AN INTRODUCTION TO BRAILLE MATHEMATICS USING UEB WITH NEMETH

A Course for Transcribers

Revised by Lindy B. Walton
Barbara Taffet, advisor

www.loc.gov/nls

National Library Service for the Blind and Physically Handicapped

The Library of Congress

I believe that I could not have reached my potential in mathematics without the Nemeth Code. With it, I am able to read and write mathematics, as well as other sciences, at all levels, limited only by my talent and my ambition.

—Dr. Abraham Nemeth, creator of the braille code for mathematics and science notation

DEDICATION

I credit my interest in the continued training of braille transcribers in the Nemeth Code to my friend and mentor, Helen Hay, whose fascination and enthusiasm about this braille code was contagious.

THANKS

I offer my gratitude to the original authors of this lesson manual, Helen Roberts, Bernard M. Krebs, and Barbara Taffet, for their insight into the learning process and for their eye for detail. Many of the excellent examples from the original book are preserved in this edition. I also wish to thank my supervisors and colleagues in the Madison Metropolitan School District for realizing the importance of the development of this curriculum.

—Lindy Walton

ABOUT THE PROGRAM

This course is designed for the UEB certified transcriber who is ready to take on the challenge of transcribing print mathematics and science materials into braille using the Nemeth Braille Code for Mathematics and Science Notation. The program operates under a contract with the National Library Service for the Blind and Print Disabled, Library of Congress (NLS). All transcribing and proofreading course lessons and tests are administered under the National Federation of the Blind Braille Certification Training Program (BCTP). The following information is copied from the (2022) cover letter that the enrolled student will receive when accepted into the course. Up-to-date instructions will be sent at the time of enrollment.

The course is based on The Nemeth Braille Code for Mathematics and Science Notation, 2022, a publication of the Braille Authority of North America (BANA). Course materials do not supersede the authority of the official BANA code book.

Eligibility

- United States citizenship or residency
- High school diploma or equivalent
- Knowledgeable in recommended braille formats for textbooks

Prerequisite

- Library of Congress certification in literary braille transcribing (UEB) for a minimum of six months

Equipment

Any of the following methods may be used in order to submit lesson exercises in braille: a forty-cell slate, a braillewriter, or a computer application that allows for direct input of 6-key braille. Use of back translation is allowed but 6-key entry is a necessity as well. A line length of forty cells is required, regardless of production method.

The Lesson Material

The lessons are available by following the "Mathematics Braille Transcribing" link at www.nfb.org/transcribers. If you are unable to utilize the material from the website, please contact us at transcribers@nfb.org or (410) 659-9314, extension 2510, and we will work with you to ensure you receive the material in a format that is usable for you.

Most lessons conclude with an exercise, which is to be submitted to your grader for evaluation. Students are encouraged to submit their work on a regular basis (at least monthly) and may submit only one lesson at a time.

Revisions to the online course lessons are occasionally necessary. The student should check the website to ensure that any lesson being completed is the current version. To report errors in this instruction manual, please send your message to transcribers@nfb.org.

Your Grader

A student must take the course by correspondence with the NFB. After receiving your transcription of the first exercise, a grader will be assigned. Your grader will evaluate your first submission and will be your point of contact for the rest of the course.

The Certification Test

Upon completion of the course, the student may apply for the certification test. When requesting the test, students who have taken the course locally must include a letter from their Library of Congress certified mathematics braille instructor attesting that the student has successfully completed the course. Certification tests are distributed and evaluated by the National Federation of the Blind. Instructions for preparation and submission will accompany the exam. The Library of Congress remains the certifying authority. Candidates scoring a passing grade will receive a Library of Congress certificate.

How to Enroll

Whether intending to take the course with a local teacher or through correspondence, prospective students must submit the application form before beginning the course. Applicants can enroll in the course in one of two ways.

- Complete the application online at https://nfb.org/programs-services/braille-certification/mathematics-braille-transcribing
- Or mail a print application to the address shown below.

    National Federation of the Blind
    Braille Certification Training Program
    200 East Wells Street at Jernigan Place
    Baltimore, MD 21230

Further Information

For more information about the braille certification training program, you may email, call, or write:

- email: transcribers@nfb.org
- call: 410-659-9314, extension 2510
- write to the address shown above.

FOREWORD TO THE 2017 EDITION

The first edition of the *Introduction to Braille Mathematics* was published in 1978 and was written by the late Helen Roberts and Bernard M. Krebs. It was my privilege to complete the text with Mr. Krebs after Helen passed away. Since that time, numerous corrections and updates have been made both to the Nemeth Code itself and to this manual. Now, however, a major change has necessitated a complete rewriting of the lessons. 2016 was the implementation year in the United States for new transcriptions to be produced using the Unified English Braille Code. Because Nemeth Code works *within* UEB, many of the rules of Nemeth Code must be modified.

After the first lesson most examples, practices, and exercises are shown in a text-like context. In this way, the student can see how the Nemeth Code works in a real setting such as found in texts of many grade levels and complexities.

The practices within each lesson are available for self checking by the student. Answers to the practices are given at the end of each lesson. Braille reading practice is offered in Appendix A. Lessons conclude with an exercise which will be graded and evaluated by your teacher or by your NFB-assigned grader.

The student should understand that the Nemeth Code itself is the authoritative source for all mathematics transcriptions. The student should also be thoroughly familiar with the sourcebooks listed in the PREREQUISITES which follow this Foreword.

It has long been my hope that this manual could be brought into the present era. Lindy Walton, an experienced transcriber who works with the NLS Nemeth certification program, led the writing of this Second Edition. Once again, it is my honor to work with an exceptional member of the braille transcriber community.

Both Lindy and I thank the following for their support and help: Mary Denault, Peggy Jackson, Bill Jackson, Kyle DeJute, Julie Sumwalt, Lynnette Taylor, the members of the BANA Nemeth Code Technical Committee, and the Grafton Braille Service Center. We would also like to thank the National Federation of the Blind which has lent support to the development and publication of this comprehensive manual.

—Barbara Taffet

PREREQUISITES

A prerequisite to the study of the Nemeth Code within UEB context is certification in Unified English Braille, adequate experience in literary braille transcription, and confidence in your production method. Before beginning this course of study the student should also be thoroughly familiar with current methods for transcribing a textbook. Rules and guidelines are found in the following sourcebooks, all of which are available from the Braille Authority of North America (BANA) at www.brailleauthority.org. Dates shown below are the editions used as a resource in this lesson manual.

The Rules of Unified English Braille, Second Edition 2013

Braille Formats: Principles of Print-to-Braille Transcription, 2016

The Nemeth Braille Code for Mathematics and Science Notation, 2022

Guidelines and Standards for Tactile Graphics, 2022

Chemical Notation Using Nemeth Braille Code, 2022

STUDY TIPS

HOW TO BECOME AN EXCELLENT NEMETH BRAILLE TRANSCRIBER

Don't race through the lesson material.

- Read carefully and deliberately as the narrative is compact and the language is exact.
- Study the examples and understand the point being made with each one but do not rely on the examples alone for an understanding of the rules. Transcribe the examples to reinforce the rule.
- Do the practice drills. Proofread them before checking the answers. See more tips below.
- Try back translating the braille examples and practices without looking at the print.
- Take special note of rules regarding spacing, punctuation, abbreviations, and format.
- Make lists to help you remember differences between Nemeth and UEB rules.
- Don't be afraid to underline, highlight, or write notes in the margins of your lesson manual.

If the braille or the print doesn't make sense to you ...

- Compare new information to similar topics learned in previous lessons.
- Some of the lesson material is grouped in "use of" and "nonuse of." Compare them and look closely at the braille examples.

THE PRACTICE MATERIAL

- Slow down. By using 6-key entry instead of a translator you will better understand the braille from the reader's point of view.
- Compare your braille transcription to the answers to the practice material found at the end of each lesson. Read each cell closely.
- At the end of each line, look at the braille cell in the line above and in the line below and compare it to the answer key. Any misalignment indicates an error on that line.
- When you identify your errors, return to the lesson to review the applicable rule.

PREPARING THE EXERCISE FOR GRADING

- Don't try to copy braille examples that look like the exercise material. Instead, understand and apply the rule.
- Don't guess. Don't rely on the proofreader's report to find your mistakes.
- Proofread carefully before turning in for grading. Your knowledge and understanding of the Nemeth Code will improve dramatically if you proofread from an embossed copy or from a simulated braille (print) copy, without looking at the print.
- Make note of items you are unsure of. If your transcription is correct, look these items over again after receiving your report to reinforce the rule.

RESEARCH/REVIEW

- Analyze the mistakes found in your exercise and make sure you understand your errors before moving ahead to the next lesson. Ask questions until you are sure of the rule.
- Return to earlier lessons. Topics will make more sense to you in retrospect.

– Read the index. Terminology used there will help you understand the language of Nemeth braille.
– Review format rules learned in earlier lessons. Study the examples.
– Go back to an earlier lesson exercise and back translate the practices or your braille exercise by writing in longhand. Don't look at the print copy until you are finished. Giving yourself some distance from the lesson material is a good review strategy.
– In later lessons, research the topic in the Nemeth Code in addition to studying the lesson book. Not only will this enrich your understanding of the current subject, you will also review material already learned in a new context.

PROOFREADING TIPS

Accuracy is crucially important in technical work. Your proofreading skills will be challenged.
– Is your lighting adequate?
– Use a magnifier when print is questionable.
– Use a straightedge when levels are in question.
– Take breaks when your concentration wanes. Then go back a few pages when resuming proofreading.
– Read the braille dots. Compare often to the print copy.
– Vary your reading medium -- don't always proofread from the screen or from simulated braille or from embossed braille.

BRAILLE TRANSLATION SOFTWARE

Many students of the Nemeth Code have been transcribing for years and have thousands of pages of braille to their credit. They also have been taking advantage of the many electronic input and proofreading aids available to transcribers and are quite adept at turning out high quality work. We expect you are one of those transcribers.

You are undertaking a serious study of one of the technical braille codes, and we would like you to consider stepping back a bit and learning the old fashioned way, using 6-key entry in your braille software program. It is our experience that the best transcribers are those that can read and write braille as the 6-dot code that it is, not solely reading a back translation or a source file and not using another input code to 'type' math problems. Using proofreading and production aids for more accurate and faster work is certainly something you will continue to use – it is important that you understand how your particular software and translation tools work in Nemeth mode – but we are convinced you will understand the Code better if you take the 6-key approach while learning.

CONTENTS

Lesson 2

Lesson 3

Lesson 4

Lesson 5

Lesson 6

Lesson 7

Lesson 8

Lesson 9

Lesson 10

Lesson 11

Lesson 12

Lesson 13

Lesson 14

Lesson 17

Lesson 18

To report errors in this instruction manual,
please send your message to transcribers@nfb.org.

PRELIMINARY LESSON

- INTRODUCTION TO NUMERALS AND THE NUMERIC INDICATOR
- THE MATHEMATICAL COMMA AND DECIMAL POINT
- INTRODUCTION TO SIGNS OF OPERATION
- INTRODUCTION TO SIGNS OF COMPARISON
- MONETARY, PERCENT, AND PRIME SIGNS
- EUROPEAN SYMBOLS

Format

- General Principles

Answers to Practice Material

LESSON PREVIEW

This lesson introduces the student to the design of the course as well as some basic Nemeth symbols. *Complete this lesson before studying Lesson 1.* Practice exercises are self scored, and a short reading exercise is offered in Appendix A.

P1 Philosophy

The Nemeth braille code is especially designed for the representation and transcription of mathematical notation encountered in educational materials on the subjects of mathematics and the sciences. Its purpose is to convey, as accurately as possible, a clear conception of the printed text to the braille reader. Using braille indicators in conjunction with the 63 braille characters, this code is capable of providing equivalent symbols for the hundreds of mathematical and scientific print signs now in use and yet to be devised. The one-to-one correspondence between braille and print symbols makes it possible to produce an accurate transference from print to braille or from braille to print.

P2 Literary vs. Technical Texts

P2.1 **Literary Texts.** Literary works which use only occasional mathematical notation are transcribed in accordance with the rules of Unified English Braille ("UEB"), using mathematical symbols and rules given in the most recent edition of *The Rules of Unified English Braille* and *Unified English Braille Guidelines for Technical Material*.

P2.2 **Technical Texts.** When mathematical notation is encountered in educational materials or in technical documents in the fields of mathematics, statistics, physics, or chemistry, the rules of the Nemeth Code are followed. Non mathematical narrative is transcribed using the symbols and rules of UEB, and mathematical notation is transcribed using the symbols and rules of the Nemeth Code. Shrewd use of code switch indicators clearly demarcate the technical material.

INTRODUCTION TO NUMERALS AND THE NUMERIC INDICATOR

Note: This section does not cover all of the rules regarding the use/nonuse of the numeric indicator.

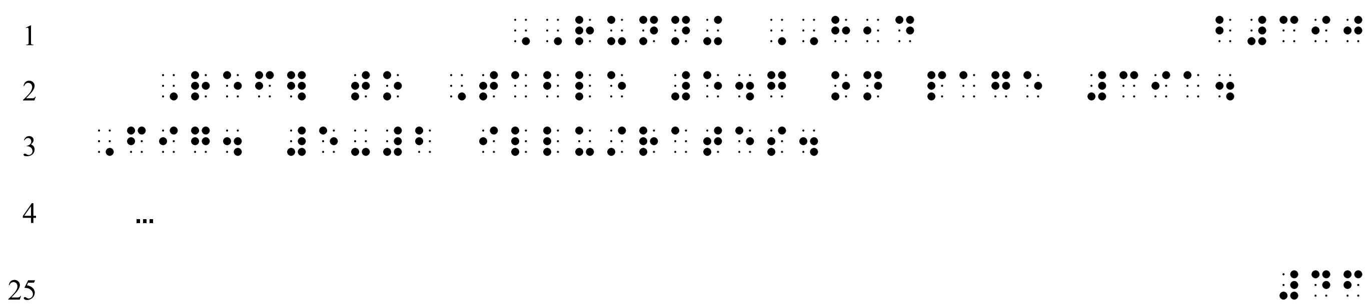
P3 Representation of Arabic Numerals

In the transcription of a technical text, digits are represented in two ways:

P3.1 **English Braille Numerals.** Numbers used to label figures, tables, sections, etc. as well as any associated punctuation are transcribed in UEB. Print page numbers, braille page numbers, and page numbers referred to within the text are transcribed in UEB.

Example P-1

Refer to Table 5.7 on page 391. Fig. 5-2 illustrates.



The number at the end of line 1 is print page number b390. The last number represents braille page 46 on line 25 of the braille page.

Numbers that have mathematical meaning within the narrative may be transcribed in UEB as long as the number is freestanding and is unmodified. "Unmodified" in this context means there is no symbol associated with the number, such as a monetary symbol or a percent sign, for example.
"Freestanding and unmodified" includes numbers with an internal comma, ordinals, and plurals. An unmodified number that touches literary punctuation is considered to be freestanding.

Example P-2

9 inches and 15 inches are 2 feet.

Example P-3

On Monday the 4th, Jamie’s step counter recorded 9,999 steps.

⠀⠀⠠⠕⠝ ⠠⠍⠕⠝⠐⠙ ⠮ ⠼⠙⠞⠓⠂ ⠠⠚⠁⠍⠊⠑⠄⠎ ⠌⠑⠏ ⠉⠨⠞⠻
⠗⠑⠉⠕⠗⠙⠫ ⠼⠊⠂⠊⠊⠊ ⠌⠑⠏⠎⠲

Example P-4

Which is greater: two 3's or three 2's?

⠀⠀⠠⠱ ⠊⠎ ⠛⠗⠞⠻⠒ ⠞⠺⠕ ⠼⠉⠄⠎ ⠕⠗ ⠹⠗⠑⠑ ⠼⠃⠄⠎⠦

P3.2 **Nemeth Digits.** Nemeth digits are used to represent Arabic numerals which occur in a mathematical expression. Nemeth digits are also required for a freestanding mathematical number within narrative that is not "unmodified," as defined in P3.1. Examples of a modified number include a negative number, a decimal, a number associated with a monetary symbol, a number associated with a percent sign.

The ten Nemeth digits are represented by the letters "a" through "j" dropped to the lower part of the braille cell.

1	2	3	4	5	6	7	8	9	0
⠂	⠆	⠒	⠲	⠢	⠖	⠶	⠦	⠔	⠴

Assume mathematical context in the isolated examples presented throughout the remainder of this lesson, even though the rules allow that unmodified, freestanding mathematical numbers can be transcribed in UEB.

P4 Numeric Indicator

Unless otherwise stated, the numeric indicator is required before a numeral that follows a space or before a numeral that begins a braille line.

⠼	Numeric Indicator

Example P-5

5 10 15 20

⠼⠢ ⠼⠂⠴ ⠼⠂⠢ ⠼⠆⠴

P4.1 **Special Case—Partitioned Numbers.** The numeric indicator is not used following a space that partitions a number into segments. Partitioned numbers must be transcribed in Nemeth because the numeric space indicator of UEB is not to be used in a Nemeth transcription.

Example P-6

987 654 321

⠼⠔⠦⠶ ⠖⠢⠲ ⠒⠆⠂

A particular book may show large numbers in this manner rather than using commas to delineate place value. Check for context clues to be sure this represents 987 million 654 thousand 321 and not three separate 3-digit numbers.

THE PRACTICE MATERIAL

By transcribing the practice material you will gain firsthand experience with the topics presented in each lesson and you will be better prepared to transcribe the exercise for grading. Many of the points discussed in the lesson are illustrated only in the practice material. The Study Tips on pages viii-ix offer more ways to get the most out of these activities.

Check your work by comparing your transcription to the simulated braille located at the end of each lesson.

PRACTICE A

Instructions: Transcribe the following numbers using the lower-cell digits of the Nemeth Code. Begin in cell 1. Leave one blank cell between each number. Begin a new line in cell 1 when you do not have room on a line to complete a number. None of these numbers are partitioned into segments—each is a new number. Check your accuracy by comparing your transcription to the practice answers at the end of this lesson.

123 456 7890 295 431 61 507 3196 15837 808 46 282 2802

61640 74 9559 404 75134 13579

THE MATHEMATICAL COMMA AND DECIMAL POINT

P5 Mathematical Comma

The mathematical comma is used for a comma occurring in a long numeral. It is also used for a comma which follows a numeral or other mathematical expression.

⠠	Mathematical Comma	,

Although numbers with commas can be transcribed in UEB, for illustrative purposes, please assume mathematical context in the isolated examples presented below.

Example P-7

987,654,321

⠼⠔⠦⠶⠠⠖⠢⠲⠠⠒⠆⠂

This represents the number 987 million, 654 thousand, 321.

Example P-8

997, 998, 999, 1,000

⠼⠔⠔⠶⠠ ⠼⠔⠔⠦⠠ ⠼⠔⠔⠔⠠ ⠼⠂⠠⠴⠴⠴

These are four individual numbers, separated by a comma and a space. The last number contains an internal comma.

Symbol Recognition: See Section P14 for a discussion of the European decimal point, which is depicted by the same print symbol as a mathematical comma.

P6 Mathematical Decimal Point

⠨	Mathematical Decimal Point	.

P6.1 **Spacing of the Decimal Point.** In a numeral, no space is left between the decimal point and the digits to which it applies.

Example P-9

3.14159

⠼⠒⠨⠂⠲⠂⠢⠔

P6.2 **The Decimal Point and the Numeric Indicator.** The numeric indicator is required before a decimal point that precedes a numeral when the decimal point follows a space or begins a braille line.

Example P-10

.25 .5 .75

⠼⠨⠆⠢ ⠼⠨⠢ ⠼⠨⠶⠢

Reminder: When a decimal is part of a numeric label to a figure, table, section, etc., UEB is used. See Section P3.1.

Format

P7 General Principles

"Format" refers to layout on the page, such as indentations (margins), line spacing (blank lines), centering, and pagination. *The Nemeth Braille Code for Mathematics and Science Notation* specifies certain formats which are covered in these lessons and are also summarized in Appendix C of this course.

When an item in a UEB transcription requires the use of Nemeth symbols, format rules of *The Nemeth Braille Code for Mathematics and Science Notation* are to be applied to the entire transcription including those portions transcribed in UEB. When a format is not specifically addressed in the Nemeth Code, the principles provided in *Braille Formats* should be followed.

PRACTICE B

Instructions: Begin the list on line 1 of the braille page. Use Nemeth numerals for all numbers in this list. Duplicate the columnar format shown. Following *Braille Formats* guidelines for the layout, you will leave a column of two blank cells between the end of the longest item in each column and the left-hand margin of the next column. These columns are unrelated therefore guide dots are not used.

592	.75	345	4.6692
206	6.4	29,254	98.6
46	59.1	1.234	3.14159
.240	0.37	1791	31,536,000
3,250	0	70.2	365.2422
8,086	987,654	.008382	273.15

INTRODUCTION TO SIGNS OF OPERATION

P8 Signs of Operation

The most common signs of operation are listed below.

⠬	Plus	$+$
⠤	Minus	$-$
⠈⠼	Multiplication Asterisk	$*$
⠈⠡	Multiplication Cross	$\times$
⠡	Multiplication Dot	$\cdot$
⠨⠌	Division (divided by)	$\div$

Since the minus sign and the hyphen are represented by the same symbol in braille, the reader determines the meaning of the symbols from context.

Symbol Recognition: See Section P13 for a discussion of the European comma, which is depicted by the same print symbol as the multiplication dot.

P8.1 **Spacing with Signs of Operation.** Unless otherwise stated, a sign of operation is unspaced from its related mathematical terms regardless of the print spacing. A numeric indicator is generally not needed within an unspaced expression. However, because the asterisk symbol includes dots 3456, a numeric indicator is required for the numeral following the asterisk.

Example P-11

$2+5$ $613-16$ 19×8 $5\cdot 3$ $98*7$ $40\div 5$

⠼⠆⠬⠢ ⠼⠖⠂⠒⠤⠂⠖ ⠼⠂⠔⠈⠡⠦ ⠼⠢⠡⠒ ⠼⠔⠦⠈⠼⠼⠶ ⠼⠲⠴⠨⠌⠢

P8.2 **Positive and Negative Numbers.** Numerals preceded by a plus sign or a minus sign must be transcribed in Nemeth. A numeral preceded by a minus sign requires a numeric indicator when the minus sign follows a space or begins a braille line. A numeral preceded by a plus sign does not require a numeric indicator even when the plus sign follows a space or begins a braille line.

Example P-12

-3 -2 -1 0 $+1$ $+2$ $+3$

⠤⠼⠒ ⠤⠼⠆ ⠤⠼⠂ ⠼⠴ ⠬⠂ ⠬⠆ ⠬⠒

Note: These are seven separate numerals.

a. **Decimals.** The numeric indicator is required between a minus sign and a decimal point that precedes a numeral when the minus sign follows a space or begins a braille line. A numeral preceded by a plus sign and a decimal point does not require a numeric indicator even when the plus sign follows a space or begins a braille line.

Example P-13

−.75, −.5, −.25, 0.0, +.25, +.5

⠤⠼⠨⠶⠢⠠ ⠤⠼⠨⠢⠠ ⠤⠼⠨⠆⠢⠠ ⠼⠴⠨⠴⠠ ⠬⠨⠆⠢⠠ ⠬⠨⠢

PRACTICE C

Instructions: Transcribe these unrelated columns using a two-column format.

+592	$9.75 + 16.22$
−7.5	$10{,}000 - 3{,}560$
404.8	19×18
−.9	$512 \cdot 63$
.708	$3{,}951 \div 9 * 7$

INTRODUCTION TO SIGNS OF COMPARISON

P9 Signs of Comparison

A few signs of comparison and their braille equivalents are listed below.

⠨⠅	Equals	=
⠨⠂	Greater Than (is greater than)	>
⠐⠅	Less Than (is less than)	<
⠰⠆	Proportion (as)	::
⠐⠂	Ratio (is to)	:

P9.1 **Spacing with Signs of Comparison.** A space is required between a sign of comparison and a sign of operation or any other expression which precedes or follows it. *Reminder*: A numeric indicator is usually required when a numeral is preceded by a space.

Example P-14

72,539 × 33.3 = 2,415,548.7

⠼⠶⠆⠠⠢⠒⠔⠈⠡⠒⠒⠨⠒⠀⠨⠅⠀⠼⠆⠠⠲⠂⠢⠠⠢⠲⠦⠨⠶

Example P-15

−3 < 0 < +3

⠤⠼⠒⠀⠐⠅⠀⠼⠴⠀⠐⠅⠀⠬⠒

Example P-16

6 : 9 :: 2 : 3

⠼⠖⠀⠐⠂⠀⠼⠔⠀⠰⠆⠀⠼⠆⠀⠐⠂⠀⠼⠒

PRACTICE D

Instructions: Begin each mathematical expression on a new line in cell 1.

8.5 < 74

85 > 9.6

29 · 3 = 3 · 29

14 : 2 :: 7 : 1

19,530 − 2,016 × 8.25 + 6.75 = 262,710.00

MONETARY, PERCENT, AND PRIME SIGNS

P10 Monetary Signs

The monetary signs of the Nemeth Code are the same as those used in UEB.

⠈⠉	Cent	¢
⠈⠎	Dollar	$
⠈⠑	Euro	€
⠈⠋	Franc	₣
⠈⠝	Naira	₦
⠈⠇	Pound Sterling	£
⠈⠺	Won	₩
⠈⠽	Yen or Yuan	¥

If a monetary sign is printed for which there is no established symbol, the transcriber should create one following the same "dot 4" pattern shown above.

P10.1 **Spacing with Monetary Signs.** No space is left between a monetary sign and its related quantity or symbol. A number which immediately follows a monetary sign does not need a numeric indicator.

Example P-17

$\$3.50 = 350¢$

⠈⠎⠒⠨⠢⠴ ⠨⠅ ⠼⠒⠢⠴⠈⠉

P11 Percent and Per Mille Signs

⠈⠴	Percent sign	%
⠈⠴⠴	Per mille sign	‰

P11.1 **Spacing with Percent and Per Mille Signs.** No space is left between these signs and their related quantities or symbols.

Example P-18

$45\% = 0.45$

⠼⠲⠢⠈⠴ ⠨⠅ ⠼⠴⠨⠲⠢

Example P-19

35‰ = .035

⠼⠒⠢⠈⠴⠴ ⠨⠅ ⠼⠨⠴⠒⠢

P12 Prime Sign

⠄	Prime Sign	'
⠄⠄	(two prime signs)	"

The braille symbol for the prime sign is used wherever the print symbol appears in mathematical context regardless of its meaning. When more than one prime sign is used in print, the equivalent number of signs are used in braille. Prime signs must be unspaced from each other and from the quantity to which they apply. In the following example, the prime sign is used to denote feet and inches.

Example P-20

4'3" > 43"

⠼⠲⠄⠒⠄⠄ ⠨⠂ ⠼⠲⠒⠄⠄

PRACTICE E

Instructions: Retain the simple vertical listing, beginning each line in cell 1.

25¢ − 5¢ = 20¢

$4.89 + 5.5% = $5.16

36‰ × 100 = 3.6

5'8" = 68"

$1 = £0.633456

EUROPEAN SYMBOLS

P13 The European Comma

⠠	American Mathematical Comma	,
⠨	European Mathematical Comma	.

The print symbol for the European comma is different from the comma used in the United States. The braille symbol follows print: dot 6 represents the American comma; dots 46 represent the European comma.

Example P-21

$27{,}000 = 27.000 = 27\ 000$

⠼⠆⠶⠠⠴⠴⠴ ⠨⠅ ⠼⠆⠶⠨⠴⠴⠴ ⠨⠅ ⠼⠆⠶ ⠴⠴⠴

P14 The European Decimal Point

⠨	American Decimal Point	.
⠠	European Decimal Point	,

The print symbol for the European decimal point is different from the decimal used in the United States. The braille symbol follows print: dots 46 represent the American decimal point; dot 6 represents the European decimal point.

Example P-22

$\$19.99 < £19{,}99$

⠈⠎⠂⠔⠨⠔⠔ ⠐⠅ ⠈⠇⠂⠔⠠⠔⠔

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

PRACTICE A

1 ⠼⠂⠆⠒ ⠼⠲⠢⠖ ⠼⠶⠦⠔⠴ ⠼⠆⠔⠢ ⠼⠲⠒⠂ ⠼⠖⠂ ⠼⠢⠴⠶ ⠼⠒⠂⠔⠖
2 ⠼⠂⠢⠦⠒⠶ ⠼⠦⠴⠦ ⠼⠲⠖ ⠼⠆⠦⠆ ⠼⠆⠦⠴⠆ ⠼⠖⠂⠖⠲⠴ ⠼⠶⠲
3 ⠼⠔⠢⠢⠔ ⠼⠲⠴⠲ ⠼⠶⠢⠂⠒⠲ ⠼⠂⠒⠢⠶⠔

PRACTICE B

1	⠼⠢⠔⠆	⠼⠨⠶⠢	⠼⠒⠲⠢	⠼⠲⠨⠖⠖⠔⠆
2	⠼⠆⠴⠖	⠼⠖⠨⠲	⠼⠆⠔⠠⠆⠢⠲	⠼⠔⠦⠨⠖
3	⠼⠲⠖	⠼⠢⠔⠨⠂	⠼⠂⠨⠆⠒⠲	⠼⠒⠨⠂⠲⠂⠢⠔
4	⠼⠨⠆⠲⠴	⠼⠴⠨⠒⠶	⠼⠂⠶⠔⠂	⠼⠒⠂⠠⠢⠒⠖⠠⠴⠴⠴
5	⠼⠒⠠⠆⠢⠴	⠼⠴	⠼⠶⠴⠨⠆	⠼⠒⠖⠢⠨⠆⠲⠆⠆
6	⠼⠦⠠⠴⠦⠖	⠼⠔⠦⠶⠠⠖⠢⠲	⠼⠨⠴⠴⠦⠒⠦⠆	⠼⠆⠶⠒⠨⠂⠢

Did You Know? The numbers in the rightmost column are significant scientific or mathematical numbers.

4.6692	the first six digits of one of Feigenbaum's constants from chaos theory
98.6	average healthy human body temperature in degrees Fahrenheit
3.14159	the first six digits of pi
31,536,000	the number of seconds in a year
365.2422	the number of days in a solar year
273.15	degrees Kelvin equivalent to zero degrees Celsius

PRACTICE C

1	⠬⠢⠔⠆	⠼⠔⠨⠶⠢⠬⠂⠖⠨⠆⠆
2	⠤⠼⠶⠨⠢	⠼⠂⠴⠠⠴⠴⠴⠤⠒⠠⠢⠖⠴
3	⠼⠲⠴⠲⠨⠦	⠼⠂⠔⠈⠡⠂⠦
4	⠤⠼⠨⠔	⠼⠢⠂⠆⠡⠖⠒
5	⠼⠨⠶⠴⠦	⠼⠒⠠⠔⠢⠂⠨⠌⠔⠈⠼⠼⠶

PRACTICE D

1 ⠼⠦⠨⠢ ⠐⠅ ⠼⠶⠲

2 ⠼⠦⠢ ⠨⠂ ⠼⠔⠨⠖

3 ⠼⠆⠔⠡⠒ ⠨⠅ ⠼⠒⠡⠆⠔

4 ⠼⠂⠲ ⠐⠂ ⠼⠆ ⠰⠆ ⠼⠶ ⠐⠂ ⠼⠂

5 ⠼⠂⠔⠠⠢⠒⠴⠤⠆⠠⠴⠂⠖⠈⠡⠦⠨⠆⠢⠬⠖⠨⠶⠢ ⠨⠅ ⠼⠆⠖⠆⠠⠶⠂⠴⠨⠴⠴

PRACTICE E

1 ⠼⠆⠢⠈⠉⠤⠢⠈⠉ ⠨⠅ ⠼⠆⠴⠈⠉

2 ⠈⠎⠲⠨⠦⠔⠬⠢⠨⠢⠈⠴ ⠨⠅ ⠈⠎⠢⠨⠂⠖

3 ⠼⠒⠖⠈⠴⠴⠈⠡⠂⠴⠴ ⠨⠅ ⠼⠒⠨⠖

4 ⠼⠢⠄⠦⠄⠄ ⠨⠅ ⠼⠖⠦⠄⠄

5 ⠈⠎⠂ ⠨⠅ ⠈⠇⠴⠨⠖⠒⠒⠲⠢⠖

This lesson has no exercise to hand in. Proceed to Lesson 1.

LESSON 1

- INTRODUCTION TO CODE SWITCHING
    - Placement of the Code Switch Indicators
- THE HYPHEN AND THE DASH
- SIGNS OF OMISSION
- INTRODUCTION TO IDENTIFIERS

Format

- Keep Together—Mathematical Expression
- Margins for Narrative (3-1)
- Margins for Itemized Material with No Subdivisions (1-3)
- FORMAT SUMMARY #1

Answers to Practice Material

LESSON PREVIEW

Introduction to the rules regarding code switching and use of Nemeth code switch indicators. The hyphen and the short dash are studied. Three signs of omission are introduced: the ellipsis, the long dash, and the general omission symbol. Two Nemeth Code formats are illustrated: 3-1 narrative and 1-3 itemized material.

Do not begin Lesson 1 until you have completed the Preliminary Lesson.

INTRODUCTION TO CODE SWITCHING

1.1 A Complete Transcription

The base code used in a "UEB with Nemeth" transcription is Unified English Braille ("UEB"). When mathematical content occurs anywhere in the transcription, the non mathematical notation follows UEB rules while the mathematical notation follows the rules of the Nemeth Code. The reader will be reading Unified English Braille unless signaled otherwise by the use of a code switch indicator.

⠸⠩	Opening Nemeth Code indicator
⠸⠱	Nemeth Code terminator

These symbols must be listed on the Special Symbols page in the transcriber-generated portion of each volume. See the Final Lesson for further details.

The opening Nemeth Code indicator is followed by a space (one blank cell). The Nemeth Code terminator is preceded by a space (one blank cell). These spaces do not represent a space in print.

The opening Nemeth Code indicator and the Nemeth Code terminator may also be referred to as "code switch indicators" or "switches".

The following principle is central to a smooth reading of UEB with Nemeth transcription: *UEB symbols are not used inside of the Nemeth code switches. Nemeth symbols are not used outside the switches.*

1.2 Use of the Code Switch Indicators

Switch to Nemeth when a mathematical symbol or a mathematical expression is encountered.

The code switch indicators are highlighted throughout this lesson.

Example 1-1

Use either the + or the × symbol to make the statements TRUE.

⠠⠥⠎⠑ ⠑⠊ ⠮ ⠸⠩ ⠬ ⠸⠱ ⠕⠗ ⠮ ⠸⠩ ⠈⠡ ⠸⠱ ⠎⠽⠍⠃⠕⠇
⠞⠕ ⠍⠁⠅⠑ ⠮ ⠌⠁⠞⠑⠰⠞⠎ ⠠⠠⠞⠗⠥⠑⠲

Recall that the space after the opening Nemeth Code indicator and the space before the Nemeth Code terminator do not represent a space in print.

Example 1-2

$2 + 2 = 22$ is Grampa's favorite joke.

Instructions: Transcribe PRACTICE 1A by beginning each sentence in cell 3, with any runovers in cell 1. Compare your transcription to the answers at the end of this lesson. (Read about the practice material in the Preliminary Lesson on page P-4.)

PRACTICE 1A

What does $16 + 4 + 100$ equal?

If $10 \times 10 \times 10 = 1{,}000$ what does $10 \times 10 \times 10 \times 10$ equal?

Does $5 \div 2$ name a whole number?

% ("percent") means parts per hundred; ‰ ("per mille") means parts per thousand.

1.3 Which Code?

1.3.1 **UEB Required.** Recall that page numbers and labels to figures, tables, sections, etc. are transcribed in UEB. Examples are shown in Section P3.1 of the Preliminary Lesson. Here is another example that does not require code switching.

Example 1-3

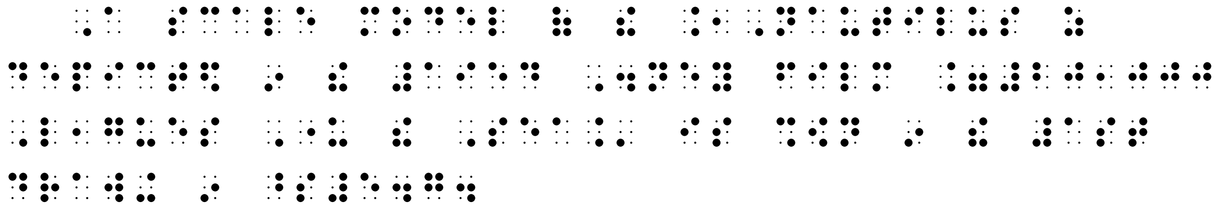
To prepare for tomorrow's quiz, review Chapters 3-4 and Table 3.1.7 on page T14.

1.3.2 **UEB Allowed.** It is best not to overdo switching just for the sake of a simple item that is easily read in either code. Recall from the Preliminary Lesson that numbers with mathematical meaning within narrative context may be transcribed in UEB as long as the number is freestanding and is unmodified. "Unmodified" in this context means there is no mathematical symbol associated with the number.

The definition of "freestanding" includes numbers with internal commas, with an ordinal ending, or with a plural ending such as "s" or "apostrophe-s". This applies to incidental numbers encountered in narrative, as well as to freestanding numbers in a word problem. When the surrounding text is UEB, there is no need to switch to Nemeth Code for such numbers. (This rule does not apply if a long number must be divided. See 1.7.1.)

Example 1-4

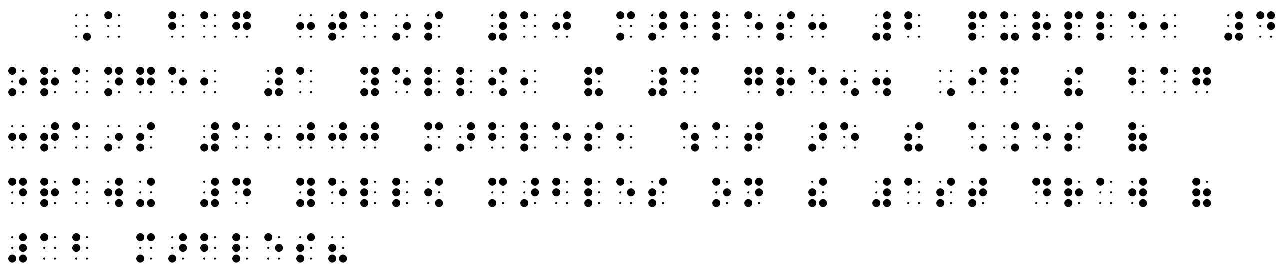
A scale model of the *Nautilus* as depicted in the 1954 Disney film *20,000 Leagues Under the Sea* is shown in the 1st drawing in §5.7.



Within this narrative sentence, the number with the internal comma is transcribed in UEB, as is the ordinal. The UEB section sign is used for the section reference.

Example 1-5

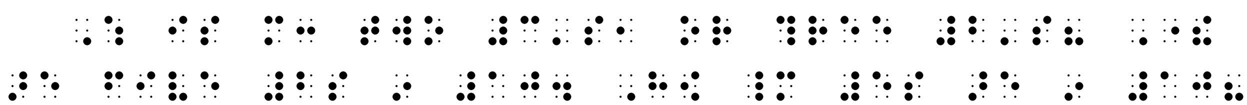
A bag contains 10 marbles: 2 purple, 4 orange, 1 yellow, and 3 green. If the bag contains 1,000 marbles, what are the chances of drawing 4 yellow marbles on the 1st draw of 12 marbles?



In this word problem, the freestanding, unmodified numbers are transcribed in UEB.

Example 1-6

Which is more: two 3's, or three 2's? There are five 2s in 10. How many 5s are in 10?



In this word problem, plural numerals are transcribed in UEB.

a. When an unmodified number is touching literary punctuation such as quotes or parentheses, it is still considered to be freestanding and may be transcribed in UEB.

Example 1-7

Is the "5" in the number "5093" in the hundreds place (500) or in the thousands place (5,000)?

Throughout this course, isolated examples which illustrate Nemeth constructions are marked with a chevron symbol. Nemeth switch indicators are omitted in order to focus on the construction itself. In the context of a complete transcription, code switch indicators are required. In many cases, the isolated construction is incorporated into the example which follows, where use of the code switch indicators is also demonstrated.

1.3.3 **Nemeth Required.** If a freestanding number or letter is combined with anything other than an ordinal, an internal comma, or a plural, it is considered to be modified and is transcribed in Nemeth. Here are some examples.

a. A numeral associated with an operation sign must be transcribed in Nemeth Code. A numeral associated with a monetary symbol or with a percent sign is included inside the switches.

⋙ 10×12

⋙ $7

⋙ 5%

Example 1-8

How many cans of paint does Rosie need to paint her 10×12 foot bedroom? At $7 per can, plus 5% tax, what will it cost to complete the job?

A switch to Nemeth Code is required for the multiplication symbol, for the monetary symbol, and for the percent sign. The numbers associated with those symbols are also transcribed in Nemeth.

b. A negative numeral must be transcribed Nemeth Code. A decimal number with mathematical meaning is transcribed in Nemeth Code.

> ⫸ –5 ⠤⠼⠢

> ⫸ 3.14159 ⠼⠒⠨⠂⠲⠂⠢⠔

Example 1-9

Which is colder, –5 degrees Celsius or 5 degrees Fahrenheit?
What does this famous decimal number represent? 3.14159

⠀⠀⠠⠱ ⠊⠎ ⠉⠕⠇⠙⠻⠂ ⠸⠩ ⠤⠼⠢ ⠸⠱ ⠙⠑⠛⠗⠑⠑⠎
⠠⠉⠑⠇⠎⠊⠥⠎ ⠕⠗ ⠼⠑ ⠙⠑⠛⠗⠑⠑⠎ ⠠⠋⠁⠓⠗⠢⠓⠑⠊⠞⠦
⠀⠀⠠⠱⠁⠞ ⠙⠕⠑⠎ ⠹ ⠋⠁⠍⠳⠎ ⠙⠑⠉⠊⠍⠁⠇ ⠝⠥⠍⠃⠻
⠗⠑⠏⠗⠑⠎⠢⠞⠦ ⠸⠩ ⠼⠒⠨⠂⠲⠂⠢⠔ ⠸⠱

A switch to Nemeth Code is required for the minus symbol and for the decimal point. The numbers associated with those symbols are also transcribed in Nemeth. The freestanding, unmodified number 5 is transcribed in UEB.

1.4 Placement of Literary Punctuation

When Nemeth is terminating and punctuation follows, the function of the punctuation mark must be determined. If the punctuation applies to the structure of the sentence and is not actually part of the math expression, the Nemeth Code terminator is transcribed first. The punctuation mark then will follow the termination indicator. There is no space between the terminator and the punctuation mark.

> ⫸ 931 684 572 ⠼⠔⠒⠂ ⠖⠦⠲ ⠢⠶⠆

Example 1-10

931684572 can be divided into groups of three digits each: 931 684 572.

⠀⠀⠼⠊⠉⠁⠋⠓⠙⠑⠛⠃ ⠉ ⠆ ⠙⠊⠧⠊⠙⠫ ⠔⠞⠕ ⠛⠗⠳⠏⠎ ⠷ ⠹⠗⠑⠑
⠙⠊⠛⠊⠞⠎ ⠑⠁⠡⠒ ⠸⠩ ⠼⠔⠒⠂ ⠖⠦⠲ ⠢⠶⠆ ⠸⠱⠲

The period is placed outside of the Nemeth Code terminator. Note that the first numeral does not require a switch to Nemeth because it is a freestanding, unmodified numeral. The second numeral requires a switch because the UEB numeric space indicator is not used in a Nemeth transcription. Review the topic of partitioned numbers in Section P4.1 of the Preliminary Lesson.

Instructions: Center the heading. Begin each problem in cell 3 with runovers in cell 1.

PRACTICE 1B

If $47 - 41 > 1$, do you suppose that $41 - 47 < 1$? Why or why not?

If $5 + 10$ equals 15, what does $10 + 5$ equal? Explain.

Format

1.5 Keep Together—General Principle Regarding Mathematical Expressions

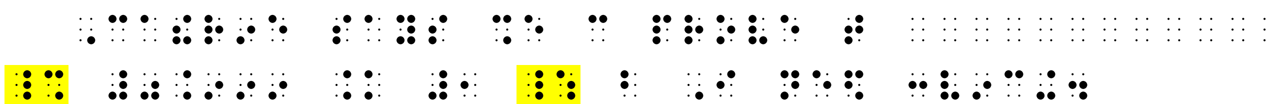
A mathematical expression that will fit on one braille line within the current margins must not be divided between lines. The entire expression is brought down to the next line.

⫸ $0.999 = 1$ 

The equation constitutes one mathematical expression.

Example 1-11

Catherine says she can prove that $0.999 = 1$ but I need convincing.



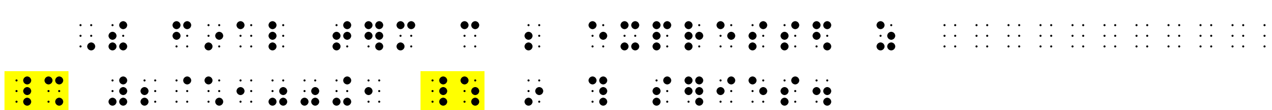
In the "keep together" examples, a dotted underscore shows the space remaining on the preceding line.

Placement of the Switch Indicators

1.5.1 **Keep Together If Possible.** Within a paragraph, the switch indicators should appear on the same line as the expression <u>if the mathematical expression and the two indicators will fit on one braille line</u> within the current margins. In the previous example as well as the next one a new line is forced to begin with the opening Nemeth Code indicator even though the indicator could fit on the previous line.

Example 1-12

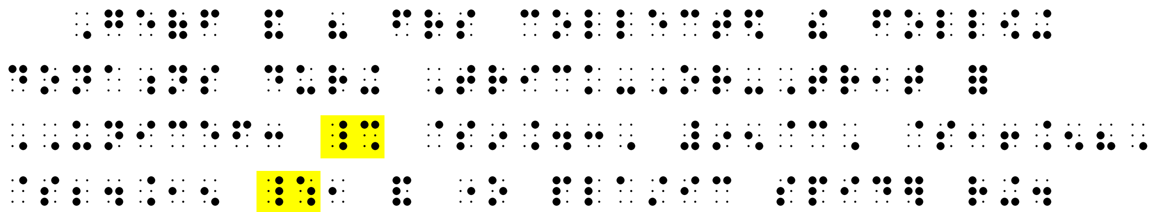
The final term can be expressed as $2 \times 100 + 1$ in this series.



a. If two or more math expressions occur between the same code switch indicators, the line may wrap at the space between the expressions, even if the entire Nemeth portion could fit on one line.

Example 1-13

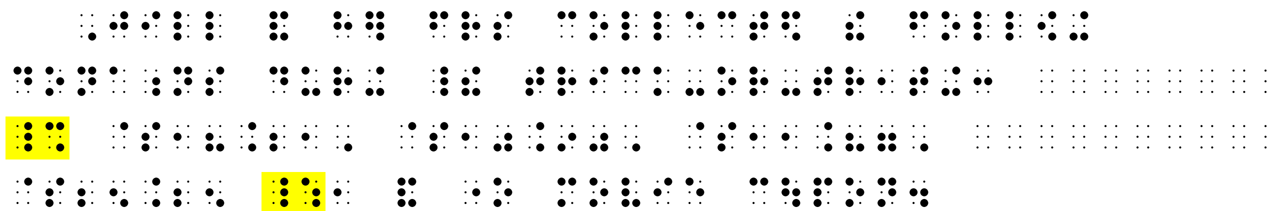
Geoff and his friends collected the following donations during Trick-Or-Treat for UNICEF: $9.43, 95¢, $16.58, $24.15, and one plastic spider ring.



b. Within a paragraph, a switch indicator should not stand alone on a line if there is room for it to fall on the line with the math expression to which it applies.

Example 1-14

Jill and her friends collected the following donations during their trick-or-treating: $18.21, $10.90, $11.87, $25.25, and one movie coupon.



The opening Nemeth Code indicator is placed on the same line as the first dollar amount, even though there is room for just the indicator on the previous line. The last dollar amount is placed on the same line as the Nemeth Code terminator, even though there is room for just the dollar amount on the previous line.

1.5.2 **A Switch Indicator May Stand Alone on a Line.** If a math expression will fit on one line but there is not room for one or both of the switch indicators, one or both switch indicators may stand alone on a line. Keeping the mathematical expression intact on one line is the priority. Several layouts are illustrated below.

Example 1-15

Multiplication can be expressed as addition. For example, 14×12 is the same as twelve fourteens, or 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14 + 14.

This opening Nemeth Code indicator falls on the line before the long expression.

Example 1-16

Similarly, 13×12 is the same as 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13.

This Nemeth Code terminator (along with the related punctuation) falls on the line after the long expression.

Example 1-17

... Reducing to prime factors, 252 = 4 · 63 = 4 · 3 · 21 = 2 · 2 · 3 · 3 · 7 .

Because the expression takes up the full available line width, both the opening Nemeth Code indicator and the Nemeth Code terminator fall on separate lines.

Note: A sign of comparison links the symbols on the left side of the comparison sign with the symbols on the right side of the comparison sign and so is considered to be one mathematical expression. The math portion in the example above is one expression, not four, and should not be divided between lines if it will fit on one line within the current margins.

1.5.3 **Switch Indicators at Braille Page Turns.** The effect of the opening Nemeth Code indicator is not terminated by transition to a new braille page. If, however, the switch to Nemeth occurs at a braille page turn, place the opening switch indicator on the same braille page as the mathematical material to which it applies. Similarly, the Nemeth Code terminator should be placed on the same braille page as the end of the mathematical material.

Example 1-18

... Reducing to prime factors, $252 = 4 \cdot 63 = 4 \cdot 3 \cdot 21 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 7$.

25 ⠲⠲⠲ ⠠⠗⠫⠥⠉⠬ ⠞⠕ ⠏⠗⠊⠍⠑ ⠋⠁⠉⠞⠕⠗⠎⠂ ⠼⠁⠁

---

1 ⠸⠩ ⠃⠼⠙⠙
2 ⠼⠆⠢⠆ ⠨⠅ ⠼⠲⠡⠖⠒ ⠨⠅ ⠼⠲⠡⠒⠡⠆⠂ ⠨⠅ ⠼⠆⠡⠆⠡⠒⠡⠒⠡⠶
3 ⠸⠱⠲

Although the opening switch indicator will fit after the word "factors," it would then fall on line 25 of the preceding braille page.

Example 1-19

... 36 reduces to four prime factors. $36 = 2 \cdot 18 = 2 \cdot 2 \cdot 9 = 2 \cdot 2 \cdot 3 \cdot 3$

24 ⠲⠲⠲ ⠼⠉⠋ ⠗⠫⠥⠉⠑⠎ ⠞⠕ ⠋⠳⠗ ⠏⠗⠊⠍⠑ ⠋⠁⠉⠞⠕⠗⠎⠲
25 ⠼⠁⠁

---

1 ⠃⠼⠙⠙
2 ⠸⠩ ⠼⠒⠖ ⠨⠅ ⠼⠆⠡⠂⠦ ⠨⠅ ⠼⠆⠡⠆⠡⠔ ⠨⠅ ⠼⠆⠡⠆⠡⠒⠡⠒ ⠸⠱

Although the opening switch indicator will fit after the word "factors," and the math would fit on line 25, the Nemeth Code terminator will not fit there because of the braille page number. Instead, the math expression and the two switch indicators are arranged so they will fall on the same braille page. This expression and its two code switch indicators will fit on one line so it begins on line 2.

Switch indicators at print page turns will be discussed in Lesson 3.

Instructions: Begin each paragraph in cell 3 with runovers in cell 1.

PRACTICE 1C

How many squares can you find on an 8 × 8 checkerboard? (*Hint*: There are more than 65 squares.) First determine how many 1 × 1 squares, then how many 2 × 2 squares, and so on, you can find.

12×11 is also the same as twelve elevens, or 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11.

1.6 Consistency with Mathematical Symbols

Although UEB provides symbols for math notation and measurement, the transcriber's objective is to maintain consistency in the appearance of the mathematical symbols throughout the transcription. A symbol that is used in mathematical context is transcribed in Nemeth, even within narrative. This includes the symbols learned in the Preliminary Lesson—signs of operation, signs of comparison, monetary, percent, and prime signs.

This example uses Nemeth symbols for prime signs representing minutes and seconds of arc. A switch to Nemeth Code is required.

⫸ 67'44'' ⠼⠖⠶⠄⠲⠲⠄⠄

Example 1-20

Convert 67'44'' to decimal degree form.

⠠⠒⠧⠻⠞ ⠸⠩ ⠼⠖⠶⠄⠲⠲⠄⠄ ⠸⠱ ⠞⠕ ⠙⠑⠉⠊⠍⠁⠇ ⠙⠑⠛⠗⠑⠑
⠿⠍⠲

The next example uses the Nemeth decimal point in a mathematical numeral. A switch to Nemeth Code is required.

⫸ 1.6180339887 ⠼⠂⠨⠖⠂⠦⠴⠒⠒⠔⠦⠦⠶

Example 1-21

1.6180339887 is known as the Golden Number, **Phi.**

⠸⠩ ⠼⠂⠨⠖⠂⠦⠴⠒⠒⠔⠦⠦⠶ ⠸⠱ ⠊⠎ ⠐⠅⠝ ⠵ ⠮ ⠠⠛⠕⠇⠙⠢
⠠⠝⠥⠍⠃⠻⠂ ⠘⠂⠠⠏⠓⠊⠲

PRACTICE 1D

Percent to Decimal. To convert 48.6% to a decimal number, divide the percent by 100. For example, to convert 48.6%, $48.6 \div 100 = 0.486$.

With a 7% sales tax added, how much will a $6 bus ticket actually cost?

Julio's lot measures 56' wide and 100' deep. The garden shed sits 20'8'' back from the street.

THE HYPHEN AND THE DASH

1.7 The Hyphen and the Dash As Punctuation

As punctuation marks, the hyphen and the dash will most often be found in non mathematical context, in which case the UEB symbols and rules are applied. When these symbols are encountered within mathematical context, the the Nemeth symbols are used and Nemeth spacing rules are followed. The hyphen is the same in either code. The dash is not. If a dash occurs in mathematical context, the "short dash" of the Nemeth Code is transcribed.

⠤	Hyphen	-
⠤⠤	Dash ("short")	—

1.7.1 **A Hyphen May Divide a Long Numeral.** If a long numeral cannot be contained on one braille line it is divided after a comma (if a comma is present) and a hyphen is inserted. A divided number must be transcribed in Nemeth, even if it is freestanding and unmodified. When a numeral is divided between braille lines the numeric indicator must be used before the first digit of the numeral on the next line.

Example 1-22

One decillion can be written as 1,000,000,000,000,000,000,000,000,000,000,000.

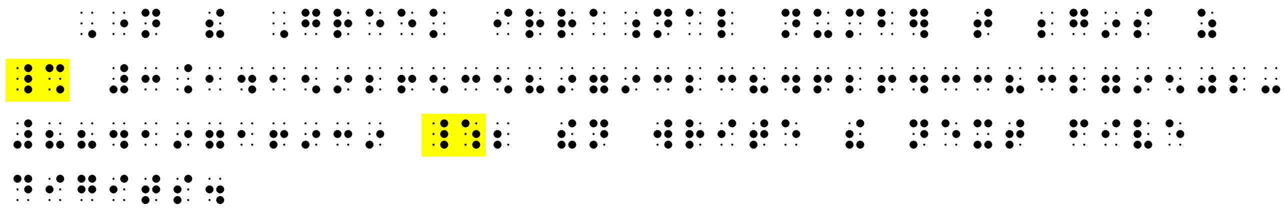
⠠⠐⠕ ⠙⠑⠉⠊⠇⠇⠊⠕⠝ ⠉ ⠆ ⠺⠗⠊⠞⠞⠢ ⠵ ⠸⠩ ⠼⠂⠠⠴⠴⠴⠠
⠼⠴⠴⠴⠠⠴⠴⠴⠠⠴⠴⠴⠠⠴⠴⠴⠠⠴⠴⠴⠠⠴⠴⠴⠠⠴⠴⠴⠠⠴⠴⠴⠠⠴⠴⠴⠠⠤
⠼⠴⠴⠴ ⠸⠱⠲

Because this long numeral must be divided, it is transcribed in Nemeth. Each line of the divided numeral begins with a numeric indicator.

a. **No Comma.** If the long numeral does not contain a comma, the hyphen may be inserted after any digit.

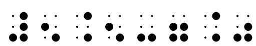
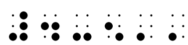
Example 1-23

Name the Greek irrational number that begins as 3.14159265358979323846264338327950288419716939; then write the next five digits.



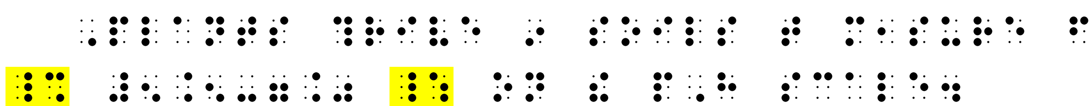
The semicolon applies to the sentence structure and is placed after the Nemeth Code terminator.

1.7.2 **A Hyphen May Connect Numerals.** Freestanding, unmodified numbers which are connected by a hyphen may be transcribed in UEB. If one or both numbers are modified, the two numbers are transcribed in Nemeth. In Nemeth Code, the numeric indicator is not used after the hyphen.

- ⋙ 5.5-7.0 
- ⋙ 4-5" 

Example 1-24

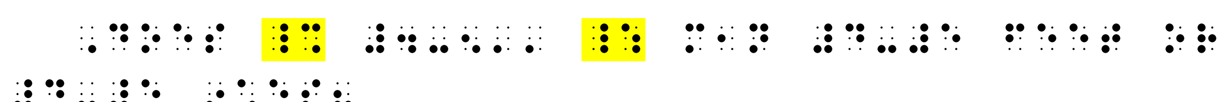
Plants thrive in soils that measure from 5.5-7.0 on the pH scale.



Because the numbers representing quantities contain a decimal point, they are transcribed in Nemeth.

Example 1-25

Does 4-5" mean 4-5 feet or 4-5 inches?



Nemeth is required for 5" because of the double prime sign. The entire hyphenated range 4-5" is transcribed in Nemeth. The other freestanding <u>un</u>modified number ranges are transcribed in UEB.

a. **UEB Context.** Non mathematical or non scientific numerals connected by hyphens such as dates, page number ranges, chapter or section ranges, etc. follow the rules of UEB.

Example 1-26

Your assignment for 3-6-17 is to read Sections 5.4-5.5 on pages 72-86.

The decimal points in the section numbers are transcribed in UEB.

1.7.3 **The Short Dash.** A dash as a mark of punctuation implies a pause or a break in thought. The numbers on either side of a dash may be transcribed in different codes. As in UEB context, follow print spacing before and after a dash. A dash may begin or end a line but the sign itself must not be divided.

In mathematical context, the Nemeth short dash is used to represent a dash used as a mark of punctuation. A numeric indicator is required following a dash even though that number may not be preceded by a space.

- ⫸ \$47,689—2.6%
- ⫸ \$47,689 — 2.6%

Example 1-27

The final cost was \$47,689 — 2.6% more than advertised.

The numbers on both sides of this dash require Nemeth Code. The Nemeth short dash is transcribed. Spacing follows print.

Example 1-28

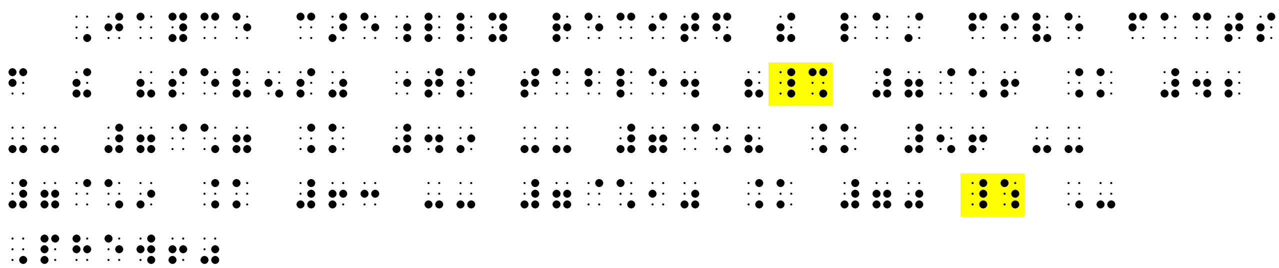
The final cost was \$47,689—3 percent more than advertised.

The first number requires Nemeth Code, but the second number does not. Nemeth Code is terminated after the dollar amount and the UEB dash is transcribed. Spacing follows print.

a. Even though the symbols differ between UEB and Nemeth, you may use the two dash forms within a sentence.

Example 1-29

Jayce carefully recited the last five facts from the "sevens" times table.
"7 × 6 = 42 — 7 × 7 = 49 — 7 × 8 = 56 — 7 × 9 = 63 — 7 × 10 = 70 — Phew!"

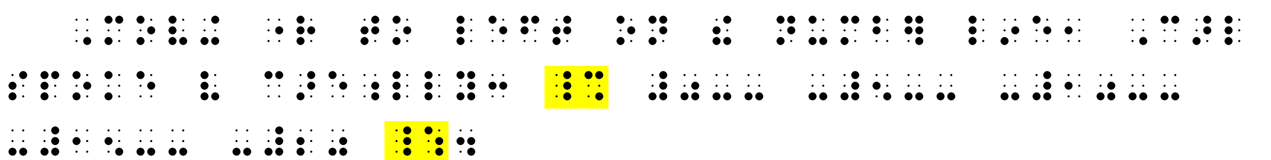


The Nemeth dash is used in the math portion, and the UEB dash is used after Nemeth is terminated. Each dash is preceded and followed by a space, as in print. Recall that the space before the Nemeth Code terminator does not represent a space in print.

1.7.4 **Hyphen, Dash, or Minus Sign?** Read carefully to determine whether a symbol is a hyphen, a dash or a minus/negative sign. A space must come between a hyphen and a minus sign or between a dash and a minus sign in order to distinguish the similar constructions.

Example 1-30

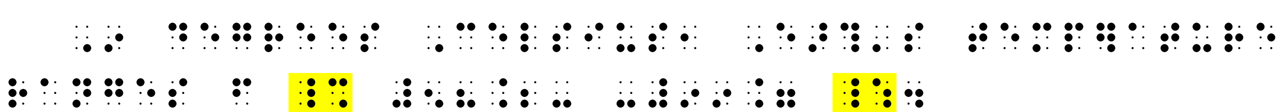
Moving right to left on the number line, Carl spoke very carefully: 0— –5— –10— –15— –20.



Although the numbers will fit all together on one line, they are five individual expressions and so the fullest available extent of the line is used.

Example 1-31

In degrees Celsius, Earth's temperature ranges from 58.2- –99.7.



Because a space must be inserted between the hyphen and the minus sign, a numeric indicator is required for –99.

PRACTICE 1E

Represent cents as decimals—$0.89—not like this—89¢.

Marcus' answer was 37¢ — 38¢ is the correct change.

In degrees Fahrenheit, Earth's temperature ranges from –126-136.

Expressed in seconds, the shortest meaningful interval of time (Planck Time) contains 46 decimal places: 0.0000000000000000000000000000000000000000000054 !

SIGNS OF OMISSION

1.8 General Use of Signs of Omission

In print, omission of mathematical or literary material may be shown by dots, a low line (underscore), a blank space, a question mark, or a combination of these or other signs devised by the author. Unless otherwise stated, the omission symbol used in braille should correspond to the print sign.

1.9 Ellipsis

An ellipsis is a series of dots which represent an omission. In UEB context, the UEB ellipsis is transcribed. In Nemeth context, the ellipsis of the Nemeth Code is transcribed. In Nemeth context, three dot 3s represent the ellipsis, regardless of how many dots are used in print. (One exceptions will be presented in Lesson 17 for ellipses within an array.)

⠄⠄⠄	Ellipsis	...

Example 1-32

What is the secret clue in this series? 5.0, 8.2, 11.4, 14.6, ...

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠎⠑⠉⠗⠑⠞ ⠉⠇⠥⠑ ⠔ ⠹ ⠎⠑⠗⠊⠑⠎⠦
⠸⠩ ⠼⠢⠨⠴⠠ ⠼⠦⠨⠆⠠ ⠼⠂⠂⠨⠲⠠ ⠼⠂⠲⠨⠖⠠ ⠄⠄⠄ ⠸⠱

The Nemeth ellipsis is used inside the switches.

Example 1-33

Marissa counted by 4s: 4, 8, 12, 16, 20, ...

⠠⠍⠜⠊⠎⠎⠁ ⠉⠨⠞⠫ ⠃⠽ ⠼⠙⠎⠒ ⠼⠙⠂ ⠼⠓⠂ ⠼⠁⠃⠂ ⠼⠁⠋⠂
⠼⠃⠚⠂ ⠲⠲⠲

The UEB ellipsis is used outside the switches.

1.10 Long Dash

When a low line (underscore) is used to denote an omission in print, within mathematical context the Nemeth "long dash" is transcribed. Outside of the code switches, the UEB underscore is used.

⠤⠤⠤⠤	Long Dash	____

Example 1-34

Multiply: 79 × 542 = ____

⠠⠍⠥⠇⠞⠊⠏⠇⠽⠒ ⠸⠩ ⠼⠶⠔⠈⠡⠢⠲⠆ ⠨⠅ ⠤⠤⠤⠤ ⠸⠱

The Nemeth long dash is used inside the switches.

Example 1-35

Two and ___ are ten.

⠠⠞⠺⠕ ⠯ ⠨⠤ ⠜⠑ ⠞⠢⠲

The UEB underscore is used outside the switches.

1.11 General Omission Symbol

An omission may be printed as a question mark, or a blank space may indicate a missing sign. The printed question mark can be standing alone, underlined, or be shown with hyphens. In all cases, the general omission symbol is transcribed.

⠿	General Omission Symbol

All four examples below are transcribed the same way.

- 92 ÷ 2 = ? ⠼⠔⠆⠨⠌⠆ ⠨⠅ ⠿
- 92 ÷ 2 = -?- ⠼⠔⠆⠨⠌⠆ ⠨⠅ ⠿
- 92 ÷ 2 = ⠼⠔⠆⠨⠌⠆ ⠨⠅ ⠿
- 92 ÷ 2 = _?_ ⠼⠔⠆⠨⠌⠆ ⠨⠅ ⠿

a. The general omission symbol follows the spacing rules of the material which it represents.

- 6 + ? = 15 ⠼⠖⠬⠿ ⠨⠅ ⠼⠂⠢

The omitted digit is unspaced from the plus sign and spaced from the equals sign.

⫸ 1,5?4 + 200 = 1,734

The omitted digit in the tens place is unspaced from the digits in the hundreds place and the ones place.

Example 1-36

Greater than or less than? 24 ÷ 3 ? 24 ÷ 4

The omitted comparison sign is preceded and followed by a space, according to the spacing rules for signs of comparison.

b. The number of general omission symbols used in braille must correspond to the number of omission signs used in print. A box or or a shaded region that does not show a specific number of places is represented by a single general omission symbol.

Example 1-37

678 × 27 = ? ? ? ? ?

A 5-digit answer is implied by showing five omission signs, which represent the five question marks shown in print.

Example 1-38

197 × ▭ = 4925

A gray rectangle indicates a missing factor.

1.12 Spacing of the Ellipsis and Long Dash

The ellipsis and the long dash are preceded and followed by a space in most circumstances, even when next to an operation sign. The print copy may or may not show a space but to avoid misreading the ellipsis or the long dash as other math symbols Nemeth spacing rules must be followed.

a. Insert a space between an ellipsis or a long dash and a symbol of operation.

⫸ ___ + 5

Example 1-39

There are many ways to write "10". Here are just a few: ____ + 5, ____ − 3, 2 × ____, 50 ÷ ____.

⠠⠐⠮ ⠜⠑ ⠸⠍ ⠺⠁⠽⠎ ⠞⠕ ⠺⠗⠊⠞⠑ ⠦⠼⠁⠚⠴⠲ ⠠⠐⠓ ⠜⠑
⠚ ⠁ ⠋⠑⠺⠒ ⠸⠩ ⠤⠤⠤⠤ ⠬⠢⠠ ⠤⠤⠤⠤ ⠤⠼⠒⠠
⠼⠆⠈⠡ ⠤⠤⠤⠤⠠ ⠼⠢⠴⠨⠌ ⠤⠤⠤⠤ ⠸⠱⠲

b. No space is left between an ellipsis or long dash and a related decimal point, monetary symbol, percent, or prime sign, even if a space is present in print.

