 + b^2}$
2. $\sqrt[3.4]{8}$
3. The mean $= \frac{\Sigma fx}{\Sigma f}$
4. $y = x^4 - x^3 - 10x^2 - 8x$
5. $\log \frac{1+x}{x}$
6. $\frac{a}{\sin 20°} = \frac{12}{\sin 60°}$
7. $y = log_a f(x)$
8. $f(L) = 2\pi\sqrt{\frac{L}{g}}$
9. $f(x) = 2\,cos\,\left(x + \frac{\pi}{2}\right) + 1$
10. $\log(x^2 - x - 2) - \log(x + 1)$
11. $\log\sqrt{x^2 - 4x + 4}$
12. $\log_a \frac{x}{\sqrt{y}}$
13. $\sin 120° = 0.87$
14. $\tan\,\theta = \frac{\sin\,\theta}{\cos\,\theta}$
15. $\cos\,\frac{\pi}{5} = \frac{1+\sqrt{5}}{4}$
16. $\log_e x$
17. $m = \frac{y_2 - y_1}{x_2 - x_1}$
18. $\frac{1}{k(k+1)(k+2)}$
19. $\frac{4.1 \times \sqrt{48.12}}{26.23}$
20. $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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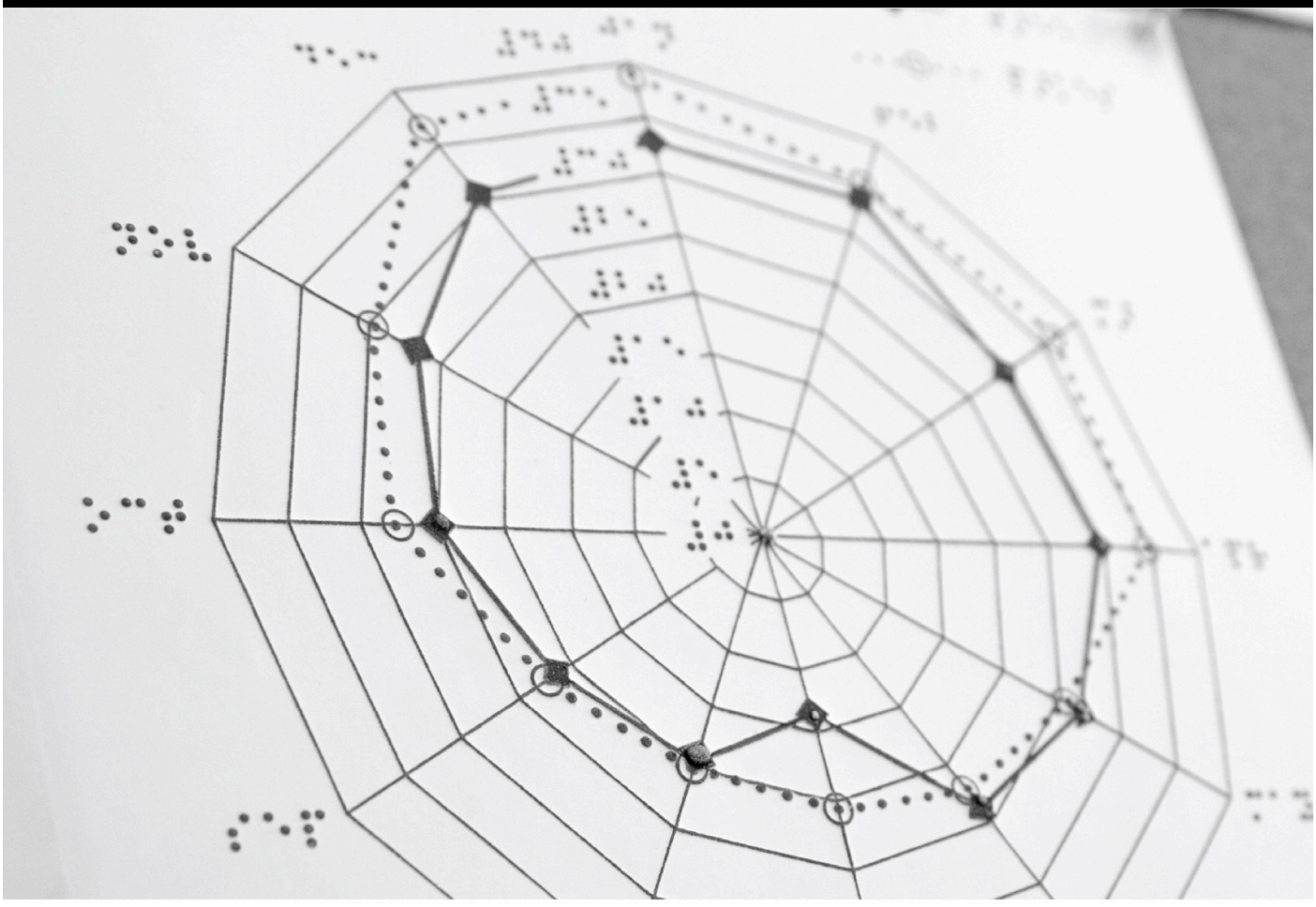
Next Sense

Advancing the RIDBC Renwick Centre experience.

Unified English Braille Training Manual

Extension Mathematics

Josie Howse



UEBOnline

Unified English Braille Training Manual: Extension Mathematics
Revision 4

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NextSense Institute is Australia's leading centre for research and professional studies in the field of education for children with sensory disabilities; offering webinars, short courses, and degree programs for parents, carers, educators, and health professionals. NextSense Institute is committed to providing high-quality teaching and learning opportunities. Our programs are conducted by leading national and international experts for education and health professionals who support people who are deaf, hard of hearing, blind or have low vision.

Table of Contents

Foreword

We live in an era where education leaders and decision makers are responding to national and international disability anti-discrimination legislation, policies and standards that prioritise equitable inclusive education for all, leaving no-one behind. However, for learners with vision impairment, the low incidence nature of their disabilities can present challenges for education systems and schools. The World Blind Union in 2019 highlighted three global challenges impeding education enrolment and achievement for children and young people with vision impairment, in particular in low and middle-income countries. These challenges are a chronic shortage of teachers who are qualified to teach braille, an absence of braille instruction in teacher training programs or offered by education systems, and the high cost of braille equipment. With these challenges in mind, NextSense has created online, open access training programs and resources in braille literacy and mathematics for education leaders, teachers, parents and caregivers – basically, for anyone who wants to learn braille.

In my experience as a lecturer in sensory impairment, professional qualifications, quality teaching and positive student outcomes are closely linked. Knowledgeable and inspiring teachers reach out to their students, building and enriching student knowledge and igniting curiosity, self-confidence, and motivation to pursue learning. In the field of mathematics, we need such teachers who can reach out to their students with vision impairment. Teacher training institutions have a responsibility to provide teachers with instruction in braille codes and braille literacy development, the mechanics of producing braille and tactile representations of visual information, and practical experience in reading and transcribing print and braille materials.

In offering this UEB Extension Mathematics training manual and online program, NextSense **aims** to address the following:

- To extend professional and parent/caregiver knowledge of braille mathematics by building upon the lessons and exercises presented in the introductory and advanced braille mathematics training programs
- To introduce mathematical symbols in braille that are commonly found in senior secondary mathematics publications; and in so doing,
- To support and promote equitable access to mathematics for learners who use braille or a combination of braille, print and digital formats.

In Australia, as in other parts of the world, braille experts who are knowledgeable in the field of mathematics are limited in number. NextSense is fortunate to have Josie Howse write the content of this Extension Mathematics training manual, in addition to the Introductory and Advanced Mathematics training manuals. During her employment and professional affiliations over the past four decades, Josie Howse has been at the forefront of braille training, assessment, and production in Australia and overseas. She has

contributed to the development of Unified English Braille (UEB) by the International Council on English Braille (ICEB), and adoption and implementation of Unified English Braille by Australia in 2005.

NextSense acknowledges the contribution of Craig Cashmore of PeppaCode in developing and maintaining the UEB Online website and translating the content of the literary and mathematics training manuals into accessible, interactive online training programs. Sincere thanks are also extended to the NextSense UEB Online design team and the foundations and trusts that have contributed to the UEB Online training programs since 2014.

I wish to reflect on a quotation from Hellen Keller, as published by the American Foundation for the Blind. Hellen Keller spoke these words 14 years before the United Nations “Universal Declaration of Human Rights” in 1948. Her words are just as relevant today and serve as a clarion call to us all to continue our efforts to promote and support the right to education for all children, leaving no-one behind.

> "Education should train the child to use his brains, to make for himself a place in the world and maintain his rights even when it seems that society would shove him into the scrap-heap."
>
> Hellen Keller, 1934

Frances Gentle, AO, PhD, D. Litt. *Honoris Causa*
Conjoint Lecturer, NextSense Institute,
President, International Council for Education of People with Visual Impairment; and Co-President, South Pacific Educators in Vision Impairment

Contributors

Author: **Josie Howse**, PSM

M.Spec. Ed. (Sensory Disability) – University of Newcastle,
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Josie Howse is an Adjunct Research Fellow with the NextSense Institute and former Manager of the Braille and Large Print Services, NSW Department of Education. The NSW Department of Education team provides all texts and examinations in braille, large print and e-text to students with vision impairment in the government sector and is the largest producer of alternate format student textbooks and examinations in Australia.

Josie has been working in the field of vision impairment for more than 40 years. She has held a number of executive positions at national and state levels and has extensive experience in braille code development at an international level. Josie was the editor of the 2006 Unified English Braille Primer: Australian Edition, and co-editor of the 2016 Unified English Braille: Australian Training Manual.

Josie was awarded the Public Service Medal (PSM) in the 2007 Queen's Birthday Honours list, has been listed annually in the Who's Who of Australian Women since 2008, and is the 2012 recipient of a Lifetime Achievement Award from the Round Table on Information Access for People with Print Disabilities, and awarded Honorary Life Membership of the South Pacific Educators in Vision Impaired (SPEVI) in 2020.

UEB Online Developer: Craig Cashmore

B. Eng. (Hons) - UTS

Craig holds an Engineering Degree in Telecommunications and has worked in the software development industry for over 30 years, holding senior software design, software architecture and technical management positions in companies including Jtec, Ericsson and LongReach Networks.

More recently Craig founded PeppaCode, a web and app development business focused on 'out-of-the-ordinary' strategic web and software development for small business, start-ups and educational institutions.

Some of Craig's achievements at PeppaCode include the successful launch of UEB Online for NextSense and a vehicle tracking and management system for a bus operator. Craig continues to work on new and innovative projects using modern web technologies.

Lesson 1 - Revision

Welcome to the Unified English Braille (UEB) Training Manual: Extension Mathematics. This training program builds on the basic principles that you have already learnt in the Introductory and Advanced level training programs but will apply the principles in an extended mathematics context. You will also be introduced to some new signs which may be required when transcribing higher level mathematics.

