Overview

For thousands of years, man has used the abacus as a counting and calculating device. No one knows with certainty when or where the abacus was invented, but best scholarship indicates that it was used in Mesopotamia 5 or 6 thousand years ago and was introduced into the Orient through trade with ancient Rome. In the Western world, the abacus gave way to written forms of arithmetic in the 16th or 17th century. It thrived in the Orient and today is common to most Japanese businesses, households, and schools. According to Buddhist tradition, odd numbers are imperfect numbers and even numbers are perfect. The soroban, as the abacus is called in Japan, is thought to be an imperfect instrument that is used in the search for perfection.

People with visual impairment or blindness cannot use the abacus that sighted people use, because it has free-moving beads and therefore cannot be read by touch. In the early 1960s, T. V. Cranmer, then director of the Division of Services for the Blind, Kentucky Department of Education, adapted an abacus that blind individuals could use. He added a foam backing to put tension on the beads and keep them stable. He also increased the length of the rods to give more distance between beads and make them easily read by touch. In the text, Using the Cranmer Abacus for the Blind, Fred L. Gissoni (at that time affiliated with the Kentucky Rehabilitation Department) was the first to explain the operation of the Cranmer abacus in 1962. This was followed in 1966 by The Abacus Made Easy by Mae E. Davidow, who was at that time teacher of mathematics at the Overbrook School for the Blind. In 1964 at the University of Kentucky, Mr. Gissoni directed the first abacus institute ever held in America. This generated much enthusiasm for the abacus among blind people and teachers of those with visual impairment or blindness; since then, many similar institutes have been held. About the same time, the Hadley School offered a correspondence course in the use of the abacus for blind people throughout the world.

The Cranmer abacus is an efficient and accurate tool that enables you to perform mathematical calculations. It affords more speed and ease of manipulation than braille writers, Taylor slates, pegboards, and other cumbersome tools. It also removes the drudgery formerly associated with arithmetic. Although calculations can be done quicker on a talking calculator or computer, the abacus is unique as it allows you to mentally perform calculations in an expedient form.

The goal of this course is to provide you with the information you need to become a proficient abacus user. This course is divided into four units. Unit 1, which includes Lessons 1-4, explains how to set, read, and add whole numbers and decimals. Unit 2, which includes Lessons 5-8, examines how to multiply whole numbers and decimals. Unit 3 includes Lessons 9 and 10; it focuses on subtraction of whole numbers and decimals. Lessons 11-15 in the final unit cover long and short division of whole numbers and decimals. This course provides the prerequisite skills that will enable you to proceed to "Abacus II," the sequel that features advanced forms of arithmetic.

To complete this course, you need a Cranmer abacus. The American Printing House for the Blind (APH) also offers an abacus coupler, which enables you to join two or more abacuses together. Most lessons in this course feature practice exercises that prepare you to complete the assignment at the end of each lesson. These exercises are for your personal development only.

Because each lesson builds on previously learned skills, submit only one lesson at a time, at least initially. If you prefer to work at a quicker pace, review this policy with your instructor after completing the first few lessons. Wait for your feedback before sending in your next assignment so that you can learn from your mistakes. Now, if you're ready to become a proficient abacus operator, begin Unit 1, Lesson 1: Setting, Reading, and Adding Numbers.

Unit 1: Addition

This unit introduces the Cranmer abacus, a tool that you can use to perform mathematical calculations. It includes four lessons, which explain direct and indirect methods of adding whole numbers and decimals.

Lesson 1: Setting, Reading, and Adding Numbers

To use the abacus more efficiently and accurately, Lesson 1 defines the special terms associated with this tool. This lesson explains how to set and read numbers, and it introduces addition. Familiarizing yourself with the information in this lesson will enable you to set, read, and add numbers using the abacus.

Objectives

After completing this lesson, you will be able to

- set and clear numbers
- read numbers
- add numbers

Setting Numbers

As with many other specialized tools, the abacus has a special language of its own. To enter a number on the abacus, you do not use the word write; instead you set a number. To erase or remove a number, use the word dear. For instance, to add 7 on a particular column, you'll be told, "Set 7." To subtract 7, you'll be told, "Clear 7." Adding is the mathematical process, whereas setting is the operation performed on each column of the abacus to accomplish the addition. For example, consider the addition of 5 plus 3, which equals 8. In order to add 3 to 5, you set three beads on the abacus. Adding refers to the mathematical process, whereas setting means moving the beads.

Occasionally it will be necessary to add or to subtract the number 1 from the column immediately to the left of that upon which you are working. You will refer to this operation as "Set 1 left" or "Clear 1 left." "Set 1 left" simply means to add the digit 1 on the column immediately left of that upon which you are working. "Clear 1 left" simply means to subtract the digit 1 from the column immediately left of that upon which you are working. If you are told, "Clear 4 and set 1 left," remove 4 from the column upon which you are working, and set the digit 1 on the column immediately to the left. If you are asked, "Clear 1 left and set 6," remove 1 from the column immediately to the left of that upon which you are working, and enter 6 on the column upon which you are working.

The beads are used to represent numbers. They take on value when moved toward the bar and lose value when moved away from it. On each column it is possible to show the digits from zero to nine. However, only one digit can be shown on a given column at one time. Each bead below the separation bar has a value of one. The single bead above the bar has a value of five. When all of the beads on a column are moved away from the bar, that column either contains the digit 0, or it is not being used in the problem and therefore has no value. The single bead above the separation bar, which stands for the digit 5, takes on value when moved down toward the separation bar. It loses its value when moved up away from the separation bar. The four beads below the separation bar each stand for the digit 1. They take on value when moved up toward the separation bar. They lose their value when moved down away from the separation bar.

Generally, numbers should be set and cleared with the right hand. All of the beads below the bar are set with the right thumb and are cleared with the right index finger. The beads above the bar are set and cleared with the right index finger. The index finger of the left hand should always trail the right hand. This will help you keep your place on the proper column and will help to set 1 left or to clear 1 left. As you learn to operate the abacus, take special care to use the correct finger motions. This will help you attain the greatest possible speed.

Now see how to set and clear the digits from 0 to 9: To set and clear 0, do not change the pattern of beads. All the beads above and below the separation bar should remain away from the separation bar.

- To set 1, slide one of the lower beads on the far right column up toward the bar with the right thumb. Clear 1 by sliding this lower bead down with the right index finger.
- To set the number 2, slide two lower beads on the far right column up to the bar with the right thumb in a single motion. Clear the number 2 with the right index finger by sliding those two lower beads on the far right column down away from the bar.
- To set 3, push three lower beads up to the bar with the right thumb. Clear 3 with the index finger by moving those three lower beads down.
- To set 4, slide all four lower beads up to the bar with the thumb. Clear 4 by sliding those four lower beads down with the index finger.
- Set 5 with the right index finger by sliding the single upper bead on the far right column down toward the bar. Clear 5 with the index finger by moving that upper bead up away from the bar.
- To set 6, use the index finger to move the upper bead down toward the separation bar. Move one lower bead on the same column up with the thumb; 5 plus 1 is 6. To clear 6, move the five bead up with the index finger. Then slide this same finger down across the bar, and move the one lower bead down away from the bar.
- To set 7, set the five bead with the index finger, and then set two lower beads with the thumb; 5 plus 2 is 7. Clear 7 by clearing the five bead above the bar with the right index

finger. Then move down across the bar and clear the two beads below the bar with the same index finger.
- To set 8, first set the five bead with the index finger and set 3 with the thumb. Clear 8 by sliding the five bead up and the three lower beads down, all with the index finger.
- To set 9, move all of the beads on the column as close to the bar as possible by sliding the five bead down with the index finger and all four lower beads up with the thumb. Clear 9 by clearing the five bead and clearing the four lower beads. With practice, the digits 6, 7, 8, and 9 can be set with a single pinching motion. That is, while the index finger is setting the upper five bead, the thumb will be setting the lower beads.

When working with whole numbers, the column to the extreme right end of the abacus is the units column. The next column to its left is the tens column. Then comes the hundreds column, followed by a unit mark or comma that separates the hundreds from the thousands column. Proceeding leftward, you have the thousands, the ten thousands, and the hundred thousands columns. Another comma follows. Then comes the millions column, and so forth, up to the trillions column, which is found on the thirteenth column to the extreme left end of the abacus.

Unit marks are the short, vertical lines found on the separation bar and also at the bottom of the abacus frame. When counting from right to left, these unit marks occur after every third column. Therefore, unit marks are found between columns 3 and 4, columns 6 and 7, columns 9 and 10, and columns 12 and 13.

Unit marks are used as commas or, when necessary, as decimal points.

One column is needed to set a one-digit number, two columns to set a two-digit number, five columns for a five-digit number, ten columns for a ten-digit number, and so on. For example, the number 284 consists of three digits: 2, 8, and 4. So the three far-right columns are needed to set this number. To do so, set the number 2 on the hundreds column (i.e., the third column from the right end), which is known as column 3. Set 8 on the tens column (i.e., the second column from the right end), also known as column 2. Set 4 on the units column (i.e., the far right column of the abacus), which is also called column 1. Numbers are always set from left to right. Therefore, anticipate the number of columns needed to set or add a number before you begin.

Try setting the number 13,507. It contains five digits; therefore, it requires five columns to set it. Locate column 5, and set 1 on this column. Set 3 on column 4, followed by a unit mark. Set 5 on column 3, 0 on column 2, and 7 on column 1. When setting or adding 0, no change in the arrangement of the beads is made on that particular column.

Reading Numbers

Numbers are read in a left-to-right direction. Generally, numbers are read using the thumb, index finger, and middle finger of your right hand. However, develop the method of reading numbers that is most comfortable for you.

Practice setting and reading many numbers. For instance, try setting and reading your phone number, street address, zip code, age, birth year, and historical dates. The current month, day, and year expressed in numerical form can also be set and read on the abacus.

Try, for example, setting and reading these numbers.

- 7
- 1,067
- 294
- 300

- 9,405
- 83
- 246,813
- 40
- 4
- 7,923
- 81,045
- 579,108
- 226
- 3,336
- 200,963
- 10
- 7,248,019
- 34,826,120
- 509
- 85,003

It is important to set and read numbers accurately and quickly; otherwise the abacus will be of little value to you. Once you can set and read numbers accurately and quickly, you are ready to proceed to addition.

Adding Numbers

Numbers can be added on the abacus by either direct or indirect means. When adding by direct means, simply set the number you want to add on the appropriate column by moving beads representing that number toward the bar. For instance, suppose you want to add 1 plus 2 plus 6. Begin by setting 1 on column 1 by moving one lower bead up to the bar with the thumb of the right hand. Next add the number 2 by moving two lower beads on the same column up toward the bar. You now have a total of 3 showing on column 1. Now add 6 by sliding the five bead down with the right index finger and by sliding one lower bead up with the thumb. Column 1 now contains the number 9 because 1 plus 2 plus 6 is 9.

Though you should always try to initially add a number directly, it is not always possible to do so. Whenever it is impossible to add a number directly, do so indirectly. Do this by setting either a 5 or a 10. This will result in your adding more than you wanted to add. So subtract the difference between the number you wanted to add and either the 5 or the 10 that you actually added.

To illustrate indirect addition, begin by adding the number 1 to itself a number of times. One is a units number containing a single digit. So all of the 1s must be entered on the units column, which is column 1. Begin by setting the number 1 on column 1 by sliding one of the lower beads up to the bar with the right thumb. (As previously mentioned, your left hand should be resting immediately to the left of the right hand.) Then, on this same column, add 1 to 1 directly. Now add another 1 directly, and yet another 1. Column 1 now contains the number 4. When you try to add another 1, however, it cannot be added directly. That is, you cannot slide one more lower bead up to the bar because there are no more lower beads to move up. All of them are being used to represent the 4.

However, the five bead on column 1 is not in use. So set 5, which is more than you wanted to add. Recall that you only wanted to add 1, but you have added 5. Since you have over-added, subtract the difference between the 1 (which you wanted to add) and the 5 (which you actually added). That is, subtract 4 because 5 minus 1 equals 4. To do so, move the right index finger above the bar to the top of column 1. Then in one continuous downward sweep of the right index finger, slide the five bead down to the bar, move under it, and slide the four lower beads to the bottom of the column. By doing this, you have indirectly added 1 to 4. Since 4 plus 1 is the same as 4 plus (5 minus 4), you now have the number 5 showing on column 1. This shows one of the indirect methods for adding 1—that is, to set 5 and clear 4.

In this last step, you had to add 1 to 4 indirectly by setting 5 and clearing 4. At first this may seem difficult to understand. But think of it in terms of an everyday money transaction. Instead of exchanging coins, you are exchanging beads. Suppose you have four pennies and a friend owes you one cent. Unfortunately he does not have a penny with him, but he does have a nickel, which he gives to you. Since he gave you 5 cents but owed you only 1 cent, you must give him 4 cents in change. By giving you 5 cents and taking back 4 cents, your friend has indirectly added 1 cent to the 4 cents that you already had. To state this money transaction in the language of the abacus, your friend has indirectly paid you 1 cent by setting 5 cents and clearing 4 cents. You now have a nickel, worth 5 cents.

Now continue to add 1s as you did before. Add 1 to 5 by sliding one lower bead up to the bar with the right thumb. You now have a 5 set above the bar and a 1 immediately below the bar; 5 plus 1 is 6. Now add 1 and you have 7. Add 1 more and you have 8. Add another and you have 9. But when you try to add one more, you cannot add it directly. There are no more inactive (i.e., unused) beads on column 1 to move toward the bar. So with your left hand, set 1 left. Do this by sliding one lower bead on the column immediately to the left (column 2) up to the bar with the index finger of your left hand.

This one bead equals 10 (i.e., ten 1s) because it is on column 2—the tens column. However, you have overadded—you wanted to add only 1, but you added 10. Therefore, subtract the difference between 10 and 1, which is 9. With the right index finger, clear 9 from column 1. By clearing 9 and setting 1 left, you have indirectly added 1 to 9. The number 10 now appears on columns 2 and 1. The digit 1 appears on column 2 followed by 0 on column 1. This illustrates another rule for the indirect addition of 1, which is to clear 9 and set 1 left.

Think of the last step, 9 plus 1 equals 10, as a money transaction. Now 9 plus 1 is the same as 9 plus (10 minus 9). Suppose you have 9 cents and a friend owes you 1 cent. She does not have a penny, but she does have a dime, which she gives to you. Since she gave you a dime but owed you only 1 cent, she has paid you 9 cents more than she owed you. You must give her 9 cents in change. Your friend has indirectly added 1 cent to your 9 cents by setting 10 cents and clearing 9 cents. You now have a dime, worth 10 cents.

Now continue to add 1s on column 1. Add 1 to 10 by sliding one lower bead up to the bar on column 1. This produces one bead immediately below the bar on both columns 2 and 1, which represent the number 11. Now continue to add 1 directly until you reach 14. To do so, add 1 to 14 indirectly. Add 1 to the 4 of 14, indirectly, setting 5 and clearing 4 with a continuous downward sweep of the right index finger. Your total then will be 15.

When you reach 19, use indirect addition to add 1 to the 9 of 19. Because the five bead is already in use, you cannot set 5 and clear 4. So clear 9 and set 1 left. Clear 9 on column 1 with the right hand, and set 1 left on column 2 with the left hand. At this point, the abacus will have a 2 on column 2, and a 0 on column 1, which together represent the number 20.

Now continue to add 1 on column 1 until you reach 49. To add 1 to 49, add 1 to the 9 of 49 on column 1. You cannot add 1 directly; nor can you set 5 and clear 4, because the five bead is in use. So clear 9 and set 1 left. Clear 9 from column 1 and set 1 left on column 2. However, column 2 already contains a 4. Because there are no more lower beads on column 2 to move up to the bar, it is not possible to set 1 left directly. So set 1 left indirectly by setting 5 and clearing 4 with a continuous downward sweep of the left index finger. In this operation, you have actually cleared 9, set 50, and cleared 40, thereby indirectly adding 1 to 49. Your abacus should now contain 50.

Continue to add 1 on column 1 until you reach 99. To add 1 to 99, add 1 to the 9 of 99 on column 1. This cannot be done directly; nor can you set 5 and clear 4, because the five bead is in use. So add 1 to 9 indirectly by clearing 9 and setting 1 left. Clear 9 from column 1 with the right hand and set 1 left on column 2. Since column 2 contains a 9, it is impossible to set 1 left directly. Therefore, with the left hand, clear 9 from column 2, move leftward to column 3, and set 1 on that column. By doing this, you have actually cleared 9, cleared 90, and set 100, thereby adding 1 to 99 indirectly. Your abacus now contains 100 on columns 3, 2, and 1.

Summary

This lesson introduced the terminology you need for using the abacus. It explained how the beads represent numbers, and how to set and read those numbers. The lesson also described how to add numbers.

Assignment 1

Indicate whether the following statements are true or false. If the statement is false, reword it to make it true.

- Blind or visually impaired people can easily and effectively use the same abacus that sighted people use.
- Numbers are set and read from right to left.
- The separation bar appears in the lower third of the abacus.
- The unit marks are the short vertical lines appearing after every fifth column.
- To set a number means to enter it on the abacus; to clear a number means to remove it.
- Beads take on their value when they are moved as close to the separation bar as possible.
- If a column already contains the digit 4, the digit 1 can be added to it directly.
- An unused column and one that contains the digit 0 look and feel the same.
- The individual bead on each column, either above or below the separation bar, has a value of 5.

Select the answer that best completes the following:

- To add 1 to 64, you must
    - add 1 to 6 directly
    - add 1 to 4 indirectly by setting 5 and clearing 4
    - add 1 to 4 indirectly by clearing 9 and setting 1 left
- How many beads are needed to show the digit 7?
    - three 2
    - one
    - seven
- How many columns are needed to set 7,205?
    - three
    - four
    - five
- You cannot add 1 directly to
    - 0
    - 5
    - 9
- Set 1 2 on the two columns to the far right. To add 1, you must
    - add it directly to the 1 of 12
    - add it directly to the 2 of 12

    - add it indirectly to the 2 of 1 2
- Set 49 on the two columns to the far right. To add 1, you must
    - add it directly to the 9 of 49
    - add 1 to the 4 in column 2
    - add 1 indirectly to the 9 in column 1 by first clearing 9, and then setting 1 left indirectly, by setting 5 and clearing 4

Lesson 2: Indirect Methods of Addition

Lesson 1 explained how to set, clear, read, and add numbers. This lesson provides additional instruction and practice in the direct and indirect methods of adding whole numbers on the abacus. It not only distinguishes between these two methods of addition, but also explains the reason for using each one. Familiarizing yourself with the information in this lesson will enable you to use direct and indirect methods when adding with the abacus.

Objectives

After completing this lesson, you will be able to

- add one-digit numbers directly and indirectly

Direct Versus Indirect Addition

As indicated in Lesson 1, numbers can be added on the abacus either directly or indirectly. Except for 5, every digit from 1 to 9 can be added in three different ways, one direct way and two indirect ways. The digit 5, however, can only be added in two ways: one direct and one indirect. Only one of these ways can be used any given time. To determine which method to use, first see if the digit can be added directly. If it cannot, such as with 4 plus 1 and 9 plus 1, apply the appropriate indirect method of addition. That is, set either a 5 or a 10; then subtract the difference between the number that you wanted to add and either 5 or 10.

To summarize: Always try to add a number by direct means first. If a number cannot be added directly, then set it indirectly. This is done in one of two ways: either by setting 5 and clearing the appropriate difference, or by setting 10 and clearing the appropriate difference.

The indirect methods of addition are based on the principle of complementary numbers. Set either a 5 or a 10; then subtract the appropriate complementary or difference number. The complementary or difference numbers for 5 are 1 and 4, and 2 and 3. The complementary or difference numbers for 10 are 1 and 9, 2 and 8, 3 and 7, 4 and 6, as well as 5 and 5. Study the following table of equivalents that shows the complementary numbers for each digit from 1 to 9 with reference to 5 and 10. This table may be helpful when you are trying to decide how a particular digit should be added.

Table of Equivalents

- 1 is the same as (5 minus 4)
- 1 is the same as (10 minus 9)
- 2 is the same as (5 minus 3)
- 2 is the same as (10 minus 8)
- 3 is the same as (5 minus 2)
- 3 is the same as (10 minus 7)
- 4 is the same as (5 minus 1)
- 4 is the same as (10 minus 6)
- 5 is the same as (10 minus 5)

- 6 is the same as (10 minus 5 plus 1)
- 6 is the same as (10 minus 4)
- 7 is the same as (10 minus 5 plus 2)
- 7 is the same as (10 minus 3)
- 8 is the same as (10 minus 5 plus 3)
- 8 is the same as (10 minus 2)
- 9 is the same as (10 minus 5 plus 4)
- 9 is the same as (10 minus 1)

Consider a few more examples of indirect addition, starting with 2 plus 4. Begin by setting 2 on column 1. If you try to add 4, you will find that there are only two inactive lower beads. Therefore, you cannot add 4 directly. You will have to add 4 indirectly by setting either a 5 or a 10. The five bead on column 1 is still available. So set 5, and clear the difference between 5 and 4. That is, clear 1 because 4 is the same as (5 minus 1). Your abacus should now contain 6 because 2 plus 4 is 6.

To review, think of this problem as a money transaction. You have two pennies, and your friend owes you four cents. Your friend does not have four pennies, but he does have a nickel, which he gives to you. How much change will you have to return to him? One cent—the difference between the nickel that he paid you and the four cents that he owed you. You now have a penny and a nickel, for a grand total of six cents.

This time add 8 plus 3. Set 8 on column 1. Because there is only one inactive lower bead to move up to the bar, you cannot add 3 directly to 8. So over-add by setting either a 5 or a 10. The five bead is already in use, so you cannot set 5 and clear 2. Therefore, clear 7 from column 1 and set 1 left on column 2. Recall that 3 is the same as (10 minus 7). You now have 11 showing on the abacus because 8 plus 3 is 11.

In terms of a money transaction, imagine that you have eight cents and your friend owes you three cents. She does not have three pennies, but she does have a dime, which she gives to you. She has now paid you seven cents more than she owed you, so you will have to give her seven of your eight cents in change. You now have a dime and a penny for a total of 11 cents.

This time, add 6 plus 7. Begin by setting 6 on column 1. When you try to add 7 to 6, you find that it cannot be added directly. (Although you can move two lower beads up to the bar, you cannot move the five bead down because it is already in use.) Therefore, add 7 indirectly by setting the two lower beads with the right thumb, clearing the five bead with the right index finger, and then setting 1 left. You now have a total of 13 because 6 plus 7 is 13.

When you find that you cannot add the higher digits 6, 7, 8, and 9 directly, and the five bead is in use, do the following: Set the difference between the number you wanted to add and 5; then move up over the separation bar, clear 5, and finally set 1 left. For example, if you are adding 6, set one lower bead; if 7, two lower beads; if 8, three lower beads; if 9, four lower beads. Then proceed to clear 5 and set 1 left; 10 minus 5 equals 5. Up to this point, you have only added 5, which is why you must set the difference between 5 and the number you wanted to add. Recall that to indirectly add a number, you over-add by setting either a 5 or a 10. Then you subtract the difference between the number you wanted to add and either 5 or 10.

How does that apply to this situation? Consider the last example, 6 plus 7 equals 13. After setting 6, you see that you cannot add 7 directly to 6. Therefore, you over-add by setting 10. This means that you must subtract the difference between 10 and 7, which is 3. However, you cannot subtract 3 directly from column 1. Therefore, clear the five bead. You wanted to subtract only 3, but instead you subtracted 5. So return the difference between 5 and 3, which is 2.