- ⫸ ____.6 ⠤⠤⠤⠤⠨⠖
- ⫸ $ ____ ⠈⠎⠤⠤⠤⠤
- ⫸ ... % ⠄⠄⠄⠈⠴
- ⫸ ____' ⠤⠤⠤⠤⠄

Example 1-40

Fill in the blank. 14.9 − 12.3 = ____.6

⠠⠋⠊⠇⠇ ⠔ ⠮ ⠃⠇⠁⠝⠅⠲
⠸⠩ ⠼⠂⠲⠨⠔⠤⠂⠆⠨⠒ ⠨⠅ ⠤⠤⠤⠤⠨⠖ ⠸⠱

Example 1-41

Subtract: $3.52 − $7.14 = $ ____

⠠⠎⠥⠃⠞⠗⠁⠉⠞⠒
⠸⠩ ⠈⠎⠒⠨⠢⠆⠤⠈⠎⠶⠨⠂⠲ ⠨⠅ ⠈⠎⠤⠤⠤⠤ ⠸⠱

Example 1-42

Multiply. 92% × .04 = ... %

⠠⠍⠥⠇⠞⠊⠏⠇⠽⠲ ⠸⠩ ⠼⠔⠆⠈⠴⠈⠡⠨⠴⠲ ⠨⠅ ⠄⠄⠄⠈⠴ ⠸⠱

Example 1-43

Convert inches to feet: 24" = ____'

⠠⠒⠧⠻⠞ ⠔⠡⠑⠎ ⠞⠕ ⠋⠑⠑⠞⠒
⠸⠩ ⠼⠆⠲⠄⠄ ⠨⠅ ⠤⠤⠤⠤⠄ ⠸⠱

c. More rules regarding the spacing of an ellipsis or a long dash will be addressed in later lessons.

1.13 Other Omission Symbols

Omissions are frequently shown in other ways besides a blank space, a question mark, a dash, or an ellipsis. A shape, such as a square or a circle, may indicate an omission. If the omission sign used in print has no braille equivalent in the code, the sign may be represented by a devised braille symbol or by a drawing. Shape symbols, devised symbols, and drawings will be discussed in Lesson 11.

Format

1.14 Paragraph Margins for Narrative Portions of Text (3-1)

The Nemeth Code states that each paragraph is to begin in cell 3, with runovers in cell 1. Nemeth formats are applied throughout a Nemeth transcription, including the UEB portions of text. If the print copy uses blocked paragraphing style, the transcriber must follow Nemeth format rules and begin each new paragraph in cell 3. There is no blank line inserted between paragraphs unless another situation requires a blank line according to *Braille Formats* or according to other Nemeth formats (yet to be studied).

Instructions: Treat the marginal heading as a cell-5 heading.

PRACTICE 1F

Numerical Prefixes Here are some examples of numeral prefixes: "Tetra-" means 4; "hexa-" means 6; "hepta-" means 7; "deca-" means 10; "dodeca-" means 12.

If a *dodecagon* is a 12-sided figure, a *dodecahedron* is a __-faced solid. A 10-faced solid is called a ________.

INTRODUCTION TO IDENTIFIERS

1.15 Terminology

When material is identified sequentially by number or letter, as in exercise material, it is referred to as *itemized material*. The number or letter is referred to as the *identifier*. Itemized subentries are referred to as *subdivisions*.

Labeled steps such as "Step 1" are not considered to be itemized material.

Format

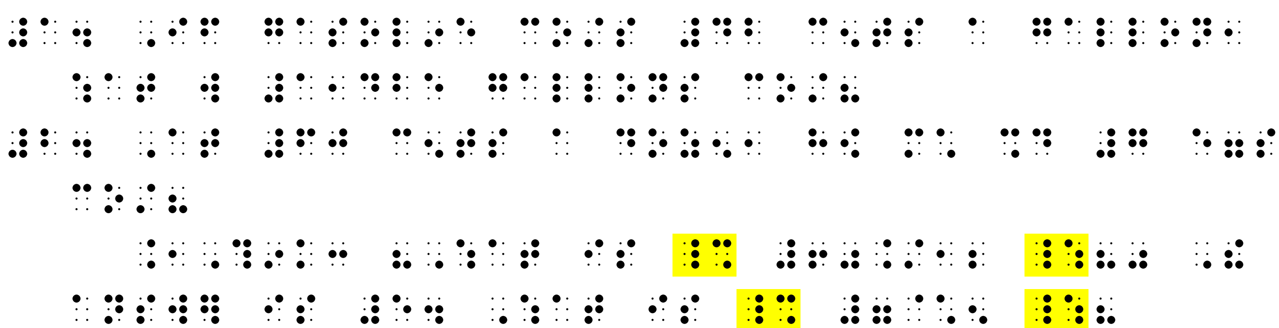
1.16 Margins for Itemized Material with No Subdivisions (1-3)

The identifier begins in cell 1; runovers begin in cell 3. If the material contains more than one paragraph, each subparagraph begins in cell 5 with runovers in cell 3.

Example 1-44

1. If gasoline costs 42 cents a gallon, what will 1,425 gallons cost?
2. At 60 cents a dozen, how much should 7 eggs cost?

   Think: "What is 60 ÷ 12 ?" The answer is 5. What is 7 × 5 ?

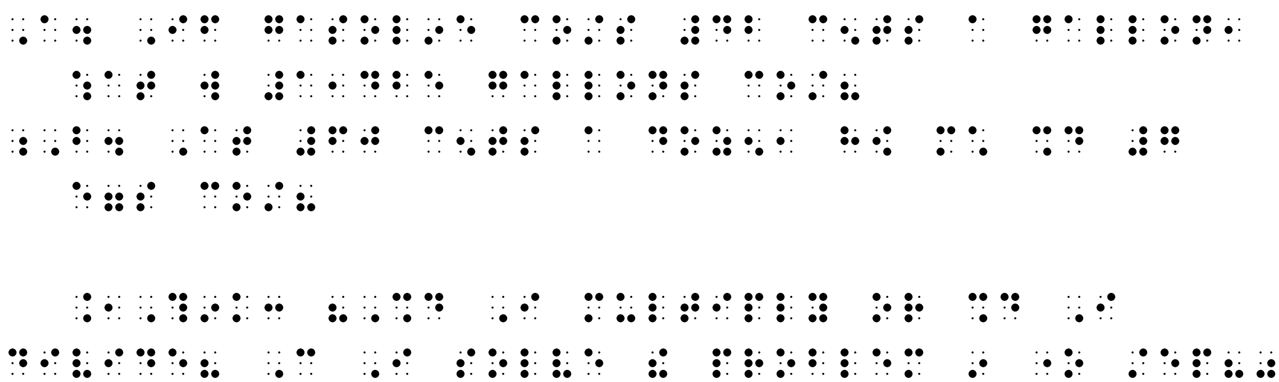


Use clues from the print layout as well as the information in the paragraph to determine if the new paragraph is indeed a continuation of the item or if it is separate material. Compare Example 1-44 above with the next example.

Example 1-45

A. If gasoline costs 42 cents a gallon, what will 1,425 gallons cost?
B. At 60 cents a dozen, how much should 7 eggs cost?

Think: "Should I multiply or should I divide? Can I solve the problem in one step?"



Notice that a blank line is left between itemized format and a 3-1 narrative paragraph.

Instructions: Format the first sentence as a narrative paragraph. Retain boldface for the paragraph heading. Insert a blank line between the narrative paragraph and the itemized material. Ignore typeface of the identifiers.

PRACTICE 1G

Refresher. These problems will test your skill with decimals.

1. First, addition: $42.6 + 37.23 + 3.215 =$
2. Now subtract: $87.6 - 51.35 =$
3. Try multiplication: $625.1 \times 2.7 =$
4. And now divide: $4.864 \div 3.2 =$

FORMAT SUMMARY #1

Here is a summary of the Nemeth formats encountered so far in this course.

General Principles   When an item in a UEB transcription requires the use of Nemeth symbols, format rules of The Nemeth Braille Code for Mathematics and Science Notation are to be applied to the entire transcription including those portions transcribed in UEB. When a format is not specifically addressed in the Nemeth Code, the principles provided in *Braille Formats* should be followed.

Mathematical Expressions—Keep Together   If a mathematical expression will fit on one braille line within the current margins, it must not be divided between lines. The entire expression is brought down to the next line.

Paragraph Margins for Narrative Portions of Text (3-1)   In a document governed by Nemeth formatting, an unitemized paragraph in explanatory portions of text begins in cell 3 and all runovers begin in cell 1. Blocked paragraphing is not used in a Nemeth transcription. A blank line is inserted after a paragraph when itemized material follows.

Margins for Itemized Material with No Subdivisions (1-3)   The identifier begins in cell 1; runovers begin in cell 3. If the material contains more than one paragraph, each subparagraph begins in cell 5 with runovers in cell 3. A blank line is inserted after itemized material when a (3-1) narrative paragraph follows.

Placement of Code Switch Indicators within Narrative   Place the entire math expression and the two code switch indicators on the same braille line if they will fit within the current margins. If the entire string will not fit on one line, a switch indicator may stand alone on a line to allow the math expression to remain undivided. It is preferable to keep the switch indicators on the same braille page as the mathematical material to which they apply.

For further practice, see Appendix A—Reading Practice.

EXERCISE 1

Prepare Exercise 1 for your grader. Instructions for preparing the lesson exercises are included in the exercise packet.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 1A

⠠⠱⠁⠞ ⠙⠕⠑⠎ ⠸⠩ ⠼⠂⠖⠬⠲⠬⠂⠴⠴ ⠸⠱ ⠑⠟⠥⠁⠇⠦
⠠⠊⠋ ⠸⠩ ⠼⠂⠴⠈⠡⠂⠴⠈⠡⠂⠴ ⠨⠅ ⠼⠂⠠⠴⠴⠴ ⠸⠱ ⠱⠁⠞
⠙⠕⠑⠎ ⠸⠩ ⠼⠂⠴⠈⠡⠂⠴⠈⠡⠂⠴⠈⠡⠂⠴ ⠸⠱ ⠑⠟⠥⠁⠇⠦
⠠⠙⠕⠑⠎ ⠸⠩ ⠼⠢⠨⠌⠆ ⠸⠱ ⠐⠝ ⠁ ⠱⠕⠇⠑ ⠝⠥⠍⠃⠻⠦
⠸⠩ ⠈⠴ ⠸⠱ ⠐⠣⠦⠏⠻⠉⠢⠞⠴⠐⠜ ⠍⠂⠝⠎ ⠐⠏⠎ ⠏⠻
⠓⠥⠝⠙⠗⠫⠆ ⠸⠩ ⠈⠴⠴ ⠸⠱ ⠐⠣⠦⠏⠻⠍⠊⠇⠇⠑⠴⠐⠜ ⠍⠂⠝⠎
⠐⠏⠎ ⠏⠻ ⠹⠳⠎⠯⠲

PRACTICE 1B

⠠⠊⠋ ⠸⠩ ⠼⠲⠶⠤⠲⠂ ⠨⠂ ⠼⠂ ⠸⠱⠂ ⠙ ⠽ ⠎⠥⠏⠏⠕⠎⠑ ⠞
⠸⠩ ⠼⠲⠂⠤⠲⠶ ⠐⠅ ⠼⠂ ⠸⠱⠦ ⠠⠱⠽ ⠕⠗ ⠱⠽ ⠝⠦
⠠⠊⠋ ⠸⠩ ⠼⠢⠬⠂⠴ ⠸⠱ ⠑⠟⠥⠁⠇⠎ ⠼⠁⠑⠂ ⠱⠁⠞ ⠙⠕⠑⠎
⠸⠩ ⠼⠂⠴⠬⠢ ⠸⠱ ⠑⠟⠥⠁⠇⠦ ⠠⠑⠭⠏⠇⠁⠔⠲

PRACTICE 1C

⠠⠓⠪ ⠸⠍ ⠎⠟⠥⠜⠑⠎ ⠉ ⠽ ⠋⠔⠙ ⠕⠝ ⠁⠝
⠸⠩ ⠼⠦⠈⠡⠦ ⠸⠱ ⠡⠑⠉⠅⠻⠃⠕⠜⠙⠦ ⠐⠣⠨⠂⠠⠓⠔⠞⠒ ⠠⠐⠮ ⠜⠑
⠍ ⠹⠁⠝ ⠼⠋⠑ ⠎⠟⠥⠜⠑⠎⠲⠐⠜ ⠠⠋⠌ ⠙⠑⠞⠻⠍⠔⠑ ⠓⠪ ⠸⠍
⠸⠩ ⠼⠂⠈⠡⠂ ⠸⠱ ⠎⠟⠥⠜⠑⠎⠂ ⠮⠝ ⠓⠪ ⠸⠍ ⠸⠩ ⠼⠆⠈⠡⠆ ⠸⠱
⠎⠟⠥⠜⠑⠎⠂ ⠯ ⠎ ⠕⠝⠂ ⠽ ⠉ ⠋⠔⠙⠲
⠸⠩ ⠼⠂⠆⠈⠡⠂⠂ ⠸⠱ ⠊⠎ ⠁⠇ ⠮ ⠎⠁⠍⠑ ⠵ ⠞⠺⠑⠇⠧⠑
⠑⠇⠑⠧⠢⠎⠂ ⠕⠗ ⠸⠩
⠼⠂⠂⠬⠂⠂⠬⠂⠂⠬⠂⠂⠬⠂⠂⠬⠂⠂⠬⠂⠂⠬⠂⠂⠬⠂⠂⠬⠂⠂⠬⠂⠂⠬⠂⠂ ⠸⠱⠲

PRACTICE 1D

⠘⠶⠠⠏⠻⠉⠢⠞ ⠞⠕ ⠠⠙⠑⠉⠊⠍⠁⠇⠲⠘⠄ ⠒⠧⠻⠞
⠸⠩ ⠼⠲⠦⠨⠖⠈⠴ ⠸⠱ ⠞⠕ ⠁ ⠙⠑⠉⠊⠍⠁⠇ ⠝⠥⠍⠃⠻⠂ ⠙⠊⠧⠊⠙⠑
⠮ ⠏⠻⠉⠢⠞ ⠃⠽ ⠼⠁⠚⠚⠲ ⠠⠿ ⠑⠭⠁⠍⠏⠇⠑⠂ ⠞⠕ ⠒⠧⠻⠞
⠸⠩ ⠼⠲⠦⠨⠖⠈⠴⠠ ⠼⠲⠦⠨⠖⠨⠌⠂⠴⠴ ⠨⠅ ⠼⠴⠨⠲⠦⠖ ⠸⠱⠲
⠠⠾ ⠁ ⠸⠩ ⠼⠶⠈⠴ ⠸⠱ ⠎⠁⠇⠑⠎ ⠞⠁⠭ ⠁⠙⠙⠫⠂ ⠓⠪ ⠍⠡
⠺ ⠁ ⠸⠩ ⠈⠎⠖ ⠸⠱ ⠃⠥⠎ ⠞⠊⠉⠅⠑⠞ ⠁⠉⠞⠥⠁⠇⠇⠽ ⠉⠕⠌⠦
⠠⠚⠥⠇⠊⠕⠄⠎ ⠇⠕⠞ ⠍⠂⠎⠥⠗⠑⠎ ⠸⠩ ⠼⠢⠖⠄ ⠸⠱ ⠺⠊⠙⠑ ⠯
⠸⠩ ⠼⠂⠴⠴⠄ ⠸⠱ ⠙⠑⠑⠏⠲ ⠠⠮ ⠛⠜⠙⠢ ⠩⠫ ⠎⠊⠞⠎
⠸⠩ ⠼⠆⠴⠄⠦⠄⠄ ⠸⠱ ⠃⠁⠉⠅ ⠋ ⠮ ⠌⠗⠑⠑⠞⠲

PRACTICE 1E

⠠⠗⠑⠏⠗⠑⠎⠢⠞ ⠉⠢⠞⠎ ⠵ ⠙⠑⠉⠊⠍⠁⠇⠎⠠⠤
⠸⠩ ⠈⠎⠴⠨⠦⠔ ⠸⠱⠠⠤⠝ ⠇ ⠹⠠⠤⠸⠩ ⠼⠦⠔⠈⠉ ⠸⠱⠲
⠠⠍⠜⠉⠥⠎⠄ ⠁⠝⠎⠺⠻ ⠴ ⠸⠩ ⠼⠒⠶⠈⠉ ⠤⠤ ⠼⠒⠦⠈⠉ ⠸⠱
⠊⠎ ⠮ ⠉⠕⠗⠗⠑⠉⠞ ⠡⠁⠝⠛⠑⠲
⠠⠔ ⠙⠑⠛⠗⠑⠑⠎ ⠠⠋⠁⠓⠗⠢⠓⠑⠊⠞⠂ ⠠⠑⠜⠹⠄⠎
⠞⠑⠍⠏⠻⠁⠞⠥⠗⠑ ⠗⠁⠝⠛⠑⠎ ⠋ ⠸⠩ ⠤⠼⠂⠆⠖⠤⠂⠒⠖ ⠸⠱⠲
⠠⠑⠭⠏⠗⠑⠎⠎⠫ ⠔ ⠎⠑⠉⠕⠝⠙⠎⠂ ⠮ ⠩⠕⠗⠞⠑⠌ ⠍⠂⠝⠬⠰⠇
⠔⠞⠻⠧⠁⠇ ⠷ ⠐⠞ ⠐⠣⠠⠏⠇⠁⠝⠉⠅ ⠠⠐⠞⠐⠜ ⠒⠞⠁⠔⠎ ⠼⠙⠋
⠙⠑⠉⠊⠍⠁⠇ ⠏⠇⠁⠉⠑⠎⠒ ⠸⠩ ⠼⠴⠨⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠤
⠼⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠴⠢⠲ ⠸⠱⠖

Line 2: Because the UEB dashes are unspaced (as in print), the code switch indicators may touch the dash.

PRACTICE 1F

⠠⠝⠥⠍⠻⠊⠉⠁⠇ ⠠⠏⠗⠑⠋⠊⠭⠑⠎
⠠⠐⠓ ⠜⠑ ⠐⠎ ⠑⠭⠁⠍⠏⠇⠑⠎ ⠷ ⠝⠥⠍⠻⠁⠇ ⠏⠗⠑⠋⠊⠭⠑⠎⠒
⠦⠠⠞⠑⠞⠗⠁⠤⠴ ⠍⠂⠝⠎ ⠼⠙⠆ ⠦⠓⠑⠭⠁⠤⠴ ⠍⠂⠝⠎ ⠼⠋⠆
⠦⠓⠑⠏⠞⠁⠤⠴ ⠍⠂⠝⠎ ⠼⠛⠆ ⠦⠙⠑⠉⠁⠤⠴ ⠍⠂⠝⠎ ⠼⠁⠚⠆
⠦⠙⠕⠙⠑⠉⠁⠤⠴ ⠍⠂⠝⠎ ⠼⠁⠃⠲
⠠⠊⠋ ⠁ ⠨⠂⠙⠕⠙⠑⠉⠁⠛⠕⠝ ⠊⠎ ⠁ ⠼⠁⠃⠤⠎⠊⠙⠫
⠋⠊⠛⠥⠗⠑⠂ ⠁ ⠨⠂⠙⠕⠙⠑⠉⠁⠓⠫⠗⠕⠝ ⠊⠎ ⠁ ⠨⠤⠤⠋⠁⠉⠫
⠎⠕⠇⠊⠙⠲ ⠠⠁ ⠼⠁⠚⠤⠋⠁⠉⠫ ⠎⠕⠇⠊⠙ ⠊⠎ ⠉⠁⠇⠇⠫ ⠁ ⠨⠤⠲

PRACTICE 1F does not require any Nemeth switches. Rules of UEB are followed for the underscores.

PRACTICE 1G

⠘⠂⠠⠗⠑⠋⠗⠑⠩⠻⠲ ⠠⠘⠮ ⠏⠗⠕⠃⠇⠑⠍⠎ ⠺ ⠞⠑⠌ ⠽⠗
⠎⠅⠊⠇⠇ ⠾ ⠙⠑⠉⠊⠍⠁⠇⠎⠲

⠼⠁⠲ ⠠⠋⠌⠂ ⠁⠙⠙⠊⠰⠝⠒
⠸⠩ ⠼⠲⠂⠨⠖⠬⠒⠶⠨⠆⠒⠬⠒⠨⠆⠂⠢ ⠨⠅ ⠿ ⠸⠱
⠼⠃⠲ ⠠⠝⠪ ⠎⠥⠃⠞⠗⠁⠉⠞⠒ ⠸⠩ ⠼⠦⠶⠨⠖⠤⠢⠂⠨⠒⠢ ⠨⠅ ⠿ ⠸⠱
⠼⠉⠲ ⠠⠞⠗⠽ ⠍⠥⠇⠞⠊⠏⠇⠊⠉⠁⠰⠝⠒
⠸⠩ ⠼⠖⠆⠢⠨⠂⠈⠡⠆⠨⠶ ⠨⠅ ⠿ ⠸⠱
⠼⠙⠲ ⠠⠯ ⠝⠪ ⠙⠊⠧⠊⠙⠑⠒ ⠸⠩ ⠼⠲⠨⠦⠖⠲⠨⠌⠒⠨⠆ ⠨⠅ ⠿ ⠸⠱

LESSON 2

- MORE ABOUT PUNCTUATION
- PUNCTUATION IN NEMETH CODE
    - The Punctuation Indicator
- INTRODUCTION TO SIGNS OF GROUPING
    - Code-Switching Considerations
    - Spacing with Signs of Grouping
- IDENTIFIERS, cont.

Format

- Keep Together—Hyphenated Expressions
- Side-by-Side Layout

Answers to Practice Material

LESSON PREVIEW

The punctuation indicator is introduced as we take a closer look at punctuation inside the switches. Summaries are given regarding the use/nonuse of the punctuation indicator. Nemeth grouping symbols are introduced. Code switching within numbered/lettered formats is discussed. Nemeth rules regarding hyphenated expressions are given. An alternate layout option for itemized material is considered.

MORE ABOUT PUNCTUATION

2.1 Punctuation Mode

Punctuation mode is determined by whether the punctuation occurs inside or outside of the Nemeth switches. The concept is simple – punctuation that occurs outside of the switch indicators is transcribed in "literary mode" according to the rules of Unified English Braille; punctuation occurring within the switch indicators is transcribed in "mathematical mode" according to the rules of the Nemeth Code. Take another look at this example from Lesson 1, noting that UEB punctuation is used for the question mark and the Nemeth comma and ellipsis are used in the mathematical portion.

Example 2-1

What is the secret clue in this series? 5.0, 8.2, 11.4, 14.6, …

  ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠎⠑⠉⠗⠑⠞ ⠉⠇⠥⠑ ⠔ ⠹ ⠎⠻⠊⠑⠎⠦
⠸⠩ ⠼⠢⠨⠴⠠ ⠼⠦⠨⠆⠠ ⠼⠂⠂⠨⠲⠠ ⠼⠂⠲⠨⠖⠠ ⠄⠄⠄ ⠸⠱

2.2 Spacing of UEB Punctuation and Code Switch Indicators

As shown in Lesson 1, punctuation that relates to the main text is placed outside of the switch indicators when the surrounding text is in UEB. There is no space between the terminator and the following punctuation.

Example 2-2

To begin, divide 64 ÷ 8, then subtract.

  ⠠⠞⠕ ⠆⠛⠔⠂ ⠙⠊⠧⠊⠙⠑ ⠸⠩ ⠼⠖⠲⠨⠌⠦ ⠸⠱⠂ ⠮⠝
⠎⠥⠃⠞⠗⠁⠉⠞⠲

Note the use of the literary comma (dot 2) outside of the Nemeth Code terminator.

Example 2-3

Divide 64 ÷ 8—then subtract.

  ⠠⠙⠊⠧⠊⠙⠑ ⠸⠩ ⠼⠖⠲⠨⠌⠦ ⠸⠱⠠⠤⠮⠝ ⠎⠥⠃⠞⠗⠁⠉⠞⠲

The unspaced dash is part of the sentence punctuation and is placed outside of the Nemeth Code terminator.

Example 2-4

Divide 64 ÷ 8 — then subtract.

The spaced dash is part of the sentence punctuation and is placed outside of the Nemeth Code terminator. The space after the opening Nemeth Code indicator does not represent a space in print.

Example 2-5

We continue ... 8–14 = –6

The ellipsis is part of the sentence punctuation and is placed outside of the opening Nemeth Code indicator. A space precedes and follows the ellipsis, as printed. The space after the opening Nemeth Code indicator does not represent a space in print.

2.3 Nemeth Punctuation

When punctuation occurs within mathematical material, excessive code switching is avoided by using Nemeth punctuation. In the example below, the Nemeth comma (dot 6) is used within the series even though the comma itself is not mathematical.

⫸ 5+5,

Example 2-6

Multiplication can be expressed as a series of addition problems: $5 \times 2 = 5 + 5$, $5 \times 3 = 5 + 5 + 5$, $5 \times 4 = 5 + 5 + 5 + 5$, and so on.

The last comma is placed outside of the switch, as a dot 2 literary comma, because UEB text follows.

Instructions: Consider carefully where to place the code switch indicators and what kind of punctuation to use in these three sentences. Apply 3-1 Nemeth paragraphing.

PRACTICE 2A

72813654, when written as 72 81 36 54, is obviously divisible by 9.

Write these numbers: 3.29, 500, –123, 2,000.88, –250,794. Now add them together.

Is the answer 4.0‰, or is it 4.0%?

PUNCTUATION IN NEMETH CODE

2.4 Background

So far we have looked at punctuation that is unambiguous in mathematical context: the comma and the short dash. These symbols are not the same as their UEB counterparts.

⠠	Mathematical comma	,
⠤⠤	Short dash	–

When other literary punctuation marks are transcribed inside the Nemeth switches, the punctuation symbols from UEB are used: the apostrophe, colon, exclamation point, period, question mark, quotation marks,* and semicolon. When a punctuation mark is not preceded by a space, clarification is required because the symbols are formed with the same braille dots as Nemeth numerals and symbols, as demonstrated in this list.

A semicolon ⠆ could be misread in Nemeth as the numeral 2.
A colon ⠒ could be misread in Nemeth as the numeral 3.
A period ⠲ could be misread in Nemeth as the numeral 4.
An exclamation point ⠖ could be misread in Nemeth as the numeral 6.
A question mark ⠦ could be misread in Nemeth as the numeral 8.
A closing “double” quotation mark ⠴ could be misread in Nemeth as the numeral 0.
A closing “single” quotation mark ⠠⠴ could be misread in Nemeth as a comma and the numeral 0.
An apostrophe ⠄ could be misread in Nemeth as a prime sign.

Clarification is achieved by use of the *punctuation indicator*.

———

* Only the one-cell “double” quotation marks ⠦ ⠴ and the two-cell "single" quotation marks ⠠⠦ ⠠⠴ are used inside the Nemeth switches.

The Punctuation Indicator

2.5 Role of the Punctuation Indicator

A punctuation indicator is placed before one or more of the punctuation marks listed in the table above when such punctuation occurs in mathematical context. Use of the punctuation indicator assures that the braille is read as punctuation and is not misread as a mathematical symbol.

⠸	Punctuation Indicator

⫸ 2 · 2 · 2; ⠼⠆⠡⠆⠡⠆⠸⠆

The punctuation indicator prevents the semicolon from being misread as the numeral 2.

Example 2-7

In this example, a comma separates member pairs; a semicolon separates sets. 2 · 2, 2 · 2 · 2; 3 · 3, 3 · 3 · 3; 4 · 4, 4 · 4 · 4.

⠀⠀⠠⠔ ⠮ ⠑⠭⠁⠍⠏⠇⠑⠂ ⠁ ⠉⠕⠍⠍⠁ ⠎⠑⠏⠜⠁⠞⠑⠎ ⠍⠑⠍⠃⠻
⠏⠁⠊⠗⠎⠆ ⠁ ⠎⠑⠍⠊⠉⠕⠇⠕⠝ ⠎⠑⠏⠜⠁⠞⠑⠎ ⠎⠑⠞⠎⠲ ⠀⠀⠀⠀⠀⠀
⠸⠩ ⠼⠆⠡⠆⠠ ⠼⠆⠡⠆⠡⠆⠸⠆ ⠼⠒⠡⠒⠠ ⠼⠒⠡⠒⠡⠒⠸⠆ ⠼⠲⠡⠲⠠
⠼⠲⠡⠲⠡⠲ ⠸⠱⠲

Although the opening Nemeth Code indicator will fit on line 2, the first math expression will not. With a paragraph, keep each switch indicator on the same line as the mathematics to which it applies, if it will fit. The final period applies to the entire sentence. It is placed <u>after</u> the Nemeth Code terminator.

2.5.1 **Two or More Punctuation Marks in a Row.** When two or more punctuation marks follow a mathematical item, only one punctuation indicator is used.

⫸ "=". ⠦ ⠨⠅ ⠸⠴⠲

a. If a comma is the second punctuation mark, the mathematical comma is transcribed.

⫸ "−", ⠦⠤⠸⠴⠠

Example 2-8

Signs for "plus", "minus", and "equals" are "+", "−", "=". ÷ means "divided by".

⠠⠎⠊⠛⠝⠎ ⠿ ⠦⠏⠇⠥⠎⠴⠂ ⠦⠍⠔⠥⠎⠴⠂ ⠯ ⠦⠑⠟⠥⠁⠇⠎⠴ ⠜⠑
⠸⠩ ⠦⠬⠸⠴⠠ ⠦⠤⠸⠴⠠ ⠦⠨⠅⠸⠴⠸⠲ ⠨⠌ ⠸⠱ ⠍⠂⠝⠎ ⠦⠙⠊⠧⠊⠙⠫
⠃⠽⠴⠲

A punctuation indicator is not needed for the opening quotation marks because they are preceded by a space and so will not be misread as numerals.

b. If the first punctuation mark is a comma, a hyphen, or a short dash, a punctuation indicator is needed before the second punctuation mark provided that a punctuation indicator would be required if the first mark was removed and the space which it occupies was not present,.

⫸ 3y,"   ⠼⠒⠽⠠⠸⠴

because, without the comma, ⠼⠒⠽⠸⠴

⫸ "$99—"   ⠦⠈⠎⠔⠔⠤⠤⠸⠴

because, without the dash, ⠦⠈⠎⠔⠔⠸⠴

2.5.2 **A Comparison Sign in Quotes.** Note that, although a space is generally left between a comparison sign and an expression which precedes or follows it, a space is <u>not</u> left between a comparison sign and a punctuation mark which applies to it.

⫸ "="   ⠦⠨⠅⠸⠴

Observe how this applies to the equals sign in <u>Example 2-8</u>, above.

2.5.3 **A Number in Quotes.** A numeric indicator is required when a numeral immediately follows an opening quotation mark.

⫸ "40+10=50"   ⠦⠼⠲⠴⠬⠂⠴ ⠨⠅ ⠼⠢⠴⠸⠴

Example 2-9

Add 48+13 in your head. Think: "40+10=50" ... "8+3=11" ... "50 + 11 = 61".

⠠⠁⠙⠙ ⠸⠩ ⠼⠲⠦⠬⠂⠒ ⠸⠱ ⠔ ⠽⠗ ⠓⠂⠙⠲ ⠠⠹⠔⠅⠒
⠸⠩ ⠦⠼⠲⠴⠬⠂⠴ ⠨⠅ ⠼⠢⠴⠸⠴ ⠄⠄⠄ ⠦⠼⠦⠬⠒ ⠨⠅ ⠼⠂⠂⠸⠴
⠄⠄⠄ ⠦⠼⠢⠴⠬⠂⠂ ⠨⠅ ⠼⠖⠂⠸⠴ ⠸⠱⠲

Even though this ellipsis is not mathematical (it indicates pausing while thinking), the ellipsis of the Nemeth Code is used because it is inside the

Nemeth switches. The final period applies to the entire sentence. It is placed after the Nemeth Code terminator.

2.5.4 **Digital Clock Time.** Digital clock time is transcribed in UEB unless the time is involved in computation or is part of a number line, in which case Nemeth Code is used. In Nemeth, a punctuation indicator precedes the colon to prevent misreading the colon as the number 3. A numeric indicator is then required to set the reading mode back to "numeric."

⋙ 3:30

UEB: ⠼⠉⠒⠼⠉⠚

Nemeth: ⠸⠩ ⠼⠒⠸⠒⠼⠒⠴ ⠸⠱

Recall from Lesson 1 that the numeric indicator is not restated when a hyphen connects Nemeth numerals.

⋙ 7:45-8:20

UEB: ⠼⠛⠒⠼⠙⠑⠤⠼⠓⠒⠼⠃⠚

Nemeth: ⠸⠩ ⠼⠶⠸⠒⠼⠲⠢⠤⠦⠸⠒⠼⠆⠴ ⠸⠱

Example 2-10

Last night, Jayquan arrived at 7:45 and left at 8:20. Use the shortcut method to figure out how many minutes he stayed. 7:45-8:20 = 15+20 = 35. Jayquan stayed for 35 minutes.

⠀⠀⠠⠇⠁⠌ ⠝⠊⠣⠞⠂ ⠠⠚⠁⠽⠟⠥⠁⠝ ⠜⠗⠊⠧⠫ ⠁⠞ ⠼⠛⠒⠼⠙⠑ ⠯
⠇⠑⠋⠞ ⠁⠞ ⠼⠓⠒⠼⠃⠚⠲ ⠠⠥⠎⠑ ⠮ ⠩⠕⠗⠞⠉⠥⠞ ⠍⠑⠹⠕⠙ ⠞⠕
⠋⠊⠛⠥⠗⠑ ⠳ ⠓⠪ ⠸⠍ ⠍⠔⠥⠞⠑⠎ ⠓⠑ ⠌⠁⠽⠫⠲
⠸⠩ ⠼⠶⠸⠒⠼⠲⠢⠤⠦⠸⠒⠼⠆⠴ ⠨⠅ ⠼⠂⠢⠬⠆⠴ ⠨⠅ ⠼⠒⠢ ⠸⠱⠲
⠠⠚⠁⠽⠟⠥⠁⠝ ⠌⠁⠽⠫ ⠿ ⠼⠉⠑ ⠍⠔⠥⠞⠑⠎⠲

It is not necessary for the digital time to be transcribed in the same code within the same word problem. UEB is used in the narrative and Nemeth is used in the computation.

2.6 Punctuation with Omission Signs

When a sign of omission represents omitted mathematical content the appropriate omission symbol of the Nemeth Code is transcribed and it is punctuated mathematically. The punctuation is unspaced from the omission symbol.

⋙ 2 × ..., ⠼⠆⠈⠡ ⠄⠄⠄⠠

⫸ 2 × ____.

Take another look at this example from Lesson 1. Notice the use of the mathematical comma with the long dash.

Example 2-11

Ways to write "10": ____ + 5, ____ − 3, 2 × ____, 50 ÷ ____.

Reminders: The long dash and the ellipsis are spaced away from the multiplication symbol. Note that the omission dash is placed on the same line as the rest of the math statement "blank minus three" even though there is room for the long dash on the first line.

Example 2-12

Fill in the missing numbers in the series: 3, 6, ?, 12, ??, 18.

Switch Decision: The general omission symbol is a Nemeth symbol. In order to avoid excess code switching, the entire series is transcribed in Nemeth even though the numerals themselves could be transcribed in either code.
Reminder: The general omission symbol is spaced according to rules of the item it represents (in this case, a numeral). The same number of omission symbols shown in print should be used in braille.

2.6.1 **Spacing Exception—The Hyphen.** Although no space is left between an ellipsis and a related punctuation mark or between a long dash and a related punctuation mark, if the punctuation mark is a hyphen then a space is required.

⫸ 40% - ___

⫸ ... -9.3

Example 2-13

Orchids thrive when humidity ranges from 40% - ___. ___ -80% is considered optimal for most varieties.

2.7 Punctuation and Spacing of Plural or Possessive Endings

When "s" or apostrophe-s is attached to a mathematical symbol, it becomes part of that expression. This means that the "s" is punctuated mathematically.

⫸ ×s: ⠈⠡⠎⠸⠒

Example 2-14

Insert +s or insert ×s: 4 ... 2 = 8; 8 ... 2 = 10.

⠠⠔⠎⠻⠞ ⠸⠩ ⠬⠎ ⠸⠱ ⠕⠗ ⠔⠎⠻⠞ ⠸⠩ ⠈⠡⠎⠸⠒ ⠀⠀⠀⠀⠀⠀
⠼⠲ ⠄⠄⠄ ⠼⠆ ⠨⠅ ⠼⠦⠸⠆ ⠼⠦ ⠄⠄⠄ ⠼⠆ ⠨⠅ ⠼⠂⠴ ⠸⠱⠲

Reminder: A mathematical expression must not be divided between braille lines if it will fit on one line within current margins. The expression "4 ... 2 = 8" must not be divided and so it begins on line 2.

A punctuation indicator is required before the apostrophe in a possessive ending "apostrophe-s". Even so, a punctuation indicator is still required before a punctuation mark that immediately follows the "s". Compare this similar example to the previous one.

⫸ ×'s: ⠈⠡⠸⠄⠎⠸⠒

Example 2-15

Insert +'s or insert ×'s: 4 ... 2 = 8; 8 ... 2 = 10.

⠠⠔⠎⠻⠞ ⠸⠩ ⠬⠸⠄⠎ ⠸⠱ ⠕⠗ ⠔⠎⠻⠞ ⠸⠩ ⠈⠡⠸⠄⠎⠸⠒
⠼⠲ ⠄⠄⠄ ⠼⠆ ⠨⠅ ⠼⠦⠸⠆ ⠼⠦ ⠄⠄⠄ ⠼⠆ ⠨⠅ ⠼⠂⠴ ⠸⠱⠲

A punctuation indicator is needed both before the apostrophe <u>and</u> before the colon in ×'s:.

The ending is unspaced from the symbol even if the symbol normally requires a space.

⫸ =s ⠨⠅⠎

⫸ ='s ⠨⠅⠸⠄⠎

Instructions: Here is a list of isolated mathematical items and punctuation marks. Transcribe an opening Nemeth Code indicator in cell 1 and continue with the first item in the list on the same line. Stay in Nemeth throughout the practice (transcribe the clock time in Nemeth). Begin each line in cell 1; begin any runovers in cell 3. Place a Nemeth Code terminator after the last item in the list.

PRACTICE 2B

+, − ; ×, ÷ .

+'s, −'s, ×'s, ÷'s; =s, >s, <s.

" :: "

5.1, 6.22, 7.333; $8.44, $9.55; $10.66.

10:45-11:25

−16 > −___ ; 16 < ___ .

$1,400 < £ _?_

5'3" ..., 6'1"—6'2" ..., 7'0".

"8 · 3 = 3 · 8"

2.8 Summary of the Use and Nonuse of the Punctuation Indicator

2.8.1 **Situations That Do Not Require a Punctuation Indicator.** A punctuation indicator is not required before any of the following punctuation marks. In these isolated examples, assume that the technical material continues after what is shown.

a. The mathematical comma never requires a punctuation indicator.

⫸ 5.0, ⠼⠢⠨⠴⠠

b. A punctuation indicator is not used before a hyphen or a dash.

⫸ 5.5-7.0 ⠼⠢⠨⠢⠤⠶⠨⠴

⫸ $47,689—2.6% ⠈⠎⠲⠶⠠⠖⠦⠔⠤⠤⠼⠆⠨⠖⠈⠴

c. A punctuation indicator is not needed if the first character following a space is a punctuation mark or if the punctuation mark begins on a new line.

⫸ "+", "−" ⠦⠬⠸⠴⠠ ⠦⠤⠸⠴

d. In a sequence of punctuation marks following a mathematical expression, the punctuation indicator precedes only the first punctuation mark.

> ⫸ "=". ⠦⠨⠅⠸⠴⠲

e. In the next lesson, another situation where the punctuation indicator is not required will be presented: after a word or abbreviation.

2.8.2 **Situations That Require a Punctuation Indicator.** A punctuation indicator is required after any symbol of the type listed below when Nemeth has not been terminated and the mark of punctuation is not a comma, hyphen, or dash. In the following isolated examples, assume that the technical material continues after the final punctuation mark.

a. After a numeric symbol.

> ⫸ 98.6. ⠼⠔⠦⠨⠖⠸⠲

> ⫸ "4.9" ⠦⠼⠲⠨⠔⠸⠴

b. After a long dash or after an ellipsis.

> ⫸ 24 = 6 + ___. ⠼⠆⠲ ⠨⠅ ⠼⠖⠬ ⠤⠤⠤⠤⠸⠲

> ⫸ 1, 3.1413, … . ⠼⠂⠠ ⠼⠒⠨⠂⠲⠂⠒⠠ ⠄⠄⠄⠸⠲

c. After a general omission symbol.

> ⫸ 15 ÷ 3 = ?. ⠼⠂⠢⠨⠌⠒ ⠨⠅ ⠿⠸⠲

d. After a grouping symbol.

> ⫸ (".8"): ⠷⠸⠦⠼⠨⠦⠸⠴⠾⠸⠒

e. After any of the miscellaneous symbols presented so far.

> ⫸ 100%. ⠼⠂⠴⠴⠈⠴⠸⠲

> ⫸ 48¢? ⠼⠲⠦⠈⠉⠸⠦

f. After a comma, hyphen, or short dash, provided that if these were removed and the space which they occupy were not present, one of the situations above would apply.

> ⫸ 3y," ⠼⠒⠽⠠⠸⠴

> ⫸ "$99—" ⠦⠈⠎⠔⠔⠤⠤⠸⠴

Other situations where the punctuation indicator is required will be presented later in this course.

INTRODUCTION TO SIGNS OF GROUPING

2.9 Definition

In mathematical context, symbols such as parentheses, braces, and brackets are not considered to be punctuation; they are classified as signs of grouping. Here are some grouping signs commonly encountered in technical material.

⠷	Left Parenthesis	(
⠾	Right Parenthesis	)
⠨⠷	Left Curly Brace	{
⠨⠾	Right Curly Brace	}
⠈⠷	Left Square Bracket	[
⠈⠾	Right Square Bracket	]
⠨⠨⠷	Left Angle Bracket	⟨
⠨⠨⠾	Right Angle Bracket	⟩
⠳	Vertical Bar	|
⠳⠳	Double Vertical Bar	‖

More signs of grouping will be presented in Lesson 7.

2.10 Signs of Grouping with Numerals

a. The numeric indicator is not used before a numeral that immediately follows a grouping symbol.

⋙ (3)(5) ⠷⠒⠾⠷⠢⠾

Example 2-16

"Three times five" can be written this way: (3)(5).

⠦⠠⠹⠗⠑⠑ ⠐⠞⠎ ⠋⠊⠧⠑⠴ ⠉ ⠆ ⠺⠗⠊⠞⠞⠢ ⠹ ⠺⠁⠽⠒
⠸⠩ ⠷⠒⠾⠷⠢⠾ ⠸⠱⠲

These parentheses function as mathematical symbols representing multiplication, therefore a switch to Nemeth is required.

b. The numeric indicator is not used after a minus sign that immediately follows a grouping symbol.

⋙ |−8| ⠳⠤⠦⠳

Example 2-17

|−8| is spoken "the absolute value of negative eight."

c. Lacking a left grouping sign, the numeric indicator is required when the numeral is preceded by a space or begins a braille line. Compare:

⋙ 1)

⋙ (1)

2.11 Punctuation with Grouping Symbols

Grouping symbols of the Nemeth Code are mathematical symbols and therefore must be punctuated mathematically. Thus, except for the mathematical comma, hyphen, and dash, a punctuation indicator must be used before a punctuation mark which follows a sign of grouping.

⋙ {−5}, (0), [+5].

⋙ ("20%"),

Example 2-18

What fractions do these percentages represent? ("20%"), ("25%"), ("50%")

Switching Decision: To avoid excessive code switching, the entire string of percentages is transcribed in Nemeth, including the parentheses, quotation marks, and commas. (Reminder: A numeric indicator is required when a numeral immediately follows an opening quotation mark.) Note that, although the third percentage will fit on line 2, this would mean the Nemeth Code terminator would be alone on the next line. Because these are three separate expressions, they do not need to be all on the same line. The third expression is placed on the same line as the Nemeth Code terminator, which is the preferred layout.

2.12 Nested Grouping Symbols

When two or more grouping signs follow one another the outer set may be printed using a taller size in order to visually distinguish the nested groupings. The braille transcription does not differentiate between the sizes—regular grouping symbols are transcribed.

Example 2-19

Perform the inner computations before subtracting. $\big((4+7)-(7+4)\big)$

In print, the first and last parentheses are taller than the others.

Code-Switching Considerations

2.13 Enclosed Technical Material

When parentheses, brackets, braces, or quotation marks enclose isolated technical material, transcribe the paired punctuation inside the code switches.

⪢ "+"

Example 2-20

"+" means *plus*, "−" means *minus*, and "=" means *equals*.

a. Recall that many UEB punctuation symbols can be used inside of the Nemeth code switches. UEB parentheses and brackets do not fall into this category. Inside the switches, Nemeth grouping symbols are transcribed, even when a grouping sign functions as a punctuation mark.

⪢ (×)

Example 2-21

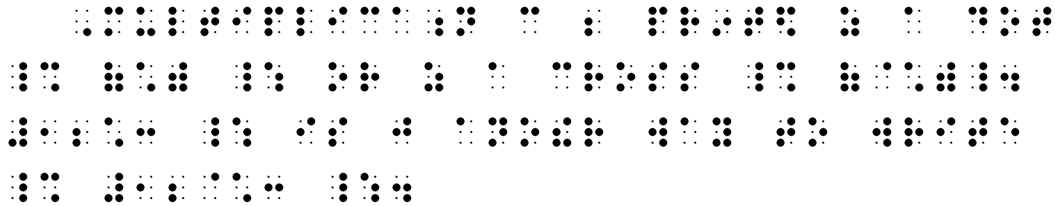
Multiplication can be printed as a dot (·) or as a cross (×).

2.13.1 **Punctuation Following a Sign of Grouping.** Grouping signs of the Nemeth Code are punctuated mathematically. This rule is illustrated by expanding the previous example, continuing in Nemeth following the period.

⪢ (×).

Example 2-22

Multiplication can be printed as a dot (·) or as a cross (×). 12 · 3 is just another way to write 12 × 3.

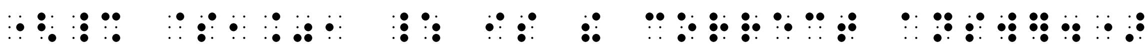


2.14 Paired Symbols

If parentheses apply to a larger phrase which begins or ends in UEB, transcribe the paired punctuation marks in UEB. Similarly, paired quotation marks should both be inside or both be outside of the switches.

Example 2-23

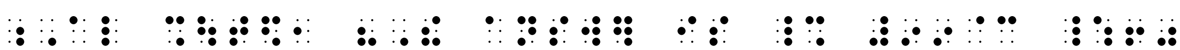
($1.01 is the correct answer.)



> *To transcribe both opening and closing parentheses in UEB, the opening Nemeth Code indicator is placed just inside the opening parenthesis.*

Example 2-24

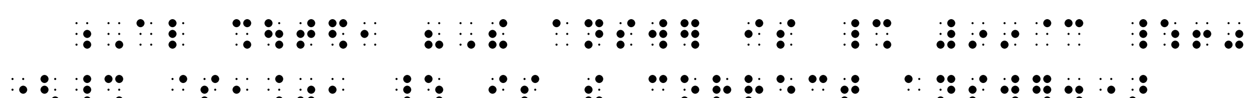
Al shouted, "The answer is 99¢!"



> *The opening quotation mark is in UEB. To match, the closing quotation mark is placed outside of the Nemeth Code terminator.*

Example 2-25

Al shouted, "The answer is 99¢!" ($1.01 is the correct answer.)



> *It would be incorrect to stay in Nemeth Code to transcribe the punctuation that occurs between these two monetary items because the quotation mark and the parenthesis are paired with UEB symbols outside of the switches.*

Spacing with Signs of Grouping

2.15 Spacing Inside of the Grouping Signs

Unless other rules apply, no space is left between an opening or a closing sign of grouping and the material which it encloses.

⫸ $[-4]$ ⠈⠷⠤⠲⠈⠾

⫸ (12¢) ⠷⠂⠆⠈⠉⠾

This includes symbols which usually require spacing—no space is left between a dash, an ellipsis, a sign of comparison, or any other symbol and its sign of grouping.

⫸ $(\_\_)(x) = 4x$ ⠷⠤⠤⠤⠤⠾⠷⠭⠾ ⠨⠅ ⠼⠲⠭

⫸ || … || ⠳⠳⠄⠄⠄⠳⠳

⫸ $(<, =, >)$ ⠷⠐⠅⠠ ⠨⠅⠠ ⠨⠂⠾

Example 2-26

Circle the correct comparison sign. $14 \div 7\ (<, =, >)\ 14 - 7$

⠠⠉⠊⠗⠉⠇⠑ ⠮ ⠉⠕⠗⠗⠑⠉⠞ ⠉⠕⠍⠏⠜⠊⠎⠕⠝ ⠎⠊⠛⠝⠲
⠸⠩ ⠼⠂⠲⠨⠌⠶ ⠷⠐⠅⠠ ⠨⠅⠠ ⠨⠂⠾ ⠼⠂⠲⠤⠶ ⠸⠱

2.15.1 **Special Case.** When a space is printed between an opening and a closing sign of grouping and that blank space does not represent an omission, the space between the grouping signs is included in the braille transcription.

Example 2-27

Angle brackets ⟨ ⟩ denote a sequence.

⠠⠁⠝⠛⠇⠑ ⠃⠗⠁⠉⠅⠑⠞⠎ ⠸⠩ ⠨⠨⠷ ⠨⠨⠾ ⠸⠱ ⠙⠢⠕⠞⠑ ⠁
⠎⠑⠟⠥⠰⠑⠲

Context will help you determine whether the print sign is an angle bracket or a "less than" or a "greater than" symbol.

2.16 Spacing Outside of the Grouping Signs

The spacing before and after an enclosed expression is subject to the spacing rules for the signs which precede or follow the enclosure.

Example 2-28

Perform the multiplication before the addition. $(4 \times 30) + (4 \times 2) = 128$

Operation signs are unspaced; comparison signs are spaced.

Example 2-29

Complete the missing values in the range (0.1) ... (0.9)

The ellipsis is spaced.

a. No space is left between an enclosed expression and a numeral when these items are part of the same expression unless other spacing rules apply. These items often appear to be spaced in print.

Example 2-30

Does $5\ (9 + 7) = (5 \cdot 9) + 7$?

b. No space is left between an enclosed expression and another sign of grouping when these items are part of the same expression unless other spacing rules apply. These items often appear to be spaced in print.

Example 2-31

Multiply, then add. $[\ (3)\ (-1)\ ] + [\ (1)\ (-3)\ ]$

Instructions: Format each line or sentence in print as a 3-1 paragraph in braille.

PRACTICE 2C

Is $3(-2.5) + (-4)$ the same as $3(-2.5 + (-4))$?

Use a number line to illustrate this addition problem: $[-4 - (-1)] + [-1 - (-3)]$.

$7 + (-3) + (-4) = ?$

$8 + |(-2) + (-3)| = ?$

$|2(-7.5)| + 3.2(2) = ?$

The **multiplicative identify** [*sic*] property is illustrated: $(83)(1) = 83$.

A **unit set** is a set containing only one element. For example, {9} is a unit set containing the element "9".

What is the meaning of the symbol "||" in "The answer is ‖3.1‖"?

A finite decimal (such as 0.152) is one that stops, whereas an infinite decimal (such as 0.9999…) repeats indefinitely.

2.17 Identifiers and Braille Page Turns

As stated earlier, itemized problems may begin at the bottom of a braille page and run over to the top of the next braille page. However, if no part of the problem will fit on line 25, place the identifier at the top of the new braille page. Do not leave an identifier standing alone at the bottom of a braille page.

2.17.1 **Print Page Number Interference.** A math expression may begin in the runover cell of the line following the identifier (line 2) if the space taken up by the print page number on line 1 will not allow it to fit there. Keeping the math expression together on one braille line takes precedence.

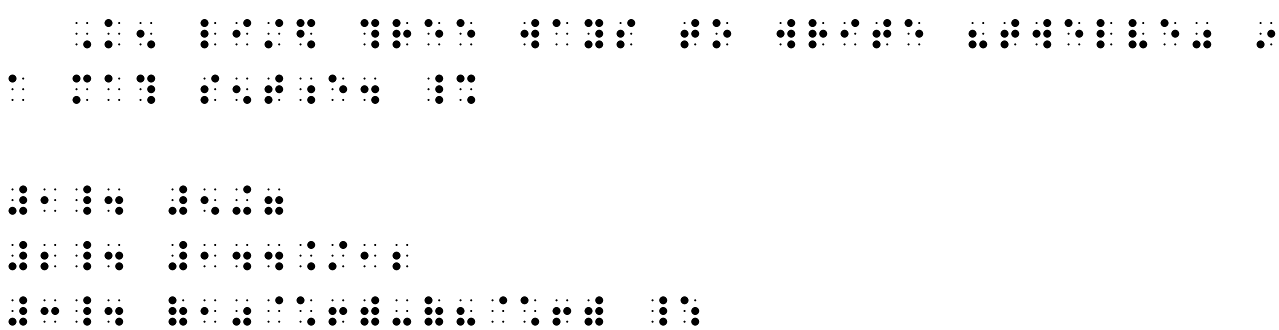
2.18 Code Switching and Identifiers

2.18.1 **A Numbered List of Nemeth Items.** You have learned one format for itemized material: 1-3. In this layout, each identifier begins in cell 1. When a numbered list of Nemeth items follows UEB text, place the opening Nemeth Code indicator at the end of the line of text that precedes the list. (See Example 2-32.) If the opening Nemeth Code indicator does not fit at the end of the line that precedes the identified Nemeth material, place it on the next line in the runover position. Note that the code switch indicator does not take the place of the blank line that must precede the list. (See Example 2-33.) Embedded identifiers follow the same guidelines. (See Example 2-34.)

Example 2-32

Ken listed three ways to write "twelve" in a math sentence.

1. $5+7$
2. $144 \div 12$
3. $(10 \times 6) - (8 \times 6)$



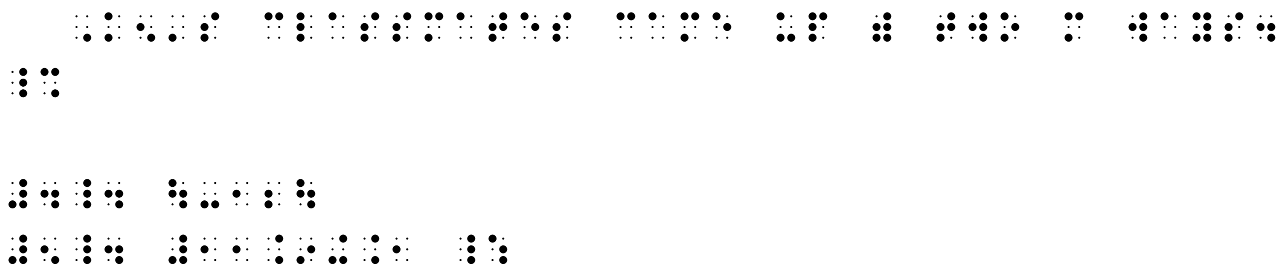
Line 2: The opening Nemeth Code indicator is placed at the end of the line of text when the following listed items are all in Nemeth.

Example 2-33

Ken's classmates came up with two more ways.

4. $|-12|$

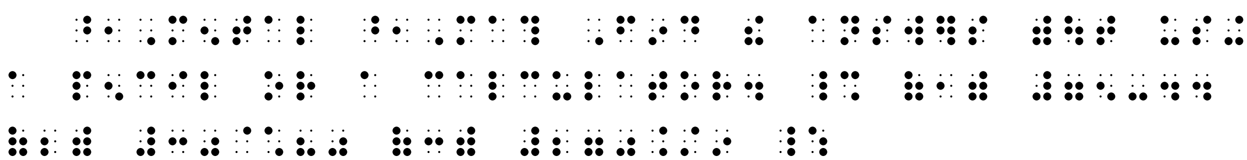
5. $11.9 + .1$



Line 2: The opening Nemeth Code indicator is placed in the runover cell of the narrative when it does not fit on the preceding line.

Example 2-34

Mental Math Find the answers without using a pencil or a calculator. (1) $75 - 44$ (2) 30×80 (3) $270 \div 9$



Line 2: Since all three problems are in Nemeth, the opening Nemeth Code indicator is placed before the first identifier.

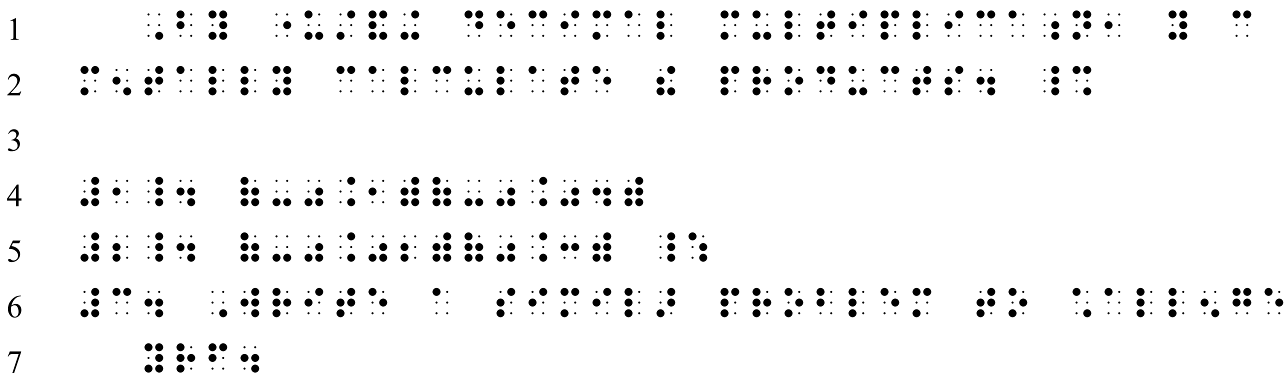
2.18.2 **A Numbered List of Mixed Items.** Even though an identifier is not part of the math, there is no need to transcribe all of the identifiers in UEB, nor is it necessary to transcribe all of the identifiers in the same code. Each identifier is transcribed according to the rules for the code in use at the time. In a mixture of itemized UEB and Nemeth, the following suggestions are offered. By following these guidelines, the switch indicators will be placed where they are least intrusive to the flow of text and will not interfere with the alignment of the identifiers.

a. If the first two or more items in the list require Nemeth exclusively, the opening Nemeth Code indicator may be placed as shown in the previous section. Nemeth is terminated after the last math item.

Example 2-35

By understanding decimal multiplication, you can mentally calculate the products.

1. (−0.1) (−0.04)
2. (−0.02) (0.3)
3. Write a similar problem to challenge yourself.



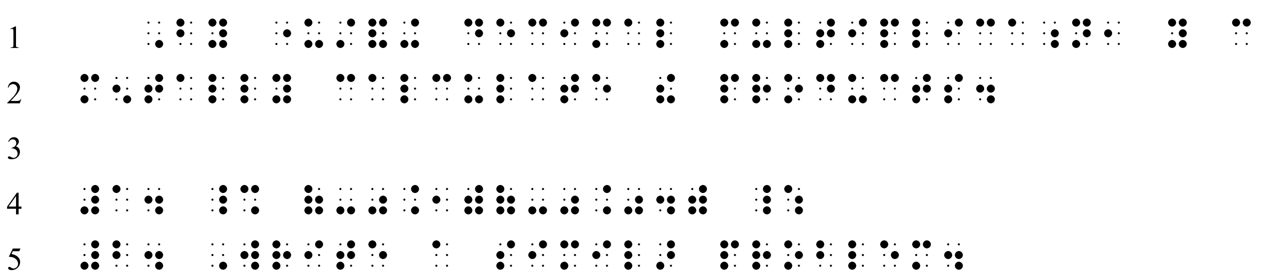
Line 2: The opening Nemeth Code switch indicator is placed at the end of the narrative.
Line 4: The first identifier is in Nemeth.
Line 5: Nemeth continues, and is then terminated before proceeding to the next item, which is in UEB.
Line 6: UEB resumes.

b. If only the first item in the list requires Nemeth, it is recommended that the switches be placed before and after the math portion only. In other words, the identifier will be in UEB, followed directly by an opening Nemeth Code indicator.

Example 2-36

By understanding decimal multiplication, you can mentally calculate the products.

1. (−0.1) (−0.04)
2. Write a similar problem.



Line 4: The identifier is in UEB. The switches are placed before and after the math portion only.
Line 5: UEB resumes.

c. Similarly, within the list, when UEB is in effect at the end of an item and the beginning of the next item is in Nemeth, transcribe the identifier in UEB and then switch to Nemeth.

Example 2-37

True or False?
(1) 98.6 is normal human body temperature expressed in degrees Celsius.
(2) 50% represents the same portion as 0.5.

Line 3: Only the decimal numeral is between the switches.
Line 6: Only the percentage is between the switches.

d. When Nemeth is in effect at the end of an item and the beginning of the next item is in UEB, place the Nemeth Code terminator at the end of the Nemeth material and transcribe the next identifier in UEB.

Example 2-38

True or False?
(1) 50% represents the same portion as 0.5.
(2) Expressed in degrees Celsius, normal human body temperature is 98.6.

Line 4: Nemeth is terminated before the period.
Line 5: The identifier is in UEB, and UEB continues.

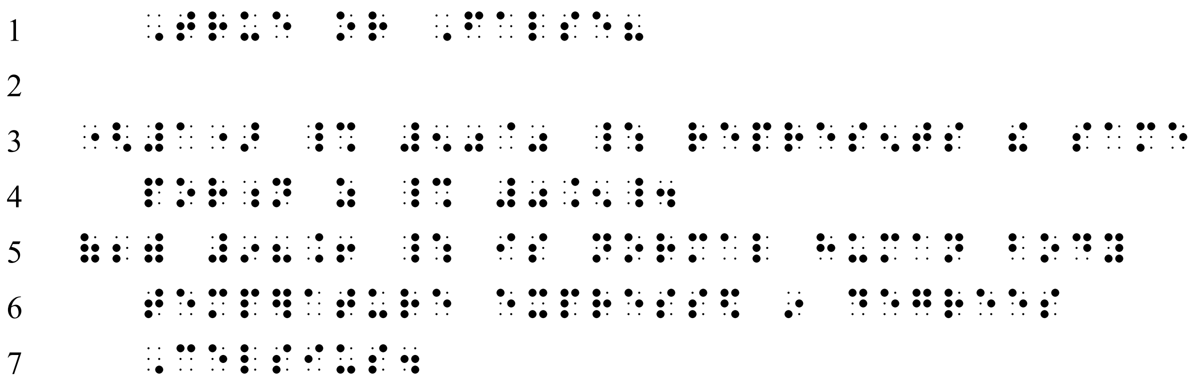
e. When Nemeth is in effect at the end of an item and the beginning of the next item is also in Nemeth, do not switch out of Nemeth code simply to transcribe the identifier. The identifier will be transcribed following Nemeth rules.

Example 2-39

True or False?

(1) 50% represents the same portion as 0.5.

(2) 98.6 is normal human body temperature expressed in degrees Celsius.



Line 4: The period requires a punctuation indicator because Nemeth continues.

Line 5: The identifier uses Nemeth parentheses and the lower-cell numeral. Nemeth is terminated after the decimal numeral.

PRACTICE 2D

1) The box is 2’4” in height.

2) $5'$ is the same as $60''$

3) $12' + 15'' = 13'3''$

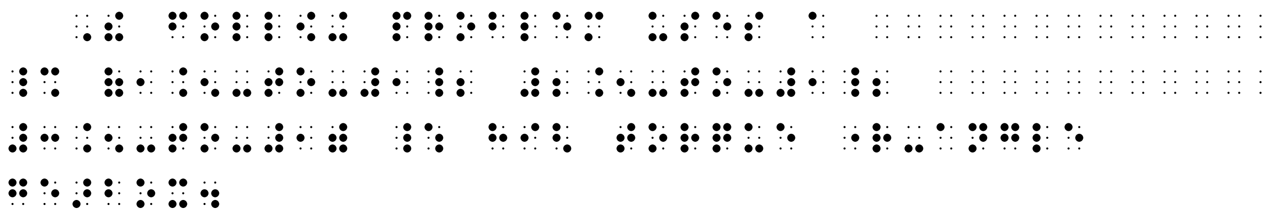
Format

2.19 Keep Together—Hyphenated Expressions

A hyphenated expression containing one or more mathematical components must not be divided between braille lines. Because Nemeth format rules are applied throughout a technical transcription, this rule also applies in UEB text when a numeral and a word are connected by a hyphen.

Example 2-40

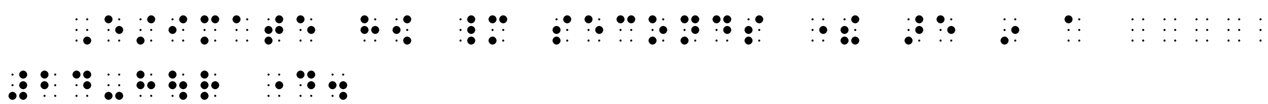
The next problem uses a (1.5-to-1; 2.5-to-1; 3.5-to-1) high torque right-angle gearbox.



The hyphenated expressions are not divided.

Example 2-41

Estimate how many seconds there are in a 24-hour day.



Do not divide "24-hour".

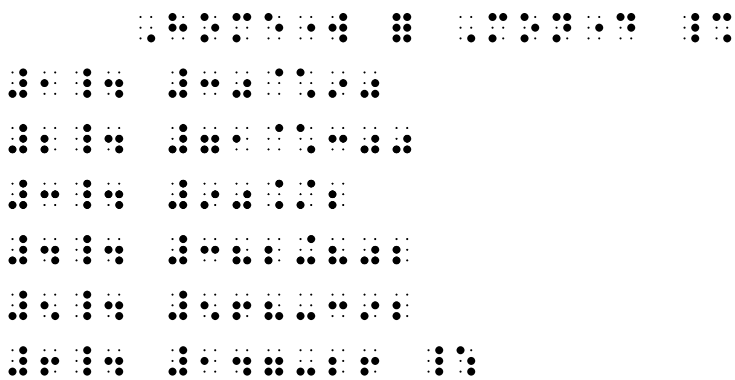
2.20 Margins for Itemized Material with No Subdivisions—Side-by-Side Layout

When the print copy arranges itemized material side by side across the page and there are no subdivisions, the braille format is changed so that all identifiers start in cell 1.

Example 2-42

Homework for Monday

1. 30×90	2. 71×300	3. $90 \div 2$
4. $382 + 802$	5. $568 - 392$	6. $147 - 26$



Note that the opening Nemeth Code indicator is placed at the end of the cell-5 heading. A discussion about code switches and headings occurs in Lesson 4.

For further practice, see Appendix A—Reading Practice.