All examples and test items presented in this training program have been taken directly from Senior Mathematics textbooks.

Lesson 1 will review some of these critical principles to reinforce what has already been delivered in the Introductory and Advanced mathematics programs and prepare you for implementing best practice in this extension training program.

Some of the topics covered in this revision lesson are as follows:

- Grade 1 Mode
- Shape Indicators
- Fractions
- Miscellaneous Symbols
- Superscripts and Subscripts

Grade 1 Mode

The rules for the use of Grade 1 mode in a literary context will also apply in a mathematical context and are summarised below.

A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.

The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

 Grade 1 Symbol Indicator (dots 5 6)

 Grade 1 Word Indicator (dots 5 6, 5 6)

 Grade 1 Passage Indicator (dots 5 6, 5 6, 5 6)

 Grade 1 Passage Terminator (dots 5 6, 3)

 Grade 1 Passage Indicator on a line of its own

 Grade 1 Passage Terminator on a line of its own

Note:

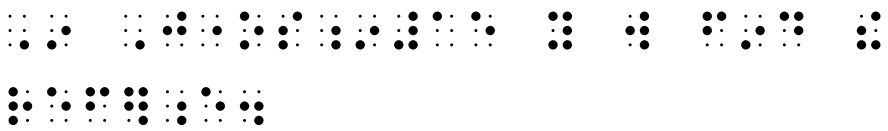
- Grade 1 indicators will not be needed for simple arithmetic problems involving numbers, operation signs, numerical fractions, and mixed numbers.

Grade 1 Symbol Indicator

- A Grade 1 symbol indicator () sets Grade 1 mode for only the next symbol.

  Example:

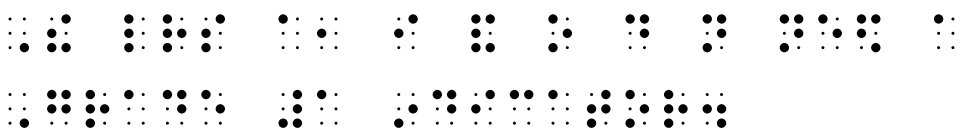
  In Jones[15] you will find the reference.

- A Grade 1 symbol indicator is NOT required before the letters a, i and o, because they do not have a contraction meaning when they “stand alone”.

  Example:

  The letters a, i and o do not need a Grade 1 indicator.

- A Grade 1 symbol indicator precedes a capitalisation indicator.

  Example:

  The letter B.

- A Grade 1 indicator will be required when a lower-case letter a – j immediately follows a digit so it will not be misread as part of the number.

  Examples:

  25b  not 252

  Trevor@take5.com

   not 5.3

- A Grade 1 symbol indicator may be required for a sequence of letters in braille that could represent an alphabetic wordsign, or a shortform such as *ab* (about) or *ac* (according), if the letter or letter sequence is “standing alone”.

  Example:

  ab = ac 

- Remember that the numeric indicator, besides setting numeric mode, also sets Grade 1 mode for the next symbols-sequence.

  Example:

  www.shopping4you

Termination of Grade 1 Mode

- When Grade 1 mode is set by a numeric indicator, it is terminated by a space, a hyphen, a dash, or a Grade 1 terminator.

  Examples:

  3-dimensional

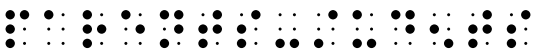
  

  3–D

  www.education4everyone/parents-students

Numeric Mode

- Remember that the following symbols may occur in numeric mode:
  - The ten digits
  - Full stop (period)
  - Comma
  - The numeric space
  - Simple numeric fraction line, and

- The continuation indicator

Termination of Numeric mode

- Therefore, the numeric mode is terminated by a space or any symbol such as the hyphen, dash, slash/oblique stroke, and colon.

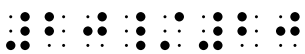
  1915-19

  1915–1919

  20/20

  16:30 pm

Standing Alone Rule

Refer to *The Rules of Unified English Braille* (Simpson, 2013) for a comprehensive definition and explanation of usage of “standing alone”.

A letter or letters-sequence is considered to be “standing alone” when:

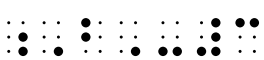
- it is **preceded** and **followed** by a space, a hyphen, or a dash (of any length); or
- common punctuation and indicator symbols intervene between the letter or letter-sequence and the **preceding** or **following** space, hyphen, or dash; or
- a word with an interior apostrophe is considered to be “standing alone” under the specific provisions stated in *The Rules for Unified English Braille*, 2013 referring to contractions, alphabetic wordsigns, strong wordsigns and shortforms.

  Examples:

  b-3

  B–3

“b”

⠃

(b)

⠐⠃⠐⠜

(B)

⠠⠃⠠⠜

“a’s and b’s”

⠁⠄⠎ ⠃⠄⠎

“A’s and B’s”

⠠⠁⠄⠎ ⠠⠃⠄⠎

“can’t”

⠉⠁⠝⠄⠞

Grade 1 Word Indicator

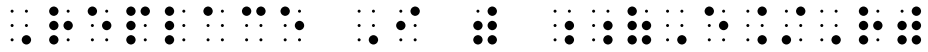
- The Grade 1 word indicator (⠰⠰) sets Grade 1 mode for the next symbols-sequence or the remainder of the current symbols-sequence.
- The effect of a Grade 1 word indicator is terminated by a space or a Grade 1 terminator.
- Remember that the numeric indicator also sets Grade 1 mode for the next symbols-sequence.
- If a complex algebraic expression does NOT include a comparison sign (such as an equal's sign) then it is unlikely to include interior spaces in braille. In such cases, a Grade 1 word indicator will be enough to ensure that superscripts, subscripts, fractions, radicals, arrows, and shape indicators are well defined without the need for Grade 1 symbol indicators.

  Examples:

- Evaluate $\sqrt{(y\ -x^2)}$

⠰⠰⠩⠽⠀⠤⠀⠭⠘⠆⠜⠱

- Replace I with $\frac{E}{R}$



Grade 1 Passage Indicator

- A Grade 1 passage indicator () sets Grade 1 mode for the next passage.
- Complex algebraic equations that include a comparison sign are best enclosed in Grade 1 passage indicators. This will ensure that isolated letters and indicators such as superscripts, subscripts, fractions, radicals, arrows, and shapes are well defined without the need for Grade 1 symbol indicators.
- A Grade 1 passage is terminated by the Grade 1 passage terminator **(** )

Grade 1 Passage Terminator

- The Grade 1 passage terminator () follows **immediately** after the last affected symbols-sequence of a Grade 1 passage.

  Example:

  $x = y - z,\ y = x + z,\ z = y - x$

Grade 1 Passage Indicator and Terminator on lines of their own

- Where it is better to preserve the natural alignment of a text, for example in a computer program or a set of equations in mathematics, place the Grade 1 passage indicator () on a separate line above the equations (consider the margin as a suitable location), and the Grade 1 terminator () on a separate line below the equations.
- Refer to the example below which shows this use with a set of quadratic equations using superscripts. When this method is used, each indicator should be preceded and followed by the dot locator for “use”  .

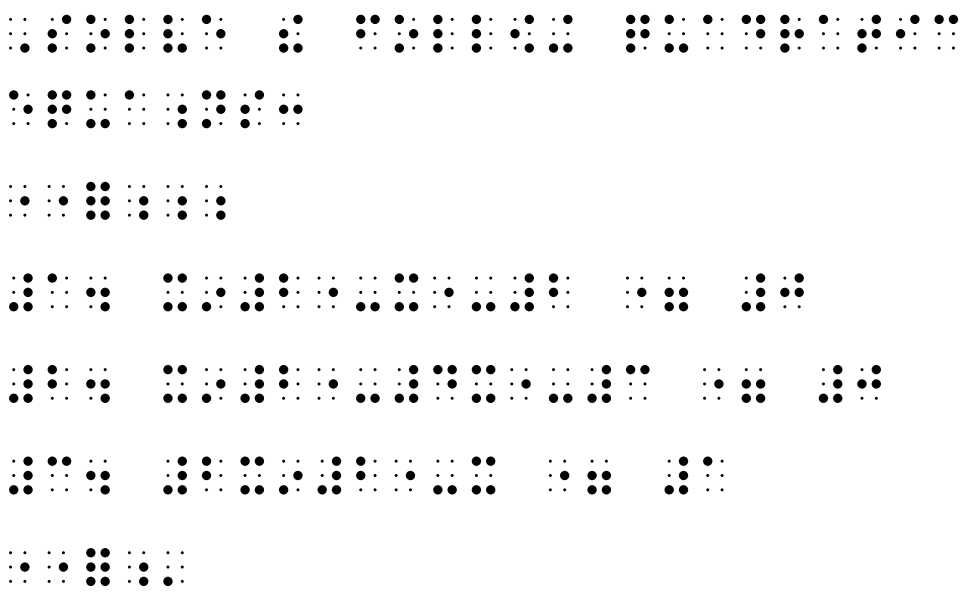
  Example:

  Solve the following quadratic equations:

  1. $x^2 - x - 2 = 0$

2. $x^2 - 4x - 3 = 0$

3. $2x^2 - x = 1$



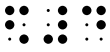
Shape Indicators

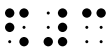
A number of shapes used in mathematics were introduced in the UEB Introductory and Advanced Mathematics training manuals. The most common shapes used in senior mathematics are shown below.

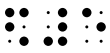
Please note that the shape indicator also has a contracted (Grade 2) meaning and so may require a Grade 1 indicator in front of it. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning and Grade 1 mode has not already been established in the sequence.

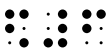
 shape indicator

 shape termination indicator (there is no print representation for this braille symbol)

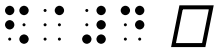
 square

 triangle (equilateral)

 pentagon

 hexagon

 octagon

 parallelogram

 circle

 visible space

Transcriber defined shapes

 transcriber-assigned shape indicator

Note: A transcriber-assigned shape indicator should **not** be used if the print symbol has already an assigned braille sign. The indicator should precede a short series of initials or a single Grade 1 word. The definitions of all the transcriber-assigned shape indicators used in the transcription should be available to the reader in either a transcriber's note or on a specific page.

Example:

A smiling face icon could be shown either of the following ways:

 or 

Use of the shape termination indicator

 Shape termination indicator (there is no print representation for this symbol in braille)

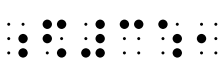
- If a shape indicator is followed by a space, then no termination sign is required.
- If the shape indicator is followed by punctuation or is unspaced from a following symbol, then the shape termination indicator must be used.
- All the initial shape indicators initiate shape mode so no further Grade 1 indicators will be needed.

  Examples:

  Δ ABC

  Δ,

  ΔABC

Omissions in technical expressions

Follow the print when omissions are indicated by shapes (such as squares) or punctuation (such as underscores or question marks).