For a further example, add 5 plus 8. Begin by setting 5 on column 1. You cannot add 8 directly to 5. (Although you can move three lower beads up to the bar, you cannot move the five bead down to the bar because it is already in use.) Therefore set 3 (i.e., the difference between 8 and 5), then clear 5, and set 1 left. The abacus now contains 13 because 5 plus 8 is 13.

To apply what you have learned, add a series of numbers. For example, add all of the numbers from 1 to 9. Begin by setting 1 on column 1. Then add 2 to this 1 directly by moving two lower beads up to the bar. The abacus now shows 3. Add 3 by setting 5 and clearing 2 with a continuous downward sweep of the right index finger. This gives a total of 6. Now add 4 by clearing 6 and setting 1 left on column 2. Since 4 is the same as (10 minus 6), you now have a subtotal of 10. Next add 5 directly by moving the five bead on column 1 down to the bar. The abacus now shows 15. Add 6 to the 5 of 15 because both 6 and the 5 of 15 are units digits. Set 1 (i.e., the difference between 6 and 5) with the right thumb. Then clear 5 with the right index finger, and set 1 left on column 2 with the left index finger. The abacus now contains 21. Add 7 directly on column 1. You now have 28. Add 8 in column 1 by clearing 2 with the right hand and setting 1 left in column 2. Since 8 is the same as (10 minus 2), you now have 36 showing on the abacus. Finally, add 9 to the 6 of 36 in column 1, by clearing 1 and setting 1 left in column 2. Since 9 is the same as (10 minus 1), the grand total is 45.

You could continue this process by adding 9 to the 5 of 45 in the units column. To do so, set 4 (i.e., the difference between 9 and 5), then clear 5, and set 1 left indirectly in the tens column. This 1 must be set left indirectly because you cannot add 1 to 4 directly. Therefore, set 1 left indirectly by setting 5 and clearing 4 with a continuous downward sweep of the left hand. The total is 54.

In working through these problems, notice that you have a running subtotal. When you complete the problem, you have a grand total but no record of each subtotal. So you cannot go back and check the accuracy. This is not a problem for skilled abacus operators, however; they can rapidly rework the problem to verify the result.

Now that you have learned the basic principles by which the abacus operates, you can add any one-digit number to another. The following is a chart of all the indirect methods of addition, expressed in statement form. If you are uncertain how a particular digit should be entered on the abacus, consult this chart. The column to the left lists all the digits from 1 to 9. The column to the right lists the indirect methods of addition. First look down the left-hand column until you find the digit you want to add. Then look to the right to read the indirect methods for adding that number. Choose the correct method that applies to your particular problem. Many beginning abacus operators find it useful to memorize these statements; then they practice adding numbers until the use of the correct statement becomes automatic and is no longer a matter of conscious effort.

To Add By Indirect Method

To Add by Indirect Methods
1 set 5, clear 4
1 clear 9, set 1 left
2 set 5, clear 3
2 clear 8, set 1 left
3 set 5, clear 2
3 clear 7, set 1 left
4 set 5, clear 1
4 clear 6, set 1 left
5 clear 5, set 1 left
6 set 1, clear 5, set 1 left
6 clear 4, set 1 left
7 set 2, clear 5, set 1 left
7 clear 3, set 1 left
8 set 3, clear 5, set 1 left
8 clear 2, set 1 left
9 set 4, clear 5, set 1 left
9 clear 1, set 1 left

Recall that to set 1 left means to set the digit 1 on the column immediately to the left of the column upon which you are working. If that column contains a 4 (which makes it impossible to set 1 left directly), then set 1 left indirectly by setting 5 and clearing 4. If the column or columns immediately to the left contain 9, then set 1 left indirectly by clearing the 9 (or 9s) and setting 1 left on the first available column. For example, to add 3 to 998: Clear 7 from column 1; then set 1 left indirectly by clearing 9 from column 2, clearing 9 from column 3, and setting 1 left on column 4. The total is 1,001.

Suppose you want to add 3 to 498. Clear 7 from column 1, and set 1 left on column 2. On column 2 you cannot set 1 directly, so you clear 9 and set 1 left on column 3. You have a 4 on column 3. Add 1 to 4 indirectly by setting 5 and clearing 4. The total is 501. Your right hand should remain on the column upon which you are working, while your left hand performs all of the various steps necessary to set 1 left. In this way, you will not lose your place, and your right hand will be in the proper position to resume entering digits.

To become proficient at operating the abacus, practice adding numbers using either the direct or indirect method of addition until you can do it automatically. For example, practice adding the same number to itself, such as 8 plus 8 plus 8, a hundred or more times. Do this for each digit from 1 to 9. Then try building pyramids of numbers and add them over and over. For instance, add 1 plus 2 plus 3, which equals 6. Then continue to add 1 plus 2 plus 3, plus 1 plus 2 plus 3, a number of times. Each new sum will be a multiple of 6. Build other pyramids, such as 3 plus 4 plus 5, or 6 plus 7 plus 8, or 8 plus 6 plus 4 plus 2, and so on. In each case, determine the sum of the first round of addition; each new sum thereafter will be a multiple of that number.

An excellent exercise is to add the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9, which total 45. For variation, add these nine numbers in any sequence; the total will still be 45. Once you have reached 45, you can continue the process by adding the numbers from 1 to 9. Your answer will be 90. Each time you finish a round of 1 through 9, your answer will increase by 45. For another variation, begin with 4 (or some other number on the units column); then add all the numbers from 1 to 9. Your answer will be 45, increased by the number with which you started.

In literary braille, the letters a through j are used to represent the numbers 1 through 0 when they follow the number sign. If you think of the letters of a given word in terms of their numerical value, you can use words as a practice exercise for addition. Many words can be formed using the letters a through j. These may be helpful as you learn to add one-digit numbers. For example, think of the word cabbage as 3 plus 1 plus 2 plus 2 plus 1 plus 7 plus 5. When added together, these numbers equal 21. Here are a few more such words to try adding together:

- abide
- acid
- aided
- ache
- aged
- bad
- badge
- bagged
- baggage
- beaded
- bee
- bed
- beef
- begged
- beach
- bib
- bid
- big
- cafe
- caged
- cab
- cad
- dad
- dabbed
- deadhead
- defaced
- did
- ebb
- faded
- fed
- feed
- fig
- fibbed
- gage
- had
- headed

- hedge
- headache
- hide
- if
- jab
- jade
- jagged
- jib

If you are using an abacus to add numbers on a printed page, try using your abacus as a straightedge in order to help you keep your position. This applies no matter whether the numbers are arranged vertically or horizontally. Simply place your abacus below or beside the number you are adding, read that number with the index finger of one hand, and enter it with the other hand. When the entire number has been entered, slide the abacus either down or across one place, so that the next number to be added becomes exposed.

Practice Exercise

Practice adding one-digit whole numbers directly and indirectly on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 8 plus 5 plus 1 plus 4 plus 7 plus 2 plus 6 plus 3
- 9 plus 3 plus 5 plus 7 plus 4 plus 1 plus 2 plus 4
- 1 plus 4 plus 6 plus 3 plus 5 plus 8 plus 9 plus 7
- 7 plus 7 plus 2 plus 8 plus 9 plus 5 plus 1 plus 8
- 12 plus 6 plus 4 plus 9 plus 3 plus 7 plus 8 plus 8
- 2 plus 9 plus 7 plus 6 plus 3 plus 8 plus 4 plus 4 plus 9
- 9 plus 1 plus 5 plus 3 plus 6 plus 4 plus 8 plus 7 plus 3
- 4 plus 6 plus 1 plus 5 plus 9 plus 2 plus 7 plus 1 plus 9
- 4 plus 2 plus 6 plus 7 plus 8 plus 1 plus 5 plus 5 plus 4
- 29 plus 3 plus 8 plus 4 plus 5 plus 9 plus 1 plus 6 plus 8

Answers

- 36
- 35
- 43
- 47
- 57
- 52
- 46
- 44
- 42
- 73

Summary

This lesson explained the difference between adding one-digit numbers directly and indirectly. After listing the complementary numbers for each digit from 1 to 9, the lesson identified when to use each particular method.

Assignment 2

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

- You must always use one of the indirect methods of addition when
    - you are adding a column containing two or more numbers
    - you cannot add a number directly
    - you want to add a number that is greater than 5
- Except for 5, how many different ways can every digit from 1 to 9 be added? (Remember that numbers can be added directly and indirectly.)
    - one
    - two
    - three
- The digit 5 can be added in how many different ways? (Remember that numbers can be added both directly and indirectly.)
    - one
    - two
    - three
- The indirect methods of addition are
    - not necessary once you gain skill in abacus operation
    - a quaint custom arising from Buddhist tradition
    - logical rules of procedure based on complementary numbers
- Your abacus contains the digit 6. In order to add 2 to 6, you must
    - enter 2 directly
    - set 5, clear 3
    - clear 6 and set 1 left
- Your abacus contains the digit 4. To add 7 to 4, you must
    - set 7 directly
    - set 5, clear 2, and set 1 left
    - clear 3 and set 1 left
- Which of these is an exact and complete method for the indirect addition of 4?
    - set 5, clear 4
    - clear 6, set 1
    - set 5, clear 1

Questions 8-10 pertain to the following problem: Enter 3 on the far right column five times, producing a total of 15.

- How many times did you enter 3 directly?
    - once
    - twice
    - three times
- How often did you have to set 5 and clear 2?
    - once
    - twice
    - three times
- How often did you have to clear 7 and set 1 left?
    - never
    - once
    - twice

Questions 11 and 12 pertain to the following problem: On column 1, enter 4 five times, which will give you a total of 20.

- How many times did you enter 4 directly?
    - never
    - once
    - twice
- How often did you enter 4 indirectly?
    - twice
    - three times
    - four times

Questions 13-15 pertain to the following problem: Set 9 on the column to the far right five times, which will give you 45.

- How many times did you enter 9 directly?
    - once
    - twice
    - three times
- How often did you set 4, clear 5, and set 1 left?
    - never
    - twice
    - four times
- How many times did you clear 1 and set 1 left?
    - twice
    - three times
    - four times

Questions 16-18 pertain to the following problem: Entering 6 on the same column five times produces 30.

- In so doing, how many times must you enter 6 directly?
    - never
    - twice
    - four times
- How often did you set 1, clear 5, and set 1 left?
    - once
    - twice
    - three times
- How often did you clear 4 and set 1 left?
    - once
    - twice
    - three times

Questions 19-21 pertain to the following problem: The number 1, entered five times, produces 5.

- How many times is the number 1 entered directly?
    - once
    - four times
    - three times
- How often must you set 5 and clear 4?

    - never
    - once
    - twice
- If you continue to add 1 on column 1, when will it first become necessary to clear 9 and set 1 left?
    - when you add 1 to 6
    - when you add 1 to the 2 of 12
    - when you add 1 to 9

Questions 22-24 pertain to the following problem: To enter 8 on column 1 five times produces 40.

- In so doing, how many times do you enter 8 directly?
    - never
    - once
    - twice
- How often must you set 3, clear 5, and set 1 left?
    - once
    - twice
    - three times
- How often is it necessary to clear 2 and set 1 left?
    - once
    - twice
    - three times

Questions 25 and 26 pertain to the following problem: The number 5 entered five times produces 25.

- How often must you enter 5 directly?
    - once
    - twice
    - three times
- The indirect method for adding 5 is used
    - once
    - twice
    - three times

Questions 27 and 28 pertain to the following problem: Adding 7 five times produces 35.

- When adding the final 7, you must
    - enter 7 directly
    - set 2, clear 5, and set 1 left
    - clear 3 and set 1 left
- When working the problem, 7 is set indirectly
    - three times
    - four times
    - five times
- Set 98 on the two far-right columns. When adding 3 to 98, you must
    - add 3 to the 8 of 98
    - add 3 to the 9 of 98
    - know how to set 1 left indirectly
- Set 64 on the two far-right columns. When adding 7 to 64, you must

    - use direct means
    - set 2, clear 5, set 1 left
    - clear 3, set 1 left

Explain the following problem. Be as thorough and detailed as possible.

- Work the problem 7 plus 4 plus 5 plus 8 plus 3. Explain where you set the problem, describe every step, and mention each method that you used and why.

Lesson 3: Adding Numbers With Two or More Digits

Lesson 1 explained how to set, clear, read, and add numbers. Lesson 2 not only distinguished between direct and indirect addition, but also explained why each method is applied. This lesson provides additional instruction and practice in the direct and indirect methods of addition. In particular, it applies the instruction to whole numbers with two or more digits. Familiarizing yourself with the information in this lesson will enable you to use the abacus to add numbers with multiple digits.

Objectives

After completing this lesson, you will be able to

- directly and indirectly add numbers with two or more digits
- practice your skills by using the doubling exercise

Directly and Indirectly Adding Numbers With Two or More Digits

Addition on the abacus is always performed in a left-to-right direction. When adding numbers that have two or more digits, you enter one entire number, setting it from left to right, before moving on to the next number. Adding numbers in a left-to-right direction eliminates the need for carrying numbers.

For example, to add 18 plus 27 plus 36, begin by setting 18 on columns 2 and 1. Set 1 on column 2 with the right hand; then move one column to the right and enter 8 on column 1, again with the right hand. Next add 27 to 18, first by adding the 2 of 27 on column 2 directly to the 1 already on that column. Then add the 7 of 27 on column 1 to the 8 already on that column. It is impossible to add 7 directly to 8; so clear 3 from column 1 with the right hand, and set 1 left on column 2 with the left hand. The abacus now shows 45. To add 36, add 3 to the 4 on column 2 by setting 5 and clearing 2 with a continuous downward sweep of the right hand. Finally, add 6 indirectly to the 5 already on column 1. To do this, set 1 and clear 5 on column 1; then set 1 left directly on column 2. This gives a grand total of 81 on columns 2 and 1.

When adding a series of numbers on the abacus, take special care to enter each digit on the proper column where it belongs. A two-digit number is entered on columns 2 and 1; a four-digit number on columns 4, 3, 2, and 1; a nine-digit number on columns 9, 8, 7, 6, 5, 4, 3, 2, and 1; and so forth. Zeros occupy columns just as any other digit, so they must be taken into account when entering numbers.

Suppose you are adding 12,045 plus 678 plus 90. Begin by setting 12,045 on columns 5, 4, 3, 2, and 1. To do so, first enter 1 on column 5, followed in sequence to the right by 2, 0, 4, and 5. Then add 678 on columns 3, 2, and 1, since 678 is a three-digit number. Add 6 to 0 on column 3; 7 to 4 on column 2; and 8 to 5 on column 1. The number 6 can be added directly to the 0 on column 3. Add 7 indirectly to the 4 on column 2 by clearing 3 from column 2 and setting 1 left on column 3. To add 8 to 5 on column 1, use indirect means. Set 3 and clear 5 on column 1; then set 1 left on column 2. At this point, the abacus shows a subtotal of 12,723. To this, add 90 as the final step. Since 90 contains two digits, enter it on columns 2 and 1. To add 9 to the 2 already on column 2, clear 1 from that column and set 1 left on column 3. To add 0 to column 1, do not change the arrangement of beads. Zero represents nothing, so do nothing to the column except lightly touch it. Your final total is 12,81 3. Notice the position of the unit mark representing the comma in 12,813—it will help you read the number.

For practice, add the same number to itself ten times. Your answer will be the same number with which you started with a 0 at the right. For example, 777 plus 777 plus 777 ten times equals 7,770. Adding 6,789 ten times equals 67,890. Adding 123,456,789 ten times produces a total of 1,234,567,890.

If you add the numbers 111, 222, 333, 444, 555, 666, 777, 888, and 999 in any sequence, your answer will be 4,995. For variation, begin with another number; then add these same nine numbers. Your answer will be 4,995 increased by the number with which you started. If you add 123 plus 456 plus 789 nine times, your total will be 12,312. An excellent exercise is to add all the numbers from 1 up to and including 100, in any sequence. Your answer will be 5,050. To help you check your work as you go along, the subtotal will be 325 once you reach 25; 1,275 when you reach 50; and 2,850 when you reach 75.

Practice Exercise

Practice adding whole numbers with two or more digits directly and indirectly on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 24 plus 36 plus 19 plus 57 plus 20 plus 86
- 4 plus 60 plus 81 plus 7 plus 26 plus 9
- 26 plus 57 plus 89 plus 30 plus 15 plus 74
- 31 plus 20 plus 73 plus 58 plus 94 plus 62
- 30 plus 62 plus 75 plus 28 plus 41 plus 64 plus 80 plus 57 plus 14 plus 93
- 64 plus 31 plus 59 plus 80 plus 63 plus 25 plus 40 plus 62 plus 28 plus 91
- 31 plus 40 plus 72 plus 96 plus 17 plus 50 plus 82 plus 95 plus 83 plus 65
- 60 plus 97 plus 56 plus 30 plus 85 plus 79 plus 64 plus 21 plus 18 plus 42
- 79 plus 65 plus 48 plus 13 plus 95
- 528 plus 304 plus 716 plus 935 plus 697
- 901 plus 506 plus 373 plus 340 plus 712
- 139 plus 420 plus 654 plus 788 plus 876
- 763 plus 804 plus 625 plus 138 plus 957
- 410 plus 632 plus 294 plus 871 plus 705
- 13,586 plus 179 plus 920 plus 453 plus 1,648
- 7,502 plus 836 plus 3,051 plus 6,428 plus 917
- 4,639 plus 157 plus 2,980 plus 316 plus 5,748
- 285 plus 6,473 plus 910 plus 7,804 plus 3,561
- 2,915 plus 806 plus 9,532 plus 198
- 6,702 plus 354 plus 8,071 plus 4,968
- 783 plus 5,927 plus 346 plus 4,851
- 8,251 plus 64,907 plus 37,582 plus 2,093 plus 16,754
- 47,182 plus 5,068 plus 23,719 plus 6,425 plus 10,938
- 370,169 plus 48,952 plus 615,370 plus 82,741 plus 927,385
- 26,362,437 plus 151,580,196 plus 555,273,658 plus 34,049,701 plus 805,936

Answers

- 242
- 187
- 291
- 338
- 544
- 543
- 631
- 552
- 300
- 3,180
- 2,832
- 2,877
- 3,287
- 2,912
- 16,786
- 18,734
- 13,840
- 19,033
- 13,451
- 20,095
- 11,907
- 129,587
- 93,332
- 2,044,617
- 768,071,928

Doubling Exercise

To wrap up the discussion on addition, consider doubling as an excellent skill for beginning and skilled abacus users to develop. Begin by setting the number 1 on column 1. Then add 1, for a total of 2. Next add 2, which produces 4. Then add 4, producing 8. Next add 8, and you have 16. To double 16, add 1 to 1, then 6 to 6. Your abacus now contains 32. Continue doubling in this manner, simply by adding the number that appears on each column to itself. After you double five more times, your abacus should show 1,024. After the next five additions, your abacus should contain 32,768. Double five more times and your abacus will show 1,048,576. After the next five additions, your abacus should contain 33,554,432. Double five more times, and your abacus should show 1,073,741,824. After the next five additions your abacus should contain 34,359,738,368. Double five more times, and your abacus should show 1,099,511,627,776. After the last three additions are made, all thirteen columns of your abacus will be full and you can no longer double. The final answer will be 8,796,093,022,208.

This problem will take considerable time and concentration. So try to work it at a time when you will not be interrupted, as it is quite easy to lose your place. Happy doubling!

Summary

This lesson examined more complex addition problems. It explained direct and indirect addition of whole numbers with two or more digits. It also introduced the doubling exercise to help you hone your skills.

Assignment 3

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

- When adding numbers on the abacus that have two or more digits, you
    - add down the entire left-hand column, then down the entire right-hand column
    - enter one entire number, setting it in a right-to-left direction, before going on to the next number
    - enter one entire number, from left to right, before going on to the next number

Questions 2-4 pertain to the following problem: Set 34 on columns 2 and 1. Add 43.

- The result is
    - 67
    - 76
    - 77
- The 4 of 43 was added to the 3 of 34
    - directly
    - by setting 5 and clearing 4
    - by setting 5 and clearing 1
- The 3 of 43 was added to the 4 of 34
    - directly
    - by setting 5 and clearing 2
    - by setting 5 and clearing 3

Questions 5-7 pertain to the following problem: On columns 2 and 1, add all the numbers from 10 to 20, including both 10 and 20.

- The result is
    - 115
    - 161
    - 165
- The first time you used the indirect method of addition was when you entered the number
    - 11
    - 13
    - 15
- Your work carried over to column 3 when you entered the number
    - 16
    - 17
    - 18
- Your abacus contains 9,81 5. To add 60 to it, you must enter 60 on the columns containing
    - 98
    - 81
    - 15
- The numbers 123 and 321 when added together
    - may be entered directly
    - produce a total of 444
    - limit your activity to the region below the bar
- In the problem 99,998 plus 4
    - you clear the 9s on columns 2, 3, 4, 5 and set1 left on column 6

    - the addition is done by direct means
    - the result is 100,002
- Set 12,345 on your abacus. Add 6,789. The result is
    - 18,134
    - 19,134
    - 80,235

Questions 12 and 13 pertain to the following problem: On columns 3, 2, and 1, set 249. Add 111.

- The result is
    - 36
    - 360
    - 350
- To add the 1 of 111 on column 2 to the 4 of 249 on column 2,you must
    - set it directly
    - set 5 and clear 4
    - set 5 and clear 1

Questions 14 and 15 pertain to the following problem: Set 429 on your abacus. Add 222.

- The result is
    - 551
    - 641
    - 651
- How do you add the final 2 of 222?
    - Add it directly.
    - Set 5 and clear 3?
    - Clear 8, and set 1 left indirectly by setting 5 and clearing 4.

Questions 16 and 17 pertain to the following problem: Set 319 to the far right and add 333 to it.

- The result is
    - 1,152
    - 652
    - 642
- How do you add the final 3 of 333?
    - Set 5 and clear 2.
    - Add it directly
    - Set 1 left indirectly

Questions 18 and 19 pertain to the following problem: Enter 2,802 on your abacus. Add 444.

- The result is
    - 3,246
    - 3,241
    - 7,242
- How do you add the first 4 of 444?
    - Set 5 and clear 1
    - Clear 6 and set 1 left
    - Use direct means

Questions 20 and 21 pertain to the following problem: Set 652 on your abacus. Add 505.

- The result is

    - 1,102
    - 1,152
    - 1,157
- How do you add the 0 of 505 on column 2 to the 5 of 652 on column 2?
    - Add 0 indirectly by clearing 5.
    - Clear 5 and set 1 left.
    - Make no change in the arrangement of beads on column 2._

Questions 22 and 23 pertain to the following problem: Set 384 on your abacus. Add 666.

- The result is
    - 950
    - 1,040
    - 1,050 ;
- When adding 384 plus 666, you
    - use direct means to add the first 6 of 666 on column 3
    - indirectly add 6 two times
    - indirectly set 1 left two times

Questions 24 and 25 pertain to the following problem: Enter 961 on your abacus. Add 777 to it.

- The result is
    - 733
    - 1,738
    - 1,638
- Which 7 of 777 can be added directly?
    - the 7 on column 3
    - the 7 on column 2
    - the 7 on column 1

Questions 26 and 27 pertain to the following problem: Set 186 on your abacus. Add 888 to it.

- The result is
    - 924
    - 1,024
    - 1,074
- How did you add the 8 of 888 on column 2 to the 8 of 1 86 in column 2?
    - Set 3, clear 5, and set 1 left.
    - Clear 2, and set 1 left indirectly by clearing 9 and setting 1 left.
    - Use direct means.

Questions 28-30 pertain to the following problem: Set 1,705 on your abacus. Add 999.

- The result is
    - 2,704
    - 2,604
    - 2,204
- Which 9 of 999 is entered by clearing 1 and setting 1 left?
    - the 9 on column 3
    - the 9 on column 2
    - the 9 on column 1
- What is your first step?