EXERCISE 2

Prepare Exercise 2 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 2A

⠀⠀⠼⠛⠃⠓⠁⠉⠋⠑⠙⠂ ⠱⠢ ⠺⠗⠊⠞⠞⠢ ⠵
⠸⠩ ⠼⠶⠆ ⠦⠂ ⠒⠖ ⠢⠲ ⠸⠱⠂ ⠊⠌ ⠕⠃⠧⠊⠳⠌⠇⠽
⠙⠊⠧⠊⠌⠊⠃⠇⠑ ⠃⠽ ⠼⠊⠲
⠀⠀⠠⠺⠗⠊⠞⠑ ⠘⠮ ⠝⠥⠍⠃⠻⠌⠒ ⠸⠩ ⠼⠒⠨⠆⠔⠠ ⠼⠢⠴⠴⠠
⠤⠼⠂⠆⠒⠠ ⠼⠆⠠⠴⠴⠴⠨⠦⠦⠠ ⠤⠼⠆⠢⠴⠠⠶⠔⠲ ⠸⠱⠲ ⠠⠝⠪ ⠁⠙⠙
⠮⠍ ⠞⠛⠗⠲
⠀⠀⠠⠊⠌ ⠮ ⠁⠝⠌⠺⠻ ⠸⠩ ⠼⠲⠨⠴⠈⠴⠴ ⠸⠱⠂ ⠕⠗ ⠊⠌ ⠭
⠸⠩ ⠼⠲⠨⠴⠈⠴ ⠸⠱⠦

PRACTICE 2B

⠸⠩ ⠬⠠ ⠤⠸⠆ ⠈⠡⠠ ⠨⠌⠸⠲
⠬⠸⠄⠌⠠ ⠤⠸⠄⠌⠠ ⠈⠡⠸⠄⠌⠠ ⠨⠌⠸⠄⠌⠸⠆ ⠨⠅⠌⠠ ⠨⠂⠌⠠
⠀⠀⠐⠅⠌⠸⠲
⠦⠰⠆⠸⠴
⠼⠢⠨⠂⠠ ⠼⠖⠨⠆⠆⠠ ⠼⠶⠨⠒⠒⠒⠸⠆ ⠈⠌⠦⠨⠲⠲⠠ ⠈⠌⠔⠨⠢⠢⠸⠆
⠀⠀⠈⠌⠂⠴⠨⠖⠖⠸⠲
⠼⠂⠴⠸⠒⠼⠲⠢⠤⠂⠂⠸⠒⠼⠆⠢
⠤⠼⠂⠖ ⠨⠂ ⠤ ⠤⠤⠤⠤⠸⠆ ⠼⠂⠖ ⠐⠅ ⠤⠤⠤⠤⠸⠲
⠈⠌⠂⠠⠲⠴⠴ ⠐⠅ ⠈⠇⠿
⠼⠢⠄⠒⠄⠄ ⠄⠄⠄⠠ ⠼⠖⠄⠂⠄⠄⠤⠤⠼⠖⠄⠆⠄⠄ ⠄⠄⠄⠠ ⠼⠶⠄⠦⠄⠄⠸⠲
⠦⠼⠦⠱⠒ ⠸⠅ ⠼⠒⠱⠦⠸⠴ ⠸⠱

PRACTICE 2C

⠠⠊⠎ ⠸⠩ ⠼⠒⠷⠤⠆⠨⠢⠾⠬⠷⠤⠲⠾ ⠸⠱ ⠮ ⠎⠁⠍⠑ ⠵
⠸⠩ ⠼⠒⠷⠤⠆⠨⠢⠬⠷⠤⠲⠾⠾ ⠸⠱⠦

⠠⠥⠎⠑ ⠁ ⠝⠥⠍⠃⠻ ⠇⠔⠑ ⠞⠕ ⠊⠇⠇⠥⠌⠗⠁⠞⠑ ⠹ ⠁⠙⠙⠊⠰⠝
⠏⠗⠕⠃⠇⠑⠍⠒ ⠸⠩ ⠈⠷⠤⠲⠤⠷⠤⠂⠾⠈⠾⠬⠈⠷⠤⠂⠤⠷⠤⠒⠾⠈⠾⠸⠲

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⠎⠑⠞ ⠒⠞⠁⠔⠬ ⠮ ⠑⠇⠑⠰⠞ ⠦⠼⠊⠴⠲

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⠠⠁ ⠋⠔⠊⠞⠑ ⠙⠑⠉⠊⠍⠁⠇ ⠐⠣⠎⠡ ⠵ ⠸⠩ ⠼⠴⠨⠂⠢⠆ ⠸⠱ ⠐⠜
⠊⠎ ⠐⠕ ⠞ ⠌⠕⠏⠎⠂ ⠐⠱⠁⠎ ⠁⠝ ⠔⠋⠔⠊⠞⠑ ⠙⠑⠉⠊⠍⠁⠇
⠐⠣⠎⠡ ⠵ ⠸⠩ ⠼⠴⠨⠔⠔⠔⠔ ⠄⠄⠄ ⠸⠱⠐⠜ ⠗⠑⠏⠂⠞⠎
⠔⠙⠑⠋⠔⠊⠞⠑⠇⠽⠲

PRACTICE 2D

⠼⠁⠐⠜ ⠠⠮ ⠃⠕⠭ ⠊⠎ ⠸⠩ ⠼⠆⠄⠲⠄⠄ ⠸⠱ ⠔ ⠓⠑⠊⠣⠞⠲
⠼⠃⠐⠜ ⠸⠩ ⠼⠢⠄ ⠸⠱ ⠊⠎ ⠮ ⠎⠁⠍⠑ ⠵ ⠸⠩ ⠼⠖⠴⠄⠄
⠼⠒⠾ ⠼⠂⠆⠄⠬⠂⠢⠄⠄ ⠨⠅ ⠼⠂⠒⠄⠒⠄⠄ ⠸⠱

LESSON 3

- WORDS
    - Introduction to Abbreviations
    - Single-Word Switch Indicator
- LETTERS
    - Introduction to the English-Letter Indicator
    - Mathematical Letter Combinations

Format

- Keep Together
- FORMAT SUMMARY #2

Answers to Practice Material

LESSON PREVIEW

Transcription of words in mathematical context requires a close look at punctuation, capitalization, and nonuse of contractions. Abbreviations require special treatment. A single narrative word may be transcribed within the code switches by using a single-word switch indicator. Code switching at page turns is examined. "Single letters" in Nemeth are defined, and the English-letter indicator is introduced.

WORDS

3.1 Words in Mathematical Context

When words are part of an equation or math expression the words are included in the technical notation—that is, the whole expression is placed inside the Nemeth switches. No contractions are used within Nemeth switches. Spacing rules of the Nemeth Code are followed.

Example 3-1

In the next problem, length=5, width=12, height=7.

⠀⠀⠠⠔ ⠮ ⠝⠑⠭⠞ ⠏⠗⠕⠃⠇⠑⠍⠂ ⠸⠩ ⠇⠑⠝⠛⠞⠓ ⠨⠅ ⠼⠢⠠
⠺⠊⠙⠞⠓ ⠨⠅ ⠼⠂⠆⠠ ⠓⠑⠊⠛⠓⠞ ⠨⠅ ⠼⠶ ⠸⠱⠲

The words are transcribed without contractions. Nemeth Code requires a space before and after the comparison signs (equals signs) regardless of print spacing.

In the next example, words are substituted for values in a formula. The words are part of the math expression and are transcribed without contractions. The division symbol is unspaced from the words according to Nemeth Code rules for spacing of operation signs.

Example 3-2

Dividend ÷ Divisor = Quotient

⠸⠩ ⠠⠙⠊⠧⠊⠙⠑⠝⠙⠨⠌⠠⠙⠊⠧⠊⠎⠕⠗ ⠨⠅ ⠠⠟⠥⠕⠞⠊⠑⠝⠞ ⠸⠱

As part of a math problem expressed in symbols and words, the words are included in the switch.

Example 3-3

Adding decimals in a recipe: .5 of a cup + .75 of a cup = ? cup.

⠀⠀⠠⠁⠙⠙⠬ ⠙⠑⠉⠊⠍⠁⠇⠎ ⠔ ⠁ ⠗⠑⠉⠊⠏⠑⠒ ⠸⠩
⠼⠨⠢ ⠕⠋ ⠁ ⠉⠥⠏⠬⠨⠶⠢ ⠕⠋ ⠁ ⠉⠥⠏ ⠨⠅ ⠿ ⠉⠥⠏ ⠸⠱⠲

The operation sign (+) *is unspaced; the comparison sign* (=) *is spaced, according to the rules of the Nemeth Code.*

3.1.1 **Capitalization.** Each fully capitalized word in mathematical context is preceded by the double capitalization indicator of the Nemeth Code. The UEB capitalized passage indicator is not used in Nemeth context.

⠠⠠	Double Capitalization Indicator

Example 3-4

TOTAL EGG COUNT = 79

3.2 Words in Narrative

When a word in UEB narrative is associated with an expression that requires Nemeth, the word is not included inside the Nemeth switches. The word and its associated expression may fall on separate braille lines, with the line wrapping at the space between them. Note that this rule differs from an abbreviation associated with a Nemeth expression. (See Section 3.4.)

a. Words Labeling a Math Item

Example 3-5

Figure 4.7 shows Shape 4 and its reflection, Shape 4′.

Example 3-6

Chris used 25.5 cans of paint.

Example 3-7

What is 5.5 percent of 72?

b. Units of Measure

Example 3-8

Logan weighed exactly 7.00 pounds at birth.

Only the decimal number is inside the switches.

Example 3-9

Marc's sister weighed 6 pounds 3.2 ounces.

The number need not fall on the same line with its associated word.

3.3 Punctuation With Words

Words are not punctuated mathematically, even when they are transcribed in mathematical context. For a comma, the dot 2 comma is used; for other punctuation marks, no punctuation indicator is used. The next example shows a set of class members using mathematical braces to enclose the set. Set notation is transcribed in Nemeth.

Example 3-10

{Richard, Daniel, Steven}

The literary comma is used when a comma follows a word, even in mathematical context.

In the next example, the math form being described is illustrated with words. Because the model is a math expression, a switch to Nemeth is recommended.

Example 3-11

A semicolon is used to separate variables from parameters in the form (variable; parameter).

The semicolon does not require a punctuation indicator because words are punctuated in literary mode, even in mathematical context.

PRACTICE 3A

A. If 1 pound of Swiss cheese costs $2.50, how much does 4.8 pounds cost?

B. JMHS's set of high-jump champions: {Terry, Leslie, Traci}

C. The parts of a subtraction problem are named as follows: minuend – subtrahend = difference.

D. Did you know that 98.6 degrees Fahrenheit is not necessarily "normal" body temperature for everyone?

Introduction to Abbreviations

This lesson examines abbreviations within narrative.

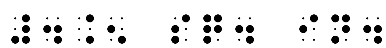
3.4 Abbreviations

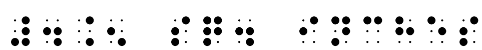
An abbreviation by itself is not mathematical and does not require a switch to Nemeth Code. However, when the value requires Nemeth, both the value and the abbreviation are placed inside the Nemeth switches.

⫸ 1.5 min 

No contractions are used in Nemeth.

The same rule applies to a two-part abbreviation, as well as a two-part unit when one part is a whole word and the other part is an abbreviation.

⫸ 4.5 sq. in. 

⫸ 4.5 sq. inches 

Example 3-12

SAS means "side angle side".

Even though SAS is a special abbreviation in the field of mathematics, as used in this example it is simply an abbreviation in the narrative and is transcribed in UEB.

Example 3-13

In hours, how long is the 8 a.m.-3 p.m. school day?

⠠⠔ ⠓⠳⠗⠎⠂ ⠓⠪ ⠇⠰⠛ ⠊⠎ ⠮ ⠼⠓ ⠁⠲⠍⠲⠤⠼⠉ ⠏⠲⠍⠲
⠎⠡⠕⠕⠇ ⠐⠙⠦

A freestanding unmodified numeral and any associated abbreviation does not require a code switch.

Example 3-14

15 mm can be expressed as 1.5 cm.

⠼⠁⠑ ⠍⠍ ⠉ ⠆ ⠑⠭⠏⠗⠑⠎⠎⠫ ⠵ ⠸⠩ ⠼⠂⠨⠢ ⠉⠍ ⠸⠱⠲

The abbreviation cm applies to the number 1.5 and so the abbreviation is included in the switches.

Example 3-15

There are 60 sec in 1 min. It follows that 1.5 min, expressed in seconds, is 90 sec.

⠠⠐⠮ ⠜⠑ ⠼⠋⠚ ⠎⠑⠉ ⠔ ⠼⠁ ⠍⠔⠲ ⠠⠭ ⠋⠕⠇⠇⠪⠎ ⠞
⠸⠩ ⠼⠂⠨⠢ ⠍⠊⠝ ⠸⠱⠂ ⠑⠭⠏⠗⠑⠎⠎⠫ ⠔ ⠎⠑⠉⠕⠝⠙⠎⠂ ⠊⠎
⠼⠊⠚ ⠎⠑⠉⠲

3.4.1 **Format—Keep Together.** An abbreviation and a preceding or following numeral to which it applies must not be divided between braille lines. Because Nemeth format rules are applied throughout a technical transcription, this rule also applies in the UEB text. Note that the print copy may not follow this format, but it must be applied in the braille transcription.

Example 3-16

One millisec is a thousandth of a second. In other words, there are 1000 ms in 1 sec, or 1 sec equals 1000 ms. How many ms in 1 min?

⠠⠐⠕ ⠍⠊⠇⠇⠊⠎⠑⠉ ⠊⠎ ⠁ ⠹⠳⠎⠯⠹ ⠷ ⠁ ⠎⠑⠉⠕⠝⠙⠲ ⠠⠔
⠕⠮⠗ ⠘⠺⠎⠂ ⠐⠮ ⠜⠑ ⠼⠁⠚⠚⠚ ⠍⠎ ⠔ ⠼⠁ ⠎⠑⠉⠂ ⠕⠗ ⠀⠀⠀
⠼⠁ ⠎⠑⠉ ⠑⠟⠥⠁⠇⠎ ⠼⠁⠚⠚⠚ ⠍⠎⠲ ⠠⠓⠪ ⠸⠍ ⠍⠎ ⠔ ⠀⠀⠀⠀
⠼⠁ ⠍⠔⠦

The number 1 is placed on the same line as its related abbreviation (1 sec and 1 min).

Example 3-17

New carpet costs $32.50 per sq. yard. What will it cost to carpet a room that measures 9.6 sq. yards?

sq. yard must not be divided between lines, even in UEB context. sq. yards is associated with a decimal value and so is included inside the code switches.

3.4.2 **Punctuation with Abbreviations.** Abbreviations are punctuated in literary mode, even when they are transcribed in mathematical context. For a comma, the dot 2 comma is used; for other punctuation marks, no punctuation indicator is used. Note in the example below that the periods following min. and sec. do not require a punctuation indicator.

⫸ 4.72 min. = 283.2 sec.

Example 3-18

Converting minutes to seconds, 4.72 min. = 283.2 sec.

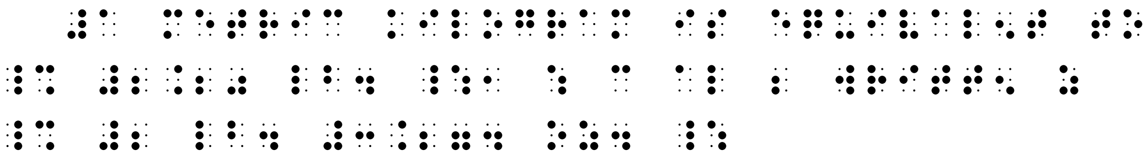
a. **Abbreviations with a Related Period.** Examine the surrounding material to determine if the abbreviations include a period. If they do, the abbreviation must not be separated from its related period. If a Nemeth Code terminator follows the abbreviation, the period that belongs to the abbreviation is placed before the terminator, unspaced from its abbreviation.

If a period functions both with an abbreviation and as punctuation at the end of a sentence, keep the period with its abbreviation.

In an isolated problem where there are no context clues to determine whether a period applies to the abbreviation or merely ends the sentence, assume that it applies to the abbreviation.

Example 3-19

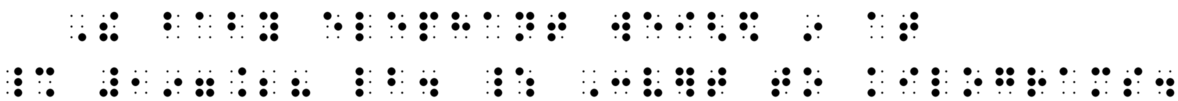
1 metric kilogram is equivalent to 2.20 lb., which can also be written as 2 lb. 3.274 oz.



The comma is part of the sentence structure and so is placed after the Nemeth Code terminator. Although 2 lb. by itself does not require a switch, because it is part of a measurement that contains a decimal, the entire weight is transcribed in Nemeth to maintain continuity.

Example 3-20

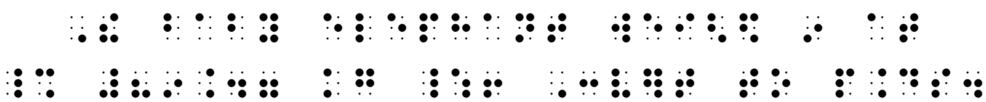
The baby elephant weighed in at 197.28 lb. Convert to kilograms.



The period after lb is treated as if it belongs to the abbreviation because there are no context clues regarding the function of the period.

Example 3-21

The baby elephant weighed in at 89.47 kg! Convert to pounds.



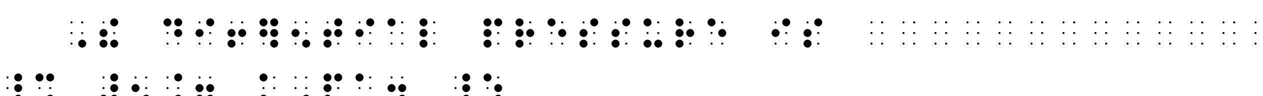
There is no period after kg. The punctuation applies to the sentence.

3.4.3 **Spacing with Abbreviations**.

a. Unless a Nemeth rule states otherwise, a space must be left between an abbreviation and the numeral to which it applies, even if the print copy shows no space. Because Nemeth format rules are applied throughout a technical transcription, this rule also applies in the UEB text.

Example 3-22

The differential pressure is 5.7kPa.



In print, there is no space between 5.7 and the abbreviation kPa.

Example 3-23

Measure the width of your desk using a 30mm ruler.

In braille, a space is inserted between 30 and mm, even though it is unspaced in print.

b. An abbreviation consisting of two or more components is transcribed as spaced or unspaced to conform with the print text. "Keep together" format applies to the entire abbreviation and its related numeral, even in the UEB text.

Example 3-24

Demonstrate to your classmates that 1 sq. in. is equivalent to 645.16 sq. mm.

In print, there is a space between sq. and mm.

Example 3-25

15.34 fl.oz. of water weighs 1 lb.

In print, there is no space between fl. and oz.

3.5 Numbers with Ordinal Endings

Ordinal endings are not abbreviations. Recall from Lesson 1 that a numeral with an ordinal ending is transcribed in UEB if it occurs in literary context. If the ordinal appears in mathematical context, the ordinal ending becomes part of the expression and is punctuated mathematically.

⫸ 1st,

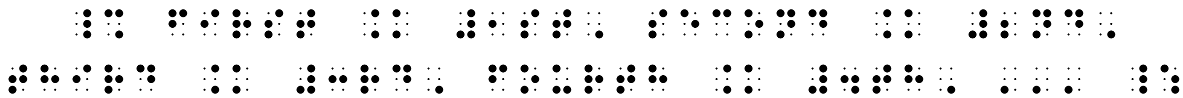
Example 3-26

Rearrange: 2nd, 4th, 1st, 3rd.

These ordinals are transcribed in UEB.

Example 3-27

first = 1st, second = 2nd, third = 3rd, fourth = 4th, ...



In Nemeth, ordinal endings are punctuated mathematically.

PRACTICE 3B

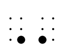
1. If 1 lb. of Gouda cheese costs $2.96, what will you pay for 2.5 lbs.?
2. If 2kg Gruyère costs £2,65, what is the cost of a wheel weighing 3kg?
3. Continuing the set of ordinals, fifth = 5th, sixth = 6th, seventh = 7th, eighth = 8th.
4. 1 kcal is equivalent to 3088.03 ft.lb.

More To Come   This does not complete the discussion of abbreviations in mathematical context. Single-letter abbreviations, abbreviations that use the same letters as a shortform, and further spacing rules within mathematical expressions will be discussed in Lesson 4.

Single-Word Switch Indicator

3.6 The Single-Word Switch Indicator

Words that do not provide mathematical meaning are transcribed in UEB. When a single word occurs between two math expressions, the single-word switch indicator is used to indicate that the following word is in UEB.

	`Single-Word Switch Indicator`

Until this symbol becomes widely recognized, we suggest that the single-word switch indicator be listed on the Special Symbols page. See the Final Lesson for details.

3.6.1 **Spacing and Contractions.** The single-word switch indicator is unspaced from the word. Contractions are used according to the rules of UEB. The switch is required on a single word even if the word contains no contractions. The effect of the single-word switch indicator is terminated by a space, and Nemeth Code resumes.

Example 3-28

Since $40 \cdot 7 = 280$ and $5 \cdot 7 = 35$, does $45 \cdot 7 = 280 + 35$ or 315?

The words Since, and, does, and or are part of the sentence structure—they are not being used mathematically—and so UEB applies.

3.6.2 **With a Hyphenated Word.** The single-word switch indicator can be used with a hyphenated compound word.

Example 3-29

Compare: $2 + 2 = 4$ vis-à-vis $2 \times 2 = 4$.

The hyphenated compound word vis-à-vis is considered to be one word. The acute accent follows UEB rules for modified letters.

3.6.3 **With Typeform.** The single-word switch indicator can be used with a word associated with a UEB typeform word indicator.

Example 3-30

Can $2 + 3 \times 4$ be both $(2 + 3) \times 4$ <u>and</u> $2 + (3 \times 4)$?

The single-word switch indicator is used on the underlined word <u>and</u>.

3.6.4 **With Lower Wordsigns.** A lower wordsign may be used with a single-word switch indicator without violating the lower sign rule.

Example 3-31

Let $2 + 3 \times 4$ be $2 + (3 \times 4)$.

The single-word switch indicator is used on the lower wordsign for be.

3.6.5 **With Abbreviations.** A single-word switch may be used for an abbreviation. But remember, an abbreviation of measurement associated with a Nemeth number is part of the Nemeth expression.

Example 3-32

Diving Scores: YWCA 6.8, YMCA 6.4.

Example 3-33

Which is longer, 4.5 in. or 4.5 mm?

The single-word switch is used for the word or.

Example 3-34

Which is longer, 4.5 inches or 4.5 millimeters?

There are two words between the Nemeth numbers. Nemeth Code is terminated before the words are transcribed.

3.6.6 **Switch Considerations.** Just because a numeral <u>can</u> be transcribed in UEB does not mean it <u>must</u> be transcribed in UEB. Consider how cumbersome it would be to read the next example if you used Nemeth only for the negative numbers.

Example 3-35

The daily high temperatures last week (in degrees Fahrenheit) were 7, 1, –3, 0, –1, 3, and –5.

Excessive switching is avoided by including all of the numbers (even the unmodified ones) inside one set of switch indicators.

3.6.7 **With Punctuation.** A single-word switch indicator cannot be placed immediately before a mark of punctuation such as an opening quotation mark or an opening parenthesis.

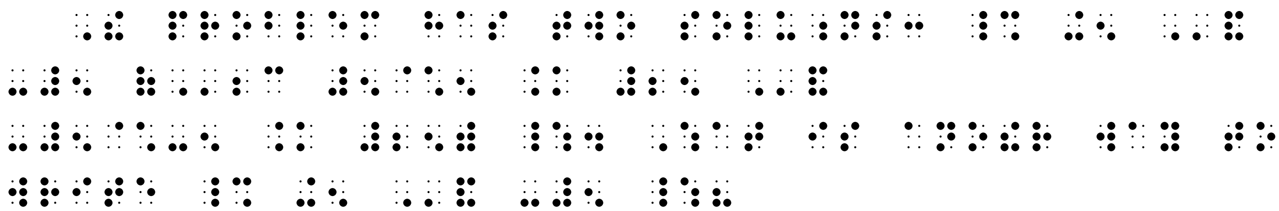
Correct:

Incorrect:

Recall from Lesson 2 that paired grouping symbols must be transcribed in the same code. Examine the treatment of the parentheses in the next two examples. Note that Example 3-37 shows two viable interpretations.

Example 3-36

The problem has two solutions: $+5$ and -5 (because $5 \times 5 = 25$ and $-5 \times -5 = 25$). What is another way to write $+5$ and -5?

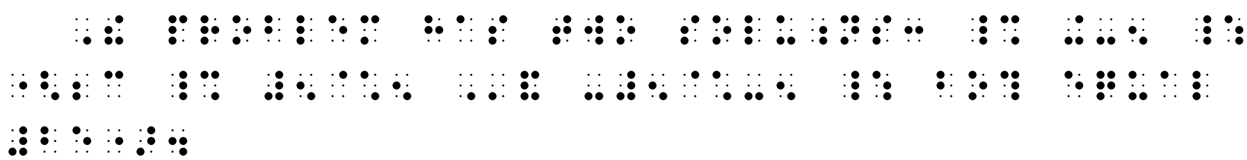


The opening and closing parentheses are in Nemeth. The single-word switch indicator immediately follows the opening parenthesis.

Example 3-37

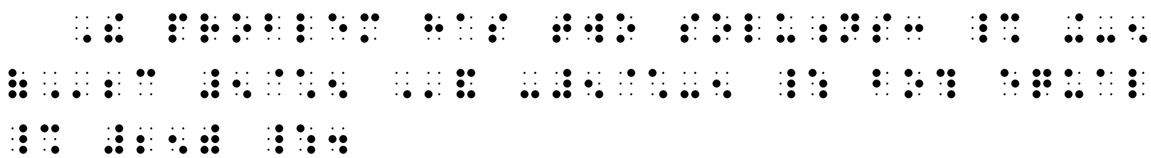
The problem has two solutions: ± 5 (because 5×5 and -5×-5 both equal 25).

Transcription A.



The closing parenthesis is in UEB. The opening parenthesis must also be in UEB. Because a single-word switch indicator cannot be used immediately before an opening parenthesis, Nemeth Code is terminated and then reopened after the word.

Transcription B.



By transcribing the numeral 25 and the closing parenthesis in Nemeth, the opening parenthesis is now done in Nemeth, similar to Example 3-36.

3.6.8 **The Word "of".** The word "of" requires a closer look. Within a narrative sentence, it is a word like any other word and may require a single-word switch indicator. However, when "of" is part of an equation, it is transcribed in Nemeth, uncontracted, without any code switching. Compare the treatment of the word "of" in these examples.

Example 3-38

Estimate the tax: 6% of $5.25

"of" is narrative and so is transcribed in UEB using a single-word switch indicator.

Example 3-39

6% of $5.25 = $.32

"of" is part of the equation ($5.25 alone does not equal $.32) and so "of" is transcribed in Nemeth, uncontracted.

Example 3-40

What is 5.5 percent of 72? 5.5% of 72 = 3.96.

3.6.9 **Two or More Words.** When more than one narrative word in succession appears within mathematical context, Nemeth Code must be terminated in order to transcribe the words in UEB.

Example 3-41

"Work the problem 2 + 3 × 4 as 2 + (3 × 4), not as (2 + 3) × 4," said Mary.

a. **Dashes and Slashes.** Two words separated by a dash or a slash are considered to be more than one word. The single-word switch cannot be applied.

Example 3-42

The lot measures 4.5 acres—not 3.5 acres.

Example 3-43

Use + and/or ×, as necessary.

⠀⠀⠠⠥⠎⠑ ⠸⠩ ⠬ ⠸⠱ ⠯⠸⠌⠕⠗ ⠸⠩ ⠈⠡ ⠸⠱⠂ ⠵ ⠝⠑⠉⠲

3.7 More About Switch Indicators at Braille Page Turns

Now that you have had more experience with switch indicators, we will consider more layout issues that occur at braille page turns. Observe the following "keep together" rules as they relate to mathematical expressions within the narrative text.

- A mathematical expression that will fit entirely on the braille line must not be divided between lines.
- If the math expression is preceded by the opening Nemeth Code indicator and followed by the Nemeth Code terminator, and if there is room on the line for both switch indicators and the expression, keep them all on the same line.
- If there is not room on the line for both switch indicators and the math expression, one of the switches will fall on a different line.
- If neither switch indicator will fit on the same line as the math expression, priority is given to keeping the math expression intact, placing each switch indicator on another line. The opening Nemeth Code indicator will be the last item on the preceding line; the Nemeth Code terminator will be the first item on the following line.
- If a math expression is preceded by the opening Nemeth Code indicator and followed by the Nemeth Code terminator and it falls at a braille page turn, place each switch indicator on the same braille page as the mathematical material to which it applies. An opening Nemeth Code indicator should not be the last item at the bottom of a braille page; a Nemeth Code terminator should not be the first item at the top of a braille page.
- If a page number on line 25 or line 1 does not allow the entire expression to fit on the line, the expression is brought down to the next line that has enough usable cells.

Different layouts are illustrated in the next four examples.

3.7.1 **Layout #1.** The first example shows a math expression that will fit on one braille line along with the code switch indicators and the ending punctuation.

Example 3-44

Expressed in words, dividend ÷ divisor = quotient.

⠀⠀⠠⠑⠭⠏⠗⠑⠎⠎⠫ ⠔ ⠘⠺⠎⠂
⠸⠩ ⠙⠊⠧⠊⠙⠑⠝⠙⠨⠌⠙⠊⠧⠊⠎⠕⠗ ⠨⠅ ⠟⠥⠕⠞⠊⠑⠝⠞ ⠸⠱⠲

(1) <u>Page Turn Adjustment</u> If the text begins on line 24, the page number on line 25 restricts the number of available cells on that line. In this case, placing the opening switch on line 24 will solve the problem.

(2) <u>Page Turn Adjustment</u> An opening Nemeth Code indicator cannot be the last item on the braille page. If the text begins on line 25, the opening Nemeth Code indicator must be moved to the next page. In a transcription without a running head in place, the print page number on line 1 restricts the number of available cells on that line. In this case, moving the entire math expression along with its two switch indicators is the best layout.

— —

Because the entire expression, its two code switch indicators, and the ending period will fit on one line, it is placed on line 2 of the page.

3.7.2 **Layout #2.** The second example shows a math expression that will fit on one line, but there is room for only one code switch indicator. One of the indicators must be placed on a different line.

Example 3-45

Expressed in words, multiplicand × multiplier = product.

(1) <u>Page Turn Adjustment</u> If the text begins on line 24, the page number on line 25 restricts the number of available cells on that line. In this case, the math expression will fit, but the Nemeth Code terminator will not**. If Nemeth ends after the expression, the following transcription is incorrect .**

The transcription, above, ***is incorrect*** *if Nemeth ends after this expression.*

(2) <u>Page Turn Adjustment</u> Since a Nemeth Code terminator cannot be the first item on a new braille page, the entire expression, including the opening Nemeth Code indicator, must be arranged to begin on the next page.

```
24     ⠠⠑⠭⠏⠗⠑⠎⠎⠫ ⠔ ⠘⠺⠎⠂
25                                       ⠼⠓
```

— —

```
1  ⠸⠩                                 ⠁⠼⠋
2  ⠍⠥⠇⠞⠊⠏⠇⠊⠉⠁⠝⠙⠈⠡⠍⠥⠇⠞⠊⠏⠇⠊⠑⠗ ⠨⠅ ⠏⠗⠕⠙⠥⠉⠞ ⠸⠱⠲
```

Code switch indicators must fall on the same braille page as the expression to which they apply.

3.7.3 **Layout #3.** The third example shows a math expression that requires a 39 cells. The code switch indicators must fall on separate lines.

Example 3-46

Expressed in words, integer + proper fraction = mixed number.

```
    ⠠⠑⠭⠏⠗⠑⠎⠎⠫ ⠔ ⠘⠺⠎⠂ ⠸⠩
⠊⠝⠞⠑⠛⠑⠗⠬⠏⠗⠕⠏⠑⠗ ⠋⠗⠁⠉⠞⠊⠕⠝ ⠨⠅ ⠍⠊⠭⠑⠙ ⠝⠥⠍⠃⠑⠗
⠸⠱⠲
```

(1) <u>Page Turn Adjustment</u> If the text begins on line 24, the following transcription follows the pagination rules.

```
24     ⠠⠑⠭⠏⠗⠑⠎⠎⠫ ⠔ ⠘⠺⠎⠂
25                                      ⠼⠓⠙
```

— —

```
1  ⠸⠩                                 ⠁⠼⠋
2  ⠊⠝⠞⠑⠛⠑⠗⠬⠏⠗⠕⠏⠑⠗ ⠋⠗⠁⠉⠞⠊⠕⠝ ⠨⠅ ⠍⠊⠭⠑⠙ ⠝⠥⠍⠃⠑⠗
3  ⠸⠱⠲
```

3.7.4 **Layout #4.** In this final example, there is more than one expression between the code switches. Each individual expression is kept together on one braille line, but there is no need to force both of the switches to be on the same page. Nemeth Code continues on the new page. There is no need to repeat the Nemeth indicator after the page turn.

Example 3-47

Find the volume of a rectangular prism with length = 2 ft, width = 4 ft, and height = 3 ft.

Each of the three expressions must not be allowed to wrap before or after the equals sign or between the numeral and the abbreviation ft. The first two occurrences of the abbreviation ft do not have a related period so you must assume that the period following the third occurrence does not relate to the abbreviation; it only ends the sentence and so is placed after the Nemeth Code terminator.

Instructions: Assume that the first paragraph begins on line 24 of a braille page. Use braille page number 55 on line 25, and use print page number a44 on line 1 of the new braille page.

PRACTICE 3C

A unit of work is the foot-pound (ft-lb). One foot-pound converts to 12.000000427771 inch-pounds. How many in-lb is 4.6 ft-lb? How many ft-lb is 247.9927443 in-lb?

If 1 joule = 10 million ergs and 1 megajoule = 1,000,000 joules, how many ergs is 1 megajoule? 1 megajoule = ? ergs

---

3.8 New Print Page

The page change indicator and page number are constructed in the same way in either code. The code in place before the page change indicator remains in effect following the page change indicator. Placement of code switch indicators is not affected by the presence of a page change indicator.

Example 3-48

Are 5 ∶ 2, 10 ∶ 4, and

[print page turn, page 24]

15 ∶ 16 equivalent ratios?

Nemeth Code remains in effect through the page change indicator.

LETTERS

3.9 Single English Letters in Narrative

The language of mathematics uses single letters as mathematical characters. Special provision is made for a single English letter that has mathematical meaning when it appears within narrative. As long as the English letter is freestanding and is unmodified, it may be transcribed in UEB. The letter may touch punctuation.

Exceptions: A freestanding letter with an ordinal ending may be transcribed in UEB. Certain mathematical letters are characterized by a special typeface and must be transcribed in Nemeth. Such letters will be studied in Lesson 7.

Example 3-49

In this equation, b must be greater than a.

Example 3-50

Graph the models of temperatures in summer (s) and in winter (w).

Example 3-51

Find the nth term of the arithmetic sequence.

⠠⠋⠔⠙ ⠮ ⠝⠹ ⠞⠻⠍ ⠷ ⠮ ⠜⠊⠹⠍⠑⠞⠊⠉ ⠎⠑⠟⠥⠰⠑⠲

This ordinal is transcribed in UEB.

3.10 Single English Letters in Nemeth Code

An English letter that has mathematical meaning and which appears in technical context—that is, between Nemeth switches—is transcribed according to the rules of the Nemeth Code. Before illustrating the rules with examples, the definition of "single letter" as used in the Nemeth Code is presented.

3.10.1 **Nemeth Definition of "Single Letter".** To be defined as a "single letter" in Nemeth, Code several criteria must be met.

i. A "single letter" must be from the English alphabet, in regular type, and unmodified.

These are "single letters" p D z R

These are not "single letters" π **D** $\overline{\text{z}}$ ℝ

The first letter is not from the English alphabet, the second and fourth letters are not in regular type, the third letter is modified.

- *Special Case:* A letter representing a mathematical variable is often printed in italics but the italics are disregarded in braille. Such a letter is considered to be a "single letter" in Nemeth. Lesson 7 discusses typeform.

ii. Furthermore, in the print copy the letter must be both preceded by a space or by one or more punctuation marks and followed by a space or by one or more punctuation marks.

These are "single letters" "y" x, "w S"

Each letter is preceded and followed by punctuation or by a space.

These are not "single letters" −x "wS" y+z

The x, z, and S are not preceded by a space or by punctuation (−x is "negative x"); the y and the w are not followed by a space or by punctuation.

- Nemeth grouping symbols, such as parentheses, are not considered to be punctuation marks. Rules for letters touching grouping symbols will be discussed in Lesson 4.

iii. Whether the leading punctuation mark is preceded by a space or not is irrelevant; whether the following punctuation mark is followed by a space or not is irrelevant.

These are "single letters" "x"+"y"

Each letter is both preceded and followed by punctuation.

iv. If the space shown in print is not shown in braille, the letter is no longer a "single letter."

These are not "single letters" r + s

Although each letter is preceded and followed by a space in print, in braille the plus sign is unspaced from the letters.

v. And finally, to be defined as a "single letter" the letter must not be an abbreviation nor can it be a word ("a", "A", "I", or "O").

These are not "single letters" I need 4.5 m of fabric.

I is a word; m is an abbreviation for meters.

- Single-letter abbreviations will be discussed in Lesson 4.

Throughout this course, when referring to the Nemeth Code's definition of a single letter, the term "single letter" is in quotation marks.

Introduction to the English-Letter Indicator

⠰	English-Letter Indicator

Several rules are in place regarding situations where the English-letter indicator is or is not used. It is important to note that the English-letter indicator does not function in the same way as the UEB grade 1 symbol indicator. The term "English-letter indicator" clearly describes the function of this indicator.

3.11 Use of the English-Letter Indicator with a "Single Letter"

Even though no contractions are used in Nemeth, a single letter from the English alphabet used in mathematical context may require an English-letter indicator for clarity. Except as noted in the next section, an English-letter indicator is required when a letter is a "single letter" as defined in 3.10.1, above.

3.11.1 **Capitalization of "Single Letters".** To indicate a single capitalized letter, the capitalization indicator is placed between the English-letter indicator and the letter. The effect of the capitalization indicator extends only to the letter which follows it.

⠠	Capitalization Indicator

3.11.2 **Punctuation of "Single Letters".** A "single letter" is punctuated mathematically if the letter and the punctuation fall within the Nemeth switch indicators.

The examples from 3.10.1 are illustrated below, assuming mathematical context. Note the placement of the capitalization indicator as well as the use of mathematical punctuation.

Example 3-52

p D z R x, "y" "w S" "x"+"y"

Instructions: Demonstrate use of the English-letter indicator and proper punctuation mode in the following series of single letters. Transcribe this practice entirely in Nemeth, using the example, above, as a model.

PRACTICE 3D

c, C; r, R; "l", "L"; "i, j, k"; "l"×"w"×"h".

3.12 Nonuse of the English-Letter Indicator with a "Single Letter"

Even though a letter meets the criteria of "single letter" above , the English-letter indicator is not used when the following conditions are present.

3.12.1 Comparison Sign

a. If the letter is immediately preceded by a sign of comparison or immediately followed by a sign of comparison, an English-letter indicator is not used.

- ⋙ $4 > x$
- ⋙ $x > 2$
- ⋙ $a : b :: c : d$
- ⋙ "$x = 3$"

Example 3-53

Prove: If x, y, and u are real numbers such that $x < y$ and $x = u$,
then $u < y$.

b. If punctuation comes between the letter and a sign of comparison, the letter is now a "single letter" by definition, and an English-letter indicator is required.

⫸ "x" = 3

3.12.2 **Enclosed Between Grouping Symbols.** If a "single letter" is entirely enclosed between signs of grouping, the English-letter indicator is omitted.

⫸ (a)

⫸ {P}

⫸ |y|

Example 3-54

"The absolute value of y" is notated |y|.

3.12.3 **An Unspaced Sequence of Terms.** The English-letter indicator is not used with one or more English letters which occur in an unspaced sequence of mathematical symbols.

⫸ d'

⫸ N%

⫸ 2z

⫸ 2 × z

⫸ 2nth

Example 3-55

Sides d' and d are similar.

Example 3-56

35 equals N% of 120.

Example 3-57

2· z can also be written as 2z.

Example 3-58

Let 5y = the smaller number.

The words "the smaller number" are part of the math expression (they show what 5y equals) and so are included within the switches.

Example 3-59

A field containing the nth roots of unity for odd n also contains the 2nth roots.

a. **Probability Notation.** In probability notation, a letter (often P which represents "the probability of") is followed, unspaced, by the "event" which is written between mathematical grouping symbols (often parentheses). In the next example, the event is "heads". Because the letter P is unspaced from the mathematical grouping sign, the letter is not a "single letter" and so an English-letter indicator is not used.

⫸ P(heads)

Example 3-60

The probability of a flipped coin landing on "heads" is written P(heads).

3.12.4 **Plural, Possessive, or Ordinal Endings.** When a "single letter" has a plural, possessive, or ordinal ending, the English-letter indicator is used or not used as though the ending was not present.

3.12.5 **Chemical Elements.** Single-letter chemical element symbols are transcribed in Nemeth and the English-letter indicator is not used. They are punctuated mathematically.

Example 3-61

The chemical symbols for carbon, oxygen, and hydrogen are C, O, and H, respectively.

3.13 Letters as Identifiers

Letters used as identifiers are constructed according to the rules of the code which is in effect at the time—UEB or Nemeth. Compare:

Print	UEB	Nemeth
a.	⠁ ⠲	⠰ ⠁ ⠸⠲
B.	⠰ ⠠⠃ ⠲	⠰ ⠠⠃ ⠸⠲
(a)	⠐⠣ ⠁ ⠐⠜	⠷ ⠁ ⠾
(B)	⠐⠣ ⠰ ⠠⠃ ⠐⠜	⠷ ⠠⠃ ⠾
c)	⠰ ⠉ ⠐⠜	⠰ ⠉ ⠾

Instructions: Demonstrate the use and the nonuse of the English-letter indicator for "single letters" by transcribing this practice entirely in Nemeth. Place the opening Nemeth Code indicator in cell 1 on the first line. Begin item (a) on the next line.

PRACTICE 3E

(a) r = rate

(b) "r" = rate

(c) x, y, z < 100

(d) n¢ = $4.95

(e) x > "3"

(f) a + b

(g) |y| = |−y|

(h) |x + y| = |x| + |y|

(i) P(red and blue)

Mathematical Letter Combinations

These rules apply to letter combinations which have mathematical meaning. A nonmathematical series of letters, as in a serial number, license plate, or postal code, is transcribed in UEB.

3.14 Mathematical Letter Sequence

The option to remain in UEB to transcribe a mathematical letter applies only to a single freestanding English letter. A mathematical sequence of letters is a mathematical expression and must be transcribed in Nemeth. A mathematical letter sequence is punctuated mathematically if the punctuation falls within the Nemeth switch indicators.

Example 3-62

Rays on, om, and op are the same length.

⠠⠗⠁⠽⠎ ⠸⠩ ⠕⠝⠠ ⠕⠍⠠ ⠠⠄⠯ ⠕⠏ ⠸⠱ ⠜⠑ ⠮ ⠎⠁⠍⠑
⠇⠢⠛⠹⠲

Example 3-63

If th equals ef, then lm equals ch.

⠠⠊⠋ ⠸⠩ ⠞⠓ ⠠⠄⠑⠟⠥⠁⠇⠎ ⠑⠋⠠ ⠠⠄⠮⠝ ⠇⠍
⠠⠄⠑⠟⠥⠁⠇⠎ ⠉⠓ ⠸⠱⠲

Example 3-64

Draw an xy-coordinate graph.

⠠⠙⠗⠁⠺ ⠁⠝ ⠸⠩ ⠭⠽⠤⠉⠕⠕⠗⠙⠊⠝⠁⠞⠑ ⠸⠱ ⠛⠗⠁⠏⠓⠲

The entire hyphenated expression is transcribed in Nemeth Code. No contractions are used.

Example 3-65

Note where the cylinder intersects the yz-plane.

⠠⠝⠕⠞⠑ ⠐⠱ ⠮ ⠉⠽⠇⠔⠙⠻ ⠔⠞⠻⠎⠑⠉⠞⠎ ⠮
⠸⠩ ⠽⠵⠤⠏⠇⠁⠝⠑ ⠸⠱⠲

3.15 Capitalized Letter Sequence

Each capitalized letter in a mathematical sequence of letters must be capitalized individually.

⫸ PQRS ⠠⠏⠠⠟⠠⠗⠠⠎

Example 3-66

Prove PQRS is a rhombus.

3.16 Shortform Letter Combinations

Because contractions are not used in Nemeth, a mathematical sequence that corresponds to a shortform of UEB will not be read as a word when it occurs between the switches. No English-letter indicator is needed.

Example 3-67

Use mathematical notation to express "ac times cd" and "cd plus de".

Math letters are punctuated in mathematical mode – a punctuation indicator is required before each closing quotation mark.

Example 3-68

Mark the abth and jkth columns.

Example 3-69

Find chords AB, AC, and EF.

Reminder: Each capitalized letter in a mathematical sequence of letters is capitalized individually.

Example 3-70

Wd means "W times d".

Example 3-71

If a = c = d, then ac = cd.

Example 3-72

What is angle acr + angle rcb?

Example 3-73

3g × 3r × 3t = 27grt.

Example 3-74

(ab) and (cd) are not equal.

Example 3-75

The chemical symbol for Aluminum is Al.

Reminder: Chemical element symbols are transcribed in Nemeth.

Instructions: Explain your decisions regarding use and nonuse of the English-letter indicator.

PRACTICE 3F

(A) Prove: If a < b and c < 0, then ac > bc. Verify your proof by determining ac and bc when a = 5, b = 7, and c = –4.

(B) j = 1, 2, ..., n

(C) 40% of N = 120

(D) 40% of "N" = 120

(E) If "rcv = rjc" does "v" = "j"?

FORMAT SUMMARY #2

Here is a summary of the Nemeth formats encountered in Lessons 2 and 3.

Side-by-Side Items in Itemized Material with No Subdivisions   When unsubdivided itemized material is arranged side by side across the page in print, the braille format must be changed so that all identifiers start in cell 1.

Keep Together—Hyphenated Expressions   A hyphenated expression containing one or more mathematical components must not be divided between braille lines.

Keep Together—Mathematical Expression   If a page number on line 25 or line 1 does not allow the entire mathematical expression to fit on the line, the expression must be brought down to the next line that has enough usable cells. If the expression will fit on one line but the code switch indicators will not, one or both of the indicators can be placed on a different line.

Keep Together—Abbreviation   An abbreviation and a preceding or following numeral to which it applies must not be divided between braille lines.

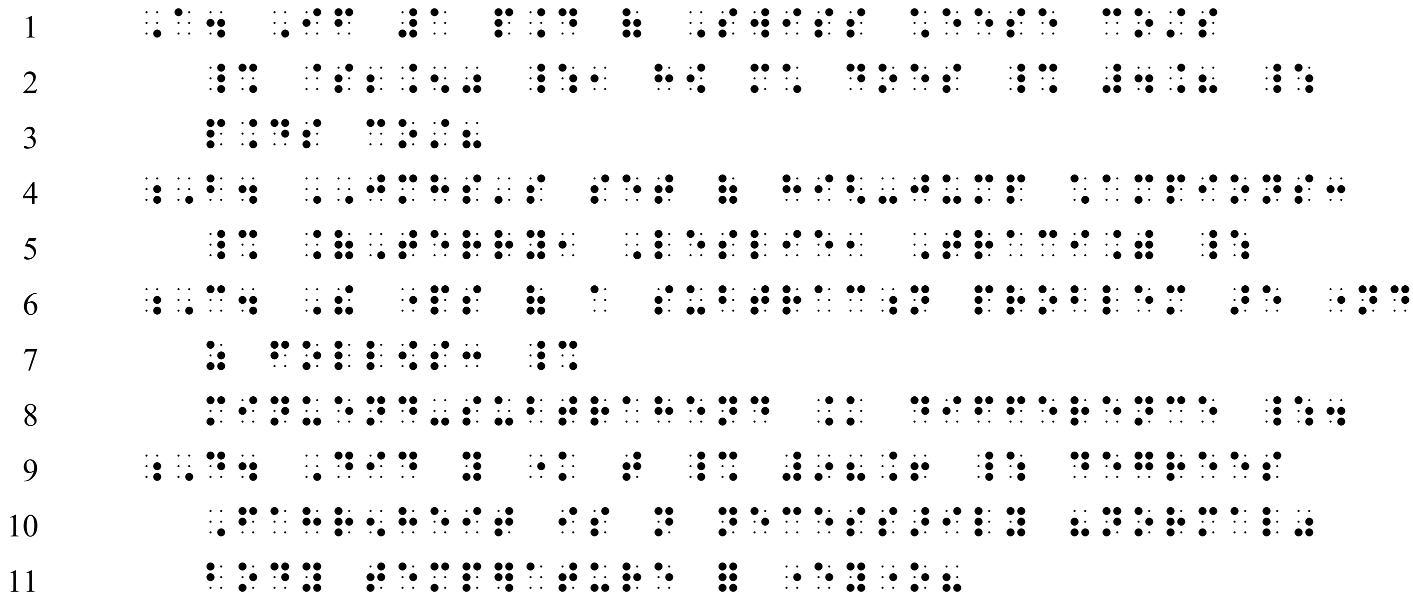
For further practice, see Appendix A—Reading Practice.

EXERCISE 3

Prepare Exercise 3 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 3A



Lines 2-3: A number and a related word (4.8 pounds) do not have to fall together on the same line.

Line 5: Words are punctuated with the dot 2 comma, even in mathematical context.

Line 8: Following Nemeth spacing rules, the operation sign is unspaced from the words minuend and subtrahend. Words are transcribed without contractions in Nemeth.

PRACTICE 3B

⠼⠁⠲ ⠠⠊⠋ ⠼⠃ ⠇⠃⠲ ⠷ ⠠⠛⠳⠙⠁ ⠡⠑⠑⠎⠑ ⠉⠕⠌⠎
⠸⠩ ⠈⠎⠆⠨⠔⠖ ⠸⠱⠂ ⠱⠁⠞ ⠺ ⠽ ⠏⠁⠽ ⠿
⠸⠩ ⠼⠆⠨⠢ ⠇⠃⠎⠲ ⠸⠱⠦
⠼⠃⠲ ⠠⠊⠋ ⠼⠃ ⠅⠛ ⠠⠛⠗⠥⠽⠘⠡⠑⠗⠑ ⠉⠕⠌⠎
⠸⠩ ⠈⠇⠆⠠⠖⠢ ⠸⠱⠂ ⠱⠁⠞ ⠊⠎ ⠮ ⠉⠕⠌ ⠷ ⠁ ⠱⠑⠑⠇
⠺⠑⠊⠣⠬ ⠼⠉ ⠅⠛⠦
⠼⠉⠲ ⠠⠒⠞⠔⠥⠬ ⠮ ⠎⠑⠞ ⠷ ⠕⠗⠙⠔⠁⠇⠎⠂
⠸⠩ ⠋⠊⠋⠞⠓ ⠨⠅ ⠼⠢⠞⠓⠠ ⠎⠊⠭⠞⠓ ⠨⠅ ⠼⠖⠞⠓⠠
⠎⠑⠧⠑⠝⠞⠓ ⠨⠅ ⠼⠶⠞⠓⠠ ⠑⠊⠛⠓⠞⠓ ⠨⠅ ⠼⠦⠞⠓ ⠸⠱⠲
⠼⠙⠲ ⠼⠁ ⠅⠉⠁⠇ ⠊⠎ ⠑⠟⠥⠊⠧⠁⠇⠰⠞ ⠞⠕
⠸⠩ ⠼⠒⠴⠦⠦⠨⠴⠒ ⠋⠞⠲⠇⠃⠲ ⠸⠱

Line 3: A number and a related abbreviation (2.5 lbs.) must not be separated between lines.
Lines 4 and 6: A space is inserted before the abbreviation kg even though there is no space in print.
Line 5: The European decimal point is transcribed as dot 6.
Lines 8 and 9: Care is taken to ensure that each equality is not divided between braille lines.
Line 8: The ordinals are punctuated mathematically within the code switches.
Line 11: The spacing of ft.lb. matches print spacing (unspaced).

PRACTICE 3C

PRACTICE 3D

PRACTICE 3E

PRACTICE 3F

1 ⠐⠣⠠⠁⠐⠜ ⠠⠏⠗⠕⠧⠑⠒ ⠠⠊⠋ ⠸⠩ ⠁ ⠐⠅ ⠃ ⠠⠄⠯

2 ⠉ ⠐⠅ ⠼⠴⠠ ⠠⠄⠮⠝ ⠁⠉ ⠨⠂ ⠃⠉ ⠸⠱⠲ ⠠⠧⠻⠊⠋⠽ ⠽⠗

3 ⠏⠗⠕⠷ ⠃⠽ ⠙⠑⠞⠻⠍⠔⠬ ⠸⠩ ⠁⠉ ⠠⠄⠯ ⠃⠉ ⠠⠄⠱⠢

4 ⠁ ⠨⠅ ⠼⠢⠠ ⠃ ⠨⠅ ⠼⠶⠠ ⠠⠄⠯ ⠉ ⠨⠅ ⠤⠼⠲⠸⠲

5 ⠷⠠⠃⠾ ⠚ ⠨⠅ ⠼⠂⠠ ⠼⠆⠠ ⠄⠄⠄⠠ ⠰⠝

6 ⠷⠠⠉⠾ ⠼⠲⠴⠈⠴ ⠕⠋ ⠠⠝ ⠨⠅ ⠼⠂⠆⠴

7 ⠷⠠⠙⠾ ⠼⠲⠴⠈⠴ ⠕⠋ ⠦⠰⠠⠝⠸⠴ ⠨⠅ ⠼⠂⠆⠴

8 ⠷⠠⠑⠾ ⠠⠄⠠⠊⠋ ⠦⠗⠉⠧ ⠨⠅ ⠗⠚⠉⠸⠴ ⠠⠄⠙⠕⠑⠎

9 ⠦⠰⠧⠸⠴ ⠨⠅ ⠦⠰⠚⠸⠴ ⠸⠱⠦

Lines 1, 2, and 4: Single letters that fall before and after signs of comparison need no English-letter indicator.

Line 3: Two-letter mathematical expressions must be transcribed in Nemeth.

Line 5: Nemeth continues and so the identifier is transcribed in Nemeth. No English-letter indicator is needed when a single letter is enclosed between grouping signs. Letter j is followed by a comparison sign—no English-letter indicator. Letter n is preceded and followed by a space— English-letter indicator required.

Line 6: Letter N is followed by a sign of comparison—no English-letter indicator.

Line 7: Letter N is preceded and followed by punctuation— English-letter indicator required even though equals sign follows.

Line 8: Nemeth continues, so the identifier is transcribed in Nemeth. The word If uses single-word switch indicator.

Line 9: Because letters v and j are each preceded and followed by punctuation, an English-letter indicator is required despite the proximity of the equals sign.

LESSON 4

- MORE ABOUT LETTERS
  - Variables
  - Roman Numerals
  - Nondecimal Bases
- OTHER ALPHABETS
- ENCLOSED LISTS
- MORE ABOUT ENGLISH LETTERS
- MORE ABOUT ABBREVIATIONS
- CODE SWITCHING, cont.

Format

- Keep Together—Abbreviation
- Keep Together—Enclosed List

Answers to Practice Material

LESSON PREVIEW

A closer look at letters used as variables. How to tell if a letter is a variable or an abbreviation. Treatment of Roman numerals. Treatment of other mathematical letter combinations as well as sequences of unspaced letters. How to handle letters used as numerals in nondecimal bases. German, Greek, Hebrew, and Russian letters used in mathematical notation. A look at mathematical constants. Rules regarding the "enclosed list". More rules about English letters and about abbreviations. Code switching considerations with headings.

MORE ABOUT LETTERS

Variables

4.1 Mathematical Variables

An alphabetic character that represents an unspecified number is called a *variable*. Variables are usually printed in italics uniformly throughout a technical document or textbook. In both UEB and Nemeth, italics applied to a variable are disregarded unless other circumstances require the typeface to be retained, a topic which will be discussed in Lesson 7.

Example 4-1

The equation of a line is $y = mx + b$ where m represents the *slope*.

⠠⠮ ⠑⠟⠥⠁⠰⠝ ⠷ ⠁ ⠇⠔⠑ ⠊⠎ ⠸⠩ ⠽ ⠨⠅ ⠍⠭⠬⠃ ⠸⠱
⠐⠱ ⠰⠍ ⠗⠑⠏⠗⠑⠎⠢⠞⠎ ⠮ ⠨⠂⠎⠇⠕⠏⠑⠲

Reminder: Line 1 "y": The English-letter indicator is not used when a "single letter" immediately precedes a sign of comparison. Line 2 "m": A freestanding letter may be transcribed in UEB.

4.1.1 **A Variable with an Associated Abbreviation—Keep Together.** When a variable is associated with an abbreviation, the letter and the abbreviation must not be divided between lines. This format rule also applies in UEB text.

Example 4-2

Write the expression for the number of miles Ann can cover in 11 days if she can walk x mi. in 3 days.

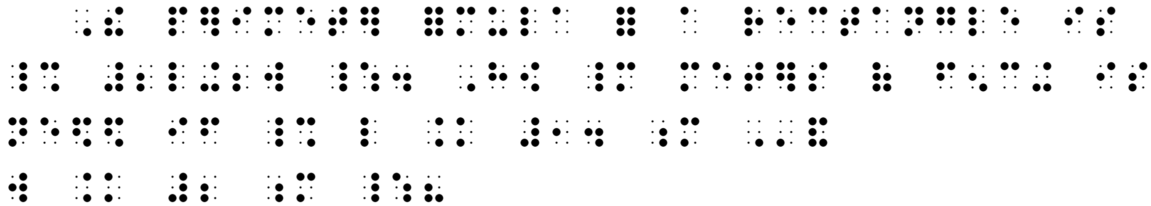
⠠⠺⠗⠊⠞⠑ ⠮ ⠑⠭⠏⠗⠑⠎⠨⠝ ⠿ ⠮ ⠝⠥⠍⠃⠻ ⠷ ⠍⠊⠇⠑⠎
⠠⠁⠝⠝ ⠉ ⠉⠕⠧⠻ ⠔ ⠼⠁⠁ ⠐⠙⠎ ⠊⠋ ⠩⠑ ⠉ ⠺⠁⠇⠅ ⠀⠀⠀⠀⠀
⠰⠭ ⠍⠊⠲ ⠔ ⠼⠉ ⠐⠙⠎⠲

"x" and "mi." must be transcribed on the same line.

4.1.2 **Abbreviation or Variable?** The letter chosen to represent a variable is often based on the subject matter. In the next example, $2l + 2w$, the variables l and w represent unknown measurements for length and width. The letters l and w are chosen to aid in recognition of the parts of the formula, they are not abbreviations for the words length and width. Keep in mind that a variable represents a numerical value. A value will be "plugged into" the formula in place of the variable to produce a solution. In contrast, an abbreviation represents a word—it has no numerical value. You can often answer the question "abbreviation or variable?" by noticing the typeform. In a formal publication, a variable will be printed in italics; an abbreviation will be in normal typeface.

Example 4-3

The perimeter formula for a rectangle is $2l + 2w$. How many meters of fencing is needed if $l = 14$ m and $w = 2$ m?



l and w are variables; m is the abbreviation for "meters".

PRACTICE 4A

1. Express y in terms of x if $2x - 3y = 12$.
2. If $A = l \times l$, what is the length (l) of a side in inches if the area (A) of a square is 7.3 sq.ft.?
3. It is much easier to remember $A = lw$ (Area = length × width) than it is to remember $B = jt$ when trying to figure out how much carpet to buy for the living room.
4. What is the area A of trapezoid T with upper base $a = 3$ m, lower base $b = 6$ m, and height $h = 13$ m?

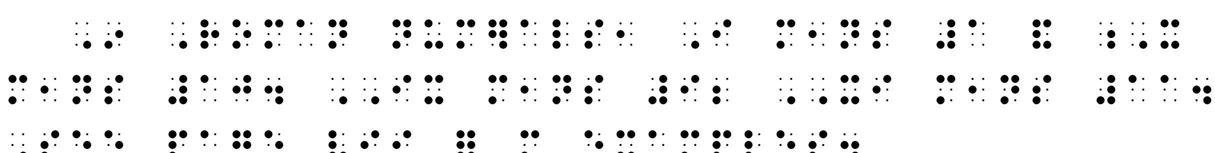
Roman Numerals

4.2 Code Switching with Roman Numerals

The rules you have learned about freestanding, unmodified mathematical numbers also apply to Roman numerals. Specifically, freestanding, unmodified Roman numerals within the narrative may be transcribed in UEB, but inside the switches they follow Nemeth rules. A Roman numeral used as an identifier may be transcribed in either code. Use context clues to decide whether or not to switch, just as you do with Arabic numerals.

Example 4-4

In Roman numerals, I means 1 and X means 10. IX means 9; XI means 11. See page vii for more examples.



Code switching is not required for these unmodified Roman numerals within the narrative.

4.3 Capital Roman Numerals

4.3.1 **Roman Numerals Consisting of One Capital Letter.** In Nemeth context, an English-letter indicator and a single capitalization indicator are used before a Roman numeral when it follows the Nemeth definition of a "single letter" – that is, if the Roman numeral is preceded by a space or by one or more punctuation marks <u>and</u> followed by a space or by one or more punctuation marks. (See 3.10.1.)

⠰	English-letter Indicator
⠠	Single Capitalization Indicator

There are seven Roman numerals consisting of a single letter.

⫸ I V X L C D M

⠰⠠⠊ ⠰⠠⠧ ⠰⠠⠭ ⠰⠠⠇ ⠰⠠⠉ ⠰⠠⠙ ⠰⠠⠍

Within a mathematical expression, the rules regarding the use or the nonuse of the English-letter indicator with a single-letter capital Roman numeral are the same as for any single English letter. (Lesson 3)

Example 4-5

Since $\mathrm{I} = 1$ and $\mathrm{X} = 10$, it follows that $\mathrm{IX} + \mathrm{I} = \mathrm{X}$.

⠠⠎⠔⠉⠑ ⠸⠩ ⠠⠊ ⠨⠅ ⠼⠂ ⠠⠠⠯ ⠠⠭ ⠨⠅ ⠼⠂⠴ ⠸⠱⠂ ⠭
⠋⠕⠇⠇⠪⠎ ⠞ ⠸⠩ ⠠⠠⠊⠭⠬⠠⠊ ⠨⠅ ⠠⠭ ⠸⠱⠲

English-letter indicators are not needed, according to Nemeth rules for letters touching a sign of operation and for letters immediately preceding or following a comparison sign. Each Roman numeral is preceded by a single or a double capitalization indicator.

4.3.2 **Roman Numerals Consisting of Two or More Capital Letters.** The double capitalization indicator of the Nemeth Code is used before a Roman numeral consisting of two or more unspaced capitalized letters.

⠠⠠	Double Capitalization Indicator

Recall that, in Nemeth, a mathematical letter sequence that corresponds to a shortform of UEB does not require an English-letter indicator because no contractions are used inside the switches. This rule also applies to Roman numerals.

⫸ IV DC MMXVII ⠠⠠⠊⠧ ⠠⠠⠙⠉ ⠠⠠⠍⠍⠭⠧⠊⠊

⫸ CD DCV

Within a mathematical expression, Nemeth spacing rules are followed.

Example 4-6

Add. CCCXX + CCLXXXV = DCV

Within the equation, the Roman numerals are unspaced from the plus sign. A space precedes and follows the equals sign. Each of these Roman numerals is preceded by a double capitalization indicator.

4.4 Lowercase Roman Numerals

In Nemeth context, an English-letter indicator is used before any lowercase Roman numeral when it follows the Nemeth definition of a "single letter" – that is, if the Roman numeral is preceded by a space or by one or more punctuation marks and followed by a space or by one or more punctuation marks.

	English-letter Indicator

A lowercase Roman numeral is treated as a "single letter" even when it consists of more than one character.

⫸ i ii iii iv v

4.5 Punctuation with Roman Numerals

A Roman numeral is punctuated mathematically if the punctuation falls inside the switches. The presence of punctuation does not change the rules regarding use of the English-letter indicator if the numeral is standing alone as a "single letter".

⫸ "I, IV, V, MMXVI"

⫸ "i", "ii", "iii".

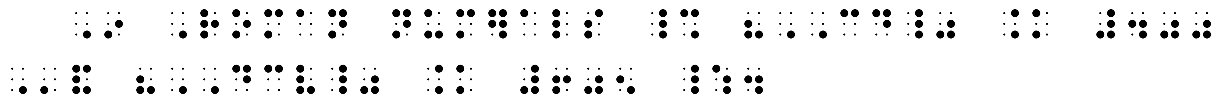
Example 4-7

In Roman numerals, "IX" = 9, "L" = 50, and "C" = 100.

An English-letter indicator is required for a single-letter Roman numeral when preceded and followed by a punctuation mark.

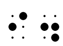
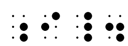
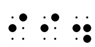
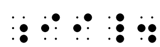
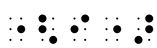
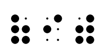
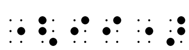
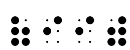
Example 4-8

In Roman numerals "CD" = 400 and "DCV" = 605.



4.6 Roman Numerals Used as Identifiers

Identifiers are transcribed according to the rules for the code in use at that location. Compare these isolated examples of Roman numeral identifiers, noting the use of indicators and the construction of the punctuation or grouping symbols.

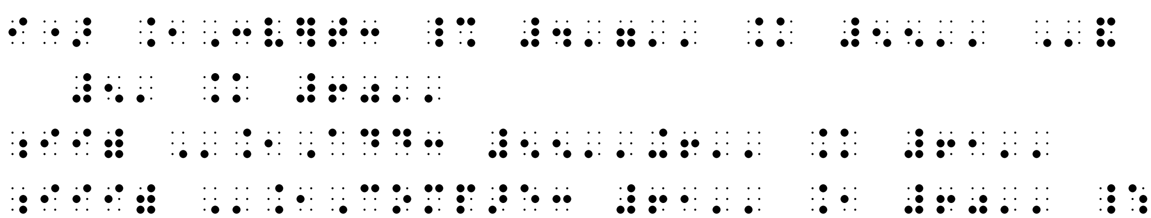
	UEB	Nemeth
i.		
ii.		
(i)		
(ii)		

Example 4-9

i) *Convert:* 4’7” = 55” and 5' = 60"

ii) *Add:* 55" + 6" = 61"

iii) *Compare:* 61" > 60"



4.7 Mathematical Letter Combinations Similar to Roman Numerals

When it is unclear whether a mathematical letter combination is a Roman numeral, the combination is treated as if it were *not* a Roman numeral. In such cases, the letter combination is transcribed in Nemeth. The letters are treated individually and the English-letter indicator is used or is not used in accordance with the rules for English letters. (Review *Mathematical Letter Combinations* in Lesson 3.)

Example 4-10

What does DC denote in the following statement?

Out of context, it is not clear whether "DC" means the Roman numeral "600" or if it is referring to a line segment. Therefore, the letters are transcribed as a mathematical letter sequence. A switch to Nemeth Code is required.

Example 4-11

div has special meaning.

Out of context, it is not clear whether "div" means the Roman numeral "504" or if it is a special mathematical term. Therefore, the letters are transcribed as a mathematical letter sequence. A switch to Nemeth Code is required.

Review: Nonuse of the English-letter Indicator

In Nemeth, the English-letter indicator is <u>not used</u> with a Roman numeral in the following circumstances.

i. When a Roman numeral consists of two or more unspaced capitalized letters in regular type, no English-letter indicator is used.

   ⋙ II III VIII XV

ii. When a Roman numeral immediately precedes or follows a sign of comparison, no English-letter indicator is used.

   ⋙ X = 10

   ⋙ 1000 = M

iii. When a Roman numeral is in an expression consisting of a sequence of unspaced mathematical symbols, no English-letter indicator is used.

   ⋙ ix − v = iv

iv. When a Roman numeral is entirely enclosed between grouping signs, no English-letter indicator is used.

   ⋙ (ii)

v. When a Roman numeral is modified, no English-letter indicator is used.

> ⫸ I' ⠠⠊⠄

Roman numerals starting with 5,000 include a line over the numeral in print. This notation will be discussed in Lesson 12.

PRACTICE 4B

i. Triangle ABC in Quadrant IV is reflected in Quadrant III as Triangle $A'B'C'$.

ii. $\text{iv} + \text{vi} = \text{x}$

iii. $X = 10$, $L = 50$, $C = 100$, and $D = 500$.

iv. Review items v and vi.

v. Explain why $MC = 1100$, but $CM = 900$.

vi. Use Formulas I' and III' to prove the statement.

Nondecimal Bases

4.8 Letters Used to Represent Numerals in Nondecimal Bases

When a system of numeration is to a base larger than 10, additional digits are devised to represent digits beyond the ten Arabic numerals. One method for providing additional digits is to use letters. For example, in base 12, t or T may represent ten and e or E may represent eleven. These letters do not function as letters – they are digits and are indicated as such by use of the numeric indicator. Only uncapitalized letters are used to represent nondecimal numerals in braille, even when the letters are capitalized in print.

⫸ t or T ⠼⠞

⫸ e or E ⠼⠑

The rules regarding the use (or nonuse) of the numeric indicator for nondecimal digits are the same as the rules for the ten Arabic numerals 0 through 9. Numerals in nondecimal bases are mathematical symbols and are punctuated accordingly.

Transcriber's Note Required. If the print copy uses capital letters, a transcriber's note is required to inform the reader of a change in capitalization in the braille transcription.

Sample note on the Transcriber's Notes page: "Letters representing nondecimal digits are capitalized in print."

Sample transcriber's note within the text:

t and e are capitalized in print.

Example 4-12

Counting in base twelve: 0 1 2 3 4 5 6 7 8 9 T E. 13T8 and T1E5 are base 12 numerals.

⠈⠨⠣⠰⠞ ⠯ ⠰⠑ ⠜⠑ ⠉⠁⠏⠊⠞⠁⠇⠊⠵⠫ ⠔
⠏⠗⠔⠞⠲⠈⠨⠜
⠠⠉⠨⠞⠬ ⠔ ⠃⠁⠎⠑ ⠞⠺⠑⠇⠧⠑⠒ ⠸⠩ ⠼⠴ ⠼⠂ ⠼⠆ ⠼⠒ ⠼⠲
⠼⠢ ⠼⠖ ⠼⠶ ⠼⠦ ⠼⠔ ⠼⠞ ⠼⠑⠸⠲ ⠼⠂⠒⠞⠦ ⠠⠄⠯
⠼⠞⠂⠑⠢ ⠸⠱ ⠜⠑ ⠃⠁⠎⠑ ⠼⠁⠃ ⠝⠥⠍⠻⠁⠇⠎⠲

4.9 Nonalphabetic Symbols Used to Represent Numerals

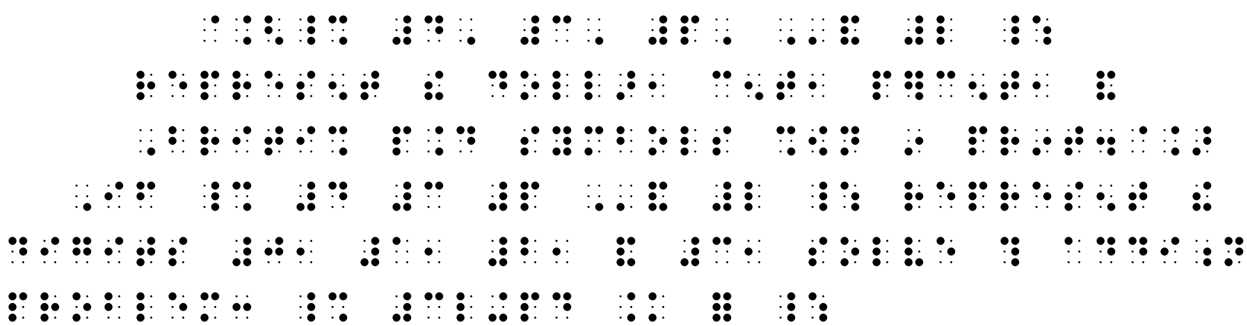
If symbols other than letters represent digits, the transcriber should choose one-cell symbols to represent the special signs. The preferred method is to select letters of the English alphabet in a similar manner as described above. A transcriber's note must specify the meanings assigned to these letters. If the print sign lacks a symbol in the Code, the transcriber's note should include a drawing or a description in order to identify it.