When print uses a blank within a technical expression, use the visible space indicator (⠸⠀) to represent the blank.

Space the sign of omission the same as the sign being omitted.

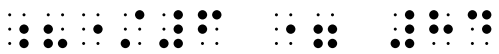
Remember a Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning and Grade 1 mode has not already been established in the sequence.

Examples:

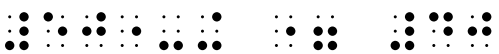
□ + 75 = 80

48 = 12 × □ + Δ

? ÷ 6 = 84



50 − = 40



Fractions

It is important to understand the definition of a **simple fraction**, which the elements and only those elements that can be considered to be a simple fraction and subsequently use the simple fraction line, (⠸⠂) between the numerator and the denominator.

If a fraction does **not** comply with the **simple fraction** definition (restated below) for whatever reason, then it will be a **general fraction** and will require a quite different approach, using different signs as described in a section that follows.

Simple numeric fraction

A simple numeric fraction is one whose numerator (top of the fraction line) and denominator (bottom of the fraction line) contain only:

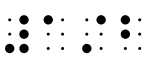
- digits,
- decimal points,
- commas or
- separator spaces,
- **and**, if the fraction line in print (often referred to as the vinculum) is drawn between the two vertically (or near vertically) arranged numbers as shown in the print.

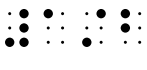
If the fraction complies fully with the definition above for a simple fraction, then a numeric fraction line symbol () should be used between the numerator and the denominator and continues the numeric mode.

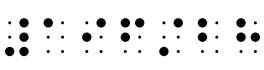
Simple numeric fraction line symbol 

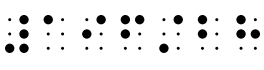
The numeric indicator for a simple fraction, will **not** need to be repeated after the fraction line as numeric mode has been continued.

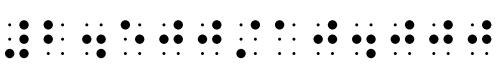
Examples:

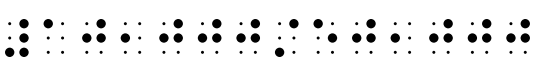
 $\frac{1}{2}$ (vertically) or

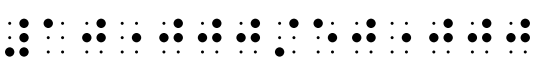
 $^{1}/_{2}$ (near vertically)

 $\frac{196}{28}$ (vertically) or

 $^{196}/_{28}$ (near vertically)

 $\frac{2.500}{10.000}$ (decimals)

 $\frac{10{,}000}{50{,}000}$ (commas)

 $\frac{10\,000}{50\,000}$ (separator spaces)

General Fractions

The braille symbols shown below for the opening and closing general fraction indicators have a contracted (Grade 2) meaning.

A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of the symbol could be misread as a contraction meaning. The extent of Grade 1 mode is determined by the Grade 1 indicator in use.

General fraction indicators

⠰⠰ Opening general fraction indicator

⠸⠸ Closing general fraction indicator

⠤⠤ General fraction line

- If the numerator or denominator of a fraction is not entirely numeric, as defined above for a simple fraction, then the general fraction indicators should be used. Write the opening general fraction indicator (⠰⠰), then the numerator (top) expression, then the general fraction line symbol (⠤⠤), then the denominator (bottom) expression and finally the closing general fraction indicator (⠸⠸). See examples below.
- The numerator and denominator may be any kind of expression, including fractions of either simple numeric or general type.
- Remember the numeric indicator (⠼) also sets Grade 1 mode for the remainder of the symbols-sequence.

Some of the following examples of General Fractions show the choice that can be made when using the Grade 1 indicator. There is usually a preferred option. Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration of such factors as simplicity or functionality.

Examples:

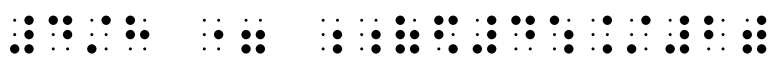
$$\frac{2/3}{6}$$

⠰⠰⠼⠃⠌⠉⠤⠤⠼⠋⠸⠸

$$\frac{20+30}{40-15}$$

⠰⠰⠼⠃⠚⠬⠼⠉⠚⠤⠤⠼⠙⠚⠤⠼⠁⠑⠸⠸

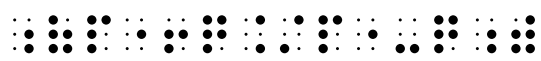
$\frac{4}{8} = \frac{\square}{2}$



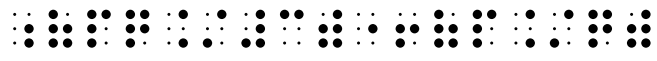
$\frac{p+q}{p-q}$

 preferred option

OR



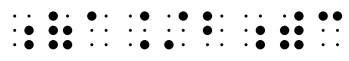
$\frac{pq}{3} + \frac{p}{q}$



$\frac{a}{b}c$

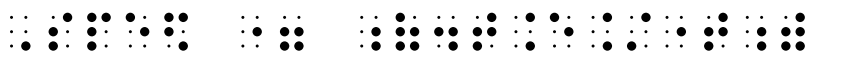
 preferred option

OR



$Speed = \frac{\boldsymbol{distance}}{\boldsymbol{time}}$

The first example shown in braille below is the preferred option so as to retain more contracted words in the literary equation.

 preferred option

OR

OR

$\frac{x \times y}{p \times q} = \frac{xy}{pq}$

The first option shown below is the preferred option, due to the comparison sign used in the sequences and Grade 1 indicators that are required on both sides of the equation.

OR

OR

Notes:

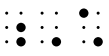
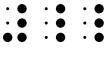
- It is important to remember that the opening and closing fraction indicators each have a Grade 2 (contracted) meaning. Therefore, if Grade 1 mode has not been established for some reason, then the opening and closing fraction indicators will require Grade 1 indicators.
- Remember the numeric indicator sets numeric mode as well as Grade 1 mode for the remainder of the symbols-sequence unless terminated for some reason.

Miscellaneous Symbols

Not all miscellaneous symbols are included in this training program. For a more extensive list of miscellaneous symbols and their equivalent braille representations, please refer to the *"Unified English Braille Guidelines for Technical Material"* (International Council on English Braille, 2014).

Specific use of many of the signs presented below will be outlined in further lessons.

	√	open radical (root) sign, with vinculum
		close radical (root) sign, (there is no print representation for this braille symbol)
	∫	integral sign
	°	degree sign
	′	foot or minute (shown as a prime sign)
	″	inch or second (shown as a double prime sign)

	|	vertical bar (absolute value)
	∞	infinity
	!	factorial sign
	∴	therefore sign
	∠	angle sign
	∥	parallel to
	⊥	perpendicular to
	≡	is congruent/equivalent to
	|||	is similar to
	[	open square bracket
	]	close square bracket
	{	open curly bracket
	}	close curly bracket
		dot 5 continuation indicator (positioned up close to the last character when the braille sequence is too long for the line and needs to be broken at a logical place)
		visible blank space (an omission, which often occurs in fractions)
	/	cancelling sign
	✓	tick sign (not to be confused with the root sign)

Notes:

- The spacing of symbols generally follows the print.

- Some of the signs listed above have a Grade 2 (contracted) meaning. A Grade 1 indicator will therefore be necessary if the sequence is not already in Grade 1 mode.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠨) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign or a fraction line symbol. For the purposes of this training program, take the remainder of the sequence to the margin. It is recognised there may be various local formatting guidelines.
- Remember the numeric indicator sets numeric mode as well as Grade 1 mode for the remainder of the symbols-sequence unless terminated for some reason.
- Further explanation of the use of some of these signs below will be explained in subsequent lessons.

  Examples:

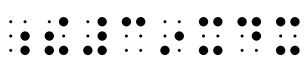
  $\sqrt{a^2}$

  OR

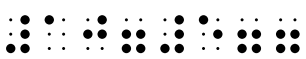
  $\int 3^x \, dx$

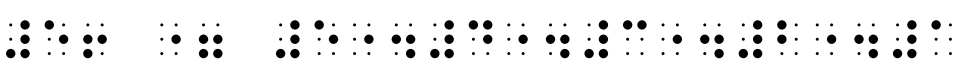
  $25^{\circ}\mathrm{C}$

  $10'5''$

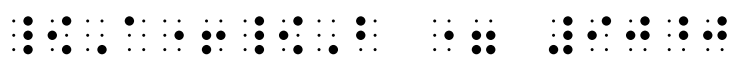
  $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ (the dot shown as a running product)

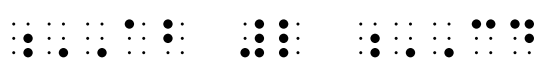
  $\therefore y = 7$

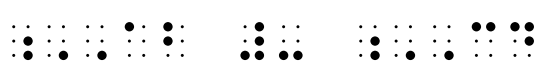
$\angle A + \angle B = 90°$



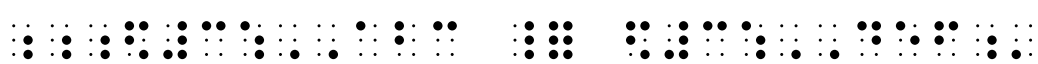
$AB \parallel CD$



$AB \perp CD$

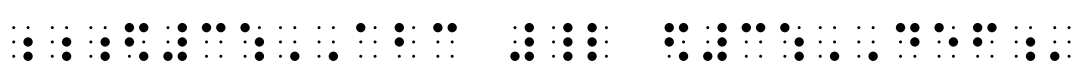


$\Delta ABC \equiv \Delta DEF$



OR

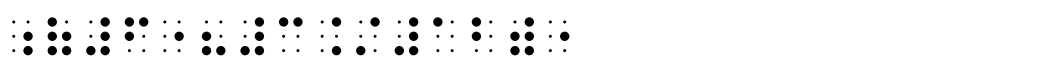
$\Delta ABC \mid\mid\mid \Delta DEF$



OR

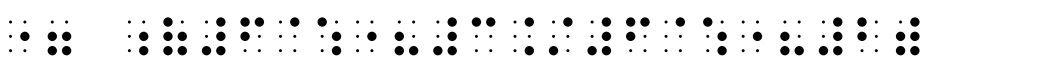
$$\frac{6\times3}{12} = \frac{\not{6}\times3}{\not{6}\times2}$$

Note the use of the dot 5 continuation indicator in the following example.



OR

Note the equals sign taken to the new line without the dot 5 continuation indicator.



Yes ✓

Superscripts and Subscripts

 Level change up

 Level change down

 Open braille grouping sign

 Closing braille grouping sign

Definition of an Item

An “item” is defined as any single symbol(s) that follows immediately after the level change indicator. It is therefore important to make clear to the reader exactly what symbol(s) must be included as a consequence of the level change indicator.

There is a defined list below of specific conditions that are considered to be the “next item”.