- Add 9 to 5 on column 1.
- Add 9 to 7 on coumn 3.
- Add 9 to 1 on column 4.

Explain the following problem. Be as thorough and detailed as possible.

- Add 6,508 plus 947. Explain where you set the problem, describe every step, and mention each method that you used and why.

Lesson 4: Adding Decimals

Lesson 1 explained how to set, clear, read, and add numbers. Lesson 2 not only distinguished between direct and indirect addition, but also explained why each method is applied. Lesson 3 applied the direct and indirect methods of addition to whole numbers with two or more digits. This final lesson in addition demonstrates how to add decimals. It particularly emphasizes where to place the decimal point.

Familiarizing yourself with the information in this lesson will enable you to add decimals using the abacus.

Objectives

After completing this lesson, you will be able to

- determine which unit mark serves as the decimal point in an addition problem
- directly or indirectly add any decimal number
- add sums of money

Determining the Decimal Point

Except for special rules of placement, adding decimals is done the same way you add whole numbers. The unit marks on the abacus represent decimal points. As explained in Lesson 1, these unit marks are the short vertical lines found after every third column when counting from right to left. First look at all the numbers being added and determine which number contains the most decimal places—that is, the most digits to the right of the decimal point. If that number has three or less decimal places, use the first unit mark from the right as the decimal point (between columns 4 and 3)—for example, 15.3, 35.78, and 4,041.912. If there are four, five, or six decimal places, use the second unit mark from the right as the decimal point (between columns 7 and 6)—for example, 1.8972, 27.56789, and 42.000275. If there are seven, eight, or nine decimal places, use the third unit mark from the right as the decimal point (between columns 10 and 9), and so forth.

The unit mark separates the whole-number digits from the decimal digits. The last whole-number digit of a number is found on the column immediately to the left of the appropriate unit mark. For example, add 1.025 plus 0.5 plus 42.0001 7. The number 42.00017 contains the most decimal places. Since it has five decimal places, the second unit mark from the right serves as your decimal point. Set 1.025 so that 1 is on the column immediately left of the second unit mark, which is column 7. Next enter 0.025 on the three columns immediately to the right of the second unit mark (i.e., columns 6, 5, and 4). There are no whole-number digits in 0.5. So add 5 to the column immediately right of the second unit mark, which is column 6. You now have 1.525 on columns 7, 6, 5, and 4. Finally, add 42.00017 by adding 42 to the two columns to the left of the second unit mark (i.e., columns 8 and 7). Then add 00017 on the five columns immediately to the right of the unit mark (i.e., columns 6, 5, 4, 3, and 2). The grand total is 43.52517.

Direct and Indirect Addition Practice With Decimals

This time, when adding 2.08 plus 19.007 plus 245.6, you find that 19.007 contains the most decimal places. Since it has three decimal places, the first unit mark from the right serves as the decimal point. Begin by setting 2.08. That is, set 2 on the column immediately to the left of the first unit mark (i.e., column 4). Then set 08 on the two columns immediately to the right of the first unit mark (i.e., columns 3 and 2). Column 1 is not yet used.

Next add 19.007 to 2.08. Enter the 1 of 19.007 on column 5, which is the second column to the left of the unit mark. Enter the 9 of 19.007 on column 4, which is immediately left of the unit mark, adding 9 to the number 2. Add 9 indirectly by clearing 1 and setting 1 left. Finally, enter 007 directly on the three columns immediately to the right of the first unit mark (i.e., columns 3, 2, and 1). The subtotal now is 21.087. Now add 245.6 to the subtotal. To do so, add 245 on the three columns immediately to the left of the first unit mark (i.e., columns 6, 5, and 4). Then add 6 on column 3. The answer is 266.687.

Adding Sums of Money

When adding sums of money, always use the first unit mark from the right as the decimal point. The dollar amounts appear to the left of the first unit mark, the cents to the right. Column 1 remains unused. For example, add $14.52 plus $3.07. Using the first unit mark from the right as the decimal point, set $14.52. To do so, set the 1 on column 5 and the 4 on column 4; then you have the decimal point. Set the 5 on column 3 and the 2 on column 2; column 1 is unused. Next add $3.07 by adding 3 to 4 on column 4. Add the 0 of $3.07 on column 3 by doing nothing. Add the 7 of $3.07 to the 2 of $14.52 on column 2. The answer, $17.59, appears on columns 5, 4, 3, and 2. Column 1 is unused.

Practice Exercise

Practice adding decimals on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 63.47 plus 98.75 plus 66.4 plus 8.8
- $1.98 plus $2.75 plus $3.15 plus $4.61
- 246.8 plus 156.974 plus 263.0676 plus 12.146
- 2.7433304 plus 4.42737 plus 2.5421
- 0.83 plus 0.495 plus 0.06
- $.85 plus $.99 plus $4.32
- 13.546 plus 8.57 plus 0.3779 plus 64.22
- 765.43 plus 21.98 plus 76.0012034 plus 17.102
- 1.002340607 plus 0.76001000123 plus 2.926 plus 0.425633
- $9,750.25 plus $231.06 plus $10.15 plus $9.73

Answers

- 237.42
- $12.49
- 678.9876
- 9.7128004
- 1.385
- $6.16
- 86.7139
- 880.5132034
- 5.11398360823
- $10,001.19

Summary

This lesson examined the addition of decimals. After explaining how to determine which unit mark serves as the decimal point, it described how to directly and indirectly add decimals. The lesson concluded with a brief explanation on adding sums of money.

Assignment 4

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

- If you add 1.02 plus 34.56000789 plus 10.1425, which unit mark would serve as the decimal point?
    - the first one from the right
    - the second one from the right
    - the third one from the right
- Suppose you want to add $1,998.50 plus $27.65 plus $1.75. Which unit mark would serve as the decimal point?
    - the first one from the left
    - the first one from the right
    - the second one from the right

Questions 3 and 4 pertain to the following problem: Suppose you want to add 28 cents plus 93 cents plus 46 cents.

- Which unit mark would you use as the decimal point?
    - the first one from the left
    - the first one from the right
    - the second one from the right
- The sum is
    - $16.70
    - $1.52
    - $1.67

Questions 5-7 pertain to the following problem: Add 12.34 plus 6.5.

- Which unit mark would you use as the decimal point?
    - the first one from the right
    - the second one from the right
    - the second one from the left
- On which columns would you set 12.34?
    - columns 4, 3, 2, and 1
    - columns 5, 4, 3, and 2
    - columns 13, 12, 11, and 10
- The 6 of 6.5 is added to the
    - 1 of 12.34
    - 2 of 12.34
    - 3 of 12.34
- If you added 17.15 plus 26.1978 plus 7.023, which unit mark would serve as the decimal point?
    - the first one from the right
    - the second one from the left
    - the second one from the right

- $246.81 plus $35.79 equals
    - $28.26
    - $282.06
    - $282.60

Explain the following problem. Be as thorough and detailed as possible.

- Work the problem 3.7692 plus 0.0419. Explain where you set the problem, describe every step and method that you used, and show how you determined where to place the decimal point.

Unit 2: Multiplication

The four lessons in Unit 1 explained direct and indirect methods of adding whole numbers and decimals on the abacus. The four lessons in Unit 2 explain how to multiply them.

Lesson 5: Multiplication of Whole Numbers

Lesson 5 introduces you to multiplication of whole numbers. This lesson defines the different parts of a multiplication problem and their proper placement. After a brief definition of key terms, the lesson provides instruction and practice in working wholenumber multiplication problems that contain one digit in the multiplier and any number of digits in the multiplicand. It also discusses the rules for positioning subproducts. Familiarizing yourself with the information in this lesson will enable you to perform multiplication using the abacus.

Objectives

After completing this lesson, you will be able to

- define multiplier, multiplicand, and product
- set up a whole-number multiplication problem containing one digit in the multiplier and any number of digits in the multiplicand
- multiply whole numbers with one digit in the multiplier and any number of digits in the multiplicand
- apply the rules for positioning subproducts

Definitions

Multiplication is a rapid form of addition. For instance, 8 plus 8 plus 8 plus 8 equals 32. This is the same as multiplying 4 times 8; except you arrive at the answer much quicker in multiplication — in one step rather than four. In multiplication, you use the basic principles of addition. Therefore, before proceeding further, be sure you can correctly do all of the operations explained so far. You must also know the multiplication tables thoroughly. Otherwise, the abacus will be of little value to you as a tool for solving multiplication problems. If you need additional help with the multiplication tables, consult your instructor—now.

Each part of a multiplication problem has a name. Since this course will refer to them constantly, you should learn them. The number being multiplied is the multiplicand. The number doing the multiplying is the multiplier. The answer is called the product. In the example 4 times 8 equals 32, 4 is the multiplier, 8 is the multiplicand, and the answer 32 is the product.

Setting Up Whole-Number Multiplication Problems

To set up an entire multiplication problem on the abacus, set the multiplier to the extreme left. To determine where to set the multiplicand, go to the far right end of the abacus. Count to the left as many columns as there are digits in the multiplier and the multiplicand combined, plus one more column for the process of multiplication. Set the first digit of the multiplicand (i.e., the far left digit of the multiplicand) on the last column counted off. The remaining digits will follow sequentially to the right. After the entire multiplicand has been entered, there will be a number of unused columns to its right. If the multiplicand has been placed correctly, the number of unused columns will equal one more than the number of digits in the multiplier.

When doing a multiplication problem on the abacus, multiply the multiplier times the multiplicand. Set the resulting subproduct to the right of the multiplicand, in a left-to-right direction. The final product will always appear at the extreme right end of the abacus with its final digit on column 1.

For example, consider the multiplication of 4 times 8. Set the multiplier 4 on column 13 (the far left column). To determine where to set the multiplicand 8, start with your index finger on column 1. Repeat the problem out loud, moving your finger one column to the left for each digit and for the word times. In this problem 4 times 8, place your index finger on column 1 and say, "four." Move your finger to the left to column 2 and say, "times," then to column 3 and say, "eight." Set the multiplicand 8 on column 3. Before going any farther, check to see how many unused columns there are to the right of the multiplicand 8. There are two. Since 2 is one more than the single digit in the multiplier 4, you know that the multiplicand 8 has been entered correctly. It is extremely important that the multiplicand be set correctly, because the length of the product is determined by its placement.

Working Whole-Number Multiplication Problems

Now you are ready to multiply 4 times 8. Since 4 times 8 is 32, set 32 on the two columns immediately right of the multiplicand 8. The 3 of 32 is set immediately to the right of the multiplicand. The 2 of 32 is set on the second column to the right of the multiplicand 8. (***Note: Subproducts are always set in a left-to-right direction***. That is why you first set the 3 of 32, and then the 2 of 32.) Clear the multiplicand 8. This leaves the product 32 at the extreme right end of the abacus.

Now see what would have happened if you had misplaced the multiplicand 8, setting it on column 4, rather than on column 3. Multiply 4 times 8, set 32 on the two columns immediately right of 8, and clear 8. Products always appear at the extreme right end of the abacus. Therefore, you would read the answer as 320, because you must include the 0 on column 1 as the final digit of the product. In this case, you can easily recognize your error. You see that you misplaced the multiplicand 8. In more complex problems, however, the error may go unrecognized, thereby resulting in a wrong product. So after setting the multiplicand, always ensure that the number of unused columns to its right equals one more than the number of digits in the multiplier. If it does not, then the multiplicand has been placed incorrectly and should be changed.

A word of caution when you find 0 in the multiplier, multiplicand, or both: The 0 counts as a digit; you must move your finger for each 0, as well. This time, try multiplying a one-digit multiplier times a three-digit multiplicand—for example, 7 times 492 equals 3,444. Set the multiplier 7 on column 13 (i.e., the far left column). To determine where to set the multiplicand 492, go to the right end of the abacus. Place your finger on column 1 and say, "seven." Then move your finger to the left to column 2 and say, "times." Move to column 3 as you say, "four." Move to column 4 as you say, "nine." Finally, move to column 5 and say, "two." Place 4 in column 5, 9 in column 4, and 2 in column 3. This leaves two unused columns to the right of the multiplicand, which is one more than the single digit in the multiplier 7.

When working with a multiplicand that has two or more digits, apply those digits in the reverse order of their occurrence—that is, from right to left. Multiply the multiplier by each digit in the multiplicand in their reverse order. First multiply the final digit of the multiplicand. Set the resulting product on the two columns immediately to its right. Then clear the final digit of the multiplicand. Shift your attention one column to the left to the next-to-last digit of the multiplicand.

In the case of 7 times 492, multiply 7 three times: first times 2, then times 9, and finally times 4. The first step is to multiply 7 times 2, which is 14. Set 14 on the two columns immediately right of 2. First set the 1 of 14, and then set the 4 of 14. Clear the 2 of 492, and shift your attention leftward to the 9. Since 7 times 9 is 63, set 63 on the two columns immediately right of 9. First set the 6 of 63, and then set the 3 of 63. The 3 of 63 will be set directly on the same column that contains the 1 of 14. Next clear the 9 of 492. The partial product is 644. Shift your attention leftward to the 4. Since 7 times 4 is 28, set it on the two columns immediately right of 4. First set the 2 of 28, and then set the 8 of 28. The 2 of 28 can be set directly; however, the 8 cannot. Since 8 is set on the column that already contains 6, set 8 indirectly by setting 3, clearing 5, and setting 1 left. Finally, clear the 4 of 492, and the job is done. The product 3,444 appears at the extreme right end of the abacus.

The Rules of Positioning Subproducts

Always think of a subproduct as having two digits. Simply place a zero before single-digit subproducts. For example, think of the subproduct of 4 times 2 as 08. The subproduct of 5 times 0 is 00. The subproduct of 0 times 0 is 00.

To apply this concept, multiply 3 times 271. Set the multiplier 3 on column 13 (i.e., the far left column). Set the multiplicand 271 on columns 5, 4, and 3. Determine this by placing your finger on column 1 while repeating the problem, "three times two seven one." Move your finger one column to the left each time you say a word. The 2 is placed in column 5, the 7 in column 4, and the 1 in column 3. There are two unused columns to the right of 271. The multiplicand has been placed correctly, because 2 is one more than the single digit in the multiplier 3. Now multiply the multiplier 3 times the far right digit of the multiplicand—that is, the 1 of 271. Set the product 03 on the two columns immediately right of 1. Pass over the first column as you say the 0 of 03. Then set 3 on the second column to the right of the multiplicand. Next clear the 1 of 271. Multiply the multiplier 3 times 7. Then set 21 directly on the two columns immediately right of 7. Clear the 7 of 271. The partial product is 213. Finally, multiply the multiplier 3 times 2. The product is 06, which you enter directly on the two columns immediately right of 2. Then clear the 2 of 271. At this point, the job is done. Clear the multiplier 3. The answer is 813.

When setting up an entire multiplication problem on the abacus, there should never be less than two unused columns between the multiplier and the multiplicand. Occasionally, there is not enough room on the abacus to set up the entire problem (e.g., in extremely lengthy problems). Nevertheless, the multiplicand should be set in all cases. The multiplier must be retained in memory, set on another abacus, or written on a piece of paper. After gaining proficiency, many people prefer not to set the multiplier in any problem but to retain it in memory. To help you increase your multiplication skills on the abacus, a useful exercise is to multiply 9 times the multiplicand 123,456,789. Then try multiplying this same multiplicand by 8, 7, 6, 5, 4, 3, and finally 2. Next try multiplying 987,654,321 by all the digits from the number 2 through 9.

Practice Exercise

Practice multiplying whole numbers on the abacus by working the following problems. Then compare your with those that follow the exercise.

- 8 times 46
- 4 times 28

- 6 times 89
- 2 times 854
- 9 times 385
- 7 times 789
- 5 times 9,674
- 3 times 2,387
- 6 times 4,386
- 5 times 41,826
- 7 times 45,829
- 4 times 57,267
- 8 times 631,492
- 3 times 217,762
- 2 times 975,831
- 6 times 912,458
- 8 times 728,651
- 3 times 2,137,689
- 7 times 1,348,765
- 9 times 1,027,688

Answers

- 368
- 112
- 534
- 1,708
- 3,465
- 5,523
- 48,370
- 7,161
- 26,316
- 209,130
- 320,803
- 229,068
- 5,051,936
- 653,286
- 1,951,662
- 5,474,748
- 5,829,208
- 6,413,067
- 9,441,355
- 9,249,192

Summary

This lesson defined the terms multiplier, multiplicand, and product. It specified how to set up and solve whole-number multiplication problems on the abacus. It also explained the rules of positioning subproducts.

Assignment 5

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

- Multiply 7 times 9. In working this problem, you set
    - the multiplier 7 on column 13 (the far left column)
    - the multiplicand 9 on column 1
    - the multiplicand 9 on column 3
- Multiply 6 times 8. After setting the multiplicand 8, how many unused columns will be to its right?
    - one
    - two
    - three

Questions 3 and 4 pertain to the following problem: 8 times 43.

- The answer is
    - 334
    - 344
    - 444
- The multiplicand 43 is set on columns
    - 3 and 2
    - 4 and 3
    - 5 and 4

Questions 5 and 6 pertain to the following problem: 3 times 32

- The answer is
    - 96
    - 906
    - 960
- How many unused columns will there be to the right after you set 32?
    - one
    - two
    - three

Questions 7 and 8 pertain to the following problem: 4 times 52.

- The answer is
    - 28
    - 208
    - 280
- Which statement is true?
    - The multiplicand is set on columns 5 and 4.
    - The first multiplication is 4 times_5.
    - The first multiplication is 4 times 2

Questions 9 and 10 pertain to the following problem: 3 times 27.

- The answer is
    - 81
    - 31

    - 621
- When you multiply 3 times the 2 of 27, the resulting subproduct is 06. The 6 of 06 is set on the column
    - immediately to the right of the 2 of 27
    - containing the 2 of 21
    - containing the 1 of 21

Questions 11-13 pertain to the following problem: 8 times 439.

- The answer is
    - 3,012
    - 3,152
    - 3,512
- You set 439 on columns
    - 4, 3, and 2
    - 5, 4, and 3
    - 6, 5, and 4
- When multiplying 8 times 439, after you finish multiplying 8 times the 3 of 439, the partial product is
    - 212
    - 312
    - 412
- Multiply 5 times 87,653. While working this problem,
    - you had to use the indirect method for adding 5
    - every product was entered directly
    - the result is 438,265

Explain the following problem. Be as thorough and detailed as possible.

- Work the problem 6 times 789. Explain where the multiplier and multiplicand are set and why. Also explain where each subproduct is set. After each round of multiplication, state the partial product.

Lesson 6: Multipliers With Two or More Digits

Lesson 5 explained how to multiply whole numbers with one digit in the multiplier and any number of digits in the multiplicand. This lesson covers how to multiply whole numbers with two or more digits in the multiplier and one digit in the multiplicand. Instruction and practice are designed to explain where each subproduct is set according to the rules of positioning, especially of overlapping positions. Familiarizing yourself with the information in this lesson will enable you to use the abacus to multiply with multipliers of two or more digits.

Objectives

After completing this lesson, you will be able to

- set up a multiplication problem with any number of digits in the multiplier and one digit in the multiplicand
- demonstrate the rules of positioning, especially the concept of overlapping positions
- Apply the rules of positioning to multiplication problems

Setting Multiplication Problems

Now you are ready to set a problem with a two-digit multiplier and a one-digit multiplicand—for example, 28 times 6 equals 168. Set the multiplier 28 on columns 13 and 12. To determine where to set the multiplicand 6, start with your index finger on column 1. As you repeat the problem out loud, move your finger one column to the left for each digit as well as for the word times. That is, place your index finger on column 1 and say, "two." Then move your finger to column 2 and say, "eight." Continue moving left to column 3 and say, "times." Then proceed to column 4 and say, "six." Set the multiplicand 6 on column 4. There are three unused columns to the right of the multiplicand because three is one more than the two digits in the multiplier 28.

The Rules of Positioning

Whenever the multiplier has two or more digits, apply them in their natural order of occurrence—that is, from left to right. In the problem 28 times 6 equals 168, first multiply the 2 of 28 times 6, then the 8 of 28 times 6. Multiply the first digit of the multiplier (i.e., the 2 of 28) times 6. The product 12 is set on the first two columns immediately to the right of 6. This is called the first position. After entering 12, do not move your right hand. Rather, let it remain resting on the 2 of 12. Now multiply the second digit of the multiplier (i.e., the 8 of 28) times 6, which produces 48. Set 48 in the second position—that is, the second and third columns to the right of 6. The 4 of 48 is set on the same column containing the 2 of 1 2, where your right hand is now resting. Set the 4 of 48 indirectly by setting 5 and clearing 1. Then move to the third column to the right of 6 (i.e., column 1) and set the 8 of 48 directly. Finally, clear the multiplicand 6. The answer is 168.

Overlapping Positions

Overlapping of positions is extremely important in abacus use. The first column of the second position overlaps the second column of the first position. Your right hand should remain on the overlapping column so that it will be in place to enter the next digit. A position always consists of two columns. The first position consists of the first two columns immediately to the right of the multiplicand. The second position consists of the second and third columns to the right. The third position consists of the third and fourth columns. The sixth position consists of the sixth and seventh columns, and so on.

There will be as many positions as there are digits in the multiplier. In the last example 28 times 6 equals 168, recall that you used two positions: the first to set 12 and the second to set 48. The multiplier 28 contains two digits; therefore there will be two multiplications of 6, resulting in two products, which require two positions. The product of the first multiplication (i.e., the first digit of the multiplier—the 2 of 28) times the multiplicand 6 is set in the first position. This consists of the first and second columns to the right of the multiplicand 6. The product of the second multiplication (i.e., the second digit of the multiplier—the 8 of 28) times the multiplicand 6 is set in the second position. That consists of the second and third columns to the right of the multiplicand 6. There are as many positions as the number of digits in the multiplier. If the multiplier contains three digits, there are three positions. If the multiplier contains six digits, there are six positions; if ten digits, then ten positions; and so on.

The product resulting from the multiplication of the first digit of the multiplier times the multiplicand is entered in the first position (i.e., the first and second columns to the right of the multiplicand). The product resulting from the multiplication of the second digit of the multiplier times the multiplicand is entered in the second position (i.e., the second and third columns to the right of the multiplicand). The product of the fifth digit times the multiplicand is entered in the fifth position (i.e., the fifth and sixth columns to the right of the multiplicand). The product of the seventh digit times the multiplicand is entered in the seventh position (i.e., the seventh and eighth columns to the right of the multiplicand), and so on. In other words: first digit, first position, first and second columns to the right of the multiplicand. Second digit, second position, second and third columns to the right of the multiplicand. Third digit, third position, third and fourth columns to the right of the multiplicand. Tenth digit, tenth position, tenth and eleventh columns to the right of the multiplicand, and so on. This enables you to predict the two columns that any given product will be entered on.

For instance, imagine yourself multiplying 987,654 times 3 without physically working the problem on the abacus. The product of the third digit of the multiplier (i.e., the 7 of 987,654) times the multiplicand 3 is set in the third position. That is, it is set on the third and fourth columns to the right of the multiplicand 3. You could also think of it as the third digit, third position, third and fourth columns to the right of the multiplicand. The product of the sixth digit of the multiplier (i.e., the 4 of 987,654) times the multiplicand 3 is entered in the sixth position. That is, it is set on the sixth and seventh columns to the right of the multiplicand. In other words, the sixth digit, sixth position, sixth and seventh columns to the right of the multiplicand. Remembering the positions concept is especially useful when you lose your place or get interrupted while working a problem. You can easily resume your work without redoing the entire problem.