Sample transcriber's note:

> #d, #c, #p, and #l represent the dollar, cent, percent, and British pound symbols shown in print.

Example 4-13

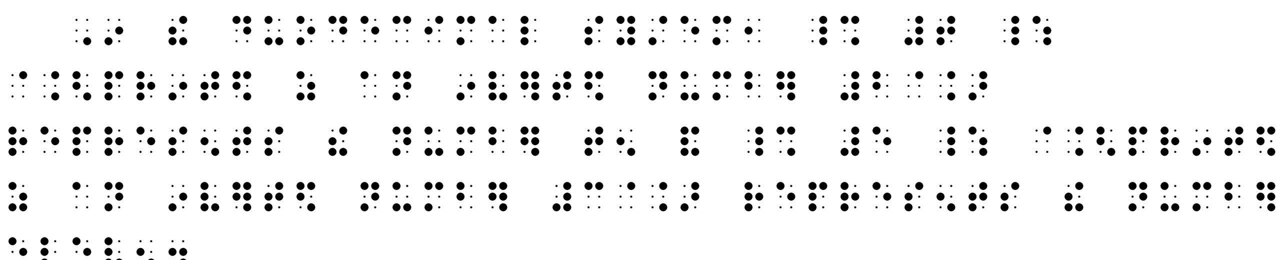
If $ ¢ % and £ represent the digits 0, 1, 2, and 3, solve this addition problem: ¢£+%$ = ?



The opening Nemeth Code indicator immediately follows the UEB opening transcriber's note indicator.

Example 4-14

In the duodecimal system, ↊ represents the number ten and ↋ represents the number eleven.



The two print symbols are described in embedded transcriber's notes: "printed as an inverted number 2" and "printed as an inverted number 3".

PRACTICE 4C

I. In the hexadecimal system (base 16), the number "one thousand" is written as 3e8.

II. Convert hex 7A1 to decimal numeration.

OTHER ALPHABETS

4.10 Alphabetic Indicators

The language of mathematics uses letters from more than just the English alphabet. Specific provision is made in the Nemeth Code for the transcription of the letters of the German, Greek, Hebrew, and Russian (Cyrillic) alphabets. Each alphabet has a unique alphabetic indicator.

4.10.1 **Code Switching and Use of Letter Indicators.** Recall that switching to Nemeth to transcribe an English letter is not always required, and that the Nemeth English-letter indicator may be omitted in certain circumstances. In contrast, an alphabetic indicator is always required to identify a letter from the German, Greek, Hebrew, or Russian alphabets and a switch to Nemeth is always required for such letters even if UEB has a symbol for the letter.

4.10.2 **Capitalization and Punctuation.** When a letter from any alphabet is capitalized in Nemeth Code, the capitalization indicator (dot 6) is placed between the alphabetic indicator and the letter. Letters are individually capitalized—the effect of the capitalization indicator extends only to the letter which follows it. In a Nemeth transcription, letters from the German, Greek, Hebrew, and Russian alphabets are mathematical symbols and so are punctuated mathematically when the punctuation falls within the Nemeth switches.

Certain letters have unique mathematical applications. If you are unsure of a letter, find an expert who can identify it. Do not guess.

4.11 The Greek Alphabet

Many letters from the Greek alphabet are used in mathematics and science. The following indicator identifies a letter as being from the Greek alphabet.

⠨	Greek-letter Indicator (standard form)

This symbol is read as the Greek-letter indicator only when immediately followed by a letter or by the capitalization indicator and a letter. The Nemeth Code table of Greek letters is reproduced below.

Greek Alphabet Table

Name of letter	Regular uncapitalized		Regular capitalized		Alternative form	
alpha	α	⠨⠁	Α	⠨⠠⠁	∝	⠨⠈⠁
beta	β	⠨⠃	Β	⠨⠠⠃	ϐ	⠨⠈⠃
gamma	γ	⠨⠛	Γ	⠨⠠⠛		
delta	δ	⠨⠙	Δ	⠨⠠⠙		
epsilon	ϵ	⠨⠑	Ε	⠨⠠⠑		
zeta	ζ	⠨⠵	Ζ	⠨⠠⠵		
eta	η	⠨⠱	Η	⠨⠠⠱		
theta	θ	⠨⠹	Θ	⠨⠠⠹	ϑ	⠨⠈⠹
iota	ι	⠨⠊	Ι	⠨⠠⠊		
kappa	κ	⠨⠅	Κ	⠨⠠⠅		
lambda	λ	⠨⠇	Λ	⠨⠠⠇		
mu	μ	⠨⠍	Μ	⠨⠠⠍		
nu	ν	⠨⠝	Ν	⠨⠠⠝		
xi	ξ	⠨⠭	Ξ	⠨⠠⠭		
omicron	o	⠨⠕	Ο	⠨⠠⠕		
pi	π	⠨⠏	Π	⠨⠠⠏		
rho	ρ	⠨⠗	Ρ	⠨⠠⠗		
sigma	σ	⠨⠎	Σ	⠨⠠⠎	ς	⠨⠈⠎
tau	τ	⠨⠞	Τ	⠨⠠⠞		
upsilon	υ	⠨⠥	ϒ	⠨⠠⠥		
phi	ϕ	⠨⠋	Φ	⠨⠠⠋	φ	⠨⠈⠋
chi	χ	⠨⠯	Χ	⠨⠠⠯		
psi	ψ	⠨⠽	Ψ	⠨⠠⠽		
omega	ω	⠨⠺	Ω	⠨⠠⠺		
sampi	ϡ	⠨⠉				
stigma	Ϛ	⠨⠮				
vau	F	⠨⠧				
koph (qoph)	Ϙ	⠨⠟				

4.11.1 **Code Switching with Greek Letters.** Even though the uncapitalized form of the Greek letters in Nemeth is identical to the uncapitalized form in UEB, you must switch to Nemeth when a Greek letter appears in a Nemeth transcription, even within the narrative. As with English letters, lowercase Greek letters are often printed in italics uniformly throughout a technical document or textbook. In both UEB and Nemeth, italics applied to a Greek letter are disregarded unless other circumstances require the typeface to be retained.

Greek letters used in the following examples and Practice are listed in the box below. Notice the placement of the capitalization indicator in the two capitalized letters. As stated in 4.10.2, the capitalization indicator is placed between the alphabetic indicator and the letter.

⠨⠠⠙	Delta	Δ
⠨⠍	mu	μ
⠨⠏	pi	π
⠨⠠⠎	Sigma	Σ
⠨⠞	tau	τ
⠨⠹	theta	θ

Example 4-15

The Greek letter θ (theta) represents a plane angle in geometry.

⠠⠮ ⠠⠛⠗⠑⠑⠅ ⠇⠗ ⠸⠩ ⠨⠹ ⠸⠱ ⠐⠣⠮⠞⠁⠐⠜
⠗⠑⠏⠗⠑⠎⠢⠞⠎ ⠁ ⠏⠇⠁⠝⠑ ⠁⠝⠛⠇⠑ ⠔ ⠛⠑⠕⠍⠑⠞⠗⠽⠲

Example 4-16

$\pi < 0 < 2\pi$

⠸⠩ ⠨⠏ ⠐⠅ ⠼⠴ ⠐⠅ ⠼⠆⠨⠏ ⠸⠱

Example 4-17

Find the button marked "π" on your calculator.

⠠⠋⠔⠙ ⠮ ⠃⠥⠞⠞⠕⠝ ⠍⠜⠅⠫ ⠸⠩ ⠦⠨⠏⠸⠴ ⠸⠱ ⠕⠝ ⠽⠗
⠉⠁⠇⠉⠥⠇⠁⠞⠕⠗⠲

Greek letters are mathematical symbols and are punctuated mathematically.

4.11.2 **Capital Greek Letters.** Several capital Greek letters look identical to capital English letters. Those Greek letters are generally not used as math symbols. You can safely assume they are English letters unless specifically identified as Greek letters in the text.

4.11.3 **Alternate Form of Greek Letters.** Some lowercase Greek letters occur in variant print forms. The Greek alphabet table in the Nemeth code book shows an alternate form for five of the Greek letters (alpha, beta, theta, sigma, and phi). The following indicator is used to identify the alternate forms.

⠨⠈	`Greek-letter Indicator (alternate form)`

The alternative form is used in braille only when both forms—standard and alternative—appear in the same print text. If a Greek letter is represented by its alternative form *instead of its standard form throughout* the print text—that is, only one form of the letter is used throughout—the symbol for the standard form is used in braille. Include a transcriber's note at the beginning of the text to inform the reader.

Sample transcriber's note on the Transcriber's Notes page: "The alternate form of the Greek letter theta is used exclusively in print. In braille, the standard form is used."

If a text shows an alternate form of a Greek letter that does not appear in the Nemeth Code table, follow the guidelines above to determine if you should substitute the regular form or if you should use the alternate Greek-letter indicator. If the alternate form is used in the braille transcription, list the symbol on the Special Symbols page, identifying it as a Nemeth symbol as you do for the Nemeth Code terminator and the single-word switch indicator.

Example 4-18

(Special Symbols page)

⠨⠿⠨⠈⠏ ⠠⠁⠇⠞⠻⠝⠁⠞⠑ ⠿⠍ ⠷ ⠠⠛⠗⠑⠑⠅ ⠇⠗ ⠏⠊
  ⠐⠣⠠⠝⠑⠍⠑⠹ ⠠⠉⠕⠙⠑ ⠎⠽⠍⠃⠕⠇⠐⠜

If the letter's identity is not clear from context, consult an expert in the field in order to determine its designation.

PRACTICE 4D

Although the handwritten form of phi (φ) may be found in source materials, only the standard form (ϕ) is used in this book.

Another circle constant, the Greek letter tau, τ, equals 2π, or approximately 6.28.

The symbol Σ indicates summation. Δ signifies change.

In statistics, μ denotes the population mean; in engineering, μ is the coefficient of friction. In number theory, μ represents the Möbius function whereas, in particle physics, μ is the symbol for the muon.

4.12 The German Alphabet

German letters used in mathematics are derived from the Gothic or "Fraktur" calligraphic style of the Latin alphabet. It is a dark font, but is not considered to be bold. These letters must be transcribed in Nemeth. The following indicator is used to identify a letter as being from the German alphabet.

⠸	German-letter Indicator

The three German Fractur letters most commonly encountered in specialized fields of mathematics and science are shown below.

⠸⠉	tseh	$\mathfrak{c}$
⠸⠛	gheh	$\mathfrak{g}$
⠸⠠⠧	Fao	$\mathfrak{V}$

If other Fraktur letters are encountered in a text, refer to the alphabet list in Rule 6 of the Nemeth code book. The German letter may be associated with the same letter from the English alphabet, which may help you identify it.

Example 4-19

Capital German $\mathfrak{V}$ looks like the English letter "B" but in fact it is a "V"!

Example 4-20

In set theory, the continuum (denoted by $\mathfrak{c}$), is an infinite cardinal number.

German letters may be encountered in the study of set theory.

Example 4-21

$\mathfrak{g} = \mathrm{Lie}(G)$

German letters may be encountered in the study of Lie algebra.

4.13 The Hebrew Alphabet

Hebrew letters used as mathematical symbols must be transcribed in Nemeth. The following indicator is used to identify a letter as being from the Hebrew alphabet.

⠠⠠	Hebrew-letter Indicator

The Hebrew alphabet has no capitalized form. The letter most commonly encountered in technical material is the aleph: ℵ The aleph is usually written with a subscript, which will be discussed in Lesson 6.

A complete list of the Hebrew letters and their braille equivalents can be found in *World Braille Usage,* which is available for download at Perkins.org.

⠠⠠⠁	aleph	ℵ

Example 4-22

Georg Cantor created the cardinal number "aleph-null" which is an aleph ℵ with a subscript zero.

⠠⠛⠑⠕⠗⠛ ⠠⠉⠁⠝⠞⠕⠗ ⠉⠗⠂⠞⠫ ⠮ ⠉⠜⠙⠔⠁⠇ ⠝⠥⠍⠃⠻
⠦⠁⠇⠑⠏⠓⠤⠝⠥⠇⠇⠴ ⠱ ⠊⠎ ⠁⠝ ⠁⠇⠑⠏⠓ ⠸⠩ ⠠⠠⠁ ⠸⠱ ⠾ ⠁
⠎⠥⠃⠎⠉⠗⠊⠏⠞ ⠵⠻⠕⠲

4.14 The Russian Alphabet

Russian (Cyrillic) letters used as mathematical symbols must be transcribed in Nemeth. The following indicator is used to identify a letter as being from the Russian alphabet.

⠈⠈	Russian-letter (Cyrillic) Indicator

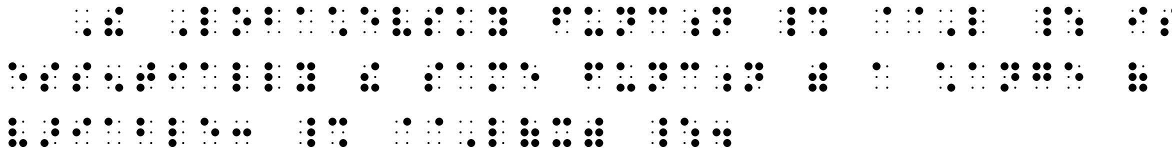
Two Cyrillic letters in common usage are Sha which is used in number theory and Ell (also the lowercase ell) which is used in hyperbolic (Lobachevskian) geometry. The Sha usually keeps company with bold and barred letters, and so will be further discussed in Lesson 7. Note the similarity between the Cyrillic letter ell and the Greek letter pi. If the letter's identity is not clear from context, consult an expert in the field in order to determine its designation.

A complete list of the Russian letters and their braille equivalents can be found in *World Braille Usage,* which is available for download at Perkins.org.

⠈⠈⠇	ell	л
⠈⠈⠠⠇	Ell	Л
⠈⠈⠠⠱	Sha	Ш

Example 4-23

The Lobachevsky function Л is essentially the same function with a change of variable: $Л(x)$.



Note: The Russian alphabet table in *World Braille Usage* shows how to transcribe the Cyrillic letters in sentence 2: dots 1, 12, 2456, 1245, 145, 15, 245, 1356, 24, and 13.

PRACTICE 4E

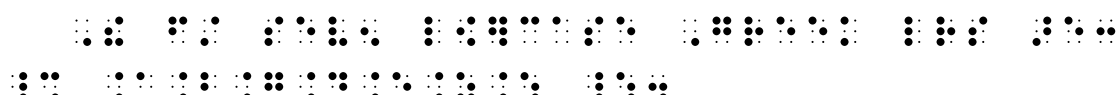
1) Be sure to differentiate between the Cyrillic letters "ell" Л and "Ell" Л and the Greek letters "pi" π and "Pi" Π.
2) The first ten uncapitalized Cyrillic letters are: ah а, beh б, veh в, gheh г, deh д, yeh е, zheh ж, zeh з, ee и, and kah к.

4.15 A Sequence of Unspaced Letters

a. **Non-English Letters.** The effect of an alphabetic indicator extends only to the letter which follows it. Thus, in a sequence of unspaced letters from non-English-letter alphabets, the appropriate alphabetic indicator is used before each letter.

Example 4-24

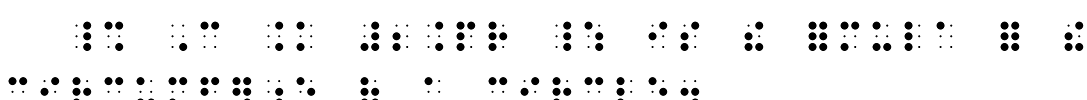
The first seven lowercase Greek letters are: $\alpha\beta\gamma\delta\varepsilon\zeta\eta$.



b. **English Letters.** An English letter in regular type which appears in an unspaced sequence of terms does not require a letter indicator. Recall that mathematical letters which are consistently *printed* in italics are transcribed in regular type.

Example 4-25

$C = 2\pi r$ is the formula for the circumference of a circle.



Example 4-26

The "change in y" is denoted as "Δy".

⠠⠮ ⠦⠡⠁⠝⠛⠑ ⠔ ⠰⠽⠴ ⠊⠎ ⠙⠢⠕⠞⠫ ⠵
⠸⠩ ⠦⠨⠠⠙⠽⠸⠴ ⠸⠱⠲

4.15.1 **Derivatives.** The English letter combinations "dx", "dy", etc. often used in differential notation are usually spaced away from surrounding characters in print in order to enhance recognition. The space is omitted in braille unless another Nemeth rule requires a space. Print may show the letter d in italics or in regular type. Either way, the letter is not italicized in braille.

Example 4-27

$(x + y)\ \mathrm{d}x\ \mathrm{d}y =$ ___

⠸⠩ ⠷⠭⠬⠽⠾⠙⠭⠙⠽ ⠨⠅ ⠤⠤⠤⠤ ⠸⠱

In print, there is a space before each "d".

4.16 Mathematical Constant

A mathematical constant is a special number whose value is nonvarying ("constant") and is represented by a certain alphabetic character. Two common examples are the Greek lowercase pi π and the English letter i. Constants are usually printed in italics uniformly throughout a document. In both UEB and Nemeth, constants are transcribed as regular type unless other circumstances require the typeface to be retained.

Example 4-28

$i(a + bi) = -b + ai$

⠸⠩ ⠊⠷⠁⠬⠃⠊⠾ ⠨⠅ ⠤⠃⠬⠁⠊ ⠸⠱

Example 4-29

$C = 2\pi r + \pi\Delta r$

⠸⠩ ⠠⠉ ⠨⠅ ⠼⠆⠨⠏⠗⠬⠨⠏⠨⠠⠙⠗ ⠸⠱

Linage and judicious placement of code switches is the challenge in this practice. After transcribing each sentence, write down your reasons for switching where you did. Then compare your decisions to the answer key and commentary at the end of the lesson.

PRACTICE 4F

1. Variables a and b are inversely related.
2. There exists a constant N such that no bit of Ω after the Nth can be proven to be 1 or 0.
3. Randall replied, "$12 - n, 11 - n, 10 - n$ ... which is correct?"
4. "$12 - n, 11 - n, 10 - n, \dots$"
5. Which is correct: "$4x + 3y$, " "$3x + 4y$, " or "$4x + 4y$"?
6. What is the remainder when 101 is divided by 3 ($101 \div 3$)?
7. The result is $(ax + by)(cx + dy)$, where all components are real.
8. Δx means "the change in x" and Δy means "the change in y". When x increases by Δx, y increases by Δy as expressed in the equation $y = \Delta y = f(x + \Delta x)$.
9. ($4x + 3y$ is the denominator.)

ENCLOSED LISTS

4.17 Special Case—Definition of an "Enclosed List"

Special provision is made for the transcription of a sequence of mathematical items enclosed within grouping signs. All of the following must be true in order to apply this rule.

i. The sequence must begin and end with a sign of grouping. The grouping signs do not have to be of the same kind.

ii. The list must have at least two items and the items must be separated by commas.

iii. An item of the list may be any sign used for omission – for example, an ellipsis or a long dash.

iv. The list cannot contain any punctuation mark other than the separating commas. (The omission ellipsis or long dash are not considered to be punctuation.)

v. The list cannot contain any words, abbreviations, ordinal endings, or plural endings. (An exception applies to set notation containing words.)

vi. The list cannot contain a sign of comparison.

An enclosed list must be transcribed in Nemeth even if the items in the list are simple numerals or letters. The enclosure symbols are part of the mathematical notation, using the appropriate Nemeth grouping symbols.

4.17.1 **Nonuse of the Numeric Indicator in an Enclosed List.** A numeric indicator is not used before a numeral or before a decimal point and a numeral in an enclosed list.

Example 4-30

{2, 4, 6, 8}

Example 4-31

(–3.1, –2, –.9, 0, .9, 2, 3.1)

Example 4-32

Consider the set {5 + 5, 10 + 10}.

Example 4-33

(5, 12] means do **not** include 5, but **do** include 12.

a. The next two examples do not satisfy the definition of an enclosed list. A numeric indicator is transcribed where required.

Example 4-34

Create two different sets from these numbers: [3, 4, 5; .3, .4, .5]

This is not an enclosed list because a semicolon is used. A numeric indicator is required before all but the first numeral.

Example 4-35

To show the probability of spinning a 3 and a 7, calculate P(3 AND 7).

⠠⠞⠕ ⠩⠪ ⠮ ⠏⠗⠕⠃⠁⠃⠊⠇⠰⠽ ⠷ ⠎⠏⠔⠝⠬ ⠁ ⠼⠉ ⠯ ⠁
⠼⠛⠂ ⠉⠁⠇⠉⠥⠇⠁⠞⠑ ⠸⠩ ⠠⠏⠷⠒ ⠠⠠⠁⠝⠙ ⠼⠶⠾ ⠸⠱⠲

This is not an enclosed list because there are no commas and because it contains a word. A numeric indicator is required before the numeral 7. The word AND has mathematical significance and is transcribed in Nemeth without contractions. (The words AND, OR, and NOT may be used as mathematical operators in the fields of probability and logic.)

4.17.2 **Nonuse of the English-letter Indicator in an Enclosed List.** In an enclosed list, the English-letter indicator is not used with any English letter or combination of English letters in regular type. This rule includes Roman numerals. (Note that a letter from a non-English alphabet must retain its appropriate alphabetic indicator.)

Example 4-36

$(a, 2x, \ldots, b, ab)$

⠸⠩ ⠷⠁⠠ ⠆⠭⠠ ⠄⠄⠄⠠ ⠃⠠ ⠁⠃⠾ ⠸⠱

Example 4-37

Write the coordinates of the points as ordered pairs (x, y).

⠠⠺⠗⠊⠞⠑ ⠮ ⠉⠕⠕⠗⠙⠔⠁⠞⠑⠎ ⠷ ⠮ ⠏⠕⠔⠞⠎ ⠵ ⠕⠗⠙⠻⠫
⠏⠁⠊⠗⠎ ⠸⠩ ⠷⠭⠠ ⠽⠾ ⠸⠱⠲

Example 4-38

(i, ii, iii, iv)

⠸⠩ ⠷⠊⠠ ⠊⠊⠠ ⠊⠊⠊⠠ ⠊⠧⠾ ⠸⠱

Example 4-39

(α, a, β, b)

⠸⠩ ⠷⠨⠁⠠ ⠁⠠ ⠨⠃⠠ ⠃⠾ ⠸⠱

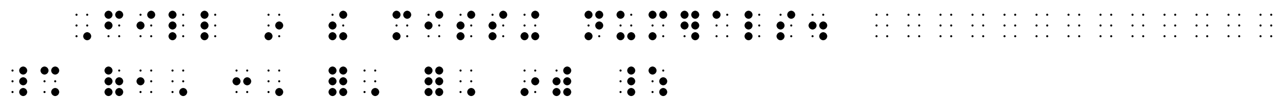
The Greek letters alpha and beta are in this enclosed list. The Greek-letter indicator is required.

Situations which require a numeric indicator or an English-letter indicator in an enclosed list will be discussed in later lessons.

4.17.3 **Format—Keep Together.** Items in an enclosed list must not be divided between braille lines if the entire list will fit on a single braille line.

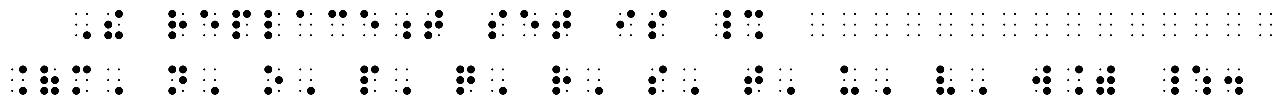
Example 4-40

Fill in the missing numerals. (1, 3, ? , ? , 9)



Example 4-41

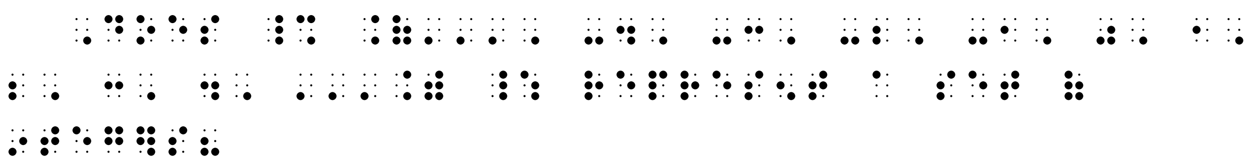
The replacement set is $\{m, n, o, p, q, r, s, t, u, v, w\}$.



a. **Division Between Lines.** If the enclosed list will not fit on a single braille line, use as much of the current line as possible and begin a runover line after a comma. When the items in an enclosed list must be divided between braille lines, neither the numeric indicator nor the English-letter indicator is used before the runover on the new line.

Example 4-42

Does $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ represent a set of integers?



Remember, if a math expression will fit on one line, do not divide it. See how this rule applies in an itemized format.

Example 4-43

Now we will discuss these three sets.
1. {25, 50, 75, 100, 125, …}
2. {100, 150, 200, 250, 300, 350 …}
3. {100, 200, 300, 400, 500, 600, 700, …}

Item 1 requires no division.
Item 2: When the enclosed list is brought down to the next line (starting in the runover cell, cell 3) no division is needed. This keeps the enclosed list together on one line.
In item 3, division is unavoidable.

b. **Set Notation Containing Words.** When words are part of set notation, the expression is treated as an enclosed list. A switch to Nemeth is required. The items in each set should not be divided between braille lines. If this is not possible, division is made after a comma.

Example 4-44

Now we must find the domain and range of the following relation.
{(Ford, –9), (Nixon, –5), (Taft, –11), (Polk, –23)}

Even though "(Polk," will fit on line 3, the pair enclosed in parentheses "(Polk, –23)" is kept together on one braille line.

Instructions: First determine if each item is or is not an enclosed list. Write YES if the item is an enclosed list and NO if it is not. Then transcribe the YES items in Nemeth.

PRACTICE 4G

$\{a, b, c, d\}$

(–1, –2, –3)

(h ft, k in)

(ab, cd, ef)

$1, i, -1, -i$

(1, i, 2, ii)

(1st, 2nd, 3rd)

(A, A', B, B', C)

{____, .13, .15, .17, ____}

$(1 + h, 2 + k, 0)$

$(x = 1, 2, \ldots, 10)$

$(a, b]$

(1 2 3)

[0, 1]

$(u, v; x, y)$

{(Denver, 19), (Utah, 27), (Minnesota, 24), (San Antonio, 28)}

$(a, b, \ldots)$

$(x + 1, x + 2, ?, ?, x + 5)$

$\langle -1,\ 0]$

(2, 4, 6, ____, 10)

$(0, a, 1, b, 2)$

{1's, 2's, 3's}

MORE ABOUT ENGLISH LETTERS

4.18 An English Letter Touching Only One Grouping Symbol

In Lesson 3, you learned that the English-letter indicator is not needed when a "single letter" is enclosed between mathematical grouping symbols.

⫸ (c) ⠷⠉⠾ compare to "c" ⠦⠰⠉⠸⠴

However, when a "single letter" is in direct contact with only one grouping sign, and the letter is not an item in an enclosed list as defined above, rules regarding the English-letter indicator are applied as though the grouping sign was not present.

Example 4-45

$(k = 1, 2, \ldots, n)$.

⠸⠩ ⠷⠅ ⠨⠅ ⠼⠂⠠ ⠼⠆⠠ ⠄⠄⠄⠠ ⠰⠝⠾ ⠸⠱⠲

Without the left parenthesis, the letter k would not need an English-letter indicator because it is followed by an equals sign. Without the right parenthesis, the letter n would need an English-letter indicator because it is preceded by a space and followed by punctuation.

Example 4-46

Consider the set $\{m$ and $n\}$.

⠠⠒⠎⠊⠙⠻ ⠮ ⠎⠑⠞ ⠸⠩ ⠨⠷⠰⠍ ⠁⠝⠙ ⠰⠝⠨⠾ ⠸⠱⠲

Without the left brace, the letter m would need an English-letter indicator because it is preceded and followed by a space. Without the right brace, the letter n would need an English-letter indicator because it is preceded by a space and followed by punctuation. Set notation is mathematical and so a switch to Nemeth Code is required. The word "and" is part of the math.

Example 4-47

If two events are mutually exclusive we write $P(A\ AND\ B) = 0$ where $P(A\ AND\ B)$ means "the probability of A and B occurring at the same time".

1 ⠠⠊⠋ ⠞⠺⠕ ⠑⠧⠢⠞⠎ ⠜⠑ ⠍⠥⠞⠥⠁⠇⠇⠽ ⠑⠭⠉⠇⠥⠎⠊⠧⠑ ⠺⠑
2 ⠺⠗⠊⠞⠑ ⠸⠩ ⠠⠏⠷⠠⠁ ⠠⠠⠁⠝⠙ ⠠⠃⠾ ⠨⠅ ⠼⠴ ⠠⠄⠐⠱
3 ⠠⠏⠷⠠⠁ ⠠⠠⠁⠝⠙ ⠰⠠⠃⠾ ⠸⠱ ⠍⠂⠝⠎ ⠦⠮ ⠏⠗⠕⠃⠁⠃⠊⠇⠰⠽ ⠷
4 ⠠⠁ ⠯ ⠰⠠⠃ ⠕⠒⠥⠗⠗⠬ ⠁⠞ ⠮ ⠎⠁⠍⠑ ⠐⠞⠴⠲

Line 2: Without the left parenthesis, the letter A would not need an English-letter indicator because it immediately follows the letter P. Without the right parenthesis, the letter B would not need an English-letter indicator because it is followed by a comparison sign.

Line 3: Without the left parenthesis, the letter A would not need an English-letter indicator because it immediately follows the letter P. Without the right parenthesis, the letter B would need an English-letter indicator because it is preceded and followed by a space.
Line 4: Letters A and B follow the rules of UEB in the narrative.

a. **Roman Numerals.** The same rule applies to a Roman numeral that is in direct contact with only an opening or closing grouping sign. The English-letter indicator is used or is not used as though the grouping sign was absent. The following example illustrates Roman numerals used as identifiers, assuming uninterrupted mathematical context.

⫸ i) ⠰⠊⠾

⫸ iv) ⠰⠊⠧⠾

⫸ v) ⠰⠧⠾

b. **Modified Grouping Sign.** If the grouping sign includes a prime or other modifying symbol, the English-letter indicator is not used with the single English letter that touches the grouping symbol.

⫸ $t]'$ ⠞⠈⠾⠄

Example 4-48

$t]'$ and $v]'$ have unique meaning.

⠸⠩ ⠞⠈⠾⠄ ⠠⠄⠯ ⠧⠈⠾⠄ ⠸⠱ ⠓ ⠥⠝⠊⠟⠥⠑ ⠍⠂⠝⠬⠲

4.19 English Letters with Plural, Possessive, or Ordinal Endings

When a "single letter" has a plural, possessive, or ordinal ending, in mathematical context the English-letter indicator rules of the Nemeth Code are applied as though such endings were not present. The following examples illustrate proper use of the English-letter indicator, assuming mathematical context. Note that the expressions are punctuated mathematically. The presence of a plural, possessive, or ordinal ending does not change the fact that the punctuation mode is mathematical.

a. Plural

⫸ ps, qs, rs ⠰⠏⠎⠠ ⠰⠟⠎⠠ ⠰⠗⠎

Think: p, q, r – English-letter indicator is required

⫸ Xs, Ys, Zs ⠰⠠⠭⠎⠠ ⠰⠠⠽⠎⠠ ⠰⠠⠵⠎

Think: X, Y, Z – English-letter indicator is required

b. Possessive

Reminders: A punctuation indicator is required before an apostrophe; otherwise dot 3 is read as a prime sign.

⫸ p’s, q’s, r’s

Think: p, q, r – English-letter indicator is required

⫸ X’s, Y’s, Z’s

Think: X, Y, Z – English-letter indicator is required

c. Ordinal

⫸ nth, 2nth

Think: n – English-letter indicator is required; 2n – English-letter indicator is not required

d. Letter Combinations

Reminder: Letter combinations require a switch to Nemeth Code, but no English-letter indicator is needed. Capital letters are individually capitalized.

⫸ ABs and GHs

⫸ AB’s and GH’s

⫸ ab’s and gh’s

⫸ abth and jkth

Instructions: Stay in Nemeth Code to transcribe items C) and E).

PRACTICE 4H

A) Find all ABs, CDs, and EFs; draw XYZs.

B) Find all AB’s, CD’s, and EF’s; draw XYZ’s.

C) (1st, 2nd, ... nth, ... 49th)

D) Does $|a| \times |b| = |ab|$?

E) If Q, then {[NOT-P] OR P}.

MORE ABOUT ABBREVIATIONS

Abbreviation Reminders

- Abbreviations are not mathematical expressions although they may be part of a mathematical expression.
- A space comes between an abbreviation and its related value, even if no space is shown in print.
- An abbreviation and its related value must not be divided between braille lines.
- Between a two-word abbreviation, follow the same spacing as used in print. Do not divide the abbreviation between lines.
- Abbreviations are punctuated in literary mode, even in mathematical context.

4.20 More Spacing Rules

4.20.1 **Spacing of Abbreviations with Operation Signs.** Spacing between an abbreviation and an operation symbol depends upon whether the abbreviation has a related value.

a. A space is required between an abbreviation and a sign of operation when the abbreviation has a related value.

Example 4-49

7 in. + 9 in. = 16 in. or 1 ft. 4 in.

⠸⠩ ⠼⠶ ⠊⠝⠲ ⠬⠔ ⠊⠝⠲ ⠨⠅ ⠼⠂⠖ ⠊⠝⠲ ⠠⠄⠕⠗
⠼⠂ ⠋⠞⠲ ⠼⠲ ⠊⠝⠲ ⠸⠱

b. No space comes between an abbreviation and a multiplication dot when the second abbreviation has no related value. No space comes between an abbreviation and a slash which applies to the abbreviation.

Example 4-50

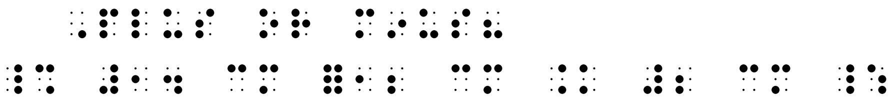
Momentum is expressed in kg·m/s.

⠠⠍⠕⠰⠞⠥⠍ ⠊⠎ ⠑⠭⠏⠗⠑⠎⠎⠫ ⠔ ⠸⠩ ⠅⠛⠡⠰⠍⠸⠌⠰⠎ ⠸⠱⠲

4.20.2 **Spacing of Omission Symbols.** If a sign of omission is used to represent an abbreviation, the omission symbol is spaced as the abbreviation which it replaces. This spacing rule is crucial to provide clarity in the braille transcription. Spacing in the print copy often does not follow this design and must be disregarded when applying spacing to the braille transcription.

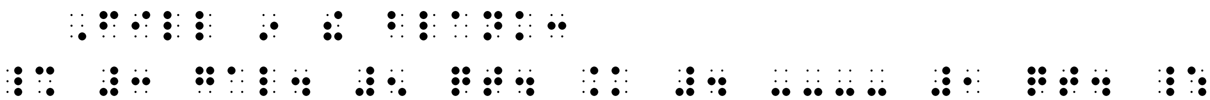
Example 4-50

Plus or minus? 14 cm _?_ 12 cm = 2 cm



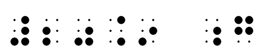
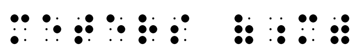
Example 4-51

Fill in the blank: 3gal.5qt. = 4___1qt.



4.21 Single-Letter Abbreviations

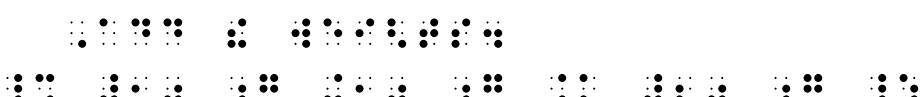
Even though contractions are not used within the switches, for clarity, a single-letter abbreviation from the English alphabet must always begin with an English-letter indicator. This rule applies regardless of the presence (or lack) of a related period. The presence of grouping signs does not change this rule. In other words, the rules regarding nonuse of the English-letter indicator with mathematical "single letters" do not apply to abbreviations.

- ⋙ 20.9 g 
- ⋙ meters (m) 
- ⋙ 3.5 c. 

Reminder: Abbreviations are punctuated in literary mode, even in mathematical context.

Example 4-52

Add the weights. 10 g + 10 g = 20 g



Even the "g" that is immediately followed by a comparison sign requires an English-letter indicator because "g" is a single-letter abbreviation.

Example 4-53

How many liters? 2 quarts (qt.) = ? liters (l)?

Even "l" enclosed within parentheses requires an English-letter indicator because "l" is a single-letter abbreviation.

Example 4-54

Teaspoons and tablespoons: 1 t. + 2 t. = 3 t. = 1 T.

4.22 Abbreviations Whose Letters Correspond to a Shortform

Because contractions are not used in Nemeth, a letter combination which corresponds to a shortform will not be read as a word when it occurs between the switches. Therefore, an abbreviation whose letters correspond to a shortform does not need an English-letter indicator.

- 1 yr. = 12 mo.
- 1 lt-yr = 9.461e + 12 km

The letter "e" is a constant and follows the rules of a "single letter".

Example 4-55

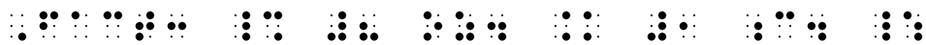
How many days? 1 year (yr.) = ? days (da.); 2 yrs. = ? da.

4.23 Context Clues

Look for context clues when an abbreviation ends a sentence. When in doubt about the function of a period at the end of a sentence, assume that the period applies to the abbreviation as well. Compare these three examples.

Example 4-56

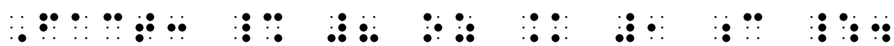
Fact: 8 oz. = 1 c.



Because the abbreviation "oz." has a related period, treat the period after "c." in the same manner. The related period is transcribed before the Nemeth Code terminator.

Example 4-57

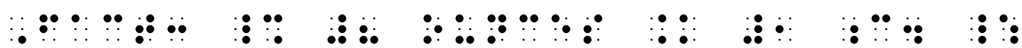
Fact: 8 oz = 1 c.



Because the abbreviation "oz" does not have a period, treat the period after "c" as an end-of-sentence period only. The period is transcribed outside of the switch.

Example 4-58

Fact: 8 ounces = 1 c.

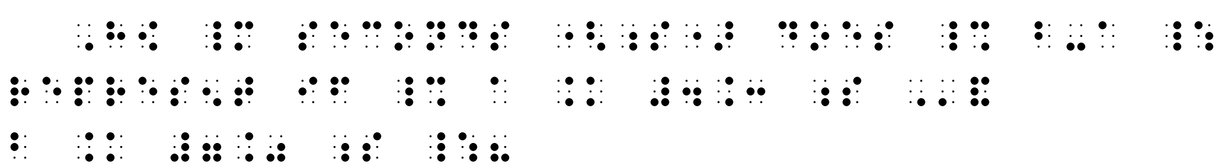


Within this example, there are no context clues to determine if this period applies to the single-letter abbreviation "c". When in doubt, assume that the period <u>does</u> apply to the abbreviation. The period is transcribed inside the switch.

In the next example, s is the abbreviation (printed in normal typeface) and a and b are the variables (printed in italics). Review <u>4.1.2,</u> "Abbreviation or Variable?"

Example 4-59

How many seconds (s) does $b - a$ represent if $a = 4.3$ s and $b = 7.0$ s?



The single-letter abbreviation requires an English-letter indicator; the variables do not because they are either touching a sign of operation or next to a sign of comparison.

4.24 Fully Capitalized Abbreviations – Acronyms and Initialisms

An abbreviation consisting of more than one capital letter is capitalized as a unit using the double capitalization indicator of the Nemeth Code. Nemeth rules for spacing and punctuation of acronyms and initialisms follow the same rules as those for abbreviations.

Example 4-60

LCM means "least common multiple." In the problem below, LCM = 12.

⠀⠀⠠⠠⠇⠉⠍ ⠍⠂⠝⠎ ⠦⠇⠂⠌ ⠉⠕⠍⠍⠕⠝ ⠍⠥⠇⠞⠊⠏⠇⠑⠲⠴ ⠠⠔ ⠮
⠏⠗⠕⠃⠇⠑⠍ ⠆⠇⠂ ⠸⠩ ⠠⠠⠇⠉⠍ ⠨⠅ ⠼⠂⠆ ⠸⠱⠲

On line 2, the three-letter abbreviation (initialism) is part of the math.

4.24.1 **UEB vs. Nemeth Code.** In literary mode, apply UEB rules regarding use of the grade 1 indicator with abbreviations. In Nemeth mode, apply Nemeth rules regarding use of the English-letter indicator with abbreviations. In both modes, the linage rules of Nemeth are followed.

Example 4-61

In this equation, SD stands for Standard Deviation: SD $\times\ c = 0$.

⠀⠀⠠⠔ ⠹ ⠑⠟⠥⠁⠰⠝⠂ ⠰⠠⠠⠎⠙ ⠌⠯⠎ ⠿ ⠠⠌⠯⠜⠙
⠠⠙⠑⠧⠊⠁⠰⠝⠒ ⠸⠩ ⠠⠠⠎⠙ ⠈⠡⠉ ⠨⠅ ⠼⠴ ⠸⠱⠲

In UEB, SD needs a grade 1 indicator in order not to be read as "SAID". In Nemeth, an English-letter indicator is not needed because contractions are not allowed inside the switches. Abbreviations—initialisms included—are spaced away from operation signs; variables are not.

4.25 Initiating Nemeth Code Before Itemized Material, Following a Heading

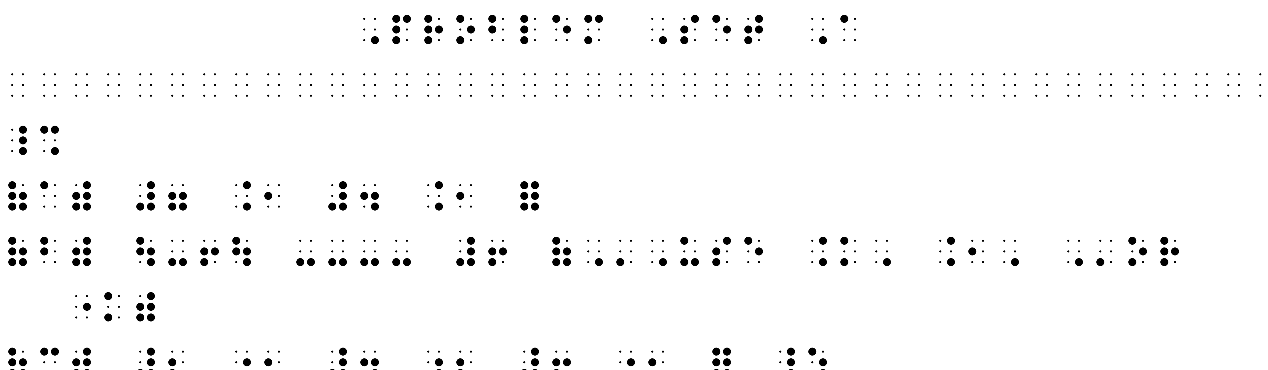
As seen in Lesson 2, in order to ensure that identifiers each begin in the same cell, the opening Nemeth Code indicator is placed at the end of the text that precedes the listed items. When an itemized set of problems immediately follows a heading, the following layouts are recommended.

4.25.1 **Centered Heading.** An opening Nemeth Code indicator cannot be placed at the end of a centered heading. When itemized material immediately follows a centered heading, place the opening Nemeth Code indicator alone on the line immediately before the first identifier. A switch indicator alone on a line does not replace a necessary blank line.

Example 4-62

Problem Set A

(a) $7 > 4 >$ -?-
(b) $|-6|$ ___ 6 (Use $=$, $>$, or $<$)
(c) $2 : 4 :: 6 : ?$



Line 2 is the blank line which is required following a centered heading.

4.25.2 **Cell-5 and Cell-7 Heading.** An opening Nemeth Code indicator may be placed at the end of a cell-5 or cell-7 heading. If the indicator does not fit on the line with the heading, it may be placed on the next line in the runover position. This is especially useful when itemized math material immediately follows the heading. The first identifier will then be transcribed in Nemeth.

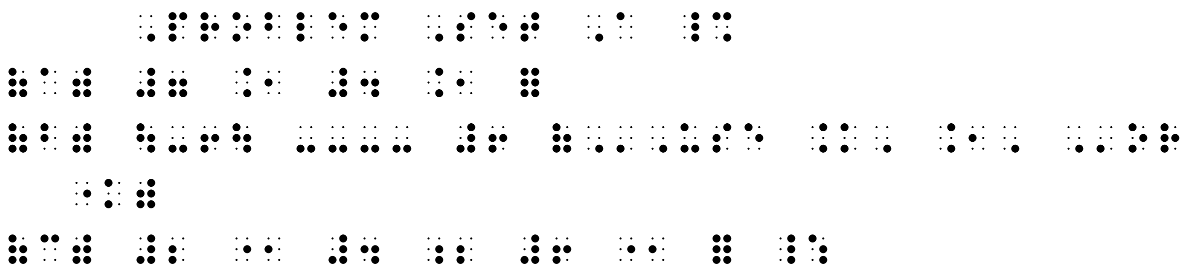
Example 4-63

Problem Set A

(a) $7 > 4 >$ -?-

(b) $|-6|$ __ 6 (Use $=$, $>$, or $<$)

(c) $2 : 4 :: 6 : ?$



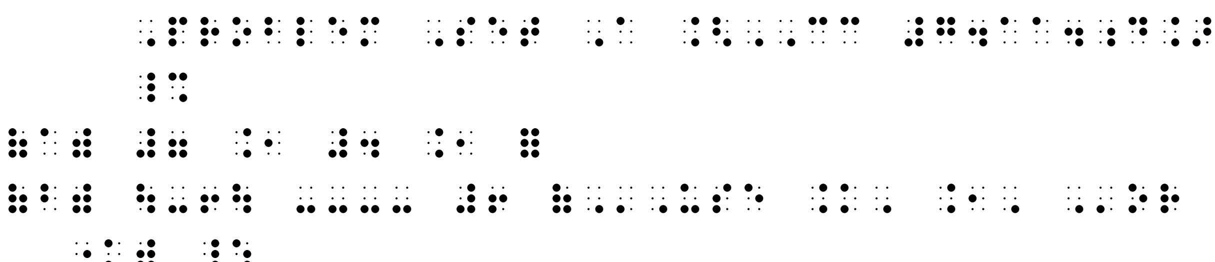
The opening Nemeth Code indicator is transcribed after the last word in the cell-5 heading.

Example 4-64

Problem Set A [CC 7.11.d]

(a) $7 > 4 >$ -?-

(b) $|-6|$ __ 6 (Use $=$, $>$, or $<$)



This opening Nemeth Code indicator is transcribed in cell 5, the runover position of the cell-5 heading.

4.26 Transcriber's Notes

Transcriber's note indicators are UEB symbols and therefore must be transcribed outside of the Nemeth switches. When the note itself contains mathematical material, code switching occurs within the note. Nemeth Code must be terminated before the closing transcriber's note indicator is transcribed. If mathematical material follows the transcriber's note, the opening Nemeth Code indicator may be placed following the closing transcriber's note indicator only if it fits on the same line.

Instructions: Review the "keep together" format rule for abbreviations and a preceding or following numeral to which it applies, (Lesson 3). In item G, assume that the letters of ARPA are pronounced individually.

PRACTICE 4I

A. 1 m = 100 cm

B. 3 yrs = 365 × 3 days

C. Draw three triangles using the given side lengths: (i) 1.5 cm, 5 cm, and 4.5 cm (ii) 4.5 cm, 5 cm, and 7.5 cm (iii) 1.5 cm, 4.5 cm, and 7 cm.

D. 1 square mile converted to acres: 1 sq mi = 640 ac

E. 5 in + 25 in = 30 in

F. **Two Types of Tons.** Compare the long ton with the short ton: 1 l. t. = 2240 lb.; 1 sh. t. = 2000 lb.

G. Fill in the missing information in the Customer Lifetime Value (CLV) Formula using Average revenue per account (ARPA). CLV = ____ × ARPA

H. Specific heat is expressed in J/kg·K.

For further practice, see Appendix A—Reading Practice.

EXERCISE 4

Prepare Exercise 4 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 4A

⠼⠁⠲ ⠠⠑⠭⠏⠗⠑⠎⠎ ⠰⠽ ⠔ ⠞⠻⠍⠎ ⠷ ⠰⠭ ⠊⠋
⠸⠩ ⠼⠆⠭⠤⠒⠽ ⠨⠅ ⠼⠂⠆⠸⠲
⠼⠆⠸⠲ ⠠⠄⠠⠊⠋ ⠠⠁ ⠨⠅ ⠇⠈⠡⠇ ⠸⠱⠂ ⠱⠁⠞ ⠊⠎ ⠮ ⠇⠢⠛⠹
⠐⠣⠰⠇⠐⠜ ⠷ ⠁ ⠎⠊⠙⠑ ⠔ ⠔⠡⠑⠎ ⠊⠋ ⠮ ⠜⠑⠁ ⠐⠣⠠⠁⠐⠜
⠷ ⠁ ⠎⠟⠥⠜⠑ ⠊⠎ ⠸⠩ ⠼⠶⠨⠒ ⠎⠟⠲⠋⠞⠲ ⠸⠱⠦
⠼⠉⠲ ⠠⠭ ⠊⠎ ⠍⠡ ⠑⠁⠎⠊⠻ ⠞⠕ ⠗⠑⠍⠑⠍⠃⠻
⠸⠩ ⠠⠁ ⠨⠅ ⠇⠺
⠷⠠⠁⠗⠑⠁ ⠨⠅ ⠇⠑⠝⠛⠞⠓⠈⠡⠺⠊⠙⠞⠓⠾ ⠸⠱ ⠹⠁⠝ ⠭ ⠊⠎
⠞⠕ ⠗⠑⠍⠑⠍⠃⠻ ⠸⠩ ⠠⠃ ⠨⠅ ⠚⠞ ⠸⠱ ⠱⠢ ⠞⠗⠽⠬ ⠞⠕
⠋⠊⠛⠥⠗⠑ ⠳ ⠓⠪ ⠍⠡ ⠉⠜⠏⠑⠞ ⠞⠕ ⠃⠥⠽ ⠿ ⠮ ⠇⠊⠧⠬
⠗⠕⠕⠍⠲
⠼⠙⠲ ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠜⠑⠁ ⠠⠁ ⠷ ⠞⠗⠁⠏⠑⠵⠕⠊⠙ ⠰⠠⠞ ⠾
⠥⠏⠏⠻ ⠃⠁⠎⠑ ⠸⠩ ⠁ ⠨⠅ ⠼⠒ ⠰⠍ ⠸⠱⠂ ⠇⠪⠻ ⠃⠁⠎⠑
⠸⠩ ⠃ ⠨⠅ ⠼⠖ ⠰⠍ ⠸⠱⠂ ⠯ ⠓⠑⠊⠣⠞
⠸⠩ ⠓ ⠨⠅ ⠼⠂⠒ ⠰⠍ ⠸⠱⠦

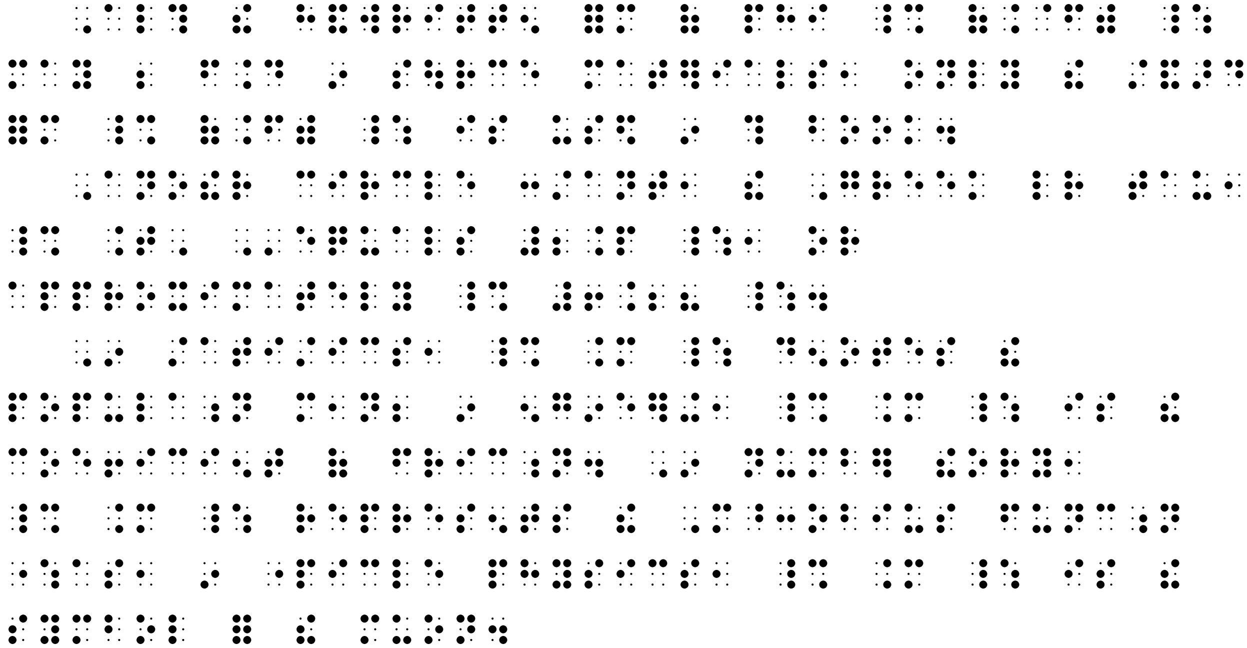
PRACTICE 4B

⠊⠲ ⠠⠞⠗⠊⠁⠝⠛⠇⠑ ⠸⠩ ⠠⠁⠠⠃⠠⠉ ⠸⠱ ⠔ ⠠⠟⠥⠁⠙⠗⠁⠝⠞
⠠⠠⠊⠧ ⠊⠎ ⠗⠑⠋⠇⠑⠉⠞⠫ ⠔ ⠠⠟⠥⠁⠙⠗⠁⠝⠞ ⠠⠠⠊⠊⠊ ⠵
⠠⠞⠗⠊⠁⠝⠛⠇⠑ ⠸⠩ ⠠⠁⠄⠠⠃⠄⠠⠉⠄ ⠸⠱⠲
⠰⠊⠊⠸⠲ ⠊⠧⠬⠧⠊ ⠨⠅ ⠭
⠰⠊⠊⠊⠸⠲ ⠠⠭ ⠨⠅ ⠼⠂⠴⠠ ⠠⠇ ⠨⠅ ⠼⠢⠴⠠ ⠠⠉ ⠨⠅ ⠼⠂⠴⠴⠠
⠠⠄⠯ ⠠⠙ ⠨⠅ ⠼⠢⠴⠴ ⠸⠱⠲
⠊⠧⠲ ⠠⠗⠑⠧⠊⠑⠺ ⠊⠞⠑⠍⠎ ⠰⠧ ⠯ ⠧⠊⠲
⠰⠧⠲ ⠠⠑⠭⠏⠇⠁⠔ ⠱⠽ ⠸⠩ ⠠⠠⠍⠉ ⠨⠅ ⠼⠂⠂⠴⠴⠠ ⠠⠄⠃
⠠⠠⠉⠍ ⠨⠅ ⠼⠔⠴⠴ ⠸⠱⠲
⠧⠊⠲ ⠠⠥⠎⠑ ⠠⠿⠍⠥⠇⠁⠎ ⠸⠩ ⠠⠊⠄ ⠠⠄⠯ ⠠⠠⠊⠊⠊⠄ ⠸⠱ ⠞⠕
⠏⠗⠕⠧⠑ ⠮ ⠌⠁⠞⠑⠰⠞⠲

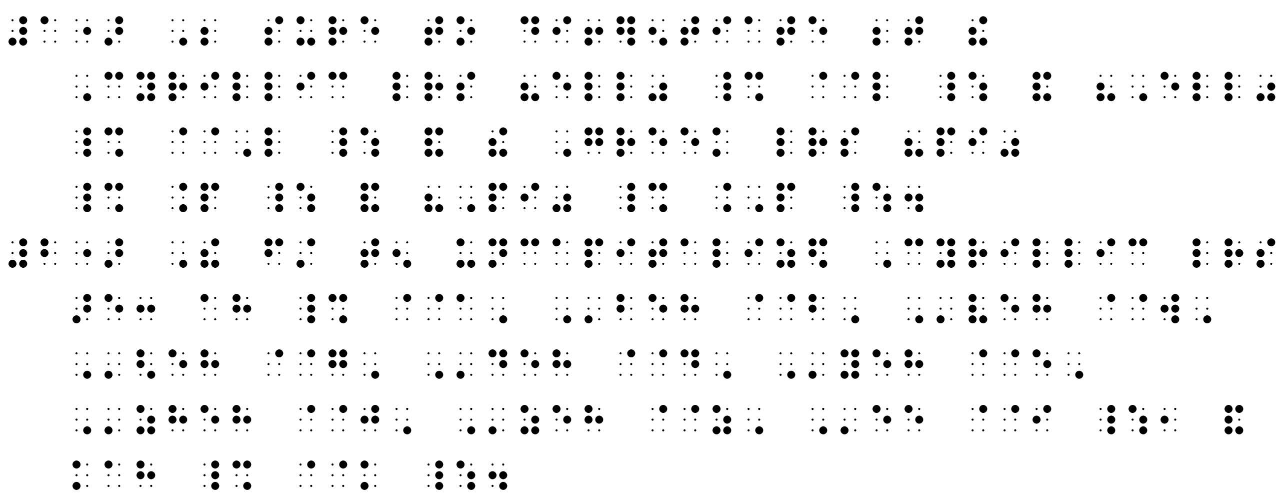
PRACTICE 4C

⠠⠊⠲ ⠠⠔ ⠮ ⠓⠑⠭⠁⠙⠑⠉⠊⠍⠁⠇ ⠎⠽⠌⠑⠍ ⠐⠣⠃⠁⠎⠑ ⠼⠁⠋⠐⠜⠂
⠮ ⠝⠥⠍⠃⠻ ⠦⠐⠕ ⠹⠳⠎⠯⠴ ⠊⠎ ⠺⠗⠊⠞⠞⠢ ⠵
⠸⠩ ⠼⠒⠑⠦ ⠸⠱⠲
⠠⠠⠊⠊⠲ ⠠⠒⠧⠻⠞ ⠓⠑⠭ ⠸⠩ ⠼⠶⠁⠂ ⠸⠱ ⠞⠕ ⠙⠑⠉⠊⠍⠁⠇
⠝⠥⠍⠻⠁⠰⠝⠲ ⠈⠨⠣⠁ ⠊⠎ ⠉⠁⠏⠊⠞⠁⠇⠊⠵⠫ ⠔ ⠏⠗⠔⠞⠲⠈⠨⠜

PRACTICE 4D



PRACTICE 4E



PRACTICE 4F

1. An unmodified, freestanding mathematical English letter in UEB context does not require a code switch. UEB rules are followed regarding use/nonuse of the grade 1 indicator.

2. Greek letters, whether capital or lowercase, require a switch to Nemeth Code. The single-letter ordinal "Nth" does not require a switch.

3. Placing the opening quotation mark before the opening Nemeth Code indicator aligns with the UEB practice of nested symbols. The commas following each math item are transcribed in Nemeth because we are still inside the switches. The three separate math items do not need to fall on the same braille line. The ellipsis follows the Nemeth Code terminator because it is not part of the mathematical expression – it indicates that the speaker is pausing.

4. This ellipsis is part of a mathematical series and so Nemeth Code is not terminated until after the ellipsis. The quotation marks are placed inside the code switch indicators along with the technical material to which they apply. There is no space between the ellipsis and the quotation mark. A punctuation indicator is required before the closing quotation mark.

5. The quotation marks enclose each expression and so are transcribed inside the switches. A punctuation indicator is required before each closing quotation mark. The question mark is placed after the Nemeth Code terminator because it applies to the whole sentence.

6. An isolated mathematical expression is enclosed in parentheses. Nemeth grouping symbols are used even though, technically, they are nonmathematical.

7. The function of the parentheses is to group the factors, specifying multiplication. Grouping symbols that are part of the mathematical expression must be transcribed as Nemeth symbols.

8. Variables x and y that are not associated with the Greek letter or within the equation are transcribed in UEB.

9. The paired grouping symbols (parentheses) must be transcribed in the same code—in this case, UEB.

PRACTICE 4G

Code switch indicators are omitted from the illustrations in this Practice.

YES

YES

NO (this list includes abbreviations)

YES

NO (this sequence does not begin and end with a sign of grouping)

YES

NO (this list contains ordinals)

YES

YES

YES

NO (this list contains a sign of comparison)

YES

NO (the items are not separated by commas)

YES ⠈⠷⠴⠠ ⠂⠈⠾

NO (there is a semicolon)

NO (this list contains words)

YES ⠷⠁⠠ ⠃⠠ ⠄⠄⠄⠾

YES ⠷⠭⠬⠂⠠ ⠭⠬⠆⠠ ⠿⠠ ⠿⠠ ⠭⠬⠢⠾

YES ⠨⠨⠷⠤⠂⠠ ⠴⠈⠾

YES ⠷⠆⠠ ⠲⠠ ⠖⠠ ⠤⠤⠤⠤⠠ ⠂⠴⠾

YES ⠷⠴⠠ ⠁⠠ ⠂⠠ ⠃⠠ ⠆⠾

NO (there are plural endings)

PRACTICE 4H

⠠⠁⠐⠜ ⠠⠋⠔⠙ ⠁⠇⠇ ⠸⠩ ⠠⠁⠠⠃⠎⠠ ⠠⠉⠠⠙⠎⠠ ⠠⠄⠯
    ⠠⠑⠠⠋⠎⠸⠆ ⠠⠄⠙⠗⠁⠺ ⠠⠭⠠⠽⠠⠵⠎ ⠸⠱⠲
⠰⠠⠃⠐⠜ ⠠⠋⠔⠙ ⠁⠇⠇ ⠸⠩ ⠠⠁⠠⠃⠸⠄⠎⠠ ⠠⠉⠠⠙⠸⠄⠎⠠ ⠠⠄⠯
    ⠠⠑⠠⠋⠸⠄⠎⠸⠆ ⠠⠄⠙⠗⠁⠺ ⠠⠭⠠⠽⠠⠵⠸⠄⠎⠸⠲
⠰⠠⠉⠾ ⠷⠂⠎⠞⠠ ⠼⠆⠝⠙⠠ ⠄⠄⠄ ⠰⠝⠞⠓⠠ ⠄⠄⠄ ⠼⠲⠔⠞⠓⠾
⠰⠠⠙⠾ ⠠⠄⠠⠙⠕⠑⠎ ⠳⠁⠳⠈⠡⠳⠃⠳ ⠨⠡ ⠳⠁⠃⠳⠸⠦
⠰⠠⠑⠾ ⠠⠄⠠⠊⠋ ⠰⠠⠟⠠ ⠠⠄⠮⠝
    ⠨⠷⠈⠷⠠⠠⠝⠕⠞⠤⠰⠠⠏⠈⠾ ⠠⠠⠕⠗ ⠰⠠⠏⠨⠾ ⠸⠱⠲

PRACTICE 4I

LESSON 5

- SIGNS OF OPERATION, cont.
- SIGNS OF COMPARISON, cont.

Format

- Instructions
- Simple Tables

Answers to Practice Material

LESSON PREVIEW

Many more operation signs and signs of comparison are explored, including negated forms. Table format is introduced, with a table consisting of mathematical symbols and their names. Mathematical use of the colon meaning "such that" is shown. The concept of symbols compounded vertically and symbols compounded horizontally is introduced with certain signs of comparison. Considerations for format of instructions are investigated.

This lesson introduces many more symbols that you will come across in all areas of mathematics. Look carefully at the print symbol as many look similar to each other. Understand the context – some symbols are used as signs of operation as well as signs of comparison. The function of the symbol must be determined in order to apply proper spacing rules.

SIGNS OF OPERATION, cont.

5.1 Review of Signs of Operation

In Nemeth, no space is left before or after a sign of operation unless it is preceded or followed by a sign of comparison, an ellipsis, a dash, an unrelated word, or an abbreviation with a related value. Signs of operation are mathematical symbols and are punctuated accordingly. The following signs of operation have already been introduced.

⠬	Plus	+
⠤	Minus	−
⠈⠡	Multiplication Cross	×
⠡	Multiplication Dot	•
⠨⠌	Division	÷

5.2 Signs of Operation Using Plus and Minus

The following signs are a combination of the plus and minus signs, written either side by side or one atop another.

⠬⠤	"Plus or Minus"	±
⠤⠬	"Minus or Plus"	∓
⠬⠐⠤	Plus followed by Minus	+−
⠤⠐⠬	Minus followed by Plus	−+
⠤⠐⠤	Minus followed by Minus	−−

Note that, in the first two symbols, the upper sign is transcribed first, followed immediately by the lower sign.

⫸ 38 ± 7 ⠼⠒⠦⠬⠤⠶

⫸ 38 ∓ 7 ⠼⠒⠦⠤⠬⠶

In the side-by-side symbols, the multipurpose indicator (dot 5) prevents these symbols from being read as "plus or minus", "minus or plus", or a dash.

⠐	Multipurpose Indicator

$10 + -5$

$10 - +5$

$10 - -5$

This is not an issue with other side-by-side operation symbols.

$-10 + \ +5$

Example 5-1

± 5 means $+5$ and -5.

Review the rules in Section P8 of the Preliminary Lesson regarding the use/nonuse of the numeric indicator with positive and negative numbers.

Example 5-2

Can $3 \pm 1 = +4$ *and* $+2$?

Example 5-3

Compare: $20 + -3;\ 20 - -3; -20 - -3;\ -20 + 3$.

5.3 Signs of Operation That Look Like Literary Symbols

Some mathematical symbols use characters also seen in literary writing. When the following signs are used in mathematical context, the Nemeth symbols shown below must be used. As mathematical operation signs, the spacing rules for operation signs must be followed.

⠸⠯	`Ampersand`	&
⠈⠼	`Asterisk`	*
⠨⠼	`Crosshatch`	#
⠸⠻	`Dagger, Single`	†
⠸⠸⠻	`Dagger, Double`	‡
⠈⠠⠏	`Paragraph Mark`	¶
⠈⠠⠎	`Section Mark`	§

a. When these symbols are used in literary context and have no mathematical meaning, the symbols and rules of UEB apply.

b. Some of these symbols have other mathematical applications, which will be discussed in later lessons.

5.3.1 **Ampersand**

⠸⠯

The ampersand functioning as a symbol of operation is commonly encountered in the study of logic, where it means "and". The symbol may be referred to as "logical conjunction".

Example 5-4

One can define F as p & $-$ p for any proposition p where "&" is *logical conjunction* and $-p$ is "not p".

⠠⠐⠕ ⠉ ⠙⠑⠋⠔⠑ ⠰⠠⠋ ⠵ ⠸⠩ ⠏⠸⠯⠤⠏ ⠸⠱ ⠿ ⠁⠝⠽
⠏⠗⠕⠏⠕⠎⠊⠰⠝ ⠰⠏ ⠐⠱ ⠸⠩ ⠦⠸⠯⠸⠴ ⠸⠱ ⠊⠎ ⠨⠂⠇⠕⠛⠊⠉⠁⠇
⠨⠂⠒⠚⠥⠝⠉⠰⠝ ⠯ ⠸⠩ ⠤⠏ ⠸⠱ ⠊⠎ ⠦⠝ ⠰⠏⠴⠲

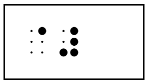
Example 5-5

Create a table to compare the price of bananas at the A & P with those at Price Chopper.

⠠⠉⠗⠂⠞⠑ ⠁ ⠞⠁⠃⠇⠑ ⠞⠕ ⠉⠕⠍⠏⠜⠑ ⠮ ⠏⠗⠊⠉⠑ ⠷
⠃⠁⠝⠁⠝⠁⠎ ⠁⠞ ⠮ ⠠⠁ ⠈⠯ ⠰⠠⠏ ⠾ ⠘⠹ ⠁⠞ ⠠⠏⠗⠊⠉⠑
⠠⠡⠕⠏⠏⠻⠲

This ampersand is used as a literary device, using the UEB symbol and following the spacing rules of UEB.

5.3.2 **Asterisk**



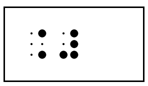
a. The asterisk as an operation symbol is commonly encountered in material about calculators where the symbol represents multiplication. When a numeral follows an asterisk, the numeric indicator is used even though the numeral is not preceded by a space.