1. An entire number expressed in braille, i.e. the initiating numeric indicator and all succeeding symbols within the numeric mode (including any interior decimal points, commas, separator spaces, or simple numeric fraction lines).
2. An entire general fraction enclosed in general fraction indicators () and (). Note that Grade 1 indicators will be required if the sequence is not already in Grade 1 mode.
3. An entire radical expression enclosed in radical indicators (such as a square root).
4. An arrow.
5. An arbitrary shape.
6. Any expression enclosed in matching pairs of round parentheses, square brackets or curly braces.

Notes:

- If none of the above conditions apply then the “item” is only the next symbol and may require braille grouping indicators, opening () and closing (), to ensure the whole of the superscript or subscript has been captured. Note that Grade 1 indicators will be required for these signs if the sequence is not already in Grade 1 mode.

- Any expression when enclosed in the braille grouping indicators described above will subsequently make it clear to the reader exactly what symbols are included as part of the level change.
- A negative superscript or subscript must be enclosed in braille grouping signs because a minus sign can be an item in its own right.
- The superscript and subscript signs each have a Grade 2 (contracted) meaning so will always require Grade 1 indicators if the sequence is not already shown in Grade 1 mode.
- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

  Examples:

  2^4

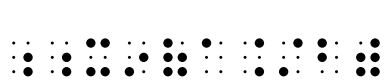
  $x^{4.4}$

  $x^{\frac{1}{4}}$

  $x^{\frac{a}{b}}$

   preferred

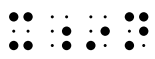
  OR

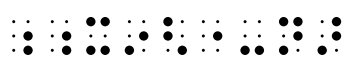
  $x^{(-5)}$

  x^n

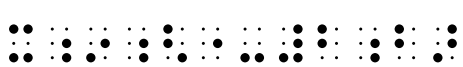
x^{-n}



x^{-2b}

 preferred

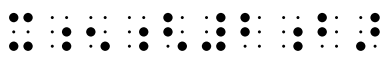
OR



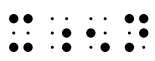
x_{2b}

 preferred

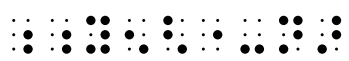
OR



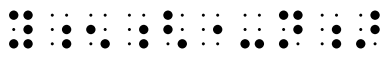
x_n



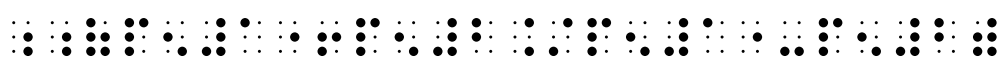
y_{-n}

 preferred

OR



$\frac{p_1 + p_2}{p_1 - p_2}$

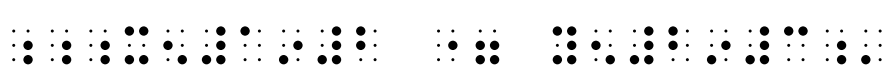


Simultaneous Superscripts or Subscripts

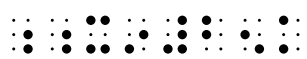
- If more than one superscript or subscript applies to the term, work from bottom to top, or left to right.

  Examples:

  $x_1^2 = y_2^3$

$x^{2}{}_{k}$



Left-displaced superscripts or subscripts

- When both a subscript and a superscript are shown together in print to the left of the base symbol, they are brailled by using the corresponding index expression prior to the base symbol, working from bottom to top.
- Note the Grade 1 indicator that is required in front of the subscript at the beginning of the example below but not required for the superscript sign, due to the numeric indicator setting Grade 1 mode for the remainder of that sequence.

  Example:

  ${}^{238}_{92}U$

General Note: There is often a choice of Grade 1 indicators in mathematical contexts, with use of any of the options for Grade 1 mode (symbol, word or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in this training program please use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember, however, the impact of Grade 1 mode on any literary elements.

Exercise 1

1. $\frac{r-3}{2} \leq -6$

2. Simplify $\frac{2}{3} - \frac{x-1}{4}$

3. $\therefore x = \frac{5+\sqrt{45}}{2}$

4. $q = \sqrt[3]{x^3 + y^3 + z^3}$

5. $\frac{m^2-8}{m+1} \geq 4$

6. $\left(\frac{9}{25}\right)^{k+3} = \sqrt{\frac{3}{5}}$

7. $\frac{y}{w} > \frac{1+\sqrt{5}}{2}$

8. $\frac{42g^3h^4}{7h^2} =$

9. $S = \sqrt{\frac{3V}{h}}$

10. $A = \frac{h}{3}(d_f + 4d_m + d_1)$

Extra Exercise 1

1. $(\frac{1}{27})^{3n+1} = \frac{\sqrt{3}}{81}$

2. $A = \frac{2 - sin\theta}{2\cos\theta}$

3. $T_n = 5^n - 1$

4. $x = t\sqrt{49 - t^2}$

5. $\log\left(\frac{x^2}{y}\right) = \log 2$

6. $(x - h)^2 = \pm 4a(y - k)$

7. Let $r = \sqrt{\frac{4p}{3}}$

8. $\frac{2b}{5} - \frac{1}{2} \geq 6$

9. $\frac{q-2}{3} < 2 + \frac{3q}{4}$

10. $(x + \frac{1}{x})^2$

Lesson 2 - Functions

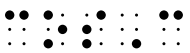
A technical definition of a function is: a **relation** from a set of inputs to a set of possible outputs where each input is related to exactly one output.

Follow the print for the spelling and capitalisation of function names.

Where letters before or after the function name are written in italics to indicate they are variables, the italics should be omitted in braille.

Example:

cos*C*



Trigonometric functions

Common trigonometric functions are Sine, Cosine, Tangent, Secant, Cosecant and Cotangent. These functions are usually abbreviated in print as sin, cos, tan, sec, cosec and cot.

- Sine (Sin), Cosine, Tangent and Cotangent may be contracted unless already in Grade 1 mode.

  Example:

  Cosine C

- Where a function name is **preceded** or **followed** by a number, then the number should be written unspaced from the function name.
- Remember the numeric indicator also sets Grade 1 mode for the next symbols-sequence.

  Examples:

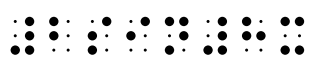
  5tan30°

  Cos15

  2sin8*x*



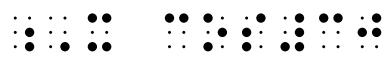
- Insert a space if a function name is **preceded** directly by a lower or upper-case Latin letter, with no intervening braille indicators or brackets.

  Examples:

  *x*cos 30

  Xcos 30

- Insert a space if a function name is **followed** directly by a lower-case letter with no intervening braille indicators or brackets.

  Example:

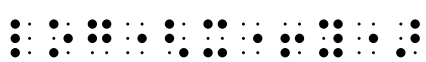
  log *x*

- The space is not needed if the function name is already separated by a bracket or by a braille indicator, such as a capitalisation indicator, a Greek letter indicator or an opening general fraction indicator.
- Remember the opening and closing general fraction indicators have a Grade 2 (contracted) meaning so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the opening and closing fraction indicator.
- We will investigate Greek letters more thoroughly in Lesson 5 of this Training Manual.

  Examples:

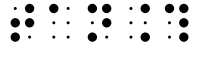
  log (*x+y)*

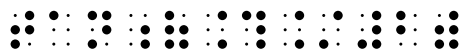
  $\cos B$

  $\mathrm{Tan}\theta$

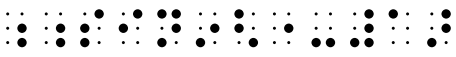
  $\tan \frac{\theta}{2}$



Inverse trigonometric functions

The examples of the inverse functions below have been shown in Grade 1 word mode, as a Grade 1 indicator is required in front of the superscript sign and the opening braille grouping indicator showing the whole of the “next item”.

The inverse functions may be written as:

 $\sin^{-1}$

 $\cos^{-1}$

 $\tan^{-1}$

 $\sec^{-1}$

 cosec^{-1}

 $\cot^{-1}$

Logarithmic functions

- The logarithmic function is usually written as log or Log and may be followed by a subscript indicating the base.

  Example:

  $log_a x = \frac{log_b x}{log_b a}$

- A logarithm to base “e” is called a natural log and is often abbreviated to ln.

  Example:

  $\ln e = 15$

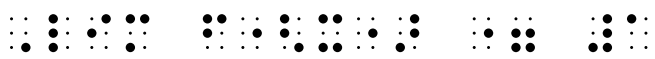
The Limit functions

Limit, lim, lm, lt are all used to indicate limit, sometimes with capitals, sometimes without.

Modifiers when shown directly above and directly below the character will be addressed in Lesson 3 and Lesson 4 of this Training Manual.

Example:

Lim f(x) = 1



Remember: As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicators in mathematical contexts, with any of the options of Grade 1 mode (symbol, word or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in this training program, please use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hint:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

Exercise 2

1. $\text{Log}\frac{y}{4}$
2. $-\frac{1}{4}\sec^4 x$
3. $\text{Log}_2\ \frac{\sqrt{2}}{4}$
4. $\theta = \cos^{-1}(\frac{5.8}{7.3})$
5. $\ln 9.8 + \log_{10} 17$
6. $\log_a(\frac{x}{y})$
7. $\theta = n\pi + \tan^{-1} a$
8. $\log_a x = \frac{\log_b x}{\log_b a}$
9. $\cot A = \frac{\cos A}{\sin A}$
10. $\sec A = \frac{1}{\cos A}$

Review Exercise 2

1. $a^{3y-5} = \frac{1}{a^2}$
2. $\ln(4x + 5)$
3. $y - y_1 = m(x - x_1)$
4. $\sin^{-1}\frac{x}{a} + C$
5. $\log_a x^n = n\log_a x$
6. $K = n\sqrt{\frac{a}{p}}$

7. $\log_a \sqrt{xy}$

8. $f(x) = 3x^4 + 4x^3 - 12x^2$

9. $\log \sqrt{x^2 - 4x + 4} - \log (x - 2)$

10. $\frac{-5 \pm \sqrt{21}}{2}$

Lesson 3 - Modifiers, Bars and Dots

Modifiers

Modifiers directly above or below

If an item is written in print directly above or directly below a term, rather than to the right or left, use the directly above indicator or directly below indicator instead of the superscript or subscript. Further explanation of a Modifier directly above or below is explained in Lesson 4.

As stated below in this lesson, special signs are provided for bars and a dot directly above and directly below.

 Expression directly above

 Expression directly below

Definition of an Item

An “item” is defined as any single symbol(s) that follows immediately after the level change indicator. It is therefore important to make clear to the braille reader exactly what symbol(s) must be included as a consequence of the level change indicator.