Applying the Positions Concept to Multiplication Problems

Now work another problem that applies the positions concept: 789 times 5 equals 3,945. Set the multiplier 789 on columns 13, 12, and 11. To determine where to set the multiplicand 5, go to the far right. Count to the left five columns as you say, "seven eight nine times five." Then set 5 on column 5. This leaves four unused columns to the right of the multiplicand. This is correct because four is one more than the three digits in the multiplier 789. In this problem, first multiply the 7 of 789 times 5, then the 8 of 789 times 5, and finally the 9 of 789 times 5.

Begin by multiplying the first digit of the multiplier (i.e., the 7 of 789) times the multiplicand 5. Enter 35 in the first position (i.e., the two columns immediately right of 5). After you have set the 5 of 35, do not move your right hand. Let it rest on the 5. This will be the overlapping column.

Next multiply the second digit of the multiplier (i.e., the 8 of 789) times the multiplicand 5. Enter 40 on the second position (i.e., the second and third columns to the right of the multiplicand). The 4 of 40 is entered on the overlapping column, adding it to the 5 of 35. After setting 4, move one column to the right to account for the 0 of 40. Since you are working with a 0, do not change the arrangement of the beads on that column. Instead, move to the right and let your right hand rest on the 0 column (i.e., column 2), because it is the column that will overlap the next position. Your right hand will then be in place to enter the first digit of the next position.

Finally, multiply the third digit of the multiplier (i.e., the 9 of 789) times 5. Set 45 on the third position (i.e., the third and fourth columns to the right of the multiplicand). Clear the multiplicand 5. The answer is 3,945.

For the last example, multiply 419 times 5. Set the multiplier 419 to the far left on columns 13, 12, and 11. To determine where to set the multiplicand 5, count from right to left five columns. That is, count three columns for the multiplier 419, one more column for times, and another column for the multiplicand 5. Set 5 on column 5. There are four unused columns to the right of the multiplicand. This is correct because 4 is one more than the three digits in the multiplier 419. In this problem, first multiply the 4 of 419 times 5, then the 1 of 419 times 5, and then the 9 of 419 times 5.

Begin by multiplying the first digit of the multiplier (i.e., the 4 of 419) times the multiplicand 5. Since 4 times 5 is 20 (which gets set in the first position), enter 2 on the first column to the right of the multiplicand and 0 on the second column to the right of the multiplicand. Your right hand should remain resting on the zero column, because it is the overlapping column.

Next multiply the second digit of the multiplier (i.e., the 1 of 419) times the multiplicand 5. Enter 05 on the second position (i.e., the second and third columns to the right of 5). Since every subproduct must contain two digits, place a 0 before 5 and think of it as 05.

This is necessary because a position always consists of two columns. Moreover, the first column of a position overlaps the second column of the preceding position. In this problem, the 0 of 05 overlaps the 0 of 20. As you say the 0 of 05, your right hand should lightly touch the overlapping column (i.e., the second column to the right of the multiplicand). Move to the right, and set 5 on the third column to the right of the multiplicand. Next multiply the third digit of the multiplier (i.e., the 9 of 419) times the multiplicand 5. Then set 45 on the third position (i.e., the third and fourth columns to the right of the multiplicand). The 4 of 45 is set on the same column containing the 5 of 05. Move to the right and set the 5 of 45. Finally, clear the multiplicand 5. The answer is 2,095.

To increase your proficiency with multipliers having two or more digits, try multiplying 123 times 9, then times each digit from 8 to 2. Then try multiplying 987 times those same numbers. To verify your result, reverse the multiplier and the multiplicand. Your answer should be the same.

Practice Exercise

Practice multiplying whole numbers on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 84 times 7
- 46 times 5
- 31 times 7
- 751 times 9
- 218 times 4
- 893 times 7
- 729 times 2
- 841 times 6
- 6,534 times 8
- 8,643 times 3
- 1,492 times 6
- 4,251 times 4
- 73,498 times 8
- 628,537 times 9
- Rework the exercises given at the end of Lesson 5 reversing the multipliers and the multiplicands.

Answers

- 588
- 230
- 217
- 6,759
- 872
- 6,251
- 1,458
- 5,046
- 52,272
- 25,929
- 8,952
- 17,004
- 587,984

- 5,656,833

Summary

This lesson described how to set multiplication problems. It explained the rules of positioning. Finally, it examined how to apply this information to multiplication problems.

Assignment 6

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

Questions 1 and 2 pertain to the following problem: 73 times 4.

- The answer is
    - 242
    - 282
    - 292
- The product of 3 times 4 is set
    - directly
    - after the product of 7 times 4
    - on the two columns immediately right of the multiplicand 4

Questions 3 and 4 pertain to the following problem: 76 times 9.

- The answer is
    - 634
    - 654
    - 684
- When multiplying 76 times 9
    - the first multiplication is to multiply the 7 of 76 times 9
    - the product of the second multiplication of 9 is set indirectly
    - the product of 6 times 9 is set on the second and third columns to the right of the multiplicand
- When multiplying 1 times 5, you set the product as
    - 01
    - 5
    - 05
- A position is made up of
    - a number of columns as determined by the size of the multiplier
    - a number of columns as determined by the size of the multiplicand
    - two columns
- There will be as many positions as the number of digits in the
    - multiplier
    - multiplicand
    - multiplier and the multiplicand combined
- If you multiply 1 23 times 4, the product of 3 times 4 will be set
    - in the first position
    - in the third position
    - on the third and fourth columns to the right of the multiplicand

Questions 9-11 pertain to the following problem: 978 times 4.

- The answer is
    - 3,812
    - 3,912
    - 4,912
- How many unused columns will there be to the right of the multiplicand 4?
    - two
    - three
    - four
- The product of 8 times 4
    - is set in the first position
    - is set in the third position
    - has both of its digits set indirectly

Questions 12 and 13 pertain to the following problem: 49 times 9.

- The answer is
    - 441
    - 449
    - 341
- When you set the product of 9 times 9, you must enter
    - 8 indirectly
    - 1 indirectly
    - 81 on the second position
- If you multiply 1,829 times 7,
    - the last digit of the product can be found on column 1
    - there will be four positions
    - the first time you multiply 7, you multiply it by the 1 of 1,829

Explain the following problem. Be as thorough and detailed as possible.

- Work the problem 943 times 7. Explain where the multiplier and multiplicand are set and why. Also explain where each subproduct is set according to the rules of positioning. After each round of multiplication, state the partial product.

Lesson 7: Multiplying Any Number of Digits

Lesson 5 explained how to multiply whole numbers with one digit in the multiplier and any number of digits in the multiplicand. Lesson 6 covered how to multiply with two or more digits in the multiplier and one digit in the multiplicand. This lesson describes how to multiply whole numbers that contain any number of digits in the multiplier or multiplicand. It places special emphasis on the treatment of zeros when they occur in the multiplier and/or the multiplicand. Familiarizing yourself with the information in this lesson will enable you to multiply any number of digits on the abacus.

Objectives

After completing this lesson, you will be able to

- multiply whole numbers with any number of digits in the multiplier, the multiplicand, or both
- work a multiplication problem where zeros occur in the middle of the multiplier, the multiplicand, or both
- work a multiplication problem with a multiplicand, a multiplier, or both ending with one or more zeros

Multiplying Whole Numbers With Any Number of Digits

Consider a problem involving a three-digit multiplier and a two-digit multiplicand—for example, 765 times 32 equals 24,480. When working problems where both the multiplier and the multiplicand contain two or more digits, combine the principles you have already learned. Apply the digits of the multiplier in their natural order of occurrence. However, apply the digits of the multiplicand in their reverse order of occurrence.

Multiply all of the digits of the multiplier (from left to right) times the final digit of the multiplicand. Set the resulting subproducts on their appropriate positions. Clear the final digit of the multiplicand; then shift your attention leftward to the next-to-last digit of the multiplicand. Then repeat the entire process, multiplying the multiplier times the second-to-the-last digit of the multiplicand. If the multiplicand contains more than two digits, repeat the process. That is, multiply the multiplier times the third-to-the-last digit of the multiplicand, then the fourth-to-the-last digit of the multiplicand, and so on. In other words, multiply the far right digit of the multiplicand by all the digits in the multiplier (from left to right). You can then remove the far right digit of the multiplicand, freeing up that column, to place the next product.

Now apply this procedure to the problem 765 times 32 equals 24,480. Set the multiplier 765 on columns 13, 1 2, and 11. To determine where to set the multiplicand 32, count six columns from right to left as you say, "seven six five times three two." Set 32 on columns 6 and 5. Four unused columns are to the right of the multiplicand 32 because 4 is one more than the three digits in the multiplier 765. In this problem, multiply 765 times the 2 of 32. Then multiply 765 times the 3 of 32.

Begin by multiplying the 7 of 765 times the 2 of 32: 7 times 2 is 14, which you set in the first position (i.e., the first and second columns immediately to the right of the multiplicand). Next multiply the 6 of 765 times 2. Set 12 in the second position (i.e., the second and third columns to the right of the multiplicand) so that the 1 of 12 overlaps the 4 of 14. Next multiply the 5 of 765 times 2. This produces 10, which you set in the third position (i.e., the third and fourth columns to the right of the multiplicand) so that the 1 of 10 overlaps the 2 of 12. You are now done multiplying the 2 of 32, so clear 2. At this point, the partial product shows 1,530.

Now turn your attention to the 3 of 32. Multiply 765 times the 3 of 32. Begin by multiplying the 7 of 765 times the 3 of 32. Set 21 on the first position (i.e., the first and second columns immediately to the right of the multiplicand). Set 2 on the first column to the right of the multiplicand. Then set 1 on the second column to the right of the multiplicand, adding it to the 1 already on that column.

Then multiply the 6 of 765 times 3. The product is 1 8, which you set on the second position (i.e., the second and third columns to the right of the multiplicand). The 1 of 18 is set on the second column to the right of the multiplicand, adding 1 to the 2 already there. Set the 8 of 18 on the third column to the right of the multiplicand, adding 8 to the 5 already on that column.

Next multiply the 5 of 765 times 3: 5 times 3 is 1 5, which is set on the third position (i.e., the third and fourth columns to the right of the multiplicand). Set the 1 of 15 on the third column to the right of the multiplicand, thereby adding 1 to 3. Then set the 5 of 15 on the fourth column to the right of the multiplicand, adding 5 to 3. Finally, clear the 3 of 32, and the job is done. Your answer is

24,480. Zero is the final digit of the product because every product falls to the extreme right end of the abacus with its final digit on the far right column (i.e., column 1).

This time consider a problem having three digits in both the multiplier and the multiplicand—for example, 321 times 718. Set the multiplier 321 on columns 13, 12, and 11. To determine where to set the multiplicand 718, begin at the far right, then count to the left seven columns as you say, "three two one times seven one eight." Set 718 on columns 7, 6, and 5. There will be four unused columns to the right of the multiplicand because four is one more than the three digits in the multiplier 321. In this problem, multiply 321 times the 8 of 718, then 321 times the 1 of 718, and finally 321 times the 7 of 718.

Begin by multiplying 321 times the 8 of 718. Multiply the 3 of 321 times the 8 of 718. Since 3 times 8 is 24, set 24 on the first position (i.e., the two columns immediately to the right of the multiplicand). Next multiply the 2 of 321 times 8, producing 16. Set 16 on the second position (i.e., the second and third columns to the right of the multiplicand) so that the 1 of 16 overlaps the 4 of 24. Move to the right and set the 6 of 16. Next multiply the 1 of 321 times 8.

Since 1 times 8 is 08, set 08 on the third position (i.e., the third and fourth columns to the right of the multiplicand). Ensure that 0 is set on the overlapping column that contains 6. Move to the right and set the 8 of 08. Now clear the 8 of 718. At this point, the partial product shows 2,568.

You are now ready to multiply 321 times the 1 of 718. Multiply the 3 of 321 times the 1 of 718. Since 3 times 1 is 03, set 03 on the two columns immediately to the right of the multiplicand. This is the first position relative to 1. Set the 0 of 03 on the first column to the right of the multiplicand. Set the 3 of 03 on the second column to the right of the multiplicand, which already contains the number 2. Next multiply the 2 of 321 times the 1 of 718.

Since 2 times 1 is 02, set 02 on the second position (i.e., the second and third columns to the right of the multiplicand). Set 0 on the second column to the right of the multiplicand, adding 0 to 5. Set the 2 of 02 on the third column to the right of the multiplicand, adding 2 to that 5. Next multiply the 1 of 321 times the 1 of 718.

Since 1 times 1 is 01, set 01 on the third position (i.e., the third and fourth columns to the right of the multiplicand). Set the 0 of 01 on the third column to the right of the multiplicand by doing nothing. Set the 1 of 01 on the fourth column to the right of the multiplicand, adding 1 to 6. You have now finished multiplying the 1 of 718, so clear it. The partial product shows 5,778. Now shift your attention to the 7 of 718. Multiply 321 times 7. Begin by multiplying the 3 of 321 times the 7 of 718. Since 3 times 7 is 21, set 21 on the two columns immediately right of the multiplicand, the first position relative to 7. Then multiply the 2 of 321 times 7. Since 2 times 7 is 14, set 14 on the second position (i.e., the second and third columns to the right of the multiplicand). Set the 1 of 14 on the second column to the right of the multiplicand, adding 1 to the 1 already on that column. Set the 4 of 14 on the third column to the right of the multiplicand, adding 4 to the 5 already on that column. Next multiply the 1 of 321 times the 7 of 718. Set 07 on the third position, the third and fourth columns to the right of the multiplicand. Set the 0 of 07 on the third column to the right of the multiplicand, adding 0 to 9 by doing nothing. Set the 7 of 07 on the fourth column to the right of the multiplicand by adding 7 to the 7 already there. You are now finished multiplying the 7 of 718, so clear 7. Your answer is 230,478.

If you have difficulty remembering how to apply the digits of the multiplier and the multiplicand, start with their outer extremities as they appear on the abacus and gradually work toward the middle. Begin with the far left digit of the multiplier and the far right digit of the multiplicand. Then gradually work your way toward the middle as if you are headed toward the other number.

Zeros in the Middle of the Multiplier or Multiplicand

Whenever zeros occur in the middle of a multiplier or multiplicand, treat them the same as any other digit as far as the positions concept is concerned. Suppose you want to multiply 4,002 times 3. Set 4,002 to the far left, and set 3 on column 6. Since 4 times 3 is 12, set 12 in the first position (i.e., the first and second columns to the right of the 3). Since 0 times 3 is 00, it is accounted for in the second position (i.e., the second and third columns to the right of the 3). Multiply 0 times 3 again, and account for the 00 in the third position (i.e., the third and fourth columns to the right of the 3). Since 2 times 3 is 06, set 06 in the fourth position (i.e., the fourth and fifth columns to the right of the 3). Ensure that 6 is set on the second column of that position. Now clear the multiplicand 3. The answer is 12,006.

Multiplicands or Multipliers Ending in One or More Zeros

When setting up a multiplicand that ends with one or more zeros, be sure to take the zeros into account when counting off columns. For example, multiply 250 times 4,000. Set 250 to the far left, and set the 4 of 4,000 on column 8. The 000 of 4,000 on columns 7, 6, and 5 automatically fall to the right of 4. Then multiply the 2 of 250 times 4, producing 08. Set 08 in the first position (i.e., the first and second columns to the right of the 4). Ensure that 8 is set on the second column of that position. Next multiply the 5 of 250 times 4.

Since 5 times 4 is 20, set 20 in the second position (i.e., the second and third columns to the right of the multiplicand). Enter the 2 of 20 on the overlapping column, which now contains 8. You need not take any steps to multiply the 0 of 250 times 4, because 0 automatically takes care of itself. Finally, clear the 4 of 4,000. The answer is 1,000,000. Notice the unit marks, or commas, in relationship to the 1 of 1,000,000 as they help you read your answer.

When setting up an entire multiplication problem on the abacus, there should never be less than two unused columns between the multiplier and the multiplicand. Occasionally, there is not enough room on the abacus to set up the entire problem (e.g., in extremely lengthy problems). Nevertheless, the multiplicand should be set in all cases. The multiplier can be retained in memory, set on another abacus, or written on a piece of paper. Once proficient, many abacus users prefer not to set the multiplier in any problem but to simply retain it in memory.

Practice Exercise

Practice multiplying whole numbers on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 67 times 52
- 15 times 39
- 40 times 1 8
- 83 times 46
- 92 times 70
- 725 times 91
- 63 times 458
- 908 times 76
- 174 times 25
- 89 times 360
- 519 times 23
- 46 times 795

- 782 times 51
- 307 times 86
- 917 times 342
- 356 times 180
- 7,092 times 54
- 8,759 times 130
- 231 times 8,607
- 1,805 times 469

Answers

- 3,484
- 585
- 720
- 3,818
- 6,440
- 65,975
- 28,854
- 69,008
- 4,350
- 32,040
- 11,937
- 36,570
- 39,882
- 26,402
- 313,614
- 64,080
- 382,968
- 1,138,670
- 1,988,217
- 846,545

Summary

This lesson examined how to multiply whole numbers with any number of digits. It explained how to handle multiplication when zeros appear in the middle of the multiplier or multiplicand. It also gave examples of multiplication problems that involve multiplicands or multipliers ending in one or more zeros.

Assignment 7

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two, or even three choices are needed to answer the question correctly.

Questions 1-3 pertain to the following problem: 679 times 43.

- The answer is
    - 28,197
    - 29,192
    - 29,197
- When setting up the problem, you place the multiplicand 43 on columns
    - 4 and 3

    - 5 and 4
    - 6 and 5
- In working the problem, your first multiplication will be
    - 6 times 4
    - 9 times 3
    - 6 times 3
- Reverse the problem in Question 1 and multiply 43 times 679. Your result should be the same. After setting 679 on the appropriate columns, how many unused columns are to its right?
    - two
    - three
    - four

Questions 5 and 6 pertain to the following problem: Multiply 73,425 times an unknown multiplicand.

- After setting the multiplicand on the appropriate columns, how many unused columns are to its right?
    - six
    - five
    - This cannot be answered because you do not know the multiplicand.
- The products from multiplying the 2 of 73,425 times any digit of the multiplicand are entered in the
    - fifth position
    - fourth position
    - second position

Questions 7-10 pertain to the following problem: 77 times 77.

- The answer is
    - 5,429
    - 5,924
    - 5,929
- When working the problem, it is necessary to use indirect means to add 9
    - once
    - twice
    - three times
- How often must you use indirect means to add 4?
    - never
    - once
    - twice
- When setting up the problem, place the multiplicand 77 on columns
    - 6 and 5
    - 4 and 3
    - 5 and 4

Questions 11-13 pertain to the following problem: 47 times 351.

- The answer is
    - 16,442
    - 16,497
    - 15,497

- What is your last multiplication?
    - 4 times 1
    - 7 times 1
    - 7 times 3
- When working the problem, the only time you set a number indirectly is when you set the
    - 2 of 21
    - 2 of 20
    - 3 of 35

Questions 14 and 15 pertain to the following problem: 24 times 13.

- The answer is
    - 312
    - 212
    - 412
- This problem requires you to
    - set every product by direct means
    - set the products of 4 times 3 and 4 times 1 in the second position
    - set the products of 4 times 3 and 4 times 1 in the first position
- 44 times 25 equals
    - 110
    - 1,110
    - 1,100
- 225 times 44 equals
    - 9,090
    - 9,900
    - 9,720
- 30,013 times 3 equals
    - 90,039
    - 9,039
    - 90,309
- 4,004 times 24 equals
    - 96,816
    - 96,096
    - 816,816
- 25 times 8,000 equals
    - 2,000,000
    - 200,000
    - 20,000
- 101 times 101 equals
    - 11,111
    - 101,101
    - 10,201
- 101 times 11 equals
    - 1,111
    - 1,102
    - 1,201
- 101 times 110 equals
    - 10,101

    - 11,110
    - 11,201
- Add the numbers 1 through 9. Multiply the answer times the result you get by adding the numbers 10 through 20. The product of this multiplication is
    - 7,425
    - 7,415
    - 7,495
- Add the single-digit even numbers 2, 4, 6, and 8. Multiply the result times the sum of the single-digit odd numbers 1, 3, 5, 7, and 9. The product of this multiplication is
    - 410
    - 500
    - 505

Questions 26 and 27 pertain to the following problem: 37 times 1,926.

- The answer is
    - 71,212
    - 71,262
    - 71,252
- When working the problem, the last time you used indirect means to enter a product,
    - it required you to clear 3 and set 1 left
    - it required you to set 2, clear 5, and set 1 left
    - it was when you multiplied the 3 of 37 times the 9 of 1,926

Questions 28-30 pertain to the following problem: 402 times 420.

- The answer is
    - 16,840
    - 168,840
    - 16,884
- When you multiply the 2 of 402 times the 2 of 420, you enter the product
    - by using indirect means for adding 4
    - on the third position
    - after you accounted for the product of the 0 of 402 times the 2 of 420
- Which of the following statements apply when you work the problem
    - You must set the multiplicand 420 on columns 7, 6, and 5
    - You must set the multiplicand 420 on columns 6, 5, and 4
    - Your first actual step is to multiply the 4 of 402 times the 2 of 420.

Explain the following problem. Be as thorough and detailed as possible.

- Work the problem 204 times 25. Explain where the multiplier and multiplicand are set. Also, explain where each subproduct is set according to the rules of positioning. After each round of multiplication, state the partial product.

Lesson 8: Multiplying Decimals and a Series of Numbers

Lesson 5 explained how to multiply whole numbers with one digit in the multiplier and any number of digits in the multiplicand. Lesson 6 covered multiplication with two or more digits in the multiplier and one digit in the multiplicand. Lesson 7 described how to multiply whole numbers that contain any number of digits in the multiplier or multiplicand. This final lesson in Unit 2 explains how to multiply decimals—particularly, where to place the decimal point in the product. It also explains how to multiply a series of numbers.

Familiarizing yourself with the information in this lesson will enable you to use the abacus to multiply decimals as well as a series of numbers.

Objectives

After completing this lesson, you will be able to

- accurately place the decimal point when multiplying decimals
- multiply a series of numbers

Accurately Placing the Decimal Point When Multiplying Decimals

The procedure for multiplying decimals is exactly the same as that for multiplying whole numbers. Ignore the decimal point when setting up the problem and when doing the actual multiplication. After completing the multiplication, point off as many decimal places in the product, from right to left, as there are decimal places in the multiplier and the multiplicand combined. The terms to count off and to point off mean the same thing. However, to count off refers to whole numbers; to point off pertains to decimals.

For example, suppose you multiply 4.75 times 9.3. Ignore the decimal points when setting up and working the problem. Therefore, treat the multiplier 4.75 like the whole number 475. Treat the multiplicand 9.3 as the whole number 93. Set 475 on columns 13, 12, and 11; then set 93 on columns 6 and 5. When you multiply 475 times 93, the product 44175 appears on columns 5, 4, 3, 2, and 1. Altogether, there are three decimal places: The first two decimal places are the two digits to the right of the decimal point in 4.75. The third decimal place is the one digit to the right of the decimal point in 9.3. So where do you place the decimal point in the product? Point off from right to left three decimal places in 44175. The answer is 44.175.

For the next few examples, use the same numbers, but shift the decimal points. In other words, the product remains 44175, but decide where to place the decimal point in each product. Begin by multiplying 47.5 times 93. There is one decimal place in 47.5 (i.e., one digit to the right of the decimal point). However, there is none in the whole number 93, because it does not contain a decimal point. So there are no digits to the right of the decimal point. Therefore, you have a total of one decimal place. Point off from right to left one decimal place in the product. The answer is 4,417.5.