⫸ $3 * 9 = 27$ 

⫸ $.3 * .9 = 0.27$ 

b. The asterisk used as a reference marker will be discussed in Lesson 13.

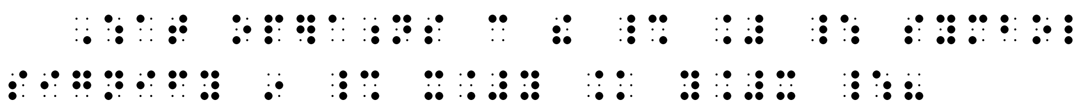
5.3.3 **Crosshatch**



a. When the crosshatch represents a symbol of operation, the Nemeth symbol is used.

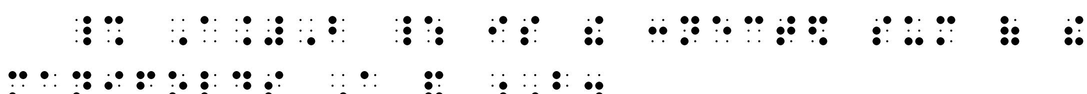
Example 5-6

What operations can the # symbol signify in $x\#y = y\#x$?



Example 5-7

$A\#B$ is the connected sum of the manifolds A and B.



b. When a numeral or a decimal point and a numeral follows a crosshatch, the numeric indicator is transcribed even though the numeral is not preceded by a space.

⫸ $1 \# .2 = .2 \# 1$ 

c. When the crosshatch is used in literary context such as "Problem #1" or as a hashtag in a social media reference or URL, the UEB symbol is used. When the crosshatch denotes pounds (weight), the Nemeth symbol is used.

Example 5-8

Notation Shortcut #4: "23 pounds" can be written "23#".

5.3.4 **Dagger and Double Dagger**

a. The dagger and double dagger may be used as operation symbols in binary operations.

⫸ $A \dagger B = B \dagger A$

⫸ $A \ddagger B = B \ddagger A$

b. The dagger used as a reference marker will be discussed in Lesson 13.

5.3.5 **Paragraph Mark**

In mathematical context, the Nemeth symbol is used. A numeric indicator is required before a numeral following a paragraph mark.

⫸ A ¶ B

⫸ 3 ¶ 4 = 4 ¶ 3

5.3.6 **Section Mark**

In mathematical context, the Nemeth symbol is used. A numeric indicator is required before a numeral following a section mark.

⫸ A § B

⫸ 3 § 4 = 4 § 3

Instructions: Review the spacing rules for operation signs before transcribing the practice. Transcribe this entire list in Nemeth Code. Place the opening switch indicator in cell 1 and continue with the first item on the same line. Terminate Nemeth after the last item in the list.

PRACTICE 5A

$4 \pm 1,\ 400 \pm 10,\ 6 \mp 1,\ 600 \mp 10$

$\mu \pm 1.645\,\sigma$

$50 - +5 = 45$

$50 + -5 = ?$

$-3 - -3 = 0$

$A \,\&\, B = B \,\&\, A$

$a * (b * c) = (a * b) * c$

$(1 + 2) * (3 + 4) = 3 * 7$

$\#A = \#B$

$.5\#.9 = .9\#.5$

$[(p \dagger p) \dagger (q \dagger q)]$

$s \P t = u \P v$

$1 \P 3 = 4 \P 3$

$m \S y = y \S m = y$

$5 \S 6 = 6 \S 5 = 6$

5.4 Signs of Operation Unique to Mathematics

Back Slash	\
Dot	·
Hollow Dot	∘
Intersection ("cap")	∩
Logical Product	∧
Logical Sum	∨
Minus with Dot Over	∸
Slash	/
Tilde, Simple	~
Tilde, Extended	$\sim\!\sim$
Union ("cup")	∪
Vertical Bar	|
Vertical Bar, Negated	∤ or ∤

Examples of each symbol are shown below. Note that, as with the other operation signs you have learned, these signs are unspaced from related mathematical terms regardless of the spacing shown in print.

5.4.1 **Back Slash**

The back slash slants upper left to lower right. In the context of operation signs, the back slash means "divides" or "is a factor of".

Example 5-9

$b \backslash a$ can be read as "b divides a."

Example 5-10

3\6 denotes "3 is a factor of 6."

5.4.2 **Dot**

⠡

In addition to operating as a multiplication sign, the dot may also be used to denote "and" in the study of logic. In either case, the symbol is transcribed without a space.

Example 5-11

In logic, $p \cdot q$ is read "p and q".

⠠⠔ ⠇⠕⠛⠊⠉⠂ ⠸⠩ ⠏⠡⠟ ⠸⠱ ⠊⠎ ⠗⠂⠙ ⠦⠰⠏ ⠯ ⠰⠟⠴⠲

Example 5-12

Torque is expressed as $\mathrm{N \cdot m}$.

⠠⠞⠕⠗⠟⠥⠑ ⠊⠎ ⠑⠭⠏⠗⠑⠎⠎⠫ ⠵ ⠸⠩ ⠰⠠⠝⠡⠰⠍ ⠸⠱⠲

5.4.3 **Hollow Dot**

⠨⠡

The hollow dot may be used as a sign of operation. It is also seen in function notation. The raised hollow dot used to represent degrees will be discussed in Lesson 6.

Example 5-13

$a \circ (b \circ c) = (a \circ b) \circ c$

⠸⠩ ⠁⠨⠡⠷⠃⠨⠡⠉⠾ ⠨⠅ ⠷⠁⠨⠡⠃⠾⠨⠡⠉ ⠸⠱

Example 5-14

$(f \circ g)(x)$ is not the same as $(g \circ f)(x)$.

⠸⠩ ⠷⠋⠨⠡⠛⠾⠷⠭⠾ ⠸⠱ ⠊⠎ ⠝ ⠮ ⠎⠁⠍⠑ ⠵
⠸⠩ ⠷⠛⠨⠡⠋⠾⠷⠭⠾ ⠸⠱⠲

Be sure to transcribe the hollow dot symbol – this is not the letter "o".

5.4.4 **Intersection**

⠨⠩

This operation symbol is also called a "cap".

⫸ $A \cap B = B \cap A$ ⠠⠁⠨⠩⠠⠃ ⠨⠅ ⠠⠃⠨⠩⠠⠁

5.4.5 **Logical Product**

⠈⠩

In the study of logic, this operation sign means "and" or "meet".

⫸ $p \wedge q \wedge r$ ⠏⠈⠩⠟⠈⠩⠗

5.4.6 **Logical Sum**

⠈⠬

In the study of logic, this operation sign means "or" or "join".

⫸ $p \vee q \vee r$ ⠏⠈⠬⠟⠈⠬⠗

5.4.7 **Minus with Dot Over**

⠨⠤

This operation sign means "proper difference".

⫸ $x \dot{-} y = 0$ ⠭⠨⠤⠽ ⠨⠅ ⠼⠴

5.4.8 **Slash**

⠸⠌

The term "slash" refers to the forward slash, which slants from lower left to upper right. In Nemeth, no space is left between the slash and any numeral, word, part of a word, or abbreviation to which it applies.

a. <u>With Numerals</u> When a slash represents a fraction line in a fraction where the numerator and denominator are printed on the baseline of writing, a switch to Nemeth Code is required. The numeric indicator is not needed for the numeral immediately following the slash.

Example 5-15

3/8 of the class are girls.

⠸⠩ ⠼⠒⠸⠌⠦ ⠸⠱ ⠷ ⠮ ⠉⠇⠁⠎⠎ ⠜⠑ ⠛⠊⠗⠇⠎⠲

b. <u>With Words</u> When a slash between words or abbreviations means "divided by", "per", or "over", the slash is a mathematical operation sign and a switch to Nemeth Code is required.

Example 5-16

1 watt = 1 joule/sec.

This slash means "per". That is, one joule per second.

Example 5-17

Slope is determined by calculating "rise" over "run" (rise/run).

This slash means "over".

c. Slash in Literary Context UEB rules for the solidus (forward slash) are followed when the slash does not mean "divided by", "per", or "over".

Example 5-18

The input/output ratio is 6-to-2.

This slash means "to".

Example 5-19

The chapter test will be given on Friday, 10/23. Bring your calculator and/or iPad to class. Practice problems can be found at https://www.Math.edu/Chapter12/Practice.html.

5.4.9 **Tilde and Extended Tilde**

⠈⠱
⠈⠠⠱

a. The simple tilde has one peak. In logic, the tilde may be used as a sign of operation meaning "not".

⫸ $\sim p$ ⠈⠱⠏

⫸ $\sim (\sim p)$ ⠈⠱⠷⠈⠱⠏⠾

b. When two symbols for the tilde follow one another, a multipurpose indicator (dot 5) is inserted between them to indicate that they are written horizontally.

⫸ $\sim \sim\ p \vee q$ ⠈⠱⠐⠈⠱⠏⠈⠬⠟

c. The extended tilde has more than one peak.

⫸ $\backsim\!\!\backsim\ s \vee t$ ⠈⠠⠱⠎⠈⠬⠞

d. When the tilde symbol is used to replace the word "approximately", "about", or "around" it is transcribed as an unspaced symbol.

Example 5-20

The test will begin in ~15 seconds.

⠠⠮ ⠞⠑⠌ ⠺ ⠆⠛⠔ ⠔ ⠸⠩ ⠈⠱⠂⠢ ⠎⠑⠉⠕⠝⠙⠎ ⠸⠱⠲

e. The tilde is also used as a sign of comparison. Consider the context to determine its meaning. (See 5.7.14.) When the tilde's meaning cannot be determined from context, follow print spacing.

5.4.10 **Union**

⠨⠬

This operation symbol is also called a "cup".

⫸ $A \cup B = B \cup A$ ⠠⠁⠨⠬⠠⠃ ⠨⠅ ⠠⠃⠨⠬⠠⠁

5.4.11 **Vertical Bar**

a. When the vertical bar means "is a factor" or "divides", it is functioning as an operation sign.

Example 5-21

In $b|a$, b is a factor of a.

Example 5-22

6|12 can be read as "6 divides 12."

b. The vertical bar is also used as a sign of grouping and as a sign of comparison. Consider the context to determine its meaning. (See Lesson 2. See also 5.6.16.) When the vertical bar's meaning cannot be determined from context, follow print spacing.

5.4.12 **Vertical Bar, Negated**

This symbol means "does not divide".

⫸ $5 \nmid n$

Format: Simple Tables

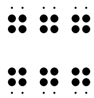
5.5 Introduction to Table Format

Guidelines for the layout of tables are given in *Braille Formats*. Study or review *Braille Formats* regarding the definition of a table, margins used, column separation lines, space between columns, use of guide dots, and considerations when a table is too wide to fit on the braille page.

The following symbols may be used in a table, in either UEB or Nemeth Code.

	Column Separation Line (width varies)
	Guide Dots (a minimum of two)

Box lines may also be used in either code.

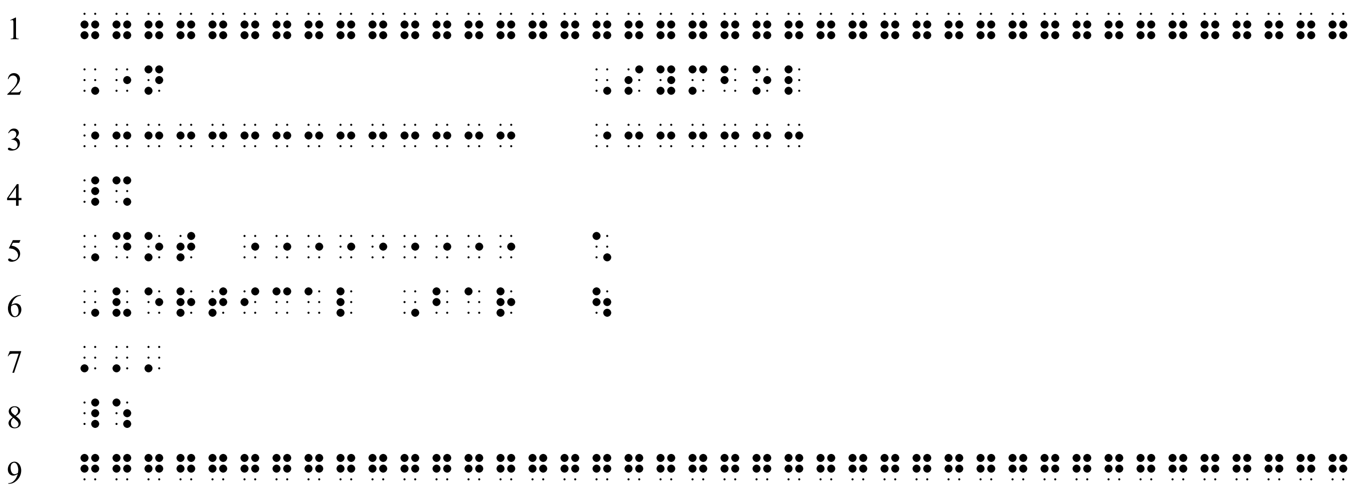


	`Top Box Line`
	`Bottom Box Line`

When table entries consist of technical material, the entire body of the table is transcribed in Nemeth Code, including any words. One opening Nemeth Code indicator precedes row 1 and one Nemeth Code terminator is placed at the end of the table. Words within the body of the table are transcribed without contractions, and the single-word switch indicator is not used. Further details regarding tables in Nemeth Code will be covered in later lessons.

The following example illustrates the layout you will use in PRACTICE 5B, which begins like this:

<u>Name</u>	<u>Symbol</u>
Dot	•
Vertical Bar	|



Line 2: The column headings are not mathematical. Contractions are used.
Line 3: Column separation lines are inserted according to Braille Formats guidelines.
Line 4: The opening Nemeth Code indicator is placed in cell 1.
Line 5: The first row is transcribed in Nemeth. Guide dots are inserted according to Braille Formats guidelines.
Line 6: Words in Nemeth are uncontracted.
Line 7: The ellipsis indicates that there will be further entries in your transcription.
Line 8: Terminate Nemeth Code on the line after the completion of the table, in cell 1.

5.5.1 **Omissions in a Simple Table.** In a table transcribed in Nemeth, when a dash, underscore, ellipsis, or other omission symbol is printed in an otherwise blank entry field, the appropriate Nemeth symbol is transcribed. (See Lesson 1.) Guide dots are inserted, as needed. When the entry field is blank, follow Braille Formats regarding the insertion of a series of guide dots across the width of a column.

Instructions: Practice transcribing these operation signs in table format. Include the box lines.

PRACTICE 5B

Name	Symbol
Dot	•
Vertical Bar	$\mid$
Logical Product	$\wedge$
Simple Tilde	$\sim$
Logical Sum	$\vee$
Extended Tilde	$\backsim\!\backsim$
Backslash	$\backslash$
Slash	/
Hollow Dot	$\circ$
Intersection	$\cap$
Union	$\cup$
Minus with Dot Over	$\dot{-}$

SIGNS OF COMPARISON, cont.

Five comparison signs were presented in the Preliminary Lesson.

⠨⠅	Equals	=
⠨⠂	Greater Than	>
⠐⠅	Less Than	<
⠰⠆	Proportion	∷
⠐⠂	Ratio	:

5.6 More Comparison Signs

⠫⠁	Arc, Concave Downward	⌢
⠫⠄	Arc, Concave Upward	⌣
⠈⠣⠠⠣	Equivalence	≎
⠨⠨⠂	Greater Than with Curved Sides	≻
⠸⠇	Identity	≡
⠸⠐⠅	Inclusion	⊂
⠨⠐⠅	Less Than with Curved Sides	≺
⠈⠑	Membership	∈
		(also ε or Ɛ)
⠫⠇	Parallel to	∥
⠫⠏	Perpendicular to	⊥
⠠⠗	Relation	R
⠸⠨⠂	Reverse Inclusion	⊃
⠈⠢	Reverse Membership	∋
⠈⠱	Tilde, Simple	~
⠈⠠⠱	Tilde, Extended	∿
⠸⠿	Variation	∝
⠳	Vertical Bar	|

Examples of each symbol are shown below. Note that, as with the other comparison signs you have learned, these signs are preceded and followed by a space regardless of the spacing shown in print.

5.6.1 Arc, Concave Upward

⠫⠁

➢ $x \frown y$ ⠭ ⠫⠁ ⠽

5.6.2 Arc, Concave Downward

⠫⠄

➢ $x \smile y$ ⠭ ⠫⠄ ⠽

5.6.3 Equivalence ("is equivalent to")

⠈⠣⠠⠣

➢ $x \asymp y$ ⠭ ⠈⠣⠠⠣ ⠽

5.6.4 Greater Than with Curved Sides ("is greater than")

⠨⠨⠂

➢ $7 \succ 4 \succ 3$ ⠼⠶ ⠨⠨⠂ ⠼⠲ ⠨⠨⠂ ⠼⠒

5.6.5 Identity (Triple Bar)

⠸⠇

This symbol is used in several different contexts. Most commonly it means "is identical with" or "is congruent to". The transcriber uses the same symbol regardless of its meaning. Do not confuse the triple bar with the Greek letter Xi or the triple bond in Chemistry.

➢ $A + B \equiv B + A$ ⠠⠁⠬⠠⠃ ⠸⠇ ⠠⠃⠬⠠⠁

5.6.6 **Inclusion**

⠸⠐⠅

This symbol is generally used for sets and their elements, meaning "is contained in" or "is a subset of".

⫸ $A \subset D$ ⠠⠁ ⠸⠐⠅ ⠠⠙

5.6.7 **Less Than with Curved Sides ("is less than")**

⠨⠐⠅

⫸ $5 \prec 9 \prec 11$ ⠼⠑ ⠨⠐⠅ ⠼⠊ ⠨⠐⠅ ⠼⠂⠂

5.6.8 **Membership**

⠈⠑

This symbol is generally used for sets and their elements, meaning "is an element of" or "belongs to". It must not be mistaken for the Greek uncapitalized epsilon even though it may be referred to as such.

⫸ $5 \in B$ ⠼⠑ ⠈⠑ ⠠⠃

5.6.9 **Parallel To ("is parallel to")**

⠫⠇

⫸ $AB \parallel CD$ ⠠⠁⠠⠃ ⠫⠇ ⠠⠉⠠⠙

5.6.10 **Perpendicular To ("is perpendicular to")**

⠫⠏

⫸ $PQ \perp RS$ ⠠⠏⠠⠟ ⠫⠏ ⠠⠗⠠⠎

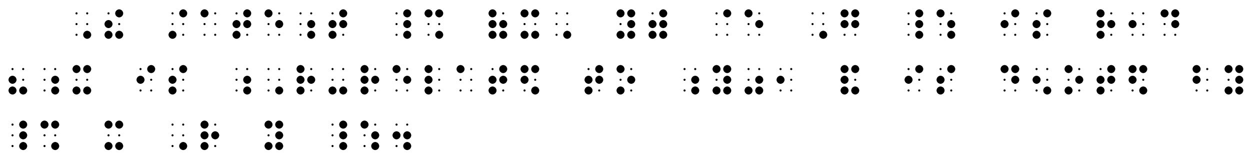
5.6.11 **Relation**

⠠⠗

When a letter R is used between two expressions to show relation ("is related to"), the letter is treated as a sign of comparison with a space before and after it regardless of print spacing. Read the surrounding narrative to realize the meaning of the letter "R" in order to transcribe it correctly. Note that other letters or signs may also be used to show relation.

Example 5-23

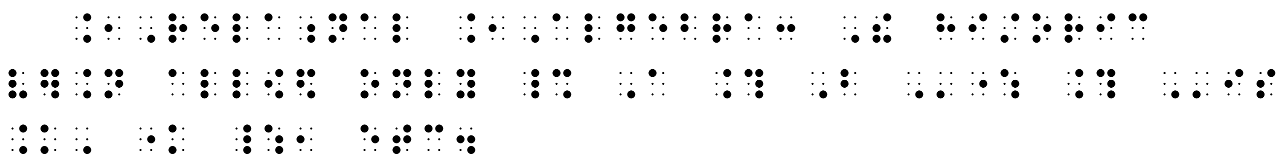
The statement $(x, y) \in G$ is read "x is R-related to y", and is denoted by $x\mathrm{R}y$.



The letter R is functioning as a comparison sign here, so it is preceded and followed by a space in the transcription.

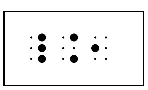
Example 5-24

Relational Algebra: The historic version allowed only $A\theta B$ where θ is $=$, $<$, etc.

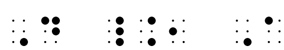


Greek letter theta is the relation symbol in this example. As a sign of comparison, it is preceded and followed by a space in the transcription.

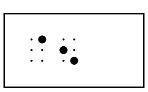
5.6.12 **Reverse Inclusion**



This symbol may mean "contains" or, in logic, "implies".

⋙ $D \supset A$ 

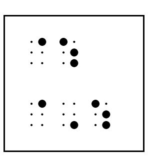
5.6.13 **Reverse Membership**



This symbol means "contains the element".

⋙ $B \ni 5$ 

5.6.14 **Tilde and Extended Tilde**



a. The simple tilde has one peak. When used as a comparison sign, the tilde means "is related to" or "is similar to".

Example 5-25

"x is related to y" is written $x \sim y$.

b. The extended tilde has more than one peak.

⫸ $x \sim y$

c. The tilde functioning as a sign of comparison is often encountered in the study of set theory and relations. The tilde is also used as a sign of operation. Consider the context to determine its meaning. (See 5.4.9.) When the tilde's meaning cannot be determined from context, follow print spacing.

5.6.15 **Variation ("varies as")**

⫸ $x \propto y$

5.6.16 **Vertical Bar**

a. When used as a sign of comparison, the vertical bar means "such that" or "given". It usually occurs in an expression within braces. The print copy may or may not show the vertical bar as a spaced symbol. In braille, however, comparison signs must be preceded and followed by a space.

⫸ $\{x \in E|\Phi(x)\}$

Example 5-26

$\{(x, y)|x + y < 6\}$ means "The set of points (x, y) such that $x + y < 6$."

b. The vertical bar functioning as a sign of comparison is often encountered in the study of conditional probability and logic. A vertical bar may also appear in other situations as a sign of comparison. Consider the context to determine its meaning. Recall that this symbol is also used as a sign of grouping and as a sign of operation. Apply proper spacing according to its function. See Lesson 2. See also 5.4.11. When the vertical bar's meaning cannot be determined from context, follow print spacing.

Example 5-27

$P(A|B)$ means "The probability of Event B **given** Event A."

⠸⠩ ⠠⠏⠷⠠⠁ ⠳ ⠠⠃⠾ ⠸⠱ ⠍⠂⠝⠎ ⠦⠠⠮ ⠏⠗⠕⠃⠁⠃⠊⠇⠰⠽
⠷ ⠠⠑⠧⠢⠞ ⠰⠠⠃ ⠘⠂⠛⠊⠧⠢ ⠠⠑⠧⠢⠞ ⠠⠁⠲⠴

5.7 Special Case: A Colon Meaning "Such That"

In set notation or mapping notation, a colon may be used to mean "such that". Although functioning as a sign of comparison, the Nemeth spacing rules governing comparison signs are not followed because a colon is a mark of punctuation.

a. Spacing in print may vary. Regardless of the spacing shown in print, in braille the colon is not preceded by a space. A punctuation indicator is required before the colon. Follow print for spacing after the colon.

b. Follow Nemeth rules for a "single letter" for the letter on either side of the colon regarding use/nonuse of the English-letter indicator. (See Lessons 3 and 4.)

⋙ $m: W$ ⠰⠍⠸⠒⠰⠠⠺

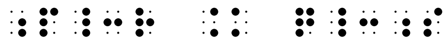
In the print copy, this colon is unspaced. Letter m needs an English-letter indicator because it is preceded by a space and is followed by punctuation. Letter W needs an English-letter indicator because it is preceded by punctuation and is followed by a space.

⋙ $\{x \ : \ x > 0\}$ ⠨⠷⠰⠭⠸⠒ ⠭ ⠨⠂ ⠼⠴⠨⠾

In the print copy, this colon is preceded and followed by a space. The first x needs an English-letter indicator because it is touching only one grouping sign and is followed by punctuation. The second x does not need an English-letter indicator because it is followed by a comparison sign.

⋙ $f: (x,\ y)$ ⠰⠋⠸⠒⠷⠭⠠ ⠽⠾

In the print copy, this colon is unspaced. Letter f needs an English-letter indicator because it is preceded by a space and is followed by punctuation. Letters x and y do not need an English-letter indicator because they are in an enclosed list.

⫸ $p:r = q:s$ 

In the print copy, these colons are unspaced. Letter p needs an English-letter indicator because it is preceded by a space and is followed by punctuation. Letters r and q do not need an English-letter indicator because they each are next to a comparison sign. Letter s needs an English-letter indicator because it is preceded by punctuation and is followed by a space.

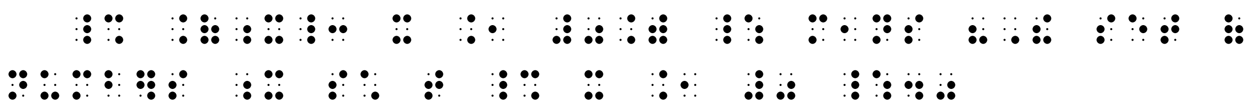
Example 5-28

$m{:}W$ means "the mapping m of W."

In the print copy, this colon is unspaced.

Example 5-29

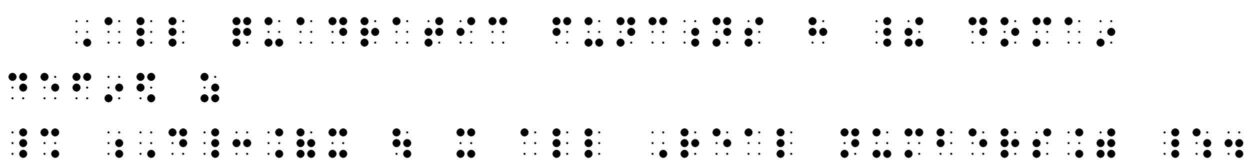
$\{x \ : \ x > 0\}$ means "The set of numbers x such that $x > 0$."



In the print copy, this colon is preceded and followed by a space.

Example 5-30

All quadratic functions have their domain defined as $D{:}\{x|x$ all Real numbers$\}$.



In the print copy, this colon is unspaced. An English-letter indicator is not needed for either letter x because they are next to a sign of comparison—the vertical bar.

c. Do not mistake the "such that" colon for a ratio symbol, which looks like a colon in print. Section P9 of the Preliminary Lesson discusses the ratio symbol.

Instructions: Review the spacing rules for comparison signs before transcribing the practice. Assume all tildes and vertical bars are comparison signs in these examples. Place the opening switch indicator in cell 1 and continue with the first item on the same line. Terminate Nemeth after the last item in the list.

PRACTICE 5C

$A \ni x$

$\{x \in A \mid x \backsim a\}$

$j \Bumpeq k$

$r \propto s$

$f(x) \equiv D(x) \cdot q(x)$

$-12 \prec -4 \prec 0$

$.9 \succ .5$

$m \smile l$

$l \frown m$

$(A \cup E) \subset (F \cup B)$

$Q \supset Z$

$PQR \sim P'Q'R'$

$AB \parallel \mathrm{MN}$

$CD \perp OP$

$EF \parallel GH \perp QR$

$\{m \mid 3(m-6) = -9\}$

$\{x : x \text{ has the property } T\}$

5.8 Signs of Comparison Compounded Vertically

When two or more simple signs of comparison are arranged one under the other, the combination becomes a single comparison sign compounded vertically. The symbol for the uppermost sign is written first, immediately followed by and unspaced from the symbol for the lower sign. Comparison signs compounded vertically not shown in the lists below are transcribed in accordance with this principle.

5.8.1 Greater Than or Equal To

⠱⠨⠂	Bar Over Greater Than	$\overline{>}$ or ⩾
⠨⠅ ⠨⠂	Equals Sign Over Greater Than	$\overset{=}{>}$ or ⋛
⠨⠂⠱	Bar Under Greater Than	≥ or ⩾
⠨⠂ ⠨⠅	Equals Sign Under Greater Than	≧ or ⩾

The "equal to" sign may be printed as an equals sign or as a single line – either a horizontal bar or an oblique line. Note that both the horizontal bar and the oblique line are represented by the same braille symbol (dots 156).

⫸ $a \overline{>} b$ ⠁ ⠱⠨⠂ ⠃

⫸ $a \overset{=}{>} b$ ⠁ ⠨⠅ ⠨⠂ ⠃

⫸ $x \geq y$ ⠭ ⠨⠂⠱ ⠽

⫸ $|x| \geqq 0$ ⠳⠭⠳ ⠨⠂ ⠨⠅ ⠼⠴

5.8.2 Inclusion ("is a subset of")

⠱⠸⠐⠅	Bar Over Inclusion	$\overline{\sqsubset}$
⠨⠅ ⠸⠐⠅	Equals Sign Over Inclusion	$\overset{=}{\sqsubset}$
⠸⠐⠅⠱	Bar Under Inclusion	⊆
⠸⠐⠅ ⠨⠅	Equals Sign Under Inclusion	⫅

⫸ $C \overline{\subset} B'$ ⠠⠉ ⠱⠸⠐⠅ ⠠⠃⠄

⫸ $C \overset{=}{\subset} B'$ ⠠⠉ ⠨⠅ ⠸⠐⠅ ⠠⠃⠄

⫸ $(D \cap E) \subseteq (E \times E)$

⠷⠠⠙⠨⠩⠠⠑⠾ ⠸⠐⠅⠱ ⠷⠠⠑⠈⠡⠠⠑⠾

⫸ $(D \cap E) \subseteqq (E \times E)$

⠷⠠⠙⠨⠩⠠⠑⠾ ⠸⠐⠅⠨⠅ ⠷⠠⠑⠈⠡⠠⠑⠾

5.8.3 **Intersection**

⠨⠩⠱	Bar Under Intersection	$\underline{\cap}$
⠨⠩⠨⠅	Equals Sign Under Intersection	$\underset{=}{\cap}$

a. The intersection sign is a sign of comparison when modified by a bar or equals sign below it. It is also called a "cap".

⫸ X $\underline{\cap}$ Y ⠠⠭ ⠨⠩⠱ ⠠⠽

⫸ X $\underset{=}{\cap}$ Y ⠠⠭ ⠨⠩⠨⠅ ⠠⠽

b. An unmodified intersection sign is a sign of operation. See 5.4.4.

5.8.4 **Less Than or Equal To**

⠱⠐⠅	Bar Over Less Than	$\overline{<}$ or ⪕
⠨⠅⠐⠅	Equals Sign Over Less Than	$\overset{=}{<}$ or ⪙
⠐⠅⠱	Bar Under Less Than	$\leq$ or $\leqslant$
⠐⠅⠨⠅	Equals Sign Under Less Than	$\leqq$ or ⫹

⫸ $v - 1 \overline{<} 5$ ⠧⠤⠂ ⠱⠐⠅ ⠼⠢

⫸ $v - 1$ ⪙ 5 ⠧⠤⠂ ⠨⠅⠐⠅ ⠼⠢

⫸ $6 \leq x \leq 9$ ⠼⠖ ⠐⠅⠱ ⠭ ⠐⠅⠱ ⠼⠔

⫸ 6 ⫹ x ⫹ 9 ⠼⠖ ⠐⠅⠨⠅ ⠭ ⠐⠅⠨⠅ ⠼⠔

5.8.5 **Logical Product**

⠱⠈⠩	Bar Over Logical Product	$\bar{\curlywedge}$
⠱⠈⠩⠱	Bar Over and Bar Under Logical Product	$\underline{\bar{\wedge}}$
⠱⠈⠩⠨⠅	Bar Over and Equals Sign Under Logical Product	$\underset{=}{\bar{\wedge}}$
⠈⠩⠱	Bar Under Logical Product	$\underline{\wedge}$
⠨⠅⠈⠩	Equals Sign Over Logical Product	$\overset{=}{\wedge}$
⠨⠅⠈⠩⠱	Equals Sign Over and Bar Under Logical Product	$\underline{\overset{=}{\wedge}}$
⠨⠅⠈⠩⠨⠅	Equals Sign Over and Equals Sign Under Logical Product	$\underset{=}{\overset{=}{\wedge}}$
⠈⠩⠨⠅	Equals Sign Under Logical Product	$\underset{=}{\wedge}$

a. The logical product sign is a sign of comparison meaning "meet" when modified by a bar or equals sign above or below it.

⫸ $ABD \mathbin{\bar{\wedge}} A'B'D'$

⠠⠁⠠⠃⠠⠙ ⠱⠈⠩ ⠠⠁⠄⠠⠃⠄⠠⠙⠄

⫸ $\{A\} \mathbin{\underset{=}{\wedge}} K$ ⠨⠷⠠⠁⠨⠾ ⠈⠩⠨⠅ ⠠⠅

⫸ $p \mathbin{\underline{\wedge}} q$ ⠏ ⠈⠩⠱ ⠟

b. An unmodified logical product sign is a sign of operation. See 5.4.5.

5.8.6 **Logical Sum**

⠱⠈⠬	Bar Over Logical Sum	$\overline{\vee}$
⠱⠈⠬⠱	Bar Over and Bar Under Logical Sum	$\underline{\overline{\vee}}$
⠱⠈⠬⠨⠅	Bar Over and Equals Sign Under Logical Sum	$\underset{=}{\overline{\vee}}$
⠈⠬⠱	Bar Under Logical Sum	$\underline{\vee}$
⠨⠅⠈⠬	Equals Sign Over Logical Sum	$\overset{=}{\vee}$
⠨⠅⠈⠬⠱	Equals Sign Over and Bar Under Logical Sum	$\underline{\overset{=}{\vee}}$
⠨⠅⠈⠬⠨⠅	Equals Sign Over and Equals Sign Under Logical Sum	$\underset{=}{\overset{=}{\vee}}$
⠈⠬⠨⠅	Equals Sign Under Logical Sum	$\underset{=}{\vee}$

a. The logical sum sign is a sign of comparison meaning "join" when modified by a bar or equals sign above or below it.

⫸ $ABC \;\overline{\vee}\; A'B'C'$

⠠⠁⠠⠃⠠⠉ ⠱⠈⠬ ⠠⠁⠄⠠⠃⠄⠠⠉⠄

⫸ $P\left(E \,\underline{\vee}\, F\right)$ ⠠⠏⠷⠠⠑ ⠈⠬⠱ ⠠⠋⠾

b. An unmodified logical sum sign is a sign of operation. See 5.4.6.

5.8.7 **Reverse Inclusion**

⠱⠸⠨⠂	Bar Over Reverse Inclusion	$\overline{\supset}$
⠨⠅⠸⠨⠂	Equals Sign Over Reverse Inclusion	$\overset{=}{\supset}$
⠸⠨⠂⠱	Bar Under Reverse Inclusion	$\supseteq$
⠸⠨⠂⠨⠅	Equals Sign Under Reverse Inclusion	$\supseteqq$

⫸ $B \,\overline{\supset}\, A$ ⠠⠃ ⠱⠸⠨⠂ ⠠⠁

⫸ $D \supseteqq C$ ⠠⠙ ⠸⠨⠂⠨⠅ ⠠⠉

5.8.8 **Tilde**

⠱⠈⠱	Bar Over Single Tilde	$\overline{\sim}$
⠨⠅⠈⠱	Equals Sign Over Single Tilde	$\overset{=}{\sim}$
⠈⠱⠈⠱	Double Tilde	≈
⠱⠈⠱⠈⠱	Bar Over Double Tilde	$\overline{\approx}$
⠨⠅⠈⠱⠈⠱	Equals Sign Over Double Tilde	$\overset{=}{\approx}$
⠈⠱⠱	Bar Under Single Tilde	≃
⠈⠱⠨⠅	Equals Sign Under Single Tilde	≅
⠈⠱⠈⠱⠱	Bar Under Double Tilde	$\underline{\approx}$
⠈⠱⠈⠱⠨⠅	Equals Sign Under Double Tilde	$\underset{=}{\approx}$

⫸ $3.14159 \approx 3.1416$

⠼⠒⠨⠂⠲⠂⠢⠔ ⠈⠱⠈⠱ ⠼⠒⠨⠂⠲⠂⠖

⫸ $ABC \cong DEF$ ⠠⠁⠠⠃⠠⠉ ⠈⠱⠨⠅ ⠠⠙⠠⠑⠠⠋

5.8.9 **Union**

⠨⠬⠱	Bar Under Union	$\underline{\cup}$
⠨⠬⠨⠅	Equals Sign Under Union	$\underset{=}{\cup}$

a. The union sign is a sign of comparison when modified by a bar or equals sign above or below it. It may also be referred to as a "cup".

⫸ $A \underline{\cup} B$ ⠠⠁ ⠨⠬⠱ ⠠⠃

b. An unmodified union sign is a sign of operation. See 5.4.10.

Instructions: Change the two-column format to a nested list by starting each phrase in cell 1, with each math example starting on a new line in cell 3. Transcribe "Signs of Comparison Compounded Vertically" as a cell-5 heading.

PRACTICE 5D

Signs of Comparison Compounded Vertically

Greater Than or Equal To	$ab \overset{=}{\geqslant} de$
	$\lvert y\rvert \geq 0$
Less Than or Equal To	$q - 7 \mathrel{\underline{\leqslant}} 5z$
	$-6 \leq x \leq -1$
Inclusion and Reverse Inclusion	$C' \mathrel{\overset{-}{\subset}} F'$ and $D \mathrel{\overset{-}{\supset}} C$
	$(B \cap E) \subseteqq (E \times E)$
Intersection and Union ("Cup")	$X \mathbin{\underline{\cap}} Y$
	$X \mathbin{\underset{=}{\cap}} Y$
	$A \mathbin{\underset{=}{\cup}} B$
Logical Product and Logical Sum	$QRS \barwedge Q'R'S'$
	$y \mathbin{\underline{\triangle}} z$ and $M(E \veebar H)$
	$ABC \mathbin{\overline{\triangledown}} A'B'C'$
Tilde	$3.14159 \approx 3.1416$
	$ABC \cong DEF$

5.9 Signs of Comparison Compounded Horizontally

When two or more signs of comparison are arranged side by side, the combination becomes a single comparison sign compounded horizontally. A multipurpose indicator (dot 5) is inserted between the unspaced symbols to indicate that they are printed horizontally, not vertically. Comparison signs compounded horizontally not shown in the lists below are transcribed in accordance with this principle.

5.9.1 Greater Than ...

⠨⠂⠐⠐⠅	Greater Than Followed by Less Than	$><$
⠨⠂⠐⠨⠅⠐⠐⠅	Greater Than Followed by Equals Followed by Less Than	$>=<$

⫸ $n><1$ ⠝ ⠨⠂⠐⠐⠅ ⠼⠂

⫸ $n>=<1$ ⠝ ⠨⠂⠐⠨⠅⠐⠐⠅ ⠼⠂

5.9.2 Less Than ...

⠐⠅⠐⠨⠂	Less Than Followed by Greater Than	$<>$
⠐⠅⠐⠨⠅⠐⠨⠂	Less Than Followed by Equals Followed by Greater Than	$<=>$

⫸ $n<>1$ ⠝ ⠐⠅⠐⠨⠂ ⠼⠂

⫸ $n<=>1$ ⠝ ⠐⠅⠐⠨⠅⠐⠨⠂ ⠼⠂

5.9.3 **A Colon Used to Define an Object.** A colon may be used with an equals sign to define an object. This notation that is borrowed from computer programming. Using the principle given above, suggested treatment is shown in the next example.

Example 5-31

y:=7x+2 means that y is defined to be 7x+2.

⠸⠩ ⠽⠸⠒⠐⠨⠅ ⠼⠶⠭⠬⠆ ⠸⠱ ⠍⠂⠝⠎ ⠞ ⠰⠽ ⠊⠎ ⠙⠑⠋⠔⠫ ⠞⠕
⠃⠑ ⠸⠩ ⠼⠶⠭⠬⠆ ⠸⠱⠲

5.10 Negated Signs of Comparison

In print, a sign of comparison may be negated by a vertical or a slanted line drawn through it. The print negation symbol may be slanted in either direction. In braille, ⠌ represents any of the print negation lines. ⠌ is placed immediately before the sign of comparison being negated.

Some examples are shown below. Negated signs of comparison not illustrated here are transcribed according to the same principle.

⠌⠨⠅	Negated Equals Sign	≠ or ǂ
⠌⠫⠇	Negated Parallel To	∦
⠌⠫⠏	Negated Perpendicular To	⊥̸
⠌⠨⠂⠱	Negated Greater Than or Equal To	≱
⠌⠈⠑	Negated Membership	∉ or ∉

⫸ $4 \times 13 \neq 14$ ⠼⠲⠈⠡⠂⠒ ⠌⠨⠅ ⠼⠂⠲

⫸ $4 \ngeq 7$ ⠼⠲ ⠌⠨⠂⠱ ⠼⠶

⫸ $9 \notin D$ ⠼⠔ ⠌⠈⠑ ⠠⠙

⫸ CD ∦ EF ⊥̸ GH

⠠⠉⠠⠙ ⠌⠫⠇ ⠠⠑⠠⠋ ⠌⠫⠏ ⠠⠛⠠⠓

Instructions: Use the principles learned in this section to construct symbols that are not shown in the examples.

PRACTICE 5E

(1) $x \notin$ A means "x is not an element of A".

(2) By typing <=, the symbol ≤ will appear. By typing /< , the symbol ≮ will appear.

(3) A ⊈ B means that at least one element of A is not an element of B.

(4) WXY ≁ VXW

(5) Since L ∦ M and M ∦ N, does it follow that L ∦ N?

(6) The domain is all $x \neq -4, 0, 5$.

Format: Instructions

5.11 Margins for Instructions Preceding Itemized Material (5-3)

Nemeth Code makes a distinction regarding instructions preceding a set of itemized problems. Following a blank line, the instructions begin in cell 5 with runovers in cell 3. The related itemized material beings on the next line.

Exceptions: Instructions may begin on line 1 if no running head is used. The blank line is unnecessary when the instructions immediately follow a cell-5 or a cell-7 heading. If the itemized material itself requires a leading blank line, this blank line must be inserted.

Instructions may be recognized with the use of a distinctive typeform in the print document. Distinctive typeform used solely as a visual enhancement is disregarded in the braille transcription, according to UEB and *Braille Formats* guidelines.

It is preferable to keep instructions on the same braille page with the exercise. To accomplish this, instructions may need to be moved to the next braille page. However, when there is not sufficient space on that page for the instructions and part of the exercise, instructions may be placed on the preceding page.

In the following example, the dashed line indicates a page turn.

Example 5-32

Problem Set 7F Tell whether the following ratios are equivalent.

1. $3:2=75:50$
2. $6:4=15:30$

Which of the following sentences are true? Which
are false?

3. $328 \div 4 = 41 \times 2$
4. $672 - 415 < 312 \div 3$

— —

54

Multiply.

5. $11{,}251.54 \times 1436$
6. $1000 \times 476{,}792$

```
      ⠠⠏⠗⠕⠃⠇⠑⠍ ⠠⠎⠑⠞ ⠼⠛⠠⠋
    ⠠⠞⠑⠇⠇ ⠱⠑⠮⠗ ⠮ ⠋⠕⠇⠇⠪⠬ ⠗⠁⠞⠊⠕⠎ ⠜⠑
  ⠟⠥⠊⠧⠁⠇⠢⠞⠲ ⠸⠩
⠼⠂⠸⠲ ⠼⠒ ⠐⠂ ⠼⠆ ⠨⠅ ⠼⠶⠢ ⠐⠂ ⠼⠢⠴
⠼⠆⠸⠲ ⠼⠖ ⠐⠂ ⠼⠲ ⠨⠅ ⠼⠂⠢ ⠐⠂ ⠼⠒⠴ ⠸⠱

    ⠠⠱ ⠷ ⠮ ⠋⠕⠇⠇⠪⠬ ⠎⠢⠞⠰⠑⠎ ⠜⠑ ⠞⠗⠥⠑⠦ ⠠⠱ ⠜⠑
  ⠋⠁⠇⠎⠑⠦ ⠸⠩
⠼⠒⠸⠲ ⠼⠒⠆⠦⠨⠌⠲ ⠨⠅ ⠼⠲⠂⠈⠡⠆
⠼⠲⠸⠲ ⠼⠖⠶⠆⠤⠲⠂⠢ ⠐⠅ ⠼⠒⠂⠆⠨⠌⠒
⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠤⠼⠑⠙

    ⠠⠄⠠⠍⠥⠇⠞⠊⠏⠇⠽⠲
⠼⠢⠸⠲ ⠼⠂⠂⠠⠆⠢⠂⠨⠢⠲⠈⠡⠂⠲⠒⠖
⠼⠖⠸⠲ ⠼⠂⠴⠴⠴⠈⠡⠲⠶⠖⠠⠶⠔⠆ ⠸⠱
```

Line 1: Cell-7 heading
Lines 2-3: Instructions (5-3) immediately following a cell-7 heading
Lines 4-5: Itemized material following instructions
Line 6: Blank line precedes instructions
Lines 7-8: Instructions (5-3)
Lines 9-10: Itemized material following instructions
Line 12: Blank line precedes instructions
Line 13: Instructions (5-3)
Lines 14-15: Itemized material following instructions

5.11.1 **Code Switching and Instructions.** The opening Nemeth Code indicator may be placed after the last word of the instructions. If there is no room on the line, place the switch indicator in the runover cell of the instructions (cell 3). (An exception applies to spatially arranged material, which will be covered in Lesson 9.)

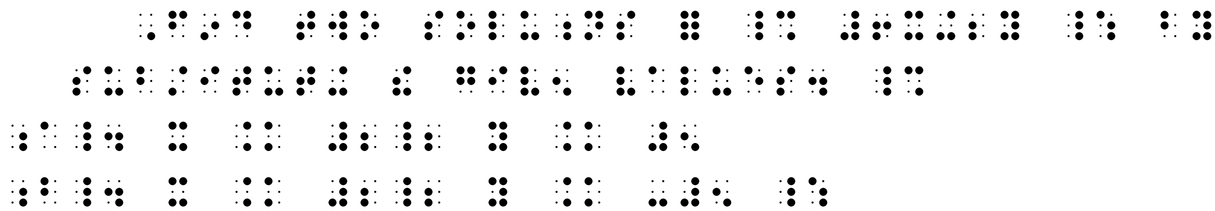
If instructions end with a Nemeth expression and the subsequent math problem starts with a Nemeth expression, Nemeth Code may be left in effect between the end of the instructions and the start of the problem.

Example 5-33

Find two solutions for $6x + 2y$ by substituting the given values.

a. $x = 2; y = 5$

b. $x = 2; y = -5$

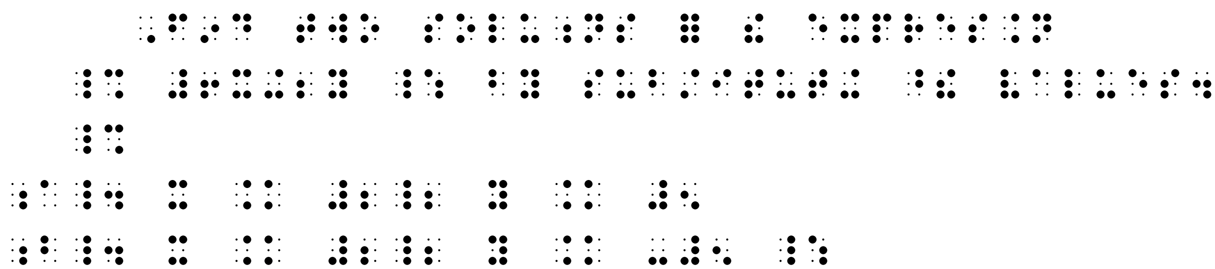


Example 5-34

Find two solutions for the expression $6x + 2y$ by substituting these values.

a. $x = 2; y = 5$

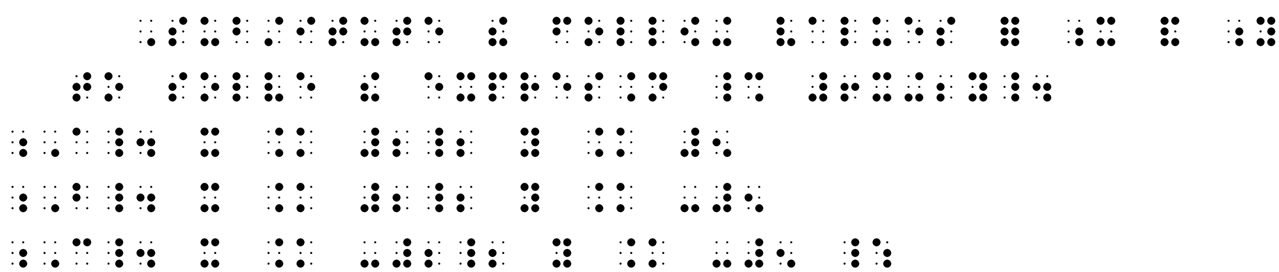
b. $x = 2; y = -5$



Example 5-35

Substitute the following values for x and y to solve the expression $6x + 2y$.

A. $x = 2; y = 5$ B. $x = 2; y = -5$ C. $x = -2; y = -5$



Reminder: When the print copy arranges itemized material side by side across the page and there are no subdivisions, Nemeth format rules dictate that all identifiers start in cell 1.

5.12 Narrative Directions

"Instruction" format (5-3) applies only to text that is followed by a set of numbered or lettered problems. If the material following the text is not numbered or lettered, if the directions themselves are numbered or lettered, or if the narrative is not giving explicit directions that apply to the following itemized material, then other established formats are followed as illustrated below. We may refer to such text as "directions".

5.12.1 **Margins for Directions Preceding Unitemized Material (3-1).** If there is no numbered or lettered exercise material following the directions, the text is treated as narrative material and is transcribed as a (3-1) paragraph. The (5-5) style of "directions" given in *Braille Formats* does not apply in a transcription that contains Nemeth.

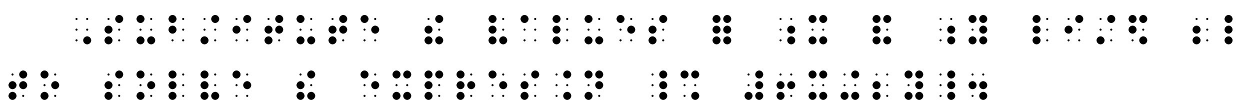
Example 5-36

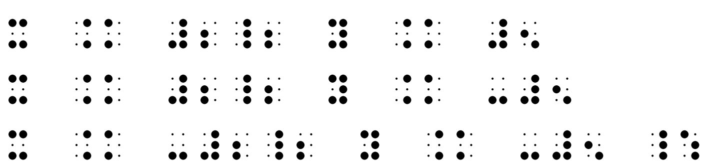
Substitute the values for x and y listed below to solve the expression $6x + 2y$.

$x = 2; y = 5$

$x = 2; y = -5$

$x = -2; y = -5$



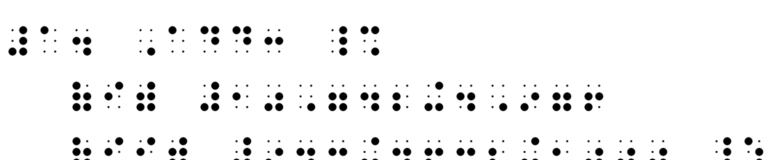


5.12.2 **Itemized Directions.** Directions that are preceded by a number or letter are formatted according to the Nemeth margin rules for itemized material.

Example 5-37

1. Add:

   (i) $10{,}742 + 4{,}976$ (ii) $943 + 4632 + 1000$

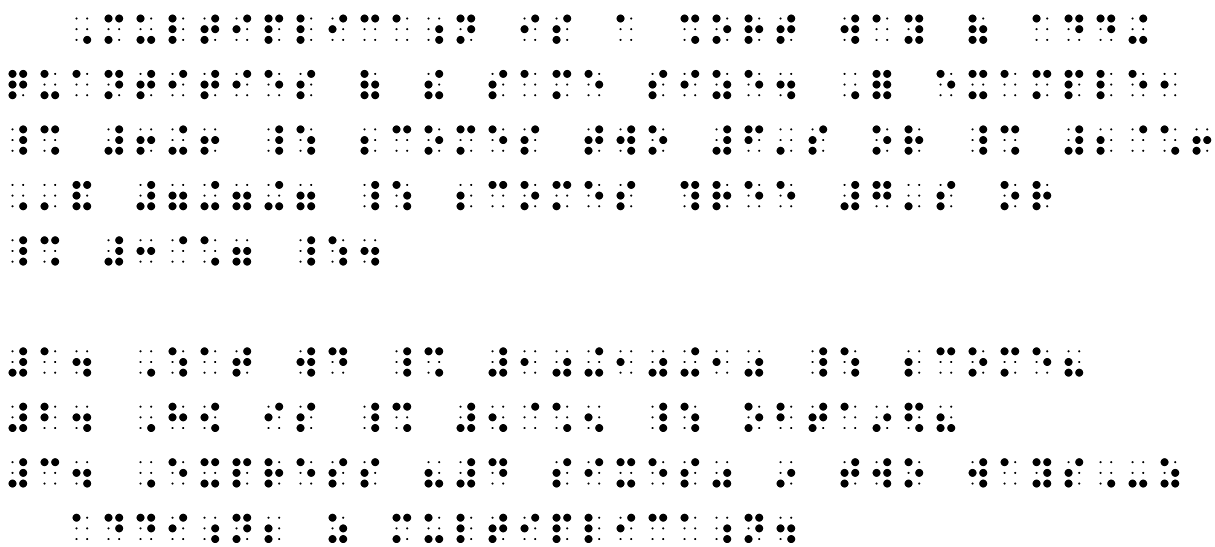


5.12.3 **Margins for Narrative Preceding Itemized Material (3-1).** Only explicit textual matter is formatted as (5-3) "instructions". If the narrative is purely explanatory, regular paragraphing is applied and a blank line precedes the itemized material.

Example 5-38

Multiplication is a short way of adding quantities of the same size. For example, $6+6$ becomes two 6's or 2×6 and $7+7+7$ becomes three 7's or 3×7.

1. What would $10+10+10$ become?
2. How is 5×5 obtained?
3. Express "4 sixes" in two ways—as addition; as multiplication.



Instructions: Treat "Signs of Comparison" and "Adding and Subtracting Integers" as cell-5 headings.

PRACTICE 5F

Signs of Comparison

These examples illustrate the basic spacing rules for comparison signs learned in this unit.

(1) $5 \prec 9 \prec 11$

(2) $11.7 \succ 1.17$

(3) $550 : 11 :: \ ? : 12$

Adding and Subtracting Integers

Find the sum or difference as indicated by the signs.

1) $-6 + -5 =$ ____

2) $5 + -19 =$ ____

3) $-7 - -13 =$ ____

4) $29 - -24 =$ ____

For further practice, see Appendix A—Reading Practice.

EXERCISE 5

Prepare Exercise 5 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 5A

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PRACTICE 5B

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PRACTICE 5C

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Notes regarding the last item: The words are part of the math and so are transcribed uncontracted without switching out of Nemeth Code. Each single letter requires an English-letter indicator because each is preceded and followed by a space and/or punctuation (or begins a new line). The presence of a single grouping symbol is ignored when determining whether an English-letter indicator is needed.

PRACTICE 5D

PRACTICE 5E

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PRACTICE 5F

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LESSON 6

- LEVEL INDICATORS
    - Superscripts
    - Baseline Indicator
    - Subscripts
    - More about Superscripts and Subscripts

Format

- Itemized Material with Subdivisions
- Tabular Format
- Margins for Exercise Sets

Answers to Practice Material

LESSON PREVIEW

This lesson begins by looking at format. Itemized material with subdivisions is found throughout math textbooks, in exercise sets, and in answer sections. The rules differ somewhat from those followed in a nontechnical transcription. The topic of superscripts and subscripts is presented. Superscript, subscript, and baseline indicators are introduced. The lesson ends with another look at grouping signs as they relate to level indicators.

The Nemeth Code Book

The Nemeth Braille Code for Mathematics and Science Notation will be your primary source once you complete this course. This is a good time to begin supplementing the lesson material by reading about the topic in the Nemeth code book. Cross references are identified with a "NC" prefix. The Nemeth code book is available for downloading at www.brailleauthority.org.

Format

6.1 Margins for Itemized Material with Subdivisions (1-5; 3-5) [NC Rule 26]

To transcribe itemized material with lettered or numbered subdivisions, begin the main item designation in cell 1. Place any runovers in cell 5. Cell 3 is reserved for the beginning of each lettered or numbered subdivision. Any runovers to subdivisions also begin in cell 5. In the Nemeth Code, "subdivision" applies to all itemized sublevels regardless of the print indentation layout. In other words, for an exercise with any number of subdivisions, use margins 1-5 for the first level, and 3-5 for all sublevels.

- The first example shows two indentation levels in the print copy. In braille, all subdivisions begin in cell 3. All runovers are in cell 5.

Example 6-1

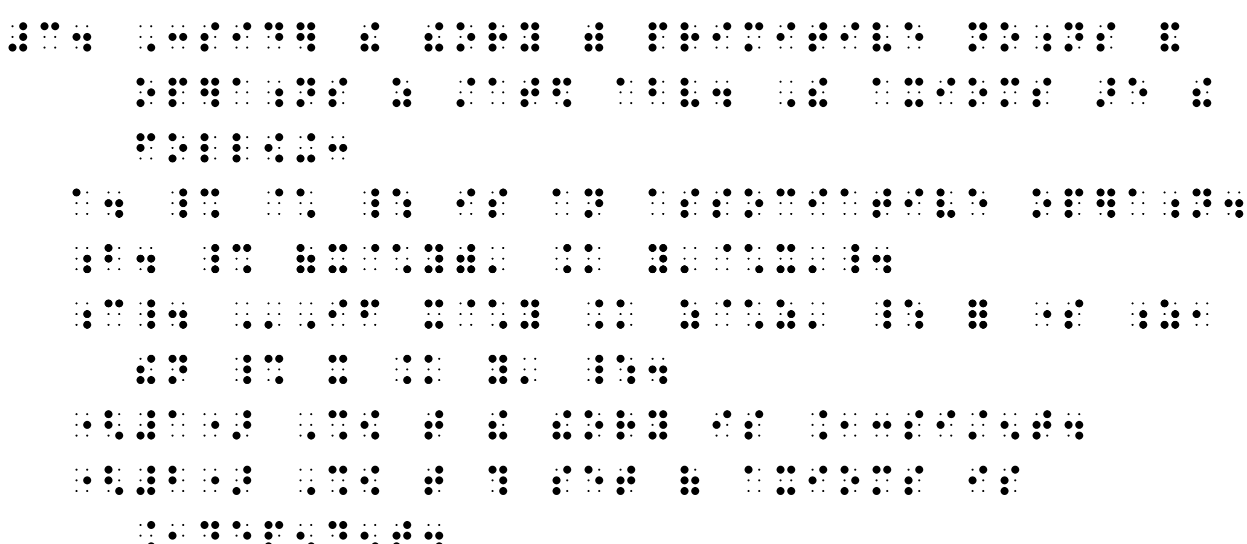
3. Consider the theory with primitive notions and operations as stated above. The axioms are the following:

   a. $\times$ is an associative operation.

   b. $(x \times y)' = y' \times x'$.

   c. If $x \times y = z \times z'$ for some z, then $x = y'$.

      (1) Show that the theory is *consistent*.
      (2) Show that this set of axioms is *dependent*.



- The next example shows no text following the main item number, with subdivision a. beginning on the same line. In braille, subdivisions must begin in cell 3 on a new line.

Example 6-2

2. a. Write each of the following as a fraction or a mixed number. .12, .88, 3.49
   b. Tell what digit is in tenth's place.

- In the next example, the subdivisions are printed without indentation. In braille, subdivisions must begin in cell 3.

Example 6-3

1. Copy and multiply.

a. $2 \times 2 \times 3 \times 3 \times 4 \times 4 \times 5 \times 5$

b. $9 \cdot 9 \cdot 8 \cdot 8 \cdot 7 \cdot 7 \cdot 6 \cdot 6$

Reminder: Each subdivision's identifier begins in cell 3. The opening Nemeth Code indicator is placed at the end of the line of text that precedes the identifier.

6.1.1 **Paragraphs Within Itemized Material with Subdivisions (7-5).** If a main item or a subdivision has more than one paragraph, each new paragraph begins in cell 7, and its runovers begin in cell 5.

- The next example shows narrative displayed to the main item. *Braille Formats* guidelines dictate the layout for that paragraph. The second paragraph in subdivision b. is recognized by the beginning of a new, indented line.

Example 6-4

5. Does the method of creating whole number x-terms work with decimals? Consider this example.

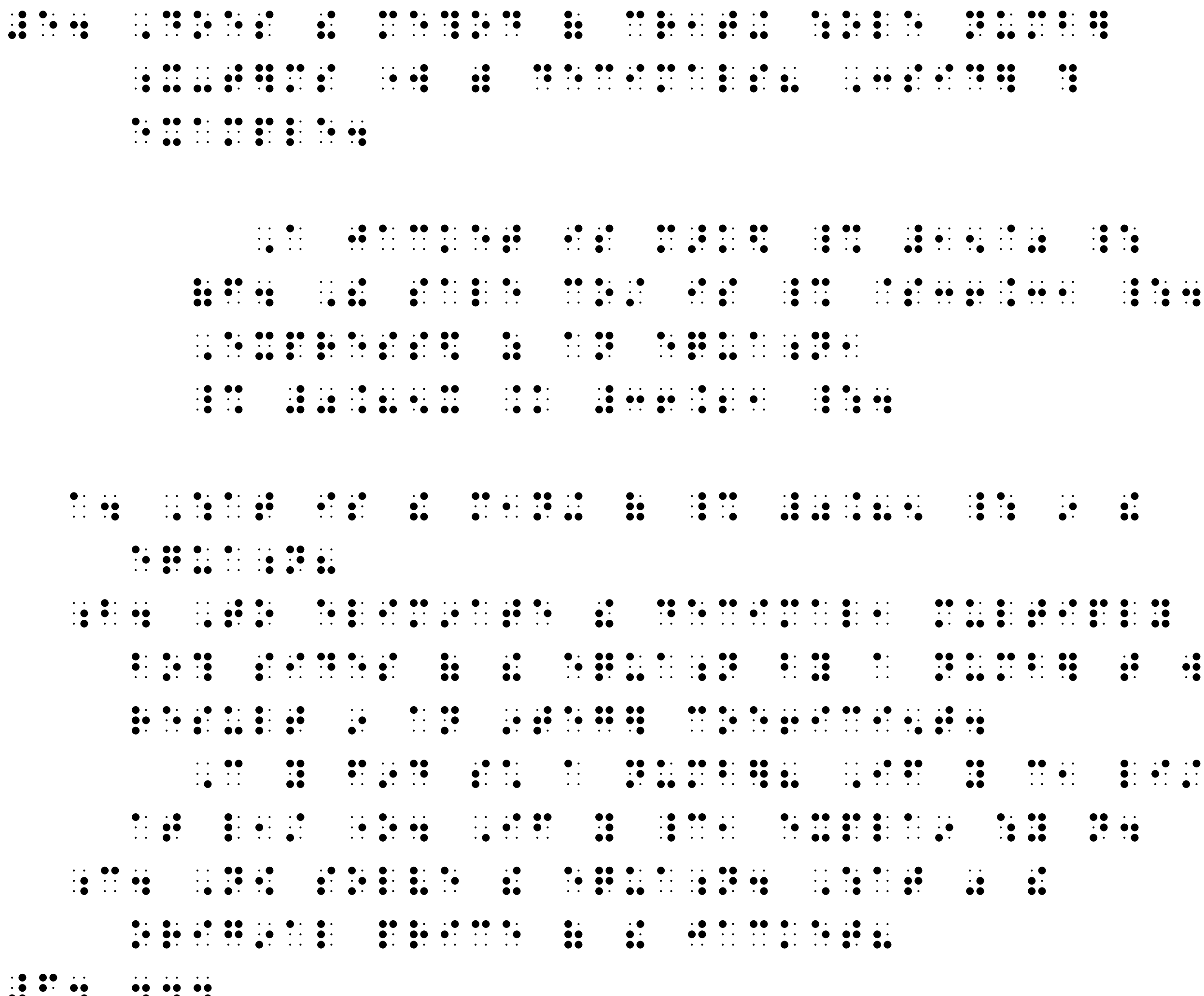
   A jacket is marked 15% off. The sale cost is \$36.31. Expressed as an equation, $0.85x = 36.21$.

   a. What is the meaning of 0.85 in the equation?

   b. To eliminate the decimal, multiply both sides of the equation by a number that will result in an integer coefficient.
      Can you find such a number? If you can, list at least one. If you cannot, explain why not.

   c. Now solve the equation. What was the original price of the jacket?

6. ...



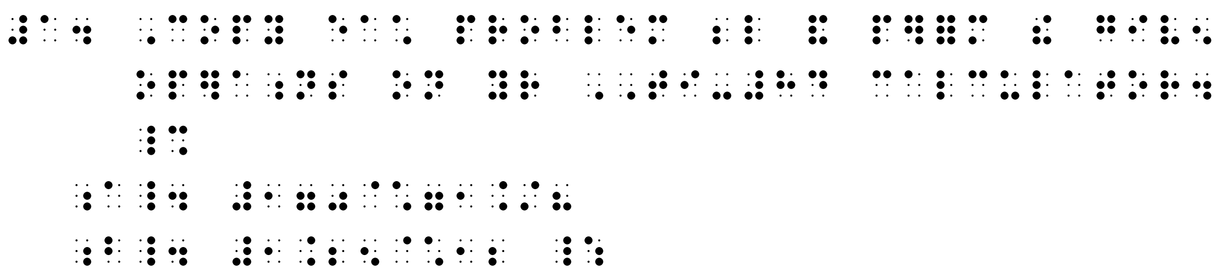
In Nemeth, indentation of the first line of each new paragraph organizes the narrative. A blank line precedes and follows the displayed word problem, as directed by Braille Formats guidelines for displayed material, but the blocked *paragraph form is changed to indented form. Within the item/subdivision Nemeth format there are no blank lines between paragraphs or subdivisions.*

6.1.2 **Side-by-Side Layout in Print.** Even when subdivisions are printed on the same line across the page, in braille each identifier must begin on a new line. (This rule does not apply to spatial material, which will be studied later in this course.)

Example 6-5

1. Copy each problem below and perform the given operations on your TI-84 calculator.

   a. $170 \times 71 \div 8$ b. 1.25×12

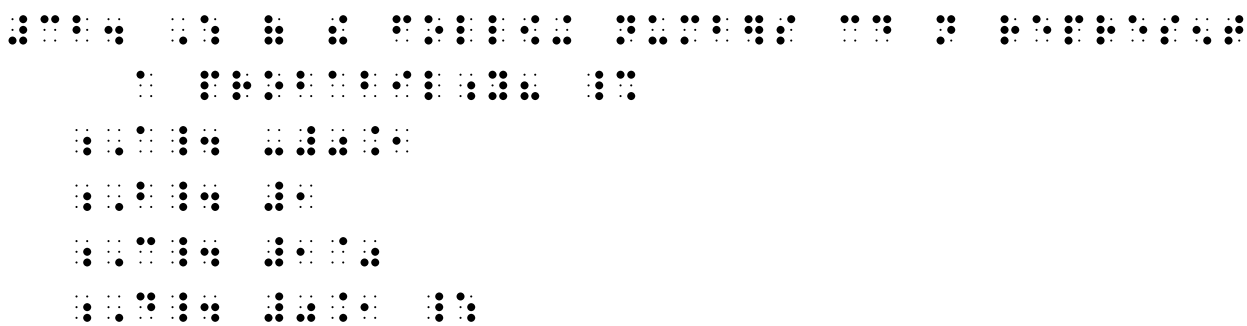


The opening Nemeth Code indicator falls in the runover cell (cell 5) of the main entry item. Each subdivision starts in cell 3 even though they are printed side by side.

Example 6-6

32. Which of the following numbers could not represent a probability?

   A. -0.1 B. 1 C. 1% D. 0.1



Notice that, although the numeral "1" (in subdivision B) can be transcribed in either code, code-switching clutter is avoided by staying in Nemeth.

PRACTICE 6A

23. Simplify and solve each equation below for x. Show your work and check your answer.

    a. $24 = 3x + 3$ b. $2(x - 6) = x - 14$

    c. $6 + 2.5x = 21$ d. $2(x + 4.5) = 32$

6.1.3 **Tabular Format.** When itemized material is arranged in tabular form so that items are numbered at the margin and subdivisions are aligned beneath lettered column headings, the material should be transcribed in one of the following ways, depending upon whether all of the columns can be accommodated across the braille page.

a. **When to Retain Column Format.** If all the columns can be accommodated across the braille page, the print columnar arrangement is followed. Each problem number begins in cell 1. The letter identifying each column is aligned with the first cell of the related column. A blank line is left above and below the lettered column headings. Two blank cells separate the columns. Guide dots are not used.

Example 6-7

	a	b	c
1.	$16 + 9$	$17 + 4$	$14 + 23$
2.	$46 + 15$	$87 + 12$	$95 + 54$
3.	$157 + 452$	$134 + 63$	$458 + 12$

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b. **When Not To Retain Column Format.** If all the columns cannot be accommodated across the braille page, each subdivision in each problem must be lettered individually and the format in Section 6.1 must be followed.