Below is a defined list of specific conditions that are considered to be the “next item”:

- An entire number expressed in braille, i.e., the initiating numeric indicator and all succeeding symbols within the numeric mode (including any interior decimal points, commas, separator spaces, or simple numeric fraction lines).
- An entire general fraction enclosed in general fraction indicators () and ().
  Note that Grade 1 indicators will be required if the sequence is not already in Grade 1 mode.
- An entire radical expression enclosed in radical indicators (such as a square root).
- An arrow.
- An arbitrary shape.
- Any expression enclosed in matching pairs of round parentheses, square brackets, or curly braces.

Notes:

- If none of the above conditions apply, then the “item” is the only the next symbol and may require braille grouping indicators, opening () and closing (), to

ensure the whole of the superscript or subscript has been captured. Note that Grade 1 indicators will be required for these signs if the sequence is not already in Grade 1 mode.

- Any expression when enclosed in the braille grouping indicators (described below) will subsequently make it clear to the reader exactly what symbols are included as part of the level change.
- The superscript (⠔⠢) and subscript (⠢⠔) signs each have a Grade 2 (contracted) meaning so will always require Grade 1 indicators if the sequence is not already shown in Grade 1 mode.
- A negative superscript or subscript must be enclosed in braille grouping signs because a minus sign can be an item in its own right.
- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

Braille Grouping Symbols

Refer to the definition of an item above to determine if the braille grouping symbols will be required.

⠢⠔ Opening braille grouping sign

⠔⠢ Closing braille grouping sign

Bars

Common modifiers such as bars are treated separately from something that is written directly above or directly below a term.

⠒⠤ bar over previous item

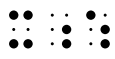
⠔⠒ ⠒⠤ bar under previous item

Note:

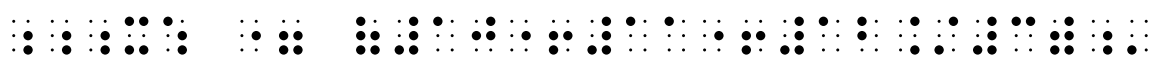
- The bar directly over a previous item (⠒⠤) and the bar directly under a previous item (⠔⠒ ⠒⠤) have a Grade 2 (contracted) meaning, so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the bar over and bar under indicators.

Examples:

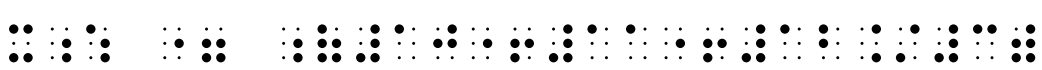
$\overline{x}$ bar over



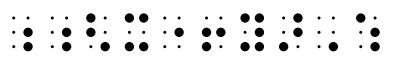
$\bar{x} = \frac{10+11+12}{3}$ bar over



OR

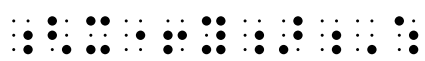


$\underline{x+y}$ bar under



preferred option as there is more than one Grade 1 indicator required in the sequence

OR

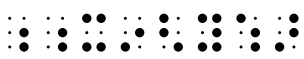


Two indicator bars applied to the same item

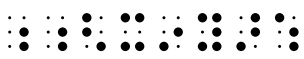
If two indicators apply to the same item, then braille grouping symbols must be used to show which applies first.

Examples:

$x^{\bar{y}}$



$\overline{x^{y}}$



Dots

Common modifiers such as a single dot are treated separately from something that is written directly above or directly below a term.

⠸⠄⠸⠄ dot over a previous item: the recurring decimal and single derivative

Recurring decimal:

⠸⠄⠸⠄ dot over a previous item: the recurring decimal

The recurring decimal (shown below with the dot over the digit/s recurring) will require braille grouping signs opening (⠸⠣) and closing (⠸⠜) otherwise the dot would refer to the entire number.

Examples:

$5.\dot{5}$

⠼⠑⠲⠸⠄⠑

$0.57\dot{1}2\dot{3}$

⠼⠚⠲⠑⠛⠸⠄⠁⠃⠸⠄⠉

OR

because the digits 123 in the above example are being implicitly grouped in print as recurring, this can also be brailled as:

$0.57\dot{1}2\dot{3}$

⠼⠚⠲⠑⠛⠸⠄⠁⠃⠉

Derivatives of $\dot{x}$

The derivative of $\dot{x}$ has its own sign as indicated below.

⠸⠄⠸⠄ dot over a previous item: the single derivative

Example:

$\dot{x}$

⠈⠄⠭

Derivatives of $\ddot{x}$

The x double dot $(\ddot{x})$ sign is treated differently to the single derivative of $\dot{x}$ which has its own specific braille sign as explained above.

⠨⠨⠭ dots over a previous item: the double derivative

The x double dot $(\ddot{x})$ will require the signs for directly above ⠨⠭⠨ or directly below ⠨⠨⠭ a term as stated at the beginning of this lesson.

Derivatives of x double dot $(\ddot{x})$, will also require braille grouping signs, opening (⠸⠣) and closing (⠸⠜), to ensure the effect of the whole of the double dot is captured.

Example:

$\ddot{x}$

⠸⠣⠨⠨⠭⠸⠜

Notes:

- The braille grouping signs have a Grade 2 (contracted) meaning. Therefore, unless you are already in Grade 1 mode, Grade 1 symbol indicators will be required before the opening and closing braille grouping signs.
- The numeric indicator also sets Grade 1 mode for the remainder of the next symbols-sequence.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠈) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign, or a fraction line symbol. For the purposes of this training program, take the remainder of the sequence to the margin as it is recognised there may be various local formatting guidelines.

Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicators in mathematical contexts, with any of the options of Grade 1 mode (symbol, word, or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in this training program, please use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.

- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hint:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

Exercise 3

1. $z = \frac{x - \bar{x}}{s}$

2. $\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$

3. $y^2 = -i\bar{y}$

4. $\bar{x} = \frac{25 + 35 + 45}{5}$

5. $\underline{PQ}$

6. $\underline{m + n}$

7. $4.1\dot{5}2\dot{5}$

8. $\dot{x} = \frac{t}{t^2 + 3} ms^{-1}$

9. $\ddot{x} = g - kv$

10. $\ddot{x} = 25e^{5t}\, ms^{-2}$

Review Exercise 3

1. $\log_x(\frac{1}{3}) = -1$

2. $\therefore \theta = 37^{\circ}23'$

3. $\frac{n^2 - n - 6}{n^2 + 5n + 6}$

4. $\sqrt{\frac{-3c}{b - a}}$

5. $\log_a x = \frac{\log_b x}{\log_b a}$

6. $\log_7 \sqrt[4]{7}$

7. $\therefore\ \Delta RST \equiv \Delta XYZ$

8. $y = 2\sqrt{x} = 2x^{\frac{1}{2}}$

9. $d = \sqrt{(x_2 - x_1)^2}$

10. $\dfrac{\log_e x}{e^{2x} + x}$

Lesson 4 - Modifiers, Arrows and Limits

Modifiers

Modifiers directly above or below

If something is written directly above or directly below a term rather than to the right or left, use the directly above indicator or directly below indicator instead of the superscript or subscript.

 Expression directly above

 Expression directly below

Comparison signs are usually unspaced when they appear in an expression placed directly above or directly below the character.

However, a common modifier such as an arrow is treated similar to bars and a dot (described in Lesson 3) and has its **own** sign instead of using the indicators for “directly above” or “directly below”. This is introduced below in this lesson.

Definition of an Item

An “item” is defined as any single symbol(s) that follows immediately after the level change indicator. It is therefore important to make clear to the braille reader exactly what symbol(s) must be included as a consequence of the level change indicator.

Below is a defined list of specific conditions that are considered to be the “next item”:

- An entire number expressed in braille, i.e. the initiating numeric indicator and all succeeding symbols within the numeric mode (including any interior decimal points, commas, separator spaces, or simple numeric fraction lines).
- An entire general fraction enclosed in general fraction indicators () and (). Note that Grade 1 indicators will be required if the sequence is not already in Grade 1 mode.
- An entire radical expression enclosed in radical indicators (such as a square root).
- An arrow.
- An arbitrary shape.
- Any expression enclosed in matching pairs of round parentheses, square brackets, or curly braces.

Notes:

- If none of the above conditions apply then the “item” is the only the next symbol and may require braille grouping indicators opening (⠸⠣) and closing (⠸⠜) to ensure the whole of the superscript or subscript has been captured. Note that Grade 1 indicators will be required for these signs if the sequence is not already in Grade 1 mode.
- Any expression when enclosed in the braille grouping indicators described below will subsequently make it clear to the reader exactly what symbols are included as part of the level change.

Braille Grouping Symbols

Refer to the Definition of an Item above to determine if the braille grouping symbols shown below will be required.

⠸⠣ Opening braille grouping sign

⠸⠜ Closing braille grouping sign

Example:

The example below using capital Greek sigma (Σ) shows an expression directly above and directly below a term with the use of a comparison sign, which is unspaced when placed directly above or below a term.

$$\sum_{n=2}^{5} n^2$$

⠸⠣⠠⠎⠨⠅⠒⠼⠃⠸⠜⠘⠣⠼⠑⠸⠜⠠⠝⠘⠣⠼⠃⠸⠜

OR

⠸⠣⠠⠎⠨⠅⠒⠼⠃⠸⠜⠘⠣⠼⠑⠸⠜⠠⠝⠘⠣⠼⠃⠸⠜

Arrows

For a complete list of Arrows, refer to the *Unified English Braille Guidelines for Technical Material* (International Council on English Braille, 2014).

The arrow indicator has a Grade 2 (contracted) meaning so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the arrow indicator.

 arrow indicator

 → simple right pointing arrow (east)

 ← simple left pointing arrow (west)

Simple Arrows

- A simple arrow has a standard barbed tip at one end (like a print letter “v” on its side) pointing away from the shaft.
- The shaft is straight, and its length and thickness are not significant in braille.
- Simple arrows are represented by an arrow indicator as shown in the list above.
- Arrows are considered signs of comparison and should usually be spaced, with the exception of when they are written immediately below an item, as in Limit Functions described later in this lesson.

  Examples:

  $n \to 0$ (n tends to zero shown using Grade 1 symbol mode)

  OR

  $n \to 0$ (n tends to zero shown using Grade 1 passage mode)

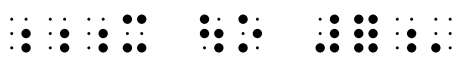
  

  $x \to \infty$ (*x* tends to infinity shown using Grade 1 symbol mode)

  OR

$x \to \infty$ (*x* tends to infinity shown using Grade 1 passage mode)



Arrow symbol directly above and below

Common modifiers such as arrows are treated similar to bars and a single dot as described in Lesson 3 and have their **own** sign instead of using the indicators for “directly above” or “directly below” as shown above in this lesson.

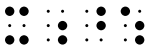
Only use these arrow indicators when a simple right pointing arrow is the only modifier directly above or directly below an item.