This time, multiply 0.475 times 0.93. In 0.475, there are three decimal places (i.e., three digits to the right of the decimal point). In 0.93, there are two decimal places (i.e., two digits to the right of the decimal point). Altogether, you have five decimal places. Therefore, point off five decimal places from right to left. The answer is 0.44175. This time, multiply 0.231 times 0.24. Ignore the decimal points when setting up the problem and doing the actual multiplication. Treat the multiplier 0.231 like the whole number 231. Set it on columns 13, 12, and 11. Treat the multiplicand 0.24 as the whole number 24. Set it on columns 6 and 5. When you multiply 231 times 24, the product 5544 appears on columns 4, 3, 2, and 1. Where should the decimal point be placed in 5544? In 0.231, there are three decimal places (i.e., three digits to the right of the decimal point). In 0.24, there are two decimal places (i.e., two digits to the right of the decimal point). Altogether, you have five decimal places. However, your answer 5544 contains only four digits. So place 0 to the left of the far left digit so that you can point off the five decimal places. This gives you a final answer of 0.05544.

The next example shows how to deal with pure decimals. A pure decimal contains no whole numbers but has zeros immediately to the right of the decimal point, such as 0.08 and 0.00028. Whenever the multiplier or the multiplicand is a pure decimal, ignore the zeros when setting up the problem and doing the actual multiplication. They will automatically take care of themselves. You must take them into consideration, however, when counting the number of decimal places in the multiplier and the multiplicand combined to determine the number of decimal places to point off in the final answer.

To multiply 0.000475 times 0.0093, treat the multiplier 0.000475 as the whole number 475. Set it on columns 13, 12, and 11. Treat the multiplicand 0.0093 as the whole number 93. Set it on columns 6 and 5. After multiplying, you arrive at the product 44175. Count all of the decimal places in the problem, including the zeros. In 0.000475, there are six decimal places (i.e., six digits to the right of the decimal point). In 0.0093, there are four decimal places (i.e., four digits to the right of the decimal point). Altogether you have ten decimal places in the product. Thus, point off ten places in the product. Your answer 44175 contains only five digits, however. Therefore, place five zeros to the left of the far left digit in your answer so that you can point off the ten decimal places. Your answer is 0.0000044175.

Multiplying a Series of Numbers

Occasionally you may need to multiply three or more numbers; for example, to find the volume of an object (length times width times height), the cost per square or cubic unit, or the tax on several items. Using the abacus, you can multiply a series of numbers without resetting the problem for each multiplication. The product still appears at the extreme right end of the abacus. But you set only the first number of the series as the multiplicand, retaining the other numbers in memory. Or you can set them on another abacus or jot them down on a piece of paper.

To determine where to set the multiplicand, begin at the far right end of the abacus. Count to the left one column for each digit in the problem plus one column for each multiplication. After setting the multiplicand, multiply the first multiplier times the multiplicand. Then set the resulting product or products to the right of the multiplicand and clear the multiplicand. Next multiply the second multiplier times the first product to get the second product. Multiply the third multiplier times the second product to get the third product. Continue multiplying the next multiplier times the last product in this manner.

For example, multiply 2 times 4 times 8 times 16. Set only the first number as the multiplicand —in this case, 2. To determine where to set the multiplicand 2, go to the far right end, and count eight columns to the left as you say, "two times four times eight times sixteen." Set the multiplicand 2 on column 8. Now multiply the first multiplier 4 times the multiplicand 2. Set the resulting product 08 on the two columns immediately right of the multiplicand 2. Clear the multiplicand 2. Next multiply the second multiplier 8 times this 08. Set 64 on the two columns immediately right of 8. Clear the multiplier 8. Finally, multiply the third multiplier 16 times 64. Multiply 16 times the 4 of 64. Set 04 in the first position to the right of 4 and 24 in the second position. Then clear the 4 of 64. Next multiply 16 times the 6 of 64. Set 06 in the first position to the right of 6, and 36 in the second position—adding 3 to 6 and 6 to 6. Then clear the 6 of 64. The product is 1,024.

Practice Exercise

Practice multiplying decimals and a series of numbers on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 6.78 times 0.4
- 0.030 times 863.2
- 87 times 0.32 times 0.67

- 468 times 2.17 times 0.9
- 0.9306 times 0.85
- 0.00089 times 0.092
- 7.9436 times 0.52
- 26.4 times 97.85
- 0.9458 times 0.306
- 32.795 times 4.8
- Rework these ten problems, reversing the multiplier and the multiplicand.

Answers

- 2.712
- 25.8960
- 18.6528
- 914.004
- 0.791010
- 0.00008188
- 4.130672
- 2583.240
- 0.2894148
- 157.4160

Summary

This lesson discussed the importance of accurately placing the decimal point when multiplying decimals. It also described how to multiply a series of numbers.

Assignment 8

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

- 7.4 times 0.6 equals
  - 44.4
  - 0.444
  - 4.44

Questions 2 and 3 pertain to the following problem: 0.00084 times 0.0007.

- The answer is
  - 0.000000588
  - 0.000000000588
  - 0.000588
- In working the problem,
  - you set only 84 and 7 on the abacus when setting up the problem
  - you set 84 on columns 10 and 9, then set 7 on column 4
  - you set 84 on columns 13 and 12, then set on column 4
- If a multiplier containing three decimal places is multiplied times a multiplicand containing four decimal places, how many decimal places will be in the product?
  - three
  - seven
  - four

- 2,551 times 0.37 equals
    - 94.387
    - 943.87J
    - 0.94387
- 1.898 times 181.2 equals
    - 343.9176
    - 34.39176
    - 3,439.176
- If you multiply 12.798 times a certain number, how many decimal places will be in the product?
    - five
    - four
    - This cannot be answered because you do not know the nature of the multiplicand.

Questions 8 and 9 pertain to the following problem: 8 times 9 times 67.

- The answer is
    - 4,824
    - 4,724
    - 4,834
- In solving the problem,
    - you set the multiplicand 8 on column 5
    - you set 8 on column 6
    - your first multiplication is 9 times 8
- Consider the problem 807 times 32 equals 25,824. Although you will need to work this problem out, do not submit your explanation for the multiplication. Instead, using these same numbers, shift the decimal points to find the answers to the following problems. Explain how you determined where to place each decimal point in each answer
    - 0.807 times 3.2
    - 0.000807 times 32
    - 80.7 times 0.32

Unit 3: Subtraction

Unit 1 explained direct and indirect ways to add whole numbers and decimals on the abacus. Unit 2 explained how to multiply them. The two lessons in Unit 3 explain direct and indirect ways to subtract whole numbers and decimals.

Lesson 9: Subtracting Whole Numbers

Lesson 9 distinguishes between the direct and indirect ways to subtract whole numbers and explains why each method is applied. Familiarizing yourself with the information in this lesson will enable you to use the abacus to subtract whole numbers.

Objectives

After completing this lesson, you will be able to

- define minuend, subtrahend, and remainder
- directly subtract whole numbers

- indirectly subtract whole numbers

Definitions

Just as with addition, subtraction on the abacus is done from left to right. Numbers can be subtracted either directly or indirectly. If you have mastered addition you should have no difficulty with subtraction, because it is simply the reverse of addition. Instead of setting or adding two or more numbers, in subtraction you remove or clear them. This means that you will have to reverse the indirect methods of addition and your finger motions.

The various parts of a subtraction problem are given names. The number from which another number is subtracted is called the minuend. The number being subtracted is the subtrahend. The resulting difference or answer is called the remainder. In the problem 987 minus 362, the minuend is 987, the subtrahend is 362, and their difference 625 is the remainder.

Direct Method of Subtracting Whole Numbers

Try working the problem 987 minus 362 on the abacus. Begin by setting 987 on columns 3, 2, and 1. To subtract 362, subtract the 3 of 362 from the 9 of 987 on column 3. You can do this directly by clearing three lower beads with the right index finger. Next subtract the 6 of 362 from the 8 of 987 on column 2 directly. That is, slide the five bead up and one lower bead down with a continuous motion of the right index finger. Finally, subtract the 2 of 362 from the 7 of 987 on column 1 directly by clearing two lower beads with the right index finger. This leaves a remainder of 625. Suppose you were to continue this process and subtract 15 from 625. Since 1 5 is a two-digit number, it must be subtracted from columns 2 and 1. Subtract the 1 of 1 5 from the 2 of 625 on column 2. Do this directly by sliding one lower bead down. Then moving to the right, subtract the 5 of 1 5 from the 5 of 625 on column 1. Do this directly by moving the bead above the separation bar away from the separation bar. The remainder is 610.

Indirect Method of Subtracting Whole Numbers

Numbers can be subtracted by either direct or indirect means. Always begin by trying to subtract the digit directly. When you cannot do so, however, use the indirect method by clearing either a 5 or a 10. In so doing, you will subtract more than you wanted to subtract. So return the difference between the number you wanted to subtract and either 5 or 10.

For example, subtract 4 from 7. Begin by setting 7 on column 1. You cannot subtract 4 directly from 7. However, you can clear the five bead. By doing this, you subtract more than you wanted to subtract. That is, you wanted to subtract 4, but you have subtracted 5. So how much will you have to return? The difference between 4 and 5, which is 1. Return 1 by setting 1. Therefore, to subtract 4 from 7, set 1 and clear 5.

This time, set 26 on columns 2 and 1 and subtract 8 from it. Subtract 8 from the 6 of 26 because they are both units digits. You cannot subtract 8 directly from 6. But you can subtract 10 by clearing 1 left on column 2. In so doing, you subtract 2 more than you wanted to subtract. That is, you wanted to remove only 8, but you cleared 10. So return the 2 by setting 2 on column 1. Thus to subtract 8 from 26, subtract 8 from 6 indirectly by clearing 1 left and setting 2. Therefore, 26 minus 8 equals 18.

Now subtract 9 from 34. Set 34 on columns 2 and 1. Since 9 cannot be subtracted directly from the 4 of 34 on column 1, subtract 10 by clearing 1 left on column 2. In doing this, you over-subtract; that is, you subtracted one more than you wanted to subtract. So return 1 to column 1. However, this cannot be added directly to the 4 of 34, because all four lower beads on column 1 are in use. Instead, return 1 indirectly by setting 5 and clearing 4. Therefore, to subtract 9 from 34, subtract 9 from the 4 of 34 by clearing 1 left with the left hand. Then in column 1, set 5 and clear 4 with a continuous downward sweep of the right index finger; 34 minus 9 equals 25.

Now work through one complete problem showing each step as you go along. Set 12,345 on columns 5, 4, 3, 2, and 1. Subtract 9,753 from this. Since 9,753 contains four digits, it must be subtracted from columns 4, 3, 2, and 1. Begin by subtracting the 9 of 9,753 from the 2 of 12,345 in column 4. Since 9 cannot be subtracted directly from 2, clear 1 from the column immediately left of 2 (i.e., column 5). Set 1 on the column containing 2 (i.e., column 4). The abacus now shows 3,345. Now you are ready to subtract the 7 of 9,753 from the 3 of 3,345 in column 3.

Since 7 cannot be subtracted directly from 3, clear 1 left from column 4. Then set 5 and clear 2 on column 3. At this point, the abacus shows 2,645. Next subtract the 5 of 9,753 from the 4 of 2,645 on column 2. This must be done indirectly by clearing 1 left from column 3 and setting 5 on column 2. The abacus now shows 2,595. Finally, subtract the 3 of 9,753 in column 1 from the 5 of 2,595 on column 1. To subtract 3 from 5, you must set 2 and clear 5. The remainder is 2,592.

To check your work, simply add the subtrahend to the remainder, which is still set on the abacus. If the result is the same as your original minuend, your subtraction is correct. In this case, add the subtrahend 9,753 to the remainder 2,592, which is still set on the abacus. The result is 12,345. This is the minuend, or the number with which you started. Therefore, your subtraction is correct.

Remember, when subtracting a number indirectly, clear either a 5 or a 10. Then set the difference between the number you wanted to subtract and either 5 or 10. This difference is called the complement. Until you feel confident about subtracting numbers indirectly, you may wish to consult the following chart, which lists all the indirect methods of subtraction.

To Subtract By Indirect Method

Subtract by Indirect Method
1 set 4, clear 5
1 clear 1 left, set 9
2 set 3, clear 5
2 clear 1 left, set 8
3 set 2, clear 5
3 clear 1 left, set 7
4 set 1, clear 5
4 clear 1 left, set 6
5 clear 1 left, set 5
6 clear 1 left, set 5, clear 1
6 clear 1 left, set 4
7 clear 1 left, set 5, clear 2
7 clear 1 left, set 3
8 clear 1 left, set 5, clear 3
8 clear 1 left, set 2
9 clear 1 left, set 5, clear 4
9 clear 1 left, set 1

Recall that "clear 1 left" means clear the digit 1 from the column immediately left of the column that you are working on. If that column contains a 5, clear 1 left indirectly by setting 4 and clearing 5. For instance, to subtract 3 from 51, clear 1 left and set 7. Because column 2 contains a 5, set 4 and clear 5 on column 2, then set 7 on column 1. The remainder is 48.

If the column or columns immediately to the left contain a zero or zeros, change each zero to a 9 and continue moving left until you find a column from which 1 can be removed. For example, if you want to subtract 8 from 3,006, clear 1 left and set 2. To clear 1 left, change the zeros in columns 2 and 3 to 9s. Then clear 1 from the 3 on column 4. Finally, set 2 on column 1 to complete the indirect subtraction of 8. The remainder is 2,998.

Suppose you were subtracting 8 from 5,006. Change the two zeros to 9s. Next subtract 1 from 5 indirectly by setting 4 and clearing 5 on column 4. Finally, set 2 on column 1 to complete the indirect subtraction of 8. The remainder is 4,998.

To gain skill in subtraction, reverse the problems given for addition. For example, set 45 on your abacus, and then subtract 1, 2, 3, 4, 5, 6, 7, 8, and 9 from it in any sequence. Your answer will be zero. Or begin by setting 4,995. Then subtract 111, 222, 333, 444, 555, 666, 777, 888, and 999 from it in any sequence. Your answer will be zero.

For an excellent subtraction exercise, set 5,050 on your abacus. Then subtract all the numbers from 1 up to and including 100, in any sequence. Your answer will be zero.

Practice Exercise

Practice subtracting whole numbers on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 2,651 minus 864
- 24,324 minus 1,342
- 7,556 minus 3,124
- 84 minus 45 minus 7
- 46,807 minus 40,206
- 74,635 minus 7,463
- 3,164 minus 1,598
- 9,238 minus 2,765
- 6,234 minus 3,889
- 50,523 minus 797
- 8,403 minus 6,71 6
- 7,961 minus 2,962
- 6,535 minus 2,164
- 4,471 minus 997
- 70,450 minus 4,011
- 27,034 minus 7,038
- 62,953 minus 5,716
- 5,307 minus 3,737
- 81,706 minus 5,522
- 1,847 minus 1,599

Answers

- 1,787
- 22,982
- 4,432
- 32
- 6,601
- 67,172
- 1,566
- 6,473
- 2,345
- 49,726
- 1,687
- 4,999
- 4,371
- 3,474
- 66,439
- 19,996
- 57,237
- 1,570
- 76,184
- 248

Summary

This lesson defined the terms for subtraction. It also demonstrated how to use the direct and indirect methods for subtracting whole numbers.

Assignment 9

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

Questions 1 and 2 pertain to the following problem: 789 minus 135.

- This problem
    - requires the use of one indirect method of subtraction
    - requires the use of two indirect methods of subtraction
    - equals 654
- When working this problem,
    - your first step is to subtract the 5 of 135 on column 1 from the 9 of 789 on column 1
    - you work from left to right
    - your first step is to subtract the 1 of 135 on column 3 from the 7 of 789 on column 3

Questions 3 and 4 pertain to the following problem: 23,948 minus 1,627.

- This problem
    - equals 22,321
    - requires the use of at least one indirect method of subtraction
    - equals 22,221
- What number is subtracted from the 4 of 23,948?
    - the 2 of 1,627
    - the 7 of 1,627
    - the 6 of 1,627

Questions 5-7 pertain to the following problem: 905 minus 111.

- The remainder is
    - 894
    - 784
    - 794
- To subtract the 1 of 111 on column 2 from the 0 of 905 on column 2, you must
    - subtract it directly
    - set 4 and clear 5
    - clear 1 left and set 9
- The first time you used indirect means to subtract a number,
    - it was to subtract 1 from 0
    - it was to subtract 1 from 5
    - it required you to set 4 and clear 5

Questions 8 and 9 pertain to the following problem: 618 minus 222.

- The remainder is
    - 296
    - 396
    - 496
- When you subtracted the 2 of 222 on column 3 from the 6 of 618 on column 3, you
    - did it directly

- set 3 and cleared 5
- set 2 and cleared 5

Questions 10 and 11 pertain to the following problem: 2,729 minus 333.

- The remainder is
    - 2,396
    - 2,346
    - 1,346
- When solving 2,729 minus 333,
    - you subtract 3 directly from 9
    - your first step requires you to set 2 and clear 5
    - you clear 1 left and set 7 to subtract the 3 on column 2
- 804 minus 444 equals
    - 360
    - 350
    - 460
- 703 minus 555 equals
    - 143
    - 168
    - 148

Questions 14 and 15 pertain to the following problem: 9,825 minus 666.

- The remainder is
    - 3,165
    - 9,159
    - 4,154
- When solving 9,825 minus 666,
    - your first step is to subtract 6 from 9
    - your first step is to subtract 6 from 5
    - you subtract the 6 on column 2 by clearing 1 left, setting 5, and clearing 1

Questions 16 and 17 pertain to the following problem: 841 minus 777.

- The remainder is
    - 64
    - 74
    - 164
- When solving 841 minus 777,
    - you subtract the first 7 by indirect means
    - your first step is to subtract the 7 in column 1
    - you subtract the 7 on column 2 by clearing left, setting 5, and clearing 2

Questions 18 and 19 pertain to the following problem: 52,864 minus 888.

- This problem
    - equals 51,976
    - requires the subtraction on column 2 to be done indirectly
    - equals 51,926
- The first time you subtract 8, it is
    - subtracted from the 2 of 52,864

    - subtracted from the 8 of 52,864
    - the only time in which 8 is subtracted directly

Questions 20 and 21 pertain to the following problem: 3,489 minus 999.

- This problem
    - equals 2,490
    - requires the last subtraction to be done directly
    - equals 2,390
- The first time you use indirect means to subtract 9 is to subtract
    - 9 from 4
    - 9 from 8
    - 9 from 9

Questions 22 and 23 pertain to the following problem: 50,036 minus 38.

- The remainder is
    - 99,998
    - 69,998
    - 49,998
- When solving 50,036 minus 38, you subtract
    - 3 from 3
    - 3 indirectly
    - 8 indirectly

Explain the following problem. Be as thorough and detailed as possible.

- Work the problem 684,007 minus 5,308. Explain each step and each method that you use.

Lesson 10: Subtracting Decimals

Lesson 9 distinguished between the direct and indirect ways of subtracting whole numbers. This lesson explains the direct and indirect methods of subtracting decimals. Particular attention is paid to accurately placing the decimal point and subtracting money.

Familiarizing yourself with the information in this lesson will enable you to use the abacus to subtract decimals.

Objectives

After completing this lesson, you will be able to

- determine the decimal point when subtracting directly or indirectly
- subtract amounts of money

Determining the Decimal Point

Except for setting up the problem, the process of subtracting decimals is the same as that for subtracting whole numbers. Just as when adding decimals, ensure that the decimal point of the number containing the most decimal places is set to coincide with the appropriate unit mark. As in addition, if the number with the most digits to the right of the decimal point has three or less decimal places, use the first unit mark from the right as the decimal point. If there are four, five, or six decimal places, use the second unit mark from the right as the decimal point. If there are seven, eight, or nine decimal places, use the third unit mark from the right as the decimal point, and so forth. The unit mark separates the whole-number digits from the decimal digits. The last whole-number digit of a number is found on the column immediately to the left of the appropriate unit mark.

To solve 27.1 minus 0.005, use the first unit mark from the right as the decimal point. Why? The number that has the most decimal places is 0.005, with three digits to the right of the decimal. First set 27.1 by setting the 2 on column 5, the 7 on column 4. This places the decimal point at the first unit mark. Set the 1 of 27.1 on column 3. Subtract 0.005. Subtract 0 from 1 (by not changing the arrangement of the beads) on column 3. Subtract 0 from 0 on column 2. Then subtract the 5 of 0.005 from the 0 on column 1 by clearing 1 left and setting 5. Because there is a 0 on column 2, set 9 and clear 1 left on column 3. Remember to set 5 on column 1 to complete the subtraction of clearing 1 left and setting 5. The remainder is 27.095.

Now subtract 7.98 from 27.095. Use the first unit mark from the right as the decimal point. Subtract 7 from 7 directly on column 4. Subtract 9 from 0 indirectly on column 3 by clearing 1 left and setting 1. When you try to clear 1 left on column 4, you find you have a 0. Set 9 on column 4, and clear 1 left from column 5. Do not forget to set 1 on column 3 to complete the subtraction of clearing 1 left and setting 1. Subtract the 8 of 7.98 from the 9 of 27.095, directly, on column 2. The final remainder is 19.115.

If you want to solve 125.12 minus 17.123456, use the second unit mark from the right as the decimal point. Since 17.123456 has six digits to the right of the decimal, it contains the most decimal places. First set 125.12 on columns 9, 8, 7, 6, and 5. Subtract 17.123456. Subtract 1 from 2, directly, on column 8. Subtract 7 from 5, indirectly, by clearing 1 left and setting 3 on column 7. Subtract 1 from 1, directly, on column 6. Subtract 2 from 2, directly, on column 5. Subtract 3 from 0, on column 4. Do this indirectly by clearing 1 left and setting 7. When you try to clear 1 left, Os are on both columns 5 and 6. Set 9s on columns 5 and 6, then clear 1 left directly from column 7. Do not forget to set 7 on column 4 to complete the subtraction of clearing 1 left and setting 7. Subtract 4 from 0 on column 3, indirectly, by clearing 1 left and setting 6. Subtract 5 from 0, indirectly, on column 2 by clearing 1 left and setting 5. Finally, subtract 6 from 0 on column 1, indirectly, by clearing 1 left and setting 4. When you try to clear 1 left, you have a 5. Clear 1 left indirectly by setting 4 and clearing 5. Do not forget to set 4 on column 1 to complete the subtraction of clearing 1 left and setting 4. The remainder is 107.996544.

Subtracting Amounts of Money

Just as you did with addition, use the first unit mark from the right as the decimal point when subtracting money. All of the dollar amounts fall to the left of the first unit mark; cents to the right. Column 1 is not used. For example, subtract $16.49 from $24.86. Using the first unit mark as the decimal point, set $24.86 on columns 5, 4, 3, and 2. Column 1 is not used. Subtract $16.49. First subtract 1 from 2, directly, on column 5. On column 4, subtract 6 from 4, indirectly, by clearing 1 left, setting 5, and clearing 1. Subtract 4 from 8 on column 3 by setting 1 and clearing 5. Subtract 9 from 6 on column 2, indirectly, by clearing 1 left and setting 1. The remainder is $8.37.

Practice Exercise

Practice subtracting decimals on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 1.4783 minus 0.6947
- 37.041 minus 8.784
- 0.6029032 minus 0.36
- 0.09263 minus 0.089
- $23.10 minus $4.87
- 736 minus 68 plus 74 minus 95 plus 736

- $79.84 minus $15.96
- 6.49 minus 1.742
- 0.74910 minus 0.6204
- 67.458 minus 3.081

Answers

- 0.7836
- 28.257
- 0.2429032
- 0.00363
- $18.23
- 1,383
- $63.88
- 4.748
- 0.12870 or 0.1287
- 64.377

Summary

This lesson explained how to determine which unit mark serves as the decimal point when subtracting directly or indirectly. It also described how to subtract amounts of money.