Example 6-8

	a	b	c	d
1.	$16+\ 9$	$17+\ 4$	$14+\ 23$	$37+\ 18$
2.	$46+\ 15$	$87+\ 12$	$95+\ 54$	$101+43$
3.	$157+\ 452$	$134+63$	$458+12$	$935+\ 298$

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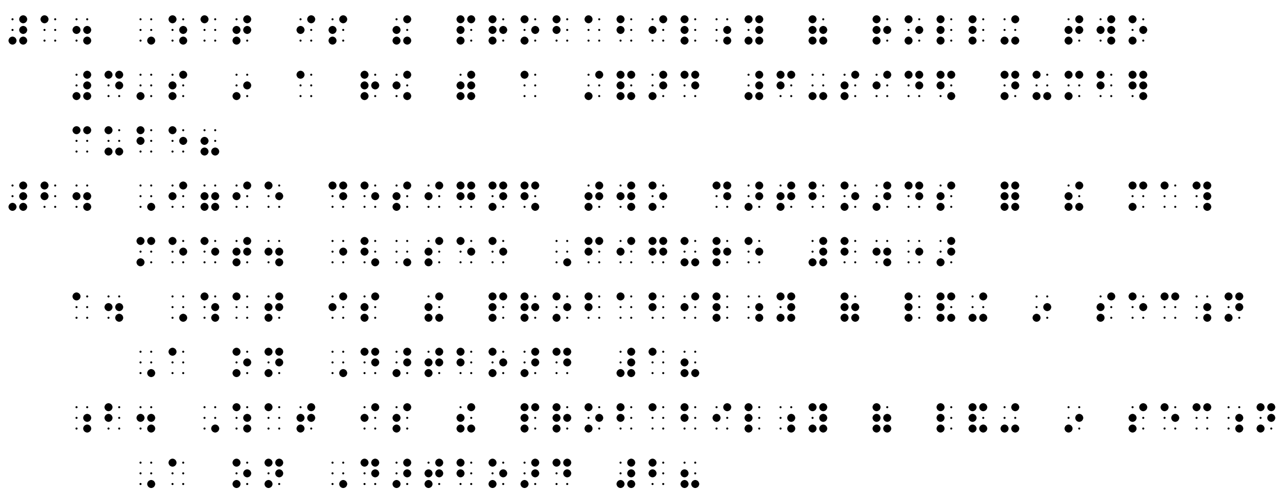
6.1.4 **Varied Margins.** Runover margins for itemized material are determined individually for each item. That is, an item with no subdivisions will be (1-3); the next item may have subdivisions and so will be (1-5; 3-5), etc.

Example 6-9

1. What is the probability of rolling two 4's in a row with a standard 6-sided number cube?
2. Iggie designed two dartboards for the math meet. (See Figure 2.)
   a. What is the probability of landing in section A on Dartboard 1?
   b. What is the probability of landing in section A on Dartboard 2?



LEVEL INDICATORS

[NC Rule 14]

6.2 Definition

A mathematical expression can contain symbols placed above or below the baseline level (the normal line of type in print). Superscripts appear above the baseline; subscripts appear below the baseline. Here are some expressions which contain superscripts and subscripts. The baseline is marked for orientation.

baseline 10^5 H_2O -4 °C πr^2 ${}^{235}_{92}U$ _baseline_

The font size is increased for most examples in this lesson to help in determining the levels.

Superscripts

6.3 Superscript Level Indicator

In braille, indicators are used to identify the level of a superscript or subscript. The superscript level indicator is used to show that the symbols immediately following it appear on the first level above the baseline of writing.

⠘	Superscript Indicator

⫸ 10^3 ⠼⠂⠴⠘⠒

⫸ $y = [g(x)]^n$ ⠽ ⠨⠅ ⠈⠷⠛⠷⠭⠾⠈⠾⠘⠝

The characters in the superscript are spaced according to the rules of the Nemeth Code.

⫸ $3^{-0.05T}$ ⠼⠒⠘⠤⠴⠨⠴⠢⠠⠞

A letter or a Roman numeral with a superscript does not need an English-letter indicator because it is not followed by a space.

⫸ x^3 ⠭⠘⠒

Example 6-10

Statements $\mathrm{i}^2$and ii^2.

⠠⠌⠁⠞⠑⠰⠞⠎ ⠸⠩ ⠊⠘⠆ ⠠⠄⠯ ⠊⠊⠘⠆ ⠸⠱⠲

Example 6-11

Theorems I^2 and V^2.

6.3.1 **Abbreviations with Superscripts**

a. When a superscript applies to an abbreviation, the level indicator is unspaced from the abbreviation.

⫸ 19 in.^3

b. The English-letter indicator is used for a single-letter abbreviation, even when unspaced from an indicator.

⫸ 5.5 m^2

c. Inside the switches, an abbreviation with a superscript is punctuated in mathematical mode.

⫸ 4 cm^3,

Instructions: Duplicate the side-by-side column format shown in print. Begin with the opening switch indicator in cell 1, followed by the blank line which is required before the list. A blank line is also required after the list. The Nemeth Code terminator will fall by itself on the next line, before beginning the paragraph text.

PRACTICE 6B

$6x^2$	$24^?$	x^{2a}
π^2	$(y-k)^3$	8^{10}
n^m	$A^{(k+1)+m}$	$q^{b(w-1)}$
$k^{0.27}$	$e^{3,000}$	e^-

Measurements of **length** are measurements in **one dimension.** They are labeled as cm, ft, m, etc. Measurements of **area** are measurements in **two dimensions.** They are labeled as cm^2, ft^2, m^2, etc. Measurements of **volume** are measurements in **three dimensions.** They are labeled as cm^3, ft^3, m^3, etc.

6.4 Returning to the Baseline Level

When a superscript is the last item in the expression, the following circumstances bring the reader back to the baseline level: a space, a comma followed by a space, or a punctuation indicator. "A space" includes the space before a Nemeth Code terminator and/or transition to a new braille line.

a. Space. A space returns the reader to the baseline.

⫸ 10^2 10^3 10^4

⫸ 10^2 and 10^3

Example 6-12

Which term of $(a+b)^{12}$ has the factor b^5 ?

Example 6-13

5.3×10^{-3} 5.3×10^{-1} 5.3×10^{0} 5.3×10^{3}

Example 6-14

Adding cubic meters: $2\ \mathrm{m}^3 + 2\ \mathrm{m}^3 = 4\ \mathrm{m}^3$

The superscript indicator is unspaced from the abbreviation. The required space which follows the abbreviation (and its superscript) returns the reader to the baseline.

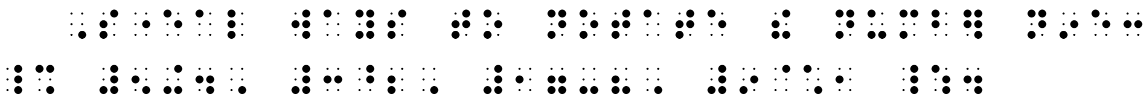
b. Comma and Space. A comma followed by a space returns the reader to the baseline.

Example 6-15

Which is the largest area? $3\ \mathrm{cm}^2$, $7\ \mathrm{m}^2$, $9\ \mathrm{ft.}^2$

Example 6-16

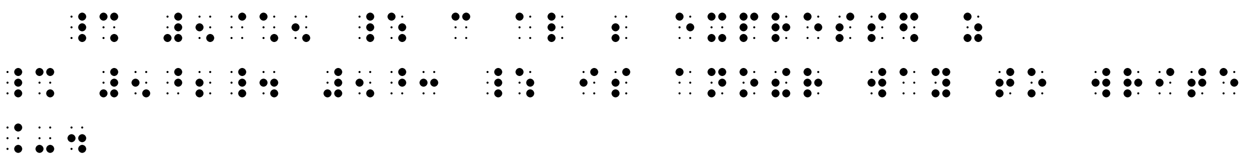
Several ways to notate the number nine: $5 + 4$, 3^2, $17 - 8$, 9×1.



c. Punctuation Indicator. A punctuation indicator returns the reader to the baseline.

Example 6-17

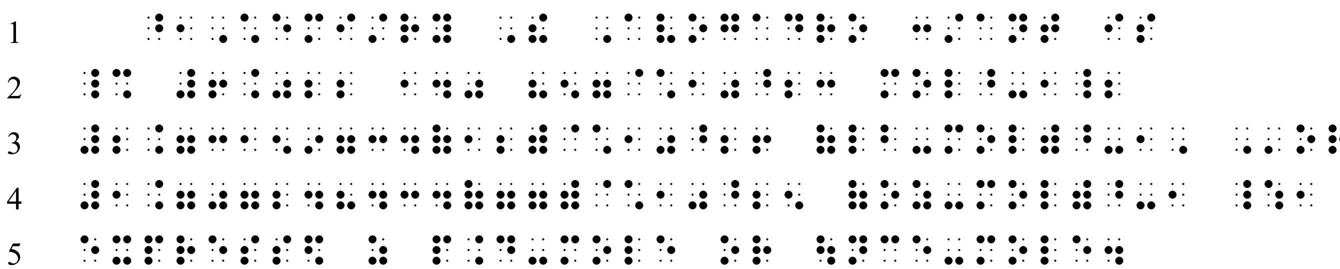
5×5 can also be expressed as 5^2. 5^3 is another way to write ____.



- Notice how the baseline level is re-established after each of the six superscripts in the next example.

Example 6-18

Chemistry The Avogadro constant is $6.022\ 140\ 857 \times 10^{23}$ mol^{-1} ;
$2.73159734(12) \times 10^{26}$(lb-mol)$^{-1}$, or $1.707248434(77) \times 10^{25}$(oz-mol)$^{-1}$,
expressed as pound-mole or ounce-mole.



The baseline is re-established as follows:

Lines 2, 3, and 4: the space before each abbreviation (mol, (lb-mol), and (oz-mol));
Line 2: the punctuation indicator before the semicolon;
Line 3: the math comma and the following space ;
Line 4: the space before the Nemeth Code terminator.

6.5 Raised Hollow Dot

The hollow dot in the superscript position may be used to represent degrees of angle or degrees of temperature. Transcription of the "degrees" symbol requires a switch to Nemeth.

⠨⠡	Hollow Dot	∘
⠘⠨⠡	Raised Hollow Dot (representing degrees)	

Example 6-19

A right triangle contains one 90° angle.

⠠⠁ ⠐⠗ ⠞⠗⠊⠁⠝⠛⠇⠑ ⠒⠞⠁⠔⠎ ⠐⠕ ⠸⠩ ⠼⠔⠴⠘⠨⠡ ⠸⠱
⠁⠝⠛⠇⠑⠲

Example 6-20

Average normal body temperature is 98.6° F.

⠠⠁⠧⠻⠁⠛⠑ ⠝⠕⠗⠍⠁⠇ ⠃⠕⠙⠽ ⠞⠑⠍⠏⠻⠁⠞⠥⠗⠑ ⠊⠎
⠸⠩ ⠼⠔⠦⠨⠖⠘⠨⠡ ⠰⠠⠋⠲ ⠸⠱

Recall from Lesson 3, Section 3.4, that an abbreviation is transcribed in the same code as its associated number, and that, in an isolated problem where there are no context clues to determine whether a period applies to the abbreviation or merely ends the sentence, to assume that it applies to the abbreviation.

PRACTICE 6C

1. Tell what number each of the following represents: 6^2, 5^{-3}, and $(3.15)^4$.
2. Which expression denotes the area of a circle?

   a. 360° b. πr^2 c. πd
3. The symbol for "feet squared" is ft^2. $1\ \mathrm{ft} \times 1\ \mathrm{ft} = 1\ \mathrm{ft}^2$
4. 6.02×10^{23} is an approximation of "Avogadro's number".
5. $144\ \mathrm{ft}^2 + 144\ \mathrm{ft}^2 = \ldots$
6. Convert 100° F to degrees Celsius.

Introduction to the Baseline Indicator

6.6 Function of the Baseline Indicator

In an unspaced expression, a return to the baseline level is brought about by the use of the baseline indicator. Notice that the baseline indicator is the same symbol as the multipurpose indicator—dot 5. The indicator's function is understood in context.

⠐	Baseline Indicator

⫸ $a^2 + b^2 + c^2$ ⠁⠘⠆⠐⠬⠃⠘⠆⠐⠬⠉⠘⠆

Example 6-21

The Pythagorean equation relates the side lengths of a right triangle as $a^2 + b^2 = c^2$.

⠠⠮ ⠠⠏⠽⠹⠁⠛⠕⠗⠂⠝ ⠑⠟⠥⠁⠰⠝ ⠗⠑⠇⠁⠞⠑⠎ ⠮ ⠎⠊⠙⠑
⠇⠢⠛⠹⠎ ⠷ ⠁ ⠐⠗ ⠞⠗⠊⠁⠝⠛⠇⠑ ⠵
⠸⠩ ⠁⠘⠆⠐⠬⠃⠘⠆ ⠨⠅ ⠉⠘⠆ ⠸⠱⠲

The baseline is re-established before each plus sign.

Example 6-22

Draw a 30°-60°-90° triangle.

⠠⠙⠗⠁⠺ ⠁ ⠸⠩ ⠼⠒⠴⠘⠨⠡⠐⠤⠖⠴⠘⠨⠡⠐⠤⠔⠴⠘⠨⠡ ⠸⠱
⠞⠗⠊⠁⠝⠛⠇⠑⠲

The baseline is re-established before each hyphen.

Example 6-23

Solve. $(x^2 + y^2) - (x^2 + y^2)$

⠠⠎⠕⠇⠧⠑⠲ ⠸⠩ ⠷⠭⠘⠆⠐⠬⠽⠘⠆⠐⠾⠤⠷⠭⠘⠆⠐⠬⠽⠘⠆⠐⠾ ⠸⠱

The baseline is re-established before each plus sign and before each right parenthesis.

6.6.1 **Degrees Fahrenheit and Degrees Celsius.** The abbreviations F (Fahrenheit) and C (Celsius) are given special consideration. Follow print spacing when these abbreviations are printed with a degree sign (raised hollow dot). A baseline indicator will be required when the degree sign is unspaced from the letter. Note that, when the letter stands alone, an English-letter indicator is required, but when the letter is unspaced from the raised hollow dot, an English-letter indicator is no longer needed. F and C are punctuated in literary mode when spaced away from the degree sign. When the symbols are unspaced, the combination is mathematical and so mathematical punctuation is applied.

⪢ 98.6° F, 99° F

⪢ 20 °C, 30 °C

Example 6-24

Expressed in degrees Celsius, normal human body temperature is 37 °C.

Example 6-25

Express normal human body temperature in °C; in °F.

PRACTICE 6D

(1) $4(x-y)^3-2(x-y)^3$

(2) $3a^3b+6a^6b^2+9a^9b^3$

(3) Convert 10° F to °C. Express 10 °C in °F.

(4) $V=2\pi^2Rr^2$

(5) x^n-dimensional system

(6) $6^2\times 6^3=6^{2+3}=6^5$

(7) $2(36x^2-1)=2(6x)^2-1^2)$, which factors into $2(6x+1)(6x-1)$.

6.7 Higher Levels of Writing

Superscripts may carry superscripts of their own. In such cases, the superscript indicator is doubled, tripled, etc. to indicate superscripts on the second, third, or higher levels of writing.

⠘⠘	Superscript with Superscript (two levels above the baseline)
⠘⠘⠘	Superscript with Superscript with Superscript (three levels above the baseline)

⫸ n^{a^b} ⠝⠘⠁⠘⠘⠃

⫸ $n^{a+1^{b+1^{c+1}}}$ ⠝⠘⠁⠬⠂⠘⠘⠃⠬⠂⠘⠘⠘⠉⠬⠂

If the print copy is difficult to analyze, a straightedge and a magnifier will help to determine the levels. In general, the print font gets smaller as the level gets higher.

6.7.1 **Combinations.** Each level indicator is read as it relates to the baseline level. The effect of one level indicator is terminated by another level indicator. Keep this in mind as you read the following examples.

⫸ $a^{(m^n)}$ ⠁⠘⠷⠍⠘⠘⠝⠘⠾

The parentheses and the letter "m" are one level above the baseline; the letter "n" is two levels above the baseline.

⫸ $10^{(^-4)}$ ⠼⠂⠴⠘⠷⠘⠘⠤⠘⠲⠾

The parentheses and the numeral "4" are one level above the baseline; the minus sign is two levels above the baseline.

Example 6-26

Will $E[x^2] = e^{2(a+b^2)}$?

⠠⠺ ⠸⠩ ⠠⠑⠈⠷⠭⠘⠆⠐⠈⠾ ⠨⠅ ⠑⠘⠆⠷⠁⠬⠃⠘⠘⠆⠘⠾ ⠸⠱⠦

The brackets are on the baseline level. The second superscript is $2(a+b^2)$ *which means that the final "2" is at the supersuperscript level.*

Example 6-27

True or false? $e^{1,000} \times e^{2,000} = e^{3,000}$

When a space brings the reader back to the baseline, a baseline indicator is not needed.

6.8 Certain Raised Signs

Some signs are printed in a raised position but are not considered to be superscripts.

6.8.1 **Raised Ordinal.** *Braille Formats* states that raised ordinal endings are not considered to be superscript. This guideline is also followed in a technical transcription.

Example 6-28

Name the 3rd and 4th item in the series.

The ordinal endings are printed in the raised position.

Example 6-29

What is the $3n^{th}$ degree?

"th" is printed in the raised position.

6.8.2 **Prime Sign.** The prime sign appears to be raised in print but it assumes the same level as the number or letter to which it applies.

	Prime Sign	'

⋙ m'^{2}

The prime sign belongs with the letter m. Only the number 2 is in the superscript position.

⋙ $A^{2''}$

The prime signs belong with the number 2 in the superscript.

Example 6-30

plane angle $\alpha = 30°2'8''$

⠏⠇⠁⠝⠑ ⠁⠝⠛⠇⠑ ⠸⠩ ⠨⠁ ⠨⠅ ⠼⠒⠴⠘⠨⠡⠐⠆⠄⠦⠄⠄ ⠸⠱

The prime signs are not superscripts. They apply to numbers at the baseline of writing.

6.8.3 **Apostrophe-s.** In an apostrophe-s ending, the apostrophe is at the same level as the "s". Because a punctuation indicator returns the reader to the baseline, a level indicator is inserted to maintain the level of the apostrophe-s.

⫸ $A^{m+m+m's}$ ⠠⠁⠘⠍⠬⠍⠬⠍⠘⠸⠄⠎

The apostrophe-s applies to m+m+m.

Compare to this example where the "s" and its apostrophe are printed on the baseline level.

⫸ A^{m+m+m}'s ⠠⠁⠘⠍⠬⠍⠬⠍⠸⠄⠎

The apostrophe-s applies to the entire expression A^{m+m+m}.

PRACTICE 6E

1. Use a calculator to find 9^{9^9}.
2. Find the r^{th} term of $(x+y)^n$.
3. Label the x^2's and x^3's.
4. What is the meaning of x''^3?
5. Simplify: $(x^3 - y^3)^2 - (x^3 + y^3)^2$.
6. $x^{y^n z}$ or $x^{y^2 z}$

Subscripts

6.9 Subscript Level Indicators

Except as stated in Section 6.11 below, the subscript level indicator is used to show that the symbols immediately following it appear on the first level below the baseline of writing. Note that the subscript indicator is the same symbol as the English-letter indicator—dots 56. The indicator's function is understood in context.

⠰	Subscript Indicator

⫸ f_n ⠋⠰⠝

⫸ $a_{(k+1)}$ ⠁⠰⠷⠅⠬⠂⠾

Subscripts may carry subscripts of their own. In such cases, the subscript level indicator is doubled, tripled, etc. to indicate subscripts on the second, third, or lower levels.

⠰⠰	Subscript with Subscript (two levels below the baseline)
⠰⠰⠰	Subscript with Subscript with Subscript (three levels below the baseline)

⫸ n_{x_y} ⠝⠰⠭⠰⠰⠽

⫸ $P_{x+1_{y+1_{z+1}}}$ ⠠⠏⠰⠭⠬⠂⠰⠰⠽⠬⠂⠰⠰⠰⠵⠬⠂

Note: The -?- in the second column is in the subscript position. The subscript of the second item in the fourth column is "minus two".

PRACTICE 6F

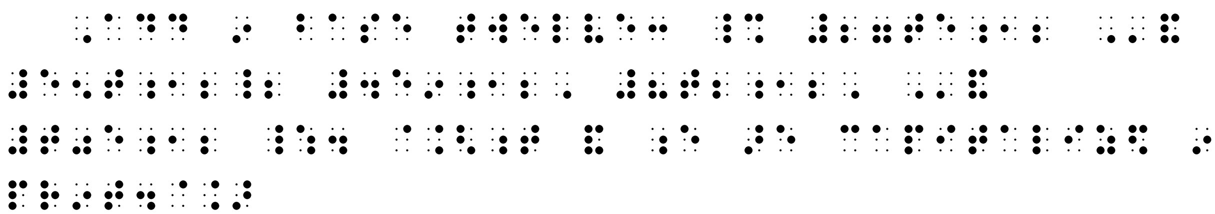
3_c	10_8	$?_3$	x_{2+k}
3_{five}	$10_{\text{-?-}}$	Ca(OH)_2	y_{-2}
y_{n_k}	P_{3n}	a_{m1}	a_{m_1}

6.10 Returning to the Baseline Level

The same circumstances discussed with superscripts bring the reader back to the baseline level from a subscript: a space, a comma followed by a space, a punctuation indicator, or a baseline indicator. "A space" includes the space before a Nemeth Code terminator and/or transition to a new braille line.

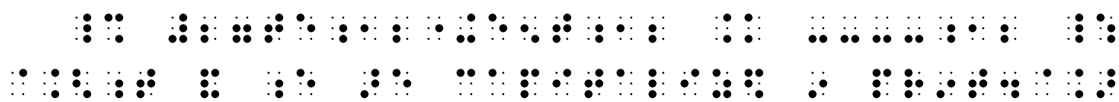
Example 6-31

Add in base twelve: $27TE_{12}$ and $E5T_{12}$; $4E9_{12}$, $8T2_{12}$, and $T0E_{12}$.



Example 6-32

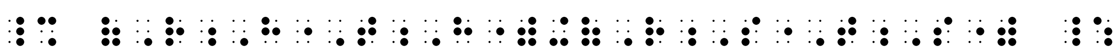
$27TE_{12} + E5T_{12} = \underline{\qquad}_{12}$



Note that the level indicator is unspaced from the long dash to which it applies.

Example 6-33

$(R_H T_H) + (R_S T_S)$



Letters H and S are subscripts. Because H and S are not "single letters" (they are unspaced), an English-letter indicator is not needed. Dots (56) is functioning as a subscript indicator throughout this expression.

PRACTICE 6G

1. Name the numeral in base ten equal to:

   a. 47_8 b. 34_6 c. 1101_2

2. $\mathrm{C}_{\mathrm{hex}} = 12_{\mathrm{dec}} = 14_{\mathrm{oct}}$

3. What do we know if $P_{n_r} = (x_{n_k}, y_{n_k})$?

4. $7_8 - 4_8$

6.11 Special Case—Nonuse of the Subscript Level Indicator

A special rule applies to certain subscripts. The subscript level indicator is not used when all of the following apply:

i. the subscript is *numeric*;

ii. the numeral is a *right subscript to a letter;*

iii. the subscript is on the *first level* below the baseline of writing.

Example 6-34

Apply Definitions i_2 and I_2 .

Each numeral 2 is printed at the subscript level.

a. When a subscript does not require a subscript indicator, a return to the baseline is implied following the numeric subscript. A baseline indicator is not needed.

⋙ $\mathrm{H}_2\mathrm{O}$

Letters H and O are on the baseline of writing. Numeral 2 is a subscript.

⋙ $a_1a_2 + b_1b_2 + c_1c_2$

All numerals are subscripts. A baseline indicator is not needed to show that the letters and the plus signs are on the baseline level because no subscript indicators are used.

⫸ $Cd(NO_3)_2$ ⠠⠉⠙⠷⠠⠝⠠⠕⠒⠾⠰⠆

The subscripts are numerals 3 and 2. Everything else is on the baseline of writing, including the parentheses. The second subscript requires a level indicator because it is not a right subscript to a letter.

6.11.1 **Further Conditions.** The definition of "numeric" and "letter" include the following.

a. **Numeric Subscript.** The numeric subscript may contain a segmenting comma or a decimal point.

⫸ $x_{1,000}$ ⠭⠂⠠⠴⠴⠴

The numeral 1,000 is a right subscript to the letter x.

⫸ $x_{5.3}$ ⠭⠢⠨⠒

The numeral 5.3 is a right subscript to the letter x.

b. **"Letter"**

(1) The letter may carry one or more primes.

⫸ x'_2 ⠭⠄⠆

The numeral 2 is a right subscript to x'.

(2) The letter may be taken from any alphabet.

⫸ $\aleph_0$ ⠠⠠⠁⠴

Hebrew aleph, subscript zero.

(3) The letter may also carry a superscript. (See <u>6.17</u>, *Simultaneous Superscripts and Subscripts*)

(4) The letter may be part of a two-letter chemical symbol.

⫸ Cl_4 ⠠⠉⠇⠲

Cl (the two-letter chemical symbol for chlorine), subscript 4.

c. **Not a Subscript.** When a number is printed on the baseline to the right of a letter, unspaced, a multipurpose indicator (dot 5) is inserted between two signs to indicate that they are printed horizontally, and that the number is not a subscript.

⫸ F1 ⠠⠋⠐⠂

6.11.2 **Restrictions.** Just because a numeral is a right subscript to a letter does not mean that the special rule can be applied. The subscript level indicator must be used if any of the following conditions apply:

(1) If the letter is functioning as a numeral in a nondecimal numeration system, a subscript indicator is required.

⋙ TE_{12} ⠼⠞⠑⠰⠂⠆

The subscript 12 means base-12 numeration.

(2) If the letter is part of a word or abbreviation, a subscript indicator is required.

⋙ five_3 ⠋⠊⠧⠑⠰⠒

(3) If the subscript contains any symbol other than a numeral with its comma or decimal point, a subscript indicator is required.

⋙ x_{2k} ⠭⠰⠆⠅

⋙ $x_{2'}$ ⠭⠰⠆⠄

⋙ x_{-2} ⠭⠰⠤⠆

(4) If the subscript carries a superscript or subscript of its own, a subscript indicator is required.

⋙ x_{2_n} ⠭⠰⠆⠰⠰⠝

(5) Numeric subscripts on the second or lower levels always require their appropriate subscript level indicators.

⋙ P_{n_1} ⠠⠏⠰⠝⠰⠰⠂

6.11.3 **Summary.** The rules regarding use/nonuse of the subscript indicator are summarized below.

a. A subscript level indicator <u>is not used</u> before a numeric subscript on the first level below the baseline of writing when the numeral is a right subscript to a letter. Furthermore, the following may also be true.

- The numeric subscript may contain a segmenting comma or a decimal point.
- The baseline letter may carry one or more primes or a superscript.
- The baseline letter may be from any alphabet.
- The baseline "letter" may be a two-letter chemical element.

b. A subscript level indicator <u>must be used</u> in the following circumstances.

- A numeral on the first level below the baseline requires a subscript indicator if the subscript contains any symbol other than a numeral with its comma or decimal point.
- A numeral on the first level below the baseline requires a subscript indicator if the subscript carries a superscript or subscript of its own.
- A subscript on the second or lower level always requires the appropriate subscript level indicator.
- If the subscript is a letter that is part of a word or an abbreviation, a subscript indicator is required.
- If the subscript is a letter functioning as a numeral in a nondecimal numeration system, a subscript indicator is required.

PRACTICE 6H

1) These expressions need subscript indicators: y_{-2}, x_{2+k}, a_{m1}, x_{3_n}, x_{y_2}.
2) These expressions do <u>not</u> need a subscript indicator: x_1, ax_2, CO_2, $z_{.4}$, β_2.
3) Decide whether these expressions require a subscript indicator and transcribe them correctly: shape_4, Q'_2, $\mathrm{C_6Fe_2O_{12}}$, n_k, x_{2k}, $P_{r_{s_t}}$, D_{56}, $\mathrm{G}_{9,999}$, and the hexadecimal number $2\mathrm{E}6\mathrm{B}_{16}$.
4) **Chemistry.** While $\mathrm{Na_2ZnCl_4}$ could be cooled in the normal way, $\mathrm{Na_2[CoCl_4]}$ had to be quenched in the liquid $\mathrm{N_2}$.
5) $f_1(x) = g(x) \cdot q_2(x) + f_2(x)$

6.12 Spaces Within Superscripts and Subscripts

A space usually returns the reader to the baseline. Various strategies are used to retain the level in effect when a space occurs within a superscript or a subscript.

6.12.1 **Commas.** *Review:* A comma followed by a space re-establishes the baseline. The return to the baseline starts at the comma. No baseline indicator is necessary.

⫸ x^2, y^2, z^2 ⠭⠘⠆⠠ ⠽⠘⠆⠠ ⠵⠘⠆

⫸ (a_1, a_{1i}, a_{2i}) ⠷⠁⠂⠠ ⠁⠰⠂⠊⠠ ⠁⠰⠆⠊⠐⠾

Example 6-35

Add: 2_{five} , 3_{five} , and 4_{five} .

⠠⠁⠙⠙⠒ ⠸⠩ ⠼⠆⠰⠋⠊⠧⠑⠠ ⠼⠒⠰⠋⠊⠧⠑⠠ ⠠⠄⠯
⠼⠲⠰⠋⠊⠧⠑ ⠸⠱⠲

The commas are on the baseline of writing. The mathematical comma is transcribed even though these subscripts are words.

a. **The Contracted Comma.** When a comma separates individual items within a superscript or subscript, the contracted comma symbol is used. This symbol represents the comma or the comma and a space. The level in effect continues through this symbol.

⠪	`Contracted Comma plus Optional Space`	`,`

⋙ $x^{1,\,2}$ ⠭⠘⠂⠪⠆

⋙ $x_{i,\,j,\,k}$ ⠭⠰⠊⠪⠚⠪⠅

The contracted comma symbol is not to be used to replace a comma on the baseline of writing. A dot 2 or dot 6 comma is used for the comma that is part of the sentence structure.

Example 6-36

If ______, then $x_{i,\,j,\,k}$, as shown below.

⠠⠊⠋ ⠨⠤⠂ ⠮⠝ ⠸⠩ ⠭⠰⠊⠪⠚⠪⠅ ⠸⠱⠂ ⠵ ⠩⠪⠝ ⠆⠇⠲

Contracted commas are used within the subscript; a dot 2 comma is used for the comma that is part of the sentence structure (on the baseline level) outside of the Nemeth switches.

Example 6-37

Consider $x_{i,\,j}$, $x_{j,\,k}$, and $x_{i,\,k}$.

⠠⠒⠎⠊⠙⠻ ⠸⠩ ⠭⠰⠊⠪⠚⠠ ⠭⠰⠚⠪⠅⠠ ⠠⠄⠯ ⠭⠰⠊⠪⠅ ⠸⠱⠲

Contracted commas are used within each subscript; a dot 6 comma is used for the commas that are part of the sentence structure (on the baseline level) within the Nemeth switches.

In the next example, the commas separate three terms at the subscript level, but each "n" has a subscript as well. The correct level of each comma is shown by shrewd use of level indicators.

> P_{n_1, n_2, n_3}

6.12.2 **Words, Abbreviations, and Letters.** If a subscript or a superscript contains a space between words, abbreviations, or letters, the level must be restated after the space. The level indicator is unspaced from the following word, abbreviation, or letter.

> $n_{\text{obtuse angles}}$

The subscript indicator before the word "angles" shows that the word is in the subscript position.

> $n_{\text{st. angles}}$

The abbreviation st. (for "straight") has a related period. No punctuation indicator is used with abbreviations.

Note that an English letter will not need an English-letter indicator in such a case because it is no longer a "single letter" as defined by the Nemeth Code. (See Section 3.10.1 in Lesson 3.) Using "surface S" as an example, as a superscript it is transcribed . As a subscript it is transcribed .

6.12.3 **Comparison Signs.** The space before a comparison sign returns the reader to the baseline, as illustrated in the next example.

> $a^2 = a \cdot a$

The superscript is 2. The equals sign is on the baseline.

If the comparison sign is within a superscript or a subscript, the level is reinstated before the comparison sign. The indicator is unspaced from the comparison sign. The space after a comparison symbol preserves the level that is already in effect.

> $S_{u = a}$

The subscript is u = a. The level in effect extends through the space following the comparison sign.

6.12.4 **Ellipsis or Long Dash in Superscripts and Subscripts**

a. **New Level.** When an ellipsis or a long dash is located at a different level from the material preceding it, the appropriate level indicator is used before the ellipsis or long dash. The level indicator is unspaced from the symbol.

Example 6-38

Fill in the blank with a simplified superscript. $10^{6+3} = 10^{-}$

b. **Same Level.** An ellipsis or a long dash <u>within</u> a superscript or a subscript assumes the level in effect. The level extends through the required spaces before and after the ellipsis or long dash.

⋙ $P_{n_1,\, n_2,\ \ldots}$

The letter n, the commas, and the ellipsis are at the subscript level. Even though a contracted comma includes the following space, there must be a space before an ellipsis unless it is immediately preceded by an indicator. The subscript level applied to the second comma extends through the space before the ellipsis.

⋙ $x^{1\,+\,3\,+\,5\,+\ \ldots\ +\,(2n-1)}$

The superscript is $1 + 3 + 5 + \ \ldots + (2n - 1)$. *The superscript level extends through the spaces before and after the ellipsis.*

c. **Exception.** If an ellipsis or long dash in a superscript or subscript is followed by a sign of comparison or by literary text, reading returns to the baseline level without the need for a baseline indicator.

⋙ $10^{7+—} = 10^{21}$

The first superscript is 7+___. The superscript level extends through the required space before the long dash. The sign of comparison after the dash is at the baseline level.

⋙ $10^{7+—}$ equals 10^{21}

The first superscript is 7+___. The superscript level extends through the required space before the long dash. The word after the dash is at the baseline level.

6.12.5 **Ellipsis or Long Dash on the Baseline of Writing.** Because the space before an ellipsis or long dash maintains the level in effect, an indicator is required to return to the baseline when the symbol is printed there. The baseline indicator takes the place of the required space.

⫸ $a^1 b^2 c^3 d^4 \ldots z^n$

⠁⠘⠂⠐⠃⠘⠆⠐⠉⠘⠒⠐⠙⠘⠲⠐⠄⠄⠄ ⠵⠘⠝

There is no need to indicate a return to the baseline after a numeric subscript that does not require a subscript indicator.

⫸ $r_1 \ldots r_n$ ⠗⠂ ⠄⠄⠄ ⠗⠰⠝

The ellipsis is printed on the baseline. The subscripts are 1 and n.

6.12.6 **Segmented Numbers.** The effect of a level indicator extends through the space inserted in a numeral for the purpose of dividing it into short regular groups of digits.

⫸ $e^{3.14159\ 26535}$ ⠑⠘⠒⠨⠂⠲⠂⠢⠔ ⠆⠖⠢⠒⠢

The superscript is 3.14159 26535

Instructions: Translate the simulated braille by writing the mathematical characters in the blank space below each line. Show spaces and levels clearly. Compare your translation to the print copy in the answer section.

PRACTICE 6I

⠸⠩

⠼⠂⠸⠲ ⠭⠰⠂⠪⠆ ⠌⠨⠅ ⠭⠘⠊⠪⠚

⠼⠆⠸⠲ ⠭⠰⠝⠤⠂⠪⠝⠤⠂⠠ ⠭⠰⠝⠤⠂⠪⠝⠠ ⠭⠰⠝⠪⠝⠤⠂

⠼⠒⠸⠲ ⠠⠁⠘⠝⠬⠝⠬⠝ ⠘⠁⠇⠇ ⠘⠝⠘⠸⠄⠎ ⠸⠱ ⠜⠑ ⠑⠟⠥⠁⠇⠲

⠼⠙⠲ ⠸⠩ ⠎⠈⠾⠰⠺ ⠰⠸⠅ ⠁

⠼⠢⠸⠲ ⠑⠘⠂⠠⠴⠴⠴

⠼⠖⠸⠲ ⠁⠘⠍⠬⠅⠐⠨⠌⠁⠘⠍ ⠨⠅ ⠁⠘⠅

⠼⠶⠸⠲ ⠠⠏⠰⠎⠰⠰⠂⠰⠄⠄⠄ ⠎⠰⠰⠆ ⠠⠄⠯ ⠠⠏⠰⠟⠰⠰⠗⠪⠎

⠼⠦⠸⠲ ⠼⠂⠴⠰⠿ ⠨⠅ ⠼⠖⠰⠦

⠼⠔⠸⠲ ⠁⠄⠂⠠ ⠁⠄⠆⠠ ⠄⠄⠄⠠ ⠁⠄⠰⠝ ⠸⠱ ⠜⠑ ⠮ ⠔⠧⠻⠎⠑⠎⠲

⠼⠁⠚⠲ ⠸⠩ ⠠⠠⠁⠴ ⠸⠱ ⠗⠑⠏⠗⠑⠎⠢⠞⠎ ⠮ ⠉⠜⠙⠔⠁⠇⠰⠽ ⠷ ⠮
⠀⠀⠎⠑⠞ ⠶ ⠁⠇⠇ ⠝⠁⠞⠥⠗⠁⠇ ⠝⠥⠍⠃⠻⠎⠲

More about Superscripts and Subscripts

6.13 Superscript and Subscript Combinations

Combinations of subscripts to superscripts or of superscripts to subscripts require level indicators composed of two or more braille symbols. Keeping in mind that level indicators relate to the baseline, transcribing these expressions is an exercise in logical thinking.

Superscripts with Subscripts

⠘⠰	Superscript With Subscript

⫸ x^{n_1} ⠭⠘⠝⠘⠰⠂

"n, subscript one" is in the superscript position. "1" is a supersubscript.

⫸ $2^{Y_0} = Y_1$ ⠼⠆⠘⠠⠽⠘⠰⠴ ⠨⠅ ⠠⠽⠂

"Y, subscript zero" is in the superscript position. "0" is a supersubscript.

Reminder: The subscript indicator is omitted for a numeric subscript to a letter only for subscripts that are located on the first level below the baseline of writing. The super/subscript indicator is needed to show a numeric subscript in the superscript position.

Subscripts with Superscripts

⠰⠘	Subscript With Superscript

⫸ x_{n^2} ⠭⠰⠝⠰⠘⠆

"n, superscript two" is in the subscript position. "2" is a subsuperscript.

⫸ P_{3^n} ⠠⠏⠰⠒⠰⠘⠝

"3, superscript n" is in the subscript position. "n" is a subsuperscript.

Reminder: The subscript indicator is required when a numeric subscript to a letter carries a superscript or subscript of its own.

Instructions: Format the bold type as cell-5 headings. Descriptions are included in a right-hand column for your benefit—do not transcribe the descriptions. Analyze the levels as you proofread.

PRACTICE 6J

Superscripts with Subscripts

$2^{\aleph_0} = \aleph_1$	$\aleph_0$ is in the superscript position.
$a = 2^{k_1}$ and $b = 2^{k_2}$	k_1 and k_2 are in the superscript position.
$(ab)^x = 2^{k_1 x} \cdot 2^{k_2 x}$	x, $k_1 x$, and $k_2 x$ are superscripts.
$\mathrm{e}^{\mathrm{i}\theta_1}$ times $\mathrm{e}^{\mathrm{i}\theta_2}$ equals $\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}$	$\mathrm{i}\theta_1$, $\mathrm{i}\theta_2$, and $\mathrm{i}(\theta_1 + \theta_2)$ are superscripts.

Subscripts with Superscripts

Z_{5^n}	5^n is in the subscript position.
$7t_{s^4}$	s^4 is in the subscript position.

6.14 Left Subscripts and Superscripts

The appropriate level indicator is transcribed before a subscript or superscript printed to the left of its related sign.

⫸ $^{14}\mathrm{C}$ ⠘⠂⠲⠐⠠⠉

⫸ $_3\mathrm{x}_1$ ⠰⠒⠐⠭⠂

⫸ $_{48}\mathrm{C}_9 \times {}_4\mathrm{C}_4$ ⠰⠲⠦⠐⠠⠉⠔⠈⠡⠰⠲⠐⠠⠉⠲

The "9" and the second "4" are numeric subscripts to the right of a letter on the baseline of writing, so no subscript indicator is needed.

6.14.1 **Raised Negative Sign.** In some texts, negative numbers are shown with a raised negative sign. The raised position of the negative sign must be shown in braille. A numeric indicator is not required when the negative sign is raised.

⫸ $^-4$ ⠘⠤⠐⠲

Compare: A numeric indicator is required when the negative sign is not raised.

⫸ -4 ⠤⠼⠲

Example 6-39

Explain why $4 + {}^{-}4 = 0$.

⠠⠑⠭⠏⠇⠁⠔ ⠱⠽ ⠸⠩ ⠼⠲⠬⠘⠤⠐⠲ ⠨⠅ ⠼⠴ ⠸⠱⠲

Example 6-40

A and B are points with coordinates $(5, 2)$ and $({}^{-}1, 4)$ respectively.

⠠⠁ ⠯ ⠰⠠⠃ ⠜⠑ ⠏⠕⠔⠞⠎ ⠾ ⠉⠕⠕⠗⠙⠔⠁⠞⠑⠎
⠸⠩ ⠷⠢⠠ ⠆⠾ ⠠⠄⠯ ⠷⠘⠤⠐⠂⠠ ⠲⠾ ⠸⠱ ⠗⠑⠎⠏⠑⠉⠞⠊⠧⠑⠇⠽⠲

Compare the subscripts in the next two examples. The first subscript is –2. A subscript indicator is required because the subscript contains a symbol other than a numeral. In the second subscript, the negative sign is raised It is a left superscript to the subscript "2".

- x_{-2} ⠭⠰⠤⠆
- $x_{{}^{-}2}$ ⠭⠰⠘⠤⠰⠆

6.15 Further Combinations

The Nemeth code book illustrates additional combinations of superscripts and subscripts. Due to the obscurity of such complex combinations, only a few examples are shown in this lesson manual. Proper interpretation of these characters will require reading the surrounding text in order to apply the correct indicators. (Notice that the letter on the baseline of writing is often a larger font.)

- ${}_{{}_{y}x}n$

n is on the baseline of writing and has a left subscript. Question: Is the subscript y^{x} or ${}_{y}x$?

This transcription shows left subscript "y" with a subsuperscript "x".

⠰⠽⠰⠘⠭⠐⠝

This transcription shows left subscript "x" with a left subsubscript "y".

⠰⠰⠽⠰⠭⠐⠝

- ${}_{x_{y}}n$

n is on the baseline of writing and has a left subscript. Question: Is the subscript x_{y} or ${}^{x}y$?

This transcription shows left subscript "x" with a subsubscript "y".

⠰⠭⠰⠰⠽⠐⠝

This transcription shows left subscript "y" with a left subsuperscript "x".

⠰⠘⠭⠰⠽⠐⠝

6.16 Consecutive Superscripts and Subscripts

When a character with right superscript or subscript is adjacent to a character with a left superscript or subscript, each superscript or subscript has its own level indicator. A multipurpose indicator is inserted between the consecutive symbols. A space may appear in print to help visually differentiate the two super/subscripts, but no space comes between the symbols braille.

⋙ $p^{b}\,{}^{c}q$ ⠏⠘⠃⠐⠘⠉⠐⠟

(There is a small space between b and c in print.)

⋙ $P_{x}\,{}_{y}Q$ ⠠⠏⠰⠭⠐⠰⠽⠐⠠⠟

(There is a small space between x and y in print.)

⋙ $P_{1}\,{}_{2}Q$ ⠠⠏⠂⠐⠰⠆⠐⠠⠟

(There is a small space between 1 and 2 in print.) The first numeric subscript does not require a subscript indicator because it is a right subscript to a letter.

6.17 Simultaneous Superscripts and Subscripts

When a superscript and a subscript are printed directly above and below each other, the subscript is transcribed first.

⋙ 10_{8}^{3} ⠼⠂⠴⠰⠦⠘⠒

⋙ ${}_{92}^{238}\mathrm{U}$ ⠰⠔⠆⠘⠆⠒⠦⠐⠠⠥

Example 6-41

An example of an I^{-} iodine anion: ${}_{53}^{127}\mathrm{I}^{-}$.

⠠⠁⠝ ⠑⠭⠁⠍⠏⠇⠑ ⠷ ⠁⠝ ⠸⠩ ⠠⠊⠘⠤ ⠸⠱ ⠊⠕⠙⠔⠑
⠁⠝⠊⠕⠝⠒ ⠸⠩ ⠰⠢⠒⠘⠂⠆⠶⠐⠠⠊⠘⠤ ⠸⠱⠲

The subscript indicator is or is not transcribed, according to the rules presented in Section 6.11. Notice that simultaneous superscripts and subscripts printed next to a letter in italics may not align exactly because of the slanted aspect of the italic typeform.

⫸ x_1^2

The numeric subscript does not require a subscript indicator because it is a right subscript to a letter.

⫸ ${}_m^n x$

The subscript is transcribed first.

⫸ $n_{a_{b_c}}^{x^{y^z}}$

The entire subscript is transcribed before transcribing the superscript.

6.18 Nonsimultaneous Superscripts and Subscripts

When a mathematical expression carries both a superscript and a subscript which are not printed directly above and below each other, the superscript and subscript are transcribed in the same order as in print, and the baseline indicator is inserted between them.

⫸ ${x_1}^2$

⫸ ${a^k}_m$

⫸ ${}_y{}^x n$

⫸ ${}^b{}_a n$

A magnifier and a straightedge can help determine whether superscripts or subscripts are simultaneous or nonsimultaneous. If in doubt whether the expression shows nonsimultaneous super/subscripts or if, instead, the super/subscripts have super/subscripts of their own, compare to the surrounding text for clues.

6.19 Detached Superscripts and Subscripts

When an entire superscript or subscript stands alone, it is transcribed without a level indicator. A transcriber's note explains its print position.

Sample transcriber's note:

The 2 is raised in print.

Example 6-42

In x^2, the ^{2} is the exponent.

6.20 Literary Symbols and Level Indicators

If an ampersand, asterisk, crosshatch, dagger, double dagger, paragraph mark, or section mark is used mathematically and appears in a superscript or a subscript, its level must be indicated. The Nemeth symbols for these literary characters are listed in Lesson 5.

Example 6-43

A dagger may be used as a superscript in quantum mechanics, for example, $A^{\dagger}$.

⠀⠀⠠⠁ ⠙⠁⠶⠻ ⠍⠁⠽ ⠆ ⠥⠎⠫ ⠵ ⠁ ⠎⠥⠏⠻⠎⠉⠗⠊⠏⠞ ⠔
⠟⠥⠁⠝⠞⠥⠍ ⠍⠑⠡⠁⠝⠊⠉⠎⠂ ⠿ ⠑⠭⠁⠍⠏⠇⠑⠂
⠸⠩ ⠠⠁⠘⠸⠻ ⠸⠱⠲

Summary

Here is a summary of the difference in print among four types of superscripts and subscripts as defined in the Nemeth Code.

Consecutive A right super/subscript belongs to the preceding character and a left super/subscript belongs to the next character.

Simultaneous A character has both a superscript and a subscript. The superscript is printed directly above the subscript.

Nonsimultaneous A character has both a superscript and a subscript. The two scripts are not printed directly above and below each other.

Detached A super/subscript stands alone without being associated with a letter or number. It is printed slightly above or below the baseline and is smaller than the rest of the text.

Note: There is no space between the subscript and the superscript in item #3.

PRACTICE 6K

1. Here are some expressions with left superscripts: ${}^{3}x,\ {}^{n}x,$

   ${}^{-}2 + {}^{-}4 = {}^{-}6,\ ({}^{-}3)^{{}^{-}2\,+\,{}^{+}2}$.

2. ${}^{12}_{6}\mathrm{C}$ and ${}^{12}\mathrm{C}$ represent the same carbon isotope.
3. $\mathrm{D_2{}^{18}O}$ is the doubly labeled water isotopologue!
4. In $\mathrm{CO_2}$, the subscript ${}_{2}$ means "two oxygen atoms".
5. ${}_{n}P_{r} = K({}_{n-1}P_{r-1})$
6. $a_1^2 + b_1^2 + c_1^2$
7. $[t]_0^4$
8. $2 \times {10_6}^2 + 3 \times {10_6}^1 + 2$
9. $P_{x\,y}Q$
10. $\mathrm{NH_4^+ + Cl^- + H_2O}$

6.21 More About Grouping Symbols and Level Indicators

As noted throughout this lesson, when a grouping symbol appears on the baseline level and a level indicator is currently in effect, the baseline indicator is placed before the sign of grouping.

⋙ $(R_H T_H) + (R_S T_S)$

If no subscript indicator is used, a return to the baseline is implied.

⋙ (a_1, a_2, a_3)

This right parenthesis does not need a baseline indicator.

Be watchful when determining the level of a grouping sign. Because the top and bottom margins of grouping signs may extend beyond the level in effect, it is better to compare the center of the symbol to the surrounding text if its printed level is in question.

a. When a grouping sign starts at a different level, the appropriate level indicator is placed before the grouping symbol to properly identify its location. Review the examples in Section 6.7.1 with this in mind.

Example 6-44

In this case, $x_{(a,\,b)} + y_a$.

The left grouping symbol begins the subscript level.

b. When a grouping symbol is printed on the baseline of writing, an associated exponent is at the superscript level, even though it may be raised to a higher level in print. This is a visual aid, not to be mistaken for a higher-degree exponent.

Example 6-45

$(a^2)^8$ means "a-squared to the 8th power," or a^{16}, whereas a^{2^8} means "a to the 2^8 power," or a^{256}.

The first exponent 8 is a superscript to (a^2). The second exponent 8 is a supersuperscript.

c. Lesson 4 explained that, when a letter touches only one grouping symbol, the English-letter indicator is applied (or is not applied) as though the grouping sign were not present. This rule is illustrated below in the context of a grouping sign that has a simultaneous subscript and superscript.

⫸ $s]_a^b$ ⠎⠈⠾⠰⠁⠘⠃

Think: s_a^b ⠎⠰⠁⠘⠃

Instructions: Treat the vertical bar in sentence (6) as an operation sign.

PRACTICE 6L

(1) $\{f_n\}$

(2) $|a_m - a_n|$

(3) $(x_1y_1 + x_2y_2)$

(4) $([\mathrm{CH}_3]_2\mathrm{CH})$

(5) $\mathrm{I}_{\mathrm{ue}}^{2''} = (\mathrm{H}'_{44}x'_{\mathrm{ve}})^{+'}$

(6) The dagger and the asterisk are used as superscripts in quantum mechanics: $A^\dagger, \left(x^\dagger\right)^\dagger = x,\ \langle\phi|\psi\rangle^* = \langle\psi|\phi\rangle$.

For further practice, see Appendix A—Reading Practice.

EXERCISE 6

Prepare Exercise 6 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 6A

```
⠼⠃⠉⠲ ⠠⠎⠊⠍⠏⠇⠊⠋⠽ ⠯ ⠎⠕⠇⠧⠑ ⠑⠁⠡ ⠑⠟⠥⠁⠰⠝ ⠆⠇ ⠿
    ⠰⠭⠲ ⠠⠩⠪ ⠽⠗ ⠐⠺ ⠯ ⠡⠑⠉⠅ ⠽⠗ ⠁⠝⠎⠺⠻⠲ ⠸⠩
  ⠰⠁⠸⠲ ⠼⠆⠲ ⠨⠅ ⠼⠒⠭⠬⠒
  ⠰⠃⠸⠲ ⠼⠆⠷⠭⠤⠖⠾ ⠨⠅ ⠭⠤⠂⠲
  ⠰⠉⠸⠲ ⠼⠖⠬⠆⠨⠢⠭ ⠨⠅ ⠼⠆⠂
  ⠰⠙⠸⠲ ⠼⠆⠷⠭⠬⠲⠨⠢⠾ ⠨⠅ ⠼⠒⠆ ⠸⠱
```