 Arrow directly above

 Arrow directly below

The symbol shown above for a simple right-pointing arrow directly above or directly below an item has a Grade 2 (contracted) meaning, so unless you are already in Grade 1 mode, a Grade 1 symbol indicator will be required before the arrow indicator.

Examples:

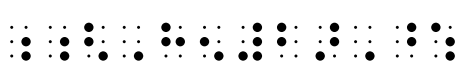
 $\vec{x}$ Simple right-pointing arrow over previous item (x)

 $\underset{\rightarrow}{x}$ Simple right-pointing arrow under previous item (x)

Notes:

- The use of braille grouping indicators in the example below make it clear to the reader exactly what symbols are affected by the arrow under the previous item (an arrow under the H_2).
- The example below has been placed in Grade 1 word mode due to more than one Grade 1 symbol indicator being required in the sequence.

  Example:

   $\underset{\rightarrow}{H_2}$

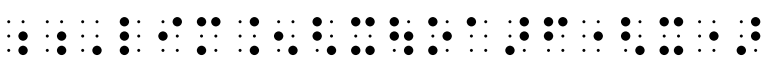
The Limit Function

Limit, lim, lm, lt are all used to indicate limit, sometimes with capitals, sometimes without.

- Signs of operation and comparison are unspaced when they appear in an expression which is not on the base line, such as directly above or directly below the character.

  Example:

  $$\lim_{x \to a} f(x)$$

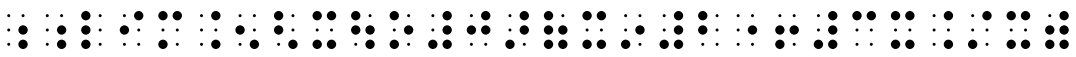
  

Notes:

- The example below contains a General Fraction.
- The example below has been placed in Grade 1 word mode due to more than one Grade 1 symbol indicator being required in the sequence.

  Example:

  $$\lim_{x \to 0} \frac{x^2 + 3x}{x}$$

Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicator, with any of the options of Grade 1 mode (symbol, word, or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in UEB Mathematics Online, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required

on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- Insert a space if a function name is **preceded** directly by a lower or upper-case Latin letter, with no intervening braille indicators or brackets.

Exercise 4

1. Input → process → output
2. $x \to y$
3. $\overrightarrow{AB}$
4. $\overrightarrow{AB} + \overrightarrow{CD}$
5. $\lim_{x \to 4} x^2 + 5$
6. $\lim_{x \to \infty} \frac{x}{x^2}$
7. $\lim_{g \to 1} \frac{g^2 - 1}{g - 1}$
8. $\lim_{t \to 4} \frac{t^2 - 16}{t - 4}$
9. $\lim_{x \to \infty} \frac{3x^2}{x^2 - 2x}$
10. $\lim_{h \to 0} \frac{2h^2x + hx^2}{h}$

Review Exercise 4

1. $a \rightarrow y = \ln(x+1)$

2. $\frac{\sqrt{2x+1}}{(x-3)^3}$

3. $\frac{2x}{(x+5)^{\frac{1}{2}}}$

4. $OC = \sqrt{r^2 - \frac{x}{2}}$

5. $c^2 = a^2 + b^2 - 2ab\cos C$

6. $\frac{48p^2q^4}{6pq^3}$

7. $A = \pi(R^2 - r^2)$

8. $n = \frac{42\sin 117^\circ}{\sin 35^\circ}$

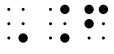
9. $\cos C = \frac{a^2+b^2-c^2}{2ab}$

10. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

Lesson 5 - Greek Letters (continued) and Probability

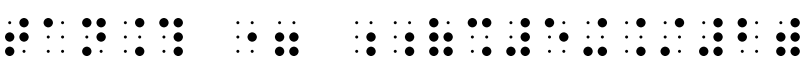
Greek Letters

Greek letters are used extensively in Mathematics. While only a small number have been used in this Training Manual, the principles for use remain the same for all. Refer to the *Unified English Braille Guidelines for Technical Material* (International Council on English Braille, 2014) for a complete list.

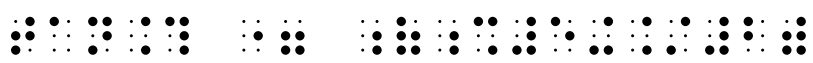
	α	alpha (lower case)		A	Capital
	β	beta (lower case)		B	Capital
	δ	delta (lower case)		Δ	Capital
	ε	epsilon (lower case)		E	Capital
	γ	gamma (lower case)		Γ	Capital
	θ	theta (lower case)		Θ	Capital
	λ	lambda (lower case)		Λ	Capital
	μ	mu (lower case)		M	Capital
	π	pi (lower case)		Π	Capital
	σ	sigma (lower case)		Σ	Capital
	ϕ	phi (lower case)		Φ	Capital

Examples:

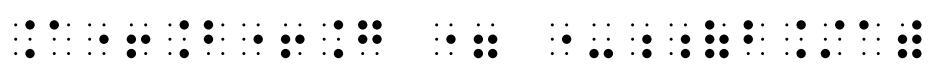
$$\tan\theta = \frac{\sqrt{5}}{2}$$

 preferred

OR

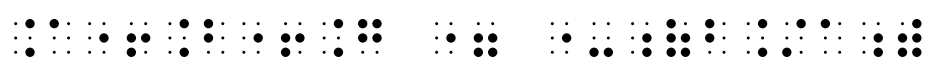


$$\alpha + \beta + \gamma = -\frac{b}{a}$$



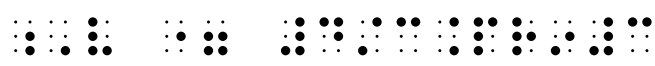
preferred

OR



$$\mu(n) = 1$$

$$V = \frac{4}{3}\pi r^3$$



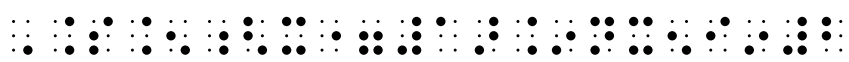
$$\Delta = b^2 - 4ac$$

Notes:

- Remember if something is written directly above or directly below a term rather than to the right or left, use the directly above indicator () or directly below indicator () instead of the superscript or subscript indicator.
- Care must be taken to ensure that the sign for capital delta (Δ) is not confused with a triangle sign.
- Any expression when enclosed in the braille grouping indicators, opening () and closing (), will subsequently make it clear to the reader exactly what symbols are included as part of the level change.
- Remember that Grade 1 indicators will be required for the braille grouping symbols if the sequence is not already in Grade 1 mode.
- It is also preferable to unspace a comparison sign when the comparison sign is not on the base line such as in sigma notation.

Example:

$$\sum_{x=1}^{n} x_{i^2}$$



Probability

Refer to the "*Unified English Braille Guidelines for Technical Material"* (International Council on English Braille, 2014) for further representations of Set Theory, Group Theory and Logic.

The following signs may be used in the topic of Probability at the level of Mathematics being presented in this training program.

 ∪ union (upright ∪ shape): unspaced

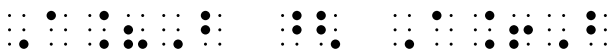
 ∩ intersection (inverted ∩ shape): unspaced

 ∈ is an element of: spaced

 ⊂ is a subset of: spaced

Examples:

A ∩ B ⊂ A ∪ B (A intersection B is a subset of A union B)



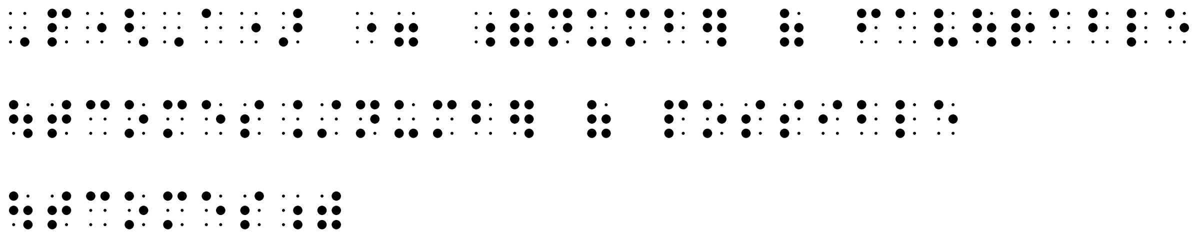
3 ∈ A ∩ B (3 is an element of A intersection B)

Probability is the study of how likely it is that something will happen.

Probability is when there is some doubt about the outcome, but the degree of doubt is different. Probability is about the doubt: it is the study of events that may or may not happen, rather than of events that will happen or that have already happened.

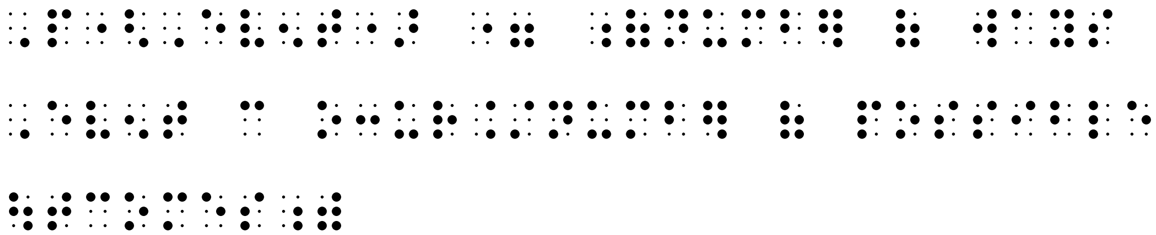
The probability of an outcome is defined as the ratio of the number of favourable outcomes to the number of possible outcomes, assuming that the outcomes are equally likely. Thus, the probability of P(A) of a particular result A is:

$$P(A) = \frac{number\ of\ favourable\ outcomes}{number\ of\ possible\ outcomes}$$



OR

$$P(\text{Event}) = \frac{\textit{number of ways Event can occur}}{\textit{number of possible outcomes}}$$



Note:

The example shown above has been brailled using Grade 1 symbol indicators where required rather than using Grade 1 word or Grade 1 passage mode due the number of literary elements in the sequence.

Venn Diagrams

Venn diagrams are a visual (or graphical) way to represent sets of data. You can name the sets with letters and talk about various properties of the sets.

Venn diagrams are an efficient tool to be used when analysing data to find the probability of events.

While it is not critical to understand the mathematics of Probability and Venn diagrams when transcribing braille, it may be helpful for the brief explanation below that was extracted from a student's textbook:

Example:

A description of a Venn diagram is as follows:

There is an outer group in a rectangle containing all the members of a set S, an inner group positioned in a circle on the left within the rectangle containing all the members of set D, an inner group positioned in a circle on the right within the rectangle containing all the members of set F.

The area where the inner groups (circles) overlap contains the members that are in both sets D and F. The region outside the groups of D and F contain all the members of set S that are in neither set D nor set F.