Assignment 10

Complete the following problems with the most appropriate answer(s). Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

- When subtracting 54.07725 from 375.8799057, which unit mark serves as the decimal point?
    - the first one from the right
    - the second one from the right
    - the third one from the right
- When subtracting 11.2 from 47,532.08, which unit mark serves as the decimal point?
    - the first one from the right
    - the second one from the right
    - the third one from the right
- When subtracting 365 from 56,378.0017, which unit mark serves as the decimal point?
    - the first one from the right
    - the second one from the right
    - the third one from the right

Questions 4 and 5 pertain to the following problem: Set $14.26 on your abacus and subtract $.08 from it.

- To solve this problem, you subtract 8 from
    - 6
    - 2
    - 4
- When solving this problem,
    - the 4 of $14.26 is set on column 4
    - column 1 is not used

- the 6 of $14.26 is set on column 1

- 2.0089 minus 0.5473
  - 1.4516
  - 4616
  - 1.4616
- 384.74 minus 18.96
  - 385.88
  - 365.78
  - 361.78
- .82341 minus 0.69408 equals
  - 0.12933
  - 0.12943
  - 0.11933
- 75.002 minus 8.006 equals
  - 66.992
  - 66.892
  - 996

Explain the following problem. Be as thorough and detailed as possible.

- Work the problem 24.00869 minus 7.309. Explain where the problem is set and why. Describe each step, and explain each method that you used.

Unit 4: Division

Unit 1 explained direct and indirect methods to add whole numbers and decimals on the abacus. Unit 2 explained how to multiply them. Unit 3 explained direct and indirect methods to subtract whole numbers and decimals. The five lessons in Unit 4 explain various aspects of dividing whole numbers, decimals, and a series of numbers.

Lesson 11: Short Division and Remainders

Lesson 11 introduces short division of whole numbers on the abacus. This lesson describes short division, its different parts, and its basic setup on the abacus. It examines the rules of quotient figure placement. Finally, the lesson provides instruction and practice in working short-division problems, with or without remainders. Familiarizing yourself with the information in this lesson will enable you to use the abacus to perform short-division problems, including those that involve remainders.

Objectives

After completing this lesson, you will be able to

- describe the components and basic setup of shortdivision problems
- apply the rules of quotient figure placement
- work whole-number short-division problems without remainders
- work whole-number short-division problems with remainders

The Components of Short-Division Problems

Division is a rapid form of subtraction. By dividing, you can quickly and accurately tell how many times a number can be subtracted from another number. For instance, 378 divided by 3 equals 126. This is the same as saying that 3 can be subtracted from 378, 126 times.

Definitions

Just as with all the other processes of arithmetic, the various parts of a division problem have names. The number being divided is called the dividend. The number by which the dividend is divided is called the divisor. The answer is the quotient. Short division is the process by which a dividend containing any number of digits is divided by a divisor containing only one digit. In the short-division problem 378 divided by 3 equals 126, 378 is the dividend, 3 is the divisor, and 126 is the quotient.

Setup

When setting up a division problem on the abacus, the divisor is set to the far left and the dividend to the far right. The quotient is formed to the left of the dividend. It is a good practice to leave at least four unused columns between the divisor and the dividend. If there is not enough room on the abacus to do this, the divisor should be retained in memory, set on another abacus, or written on a piece of paper.

Quotient Figure Placement in Short-Division Problems

To work a short-division problem, divide the first digit of the dividend by the one-digit divisor. If the first digit of the dividend is smaller than the divisor, however, divide the first two digits of the dividend by the one-digit divisor. This results in a quotient figure that is either set on the column immediately left of the dividend or on the second column left of the dividend. This separates it from the first digit of the dividend by one unused column.

The rule for determining where to place the quotient figure is as follows: If you divide the first digit of the dividend only by the first digit of the divisor, you are dividing a one-digit number by a one-digit number. Refer to this as SAME. Why? Both the number of digits being divided and the number of digits doing the dividing have the same number of digits. In other words, the first digit of the dividend and the first digit of the divisor (and in some cases, the only digit of the divisor) have the same number of digits.

If the number of digits is the **SAME, SKIP** a column to the left of the dividend before placing the quotient figure. This means that the quotient figure is separated from the first digit of the dividend by one unused column (i.e., on the second column to the left of the dividend). This course will refer to this procedure from now on as **SAME, SKIP**.

If you divide the first two digits of the dividend by the first digit of the divisor, you are dividing a two-digit number by a one-digit number. Henceforth, this course will refer to this procedure as **NOT THE SAME**. Why? The number of digits being divided is different than the number of digits doing the dividing. In other words, the first two digits of the dividend and the first digit of the divisor (and in some cases, the only digit of the divisor) do not have the same number of digits. If the number of digits is not the same, set the quotient figure immediately left of the dividend. In other words, do not skip a column. Since it is **NOT THE SAME, DO NOT SKIP**.

Set the quotient figure on the appropriate column. Then multiply it times the divisor. Subtract the resulting product from the dividend in keeping with the rules of positioning that you learned in the multiplication lessons. After completing a round of division, multiplication, and subtraction, repeat the process with the new dividend to develop the next quotient figure.

Working Short-Division Problems Without Remainders

Now solve the problem 378 divided by 3 equals 126. Set the divisor 3 on column 13 and the dividend 378 on columns 3, 2, and 1. You can divide the first digit of the dividend, the 3 of 378, by the divisor 3. Since 3 divided by 3 equals 1, the first quotient figure is 1. Where should you set this 1 ? It is possible to divide the first digit of the dividend, the 3 of 378, by the first digit of the divisor 3. In this case, the 3 is the only digit of the divisor. Therefore, both the number of digits being divided (the first digit of the dividend) and the number of digits doing the dividing (the first and only digit of the divisor) have the same number of digits. According to the rule for quotient figure placement, **SAME, SKIP**. This means, skip the column immediately left of 378. Then set the first quotient figure 1 on the second column to the left, separating it from the dividend by one unused column. That is, place the first quotient figure on column 5. Next multiply the divisor 3 times the first quotient figure 1. In so doing, the divisor serves as the multiplier, and the quotient serves as the multiplicand.

Since 3 times 1 is 03, subtract 03 from the first position—that is, from the first two columns to the right of 1. Remember that subproducts always have two digits. Subtract 0 from 0 and 3 from 3. At this point, you have completed the first round of division, multiplication, and subtraction. The new dividend is 78.

You are now ready for the second round of division. You can divide the first digit of the dividend, the 7 of 78, by the divisor 3. This results in a second quotient figure of 2. On which column should this 2 be set? It is possible to divide the first digit of the dividend, the 7 of 78, by the first digit of the divisor 3. In this case, 3 is the only digit of the divisor. Therefore, both the number of digits being divided (i.e., the first digit of the dividend) and the number of digits doing the dividing (i.e., the first and only digit of the divisor) have the same number of digits. According to the rule for quotient figure placement, **SAME, SKIP**. That is, skip one column. Set the second quotient figure 2 on the second column to the left of 78, separating it from the dividend by one unused column. Place this second quotient figure on column 4. Next multiply the divisor 3 times the second quotient figure 2.

Since 3 times 2 is 06, subtract 06 from 07 in the first position to the right of 2. The dividend portion of the abacus now shows 18. At this point, you have completed the second round of division, multiplication, and subtraction.

You are now ready for the third round of division. You cannot divide the 1 of 18, the first digit of the dividend, by the divisor 3. Therefore, divide the two digits of the dividend, the 1 and the 8, by the divisor 3.
Since 18 divided by 3 is 6, the third quotient figure will be 6. Where should this 6 be set? You cannot divide the first digit of the dividend, the 1 of 18, by the first digit of the divisor 3. In this case, 3 is the only digit of the divisor. You divided the two digits of the dividend, the 1 and the 8, by the first (and only) digit of the divisor 3. Therefore, you divided a two-digit number by a one-digit number. Refer to this as **NOT THE SAME**. Why? The number of digits being divided (i.e., the two digits of the dividend) and the number of digits doing the dividing (i.e., the first, and in this case the only, digit of the divisor) do not have the same number of digits. According to the rule for quotient figure placement, **NOT THE SAME, DO NOT SKIP**. Set the quotient figure 6 immediately left of the dividend 18, on column 3. Now multiply the divisor 3 times this third quotient figure 6. Since 3 times 6 is 18, subtract 18 from 18. This clears the dividend.

You are now ready to read the quotient. Go to the far right end of the abacus. Count one column to the left for each digit in the divisor, plus one more column for the process of division. Everything to the left of that is the quotient. In this case, count off two columns: one for the single digit in the divisor 3, plus one additional column for the process of division. The answer is 126.

To verify the answer, multiply the divisor 3 times the quotient 126. The product you obtain is the dividend with which you started, 378. In doing this multiplication, the divisor serves as the multiplier and the quotient as the multiplicand. Because the quotient is already in the proper position for a multiplicand, it is not necessary to rearrange it.

To fix the rule for quotient figure placement in your mind, simply remember to compare the number of digits in the dividend being divided, with the number of digits in the divisor doing the dividing. If they are the **SAME, SKIP**. If they are **NOT THE SAME, DO NOT SKIP**.

For a second example of short division, work the problem 28,456 divided by 4. Set the divisor 4 on column 13. Set the dividend 28,456 on columns 5, 4, 3, 2, and 1. You cannot divide the first digit of the dividend, the 2 of 28,456, by the divisor 4. Therefore, divide the first two digits of the dividend, the 2 and the 8 of 28,456, by 4. Since 28 divided by 4 equals 7, the first quotient figure is 7. Set this first quotient figure 7 immediately to the left of the dividend on column 6. Why? You divided two digits of the dividend by one digit of the divisor. **NOT THE SAME, DO NOT SKIP**. Next multiply the divisor 4 times this first quotient figure 7. Since 4 times 7 is 28, subtract 28 from 28. This completes the first round of division, multiplication, and subtraction, leaving 456 as your new dividend.

You are now ready for the second round of division. Divide the first digit of the dividend, the 4 of 456, by the divisor 4. Since 4 divided by 4 is 1, set this second quotient figure 1 on the second column to the left of 456 (i.e., column 5). Why? You divided one digit of the dividend by one digit of the divisor. **SAME, SKIP**. Multiply the divisor 4 times the second quotient figure 1. The product is 04. Subtract 04 from 04. At this point, the new dividend is 56. You have completed the second round of division, multiplication, and subtraction. You are now ready for the third round of division. Divide the first digit of the dividend, the 5 of 56, by the divisor 4. Set the resulting quotient figure 1 on the second column to the left of 56 (i.e., column 4) because you divided one digit of the dividend by one digit of the divisor. **SAME, SKIP**. Then multiply the divisor 4 times the third quotient figure 1, and subtract 04 from 05. To subtract 4 from 5, set 1 and clear 5. The new dividend is 16. You have now completed the third round of division, multiplication, and subtraction.

You are now ready for the fourth round of division. You cannot divide the first digit of the dividend, the 1 of 16, by the divisor 4. Therefore, divide the two digits of the dividend, the 1 and the 6, by 4. Since 16 divided by 4 equals 4, the fourth quotient figure is 4. Set this quotient figure immediately left of 16 on column 3. Since you divided two digits of the dividend by one digit of the divisor, it is **NOT THE SAME, DO NOT SKIP**. Now multiply the divisor 4 times the fourth quotient figure 4, and subtract 16 from 16. This clears the dividend. Count off two unused columns at the right end of the abacus: one for the divisor 4, and one more for the process of division. The answer is 7,114. Verify this answer by multiplying it times the divisor 4. The product will be 28,456.

Working Short-Division Problems With Remainders

Sometimes a dividend cannot be divided evenly by a divisor. When this occurs, there will be a remainder. The following example shows how remainders are handled on the abacus: Divide 735 by 4. Set the divisor 4 on column 1 3. Set the dividend 735 on columns 3, 2, and 1. Divide the first digit of the dividend, the 7 of 735, by the divisor 4. This produces the first quotient figure 1. Set this first quotient figure on the second column to the left of 7 (i.e., column 5). Since you divided one digit of the dividend by one digit of the divisor, it is the **SAME, SKIP**. Next multiply the divisor 4 times the first quotient figure 1, and subtract 04 from 07. This leaves 335 as the new dividend. You have now completed the first round of division, multiplication, and subtraction.

You are now ready for the second round of division. You cannot divide the first digit of the dividend, the 3 of 335, by the divisor 4. Therefore, divide the first two digits of the dividend, the 3 and the 3, by 4. Since 33 divided by 4 equals 8, set this second quotient figure 8 immediately left of the dividend on column 4. You divided two digits of the dividend by one digit of the divisor; therefore, **NOT THE SAME, DO NOT SKIP**. Next multiply the divisor 4 times this second quotient figure 8. Subtract 32 from 33, leaving 1 5 as the new dividend. You have now completed the second round of division, multiplication, and subtraction.

You are now ready for the third round of division. You cannot divide the first digit of the dividend, the 1 of 1 5, by the divisor 4. Therefore, divide the two digits of the dividend, the 1 and the 5, by 4. Since 15 divided by 4 equals 3, set this third quotient figure 3 immediately left on column 3. You divided two digits of the dividend by one digit of the divisor. Therefore, **NOT THE SAME, DO NOT SKIP**. Multiply the divisor 4 times this third quotient figure 3. Subtract 1 2 from 15. This leaves 3 in the dividend on column 1.

Since the dividend 3 cannot be divided by the divisor 4, your remainder is 3. To determine the final answer, count off two columns at the right end of the abacus: one column for the divisor 4 and a second column for the process of division. Everything to the left of these two columns is the whole-number portion of the quotient 183. Everything to the right is the remainder 3. The final answer is 183 with a remainder of 3.

To verify the answer, multiply the divisor 4 times the whole-number portion of the quotient, 183 —but not times the remainder. Multiply 4 times 183 in the usual manner. After multiplying 4 times 183, add the remainder 3 to your product. The answer 735 is the same as the dividend 735. This proves that the quotient 183 with the remainder of 3 is correct.

Suppose you needed to carry the quotient out to several decimal places. In most cases, this is not possible. Because the dividend is set to the far right end of the abacus, there may not be enough columns. You would run out of room. Therefore, whenever you need to carry the quotient out to any number of decimal places, enter the dividend three, six, or nine columns farther to the left, depending on the number of places you want to carry out the quotient. The last digit of the dividend falls on the column immediately to the left of a unit mark. Call this column the unit column because it contains the final (or units digit) of the dividend.

To apply this to the previous example, 735 divided by 4, set 735 immediately to the left of the first unit mark on columns 6, 5, and 4. Why? You want to carry the quotient out to three decimal places. Therefore, you actually set the dividend as 735.000. Do the division as already described until you reach the point where the quotient shows 183 with a remainder of 3. This 3 appears on column 4. Divide 30 by 4 and set the resulting quotient figure 7 immediately left of 30. You divided a two-digit dividend by a one-digit divisor. Therefore, **NOT THE SAME, DO NOT SKIP**. Next multiply 4 times 7. Then subtract 28 from 30, leaving 2 in the dividend. Now divide 20 by 4 and set the resulting quotient figure 5 immediately to the left of 20. You divided a two-digit dividend by a one-digit divisor. Therefore, **NOT THE SAME, DO NOT SKIP**. Multiply 4 times 5. Then subtract 20 from 20, thereby clearing the dividend.

To determine the length of the quotient in earlier examples, you began at the extreme right end of the abacus. You counted to the left one column for each digit in the divisor plus one more column for the process of division. Everything to the left of the last column counted off was the quotient; everything to the right was the remainder. Do essentially the same thing in this problem; except your starting point is the first unit mark, rather than the far right end of the abacus. So, from the first unit mark, count to the left one column for the divisor 4, plus one more column for the process of division. Place a decimal point between columns 5 and 6. Everything to the left is the whole-number portion of the quotient 183. Everything to the right is the decimal portion 0.75. Thus, your answer is 183.75.

For practice in short division, you can make up your own exercises by dividing any number you think of by a one-digit number. To verify your result, simply multiply the divisor times the quotient and add the remainder. As your product, you should obtain the very same dividend with which you started. Try using your phone number, address, zip code, or year of birth as dividends and divide them by the numbers 1 through 9.

For additional practice in short division, set the dividends 123,456,789 and 987,654,321 to the far right. Divide them, in turn, by all the single digits from 1 to 9.

Before going on to long division, be sure you thoroughly understand short division and can comfortably work any problem involving a one-digit divisor. Also, be sure that you can apply the rules of quotient figure placement.

Practice Exercise

Practice dividing whole numbers on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 1,251 divided by 3
- 8,370 divided by 9
- 7,494 divided by 6
- 94,422 divided by 4 (carried out to one decimal place)
- 3,205 divided by 5
- 9,848 divided by 8
- 59,211 divided by 2 (carried out to one decimal place)
- 4,208 divided by 8
- 5,614 divided by 7
- 923,856 divided by 3
- 41,630 divided by 5
- 975,312 divided by 9
- 674,807 divided by 7
- 864,534 divided by 6
- 161,838 divided by 9
- 95,227,797 divided by 7
- 649,608 divided by 8
- 23,138,618 divided by 4 (carried out to one decimal place)
- 72,924,477 divided by 3
- 70,060,091 divided by 5 (carried out to one decimal place)

Answers

- 417
- 930
- 1,249
- 23,605.5
- 641
- 1,231
- 29,605.5
- 526
- 802
- 307,952
- 8,326
- 108,368
- 96,401
- 144,089
- 17,982
- 13,603,971
- 81,201
- 5,784,654.5

- 24,308,159
- 14,012,018.2

Summary

This lesson defined the terms used for division and demonstrated how to set up short-division problems. It explained how to apply the rules of quotient figure placement. It also described how to work short-division problems, with or without remainders.

Assignment 11

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

- When setting up a whole division problem on the abacus,
    - the dividend must be set to the extreme right end of the abacus
    - the divisor must be set to the extreme right end of the abacus
    - it is recommended that there be at least four unused columns between the divisor and the dividend; otherwise, the divisor should be retained in memory, set on another abacus, or written on a piece of paper
- The quotient of a division problem is formed
    - to the left of the divisor
    - to the right of the dividend
    - to the left of the dividend

Questions 3-6 pertain to the following problem: 828 divided by 3.

- The final quotient is
    - 278
    - 276
    - 268
- Your first step is to divide the 8 of 828 by the divisor 3. After the first complete round of division, multiplication, and subtraction, what number is left in the dividend?
    - 228
    - 128
    - 218
- The second quotient figure is set
    - immediately to the left of the dividend
    - on the second column to the left of the dividend
    - on the second column to the left of the divisor
- When the divisor 3 is multiplied times this second quotient figure, it produces the product
    - 24
    - 21
    - 27

Questions 7-10 pertain to the following problem: 412 divided by 4.

- The final quotient is
    - 13
    - 130
    - 103
- The first quotient figure is set on

- the column immediately left of 412
    - the second column to the left of 41 2
    - the second column to the left of the divisor 4
- After the first complete round of division, multiplication, and subtraction, the dividend shows
    - 32
    - 42
    - 12
- The second quotient figure is set on
    - the column immediately left of the dividend
    - the second column to the left of the dividend
    - either of the above

Questions 11-13 pertain to the following problem: 4,644 divided by 6.

- The final quotient is
    - 774
    - 724
    - 729
- What number remains in the dividend after the first complete round of division, multiplication, and subtraction?
    - 344
    - 444
    - 434
- What number is left in the dividend after the second complete round of division, multiplication, and subtraction?
    - 44
    - 24
    - 22

Questions 14-16 pertain to the following problem: 8,617 divided by 7.

- The final quotient is
    - 1,231
    - 1,241
    - 12,301
- After the second complete round of division, multiplication, and subtraction, what number remains in the dividend?
    - 312
    - 212
    - 217
- In this problem, you set
    - the first and the final quotient figures on the second column to the left of the dividend and the second and third quotient figures immediately left of the dividend
    - the second quotient figure on the second column to the left of the dividend
    - the first quotient figure on the column immediately to the left of the dividend

Questions 17 and 18 pertain to the following problem: Divide 962 by 8 and carry your answer out to three decimal places.

- The quotient is

    - 12.025.
    - 120.25
    - 120
- When solving 962 divided by 8,
    - the first quotient figure is set on the second column to the left of the dividend
    - the second quotient figure is set immediately left of the dividend
    - the second quotient figure is set on the second column to the left of the dividend

Explain the following two problems. Be as thorough and detailed as possible.

- Work the problem 375 divided by 4 with a remainder. Explain where the divisor and dividend are set. Also explain where each quotient figure should be set and why. State the dividend after each round of division, multiplication, and subtraction. Explain how to determine the final answer.
- Explain the problem, 375 divided by 4. Carry the answer out to three decimal places. Explain where the divisor and dividend are set. Also explain where each quotient figure should be set and why. State the dividend after each round of division, multiplication, and subtraction. Explain how to determine the final answer.

Lesson 12: Long Division

Lesson 11 introduced short division of whole numbers. Lesson 12 covers long division. This lesson expands on the rule for quotient figure placement and applies it to long division. It provides instruction and practice in working whole-number long-division problems, with or without remainders. Familiarizing yourself with the information in this lesson will enable you to use the abacus to complete long-division problems.

Objectives

After completing this lesson, you will be able to

- apply the rules of quotient figure placement to long division
- work long-division whole-number problems with or without remainders

Quotient Figure Placement in Long-Division Problems

Long division is the process by which a dividend containing any number of digits is divided by a divisor with two or more digits. Except for a few additional steps, long division is treated in the same way as short division: Divide the first one or two digits of the dividend by the first digit of the divisor only-regardless of the number of digits in the divisor. This results in a quotient figure that is set on the appropriate column to the left of the dividend, according to the rules of quotient figure placement. Multiply this quotient figure times each digit of the divisor. Subtract each resulting product from the appropriate columns in the dividend in keeping with the rules of positioning. After finishing one complete round of division, multiplication, and subtraction, repeat the process to develop the next quotient figure.

Working Long-Division Problems

For example, consider the division of 258 by 43. Set the divisor 43 on columns 13 and 12. Set the dividend 258 on columns 3, 2, and 1. The first digit of the dividend, the 2 of 258, cannot be divided by the first digit of the divisor, the 4 of 43. Therefore, divide the first two digits of the dividend (i.e., the 2 and the 5 of 258) by the first digit of the divisor (i.e., the 4 of 43). Since 25 divided by 4 equals 6, the first quotient figure is 6. Where should this 6 be set? You could not divide the first digit of the dividend, the 2 of 258, by the first digit of the divisor, the 4 of 43. So you divided the first two digits of the dividend, the 2 and the 5 of 258, by the first digit of the divisor, the 4 of 43. In so doing, you divided a two-digit number by a one-digit number. Therefore, **NOT THE SAME, DO NOT SKIP**. According to the rules of quotient figure placement, set your first quotient figure 6 immediately left of 258 on column 4.

Now multiply 43 times the first quotient figure 6. Begin by multiplying the 4 of 43 times 6. Subtract 24 from 25 in the first position. Next multiply the 3 of 43 times 6. Subtract 18 from 18 in the second position. This clears the dividend. You are now ready to read the quotient. Go to the far right end of the abacus and count off three unused columns; that is, two for the two digits in the divisor 43 and one more for the process of division. The answer is 6.

This time, divide 8,833 by 73. Set the divisor 73 on columns 13 and 12. Set the dividend 8,833 on columns 4, 3, 2, and 1. Divide the first digit of the dividend, the first 8 of 8,833, by the first digit of the divisor, the 7 of 73. Since 8 divided by 7 is 1, your first quotient figure is 1. Set the 1 on the second column to the left of 8 on column 6. Why? You only divided the first digit of the dividend (i.e., the first 8 of 8,833) by the first digit of the divisor (i.e., the 7 of 73). In other words, you divided a one-digit number by a one-digit number. Therefore, **SAME, SKIP**. Next multiply the divisor 73 times the quotient figure 1. Since 7 times 1 is 07, subtract 07 from 08 in the first position—that is, the first and second columns to the right of 1. Then multiply the 3 of 73 times 1. Since 3 times 1 is 03, subtract 03 from 18 in the second position (i.e., the second and third columns to the right of 1). This leaves 1,533 in the dividend.