PRACTICE 6B

```
⠸⠩
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠼⠖⠭⠘⠆   ⠼⠆⠲⠘⠿       ⠭⠘⠆⠁
⠨⠏⠘⠆    ⠷⠽⠤⠅⠾⠘⠒     ⠼⠦⠘⠂⠴
⠝⠘⠍     ⠠⠁⠘⠷⠅⠬⠂⠾⠬⠍  ⠟⠘⠃⠷⠺⠤⠂⠾
⠅⠘⠴⠨⠆⠶  ⠑⠘⠒⠠⠴⠴⠴     ⠑⠘⠤
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠸⠱
  ⠠⠍⠂⠎⠥⠗⠑⠰⠞⠎ ⠷ ⠘⠂⠇⠢⠛⠹ ⠜⠑ ⠍⠂⠎⠥⠗⠑⠰⠞⠎ ⠔
⠘⠂⠐⠕ ⠘⠂⠙⠊⠍⠢⠨⠝⠲ ⠠⠮⠽ ⠜⠑ ⠇⠁⠃⠑⠇⠫ ⠵ ⠉⠍⠂ ⠋⠞⠂
⠰⠍⠂ ⠑⠞⠉⠲ ⠠⠍⠂⠎⠥⠗⠑⠰⠞⠎ ⠷ ⠘⠂⠜⠑⠁ ⠜⠑ ⠍⠂⠎⠥⠗⠑⠰⠞⠎
⠔ ⠘⠂⠞⠺⠕ ⠘⠂⠙⠊⠍⠢⠨⠝⠎⠲ ⠠⠮⠽ ⠜⠑ ⠇⠁⠃⠑⠇⠫ ⠵
⠸⠩ ⠉⠍⠘⠆⠠ ⠋⠞⠘⠆⠠ ⠰⠍⠘⠆ ⠸⠱⠂ ⠑⠞⠉⠲ ⠠⠍⠂⠎⠥⠗⠑⠰⠞⠎
⠷ ⠘⠂⠧⠕⠇⠥⠍⠑ ⠜⠑ ⠍⠂⠎⠥⠗⠑⠰⠞⠎ ⠔ ⠘⠂⠹⠗⠑⠑
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PRACTICE 6C

Runover margins for itemized material are determined individually for each question. Item 2 has subdivisions—its runover position (line 5) is cell 5.

PRACTICE 6D

PRACTICE 6E

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PRACTICE 6F

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PRACTICE 6G

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PRACTICE 6H

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PRACTICE 6I

1. $x_{1,\,2} \neq x^{i,\,j}$
2. $x_{n-1,\,n-1}, x_{n-1,\,n}, x_{n,\,n-1}$
3. $\mathrm{A}^{\mathrm{n+n+n\ all\ n's}}$ are equal.
4. $s]_{t\,=\,a}$
5. $e^{1{,}000}$
6. $a^{m+k} \div a^{m} = a^{k}$
7. $P_{s_1 \ldots\, s_2}$ and $P_{q_{r,\,s}}$
8. $10_{-?-} = 6_8$
9. $a'_1, a'_2, \ldots, a'_n$ are the inverses.
10. $\aleph_0$ represents the cardinality of the set of all natural numbers.

PRACTICE 6J

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PRACTICE 6K

PRACTICE 6L

LESSON 7

- TYPEFORM
  - The Five Mathematical Typeform Indicators
  - Typeform of Letters
  - Typeform of Numerals
  - Boldface Mathematical Symbols
  - Other Signs of Grouping
  - Further Details Regarding Typeform of Letters and Numerals

Format

- DISPLAYED FORMATS

Answers to Practice Material

LESSON PREVIEW

This lesson begins by defining displayed mathematical material and illustrating the format in braille. The topic of typeform in mathematical context takes up the rest of the lesson.

DISPLAYED FORMATS

7.1 Displayed Mathematical Material [NC 26.4]

Up to this point in the lesson material, mathematical expressions in the examples have appeared in line with the narrative. This is referred to as an *embedded expression*. When mathematical material is set apart from the body of the text in the print copy, it is referred to as a *displayed expression*. Various layouts in the print copy are used to set the material apart, for example, skipped lines, centering or other indentation, or off to the side. In braille, margins for displayed mathematical material depend upon the margins of the surrounding text and are transcribed in one of the following formats.

- In unitemized explanatory portions of the text (3-1), displayed mathematical material begins in cell 3. Runovers begin in cell 5. **(3-5)**
- In itemized text without subdivisions (1-3), displayed mathematical material begins in cell 5. Runovers begin in cell 7. **(5-7)**
- In itemized text with subdivisions (1-5; 3-5), displayed mathematical material begins in cell 7. Runovers begin in cell 9. **(7-9)**
- Within or following instructions (5-3), displayed mathematical material begins in cell 5. Runovers begin in cell 7. **(5-7)**

Notice that in all four layouts, the first cell of the displayed material is indented two cells to the right of the runover cell of the preceding material. These margins apply regardless of the presence or absence of a runover in the preceding material. A line is <u>not</u> skipped above or below displayed mathematical material unless the preceding or following material requires a blank line.

7.1.1 **Placement of Code Switch Indicators.** There is not one formula that can be applied to all situations when it comes to judicious placement of code switch indicators. Use the following points as guidelines when displayed mathematical material is preceded and/or followed by UEB text.

- If the displayed math and its switch indicators will fit on one braille line, that is the preferred layout.
- If the displayed math and its switches will not fit on one braille line, the opening indicator is placed at the end of the previous line of text. If the indicator will not fit on that line, it is placed on the following line in the runover position of the text. This approach can be applied to displayed material following instructions as well. When displayed material follows a transcriber's note, the opening Nemeth Code indicator must fit on the same line as the closing transcriber's note indicator. If it does not, place the switch on the same line as the displayed math.
- The Nemeth Code terminator is placed at the end of the displayed material. If the terminator will not fit on that line, it is placed on the following line in the runover position for the displayed material.

For the remainder of the course, Nemeth format summaries can be found in Appendix C. You may find the format diagrams in Appendix C helpful.

Displayed Math Using 3-5 Margins

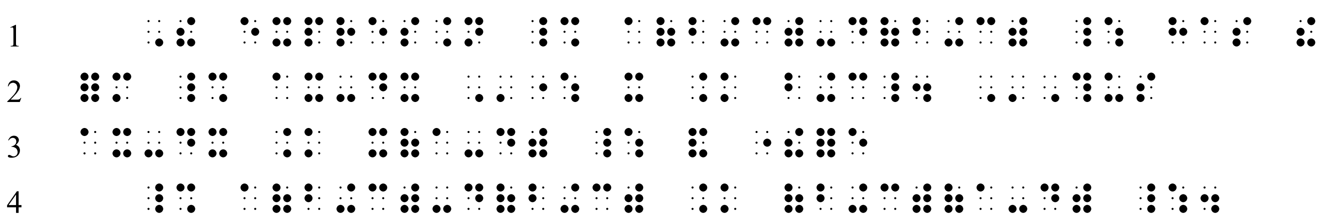
7.1.2 **Math Displayed to Unitemized Text.** Math displayed to unitemized text starts in cell 3. Runovers are in cell 5.

Notice layout patterns in the print copy to determine whether the narrative following displayed material is a continuation of the preceding text or if it is the start of a new paragraph. When the text following the displayed expression is a continuation of the same paragraph, it begins in the runover cell of that narrative, cell 1. When the text following the displayed expression starts a new paragraph, it begins in cell 3.

Example 7-1

The expression $a(b+c)-d(b+c)$ has the form $ax-dx$ where $x=b+c$. Thus $ax-dx=x(a-d)$ and therefore

$$a(b+c)-d(b+c)=(b+c)(a-d).$$



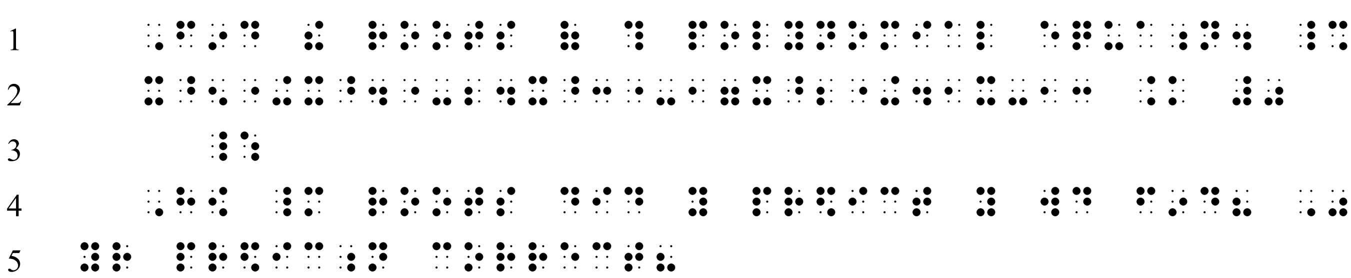
Lines 1-3: Narrative paragraph (3-1) with embedded math.
Line 4: The entire displayed expression will fit on one line with its opening switch indicator and terminator. The opening switch indicator is the first character in the display cell.

Example 7-2

Find the roots of this polynomial equation.

$$x^5+x^4-24x^3-17x^2+41x-13=0$$

How many roots did you predict you would find? Was your prediction correct?



Line 1: The narrative paragraph begins in cell 3. There is no runover. The opening Nemeth Code indicator is placed at the end of the sentence because the switch indicators do not fit on one line with the displayed math.

Lines 2-3: The displayed math begins in cell 3. The Nemeth Code terminator is placed in the runover position, cell 5. These margins apply regardless of the presence or absence of a runover in the preceding material.
Lines 4-5: New narrative paragraph (3-1).

7.1.3 **More Than One Displayed Math Item.** When more than one item is displayed, consider the following details before formatting.

a. If each expression is printed on a new line, or if short expressions are widely spaced on one line, transcribe each expression on a new line in the initial display cell.

Example 7-3

A statement with an equals sign is called an **equation.** Three examples are provided below.

$$9\times(4 - 1) = 27 \qquad 1 + 1 = 3 \qquad x + 7 = 50$$

Which equation is true? Which equation is false? Which equation may be either true or false?

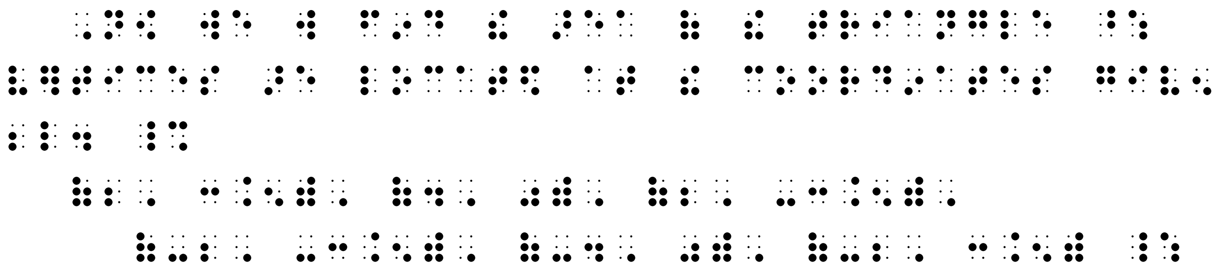
Line 3: The opening switch indicator is placed on the same line as the first displayed expression.
Lines 3-5: Although printed all on one line, each expression is transcribed on a new line.
Lines 6-7: The narrative paragraph continues in cell 1 (the runover cell to a 3-1 paragraph).

b. If displayed items are separated by sentence punctuation, such as commas, the displayed material may be formatted as one item with runovers. (See also item c, below.) Be sure to follow "keep together" linage rules when determining where to begin the runover line.

Example 7-4

Now we will find the area of the hexagon whose vertices are located at the coordinates given below.

(2, 3.5), (4, 0), (2, -3.5), (-2, -3.5), (-4, 0), (-2, 3.5)



Lines 1-3: Narrative paragraph (3-1).
Line 4: Cell 3 is the starting cell for math displayed to narrative.
Line 5: Cell 5 is the runover cell for math displayed to narrative. Notice that, although (-2, will fit at the end of line 4, an enclosed list must not be divided between braille lines.

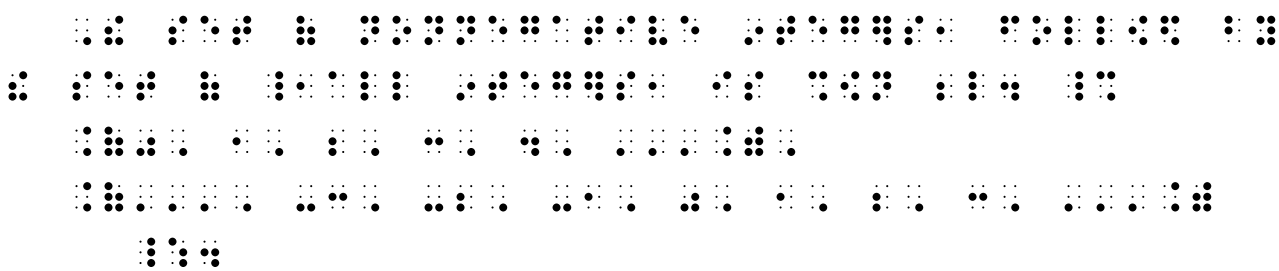
c. When sentence-structure punctuation is included in the displayed math, each expression may begin in the primary display cell if doing so more closely matches the print layout.

Example 7-5

The set of nonnegative integers, followed by the set of <u>all</u> integers, is shown below.

$\{0, 1, 2, 3, 4, \dots\},$

$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}.$



Lines 1-2: Narrative paragraph (3-1).
Line 3: Math displayed to unitemized text begins in cell 3.
Lines 4-5: The second displayed math item also begins in cell 3. The Nemeth Code terminator is in the runover cell, cell 5.

PRACTICE 7A

Polynomials

Solve this polynomial using basic algebra. *Hint:* First factor out "x" to make it a quadratic equation.

$$x^3 + 2x^2 - x = x(x^2 + 2x - 1)$$

Do you notice a familiar pattern?

Sequences

A sequence $a_1, a_2, a_3, \ldots, a_n$ is said to *converge* if there exists a positive number M such that, for each $h > 0$,

$$|a_n - A| < h \text{, for all } n > M.$$

A sequence that does not converge is said to *diverge*.

Inequalities

Now we will use number lines to illustrate the following inequalities.

$-6 < -5$ $\qquad 0 < +6$ $\qquad -8 < +2$ $\qquad -1 > -5$

Displayed Math Using 5-7 Margins

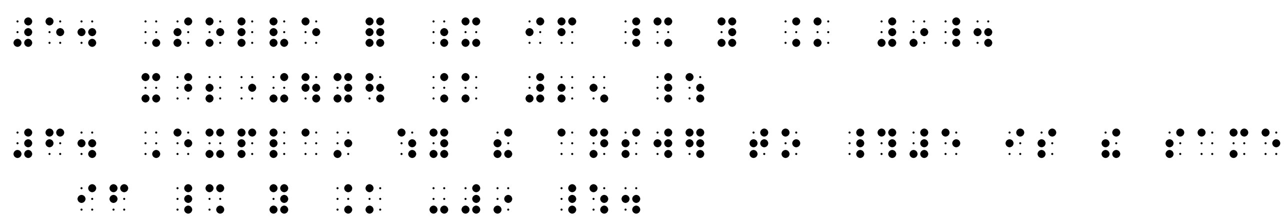
7.1.4 **Math Displayed to Itemized Text.** Math displayed to itemized text (with no subdivisions) starts in cell 5. Runovers are in cell 7.

Example 7-6

5. Solve for x if $y = 9$.

$$x^2 + |y| = 25$$

6. Explain why the answer to #5 is the same if $y = -9$.



Line 1: Itemized material begins in cell 1.
Line 2: Displayed math begins in cell 5 even though the related text does not have a runover.
Lines 3-4: Margins for itemized material (with no subdivisions) are 1-3.

PRACTICE 7B

1. Fred took his sister out to dinner. The total bill came to \$39. Fred's sister offered to pay the 15% tip. How much did she contribute?

$$0.15 \times \$39.00 = \$5.85$$

2. A pair of boots, originally priced at \$175, is marked down 20%. How much will the boots cost? Be sure to add 6.5% sales tax to the discounted price.

   Here is how Maya found the answer. Can you explain her steps?

$$0.20 \times \$175 = \$35 \text{ (discount)}$$
$$\$175 - \$35 = \$140 \text{ (price)}$$
$$0.065 \times \$140 = \$9.10 \text{ (tax)}$$
$$\$140 + \$9.10 = \$149.10 \text{ (total cost)}$$

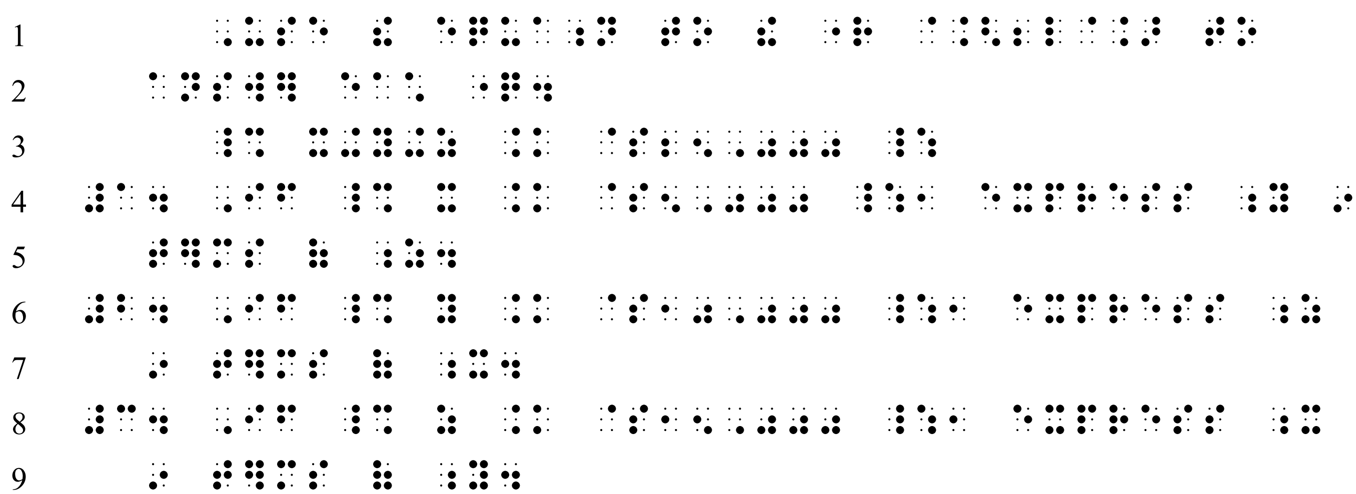
   There is a way to solve this problem using algebra. Write an equation that combines all steps into one.

7.1.5 **Displayed Material Associated with Instructions.** Nemeth instructions begin in cell 5, with runovers in cell 3. (See Lesson 5.) If displayed mathematical material appears within or immediately following instructions, the displayed material starts in cell 5 with runovers in cell 7.

Example 7-7

Use the equation to the right to answer each question. $x + y + z = \$25{,}000$

1. If x = \$5,000, express y in terms of z.
2. If y = \$10,000, express z in terms of x.
3. If z = \$15,000, express x in terms of y.



Lines 1-2: Margins for Nemeth instructions are 5-3. The boldface print used for instructions is a visual device and so is disregarded in braille. An embedded transcriber's note points the reader to the equation "below".
Line 3: The displayed math begins two cells in from the runover cell of the previous text. The two switch indicators fit on the same line as the displayed math.
Lines 4-9: Itemized problems (1-3)

Displayed Math Using 7-9 Margins

7.1.6 **Math Displayed to Itemized Text with Subdivisions.** Math displayed to itemized text with subdivisions starts in cell 7. Runovers are in cell 9.

Example 7-8

5. Give two examples illustrating

   i. the associative law for addition.

   $$(a+b)+c=a+(b+c)$$

   ii. the associative law for multiplication.

   $$(a\times b)\times c=a\times(b\times c)$$

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Line 1: Main item begins in cell 1.
Lines 2 and 4: Each subdivision begins in cell 3.
Lines 3 and 5: Displayed math is in cell 7, which is two cells in from the runover of subdivisions, whether or not runovers occur.

Example 7-9

2. Now we solve each of the following equations.

   a. $3(x+5)=6x+6$ b. $x^2-25=0$

   x = ____ x = ___ and ___

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5 ⠰⠃⠸⠲ ⠭⠘⠆⠐⠤⠆⠢ ⠨⠅ ⠼⠴
6 ⠭ ⠨⠅ ⠤⠤⠤⠤ ⠠⠄⠯ ⠤⠤⠤⠤ ⠸⠱

Lines 1-2: Main item begins in cell 1. The opening Nemeth Code indicator is placed at the end of the line of text preceding the math items. Because there is no room for the switch indicator on line 1, it falls in the runover position of the current text (cell 5).
Lines 3 and 5: Each subdivision begins in cell 3, not side by side as printed.
Lines 4 and 6: Displayed math begins in cell 7, which is two cells in from the runover of subdivisions, whether or not runovers occur. (Subdivisions are 3-5.)
Line 6: Nemeth is terminated at the end of the displayed material.

PRACTICE 7C

Use the Pythagorean formula to answer the questions.

$$a^2 + b^2 = c^2$$

3. Emma is flying a kite. The kite is 14 feet in front of her (distance a).

a. How high is the kite (distance b) if she has let out 39 feet of line (distance c)?

Solve for b: $14^2 + b^2 = 39^2$

b. How many feet of line is let out (distance c) if the kite is only 12 feet in the air (distance b)?

Solve for c: $14^2 + 12^2 = c^2$

7.1.7 **An Alternate Layout.** The next example shows a polynomial equation that will not fit on one line with both of its switch indicators. The first transcription places the opening switch in the runover position of the text, as recommended.

Example 7-10

Solve the following polynomial equation. How many roots do you predict you will find? Show your work:

$$x^5 + x^4 - 24x^3 - 7x^2 + 41x - 13 = 0.$$

An alternate layout saves a line if the displayed expression takes up most of the line <u>and</u> if the Nemeth Code terminator is followed by punctuation. It is acceptable to begin the displayed math with the opening switch indicator, with the terminator and the punctuation mark on the next line.

Alternate transcription

7.2 Displayed Material with Labels [NC 26.4.4]

Displayed mathematical expressions may be labeled in print with a unique number or letter printed in a location that is visually conspicuous, often in the right margin. That label is then used in place of the actual expression later in an example or in the narrative. This allows for compact presentation of a problem.

7.2.1 **Braille Layout.** When a number or letter is used to identify a displayed mathematical expression, the label (identifier) is placed at the left of the expression in braille regardless of the location of the label in the print copy. The label begins in the appropriate cell for displayed material in accordance with the margin rules presented in this lesson.

The label may be transcribed in either UEB or Nemeth depending on context. Labels do not need to be transcribed in the same code, although the transcriber may prefer to show a sequence of labels in the same code. Nonstandard typeform is disregarded. When the label is referred to in discussion, it may be transcribed in UEB or in Nemeth, as appropriate, and need not be in the same code as the original label.

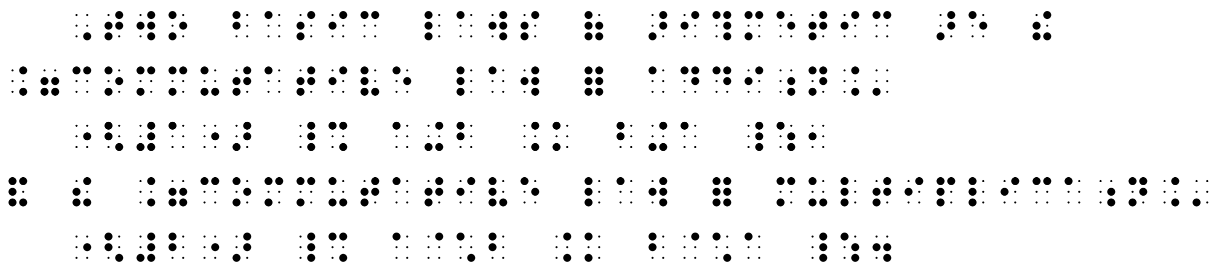
Example 7-11

Two basic laws of arithmetic are the *commutative law for addition*

$$a + b = b + a, \qquad (1)$$

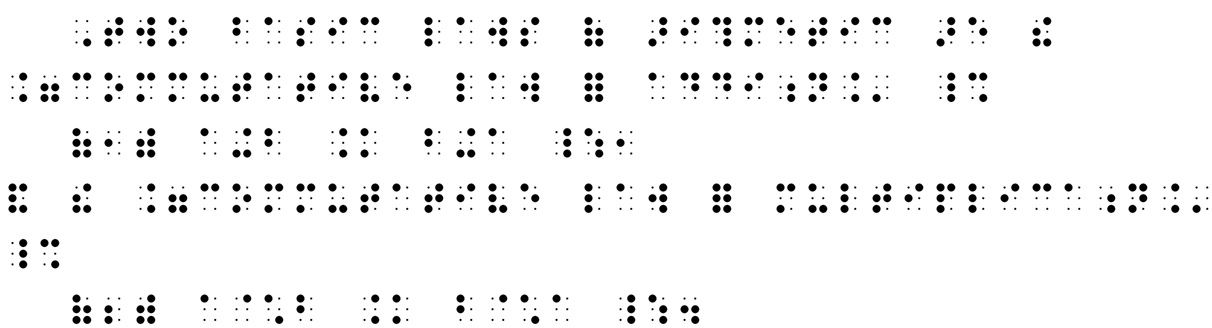
and the *commutative law for multiplication*

$$a \times b = b \times a. \qquad (2)$$



This is a narrative paragraph interrupted twice by displayed math.
Lines 1-2: The paragraph begins in cell 3 and runs over in cell 1.
Line 3: Displayed math begins in cell 3. The label (identifier) is transcribed first. The label is bold in print but the typeform is disregarded in braille. The label is transcribed in the same code as the preceding text, which is UEB in this case.
Line 4: The paragraph continues in the runover cell (cell 1).
Line 5: Same as line 3.

Alternate transcription

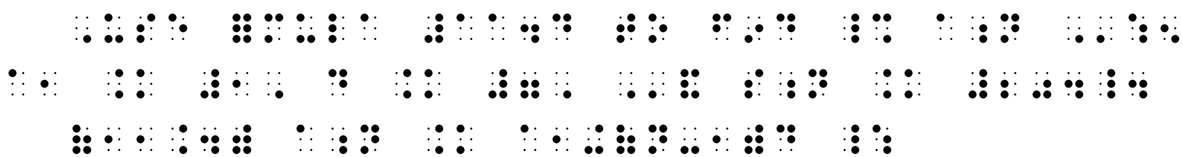


Line 2: The opening Nemeth Code indicator is placed at the end of the text.
Line 3: The label is transcribed in Nemeth.
Line 5: The opening Nemeth Code indicator will not fit at the end of the text on line 4 and so is placed in cell 1, the runover position.
Line 6: The label is transcribed in Nemeth.

Example 7-12

Use formula 11.4 to find a_n when $a_1 = 1, d = 7,$ and $s_n = 204$.

$$a_n = a_1 + (n - 1)d \qquad (11.4)$$



7.2.2 **Transcriber's Note Required.** A transcriber's note concerning the change in position of the label in the braille copy is required. Sample transcriber's note:

An identifier printed to the right of an expression is transcribed on the left.

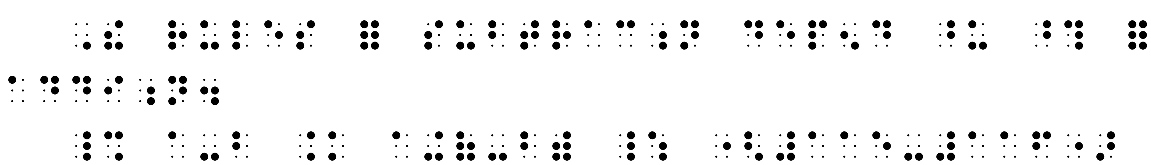
7.2.3 **Page Number Citation.** The number printed beside a displayed mathematical expression may actually be a page citation, in which case the cross reference immediately follows the expression, as printed. If a range of numbers is shown, you can be fairly confident that they are page numbers, especially in a review section. Look for context clues to determine if the label is a page number citation in order to place it in its proper location.

Since the location of the citation is not changed from its location in print, a transcriber's note is not needed.

Example 7-13

The rules for subtraction depend upon those for addition.

$$a - b = a + (-b) \qquad (115\text{-}116)$$



7.3 Displayed Narrative Material

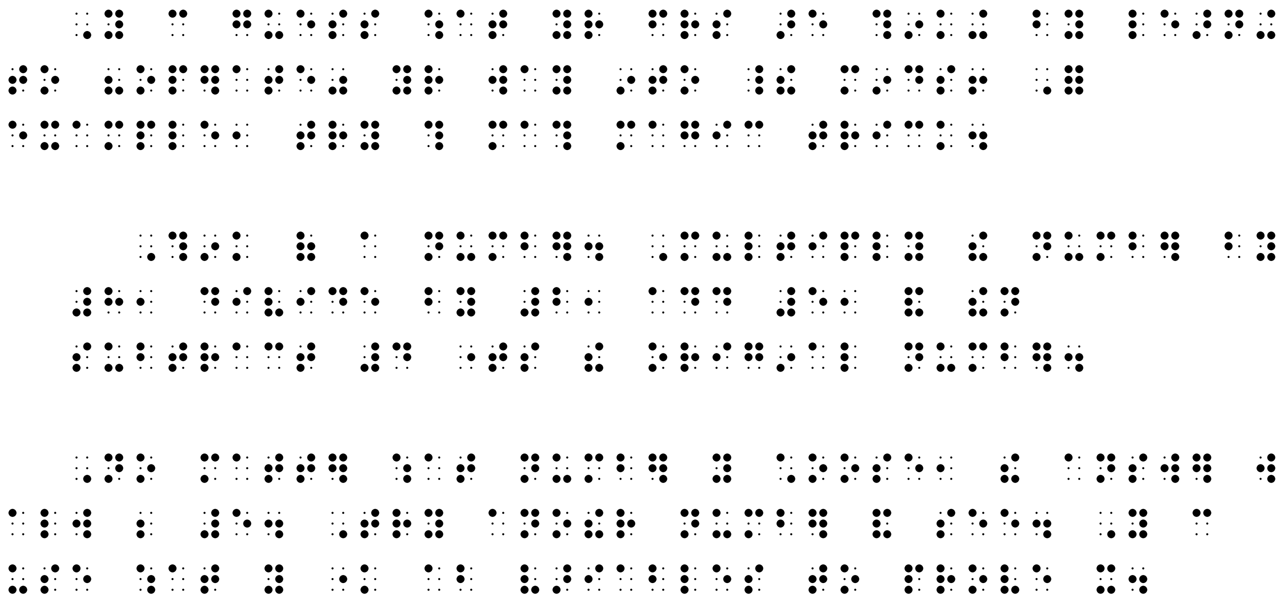
Recall that displayed narrative text follows the guidelines in *Braille Formats* with the exception that blocked paragraphs are not used throughout a technical document.

Example 7-14

You can guess what your friends are thinking by learning to "operate" your way into their minds! For example, try this math magic trick.

> **Think of a number. Multiply the number by 8, divide by 2, add 5, and then subtract 4 times the original number.**

No matter what number you choose, the answer will always be 5. Try another number and see. You can use what you know about variables to prove it.



Lines 1, 5, and 9: According to Nemeth rules, the first line of each paragraph is indented two cells from the paragraph's left margin.
Lines 4 and 8: According to Braille Formats guidelines, displayed literary material is preceded and followed by a blank line.
Lines 5-7: According to Braille Formats guidelines, cell 3 is the adjusted left margin for this displayed material.

Instructions: Create a facsimile of a Transcriber's Notes page as follows. After the centered heading, TRANSCRIBER'S NOTES, include this required statement: "Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 2022*." The next paragraph should state: "Identifying numbers printed to the right of mathematical expressions are transcribed to the left." For the purpose of this exercise, you may begin transcribing the text after one blank line.

PRACTICE 7D

This is the quadratic equation, where x is the variable and a, b, and c are constants ($a \neq 0$).

$$ax^2 + bx + c = 0 \qquad (1)$$

This is the Pythagorean Theorem:

$$a^2 + b^2 = c^2 \qquad (2)$$

Which equation, (1) or (2), is used to find the length of the sides of a right triangle?

TYPEFORM

[NC Rule 7]

In this lesson, we look at typeform as it applies to letters, numbers, and mathematical symbols. Typeform applied to words in mathematical context will be addressed in Lesson 11.

7.4 General Guidelines Regarding Typeform

When the typeform of a letter or number has mathematical significance, a typeform indicator of the Nemeth Code is used. This rule applies regardless of the existence of a similar typeform indicator in UEB.

When such a letter or number is referred to within narrative, a switch to Nemeth is required in order to show the letter or number associated with its appropriate Nemeth typeform indicator. Note that UEB typeform indicators are not used inside the switches and that Nemeth typeform indicators are not used outside the switches.

7.4.1 **Determining Significance of a Variant Typeform.** The decision whether to retain a variant typeform can be difficult. The transcriber needs to determine if the typeform has mathematical meaning (i.e., for "distinction"), if the typeform is for instructional purposes (i.e., for "emphasis"), or whether the typeform does not add any information or is merely decorative. The general rule of thumb is that, when technical material is printed in nonregular type that has no mathematical or instructional significance, the variant typeform is disregarded in the transcription.

a. **Typeform Showing Distinction**

- Significant: Various fonts often have fixed meanings in particular areas of mathematics and science. Such letters, numbers, and symbols must retain their significant typeform in the braille transcription, and must be transcribed following Nemeth Code rules.

  Examples: $\mathbb{R}$ signifies the set of real numbers.
  The null vector is denoted with a boldface $\mathbf{0}$.
  $\mathcal{S}$ represents a system's action in physics.

- Insignificant: It is standard print practice to show math variables using an italic font throughout a publication. This use of italics is not mathematically significant and so is not retained in the braille transcription.

  Examples: The variables $x, y,$ and z are real numbers.
  π is used to determine the circumference of a circle: $2\pi r$.

b. **Typeform Showing Emphasis**

- Significant: An author may use a variant typeform to focus on a teaching point or topic. Such letters or numbers may lose their meaning if their significant typeform is not retained in the braille transcription. If the typeform is mentioned in the narrative, it should either be retained or explained in a transcriber's note.

Example: Are the boldface numbers even or odd? 19, **28**, 37, **44**, 51, 67, **72**, **80**.

- Insignificant: A variant typeform is often used for the sole purpose of attracting the reader's attention. This is particularly common at the lower grade levels. Such variant typeforms are disregarded in the braille transcription.

Examples: Let **x** be the smaller number, and **9 + x** be the larger number.

A function with degree *5* has *5* zeros.

7.5 The Five Mathematical Typeform Indicators

Specific provision is made in the Nemeth Code for the transcription of five print typeforms: barred, boldface, italic, sans serif, and script. (In other publications, the barred font may be called blackboard bold or double struck.) The various typeforms may be applied to the letters of the English, German, Greek, Hebrew, and Russian alphabets as well as to numerals and mathematical symbols. (Note that underlining is not a typeform in the Nemeth Code.)

⠠⠸	Barred Type
⠸	Boldface Type
⠨	Italic Type
⠠⠨	San Serif Type
⠈	Script Type

7.6 Typeform of Letters

Certain specific mathematical letters are identifiable by their variant typeform. Common examples include the letter ℝ for "the set of real numbers" and boldfaced letters that represent vectors. In this lesson, after practicing the application of the rules regarding typeform of letters, only the variant letters in common practice will be studied.

Typeform applied to a mathematical letter is considered to be a modification. A switch to Nemeth is required when such a letter appears in the narrative, even if UEB has a typeform indicator for the font. A Nemeth typeform indicator applied to a letter must always be followed by an alphabetic indicator.

a. **Typeform Indicators with One Letter.** Here is the capital English letter R in regular type, followed by the same letter in each of the five Nemeth typeforms.

R	⠰⠠⠗	(regular English capital R)
ℝ	⠠⠸⠰⠠⠗	(barred English capital R)
R	⠸⠰⠠⠗	(boldface English capital R)
R	⠨⠰⠠⠗	(italic English capital R)

R	⠠⠨⠰⠠⠗	(sans serif English capital R)
$\mathcal{R}$	⠈⠰⠠⠗	(script English capital R)

Notes: Sans serif typeform is recognized by the lack of small lines or serifs at the ends of the letter parts. Only the English alphabet has a sans serif style of type. Script typeform looks like cursive handwriting. Publishers have different styles for this font.

Here are isolated examples of capital and lowercase letters from the other four alphabets, in various typeforms. The first indicator names the typeform; the second indicator names the alphabet; a capital letter then shows a capitalization indicator; and, finally, the letter is identified. You may wish to review the five alphabetic indicators of the Nemeth Code in Lessons 3 and 4.

$\boldsymbol{\alpha}$	⠸⠨⠁	(boldface Greek alpha, lowercase)
Ш	⠸⠈⠈⠠⠱	(boldface Russian capital Sha)
$\underline{\Sigma}$	⠠⠸⠨⠠⠎	(barred Greek capital Sigma)
c	⠨⠸⠉	(italic German tseh, lowercase)
א	⠈⠠⠠⠁	(script Hebrew alef)

This Practice is an exercise in applying the rules regarding order of indicators: typeform, alphabet, and capitalization. Since these letters are out of context, the alphabet and the individual letter name is given. The braille character of the lowercase German, Hebrew, and Russian letters are provided. Note that capital letter names are capitalized in the description.

Instructions: Transcribe only the 44 letters, using the typeform indicated before each set: boldface, barred, script, or sans serif. Do not transcribe the directions or the names—just transcribe four letters on each line, with one blank cell between each of the letters. The first line in the practice is shown below to get you started.

⠸⠩ ⠠⠸⠰⠠⠓ ⠠⠸⠰⠊ ⠠⠸⠰⠠⠟ ⠠⠸⠰⠙

PRACTICE 7E

Use BARRED typeform for these English and Greek letters.

ℍ 𝕚 ℚ 𝕕 — English letters

∑ ℽ ℾ ℼ — Greek letters (Sigma, gamma, Gamma, pi)

Use BOLDFACE typeform for these letters.

a B c D — English letters

ρ Ψ Φ χ — Greek letters (rho, Psi, Phi, chi)

Ч з г Ж — Russian letters (Cheh ⠟, zeh ⠵, gheh ⠛, Zheh ⠚)

Use ITALIC typeform for these letters.
(Disregard the dark typeface. None are bold.)

𝔳 𝔍 𝔶 ℭ — German letters (fao ⠧, Yaht ⠚, ypsilon ⠽, Tseh ⠉)

э Е щ Я — Russian letters (eh ⠪, Yeh ⠑, sha ⠱, Yah ⠫)

Use SANS SERIF typeform for these English letters.

K R h p

Use SCRIPT typeform for these *letters.*
(Disregard the dark typeface. None are bold.)

𝒻 𝒢 𝒽 𝓏 — English letters

ז ע ג ק — Hebrew letters (zayin ⠵, ayin ⠫, gimel ⠛, qof ⠟)

b. **Typeform Indicators with More Than One Letter.** The effect of a typeform indicator extends only to the letter which immediately follows it. Thus, in a sequence of unspaced letters, a typeform indicator must be used before each letter that is not in regular type. Here are some isolated examples.

𝔸𝔹	⠠⠸⠰⠠⠁⠠⠸⠰⠠⠃	(barred English A and B)
AB	⠸⠰⠠⠁⠸⠰⠠⠃	(boldface English A and B)
ab	⠸⠰⠁⠸⠰⠃	(boldface English a and b)
αβ	⠸⠨⠁⠸⠨⠃	(boldface Greek alpha and beta)
𝒜ℬ	⠈⠰⠠⠁⠈⠰⠠⠃	(script English A and B)

In a sequence of unspaced letters, an English letter in regular type does not need an alphabetic indicator.

βb	⠨⠃⠃	(regular Greek beta, regular English b)
B𝔹	⠠⠃⠠⠸⠰⠠⠃	(regular English B, barred English B)
𝖧H	⠠⠨⠰⠠⠓⠠⠓	(sans serif English H, regular English H)
p**qr**s	⠏⠸⠰⠟⠸⠰⠗⠎	(English letters: regular p, boldface q, boldface r, regular s)
x**i**y**j**	⠭⠸⠰⠊⠽⠸⠰⠚	(English letters: regular x, boldface i, regular y, boldface j)

Instructions: Practice applying typeform to English and Greek unspaced letter groupings. No italics are used in this list. Only English letters are showing a sans serif and a script typeform.

PRACTICE 7F

M 𝕄ℳM𝖬

𝗒𝓎𝕪 **y** y

⅀Σ**Σ**

ℼπ**π**

Δ𝖽d**D**λ𝒩

7.6.1 **Barred Letters.** Certain important constructs are sometimes represented by barred letters. Frequently, a barred capital letter may be used to denote the number sets. For example, some authors denote the set of natural numbers by $\mathbb{N}$.

⫸ $\mathbb{N}$

Example 7-15

$\mathbb{Q}$ denotes the set of rational numbers and $\mathbb{R}$ denotes the set of real numbers.

The barred typeform has mathematical significance and so is retained.

Note that a publication may represent these number sets using script rather than barred typeform. 7.6.5 discusses the script typeform indicator.

7.6.2 **Boldface Letters.** Boldface is used to distinguish certain specialized mathematical letters. The text may use bold italics or simply bold. The italic typeform is disregarded, but the bold must be preserved. (See 7.6.3 regarding italics.) Within narrative, a boldface mathematical letter requires a switch to Nemeth. The UEB boldface symbol indicator is not used when the letter has mathematical significance.

Recall that the German letter indicator is dots 456. For 456 to mean "boldface", it must be followed immediately by an alphabetic indicator. Thus, and are German letters fao and Fao, and and are boldface English letters v and V.

a. **Boldface Letters of Significance—Vectors and Matrices.** Bold lowercase letters are often used to identify vectors. Bold capital letters usually represent matrices. Bold letters may also be found in certain topics in engineering and physics.

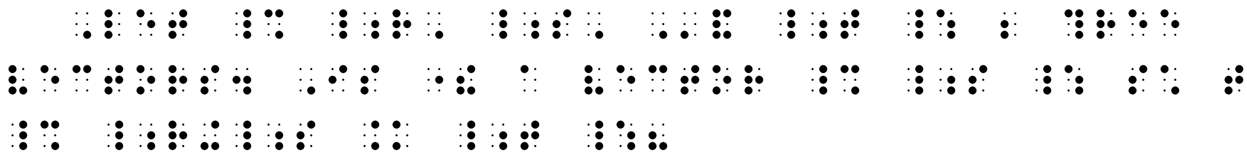
⫸ $\boldsymbol{c}^T\Sigma\boldsymbol{c}$

⫸ $\boldsymbol{c}_0(s) = \boldsymbol{\delta}$

⫸ $\boldsymbol{J} = \Delta\boldsymbol{p} = m\Delta\boldsymbol{v}$

Example 7-16

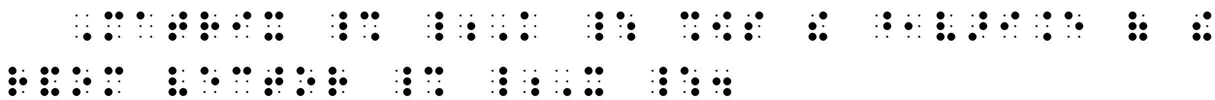
Let **r**, **s**, and **t** be three vectors. Is there a vector **s** such that **r** + **s** = **t**?



A switch to Nemeth is required within narrative for the boldface mathematical letters. An English-letter indicator is required following each typeform indicator.

Example 7-17

Matrix $\mathbf{K}$ shows the **variance** of the random vector $\mathbf{X}$.



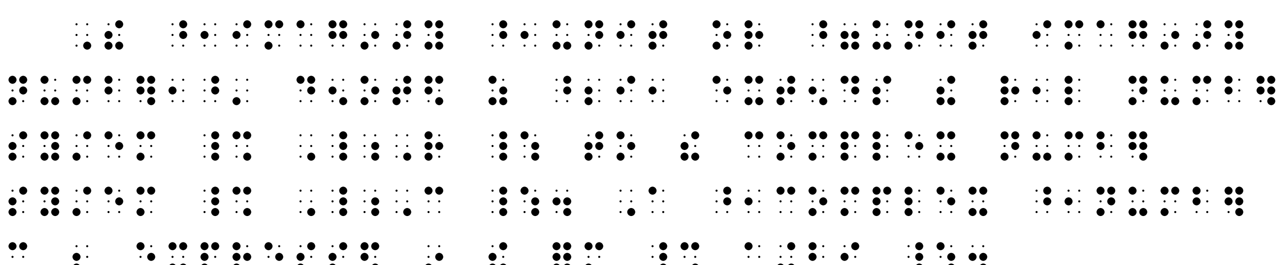
The boldfaced word in the narrative is transcribed in UEB.

b. **Context Clues.** The boldfaced words in the next example indicate to the reader that they are defined in a glossary. But what about the boldfaced letter "i"? Search the surrounding text to determine whether the letter "i" retains the bold typeform within a mathematical expression. If it does, the boldface is mathematical and a switch to Nemeth Code is required. However, you can see in the expression $a + bi$ that the imaginary unit is not printed in bold. Therefore, the bold font does not have mathematical significance. Checking the glossary, you find that "i" is a glossary entry, so the bold is retained for that reason but is transcribed in UEB, using the UEB boldface symbol indicator. The italic typeform is disregarded in both cases. (See 7.6.3 regarding italics.)

The Sha from the Cyrillic alphabet often keeps company with bold and barred letters, as seen in the second example.

Example 7-18

The **imaginary unit** or **unit imaginary number**, denoted as ***i***, extends the real number system $\mathbb{R}$ to the complex number system $\mathbb{C}$. A **complex number** can be expressed in the form $a + bi$.



Example 7-19

Ш$(\boldsymbol{E}/\mathbb{Q})[\boldsymbol{p}]$ is trivial for $\boldsymbol{p} \neq 2, 3, 5, 7$.

c. **Visual Significance Only.** Boldface type of a mathematical letter used only to draw visual attention is disregarded.

Example 7-20

A nonzero number written in **scientific notation** is written in the form $m \times 10^{n}$ where $\boldsymbol{m}$ is a real number greater than 1 and less than 9.99 and $\boldsymbol{n}$ is an integer.

Letters m and n are not bold in the formula. The boldface used for these letters in the narrative is insignificant and so is disregarded.

d. **The German Fraktur Font.** The letters of the German "fraktur" alphabet may appear to be printed in boldface, but when <u>all</u> German letters in the document are dark, the bold typeform is disregarded in the transcription.

Example 7-21

These German letters have special mathematical meaning in certain disciplines: $\mathfrak{a}$, $\mathfrak{c}$, $\mathfrak{g}$, $\mathfrak{m}$, $\mathfrak{p}$, and $\mathfrak{B}$.

The German letters are printed in a dark font. They are not bold.

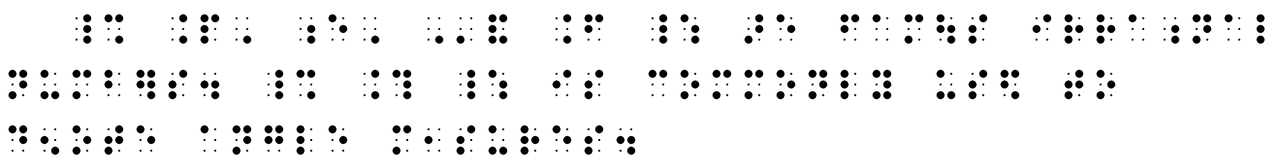
7.6.3 **Italic Letters.** In the braille transcription, italics are disregarded when mathematical letters are printed in italics consistently throughout the document. If a letter is italicized for other reasons, the transcriber must decide whether the typeface is significant. If the italic typeface is retained, an English-letter indicator is required.

It is helpful for the transcriber to notice the typographical conventions in mathematical notation, particularly that variables are printed in italics and abbreviations are not. It is also customary to print all lowercase Greek letters in italics. Constants may be seen either upright or in italics, but will be printed in a consistent manner throughout a publication. Sometimes, variables in the superscript or subscript position will be printed in regular (upright) type for clarity. Unless a specific, unique meaning is assigned to an italicized letter, it can be concluded that italics are not a mathematically significant typeface. The distinction between variables and abbreviations is not an issue in braille because Nemeth spacing rules adequately differentiate between them.

Recall that the Greek letter indicator is dots (46). For (46) to mean "italics", it must be followed immediately by an alphabetic indicator. Thus, ⠨⠏ is the Greek letter pi, and ⠨⠰⠏ is the italicized English letter p.

Example 7-22

π, e, and ϕ are famous irrational numbers. θ is commonly used to denote angle measures.



Greek letters pi, phi, and theta, as well as English letter e, are printed in italics. These letters are not italicized in braille, according to the general guidelines regarding italicized mathematical letters.

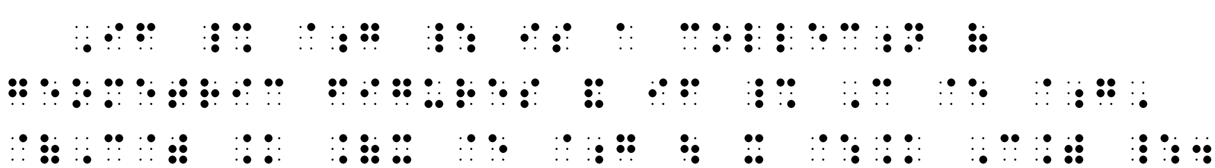
7.6.4 **Sans Serif Letters.** Sans serif letters are mainly used to differentiate computer program language from the surrounding text. Transcription of computer code is not addressed in the Nemeth code book and is beyond the scope of this course.

7.6.5 **Script Letters.** When a script letter is assigned specific mathematical significance, the typeform is retained in the braille transcription. Although there is a script typeform indicator in UEB, a mathematically significant script letter in narrative requires a switch to Nemeth Code.

Examples: $\mathcal{S}$ represents a system's action in physics.
Let $\mathcal{T}$ be a topological space.

Example 7-23

If $\mathcal{G}$ is a collection of geometric figures and if $C \in \mathcal{G}$, $[C] = \{x \in \mathcal{G} \mid x \cong C\}$.



a. **Script Letter "ell".** Print publishers often use the script form of the lowercase English letter "ell" simply to differentiate it visually from the numeral 1 (one). Since the letter and the numeral cannot be confused in braille, there is no reason to retain the script typeform.

Example 7-24

Assume that $\ell(AB) < \ell(DE)$, where ℓ is used to denote *length*.

The letter l is printed with a script font.

b. **Partial Derivative Symbol.** The symbol for "partial derivative", ∂, is its own symbol. This is not a script letter d. This symbol will be discussed in Lesson 13.

PRACTICE 7G

i. The perimeter of a rectangle is obtained by adding the measurements of the sides—two lengths and two widths—expressed as

$$P = 2\ell + 2w.$$

What is P if $\ell = 5.5$ mi and $w = 3.2$ mi ?

ii. The 1-D coordinate system is denoted by $\mathcal{R}$. The 2-D coordinate system is often denoted by $\mathcal{R}^2$. A general n-dimensional coordinate system can be denoted by $\mathcal{R}^n$.

iii. Use $\alpha_1, \beta_1, \gamma_1$ and $\alpha_2, \beta_2, \gamma_2$ to denote the direction vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$.

iv. **Two Number Sets.** $\mathbb{N}$ denotes the set of *natural numbers* – that is, the set of nonnegative integers $\{0, 1, 2, \ldots\}$. The set of <u>all</u> integers is denoted by $\mathbb{Z}$.

7.7 Typeform of Numerals

Typeform applied to a number is considered a modification if the typeform is mathematically significant or is considered to be printed in a variant typeform for instructional purposes. (See 7.4.1.b.) A switch to Nemeth is required when such a number appears in the narrative.

7.7.1 **Typeform Indicators with One Numeral.** The appropriate Nemeth typeform indicator is used when it is determined that the nonregular type is mathematically significant. A numeric indicator is required between a typeform indicator and a numeral. Here are isolated examples of a numeral in various typeforms.

$\mathbb{4}$		(barred 4)
$\mathbf{4}$		(boldface 4)

4	⠨⠼⠲	(italic 4)
4	⠈⠼⠲	(script 4)
+**8**	⠬⠸⠼⠦	(ordinary plus, boldface 8)
−**8**	⠤⠸⠼⠦	(ordinary minus, boldface 8)

7.7.2 **Typeform Indicators with More Than One Numeral.** The effect of a typeform indicator with numerals extends until there is a change in type. Thus, when numerals contain digits in more than one typeform, the appropriate typeform indicator and the numeric indicator must be used before each change in type. When the change is to regular type, only the numeric indicator is used. Here are some isolated examples.

𝟙𝟚𝟛	⠠⠸⠼⠂⠆⠒	(all three numerals are barred)
123	⠸⠼⠂⠆⠒	(all three numerals are boldface)
4*5*6	⠸⠼⠲⠨⠼⠢⠼⠖	(boldface 4, italic 5, regular 6)
4567	⠸⠼⠲⠢⠼⠖⠶	(boldface 4 and 5, regular 6 and 7)
12**34**	⠼⠂⠆⠸⠼⠒⠲	(regular 1 and 2, boldface 3 and 4)
28-**571**	⠨⠼⠆⠦⠤⠸⠼⠢⠶⠂	(italic 28, hyphen, boldface 571)
47-653	⠨⠼⠲⠶⠤⠼⠖⠢⠒	(italic 47, hyphen, regular 653)

100 + **2**00 = **3**00

⠸⠼⠂⠼⠴⠴⠬⠸⠼⠆⠼⠴⠴ ⠨⠅ ⠸⠼⠒⠼⠴⠴

(boldface 1, 2, and 3; all zeros in regular type)

7.7.3 **Barred Numerals.** The use of this typeform with numerals tends to be an author's choice for clarity, rather than using regular boldface. The transcription follows print.

⫸ 𝟝 ⠠⠸⠼⠢

7.7.4 **Boldface Numerals**

a. **Boldface Numeral of Significance—The Null Vector.** The boldface zero is defined as the "null vector" and therefore the typeform has mathematical significance. A switch to Nemeth Code is required, even in literary context.

⫸ **0** ⠸⠼⠴

Example 7-25

In $P\mathbf{v} = \mathbf{0}$, $\mathbf{v}$ is a vector and $\mathbf{0}$ is the **null vector.**

⠀⠀⠠⠔ ⠸⠩ ⠠⠏⠸⠰⠧ ⠨⠅ ⠸⠼⠴⠠ ⠸⠰⠧ ⠸⠱ ⠊⠎ ⠁ ⠧⠑⠉⠞⠕⠗
⠯ ⠸⠩ ⠸⠼⠴ ⠸⠱ ⠊⠎ ⠮ ⠘⠂⠝⠥⠇⠇ ⠘⠂⠧⠑⠉⠞⠕⠗⠲

b. Otherwise, bold numerals may simply indicate distinction, as in the next example.

Example 7-26

Are the boldface numbers even or odd? 19, **28**, 37, **44**, 51, 67, **72**, **80**.

⠀⠀⠠⠜⠑ ⠮ ⠃⠕⠇⠙⠋⠁⠉⠑ ⠝⠥⠍⠃⠻⠎ ⠑⠧⠢ ⠕⠗ ⠕⠙⠙⠦
⠸⠩ ⠼⠂⠔⠠ ⠸⠼⠆⠦⠠ ⠼⠒⠶⠠ ⠸⠼⠲⠲⠠ ⠼⠢⠂⠠ ⠼⠖⠶⠠ ⠸⠼⠶⠆⠠
⠸⠼⠦⠴ ⠸⠱⠲

7.7.5 **Typeform Indicators with Numeral/Symbol Combinations.** Rules regarding significant typeform applied to a numeral/symbol combination such as 49% or $5 will be discussed in Lesson 11.

PRACTICE 7H

1) The following math sentence represents pairs of socks in Nate's sock drawer. Barred numbers indicate pairs of blue socks; bold numbers indicate pairs of red socks.

$$\mathbb{4} + \mathbf{1} - \mathbb{1} + \mathbf{3} - \mathbf{1} - \mathbb{1} + \mathbb{2} - \mathbf{1}$$

Clean socks *added* from the wash are after a plus sign ("+"). Socks which Nate wore and put into the laundry hamper are after a minus sign ("−"). How many pairs of red socks are in Nate's drawer today?

2) If the boldface number signifies a **withdrawal** from your account, can you explain why $250 + \mathbf{250} = 0$?

7.8 Nonregular Typeform in Contact with a Grouping Symbol

Because a letter or numeral in nonregular type must include a numeric or alphabetic indicator, rules regarding such letters or numerals when in contact with grouping signs are different from rules regarding letters or numerals in regular type. A numeric or alphabetic indicator is required for numbers or letters in nonregular type when touching or enclosed between grouping symbols.

Example 7-27

A boldface zee ($\mathbf{Z}$) indicates significant decline. The boldface zero ($\mathbf{0}$) indicates the null vector.

Example 7-28

Collecting typically $m = 10^4$ such spectra, $\mathbf{X}_j(t)$ produces a smooth spectrum $\langle \mathbf{X}(t) \rangle$.

Compare the letter "t" enclosed in parentheses, which does not need an alphabetic indicator because it is in regular type, to the second letter "X" enclosed in angle brackets, which requires an English-letter indicator because it is in boldface.

Example 7-29

... where $\mu = \mathrm{E}[\mathbf{X}]$ and $|\mathbf{\Sigma}|$ is the determinant of $\mathbf{\Sigma}$.

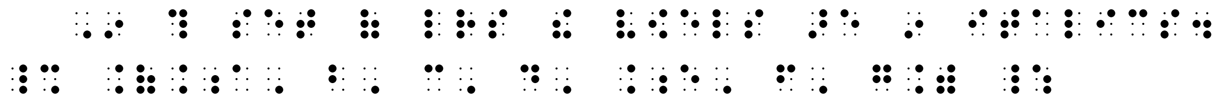
7.8.1 **Nonregular Typeform in an Enclosed List.** Recall that English letters in an enclosed list do not use an English-letter indicator. (See Lesson 4.) When letters or numerals in an enclosed list are printed in nonregular typeform and the variant typeform is retained, each typeform indicator is followed by an appropriate letter indicator or numeric indicator.

⫸ $\langle \mathbf{u}, \mathbf{v} \rangle$

⫸ $(\mathbf{0}, \mathbf{0})$

Example 7-30

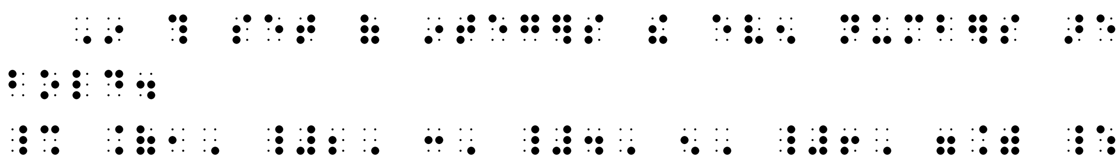
In this set of letters the vowels are in italics. {a, b, c, d, *e*, f, g}



In an enclosed list, English letters in regular type do not require an English-letter indicator.

Example 7-31

In this set of integers the even numbers are bold. $\{1, \mathbf{2}, 3, \mathbf{4}, 5, \mathbf{6}, 7\}$



In an enclosed list, numerals in regular type do not require a numeric indicator.

PRACTICE 7I

(1) For vectors $(\mathbf{a}, \mathbf{b}, \mathbf{c})$, can it be said that $\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$?

(2) $c(\mathbf{a}, \mathbf{b})=(c\mathbf{a}, \mathbf{b})$ as well as $(\mathbf{a}, c\mathbf{b})$. **a** and **b** are **vectors.** Define $\mathbf{ab}$.

7.9 Boldface Mathematical Symbols [NC 7.5]

Dots 456 can be applied only to certain specific math symbols. Each symbol consists of dots 456 followed by the appropriate symbol. (456) is considered to be an actual part of the symbol and must not be considered to be a boldface typeform indicator. As such, do not use dots 456 with any sign other than those shown in this section.

7.9.1 **Signs of Operation in Boldface Type.** The signs of operation listed below are to be used to show boldface type only when the distinction between the regular and the boldface forms of the same sign has mathematical significance. The surrounding text should be examined to determine if this is the case.

⠸⠬	Boldface Plus	**+**
⠸⠤	Boldface Minus	**−**
⠸⠬⠐⠸⠤	Bold Plus Followed by Bold Minus	**+−**
⠸⠬⠐⠤	Bold Plus Followed by Regular Minus	**+**−
⠬⠐⠸⠤	Regular Plus Followed by Bold Minus	+**−**
⠸⠤⠐⠸⠬	Bold Minus Followed by Bold Plus	**−+**
⠸⠤⠐⠬	Bold Minus Followed by Regular Plus	**−**+
⠤⠐⠸⠬	Regular Minus Followed by Bold Plus	−**+**

7.9.2 **Equals Sign in Boldface Type.** When it is necessary to show that an equals sign is printed in boldface type, dots 456 are placed before the equals symbol. Boldface equals signs are used only when the distinction between the regular and boldface forms of the same sign has mathematical significance. The surrounding text should be examined to determine if this is the case.

⠸⠨⠅	Boldface Equals	**=**

Boldface signs are used in vector equations to emphasize the distinction between vector and scalar mathematical operations, as the following examples illustrate.

Example 7-32

Vector Addition In **C = A + B**, the boldface plus sign emphasizes that adding two vector quantities requires a geometrical process and is not the same operation as adding two scalar quantities such as $2+3=5$.

Example 7-33

We define the *difference* of two vectors **A** and **B** to be the vector sum of **A** and **−B**:

A − B = A + (−B)

7.9.3 **Grouping Signs in Boldface Type.** When brackets or vertical bars are printed in mathematically significant boldface, dots 456 are placed before the grouping symbol.

⠸⠈⠷	Boldface Left Bracket	**[**
⠸⠈⠾	Boldface Right Bracket	**]**
⠸⠳	Boldface Vertical Bar	**|**
⠸⠳⠸⠳	Boldface Double Vertical Bar	**||**

Double boldface vertical bars are usually read as "the norm of."

⫸ **||**f**||** ⠸⠳⠸⠳⠋⠸⠳⠸⠳

Boldface brackets are often used to designate the "integer function".

⫸ **[**x**]** ⠸⠈⠷⠭⠸⠈⠾

Instructions: Follow the print format of a 3-column list. Place the opening Nemeth Code indicator in cell 1. After one blank line, begin the first row in cell 1. After the third row, another blank line must precede the Nemeth Code terminator, which is also placed in cell 1. Sentence A will then begin on the next line.

PRACTICE 7J

=	**+** **−**	**−** **+**
+	**+** −	**−** +
−	+ **−**	− **+**

A. In older texts, the greatest integer function may be notated with a bold bracket: **[**x**]**.

B. **||**Y**||** means "the norm of Y".

7.10 Barred Grouping Symbols and Other Signs of Grouping [NC Rule 19]

While we are on the topic of barred typeform, this is a good time to introduce the rest of the grouping signs for which the Nemeth Code has devised symbols, since four of them are barred.

7.10.1 **Barred Brackets and Barred Braces.** Use the symbols in the box below when barred brackets or barred braces are encountered. Notice that the barred grouping symbols are formed by inserting dots 456 before the second cell of the normal grouping symbol.

⠈⠸⠷	Left Barred Bracket	$\llbracket$
⠈⠸⠾	Right Barred Bracket	$\rrbracket$
⠨⠸⠷	Left Barred Brace	$\lBrace$
⠨⠸⠾	Right Barred Brace	$\rBrace$

⪢ $\llbracket x \rrbracket$ ⠈⠸⠷⠭⠈⠸⠾

⪢ $\lBrace abc \rBrace$ ⠨⠸⠷⠁⠃⠉⠨⠸⠾

7.10.2 **Half Brackets.** Use the symbols in the box below when half brackets are encountered. Notice that these symbols are formed by inserting dots 45 (upper) or dots 56 (lower) before the second cell of the normal bracket symbols.

⠈⠘⠷	`Upper Left Half Bracket`	⌈ or ⌜
⠈⠘⠾	`Upper Right Half Bracket`	⌉ or ⌝
⠈⠰⠷	`Lower Left Half Bracket`	⌊ or ⌞
⠈⠰⠾	`Lower Right Half Bracket`	⌋ or ⌟

⋙ ⌞ x ⌟ or $\lfloor x \rfloor$ ⠈⠰⠷⠭⠈⠰⠾

⋙ $A_{n\rceil i}$ ⠠⠁⠰⠝⠈⠘⠾⠊

Example 7-34

Integer division can be defined as $a \backslash b \equiv \lfloor a / b \rfloor$, where "/" denotes normal division and $\lfloor x \rfloor$ is the floor function.

Example 7-35

The result of nested floor or ceiling functions is the innermost function:

$$\lfloor \lceil x \rceil \rfloor = \lceil x \rceil,$$
$$\lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor.$$

Lines 1-2: Narrative paragraph (3-1).
Line 3: Math displayed to unitemized text begins in cell 3.
Line 4: The second displayed item begins in cell 3, matching the print layout. See 7.1.3.c.

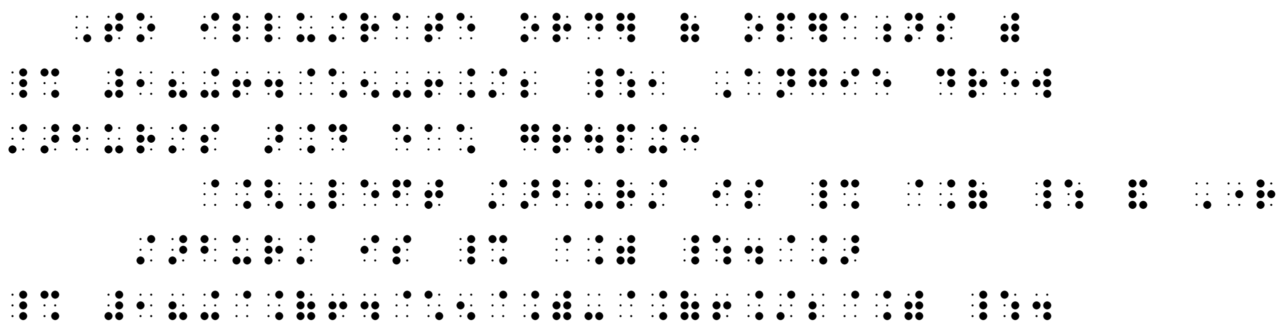
7.10.3 **Transcriber-Devised Grouping Symbols.** If a mathematical sign of grouping is not represented in the Nemeth Code, the transcriber devises one, using two or more braille symbols whose last cell is dots 12356 for the left symbol and whose last cell is dots 23456 for the right symbol. The transcriber-devised symbol must be identified in a transcriber's note or listed on the Special Symbols page at the beginning of the volume according to the guidelines given in *Braille Formats*.

Sample transcriber's note:

Left starbust is ⠈⠨⠯ and right starbust is ⠈⠨⠾.

Example 7-36

To illustrate order of operations with $18 + 64 \times 5 - 6 \div 2$, Angie drew starbursts around each grouping: 18 + ⪾64×5⪽ – ⪾6÷2⪽



PRACTICE 7K

1) Describe how the functions $f(x) = 3 + [\![x]\!]$ and $g(x) = 3 - [\![-x]\!]$ differ.

2) $\lfloor x \rfloor = m$ if and only if $m \leq x < m + 1$; $\lceil x \rceil = n$ if and only if $n - 1 < x \leq n$.

7.11 Further Details Regarding Typeform of Letters and Numerals

7.11.1 **Typeform with Subscripts.** Regarding the special subscript rule where the subscript indicator is not used for a numeral that is a right subscript to a letter, the letter may be in any typeform. (Review 6.11 in Lesson 6.)

⫸ $\boldsymbol{i}_1$ ⠸⠰⠊⠂ (Bold English letter i, subscript one)

7.11.2 **Typeform with Unspaced Mathematical Expressions.** Recall that an English-letter indicator is not used in an unspaced mathematical expression. (See 3.12.3 in Lesson 3.) The rule applies only to an English letter in regular type, or an italicized letter when the italics are disregarded in braille. If an English letter is printed in a mathematically significant typeform, an alphabetic indicator is always required.

Compare these transcriptions of the letter "i" in regular type and bold type.

⫸ $3i$ *(Spoken: three i)* ⠼⠒⠊

⫸ $3\boldsymbol{i}$ *(Spoken: three bold i)* ⠼⠒⠸⠰⠊

⫸ 3_i *(three, subscript i)* ⠼⠒⠰⠊

⫸ 3_i *(three, subscript bold i)* ⠼⠒⠰⠸⠰⠊

⫸ δ_{ij} *(delta, subscripts bold i and bold j)* ⠨⠙⠰⠸⠰⠊⠸⠰⠚

7.11.3 **Underlining and Other Typeforms.** There is no underline indicator in the Nemeth Code. Underlining of letters, numbers, and mathematical symbols will be discussed in Lesson 12.

Typeforms for which there are no provisions in the Nemeth Code may use one of the five typeform indicators that is not used elsewhere in the document. A transcriber's note should explain the substitution.

Sample transcriber's note:

> ⠈⠼ indicates red numbers. ⠠⠨⠼ indicates blue numbers.

Here is Nate's sock drawer again, substituting the script and sans serif typeform indicators for the colored type.

> *Note to students reading from a printout: Colored type appears in the next example. Some numbers are blue and some are red.*

Example 7-37

This expression represents pairs of socks in Nate's sock drawer: $4+1-1+3-1-1+2-1$.
... How many pairs of red socks are in Nate's drawer today?

The UEB underlined word indicator is used in the narrative text.

7.11.4 **Termination of a UEB Typeform Passage.** UEB typeform is terminated by the opening Nemeth indicator. An explicit UEB typeform terminator is not needed. If the nonregular typeform continues after the termination of the mathematical portion, a UEB typeform indicator must be re-entered.

Example 7-38

The null vector 0 is also called the *isotropic vector*.

A UEB boldface terminator is not needed after the bold paragraph heading because it is immediately followed by an opening Nemeth Code indicator. Typeform is retained for the numeral zero because it is mathematically significant—the bold typeform identifies it as the null vector.

Example 7-39

In *three-dimensional Euclidean space,* ***R*** *vectors* are identified with triples of scalar components.

The UEB italics are terminated by the opening Nemeth indicator. Boldface is retained for the letter R because it is mathematically significant—the bold typeform identifies it as a vector. However, the italic typeform is disregarded, *according to normal practice for italicized letters.UEB italics are reapplied on the word "vectors".*

Example 7-40

In a complex number of the form $z = a + bi$, a *is the real part of the complex number* z.

Italics are disregarded for the math equation, according to normal practice. The UEB italic passage indicator and terminator are used for the italicized phrase.

Example 7-41

Energy and mass are equivalent, *which is the message of* $E = mc^2$.

The UEB italics are terminated by the opening Nemeth indicator. Italics are disregarded for the math equation, according to normal practice.

a. **Capitalization.** Capitalization is not a typeform and must be explicitly terminated.

Example 7-42

THE NULL VECTOR **0** is known as the *null vector*.

A capitals terminator is required at the end of this paragraph heading.

PRACTICE 7L

Assume that the first pair to be repeated is (r_k, r_{k+1}) for $k \geq 0$. In the sequence of pairs, there is a later pair (r_n, r_{n+1}) equal to (r_k, r_{k+1}) with $m^2 + 1 \geq n + 1 > k + 1$. But, *since the pairs are equal,* $r_k = r_n$ *and* $r_{k+1} = r_{n+1}$.

For further practice, see Appendix A—Reading Practice.

EXERCISE 7

Prepare Exercise 7 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 7A

1 ⠠⠏⠕⠇⠽⠝⠕⠍⠊⠁⠇⠎

2 ⠠⠎⠕⠇⠧⠑ ⠹ ⠏⠕⠇⠽⠝⠕⠍⠊⠁⠇ ⠥⠎⠬ ⠃⠁⠎⠊⠉ ⠁⠇⠛⠑⠃⠗⠁⠲

3 ⠨⠂⠠⠓⠔⠞⠒ ⠠⠋⠌ ⠋⠁⠉⠞⠕⠗ ⠳ ⠦⠰⠭⠴ ⠞⠕ ⠍⠁⠅⠑ ⠭ ⠁

4 ⠟⠥⠁⠙⠗⠁⠞⠊⠉ ⠑⠟⠥⠁⠰⠝⠲

5 ⠸⠩ ⠭⠘⠒⠐⠬⠆⠭⠘⠆⠐⠤⠭ ⠨⠅ ⠭⠷⠭⠘⠆⠐⠬⠆⠭⠤⠂⠾ ⠸⠱

6 ⠠⠙ ⠽ ⠝⠕⠞⠊⠉⠑ ⠁ ⠋⠁⠍⠊⠇⠊⠜ ⠏⠁⠞⠞⠻⠝⠦

7

8 ⠠⠎⠑⠟⠥⠢⠉⠑⠎

9 ⠠⠁ ⠎⠑⠟⠥⠰⠑ ⠸⠩ ⠁⠂⠠ ⠁⠆⠠ ⠁⠒⠠ ⠄⠄⠄⠠ ⠁⠰⠝ ⠸⠱

10 ⠊⠎ ⠎⠙ ⠞⠕ ⠨⠂⠒⠧⠻⠛⠑ ⠊⠋ ⠐⠮ ⠑⠭⠊⠌⠎ ⠁ ⠏⠕⠎⠊⠞⠊⠧⠑

11 ⠝⠥⠍⠃⠻ ⠰⠠⠍ ⠎⠡ ⠞⠂ ⠿ ⠑⠁⠡ ⠸⠩ ⠓ ⠨⠂ ⠼⠴⠠

12 ⠳⠁⠰⠝⠐⠤⠠⠁⠳ ⠐⠅ ⠓ ⠸⠱⠂ ⠿ ⠁⠇⠇ ⠸⠩ ⠝ ⠨⠂ ⠠⠍

13 ⠸⠱⠲

14 ⠠⠁ ⠎⠑⠟⠥⠰⠑ ⠞ ⠙⠕⠑⠎ ⠝ ⠒⠧⠻⠛⠑ ⠊⠎ ⠎⠙ ⠞⠕

15 ⠨⠂⠙⠊⠧⠻⠛⠑⠲

16

17 ⠠⠔⠑⠟⠥⠁⠇⠊⠞⠊⠑⠎

18 ⠠⠝⠪ ⠺⠑ ⠺ ⠥⠎⠑ ⠝⠥⠍⠃⠻ ⠇⠔⠑⠎ ⠞⠕ ⠊⠇⠇⠥⠌⠗⠁⠞⠑ ⠮

19 ⠋⠕⠇⠇⠪⠬ ⠔⠑⠟⠥⠁⠇⠊⠞⠊⠑⠎⠲ ⠸⠩

20 ⠤⠼⠖ ⠐⠅ ⠤⠼⠢

21 ⠼⠴ ⠐⠅ ⠬⠖

22 ⠤⠼⠦ ⠐⠅ ⠬⠆

23 ⠤⠼⠂ ⠨⠂ ⠤⠼⠢ ⠸⠱

NARRATIVE FORMAT (3-1)
Line 5: Math displayed to (3-1) narrative begins in cell 3. This displayed expression has no runover.
Line 6: The (3-1) paragraph continues in cell 1.
Line 12: Math displayed to (3-1) narrative begins in cell 3. Nemeth Code is already in effect.
(The commentary continues on the next page.)

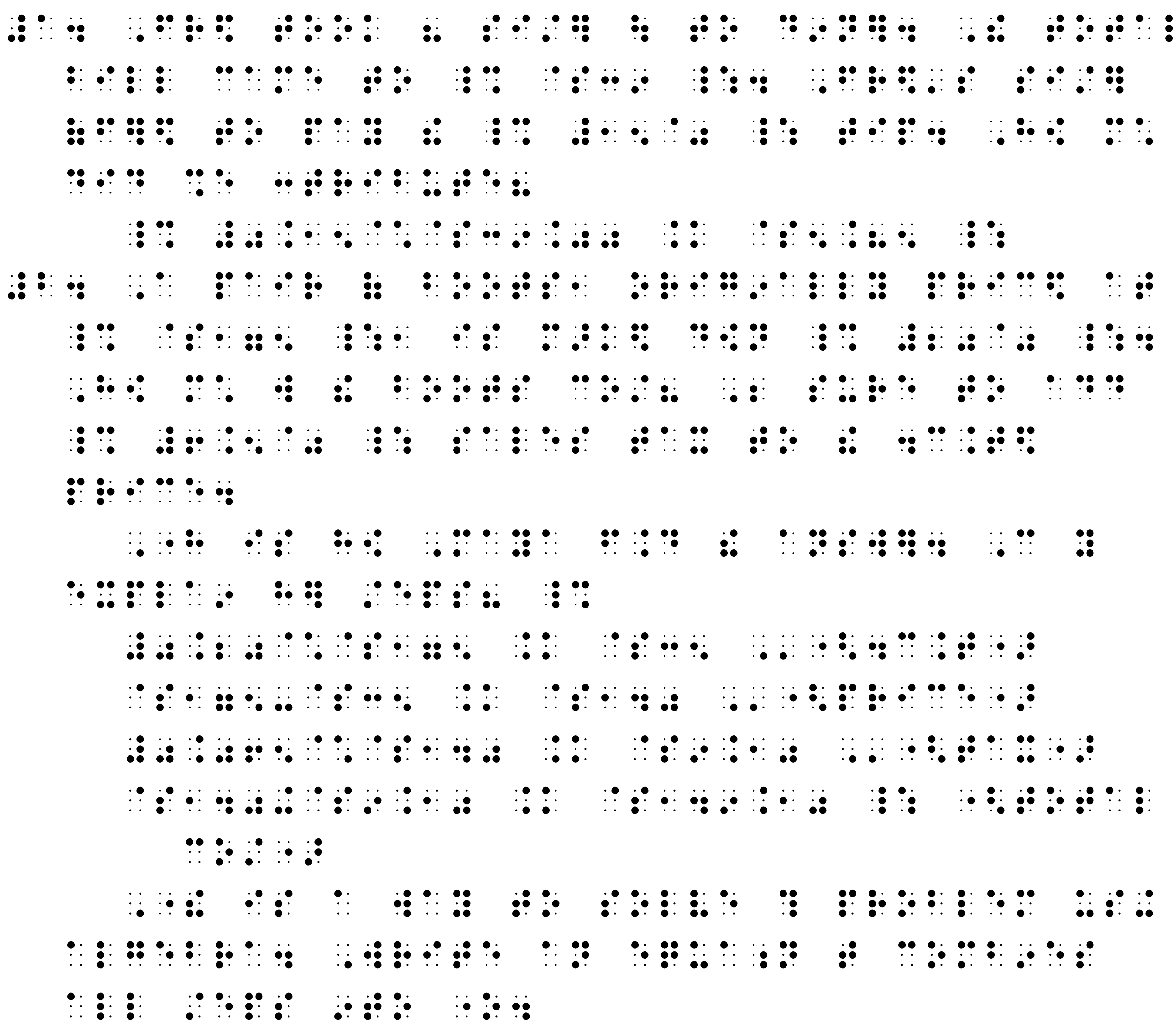
Practice 7A, continued

Line 13: The Nemeth Code terminator does not fit on the previous line. It is placed in the runover cell of the displayed expression, cell 5.

Lines 14-15: A new narrative paragraph (3-1).

Lines 20-23: Each displayed expression begins in cell 3. Note that, in print, the four expressions are printed widely spaced on one line.

PRACTICE 7B



ITEMIZED FORMAT (1-3)

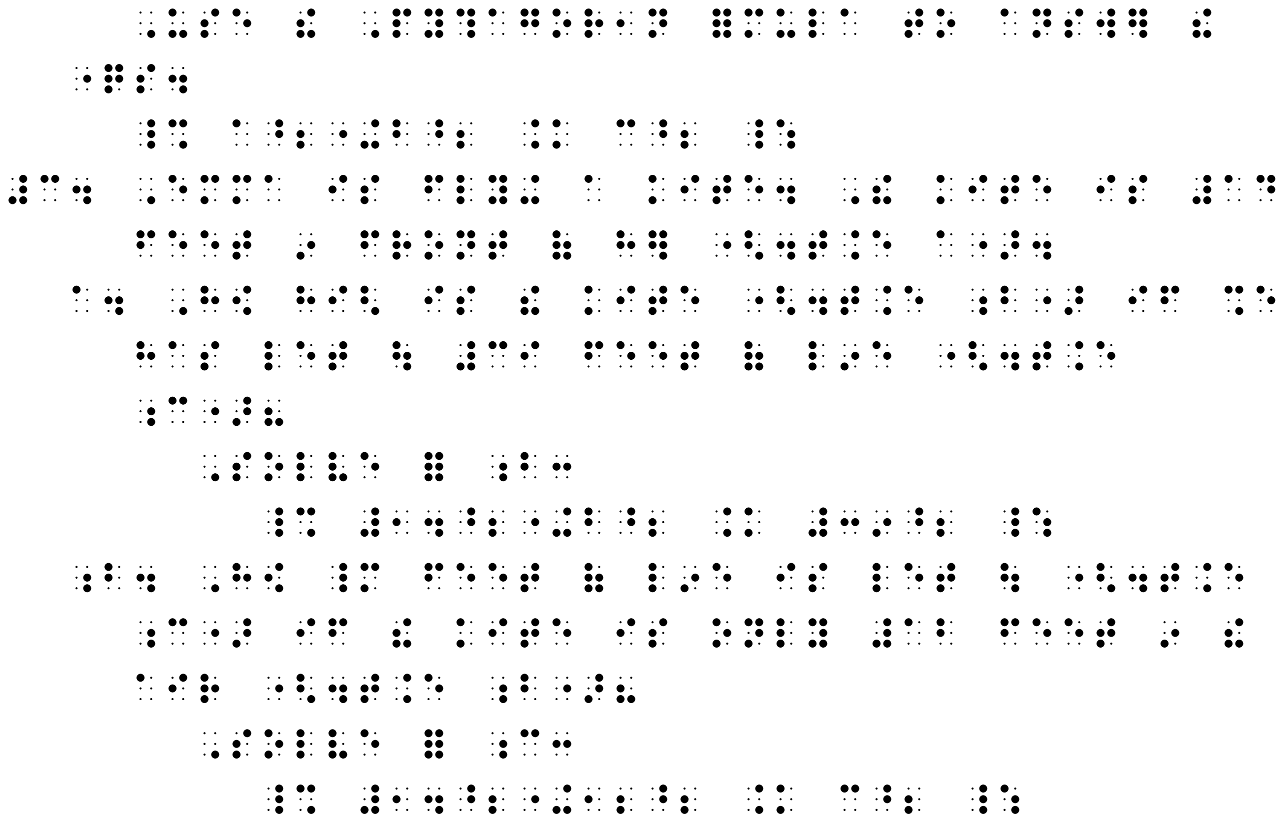
Line 5: Math displayed to (1-3) itemized material begins in cell 5. This displayed expression has no runover.

Lines 11-12: The subparagraph begins in cell 5, with runovers in cell 3.

Lines 13-17: Each displayed step begins in cell 5. The last step has a runover, in cell 7.

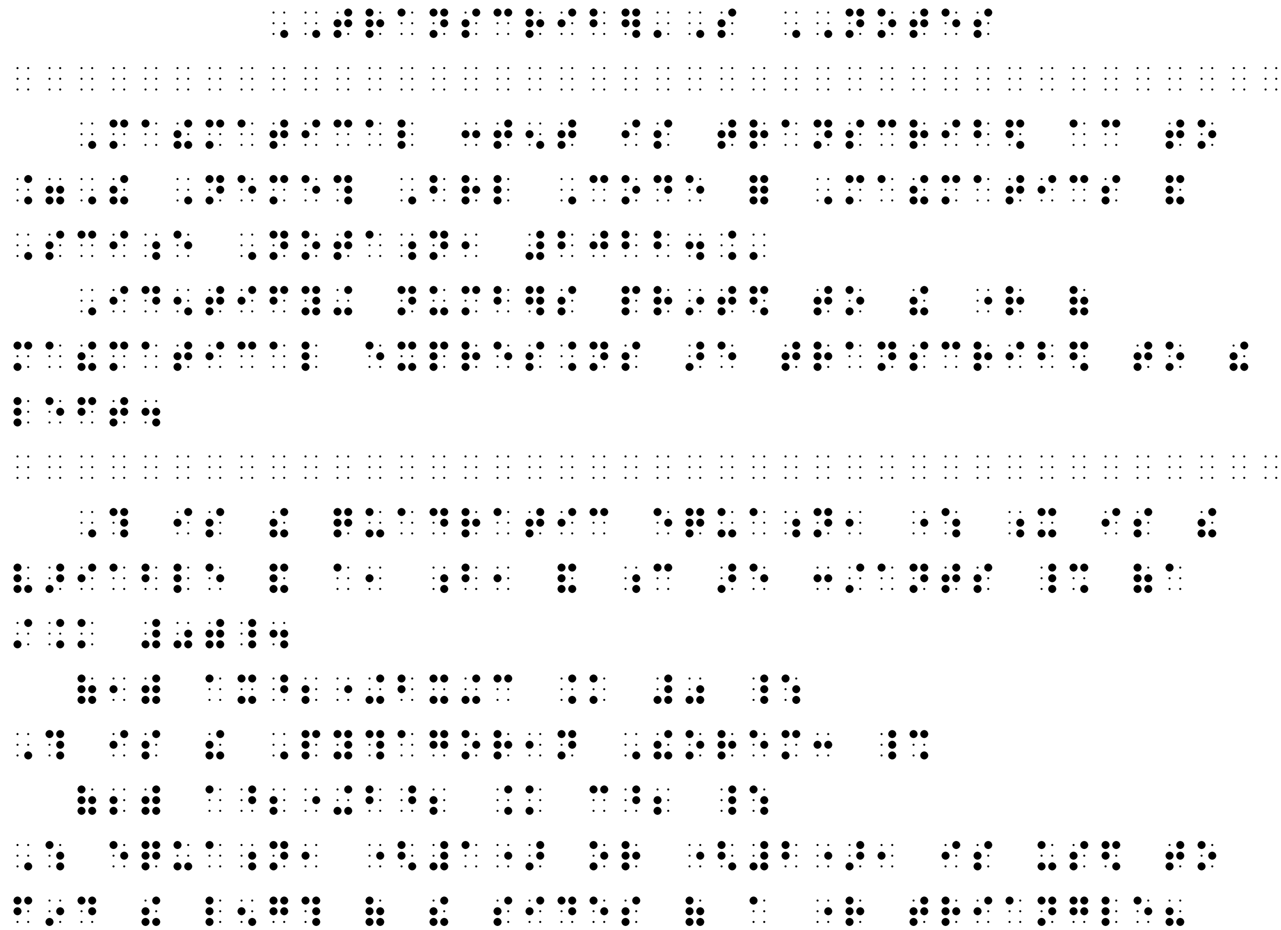
Lines 18-20: A new subparagraph begins in cell 5, with runovers in cell 3.

PRACTICE 7C



INSTRUCTIONS (5-3) and ITEMIZED FORMAT WITH SUBENTRIES (1-5; 3-5)
Lines 1-2: Instructions (5-3). The italic typeform is disregarded.
Line 3: Math displayed to instructions begins in cell 5. This expression has no runovers.
Lines 4-5: The item identifier begins in cell 1. Runovers are in cell 5 because this item has subentries.
Lines 6-8: The subdivision identifier begins in cell 3. Runovers are in cell 5.
Lines 9-10: Math displayed to a subentry begins in cell 7 with runovers in cell 9. The math and its code switches fit on one line (line 10).
Lines 11-13: A new subdivision identifier begins in cell 3. Runovers are in cell 5.
Lines 14-15: Math displayed to a subentry begins in cell 7 with runovers in cell 9. The math and its code switches fit on one line (line 15).

PRACTICE 7D



Lines 10-12: Narrative paragraph begins in cell 3 with runovers in cell 1.

Line 13: The displayed math expression begins with its label in cell 3. The label is in Nemeth because the preceding material is in Nemeth.

Line 14: The paragraph continues in cell 1. This transcription shows the opening Nemeth Code indicator at the end of this line in order to transcribe the following label in Nemeth. The decision to show both labels in the same code is not a requirement.

Line 15: The displayed math expression begins with its label in cell 3.

Lines 16-17: The paragraph continues in cell 1. The equation labels are transcribed in UEB because the context is UEB—they do not need to match the code in which the labels are first shown.

PRACTICE 7E

1 ⠸⠩ ⠠⠸⠰⠠⠓ ⠠⠸⠰⠊ ⠠⠸⠰⠠⠟ ⠠⠸⠰⠙

2 ⠠⠸⠨⠠⠎ ⠠⠸⠨⠛ ⠠⠸⠨⠠⠛ ⠠⠸⠨⠏

3 ⠸⠰⠁ ⠸⠰⠠⠃ ⠸⠰⠉ ⠸⠰⠠⠙

4 ⠸⠨⠗ ⠸⠨⠠⠽ ⠸⠨⠠⠋ ⠸⠨⠯

5 ⠸⠈⠈⠠⠟ ⠸⠈⠈⠵ ⠸⠈⠈⠱ ⠸⠈⠈⠠⠫

6 ⠨⠸⠧ ⠨⠸⠠⠚ ⠨⠸⠽ ⠨⠸⠠⠉

7 ⠨⠈⠈⠪ ⠨⠈⠈⠠⠑ ⠨⠈⠈⠱ ⠨⠈⠈⠠⠫

8 ⠠⠨⠰⠠⠅ ⠠⠨⠰⠠⠗ ⠠⠨⠰⠓ ⠠⠨⠰⠏

9 ⠈⠰⠋ ⠈⠰⠠⠛ ⠈⠰⠓ ⠈⠰⠠⠵

10 ⠈⠠⠠⠵ ⠈⠠⠠⠫ ⠈⠠⠠⠛ ⠈⠠⠠⠟ ⠸⠱

PRACTICE 7F

1 ⠸⠩ ⠸⠰⠠⠍⠠⠸⠰⠠⠍⠈⠰⠠⠍⠠⠍⠠⠨⠰⠠⠝

2 ⠠⠨⠰⠽⠈⠰⠽⠠⠸⠰⠽⠸⠰⠽⠽

3 ⠠⠸⠨⠠⠎⠸⠨⠠⠎⠨⠠⠎

4 ⠠⠸⠨⠏⠨⠏⠸⠨⠏

5 ⠨⠙⠙⠈⠰⠙⠸⠰⠠⠙⠨⠇⠈⠰⠠⠝ ⠸⠱

PRACTICE 7G

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PRACTICE 7H

PRACTICE 7I

PRACTICE 7J

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PRACTICE 7K

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⠼⠆⠾ ⠈⠰⠷⠭⠈⠰⠾ ⠨⠅ ⠍ ⠸⠱ ⠊⠋ ⠭ ⠕⠝⠇⠽ ⠊⠋
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PRACTICE 7L

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Line 8: Recall from Lesson 3 that the single-word switch indicator can be used with a word associated with a UEB typeform word indicator.

LESSON 8

- INTRODUCTION TO FRACTIONS
  - Simple Fractions
  - Mixed Numbers
  - Complex Fractions
  - More Fraction Rules
- RADICAL EXPRESSIONS

Format

- LINKED EXPRESSIONS
  - Division of Linked Expressions
  - Nested Linked Expression

Answers to Practice Material

LESSON PREVIEW

Fractions, mixed numbers, and radical expressions are studied. The "linked expression" is defined and its format is discussed in detail, including the special case of the nested linked expression.

Note: The font size in some of the examples is larger than standard, for clarity. Some of the larger symbols may appear to be bold. Do not consider the size difference or the darker image to be a variant typeform.

INTRODUCTION TO FRACTIONS

Two types of fractions are presented in this lesson: simple fractions (including mixed numbers) and complex fractions.

A fraction is composed of three parts: a numerator, a denominator, and a fraction line.

numerator ↘

$$\frac{3}{4} \leftarrow \textit{fraction line}$$

denominator ↗

Fractions are printed in a variety of ways. The numerator may be printed above the denominator or they may be printed on the same level. The fraction line may be horizontal or diagonal. Here are three examples of the fraction "three fourths" printed in different styles.

$$\frac{3}{4} \qquad {}^{3}/_{4} \qquad 3/4$$

The numerator and/or denominator may also consist of or contain words or abbreviations. Here are four examples.

m/s	(*Spoken:* meters per second)
$^{\text{ft.}}/_{\text{sec.}}$	(*Spoken:* feet per second)
$\frac{\text{rise}}{\text{run}}$	(*Spoken:* rise over run)
3.5%/year	(*Spoken:* 3.5 percent per year)

In a technical transcription, fractions are transcribed in Nemeth.

Simple Fractions

[NC 13.1]

8.1 Definition

For the purposes of the Nemeth Code, a simple fraction is one whose numerator and denominator contain no fractions, except possibly at the superscript or subscript level. These are simple fractions:

$$\frac{1}{2} \qquad \frac{a^2}{b^2} \qquad \frac{72\text{ mi.}}{4\text{ hr.}} \qquad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}} \qquad {}^{5}/_{12}$$

This is <u>not</u> a simple fraction because the numerator contains a fraction: $\frac{1/3}{5}$

8.2 Simple Fraction Indicators

Regardless of print layout, unless otherwise stated, the fraction is transcribed linearly so that the numerator, the fraction line, and the denominator are written horizontally across one braille line. When the numerator is printed at a higher level than the denominator, braille fraction indicators are used.

⠹	Opening Simple Fraction Indicator
⠼	Closing Simple Fraction Indicator

These fractions will use fraction indicators. $\frac{3}{4}$ $^{3}/_{4}$

This fraction will <u>not</u> use fraction indicators. 3/4 (See <u>8.4.2</u>, below.)

8.3 The Horizontal Simple Fraction Line

The type of fraction line used in the print copy (horizontal or diagonal) is replicated in the braille transcription. In a simple fraction, the horizontal fraction line is transcribed as the symbol shown below. Note that the horizontal fraction line symbol consists of one braille cell.

⠌	Horizontal Simple Fraction Line	—

⫸ $\frac{3}{4}$ ⠹⠒⠌⠲⠼

⫸ $\frac{d}{t}$ ⠹⠙⠌⠞⠼

a. **Spacing.** The numerator and denominator are unspaced from the fraction indicators and from the fraction line. Spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction.

b. **Keep Together.** A fraction must not be divided between braille lines. Fractions which do not fit on the line will be discussed in Lesson 15.

Example 8-1

Terry has 32 candy bars. She shares $\frac{3}{4}$ of them with her class.

⠀⠀⠠⠞⠻⠗⠽ ⠓⠁⠎ ⠼⠉⠃ ⠉⠯⠽ ⠃⠜⠎⠲ ⠠⠩⠑ ⠩⠜⠑⠎
⠸⠩ ⠹⠒⠌⠲⠼ ⠸⠱ ⠷ ⠮⠍ ⠾ ⠓⠻ ⠉⠇⠁⠎⠎⠲

Example 8-2

Rate formula: rate $= \frac{\text{distance}}{\text{time}}$ or $r = \frac{d}{t}$.

⠠⠗⠁⠞⠑ ⠿⠍⠥⠇⠁⠒
⠸⠩ ⠗⠁⠞⠑ ⠨⠅ ⠹⠙⠊⠎⠞⠁⠝⠉⠑⠌⠞⠊⠍⠑⠼ ⠠⠄⠕⠗
⠗ ⠨⠅ ⠹⠙⠌⠞⠼ ⠸⠱⠲

Reminder: Words in Nemeth are transcribed without contractions.

Example 8-3

Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{\Delta y}{\Delta x}$.

⠠⠎⠇⠕⠏⠑ ⠿⠍⠥⠇⠁⠒ ⠸⠩ ⠍ ⠨⠅ ⠹⠽⠆⠤⠽⠂⠌⠭⠆⠤⠭⠂⠼
⠠⠄⠕⠗ ⠍ ⠨⠅ ⠹⠨⠠⠙⠽⠌⠨⠠⠙⠭⠼ ⠸⠱⠲

Reminder: A numeric subscript does not need a subscript indicator when it is a right subscript to a letter.

Example 8-4

Multiplying fractions is easy! $\frac{3}{4} \cdot \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$

⠠⠍⠥⠇⠞⠊⠏⠇⠽⠬ ⠋⠗⠁⠉⠰⠝⠎ ⠊⠎ ⠑⠁⠎⠽⠖
⠸⠩ ⠹⠒⠌⠲⠼⠡⠹⠂⠌⠆⠼ ⠨⠅ ⠹⠒⠡⠂⠌⠲⠡⠆⠼ ⠨⠅ ⠹⠒⠌⠦⠼ ⠸⠱

Spacing: There is no space before or after the operation signs (multiplication dots); there is a space before and after the comparison signs (equals signs).

Example 8-5

Use the reciprocal of` the coefficient to solve for x in $\frac{3}{8}x = 72$.

⠠⠥⠎⠑ ⠮ ⠗⠑⠉⠊⠏⠗⠕⠉⠁⠇ ⠷ ⠮ ⠉⠕⠑⠖⠊⠉⠊⠢⠞ ⠞⠕
⠎⠕⠇⠧⠑ ⠿ ⠰⠭ ⠔ ⠸⠩ ⠹⠒⠌⠦⠼⠭ ⠨⠅ ⠼⠶⠆ ⠸⠱⠲

Spacing: The coefficient (fraction) is unspaced from the variable (x).

Example 8-6

Anderson sprinted $\frac{2}{3}$ of the $\frac{1}{4}$–mile track.

⠠⠯⠻⠎⠕⠝ ⠎⠏⠗⠔⠞⠫ ⠸⠩ ⠹⠆⠌⠒⠼ ⠸⠱ ⠷ ⠮
⠸⠩ ⠹⠂⠌⠲⠼⠤⠍⠊⠇⠑ ⠸⠱ ⠞⠗⠁⠉⠅⠲

Spacing: The fraction in this hyphenated expression is unspaced from the hyphen.

Instructions: Transcribe the first two lines as a paragraph, with one space between expressions. A blank line must precede the itemized portion. When you proofread, check that you closed each fraction, that you returned to the baseline after each superscript, that displayed expressions are placed in the proper cell, and that you terminated Nemeth Code where appropriate.

PRACTICE 8A

Horizontal Simple Fraction Line

Here are some examples of simple fractions. $\frac{1}{2}$ $\frac{15}{16}$ $\frac{x}{y}$ $\frac{a+b}{c+d}$ $\frac{\Delta y}{\Delta x}$ $\frac{(x+y)}{(x-y)}$ $\frac{9}{12}$ $\left(\frac{3}{2}a+\frac{1}{2}b\right)$ $\frac{3x}{17y}$ $x-\frac{1}{4}(x-2x)$

1. $V=\frac{1}{3}\pi r^2 h$
2. $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$
3. $\left|\frac{a}{b}\right|=\frac{|a|}{|b|}$
4. Write an equation to show that $\frac{3}{4}$ of $\frac{1}{2}$ is $\frac{3}{8}$.
5. $x^2\,\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4x^2-x-2}{(x+1)(y+1)}$
6. Solve this differential equation:

   $$x\frac{\mathrm{d}y}{\mathrm{d}x}+2y=e^{x^2}$$

7. The number π is the ratio of the circumference of a circle to its diameter. That is,

   $$\pi=\frac{\text{circumference}}{\text{diameter}}.$$

8. $\frac{12}{33}=\frac{m}{11}$
9. $\frac{4}{32}=\frac{10.5}{x}$
10. $\frac{1}{4}+\frac{3}{4}-\frac{1}{2}=\frac{1}{2}$

8.4 The Diagonal Simple Fraction Line

The type of fraction line used in the print copy (horizontal or diagonal) is replicated in the braille transcription. In a simple fraction, the diagonal fraction line is transcribed as the symbol shown below. Note that the diagonal simple fraction line consists of two braille cells.

⠸⠌	Diagonal Simple Fraction Line	/

When a diagonal fraction line is printed, it may not be clear where the fraction begins and where it ends. The transcriber must not attempt to analyze the math. Instead, application of the following rules will prevent misinterpretation of the expression.

8.4.1 **Use of Simple Fraction Indicators with the Diagonal Simple Fraction Line.** When the numerator and denominator are printed at different levels of writing on either side of the diagonal line, the construction is a fraction and so simple fraction indicators are used. Do not confuse this type style with superscripts and subscripts. In this example, the numeral 3 is the numerator and the numeral 4 is the denominator.

⫸ $^{3}/_{4}$ ⠹⠒⠸⠌⠲⠼

Example 8-7

Terry has 32 bags of M&Ms. She shares $^{3}/_{4}$ of the bags with her class.

⠀⠀⠠⠞⠻⠗⠽ ⠓⠁⠎ ⠼⠉⠃ ⠃⠁⠛⠎ ⠷ ⠠⠍⠈⠯⠠⠍⠎⠲ ⠠⠩⠑ ⠩⠜⠑⠎
⠸⠩ ⠹⠒⠸⠌⠲⠼ ⠸⠱ ⠷ ⠮ ⠃⠁⠛⠎ ⠾ ⠓⠻ ⠉⠇⠁⠎⠎⠲

8.4.2 **Type Size on the Baseline of Writing.** If the numerator and denominator are printed at the same level of writing on either side of the diagonal line, the transcriber must notice the type size.

a. **Fraction in Smaller Type.** If the type is in a different size from that normally used for similar expressions throughout the text, identify this as a fraction by using simple fraction indicators. In the example below, note that the fraction is printed on the baseline of writing—it is not a subscript.

⫸ 2 $1/y$ ⠼⠆⠹⠂⠸⠌⠽⠼

The numeral 1 is smaller than the numeral 2. The space between the coefficient and the fraction is not shown in braille.

Example 8-8

In the expression 2 $1/y$, 2 is the coefficient of the fraction.

⠀⠀⠠⠔ ⠮ ⠑⠭⠏⠗⠑⠎⠨⠝ ⠸⠩ ⠼⠆⠹⠂⠸⠌⠽⠼ ⠸⠱⠂ ⠼⠃ ⠊⠎ ⠮
⠉⠕⠑⠖⠊⠉⠊⠢⠞ ⠷ ⠮ ⠋⠗⠁⠉⠰⠝⠲

b. **Fraction in Normal Sized Type.** When the numerator and denominator are printed at the same level of writing on either side of the diagonal line and the type size is normal when compared to similar expressions, fraction indicators are not used. This is a straightforward transcription of number-slash-number. A switch to Nemeth Code is required. (Review 5.4.8 "Slash".)

⋙ $3/4$

⋙ $1/y$

Example 8-9

Jean has 16 bags of Skittles. She shares $3/4$ of the bags with her class.

Compare the next example to Example 8-8. The first fraction is printed on the baseline but is in smaller type. Fraction indicators are used. The second fraction, however, is printed in normal type size and the numerator and denominator are at the same level of writing on either side of the diagonal line. This meets the requirements for nonuse of simple fraction indicators, even though the text refers to it as a "fraction."

Example 8-10

In the expression 2 $1/y$, 2 is the coefficient of the fraction $1/y$.

c. **Fraction in a Subscript.** In the next example, the fraction is a subscript. A subscript is printed in a smaller type. This fraction meets the requirements for nonuse of simple fraction indicators because (a) the numerator and denominator are printed at the same level of writing on either side of the diagonal line; and (b) the type size is normal for a subscript. Simple fraction indicators are not used.

⋙ $3_{x/y}$

Example 8-11

Notice the fraction in the subscript. $3_{x/y}$

According to Nemeth rules, fraction indicators are not used here, even though the text refers to the subscript as a "fraction".

8.4.3 **A Reminder About the UEB Slash.** When the slash is serving as a separator between words or numbers in literary context, it is not mathematical.

and/or, c/o	⠯⠸⠌⠕⠗⠂ ⠉⠸⠌⠕	(UEB)
true/false test	⠞⠗⠥⠑⠸⠌⠋⠁⠇⠎⠑ ⠞⠑⠌	(UEB)
6/6/44 (date)	⠼⠋⠸⠌⠼⠋⠸⠌⠼⠙⠙	(UEB)

Example 8-12

On 11/15/59 U.S. Patent #14–5/2806ds-XRX was renamed *xerography* and Haloid/Xerox launched the world of photocopying into a new era with the Xerox 914/01.

⠠⠕⠝ ⠼⠁⠁⠸⠌⠼⠁⠑⠸⠌⠼⠑⠊ ⠠⠥⠲⠠⠎⠲ ⠠⠏⠁⠞⠢⠞
⠸⠹⠼⠁⠙⠠⠤⠼⠑⠸⠌⠼⠃⠓⠚⠋⠰⠙⠎⠤⠠⠠⠭⠗⠭ ⠴ ⠗⠑⠐⠝⠙
⠨⠂⠭⠻⠕⠛⠗⠁⠏⠓⠽ ⠯ ⠠⠓⠁⠇⠕⠊⠙⠸⠌⠠⠭⠻⠕⠭ ⠇⠁⠥⠝⠡⠫ ⠮ ⠸⠺
⠷ ⠏⠓⠕⠞⠕⠉⠕⠏⠽⠬ ⠔⠞⠕ ⠁ ⠝⠑⠺ ⠻⠁ ⠾ ⠮ ⠠⠭⠻⠕⠭
⠼⠊⠁⠙⠸⠌⠼⠚⠁⠲

Numerals printed with slashes such as dates, model numbers, etc. are not mathematical and therefore the rules of UEB are followed.

a. **Which Code?** When a slash represents a mathematical operation meaning *per*, *over*, or *divided by*, Nemeth fraction rules are followed. When a slash does not mean *per*, *over*, or *divided by*, the construction is transcribed in UEB unless it is part of a larger mathematical expression.

Example 8-13

The input/output ratio is equal.

⠠⠮ ⠔⠏⠥⠞⠸⠌⠳⠞⠏⠥⠞ ⠗⠁⠞⠊⠕ ⠊⠎ ⠑⠟⠥⠁⠇⠲

The slash means "to" and so is transcribed in UEB.

Example 8-14

The red/white probabilities are the same.

⠠⠮ ⠗⠫⠸⠌⠱⠊⠞⠑ ⠏⠗⠕⠃⠁⠃⠊⠇⠊⠞⠊⠑⠎ ⠜⠑ ⠮ ⠎⠁⠍⠑⠲

The slash means "or" and so is transcribed in UEB.

8.4.4 **Spacing.** Words or abbreviations in the numerator or denominator are unspaced from the fraction indicators and are also unspaced from the fraction line.

Example 8-15

Express m/s in mph.

⠠⠑⠭⠏⠗⠑⠎⠎ ⠸⠩ ⠰⠍⠸⠌⠰⠎ ⠸⠱ ⠔ ⠍⠏⠓⠲

Because the slash means "per" ("meters per second"), Nemeth is required.

Example 8-16

Express $^{\text{ft.}}/_{\text{sec.}}$ in mph.

⠠⠑⠭⠏⠗⠑⠎⠎ ⠸⠩ ⠹⠋⠞⠲⠸⠌⠎⠑⠉⠲⠼ ⠸⠱ ⠔ ⠍⠏⠓⠲

Because the numerator is printed higher than the denominator, fraction indicators are used.

Example 8-17

Express the rate in miles/hour.

⠠⠑⠭⠏⠗⠑⠎⠎ ⠮ ⠗⠁⠞⠑ ⠔ ⠸⠩ ⠍⠊⠇⠑⠎⠸⠌⠓⠕⠥⠗ ⠸⠱⠲

Because the slash means "per" ("miles per hour"), Nemeth is required. No contractions are used.

8.4.5 **Code Switch Reminders.** Review these points from Lesson 3.

a. **Words.** As part of a math problem expressed in symbols and words, the words are included in the switch. Compare the next two examples.

Example 8-18

$^1/_4$ of 24 is 6

⠸⠩ ⠹⠂⠸⠌⠲⠼ ⠸⠱ ⠷ ⠼⠃⠙ ⠊⠎ ⠼⠋

Only the fraction requires a switch to Nemeth Code because the math statement is expressed in words.

Example 8-19

$^1/_4$ of 24 = 6

⠸⠩ ⠹⠂⠸⠌⠲⠼ ⠕⠋ ⠼⠆⠲ ⠨⠅ ⠼⠖ ⠸⠱

The presence of the equals sign turns the entire expression into a Nemeth construction. The word "of" is part of the equation and is uncontracted in Nemeth.

b. **Abbreviations.** An abbreviation associated with a Nemeth item is included inside the switches.

Example 8-20

Now add 2/3 c. cornstarch—Oobleck!

⠠⠝⠪ ⠁⠙⠙ ⠸⠩ ⠼⠆⠸⠌⠒ ⠰⠉⠲ ⠸⠱ ⠉⠕⠗⠝⠌⠜⠡⠠⠤
⠠⠕⠕⠃⠇⠑⠉⠅⠖

The abbreviation is included inside the switch, along with its associated fraction. Fraction indicators are not used because the numerator and denominator are printed on the baseline of writing.

Example 8-21

To convert feet to inches, change $^1/_2$ ft. to 6 in. Now you can add 6 in. to 3 in. to get 9 in.

⠠⠞⠕ ⠒⠧⠻⠞ ⠋⠑⠑⠞ ⠞⠕ ⠔⠡⠑⠎⠂ ⠡⠁⠝⠛⠑
⠸⠩ ⠹⠂⠸⠌⠆⠼ ⠋⠞⠲ ⠸⠱ ⠞⠕ ⠼⠋ ⠊⠝⠲ ⠠⠝⠪ ⠽ ⠉ ⠁⠙⠙
⠼⠋ ⠊⠝⠲ ⠞⠕ ⠼⠉ ⠊⠝⠲ ⠞⠕ ⠛⠑⠞ ⠼⠊ ⠊⠝⠲

Recall that, in braille, the abbreviation must fall on the same line as its associated value.

Example 8-22

Equivalent ratios of $^y/_x$ (or $^x/_y$) can be seen in Table 4.1.

⠠⠑⠟⠥⠊⠧⠁⠇⠢⠞ ⠗⠁⠞⠊⠕⠎ ⠷ ⠸⠩ ⠹⠽⠸⠌⠭⠼ ⠷⠠⠄⠕⠗
⠹⠭⠸⠌⠽⠼⠾ ⠸⠱ ⠉ ⠆ ⠎⠑⠢ ⠔ ⠠⠞⠁⠃⠇⠑ ⠼⠙⠲⠁⠲

A single-word switch indicator is used for the word "or". Nemeth grouping symbols are used for the parentheses here because we have not switched out of Nemeth Code.

Instructions: Determine whether the slash is used mathematically, that is, does it require a switch to Nemeth? If it does, then determine if fraction indicators are required.

PRACTICE 8B

Diagonal Simple Fraction Line

A) How many ${}^{2}/_{3}$'s are there in ${}^{5}/_{6}$?

B) Energy is absorbed at the rate of 880 J/s for each square meter of the surface.

C) $y(0) = \pi/4$

D) 1 ${}^{\text{ft}}/_{\text{sec}} \approx$ 0.6818 mph

E) In y 1/5, y is the coefficient of the fraction 1/5.

F) True/False: The rise/run ratio is 5 in graph A.

G) ${}^{a}/_{b} \cdot {}^{c}/_{d} = {}^{ac}/_{bd}$

H) A 5-year CD went from earning interest at the rate of 12.06%/year in 1984 to earning less than 0.87%/year in 2015.

Mixed Numbers

[NC 13.4]

8.5 Definition of Mixed Number

A mixed number is an expression composed of a whole number followed by a simple fraction whose numerator and denominator are both numerals. Numerals in a mixed number may be represented by omission signs. An expression is not a mixed number if it contains any letters, even though the expression appears to be in the form of a mixed number. Here are some examples.

$1\frac{2}{3}$ (*Spoken:* one and two-thirds)

$99\ {}^{15}/_{16}$ (*Spoken:* ninety-nine and fifteen-sixteenths)

8.5.1 **Use of Mixed Number Fraction Indicators.** The opening and closing mixed number fraction indicators encase the fractional part of a mixed number.