The set made by combining sets D and F is called **union** of the two sets, written as D ∪ F or ⠄⠙⠄⠬⠄⠋ in braille. It consists of all the elements that are in D or in F or in both D and F.

The set made up of the elements common to sets D and F is called the **intersection** of the two sets, written as D ∩ F or ⠄⠙⠄⠩⠄⠋ in braille. The intersection contains numbers that belong to both sets.

Complementary Events

The **complement of event** A is the event "A **does not occur"** and can be denoted by $\bar{A}$.

If an experiment has n possible outcomes, ***m*** of which are associated with event ***A***, then (***n*** – ***m)*** outcomes are associated with event $\bar{\mathbf{A}}$.

Examples:

$P(A) = \frac{m}{n}$

⠄⠏⠄⠣⠄⠁⠄⠜⠄⠶ ⠄⠍ ⠄⠝

OR

⠄⠏⠄⠣⠄⠁⠄⠜⠄⠶ ⠄⠍ ⠄⠝

AND

$P(\bar{A}) = \frac{n-m}{n}$

⠄⠏⠄⠣⠄⠁⠄⠻⠄⠶ ⠄⠝ ⠄⠤ ⠄⠍

⠄⠏⠄⠣⠄⠁⠄⠜⠄⠶ ⠄⠝ ⠄⠤ ⠄⠍

⠄⠏⠄⠣⠄⠁⠄⠜⠄⠶ ⠄⠝ ⠄⠤ ⠄⠍

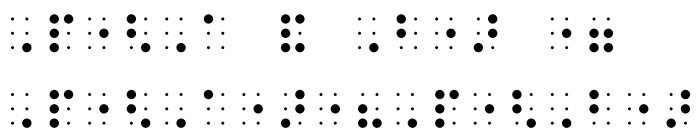
Independent Events

Two events are **independent** if the outcome of each event is not affected by the outcome of the other event. This means that the probability of both events together is the same as the probability of both events separately, and so their probabilities can be multiplied together as if they are separate successful outcomes. In other words:

Example:

If events A and B are independent, then:

$$\mathrm{P(A\ and\ B)} = \mathrm{P(A)} \times \mathrm{P(B)}$$

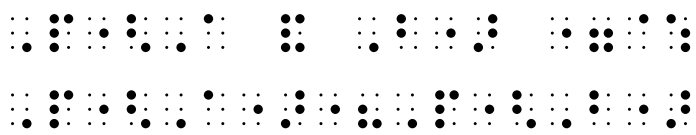


Dependent Events

Two events are **dependent** if the outcome of one event is affected by the outcome of the other event. This means that the probability of both events together is ***not*** the same as the probability of both events separately. Their probabilities cannot be simply multiplied together as if they are separate successive outcomes (as for independent events). In other words:

Example:

If events A and B are dependent, then:
$$\mathrm{P(A\ and\ B)} \neq \mathrm{P(A)} \times \mathrm{P(B)}$$



Remember:

As referred to in the General Note in Lesson 1, there is often a choice of Grade 1 indicator, with any of the options of Grade 1 mode (symbol, word, or passage) being equally correct.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the exercises in UEB Mathematics Online, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠸) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign, or a fraction line indicator. For the purposes of this training program take the remainder of the sequence to the margin as it is recognised there may be various local formatting guidelines.
- Signs of operation and comparison are unspaced when they appear in an expression which is not on the base line such as directly above or directly below the character.

Exercise 5

1. $A = \frac{\theta}{360} \times \pi r^2$
2. $z = \frac{x-\mu}{\sigma}$
3. $(\cos\frac{\pi}{4} - \tan\frac{\pi}{4})^2$
4. $\theta = 2n\pi \pm cos^{-1}a$
5. $\tan(\theta + \phi) = \frac{\tan\theta+\phi}{1-\tan\theta\tan\phi}$
6. $n(V \cup C) = n(V) + n(C)$
7. $X \cap Y = \{1, 11, 12\}$
8. $n(D \cup F) = n(C) + n(F) - n(D \cap F)$
9. $P(\bar{A} \text{ or } B) = 1$
10. $n(E \cap F) = 0$

Review Exercise 5

1. $\frac{pq}{5} \times \frac{p}{q}$
2. Roots of $P(z)$ are $\alpha, \overline{\alpha}, \beta, \bar{\beta}$
3. $\sin\theta = \frac{o}{h}$
4. $\overline{x} = \frac{\Sigma fx}{\Sigma f}$
5. $\lim_{x\to\infty} \frac{1}{x} = 0$
6. $\lim_{\delta\to 0} \frac{\delta y}{\delta x}$
7. $\frac{y+1}{y-1} > 2$

8. $\sum_{x=1}^{n}(2^r x)$

9. $\sin(\pi - \frac{\pi}{6})$

10. $\frac{1}{2}[cos\ (A) + \cos\ (B)]$

Lesson 6 - Calculus: Differentiation

As Calculus is a defined topic in senior secondary level mathematics it has been introduced specifically into the lessons for this training program. The same rules and principles of braille transcription should be observed that have been introduced previously.

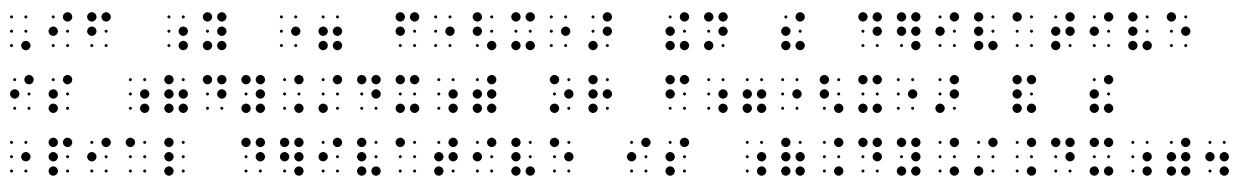
There are two main branches of Calculus: Differentiation and Integration. The topic of Integration will be introduced in Lesson 7.

Differentiation is used to calculate the rate at which two variables change in relation to one another.

The process of finding the gradient of a tangent is called **differentiation.** The resulting function is called the **derivative.**

Examples:

If *y = f(x)* then the derivative is $\frac{dy}{dx}$ or $f'(x)$ and the partial derivative is $\frac{\delta y}{\delta x}$.



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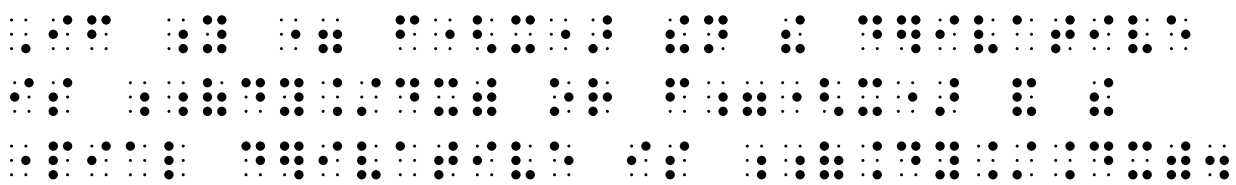
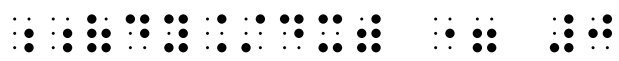


AND

$\frac{dy}{dx} = 0$



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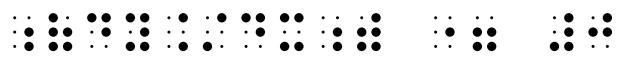
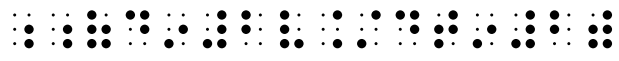


$\frac{d^2v}{dt^2}$



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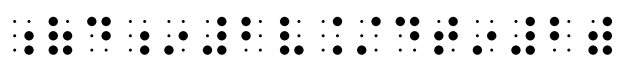
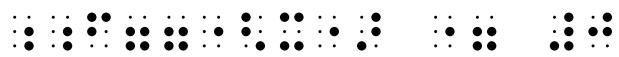


$f'(x) = 0$

$f''(x) = 0$



$\frac{d}{dx}(kx) = k$

OR

OR

Remember:

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- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- Insert a space if a function name is **preceded** directly by a lower or upper-case Latin letter, with no intervening braille indicators or brackets.
- Insert a space if a function name is **followed** directly by a lower-case Latin letter, with no intervening braille indicators or brackets.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠨) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign, or a fraction line indicator. For the purposes of this training program take the remainder of the sequence to the margin as it is recognised there may be various local formatting guidelines.

Exercise 6

1. $\frac{dy}{dx} > 0$
2. $\frac{dy}{dx} f^{'}(x)$
3. $\frac{dy}{dx} f^{'}(x) e^{f(x)}$
4. $\frac{d}{dx}(x^2 - 2x)$
5. $\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$
6. $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
7. $\frac{d}{dx}(kx^n) = knx^{n-1}$
8. $\frac{dy}{dx} = 4x^3 - 9x^2 + 7$
9. $\frac{dy}{dx} = f'(x)\cos f(x)$
10. $\frac{d}{dx}[f(x)]^2 =$

Review Exercise 6

1. $\log_b x = \frac{\log_a x}{\log_a b}$
2. $\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u} \times \frac{\delta u}{\delta x}$
3. $\frac{dy}{dx} = nx^{nx-1}$
4. $\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$
5. $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

6. $\frac{dy}{dx} = 5x^4 - 9$

7. $\left(\frac{ab^5c}{abc^4}\right)^0$

8. $\lim_{\theta \to 0} \frac{tan\frac{\theta}{3}}{\theta}$

9. $\lim_{x \to \infty} \frac{3\sqrt{x}}{\sqrt{x} - 1}$

10. $\lim_{x \to -1} \frac{x^2 + 4x + 1}{x^2 - 1}$

Lesson 7 - Calculus: Integration

As Calculus is a defined topic in senior secondary level mathematics it has been introduced specifically into the lessons for this training program. The same rules and principles of braille transcription should be observed that have been introduced previously.

There are two main branches of Calculus: Differentiation and Integration. The topic of Differentiation was introduced in Lesson 6.

Integration as presented in this Lesson is the inverse of differentiation and uses information about rates of change to go back and examine the original variables. Integration can also be used to find areas of curved objects.

This training program will only be introducing the standard integral sign ($\int$) at the level of Mathematics being presented in this training program and is shown in braille as (⠮).

Notes:

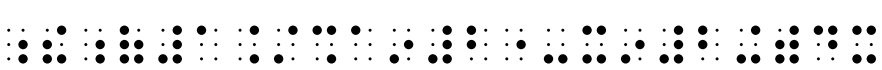
- The spacing of the integral sign in print ($\int$) may be unclear or inconsistent. In braille it is best to have the integral sign (⠮) unspaced from the function and treat its limits as subscripts and superscripts. The *dx* at the end means “integrate with respect to *x*” and can also be written unspaced.
- While both of the examples below are correct, the first example in each using Grade 1 word mode is the preferred option due to requiring more than one (1) Grade 1 symbol indicator in the sequence.