You are now ready for the second round of division. The first digit of the dividend, the 1 of 1,533, cannot be divided by the first digit of the divisor, the 7 of 73. So divide the first two digits of the dividend, the 1 and the 5, by the first digit of the divisor, the 7 of 73. This results in a second quotient figure of 2. Set the 2 immediately left of 1 5 on column 5. Why? It is not possible to divide the first digit of the dividend, the 1 of 1,533, by the first digit of the divisor, the 7 of 73. Instead, you divided the first two digits of the dividend, the 1 and the 5, by the first digit of the divisor, the 7 of 73. Therefore, **NOT THE SAME, DO NOT SKIP**. Now multiply the divisor 73 times the second quotient figure 2. Since 7 times 2 is 14, subtract 14 from 15 in the first position.

Next multiply 3 times 2. Then subtract 06 from 1 3 in the second position by subtracting 0 from 1 and 6 from 3. The dividend now contains 73. The divisor is also 73. Since 73 divided by 73 is 1, the third quotient figure is 1. Set this 1 on the second column to the left of the dividend on column 4. By applying the rule for quotient figure placement, divide the first digit of the dividend (i.e., the 7 of 73) by the first digit of the divisor (i.e., the 7 of 73). **SAME, SKIP**. Multiply the divisor 73 times the third quotient figure 1. Then subtract 07 from 07 and 03 from 03, thereby clearing the dividend. To determine the final answer, start from the right end of the abacus. Count off to the left three unused columns (i.e., two columns for the two digits in 73 plus one additional column for the process of division). Everything to the left of these three columns is the quotient 121. To check this answer, multiply 73 times 121. The product will be 8,833.

Practice Exercise

Practice dividing whole numbers on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 7,826 divided by 86
- 3,913 divided by 91
- 1,344 divided by 64
- 1,085 divided by 31
- 7,650 divided by 75
- 8,091 divided by 93

- 7,705 divided by 67
- 59,718 divided by 74
- 26,643 divided by 321
- 78,288 divided by 932
- 7,812 divided by 217
- 52,852 divided by 724
- 5,041 divided by 71
- 49,913 divided by 71
- 20,774 divided by 611
- 894,447 divided by 4,321
- 829,488 divided by 3,142
- 44,996 divided by 6,428
- 384,300 divided by 525
- 979,242 divided by 814

Answers

- 91
- 43
- 21
- 35
- 102
- 87
- 115
- 807
- 83
- 84
- 36
- 73
- 71
- 703
- 34
- 207
- 264
- 7
- 732
- 1,203

Summary

This lesson explained how to apply the rules of quotient figure placement to long division. It also described the steps to solve long-division problems, with or without remainders.

Assignment 12

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly. Questions 1-3 pertain to the following problem: 736 divided by 32.

- The quotient is
    - 2.3
    - 203
    - 23

- In solving the problem,
    - the first quotient figure was set on the column immediately left of 736
    - the first quotient figure was set on the second column to the left of 736
    - the second quotient figure was set immediately to the left of the dividend
- What number remains in the dividend after the first complete round of division, multiplication, and subtraction?
    - 96
    - 46
    - 86

Questions 4-6 pertain to the following problem: 2,688 divided by 64.

- The quotient is
    - 42
    - 402
    - 420
- When solving the problem, you set
    - the first quotient figure immediately to the left of the dividend
    - the first quotient figure on the second column to the left of the dividend
    - the second quotient figure on the second column to the left of the dividend
- What number remains in the dividend after the first complete round of division, multiplication, and subtraction?
    - 123
    - 138
    - 128
- 13,992 divided by 424 equals
    - 33
    - 43
    - 303
- Divide 7,488 by 936.
    - The answer is 8.
    - The quotient figure is set on the second column to the left of the dividend.
    - The quotient figure is set on the column immediately left of the dividend.
- 22,062 divided by 63 equals
    - 35 with a remainder of 1 2
    - 350 with a remainder of 1 2
    - 35 with a remainder of 62

Explain the following problem. Be as thorough and detailed as possible.

- Work the problem 47,120 divided by 62. Explain where the divisor and dividend are set. Also explain where each quotient figure should be set and why. State the dividend after each round of division, multiplication, and subtraction. Finally, explain how to determine the final answer.

Lesson 13: Trial Divisor and Upward Correction

Lesson 11 introduced short division of whole numbers; Lesson 12 covered long division. This lesson explains when and how to use the trial divisor and upward correction in any whole-number long-division problem. Familiarizing yourself with the information in this lesson will enable you to apply the concepts of trial divisor and upward correction when using the abacus to divide numbers.

Objectives

After completing this lesson, you will be able to

- determine the trial divisor, if any, for whole-number division problems
- use upward correction, if necessary, when working whole-number long-division problems

Determining the Trial Divisor

Recall that in long division, the divisor can have more than one digit. Sometimes when dividing a wholenumber long-division problem, a quotient figure that is too high can result. To prevent this, you often use a trial divisor to estimate the quotient figure. As you know, you divide the first one or two digits of the dividend by the first digit of the divisor. For example, when dividing with a divisor such as 20, 61, or 529, you divide the first one or two digits of the dividend by the first digit of the divisor. If the second digit of the divisor is 3 or more, however, you can round off the first digit to the next number. Then use the first digit of the rounded-off number as the trial divisor.

For example, the second digit in the divisor 25 is 5—which is 3 or greater. Therefore, round off the first digit 2 to 3; then use 3 as the trial divisor. When dividing with a divisor of 378, the second digit is 7—which is 3 or more. Therefore, round off the first digit 3 to 4, and use 4 as the trial divisor. When dividing 1,376, the second digit is 3. Therefore, round off the first digit 1 to 2, and use 2 as the trial divisor. Moreover, if the second digit of the divisor is 3 or more, and the first digit of that divisor is 9, round off to 10; then use 1 as the trial divisor. For instance, if the divisor is 98, the second digit is 8—which is 3 or greater. Round off the first digit 9 to 10, and use 1 as the trial divisor.

Caution: The trial divisor is only used to estimate the quotient figure and determine its placement. The original divisor is used to do the multiplication.

Using Trial Divisors and Upward Correction in Long-Division Problems

When working with a trial divisor, it is most important to follow, in every instance, the rule for quotient figure placement. Sometimes that rule results in the newest quotient figure being placed on the same column that contains the previous quotient figure. This is called upward correction; that is, you can raise a low quotient figure upward. When placing a second quotient figure on a column that already contains a previous quotient figure, multiply only the newest quotient figure times the original divisor. Then subtract.

Divide 912 by 24. Set the divisor 24 on columns 13 and 12 and the dividend 912 on columns 3, 2, and 1. The second digit of the divisor 24 is 4, which is 3 or more. So round off the first digit 2 to 3, and use 3 as the trial divisor. Divide the 9 of 912 by the trial divisor 3. Since 9 divided by 3 is 3, the first quotient figure is 3. Set this 3 on the second column to the left of the 9 of 912 on column 5. It is possible to divide the first digit of the dividend, the 9 of 912, by the trial divisor 3, which contains only one digit. Since you divided a onedigit number by a one-digit number, **SAME, SKIP**. Next multiply the original divisor 24 times the first quotient figure 3. Then subtract 06 from 09 and 12 from 31. The dividend now shows 192.

You are now ready for the second round of division. Divide the first two digits of the dividend, the 19 of 192, by the trial divisor 3. The second quotient figure is 6. Set this 6 immediately to the left of 1 9 on column 4. You divided the first two digits of the dividend, the 1 and 9 of 192, by the trial divisor 3, which only contains one digit. **NOT THE SAME, DO NOT SKIP**. Then multiply the original divisor 24 times the second quotient figure 6. Subtract 12 from 19 and 24 from 72. This leaves 48 in the dividend. You are now ready for the third round of division. Divide the first digit of the dividend, the 4 of 48, by the trial divisor 3. The third quotient figure is 1. You divided the first digit of the dividend, the 4 of 48, by the trial divisor 3, which contains only one digit. **SAME, SKIP**. Set this 1 on the second column to the left of 48 on column 4. That column already contains the second quotient figure 6. So, using upward correction, raise the low quotient figure 6 to 7. Now multiply the original divisor 24 times this third quotient figure 1. Why not 7? Because you have already multiplied the original divisor 24 times the low quotient figure 6. Now subtract 02 from 04 and 04 from 28. This leaves 24 in the dividend.

You are now ready for the fourth round of division. The dividend 24 divided by the original divisor 24 gives you a fourth quotient figure of 1. At this point, you can dispense with the trial divisor. Set this fourth quotient figure 1 on the second column to the left of 24 on column 4. It is possible to divide the first digit of the dividend, the 2 of 24, by the first digit of the divisor, the 2 of 24. Therefore, **SAME, SKIP**. The second column to the left of the dividend already contains the quotient figure 7. So once again, using upward correction raise the low quotient figure 7 to an 8. Next multiply the original divisor 24 times the fourth quotient figure 1. (Not 7, because you have already multiplied the original divisor 24 times the low quotient figure 7.) Subtract 02 from 02 and 04 from 04, thereby clearing the dividend. To read the answer, count off three columns at the right. The answer is 38.

Now rework a part of this last example to make an observation. Set up your abacus with 24 to the far left, 48 to the far right, and 36 on columns 5 and 4. This is the point at which you finished the second complete round of division, multiplication, and subtraction.

Examine the divisor 24 and the dividend 48. To work the problem 48 divided by 24 is 2, you divided the first digit of the dividend (i.e., the 4 of 48) by the first digit of the divisor (i.e., the 2 of 24). **SAME, SKIP**. Therefore, set 2 on the second column to the left of 48. By adding 2 to the 6 already on that column, you automatically raise the low quotient figure 6 to 8 in a single step. This eliminates the two steps needed in the last example to change 6 to 7, then 7 to 8. Now multiply 24 times the third quotient figure 2. Subtract 04 from 04 and 08 from 08. This clears the dividend and leaves the answer 38.

Work another problem 9,848 divided by 182. Set the divisor 182 on columns 13, 12, and 11. Set the dividend 9,848 on columns 4, 3, 2, and 1. The second digit of the divisor 182 is 8 —which is 3 or more. So round off the first digit 1 to 2, and use 2 as the trial divisor. Divide the 9 of 9,848 by the trial divisor 2. The first quotient figure is 4. Set this 4 on the second column to the left on column 6. It is possible to divide the first digit of the dividend, the 9 of 9,848, by the trial divisor 2, which contains only one digit. Therefore, **SAME, SKIP**. Next multiply the original divisor 182 times the first quotient figure 4. Then subtract 04 from 09, 32 from 58, and 08 from 64. This leaves 2,568 in the dividend.

For the second round of division, divide the first digit of the dividend, the 2 of 2,568, by the trial divisor 2. The second quotient figure is 1. Set this 1 on the second column to the left of 2,568 on column 6. Again, it is possible to divide the first digit of the dividend, the 2 of 2,568, by the trial divisor 2, which contains only one digit. Therefore, **SAME, SKIP**. This column already contains the first quotient figure 4. So using upward correction, raise the low quotient figure 4 to 5. Now multiply the original divisor 182 times the second quotient figure 1. Why not 5? Because you have already multiplied the original divisor 182 times the low quotient figure 4. Subtract 01 from 02, 08 from 15, and 02 from 76. At this point, the dividend contains 748.

You are now ready for the third round of division. Next divide the first digit of the dividend, the 7 of 748, by the trial divisor 2. Since 7 divided by 2 is 3, skip a column and set this third quotient figure 3 on the second column to the left of the dividend on column 5. It is possible to divide the first digit of the dividend, the 7 of 748, by the trial divisor 2, which contains only one digit. Therefore, **SAME, SKIP**. Then multiply the original divisor 182 times this third quotient figure 3. Subtract 03 from 07, 24 from 44, and 06 from 08. This leaves 202 in the dividend. You are now ready for the fourth round of division. Divide the first digit of the dividend, the 2 of 202, by the trial divisor 2. Since 2 goes into 2 once, 1 is the fourth quotient figure. Skip a column and set the fourth quotient figure 1 on the second column left of the dividend on column 5. It is possible to divide the first digit of the dividend, the 2 of 202, by the trial divisor 2, which contains only one digit. Therefore, **SAME, SKIP**. This column already contains the low quotient figure 3. So using upward correction, raise the low quotient figure 3 to 4. Now multiply the original divisor 182 times the fourth quotient figure 1. Why not 4? Because you have already multiplied the original divisor 182 times the low quotient figure 3. Subtract 01 from 02, 08 from 10, and 02 from 02. The dividend now contains 20. You cannot divide 20 by the original divisor 182. So your division is done; 20 is the remainder.

To determine the final answer, go to the far right end of the abacus and count three columns for the three digits in the divisor 182, plus an additional column for the process of division. Everything to the left is the whole-number answer 54; everything to the right is the remainder 20.

Exception When Trial Divisor Is 10

Here is an example of a problem that illustrates the exception that occurs when the trial divisor is 10. Divide 2,208 by 96. Set the divisor 96 on columns 13 and 12. Set the dividend 2,208 on columns 4, 3, 2, and 1. The second digit of the divisor 96 is 6 (which is 3 or more). Therefore, round off the first digit of the divisor 9 to 10, and use 10 as the trial divisor. Divide the first digit of the dividend, the 2 of 2,208, by the first digit of the trial divisor 1. The first quotient figure is 2.

Ordinarily, you would set this first quotient figure 2 on the second column left of the dividend. Why? It is possible to divide the first digit of the dividend, the 2 of 2,208, by the first digit of the trial divisor, the 1 of 10. Therefore, **SAME, SKIP**. However, there is an exception to this rule. This exception occurs when you divide using a trial divisor of 10, and it is possible to divide the first digit of the dividend by the first digit of the trial divisor, which is 1. Ordinarily, skip a column and set the quotient figure on the second column left. **SAME, SKIP**. Exception: Do not skip a column and set the quotient figure on the second column left. Instead, set it immediately left. Therefore, in this case, set your first quotient figure 2 immediately left on column 5. Next multiply the original divisor 96 times the first quotient figure 2. Subtract 18 from 22 and 12 from 40. This leaves 288 in the dividend.

For the second round of division, divide the first digit of the dividend, the 2 of 288, by the trial divisor 1. The second quotient figure is 2. According to the exception covering the division of a trial divisor of 10, set this 2 immediately left of 288 on column 4. Now multiply the original divisor 96 times the second quotient figure 2. Subtract 18 from 28 and 12 from 08. This leaves 96 in the dividend. Your original divisor is 96. Since you know that 96 divided by 96 is 1, dispense with the trial divisor. Set the newest quotient figure 1 on the second column to the left of 96 on column 4. Why? By dispensing with the trial divisor, you can divide the first digit of the dividend, the 9 of 96, by the first digit of the divisor, the 9 of 96. Therefore, **SAME, SKIP**. Set the newest quotient figure 1 on the column containing the previous quotient figure 2. This automatically raises the low quotient figure 2 to 3. Next multiply the original divisor 96 times the newest quotient figure 1. (Not 3, because you have already multiplied the original divisor 96 times the low quotient figure 2.) Subtract 09 from 09 and 06 from 06. This clears the dividend. From the right end of the abacus, count off three columns. The answer is 23.

Practice Exercise

Practice dividing whole numbers on the abacus by working the following problems. First indicate the trial divisor, then the answer. Compare your answers with those that follow the exercise.

- 2,975 divided by 35
- 5,824 divided by 26
- 1,884 divided by 28
- 7,050 divided by 75
- 61,880 divided by 68
- 48,066 divided by 74
- 1,204 divided by 43
- 1,625 divided by 25
- 7,917 divided by 87
- 20,414 divided by 346
- 103,984 divided by 194
- 495,040 divided by 68

Answers

- trial divisor 4; answer 85
- trial divisor 3; answer 224
- trial divisor 3; answer 67 with a remainder of 8
- trial divisor 8; answer 94
- trial divisor 7; answer 910
- trial divisor 8; answer 649 with a remainder of 40
- trial divisor 5; answer 28
- trial divisor 3; answer 65
- trial divisor 9; answer 91
- trial divisor 4; answer 59
- trial divisor 2; answer 536
- trial divisor 7; answer 7,280

Summary

This lesson explained how to determine the trial divisor. It then described how to use trial divisors and upward correction in long-division problems.

Assignment 13

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

Questions 1-7 pertain to the following problem: Set 449 on columns 3, 2, and 1 of your abacus and divide it by 47.

- When working this problem, what number did you use as the trial divisor?
    - 5
    - 4
    - 7
- Your first quotient figure is
    - 7

    - 8
    - 10
- This first quotient figure will be set on
    - the third column to the left of 449
    - the second column to the left of 449
    - the column immediately to the left of 449
- What does the dividend portion of the abacus show after the first complete round of division, multiplication, and subtraction?
    - 23
    - 63
    - 73
- What is the second quotient figure?
    - 0
    - 1
    - 2
- The second quotient figure must be set
    - on the second column to the left of the dividend
    - on the column immediately to the left of the dividend
    - on the column containing 8
- What is the final quotient?
    - 9 with a remainder of 26
    - 81 with a remainder of 26
    - 90 with a remainder of 26

Questions 8-12 pertain to the following problem: Set 237 on your abacus and divide it by 24.

- Your trial divisor is
    - 2
    - 3
    - 4
- The first quotient figure is
    - 8
    - 6
    - 7
- The first quotient figure is set on
    - the third column to the left of 237
    - the second column to the left of 237
    - the column immediately to the left of 237
- The second quotient figure is
    - 2 set on the second column to the left of the dividend
    - 2 set on the column immediately to the left of the dividend
    - 2 set on the column containing the first quotient figure
- What is the final quotient?
    - 81 with a remainder of 21
    - 9 with a remainder of 21
    - 8 with a remainder of 21
- When dividing 42,357 by 689, the trial divisor is
    - 6
    - 7

    - 8
- Set the dividend 864 on your abacus and divide it by 96.
    - Your trial divisor is 10
    - The answer is 9.
    - The answer is 81

Questions 15 and 16 pertain to the following problem: 87,156 divided by 27.

- The final quotient is
    - 3,229
    - 3,227 remainder 21
    - 3,228
- You used upward correction
    - once
    - twice
    - not at all

Questions 17 and 18 pertain to the following problem: 3,082 divided by 46.

- After the first complete round of division, multiplication, and subtraction, your abacus shows
    - 372
    - 322
    - 222
- In which round of division do you use upward correction?
    - first
    - second
    - third

Explain the following problem. Be as thorough and detailed as possible.

- Work the problem 1,792 divided by 28. Explain how the trial divisor was determined. Also explain where each quotient figure should be set and why. Finally, explain how to determine the final answer.

Lesson 14: Downward Correction and Treatment of Zeros

Lesson 11 introduced short division of whole numbers; Lesson 12 covered long division. Lesson 13 explained when and how to use the trial divisor and upward correction in whole-number long-division problems. Lesson 14 explains downward correction. It also describes how to treat zeros when they occur in the divisor, the dividend, or both. Familiarizing yourself with the information in this lesson will enable you to use downward correction and treat zeros correctly when using the abacus for division problems.

Objectives

After completing this lesson, you will be able to

- use downward correction, if necessary, in whole number long-division problems
- use the exception to the rules of downward correction
- treat zeros when they appear in the divisor, the dividend or both

Using Downward Correction in Whole-Number Long-Division Problems

Downward correction is the process you use to reduce a quotient figure that is too high. If you use a trial divisor, this rarely will occur. Nevertheless, you should know how to perform downward correction so that you are prepared for those rare occurrences. Here is the procedure for using downward correction.

Recall that in long division, you divide the first one or two digits of the dividend by the first digit of the divisor, then set the resulting quotient figure on the abacus. Next you multiply the quotient figure times each digit of the divisor. Then you subtract each resulting product from the dividend in accordance with the rules of positioning. If the estimated quotient figure is too high, however, you cannot subtract. This might not occur until you have multiplied one or more digits of the divisor times the incorrect high quotient figure, then subtracted the resulting product or products from the dividend. When the incorrect high quotient figure is discovered, you must reduce it by 1 or more. The number you reduce it by is called the corrective figure. Next return to the dividend the portion of the product or products already subtracted. Generally, you reduce the quotient figure by a corrective figure of 1, although it can be by 2, or even 3. Then multiply this corrective figure—usually 1—times those digits of the divisor that have already been multiplied by the incorrect high quotient figure and for which subtractions have been made. Return the products resulting from this multiplication to the dividend in accordance with the rules of positioning. Once this has been done, continue where you left off. Multiply the remaining digits of the divisor times the reduced quotient figure. Then proceed in the usual manner.

Here is an example to clarify the procedure: Divide 4,248 by 72. Set 72 on columns 13 and 12. Set 4,248 on columns 4, 3, 2, and 1. Divide the 42 of 4,248 by the 7 of 72. The first quotient figure is 6. Set this 6 immediately to the left of 42. Next multiply the 7 of 72 times 6. Subtract 42 from 42 in the first position. Now multiply the 2 of 72 times 6. 2 times 6 is 12. This 12 must be subtracted from the second position—that is, from the 04 on columns 3 and 2. However, 12 is larger than 04, so this subtraction cannot be made. At this point, you realize that your quotient figure 6 is too high. Therefore, reverse part of your work. Reduce the high quotient figure 6 to 5 by subtracting the corrective figure 1. You already multiplied the 7 of 72 times the high quotient figure 6, then subtracted 42 from the first position in the dividend. So multiply this 7 of 72 times the corrective figure 1. Since 7 times 1 is 07, return 07 to the first position. The dividend now shows 748. Before discovering the incorrect high quotient figure 6, you had already multiplied the 7 of 72 by 6, but not by the 2 of 72. Since you have now corrected the error, multiply the 2 of 72 times the reduced quotient figure 5.

Multiplying the reduced quotient figure 5 times 2 is 10. Subtract 10 from 74 in the second position. This leaves 648 in the dividend. Next divide the 64 of 648 by the 7 of 72. This results in a second quotient figure of 9. Set 9 immediately to the left of 64. Now multiply 72 times 9. Then subtract 63 from 64 in the first position and 18 from 18 in the second position. This clears the dividend, so the job is done. Count off three columns at the right. The answer is 59.

What is the important thing to remember in downward correction? Multiply the corrective figure, times only those digits of the divisor that have already been multiplied by the incorrect high quotient figure and for which subtractions have been made. Observe the rules of positioning when returning the products of this multiplication to the dividend.

For your next example, divide 290,720 by 32. Set 32 on columns 13 and 12. Set 290,720 on columns 6, 5, 4, 3, 2, and 1. Divide the 29 of 290,720 by the 3 of 32. The first quotient figure is 9. Set this 9 immediately to the left of 29. Next multiply the 3 of 32 times 9. Then subtract 27 from 29 in the first position. Now multiply the 2 of 32 times 9. Since 2 times 9 equals 18, subtract 18 from 20 in the second position. The dividend now shows 2,720. Divide the 27 of 2,720 by the 3 of 32. This results in a second quotient figure of 9. Set this 9 immediately to the left of 27. Now multiply the 3 of 32 times 9. Then subtract 27 from 2.1 in the first position. Next multiply the digit 2 of 32 times 9. Since 2 times 9 equals 18, subtract this 18 from the second position—that is, from the 02 on columns 3 and 2. Since 18 is larger than 02, this subtraction cannot be made.