  Examples:

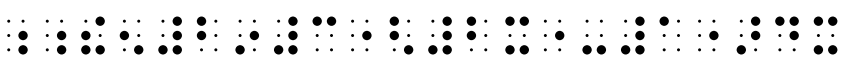
  $$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx$$

   preferred

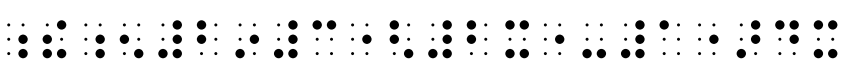
  OR

  $$\int_2^3 (2x - 1)dx$$

   preferred

  OR

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Exercise 7

1. $\int x^2\, dx$
2. $\int_0^2 \frac{3}{4+x^2}\, dx$
3. $\int 2x^4\, dx$
4. $\int_3^{\pi} \cos\left(\frac{x}{2} + \pi\right) dx$
5. $\int_0^5 \frac{dx}{x+3}$
6. $\int_a^b f(x) dx$
7. $\int \sin^2 x\, dx$
8. $\int e^{ax+b}\, dx$
9. $\int \tan x dx = \ln\cos x + c$
10. $\pi \int_0^{\frac{\pi}{4}} sec^2\, x\, dx$

Review Exercise 7

1. $f''(x) = 6x - 8$
2. $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$
3. $\frac{d^2x}{dt^2}$
4. $P(E') = 1 - p(E)$
5. $\frac{dy}{dx} = F'(u)\frac{du}{dx}$
6. $\int sec^2\,(ax+b) dx$

7. $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1$

8. $\int \cos(ax + b)\, dx$

9. $\int_1^2 \frac{x^3 - x^2 + 5x + 3}{x}\, dx$

10. $\int (ax + b)^n$

Lesson 8 - Vertical Bars

One of the underlying principles of UEB is that each print symbol should have one and only one braille equivalent. For example, the vertical bar is used in print to represent absolute value, conditional probability, and the words “such as”. The same braille symbol should be used in all these cases.

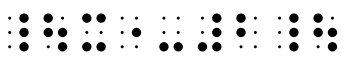
Bars are usually unspaced in print from the items they enclose and the same should be done in braille.

The absolute value of a number, for example, is symbolised in print by two vertical lines | | often referred to as bars and shown in braille as

opening ⠸⠳ and closing ⠸⠻.

Examples:

$|x-2|$

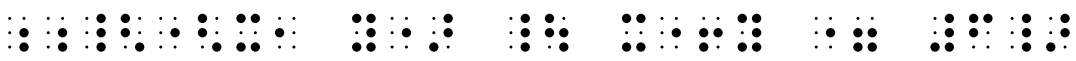


$|x|^3 = x^3$

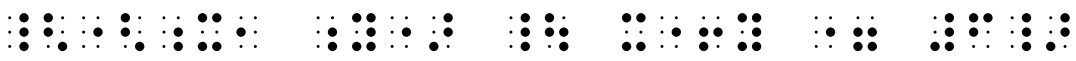
OR

$\{(x,\ y)\ |\ x + y\ =\ 6\}$

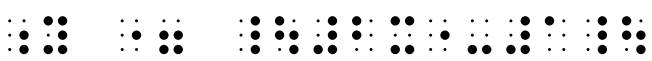
OR



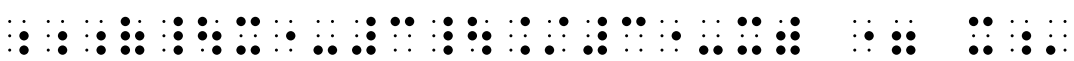
OR



$y = |2x - 1|$

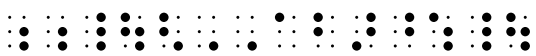


$$\frac{|x-3|}{3-x} = x$$



OR

$\left|\overrightarrow{AB}\right|$



Remember:

As referred to in the General Note in Lesson 1, the options of Grade 1 mode (symbol, word, or passage) are equally correct in mathematical contexts.

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Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.
- Signs of operation and comparison are unspaced when they appear in an expression which is not on the base line such as directly above or directly below the character.

Exercise 8

1. $|y-4|$
2. $\frac{|x|}{x^2}=1$
3. $|x|\geq 0$
4. $\sqrt{a^2}=|a|$
5. $\frac{|3x-2|}{3x-2}=1$
6. $|3y-1|+|2y+3|>5$
7. $y^2=2\sin|x|$
8. $|z-2|+|z+1|=7$
9. $|3x+2|=2x-3$
10. $\left|\overrightarrow{O_1P}\right|$

Review Exercise 8

1. $A=\frac{h}{2}(d_f+d_1)$
2. $(x-h)^2=\pm 4a(y-k)$
3. $y=e^{f(x)}$
4. $\frac{p^{\frac{1}{2}}\times p^{\frac{1}{2}}}{p^2}$
5. $S_n=\frac{a(r^n-1)}{r-1}$
6. $x^{\frac{1}{n}}=\sqrt[n]{x}$
7. $\frac{1}{\sqrt[4]{(7-x)^2}}$

8. $\left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

9. $f'(x)n[f(x)]^{n-1}$

10. $\lim_{h \to 0} \frac{2xh - h^2 - 5h}{h}$

Lesson 9 - Review Test

Congratulations on reaching this Review Test. The content of each lesson has built upon preceding lessons, with the overall structure of the training manual designed to reinforce several foundational principles, including the following:

1. The numeric indicator sets numeric mode **and** Grade 1 mode for the remainder of the symbols-sequence unless terminated for any reason.
2. A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.
3. When is a fraction considered a “simple fraction”? Note, if the sequence does not satisfy the definition of a simple fraction, then it must be treated as a general fraction.
4. The rules associated with “the next item” should be considered and are particularly relevant in the use of level change indicators.

Remember:

As referred to in the General Note in Lesson 1 in this UEB Training Manual: Extension Mathematics and throughout the lessons, the choice of options for Grade 1 mode (symbol, word, or passage) may be equally correct in mathematical contexts.

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Review Test

1. $8.754\dot{7}5\dot{9}$
2. $T_n = a\ + (n-1)$
3. $\bar{y} = -10$
4. $\overline{AB}$
5. $\ln|f(x)| + c$
6. $\bar{x} = \frac{\Sigma x}{n}$
7. $|5x - 3| \geq 2$
8. $\frac{(2x^7)^3 y^2}{x^{10} y}$
9. $[2(x+y)] \div 4 < 10$
10. $|-8| = |8| = 8$
11. $\cos C = \frac{a^2 + b^2 - c^2}{2a}$
12. $A = \int_{-1}^{2} (2 + x)\, dx$
13. $\int_b^c f(x) dx$
14. $\sin(\pi - \frac{\pi}{8})$
15. $\frac{n!}{r!(n-r)!}$
16. $\theta = n\pi\ + (-1)^n \sin^{-1} a$
17. $\lim_{x \to 0} f(x)$
18. $f_1(x) + f_2(x)$

19. $S_n = \frac{n}{2}[2a + (a - l)d]$

20. $f''(x) = k(b^2 - x^2)$

Lesson 10 - Extension Test

Congratulations on reaching this Extension Test. As noted in Lesson 9, the content of each lesson has built upon preceding lessons, with the overall structure of the training manual designed to reinforce several foundational principles, including the following:

1. The numeric indicator sets numeric mode **and** Grade 1 mode for the remainder of the symbols-sequence unless terminated for any reason.
2. A braille symbol may have both an uncontracted (Grade 1) meaning and a contracted (Grade 2) meaning. A Grade 1 indicator is used to set Grade 1 mode when the Grade 1 meaning of a symbol could be misread as a contraction meaning or a numeric meaning.
3. When is a fraction considered a “simple fraction”? Note: If the sequence does not satisfy the definition of a simple fraction, then it must be treated as a general fraction.
4. The rules associated with “the next item” should be considered and are particularly relevant in the use of level change indicators.

Remember:

As referred to in the General Note in Lesson 1 in this UEB Training Manual: Extension Mathematics and throughout the previous lessons, the choice of options for Grade 1 mode (symbol, word, or passage) may be equally correct in mathematical contexts.

Decisions about option selection are generally associated with user and transcriber preferences, or local production formatting guidelines, and include consideration for simplicity or functionality.

Therefore, for the purpose of the Extension Test in this UEB Training Manual, use the following criteria for implementing Grade 1 mode:

- Use the Grade 1 **symbol** indicator when there is only one symbol in the sequence requiring a Grade 1 indicator.
- Use the Grade 1 **word** indicator when there are two or more symbols in the sequence requiring Grade 1 mode except in a context whereby any literary elements will be affected and as a consequence will also be uncontracted.
- Use Grade 1 **passage** indicator (with Grade 1 terminator) when a comparison indicator or a space is used in the sequence and Grade 1 indicators are required on both sides of the equation. Remember however, the impact of Grade 1 mode on any literary elements.

Hints:

- The numeric indicator also sets Grade 1 mode for the remainder of the symbols-sequence.

- Insert a space if a function name is **preceded** directly by a lower or upper-case Latin letter, with no intervening braille indicators or brackets.
- Insert a space if a function name is **followed** directly by a lower-case Latin letter, with no intervening braille indicators or brackets.
- Signs of operation and comparison are unspaced when they appear in an expression which is not on the base line such as directly above or directly below the character.
- If the braille representation for a print sequence does not fit on the line, then a dot 5 (⠨) continuation indicator placed at a logical place immediately following the last character may be required to show the braille is continuing to the next line. Usually the preferred place to break is before a comparison sign, an operation sign, or a fraction line indicator. For the purposes of this training program take the remainder of the sequence to the margin as it is recognised there may be various local formatting guidelines.

Extension Test

1. $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$
2. $\int x^2 dx$
3. $\int \frac{\cos x}{\sin x}\, dx$
4. $\int_2^3 \frac{dx}{x}$
5. $d = \sqrt{(x_2 - x_1)}$
6. $\operatorname{cosec} A = \frac{1}{\sin A},\ sin\ A \neq 0$
7. $(Re(\alpha))^2 + (Re(\beta))^2 = 1$
8. $\int \frac{6x^2}{x^3}$
9. $d = \frac{|ax_1|}{\sqrt{a^2}}$
10. $\frac{d}{dx}\left(\frac{x^2}{x-1}\right)$
11. $\bar{x} = \frac{\sum x}{n}$
12. $\sum_2^4 |3p - 2|$
13. $\ddot{x} = -n^2(x - c)$
14. $y = \tan^{-1} f(x)$
15. $A_n = P\left(1 + \frac{r}{100}\right)^n$
16. $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$
17. $\max_{0 \leq x \leq 1} xe^{-x^2}$
18. $\sum_{i=1}^n x_1 p_1$

19. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

20. $\int \frac{f'(x)}{f(x)}\,dx$

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