Since the second quotient figure 9 is too high, reverse part of your work. Reduce the high second quotient figure 9 to 8 by subtracting the corrective figure 1. You have already multiplied the 3 of 32 times the high second quotient figure 9, then subtracted 27 from the first position in the dividend. So multiply this 3 of 32 times the corrective figure 1. Since 3 times 1 is 03, return 03 to the first position. The dividend now shows 320. Before discovering the incorrect high quotient figure 9, you had already multiplied the 3 of 32 by 9, but not by the 2 of 32. Since you have now corrected the error, multiply the 2 of 32 times the reduced quotient figure 8. Because 2 times 8 equals 16, subtract 16 from 32 in the second position. This leaves 160 in the dividend. Next divide the 16 of 160 by the 3 of 32. This results in a third quotient figure of 5. Set this third quotient figure 5 immediately left of 160. Next multiply the 3 of 32 times 5. Then subtract 15 from 16 in the first position. Next multiply the 2 of 32 times the third quotient figure 5. Subtract 10 from 10 in the second position. This clears the dividend, so the job is done. Count off 3 columns at the right. The answer is 9,085.

Exception to the Rules of Downward Correction

An exception occurs when the quotient 1 has been set on the second column to the left of the dividend and eventually needs to be reduced by downward correction. The exception tells you to think of this 1 as a 10. Immediately reduce the quotient 10 to 9 by subtracting the corrective figure 1. Set the reduced quotient figure 9 immediately to the left of the dividend.

To illustrate this exception, divide 401,036 by 428. The exception occurs while determining the first quotient figure. When dividing 401,036 by 428, set 428 on columns 13, 12, and 11. Set 401,036 on columns 6, 5, 4, 3, 2, and 1. Divide the 4 of 401,036 by the 4 of 428. The first quotient figure is 1. Set this 1 on the second column to the left of 4. Why? You divided the first digit of the dividend, the 4 of 401,036, by the first digit of the divisor, the 4 of 428. Therefore, **SAME, SKIP**. Next multiply the 4 of 428 times 1. Then subtract 04 from 04 in the first position. Now multiply the 2 of 428 times the first quotient figure 1. Since 2 times 1 is 02, it must be subtracted from the second position—that is, from the 00 on columns 6 and 5. 02 is larger than 00, however, so this subtraction cannot be made. Since the first quotient figure 1 is too high, reverse part of your work.

This is where the exception occurs. Think of your first quotient figure 1, which is set on the second column left, as a 10. The exception tells you to reduce this high quotient figure 10 to a 9, by subtracting the corrective figure 1. Set this reduced quotient figure 9 immediately to the left on column 7. You have already multiplied the 4 of 428 times the high quotient figure 1, and subtracted 04 from the first position in the dividend. So multiply this 4 of 428 times the corrective figure 1. Since 4 times 1 equals 04, return 04 to the first position. The dividend now shows 41,036. Before discovering the incorrect high quotient figure, you had already multiplied it by the first digit of the divisor, the 4 of 428. However, you had not multiplied it by the second or third digits of the divisor. Since you have now corrected the error, multiply the 2 of 428 times the reduced quotient figure 9. Multiply the 2 of 428 times 9. Subtract the resulting 18 from 41 in the second position. Now multiply the 8 of 428 times the reduced quotient figure 9. Then subtract 72 from the 30 in the third position. The dividend now shows 15,836.

Divide the 15 of 15,836 by the 4 of 428. This gives you a second quotient figure of 3, which you set immediately left of the dividend. Multiply the 4 of 428 times the second quotient figure 3. Then subtract 12 from 1 5 in the first position. Now multiply the 2 of 428 times the second quotient figure 3. Then subtract 06 from 38 in the second position. Multiply the 8 of 428 times the third quotient figure 3. Then subtract 24 indirectly from 23 in the third position. The dividend now shows 2,996. Divide the 29 of 2,996 by the 4 of 428. The third quotient figure is 7. Set this 7 immediately left of the dividend. Now multiply the 4 of 428 times the third quotient figure 7. Subtract 28 from 29 in the first position. Next multiply the 2 of 428 times the third quotient figure 7. Subtract 14 from 19 in the second position. Multiply the 8 of 428 times the third quotient figure 7. Subtract 56 from 56 in the third position. Your dividend is cleared and the job is done. Finally, count off four columns at the right. The answer is 937.

Treatment of Zeros

It is important to note the treatment of zeros whenever they occur in the middle of the divisor. Each digit in the divisor, whether it is a zero or not, must be multiplied times any quotient figure, then subtracted from the correct corresponding position in the dividend.

For example, divide 92,069 by 4,003. Set 4,003 on columns 13, 12, 11, and 10. Set 92,069 on columns 5, 4, 3, 2, and 1. Divide the 9 of 92,069 by the 4 of 4,003. This results in a first quotient figure of 2, which is set on the second column to the left of 9. Multiply the 4 of 4,003 times 2. Then subtract 08 from 09 in the first position. Multiply the first zero of 4,003 times 2. Then subtract 00 from 12 in the second position. Now multiply the second zero of 4,003 times 2. Then subtract 00 from 20 in the third position. Next multiply the 3 of 4,003 times the first quotient figure 2. Then subtract 06 from 06 in the fourth position. At this point, the dividend contains 12,009. Now divide the 12 of 12,009 by the 4 of 4,003. The second quotient figure is 3. Set this 3 immediately left of 12. Multiply 4,003 times 3. Then subtract 12 from 12 in the first position, 00 from 00 in the second position, 00 from 00 in the third position, and 09 from 09 in the fourth position. The dividend is now cleared. To read the answer, count off five columns at the right. That is, four columns for the four digits in 4,003, plus one more for the process of division. The answer is 23.

For another example of the proper treatment for zeros, divide 581,160 by 20,040. Set 581,160 on columns 6, 5, 4, 3, 2, and 1. Retain the divisor in memory, write it down, or set it on another abacus because of the length of this problem. Divide the 5 of 581,160 by the 2 of 20,040. The first quotient figure becomes 2. Set this 2 on the second column left and multiply it times 20,040. The 2 of 20,040 times the first quotient figure 2 is 04. Subtract 04 from the 05 in the first position. Multiply the first zero of 20,040 times the first quotient figure 2. Subtract 00 from 18 in the second position. Similarly, multiply the second zero of 20,040 times the first quotient figure 2. Then subtract 00 from 81 in the third position. Multiply the 4 of 20,040 times the first quotient figure 2. Then subtract 08 from 11 in the fourth position. Multiply the last 0 of 20,040 times the first quotient figure 2. Then subtract 00 from 36 in the fifth position. The dividend now shows 180,360. Divide the 18 of 180,360 by the 2 of 20,040. The second quotient figure is 9.

Set this 9 immediately to the left of the dividend. Now multiply the 2 of 20,040 times 9. Subtract 18 from 18 in the first position. Multiply the first zero of 20,040 times 9. Then subtract 00 from 00 in the second position. Multiply the second zero of 20,040 times 9. Then subtract 00 from 03 in the third position. Now multiply the 4 of 20,040 times 9. Then subtract 36 from 36 in the fourth position. Multiply the last zero of 20,040 times 9. Then subtract 00 from 00 in the fifth position. The dividend is cleared. To determine your final answer, go to the far right end of the abacus and count six columns. That is, five for the five digits in the divisor, plus one additional column for the process of division. Your answer is 29.

Practice Exercise

Practice dividing whole numbers on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 84,976 divided by 904
- 1,184,225 divided by 505
- 5,658 divided by 82

- 748,872 divided by 20,802
- 3,278,232 divided by 60,708
- 1,566,169 divided by 1,903 (use a trial divisor)
- 61,173 divided by 21
- 11,330,964 divided by 4,041

You would normally use a trial divisor for the next two problems. Imagine, however, that you had failed to do so. Divide using the first digit of the divisor instead of a trial divisor.

- 34,800 divided by 48
- 32,604 divided by 57

Answers

- 94
- 2,345
- 69
- 36
- 54
- 823
- 2,913
- 2,804
- 725
- 572

Summary

This lesson explained how to use downward correction in whole-number long-division problems. It also examined the exception to the rules of downward correction. Finally, it described the treatment of zeros when they occur in the middle of the divisor, the dividend, or both.

Assignment 14

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

Questions 1 and 2 pertain to the following problem: Set the dividend 5,428 on your abacus and divide it by 92.

- As you work through the problem, you will find that the first quotient figure 6 is too high. It should be reduced to 5. When this is done, what number should you return to the dividend portion of the abacus?
    - 01
    - 09
    - 05
- Your immediate next step is to
    - divide the remaining portion of the dividend by the divisor in order to determine the next quotient figure
    - multiply the 9 of 92 times 5, then subtract the resulting product from the dividend in the first position
    - multiply the 2 of 92 times 5, then subtract the resulting product from the dividend in the second position

Questions 3-5 pertain to the following problem: Divide 196 by 28.

- You would normally use a trial divisor of 3. Imagine that you used the 2 of 28 as your divisor instead. In this case, the first quotient figure would be 9. When the 2 of 28 is multiplied times 9, and the resulting product is subtracted from the dividend, the dividend portion of the abacus contains
    - 16
    - 86
    - 106
- The quotient figure 9 is too high. Suppose you reduce it by 2—that is, from 9 to 7. When this is done, what number must you return to the dividend?
    - 02
    - 14
    - 04
- If you continue to work the problem to its solution, your immediate next step would be to
    - multiply the 2 of 28 times 7, then subtract 14 from the dividend in the first position
    - multiply the 8 of 28 times 7, then subtract 56 from the dividend in the second position
    - return 56 to the dividend in the second position

Questions 6-9 pertain to the following problem: Divide 4,539 by 51. The first quotient figure is 9.

- When you multiply the 5 of 51 times 9, then subtract the resulting product from the dividend, the dividend portion of the abacus contains
    - 39
    - 139
    - 539
- When you realize that the quotient figure 9 is too high, you reduce it by 1, from 9 to 8. When this is done, what number must you return to the dividend?
    - 08
    - 05
    - 09
- Your immediate next step is to
    - divide the remaining portion of the dividend by the divisor to determine the next quotient figure
    - multiply the 5 of 51 times 8, then subtract the resulting product from the dividend in the first position
    - multiply the 1 of 51 times 8, then subtract the resulting product from the dividend in the second position
- The dividend portion of the abacus shows which number after the first complete round of division, multiplication, and subtraction?
    - 454
    - 459
    - 559

Questions 10-12 pertain to the following problem: Divide 20,041 by 409. When you divide the 20 of 20,041 by the 4 of 409, the first quotient figure is 5.

- When the 40 of 409 is multiplied times this 5, and the resulting products are subtracted from the first and second positions of the dividend, the dividend shows
    - 1
    - 441

    - 41
- The quotient figure 5 is too high. Reduce it by 1. When this is done, what number must you return to the dividend?
    - 04 to the first position
    - return 36 to the dividend in the third position
    - 04 to the first position and 09 to the second position
- In this division problem,
    - your immediate next step is to divide the remaining portion of the dividend by 409
    - your immediate next step is to multiply the 9 of 409 times the quotient figure 4, then to subtract 36 from the third position in the dividend
    - after the first complete round of division, multiplication, and subtraction, the dividend shows 3,681
- If you were to divide 70,109 by 709,
    - after the first complete round of division, multiplication, and subtraction, the dividend shows 6,299
    - after the first complete round of division, multiplication, and subtraction, the dividend shows 109
    - the first quotient figure would have to be reduced from 10 to 9
- An unknown dividend is to be divided by 60,093. When the 3 of the divisor 60,093 is multiplied by the second quotient figure, the resulting product is subtracted from which position?
    - first
    - second
    - fifth

Explain the following problem. Be as thorough and detailed as possible.

- Work the problem 7,268 divided by 92. Explain downward correction, where each quotient figure should be set, and how to determine the final answer.

Lesson 11 introduced short division of whole numbers; Lesson 12 covered long division. Lesson 13 explained when and how to use the trial divisor and upward correction in whole-number division problems. Lesson 14 explained downward correction and the treatment of zeros. This lesson explains how to divide decimals and a series of numbers. It emphasizes setting up a problem and properly placing the decimal point in the answer. Familiarizing yourself with the information in this lesson will enable you to use the abacus to divide decimals as well as a series of numbers.

Objectives

After completing this lesson, you will be able to

- determine where to place the decimal point when dividing decimals
- divide a series of numbers

Placing the Decimal Point

Problems involving the division of decimals are basically worked in the same way as those involving whole numbers. When dividing decimals, ignore the decimal point when setting up and working the problem. Set up and work the problem as you would a whole-number division problem. After completing the division, count off as many columns at the right end of the abacus as there are digits in the divisor, plus one more for the process of division. This brings you to the zero point. Only then do you account for the decimal points.

Recall that in multiplication, you determine the total number of decimal places in the multiplier and the multiplicand, then point off the total number in the product. Division is the opposite of multiplication. So do just the opposite to determine where to place the decimal point in the quotient. That is, subtract the number of decimal places in the divisor from the number of decimal places in the dividend. This brings you to the zero point. If this difference is a positive number, point off that number of columns to the left from the zero point. If there is zero difference, place the decimal point at the zero point—your answer will be a whole number. If this difference is a negative number, point off that number of columns to the right of the zero point. To summarize: Divide the decimals. Then count off as many columns as there are digits in the divisor, plus one more, to determine the zero point. Finally, point off columns to place the decimal point.

Some decimal division problems involve a divisor and/or a dividend that is a pure decimal fraction. A pure decimal fraction is a decimal number with no digits to the left of the decimal point and one or more zeros immediately to the right of the decimal point. Some examples of pure decimal fractions are 0.07, 0.004, and 0.0036. When working with pure decimal fractions, disregard the zeros when setting up the problem, doing the actual division, and counting off columns at the right. Account for them, however, when subtracting the number of decimal places in the divisor from those in the dividend. For instance, 0.07 would be treated as the whole number 7 when setting up, working, and counting off columns at the right. When subtracting the number of decimal places in the divisor from those in the dividend, however, take both decimal places back into account. Similarly, 0.004 would be treated as the whole number 4, and 0.0036 as the whole number 36.

To illustrate this, work the problem 144 divided by 12 equals 12, using three variations. First divide 0.0144 by 1.2. Ignore the decimal points when setting up and working the problem. Divide as if you were dividing the whole number 144 by the whole number 12. The quotient 12 appears on columns 5 and 4. Count off three columns—two for the two digits in the divisor 12, plus one more for the process of division. This brings you to the zero point. Now take the decimal points into account. The dividend 0.0144 contains four decimal places. The divisor 1.2 contains one decimal place. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend: 4 minus 1 equals 3. Since this is a positive difference, point off three decimal places to the left, starting at the zero point, then place the decimal point. The answer is 0.012.

To illustrate the second variation using these digits, try dividing 1.44 by 0.12 next. As with all decimal division problems, ignore the decimal points when setting up and working the problem. Divide as if you were dividing the whole number 144 by the whole number 12. The quotient 12 will appear on columns 5 and 4. Count off three columns—two for the two digits in the divisor 12, plus one more for the process of division. This brings you to the zero point. Now take the decimal points into account. The dividend 1.44 contains two decimal places. The divisor 0.12 also contains two decimal places. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend. 2 minus 2 equals 0. Therefore, do not point off any decimal places. Simply place the decimal point at the zero point. The answer is the whole number 12.

For the last variation using these same digits, divide 14.4 by 0.00012. Before you begin, note that when pointing off a negative difference to the right, sometimes you will run off the end of the abacus. There are two ways to handle this. Either imagine zeros to the right, or set the dividend at the first unit mark. In other words, use the first unit mark as the right-hand margin. To illustrate the first way to handle running off the end of the abacus, begin by imagining an additional column containing 0 off the right end of the abacus. To divide 14.4 by 0.00012, ignore the decimal points when setting up and working the problem. Divide as if you were dividing the whole number 144 by the whole number 12. The quotient 12 appears on columns 5 and 4. Count off three columns—two for the two digits in the divisor 12, plus one more for the process of division. This brings you to the zero point. Now take the decimal points into account. The dividend 14.4 contains one decimal place. The divisor 0.00012 contains five decimal places. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend. Since 1 minus 5 equals negative 4, point off four decimal places to the right. As you can see, this takes you off the end of the abacus. So imagine that there is one more 0 off the end of the abacus. Your answer is the whole number 120,000.

To illustrate the second way to handle running off the end of the abacus, use the first unit mark from the right as your margin. Set the dividend 144 on columns 6, 5, and 4. The quotient 12 appears on columns 8 and 7. Count off three columns from the first unit mark to determine the zero point. Again, subtract 1 minus 5, which equals negative 4. Point off four decimal places to the right. Your answer is 120,000.

To get additional practice dividing decimals, try one more set of variations using the digits 475 divided by 25 equals 19. Begin the first variation by dividing 0.000475 by 0.25. Ignore the decimal points when setting up and working the problem. Divide as if you were dividing the whole number 475 by the whole number 25. The quotient 19 appears on columns 5 and 4. Count off three columns —two for the two digits in the divisor 25, plus one more for the process of division. This brings you to the zero point. Now take the decimal points into account. The dividend 0.000475 contains six decimal places. The divisor 0.25 contains two decimal places. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend: 6 minus 2 equals 4. Since this is a positive difference, point off four places to the left starting at the zero point. Then place the decimal point. The answer is 0.0019.

To illustrate the second variation using these digits, try dividing 4.75 by 0.25 next. As with all decimal division problems, ignore the decimal points when setting up and working the problem. Divide as if you were dividing the whole number 475 by the whole number 25. The quotient 19 appears on columns 5 and 4. Count off three columns—two for the two digits in the divisor 25, plus one more for the process of division. This brings you to the zero point. Now take the decimal points into account. The dividend 4.75 contains two decimal places, and the divisor 0.25 contains two decimal places. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend: 2 minus 2 equals 0. Since there will be no decimal places to be pointed off, simply place the decimal point at the zero point. The answer is the whole number 19.

For the last example using these same digits, divide 475 by 0.00025. In this example, imagine two additional columns containing zeros off the right end of the abacus. As with all decimal division problems, ignore the decimal point when setting up and working the problem. Divide as if you were dividing the whole number 475 by the whole number 25. The quotient 19 appears on columns 5 and 4. Count off three columns—two for the two digits in the divisor 25, plus one more for the process of division. This brings you to the zero point. Now take the decimal points into account. The dividend 475 contains no decimal places; the divisor 0.00025 contains five decimal places. Since 0 minus 5 equals a negative 5, point off five decimal places to the right. As you can see, this takes you off the end of the abacus. Imagine that there are two more zeros off the end of the abacus. Your answer is the whole number 1,900,000.

Want to work the same problem using the first unit mark from the right as your margin? Set the dividend 475 on columns 6, 5, and 4. The quotient 19 appears on columns 8 and 7. Count off three columns from the first unit mark to determine the zero point. Again, 0 minus 5 equals negative 5. So point off five decimal places to the right. Your answer is 1,900,000.

Dividing a Series of Numbers

As with multiplication, you can divide a series of numbers without resetting the problem each time. Simply set the first dividend to the extreme right end of the abacus. Then divide it by the first divisor, which will produce the first quotient. Divide this first quotient by the second divisor, which will produce the second quotient, and so forth. How do you determine the length of the final quotient? From the extreme right end of the abacus, count one column to the left for each digit in each divisor, plus one column for each process of division. Everything to the left is the quotient.

For example, first divide 43,524 by 31. Then divide the quotient resulting from this division by 52. After dividing 43,524 by 31, the quotient 1,404 appears on columns 7, 6, 5, and 4. Your next step is to divide this quotient 1,404 by the second divisor 52. Leave the quotient 1,404 where it is on columns 7, 6, 5, and 4. It now becomes the new dividend. Divide this new dividend 1,404 by the second divisor 52. The number 27 appears on columns 8 and 7. To determine the final answer, count off six columns from the right—two for the two digits in the first divisor 31, two more for the two digits in the second divisor 52, and two more for the two processes of division. The final answer is 27.

Practice Exercise

Practice dividing decimals on the abacus by working the following problems. Compare your answers with those that follow the exercise.

- 1,884 divided by 0.12
- 9.315 divided by 3.45
- 202.996 divided by 5.342
- 241.2 divided by 5.36 (set this dividend at the first unit mark)
- 1,998.1 divided by 2.9
- 710.1 divided by 0.09
- 7,599 divided by 8.5 (set this dividend at the first unit mark)
- 1,565.2 divided by 60.2
- 2,489.52 divided by 253
- 20,001.5 divided by 8.0006
- 205,534 divided by 1.06
- 2.010 divided by 80.4 (set this dividend at the first unit mark)
- 59,904 divided by 78 and then divided by 64
- 78,584 divided by 4 and then divided by 893

Answers

- 15,700
- 2.7
- 38
- 45
- 689
- 7,890
- 894
- 26
- 9.84
- 2,500
- 193,900
- 0.025
- 12 (the first quotient is 768)
- 22 (the first quotient is 19,646)

Summary

This lesson demonstrated how to divide numbers with decimals, and where to place the decimal point. It also described how to divide a series of numbers.

Assignment 15

Complete the following problems with the most appropriate answers. Don't assume that only one answer is correct. Sometimes two or even three choices are needed to answer the question correctly.

Questions 1 and 2 pertain to the following problem: Divide 23.4567 by 0.89.

- How many columns would you count off at the extreme right end of the abacus to determine the zero point?
    - two
    - three
    - four
- How many decimal places would you have to point off after determining the zero point?
    - two
    - three
    - four

Questions 3 and 4 pertain to the following problem: Divide 823.1903 by 0.0068.

- How many columns must you count off at the right end of the abacus to determine the zero point?
    - three
    - four
    - five
- How many decimal places must you point off after determining the zero point in this problem?
    - eight
    - four
    - zero
- Divide 12.211929 by 4.2. How many decimal places in the quotient will you point off after determining the zero point?
    - six
    - five
    - seven

Questions 6 and 7 pertain to the following problem: Divide 8.06 by 0.1934.

- Two ways to set the dividend include
    - setting 8060 on columns 4, 3, 2, and 1
    - setting 806 on 3, 2, and 1
    - using the first unit mark as the right-hand margin, set 806 on columns 6, 5, and 4
- After dividing and determining the zero point, you will point off
    - zero decimal places
    - two decimal places to the right
    - four decimal places to the right
- When dividing 190.5 by 11.8,
    - you count off four columns at the right end of your abacus to determine the zero point
    - there are no decimal places to point off in the quotient
    - you point off two decimal places in the quotient
- Divide 427 by 0.01937. Two ways to set up this problem include
    - using the first unit mark from the right as the right-hand margin, set 427 on columns 6, 5, and 4

    - setting 4270 on columns 4, 3, 2, and 1
    - setting 427 on columns 3,2, and 1
- When dividing 44.164 by 0.061,
    - there are no decimal places to point off in the quotient
    - point off one decimal place in the quotient
    - the quotient is 724
- When dividing 839.424 by 6.4,
    - count off three columns at the right end of the abacus to determine the zero point
    - point off two decimal places in the quotient
    - the quotient is 131.16

Questions 12-14 pertain to this problem: Divide 7,968 by 32; then divide the quotient by 3.

- After completing the first division (i.e., 7968 divided by 32), the first quotient is
    - 239
    - 2409
    - 249
- After dividing 7,968 by 32 and then by 3, how many columns at the right end of your abacus must you count off to read the final quotient?
    - four
    - six
    - five
- The final answer is
    - 830
    - 83
    - 803
- Consider the problem 33,512 divided by 59 equals 568. Although you will need to work this problem out, do not submit your explanation for the division. Instead, using these same numbers, shift the decimal points to find the answers to the following problems. Explain how to determine the decimal point in the final answer by counting off and pointing off. State what the final answer in each case will be.
    - 3.3512 divided by 0.059
    - 3351.2 divided by 0.0059 (Use the first unit mark from the right as your margin in this problem)