```
Mixed Number Fraction Indicators
  ⠸⠹    Opening
  ⠸⠼    Closing
```

The fractional part of the mixed number uses simple fraction lines, either horizontal or diagonal, according to the fraction line style used in print.

```
  ⠌     Horizontal Simple Fraction Line  ——
  ⠸⠌    Diagonal Simple Fraction Line    /
```

The examples shown above are transcribed as follows.

⫸ $1\frac{2}{3}$ ⠼⠂⠸⠹⠆⠌⠒⠸⠼

⫸ $99\ {}^{15}/_{16}$ ⠼⠔⠔⠸⠹⠂⠢⠸⠌⠂⠖⠸⠼

Example 8-23

Russ is making a small flowerbed that is $3\frac{1}{2}$ feet by $1\frac{1}{2}$ feet.

⠀⠀⠠⠗⠥⠎⠎ ⠊⠎ ⠍⠁⠅⠬ ⠁ ⠎⠍⠁⠇⠇ ⠋⠇⠪⠻⠃⠫ ⠞ ⠊⠎
⠸⠩ ⠼⠒⠸⠹⠂⠌⠆⠸⠼ ⠸⠱ ⠋⠑⠑⠞ ⠃⠽ ⠸⠩ ⠼⠂⠸⠹⠂⠌⠆⠸⠼ ⠸⠱
⠋⠑⠑⠞⠲

a. **Linear Print Representation of a Mixed Number.** When the fractional part of a mixed number is printed on the baseline, the mixed number fraction indicators are used. Read the surrounding text to be sure this is a mixed number.

⋙ 1 1/2 ⠼⠂⠸⠹⠂⠌⠆⠸⠼

b. **Keep Together.** A whole number must not be divided from its fractional part.

8.5.2 **Mixed Numbers and Omissions.** If any part of a mixed number contains a sign of omission, the mixed number fraction indicators are used.

⋙ $\frac{7}{4} = 1\frac{?}{4}$ ⠹⠶⠌⠲⠼ ⠨⠅ ⠼⠂⠸⠹⠿⠌⠲⠸⠼

⋙ $1^{15}/_{25} = ?\,^{3}/_{5}$ ⠼⠂⠸⠹⠂⠢⠸⠌⠆⠢⠸⠼ ⠨⠅ ⠿⠸⠹⠒⠸⠌⠢⠸⠼

8.5.3 **Nonuse of Mixed Number Fraction Indicators.** If the fractional part of the expression contains a letter, it no longer suits the definition of "mixed number." Appropriate fraction indicators are used (or are not used) according to the rules.

⋙ $\frac{7}{4} = 1\,\frac{x}{4}$ ⠹⠶⠌⠲⠼ ⠨⠅ ⠼⠂⠹⠭⠌⠲⠼

⋙ $3\,x/y$ ⠼⠒⠹⠭⠸⠌⠽⠼

PRACTICE 8C

Mixed Numbers

1. Find the premium for a 1½–yr. policy at the yearly rate of 24¢ per $100.

2. $2\frac{1}{2}\,\text{ft} + 8\text{ in} = ?\text{ inches}$

3. $\left(\frac{1}{2} \times 3\frac{1}{2}\right) + \left(3\frac{1}{2} \times 2\right)$

4. $13\,^{1}/_{2} + 2\,^{2}/_{3} = 16\,^{1}/_{6}$

5. $7/4 = 1\,^{?}/_{4}$

6. $\frac{9}{4} = 2\,\frac{x}{4}$

Fraction Review

Compute each **unit rate** (price/pound).

a. \$1.50 for 2/3 pound of potatoes

b. \$4.20 for $^{1}/_{2}$ pound of Edam cheese

c. \$6.00 for $\frac{3}{4}$ pound of deli smoked turkey

d. \$12.50 for $1\frac{1}{2}$ pounds of sliced ham

Complex Fractions

[NC 13.5]

8.6 Definition of Complex Fraction

A complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers.

numerator $\dfrac{4\frac{3}{4}}{5}$ ← *complex fraction line*

denominator

Here are more examples of complex fractions.

$$\frac{\frac{4}{3}}{12} \qquad \frac{\frac{a}{b}-\frac{c}{d}}{\frac{a}{b}+\frac{c}{d}} \qquad \frac{1}{3/8} \qquad \frac{1}{2}\Big/\frac{3}{4}$$

Reminder: A fraction is not a complex fraction if the only fractions it contains are at the superscript or subscript level. Such a fraction is a simple fraction.

This is a simple fraction, not a complex fraction: $\dfrac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

8.6.1 **Use of Complex Fraction Indicators and Complex Fraction Lines.** The opening and closing complex fraction indicators are used to enclose a complex fraction.

Complex Fraction Indicators	
⠠⠹	Opening
⠠⠼	Closing

The main complex fraction line is represented by its appropriate braille symbol—either horizontal or diagonal.

⠠⠌	Horizontal Complex Fraction Line	—
⠠⠸⠌	Diagonal Complex Fraction Line	/

The examples shown above are transcribed as follows. Simple fraction indicators enclose each simple fraction when required; complex fraction indicators enclose the entire complex fraction. We suggest that you underline the complex fraction indicators and the complex fraction line in each example in order to analyze each transcription.

➢ $$\frac{4\frac{3}{4}}{5}$$ ⠠⠹⠲⠸⠹⠒⠸⠌⠲⠸⠼⠠⠌⠢⠠⠼

➢ $$\frac{\frac{4}{3}}{12}$$ ⠠⠹⠹⠲⠌⠒⠼⠠⠌⠂⠆⠠⠼

➢ $$\frac{1}{2}\Big/\frac{3}{4}$$ ⠠⠹⠹⠂⠌⠆⠼⠠⠸⠌⠹⠒⠌⠲⠼⠠⠼

➢ $$\frac{\frac{a}{b}-\frac{c}{d}}{\frac{a}{b}+\frac{c}{d}}$$ ⠠⠹⠹⠁⠌⠃⠼⠤⠹⠉⠌⠙⠼⠠⠌⠹⠁⠌⠃⠼⠬⠹⠉⠌⠙⠼⠠⠼

Although the denominator in the next example does not require simple fraction indicators (see 8.4.2.b), it is still a fraction and so the overall construction is a complex fraction.

➢ $$\frac{1}{3/8}$$ ⠠⠹⠂⠠⠌⠒⠸⠌⠦⠠⠼

Instructions: Begin each complex fraction on a new braille line, not side by side as printed. Read left to right.

PRACTICE 8D

Complex Fractions

$$\frac{\frac{1}{8}+\frac{3}{4}}{7} \qquad \frac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}-\frac{7}{9}} \qquad \frac{1/3+1/4}{4/5-1/2} \qquad \frac{\frac{\pi}{8}}{2\pi}$$

$$\frac{a}{\frac{b}{c}} \qquad \frac{33\frac{1}{3}}{100} \qquad \frac{3/5}{6} \qquad \frac{3}{5}\Big/\frac{7}{9}$$

More Fraction Rules

8.7 Fractions and the Baseline Indicator

When a fraction is on the baseline level, assure that the components of the fraction (the fraction indicators and the fraction line) are notated on the baseline of writing. When there is a level in effect at the end of a numerator or a denominator, correct placement of the baseline indicator assures accurate reading.

⫸ $\frac{a^2}{b}$ ⠹⠁⠘⠆⠐⠌⠃⠼

The baseline indicator precedes the fraction line following the superscript in the numerator.

⫸ $\frac{a}{b^2}$ ⠹⠁⠌⠃⠘⠆⠐⠼

The baseline indicator precedes the closing fraction indicator following the superscript in the denominator.

⫸ $$\frac{\frac{x}{x-1}-1^2}{\frac{x}{x+1}+1^2}$$

⠠⠹⠹⠭⠌⠭⠤⠂⠼⠤⠂⠘⠆⠐⠠⠌⠹⠭⠌⠭⠬⠂⠼⠬⠂⠘⠆⠐⠠⠼

These fractions are all on the baseline.

⫸ $x^{\frac{1}{2}} \times y$ ⠭⠘⠹⠂⠌⠆⠼⠐⠈⠡⠽

This fraction is in the superscript position.

⫸ $\frac{y_2 - y_1}{x_2 - x_1}$ ⠹⠽⠆⠤⠽⠂⠌⠭⠆⠤⠭⠂⠼

Recall that a return to the baseline after each numeric subscript is assumed when a baseline indicator is not used.

8.8 Further Observations Regarding Spacing

Spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction. No space is left between a fraction and a letter, a numeral, a sign of grouping, a braille indicator, or another fraction when these items are part of the same expression.

Example 8-24

Explain why $\frac{1}{2}+\frac{3}{4} = 1\frac{1}{4}$.

The plus sign is unspaced from the fractions before and after it; there is a space before and after the equals sign. There is no space between the components of a mixed number.

Example 8-25

Multiply the fractions. $\left(\frac{5}{12}\right)\left(\frac{4}{12}\right) = \_\_\_$

No space is left between factors even though one may appear in print.

Example 8-26

Differentiating the first two terms, $1/2\, x^{1/2} + 5/6\, x^{-3/2}$.

In the braille transcription, no spaces occur in this long math expression.

8.9 Fractions and the Ellipsis and Long Dash

a. **Spacing Next to a Fraction Indicator.** No space is left between an opening or closing fraction indicator and an ellipsis or long dash in the numerator or denominator of a fraction.

⫸ $\frac{\ldots\times 5}{2\times 10} = \frac{15}{20}$

The space following this ellipsis is required. Review 1.12 in Lesson 1.

⫸ $\dfrac{3\times 5}{2\times \_\_} = \dfrac{15}{20}$

The space preceding this long dash is required. Review 1.12 in Lesson 1.

b. **Spacing Next To a Fraction Line.** A space is left between a fraction line and an ellipsis or long dash.

⫸ $\dfrac{2+4+6+\ldots}{1+3+5+\ldots}$

c. **Spacing Between Fractions.** A space is left between a fraction and an ellipsis or long dash preceding or following the fraction.

⫸ $\dfrac{1}{10} \ldots \dfrac{10}{10}$

Example 8-27

Fill in the blanks with the missing fractions. $\dfrac{1}{10}$ ___ ___ $\dfrac{4}{10}$

8.10 Fractions in an Enclosed List

Fractions and mixed numbers may be part of an enclosed list. (Review the rules for "enclosed list" in Lesson 4.)

⫸ $\left\{0,\ \dfrac{1}{2},\ 1,\ 1\dfrac{1}{2}\right\}$

Instructions: Determine the formatting before beginning your transcription. Where does each paragraph begin? Which expressions are embedded and which are displayed? What is the proper cell placement for the displayed expressions?

PRACTICE 8E

These are simple fractions:

$$\frac{1}{2} \qquad \frac{a^2}{b^2} \qquad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$$

This is not a simple fraction: $\frac{1/3}{2/3}$

Review the rules in 6.12.5 regarding an ellipsis on the baseline of writing when it follows a superscript.

$$x^{\frac{1}{2}} \ldots \; x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}} \ldots \frac{x^{\frac{1}{2}}+1}{y^{\frac{1}{2}}-1}$$

Plot the points $\left(-\frac{1}{2},\ 4\right)$, $\left(3,\ 4\frac{1}{4}\right)$, and $\left(-9,\ \frac{3}{4}\right)$. Then express $\frac{a^{3/4}}{b^{5/4}}$ in radical form.

RADICAL EXPRESSIONS

[NC Rule 16]

8.11 Terminology

Here are the parts of a radical expression.

$\sqrt{144}$ — √ is the *radical sign*.

144 is the *radicand*.

The horizontal bar above the radicand is the *vinculum*. The vinculum shows the extent to which the radical sign applies.

There may be a figure placed to the left and slightly above the radical sign, called the *index* of the radical. For example, this radical sign has an index of three. ∛ When there is no index, the radical sign may be referred to as the "square root" sign.

8.12 The Termination Indicator

When a radical expression has a vinculum, the radical sign is placed before the radicand and the termination indicator is placed after the radicand.

⠜	Radical Sign	√
⠻	Termination Indicator	

⫸ $\sqrt{x}$ ⠜⠭⠻

⫸ $\sqrt{64}$ ⠜⠖⠲⠻

Reminders: An English-letter indicator is not needed for an English letter (in regular type) which occurs in an unspaced sequence of mathematical symbols. A numeric indicator is not used when a numeral is not preceded by a space.

a. **No Vinculum.** When a vinculum is not shown in print, or when the radical sign occurs without a radicand, a termination indicator is not used.

⫸ $\surd(x-1)$ ⠜⠷⠭⠤⠂⠾

⫸ √ ⠜

Example 8-28

The √ is called a "radical sign."

⠠⠮ ⠸⠩ ⠜ ⠸⠱ ⠊⠎ ⠉⠁⠇⠇⠫ ⠁ ⠦⠗⠁⠙⠊⠉⠁⠇ ⠎⠊⠛⠝⠲⠴

8.13 Spacing

The spacing before and after a radical expression is subject to the spacing rules for the signs preceding or following the radical expression.

⫸ $\sqrt{9} - \sqrt{4} = 1$

No space is left between a radical expression and a letter, a numeral, a fraction, a sign of grouping, a braille indicator, or another radical expression when these items are unspaced in print and belong to the same expression.

⫸ $\sqrt{5}y$

⫸ $\sqrt{x}^{2}$

⫸ $2a\sqrt{4ab}$

⫸ $\sqrt{4}\sqrt{87} = 2\sqrt{87}$

⫸ $\sqrt{y}\,\mathrm{d}x + (1 + x)\,\mathrm{d}y = 0,\ y(0) = 1$

Reminder: In print, derivative notation dx, dy, etc. is often preceded and followed by a space within an expression, for clarity. In braille, the terms are not spaced unless a space is required with the item preceding or following them. (Review 4.15.1 in Lesson 4.)

Example 8-29

Simplify. $\dfrac{2-\sqrt{\frac{1}{4}}}{3-\sqrt{\frac{1}{2}}}$

8.14 Index of Radical

A small number or letter that may appear next to the radical sign is the *index* of the radical. This print example shows an index "3".

$$\sqrt[3]{27}$$

In braille, the index-of-radical indicator and the index precede the radical sign.

⠣	Index-of-Radical Indicator

⫸ $\sqrt[3]{27} = 9$ ⠣⠒⠜⠆⠶⠻ ⠨⠅ ⠼⠔

⫸ $\sqrt[m]{\frac{a}{b}} = \frac{\sqrt[m]{a}}{\sqrt[m]{b}}$

⠣⠍⠜⠹⠁⠌⠃⠼⠻ ⠨⠅ ⠹⠣⠍⠜⠁⠻⠌⠣⠍⠜⠃⠻⠼

Example 8-30

Simplify as follows. $\frac{\sqrt{9}}{\sqrt[3]{27}} = \frac{3}{3} = 1$

⠠⠎⠊⠍⠏⠇⠊⠋⠽ ⠵ ⠋⠕⠇⠇⠪⠎⠲
⠸⠩ ⠹⠜⠔⠻⠌⠣⠒⠜⠆⠶⠻⠼ ⠨⠅ ⠹⠒⠌⠒⠼ ⠨⠅ ⠼⠂ ⠸⠱

PRACTICE 8F

Radical Expressions

1. $\sqrt{\frac{b}{a} + \frac{a}{b}}$
2. $\sqrt{c/d}$
3. $\frac{1}{4}\sqrt{\frac{1}{2}}$
4. $\sqrt{\frac{10}{16}} = \sqrt{10}/4$
5. $(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$
6. $2\sqrt{2} + 7\sqrt{2} = (2 + 7)\sqrt{2} = 9\sqrt{2}$
7. $\frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{15}}{2}$
8. $\frac{\sqrt{2} - \sqrt{\frac{1}{3}}}{\sqrt{3} - \sqrt{\frac{1}{2}}}$
9. $\sqrt{48x^3y}$
10. $\surd(y - 1) + \surd(2y) = 1$
11. $\sqrt[n]{d}$
12. $\sqrt[a+b]{z - y}$
13. $\sqrt[4]{729} + \sqrt[6]{27} = \sqrt[?]{?}$
14. $7\sqrt[3]{125} \cdot 7\sqrt[5]{2}$
15. $\sqrt[5]{m}\,\sqrt[5]{n} = \sqrt[5]{mn}$

8.15 Nested Radical Expressions [NC 16.3]

When radical expressions are nested one within the other, the appropriate number of order-of-radical indicators shows the depth of each inner radical expression.

Order-of-Radical Indicators	
⠨	First Inner Radical
⠨⠨	Second Inner Radical
⠨⠨⠨	Third Inner Radical
⠻	Termination Indicator

The appropriate order-of-radical indicator is placed before its radical sign and before its termination indicator.

⫸ $\sqrt{x+\sqrt{x+y}+z}$ ⠜⠭⠬⠨⠜⠭⠬⠽⠨⠻⠬⠵⠻

When more than one radical expression is completed at the same point, they are terminated beginning with the innermost expression.

⫸ $\sqrt{x+\sqrt{y+\sqrt{z}}}$ ⠜⠭⠬⠨⠜⠽⠬⠨⠨⠜⠵⠨⠨⠻⠨⠻⠻

8.15.1 **Nested Radical Expression with an Index.** If an inner radical expression has an index, the appropriate order-of-radical indicator is placed before the index-of-radical indicator as well as before its termination indicator. When more than one radical expression is completed at the same point, they are terminated beginning with the innermost expression.

⫸ $\sqrt{\sqrt[3]{16}}=\sqrt[3]{\sqrt{16}}$

⠜⠨⠣⠒⠜⠂⠖⠨⠻⠻ ⠨⠅ ⠣⠒⠜⠨⠜⠂⠖⠨⠻⠻

⫸ $\sqrt{\sqrt[3]{\sqrt[4]{10}}}$

⠜⠨⠣⠒⠜⠨⠨⠣⠲⠜⠂⠴⠨⠨⠻⠨⠻⠻

8.16 Radical Expressions and the Baseline Indicator

When a radical expression is on the baseline level, assure that the components of the construction (the radical symbol or the indicators) are notated on the baseline of writing. Place the baseline indicator before the component when it follows a superscript or a subscript.

⫸ $(r^2\sqrt{r})^2$ ⠷⠗⠘⠆⠐⠜⠗⠻⠾⠘⠆

⫸ $\sqrt{x^2+y^2}$ ⠜⠭⠘⠆⠐⠬⠽⠘⠆⠐⠻

The baseline indicator is not used after a numeric subscript that does not require a subscript indicator.

⫸ $\sqrt{x_1 + y_2}$ ⠜⠭⠂⠬⠽⠆⠻

8.17 Radical Expressions and the Ellipsis and Long Dash

When an ellipsis or a long dash occurs within a radical, no space should be left between the ellipsis or long dash and the termination indicator. However, a space must be left between the radical sign and an ellipsis or long dash.

⫸ $\sqrt{a + b + c + \cdots}$

⠜⠁⠬⠃⠬⠉⠬ ⠄⠄⠄⠻

⫸ $\sqrt{x + \sqrt{x + \sqrt{\ldots}}}$

⠜⠭⠬⠨⠜⠭⠬⠨⠨⠜ ⠄⠄⠄⠨⠨⠻⠨⠻⠻

A space is required between a radical expression and an ellipsis or long dash preceding or following the expression.

⫸ $\sqrt{4}$ ... $\sqrt{64}$ ⠜⠲⠻ ⠄⠄⠄ ⠜⠖⠲⠻

8.18 Radical Expressions and Abbreviations

When an abbreviation occurs within a radical, no space is left between the abbreviation and the termination or order-of-radical indicator following it.

⫸ $\sqrt{9\text{ ft}}$ ⠜⠔ ⠋⠞⠻

However, a space is required between a radical sign and an abbreviation.

⫸ $\sqrt{\text{ft.}}$ ⠜ ⠋⠞⠲⠻

A space is also required between a radical expression and an abbreviation preceding or following the expression.

⫸ $2\sqrt{12}$ sq. in. ⠼⠆⠜⠂⠆⠻ ⠎⠟⠲ ⠊⠝⠲

8.19 Enclosed Lists with Radical Expressions

Radical expressions may be part of an enclosed list.

⫸ $(\sqrt{9},\ 3,\ \sqrt{4},\ 2\sqrt{6})$

⠷⠜⠔⠻⠠ ⠒⠠ ⠜⠲⠻⠠ ⠆⠜⠖⠻⠾

PRACTICE 8G

Nested Radical Expressions

(1) $\sqrt{-\frac{1}{2} - i\frac{\sqrt{3}}{2}}$

(2) $\sqrt{\sqrt{13} + \sqrt{15} + \sqrt{117}}$

(3) $\sqrt{1 - \sqrt{a-b}} \times \sqrt{1 + \sqrt{a-b}}$

(4) $\sqrt{b\sqrt[3]{b\sqrt{b}}}$

(5) $\sqrt[a]{\sqrt[b]{\sqrt[c]{abc}}}$

(6) $\sqrt{a^2}\sqrt{b^4}\sqrt{c} = ab^2\sqrt{c}$

(7) $\left(s^2\sqrt[3]{s^4}\right)^2$

(8) $\sqrt[3]{x^2\sqrt{64x^6}}$

(9) $\sqrt{\sqrt[3]{\sqrt[4]{\sqrt[5]{b^{48}}}}}$

(10) $\sqrt{x_1 + \sqrt{x_2}}$

(11) $q^{\sqrt{r}} + s$

(12) $\sqrt{c + d + e + \cdots}$

LINKED EXPRESSIONS

[NC 26.5]

8.20 Definition of Linked Expression

A linked expression contains at least one sign of comparison. The part preceding the first sign of comparison is called the *anchor*. Each remaining part, beginning with a sign of comparison and ending before the next sign of comparison, is called a *link*. In its simplest form, $x = y$ is a linked expression where x is the anchor and $= y$ is the link.

$12.5\% > \frac{1}{10}$

The anchor is 12.5% and the link is $> \frac{1}{10}$

$6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5) = 1200 + 240 + 30 = 1470$

The anchor is 6×245*, followed by three links each beginning with an equals sign.*

8.21 Division of Linked Expressions

Recall that a mathematical expression must not be divided between lines if it will fit on one braille line within the current margins. A linked expression is considered to be one expression. If the anchor and its link will fit on one line, do not divide it. When there is more than one link, if the anchor and all of its links will fit on one line, do so. However, it is often the case that an anchor and more than one link will not fit on one line. There are rules to follow, which result in smoother reading when a mathematical expression must be divided.

(1) If the entire embedded linked expression will fit on one braille line, do not divide it. See Example 8-31.

(2) If an anchor and its link not fit on one braille line, begin the runover line with the link. See Example 8-32.

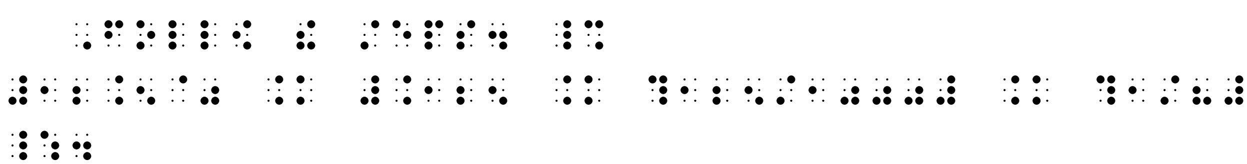
(3) If the expression contains more than one link, use as much of the line as possible before dividing. It is not necessary to divide at every link. See Example 8-33.

(4) If the anchor or a link will not fit on one braille line, further rules apply. Those rules will be covered in Lesson 15.

(5) If the linked expression is printed in the nested layout discussed in the next section, other rules apply. (See 8.22.)

Example 8-31

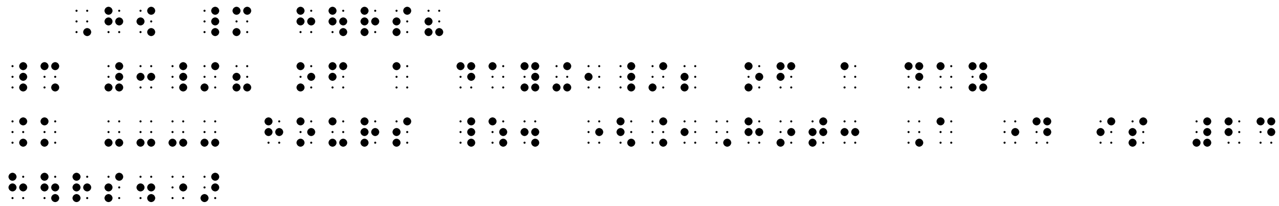
Follow the steps. $12.5\% = .125 = \dfrac{125}{1000} = \dfrac{1}{8}$.



Although the anchor and the first link will fit on line 1, since the entire linked expression fits on one line, the anchor begins on line 2.

Example 8-32

How many hours? 3/8 of a day + 1/2 of a day = ___ hours. (*Hint*: A day is 24 hours.)

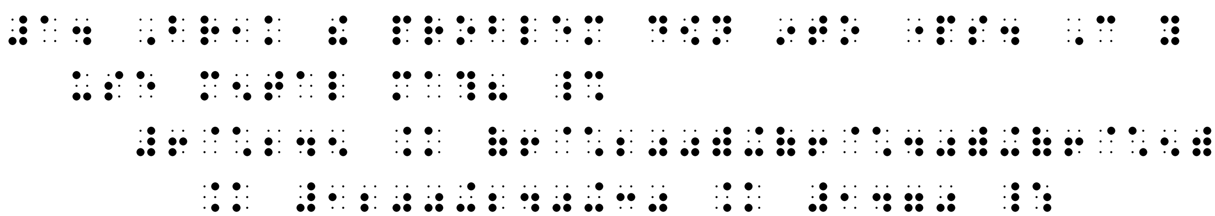


This is a narrative paragraph (3-1) with an embedded equation. The opening switch indicator will fit on the line with the anchor, in the runover position, cell 1. The link begins on the next line with its comparison sign.

Example 8-33

Break the problem down into parts. Can you use mental math?

$$6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5) = 1200 + 240 + 30 = 1470$$

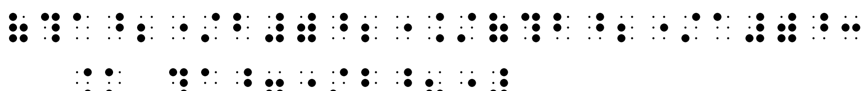


The anchor starts in cell 5, displayed to itemized text. The first link fits on the same line. The second link starts on a new line in cell 7, the runover cell in this displayed format. The third link fits on this line as well.

8.21.1 **Restrictions.** In order to divide a long expression before a comparison sign, the comparison sign must be on the baseline of writing. Furthermore, do not divide before a comparison sign that is part of an item enclosed in grouping symbols, between fraction indicators, or within radical signs. This topic will be explored in Lesson 15.

8.21.2 **Other Considerations.** A transition to a new braille line made before a sign of comparison terminates the effect of any level indicator used on the line above, just as it would if it were not divided between lines.

≫ $\left(\frac{a^2}{b}\right)^2 \div \left(\frac{b^2}{a}\right)^3 = \frac{a^7}{b^8}$



A return to the baseline after the superscript "3" is triggered by the presence of the following comparison sign even though it is on the next line.

In order to save a line, if the opening Nemeth Code indicator will not fit on the same line as the preceding text it may be placed in front of the anchor in a displayed linked expression even if the entire linked expression will not fit on one line. Compare two transcriptions of the next example. Either transcription is acceptable.

Example 8-34

1. This simple linear equation has 3 unknowns.

$$\frac{x}{2}+\frac{1}{3}\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{6}\right)=\frac{1}{2}(x+y+z)$$

⠼⠁⠲ ⠠⠹ ⠎⠊⠍⠏⠇⠑ ⠇⠔⠑⠜ ⠑⠟⠥⠁⠰⠝ ⠓⠁⠎ ⠼⠉ ⠥⠝⠅⠝⠪⠝⠎⠲
⠀⠀⠸⠩
⠀⠀⠀⠀⠹⠭⠌⠆⠼⠬⠹⠂⠌⠒⠼⠷⠹⠭⠌⠆⠼⠬⠹⠽⠌⠒⠼⠬⠹⠵⠌⠖⠼⠾
⠀⠀⠀⠀⠀⠀⠨⠅ ⠹⠂⠌⠆⠼⠷⠭⠬⠽⠬⠵⠾ ⠸⠱

The opening switch does not fit at the end of the line of text. It is placed in the runover position for itemized material, cell 3. The displayed expression begins on the next line in cell 5. The link begins in the runover position on the following line (cell 7).

Alternate Transcription

⠼⠁⠲ ⠠⠹ ⠎⠊⠍⠏⠇⠑ ⠇⠔⠑⠜ ⠑⠟⠥⠁⠰⠝ ⠓⠁⠎ ⠼⠉ ⠥⠝⠅⠝⠪⠝⠎⠲
⠀⠀⠀⠀⠸⠩ ⠹⠭⠌⠆⠼⠬⠹⠂⠌⠒⠼⠷⠹⠭⠌⠆⠼⠬⠹⠽⠌⠒⠼⠬⠹⠵⠌⠖⠼⠾
⠀⠀⠀⠀⠀⠀⠨⠅ ⠹⠂⠌⠆⠼⠷⠭⠬⠽⠬⠵⠾ ⠸⠱

This transcription begins the displayed material with the opening switch, even though the expression requires a runover.

PRACTICE 8H

Linked Expressions

1. Is the following inequality true? $\frac{3}{5}\left(\frac{2}{3}x-\frac{1}{2}\right)>\frac{2}{5}\left(\frac{1}{4}x+\frac{1}{3}\right)$
2. $33\frac{1}{3}\%+40\%+61\frac{2}{3}\%=134\frac{3}{3}\%=135\%$
3. In multiplying $5\frac{3}{4}\times 46$, recall that $5\frac{3}{4}=5+\frac{3}{4}$. Therefore, $46\times\left(5+\frac{3}{4}\right)=(46\times 5)+\left(46\times\frac{3}{4}\right)=264\frac{1}{2}$.

8.22 Special Case—Nested Linked Expressions [NC 26.5.3.b]

An expression with two or more links may be subject to special Nemeth format rules if it appears in print in a particular arrangement as described in this section.

Print Layout

- The expression is displayed.
  - There is an exception regarding itemized problems – see b, below.
- The first line contains only the anchor or only the anchor and the first link.
- Each following link begins on a new line, and the comparison signs beginning each link are vertically aligned.
  - An exception applies to the last line – see a, below.

The following linked expression meets the three requirements.

> To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.
>
> $$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c). \end{aligned}$$

a. It is common for the last line of the expression to contain more than one link. As long as the other conditions are met, this layout meets the requirements for this nested format.

> We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:
>
> $$\begin{aligned} 12\tfrac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \tfrac{125}{1000} = \tfrac{1}{8} \end{aligned}$$

b. The next example shows an itemized problem with no narrative. Although the expression is not displayed, this arrangement follows all of the other print layout specifications: the comparison signs are vertically aligned, and – other than on the first and last line – no sign of comparison is preceded by any expression on its left. Rules regarding this layout are outlined in Section 8.22.4.

1. $12\frac{1}{2}\% = 12.5\%$

$= .125$

$= \frac{125}{1000} = \frac{1}{8}$

When the print layout meets the definition of a nested linked expression, one of the following Nemeth formats is applied.

8.22.1 **Margin Requirements for a Nested Linked Expression.** The margins which are applied to a nested linked expression follow a reliable pattern, which can be generalized as follows.

- The anchor begins two cells to the right of the runover margin of the material to which it is displayed.
- Each link that starts on a new line begins two cells to the right of the anchor cell.
- Runovers to anchor or links begin four cells to the right of the anchor cell.

Note: Rules regarding how to divide a link that will not fit on the line will be discussed in Lesson 15. In this lesson, in order to illustrate runovers within a nested linked expression, a runover line will begin with a sign of operation.

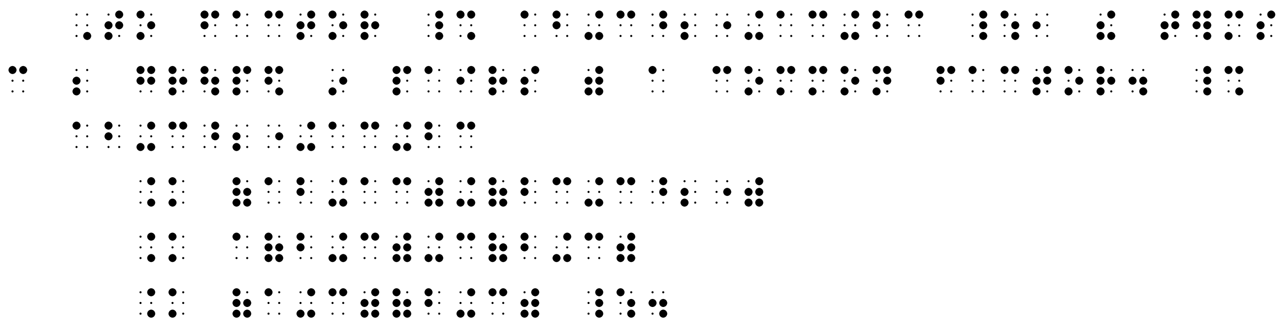
8.22.2 **Nested Linked Expression Displayed to Narrative.** When a nested linked expression occurs in unitemized explanatory portions of the text (3-1), the anchor begins in cell 3 and each link begins in cell 5. In braille, each link begins on a new line, even when the print copy shows more than one link on the last line.

Reminder: A line is not skipped above or below displayed mathematical material unless a blank line is required under other rules or guidelines.

Example 8-35

To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.

$$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c)\,. \end{aligned}$$

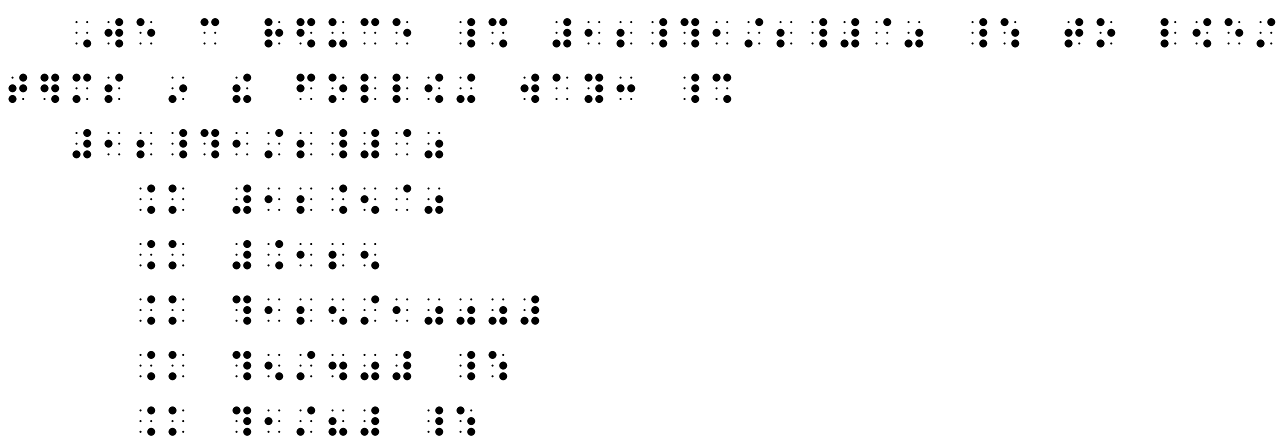


Lines 1-2: Narrative paragraph (3-1). The opening switch indicator is placed at the end of the narrative.
Lines 3-6: Nested linked expression.
Line 3: The anchor is in cell 3 (two cells to the right of the runover cell of the preceding material).
Lines 4-6: Each link begins in cell 5 (two cells to the right of the anchor), regardless of the amount of available space on the preceding line.

Example 8-36

We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:

$$\begin{aligned} 12\tfrac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \frac{125}{1000} = \frac{5}{40} = \frac{1}{8} \end{aligned}$$



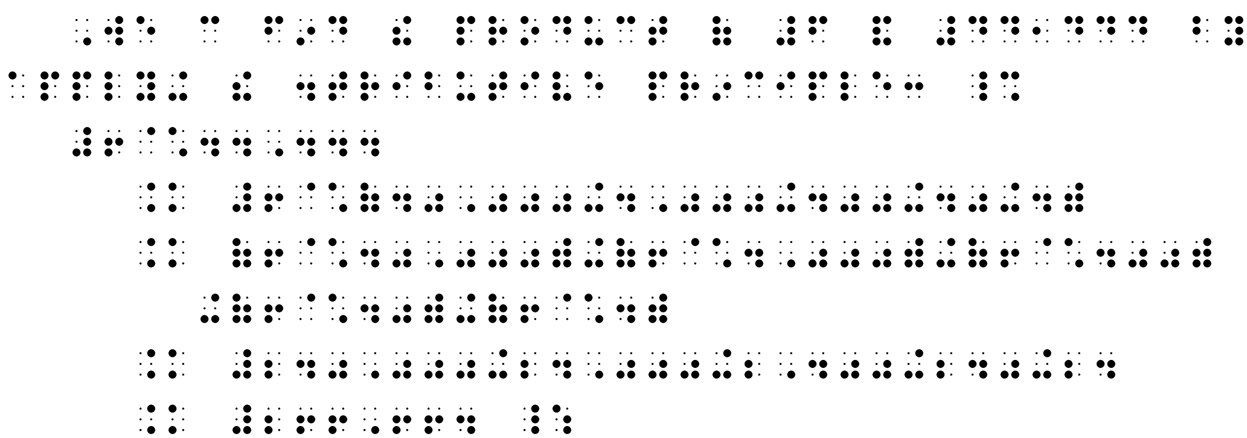
Lines 1-2: Narrative paragraph (3-1). The opening switch indicator is placed at the end of the narrative.
Lines 3-7: Nested linked expression.
Line 3: T*he anchor is in cell 3 (two cells to the right of the runover cell of the preceding material).*
Lines 4-8: Each link begins in cell 5 (two cells to the right of the anchor), regardless of the amount of available space on the preceding line and despite the fact that the last line in print shows three links.

a. **Runovers Within a Nested Linked Expression.** If the anchor or a link will not fit on the current line, the examples in this lesson divide the expression before a sign of operation. The runover line is indented four cells from the anchor, or two cells from the link. In this case, that is cell 7. Anchor: 3-7; link: 5-7.

Example 8-37

We can find the product of 6 and 44,444 by applying the distributive principle:

$$\begin{aligned} 6 \times 44{,}444 &= 6 \times (40{,}000 + 4{,}000 + 400 + 40 + 4) \\ &= (6 \times 40{,}000) + (6 \times 4{,}000) + (6 \times 400) + (6 \times 40) + (6 \times 4) \\ &= 240{,}000 + 24{,}000 + 2{,}400 + 240 + 24 \\ &= 266{,}664 \end{aligned}$$



Lines 1-2: Narrative paragraph (3-1).
Lines 3-8: Nested linked expression.
Line 3: The anchor is in cell 3.
Lines 4, 5, 7, 8: Each link begins in cell 5.
Line 6: The second link requires a runover. The runover begins in cell 7. The runover begins with a plus sign outside of the grouped expresssions.

PRACTICE 8I

Recognizing a Nested Linked Expression

To test the equation $R_t = \frac{R}{n}$, use four 1000-Ω resistors wired in series to predict a total resistance of 250 Ω .

$$R_t = \frac{R}{n} = \frac{1000\ \Omega}{4}$$

$$\frac{1000\ \Omega}{4} = 250\ \Omega$$

Then, by using the resistance theory equation

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n},$$

plug the 150-Ω and 1000-Ω resistors into the equation as R_1 and R_2.

$$\begin{aligned} \frac{1}{R_t} &= \frac{1}{150\ \Omega} + \frac{1}{1000\ \Omega} \\ &= 0.007 + 0.001 \\ &= 0.008 \end{aligned}$$

$$R_t = \frac{1}{0.008} = 125\ \Omega$$

8.22.3 **Nested Linked Expression Displayed to Itemized Material.** Apply the general pattern when a nested linked expression is displayed to itemized material: begin the anchor two cells to the right of the current runover margin; begin each two cells to the right of the anchor cell; begin runovers four cells to the right of the anchor cell.

a. **Itemized Text with No Subdivisions.** When a nested linked expression occurs in itemized text containing no subdivisions (1-3), the anchor begins in cell 5 and each link begins in cell 7. Runovers begin in cell 9.

b. **Itemized Text with Subdivisions.** When a nested linked expression occurs in itemized text containing subdivisions (1-5; 3-5), the anchor begins in cell 7 and each link begins in cell 9. Runovers begin in cell 11.

Example 8-38

1. The example below shows a way of finding 6×245 .

$$\begin{aligned} 6 \times 245 &= (6 \times 200) + (6 \times 40) + (6 \times 5) \\ &= 1200 + 240 + 30 \\ &= 1470 \end{aligned}$$

Is there another way to find 6×245 ?

```
⠼⠁⠲ ⠠⠮ ⠑⠭⠁⠍⠏⠇⠑ ⠆⠇ ⠩⠪⠎ ⠁ ⠺⠁⠽ ⠷ ⠋⠔⠙⠬
  ⠸⠩ ⠼⠖⠈⠡⠆⠲⠢⠸⠲
    ⠼⠖⠈⠡⠆⠲⠢
      ⠨⠅ ⠷⠖⠈⠡⠆⠴⠴⠾⠬⠷⠖⠈⠡⠲⠴⠾⠬⠷⠖⠈⠡⠢⠾
      ⠨⠅ ⠼⠂⠆⠴⠴⠬⠆⠲⠴⠬⠒⠴
      ⠨⠅ ⠼⠂⠲⠶⠴ ⠸⠱
  ⠠⠊⠎ ⠐⠮ ⠁⠝⠕⠮⠗ ⠺⠁⠽ ⠞⠕ ⠋⠔⠙ ⠸⠩ ⠼⠖⠈⠡⠆⠲⠢ ⠸⠱⠦
```

Observation: The nested linked expression is displayed to 1-3 itemized text.
Lines 1-2: Itemized text with no subdivisions. (1-3)
Lines 3-6: Nested linked expression.
Line 3: The *anchor is in cell 5.*
Lines 4-6: Each link begins in cell 7, regardless of the amount of available space on the preceding line.
Line 7: Text continues in the runover cell to itemized text, cell 3.

Example 8-39

2. What is the function of the parentheses in the following problem?

$$\begin{aligned} (4 \times 10{,}000) + (4 \times 1{,}000) + (4 \times 100) + (4 \times 10) + (4 \times 1) \\ = 40{,}000 + 4{,}000 + 400 + 40 + 4 \\ = 44{,}444 \end{aligned}$$

```
⠼⠃⠲ ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠋⠥⠝⠉⠰⠝ ⠷ ⠮ ⠏⠜⠢⠮⠎⠑⠎ ⠔ ⠮
  ⠋⠕⠇⠇⠪⠬ ⠏⠗⠕⠃⠇⠑⠍⠦ ⠸⠩
    ⠷⠲⠈⠡⠂⠴⠠⠴⠴⠴⠾⠬⠷⠲⠈⠡⠂⠠⠴⠴⠴⠾⠬⠷⠲⠈⠡⠂⠴⠴⠾
        ⠬⠷⠲⠈⠡⠂⠴⠾⠬⠷⠲⠈⠡⠂⠾
      ⠨⠅ ⠼⠲⠴⠠⠴⠴⠴⠬⠲⠠⠴⠴⠴⠬⠲⠴⠴⠬⠲⠴⠬⠲
      ⠨⠅ ⠼⠲⠲⠠⠲⠲⠲ ⠸⠱
```

Observation: The anchor has a runover.
Lines 1-2: Itemized text with no subdivisions. (1-3)
Line 2: The opening switch indicator is placed at the end of the narrative.

Lines 3-6: Nested linked expression.
Line 3: The anchor is in cell 5.
Line 4: The anchor requires a runover. The runover begins in cell 9. The runover begins with a plus sign outside of the grouped expressions.
Lines 5 and 6: Each link begins in cell 7.

Example 8-40

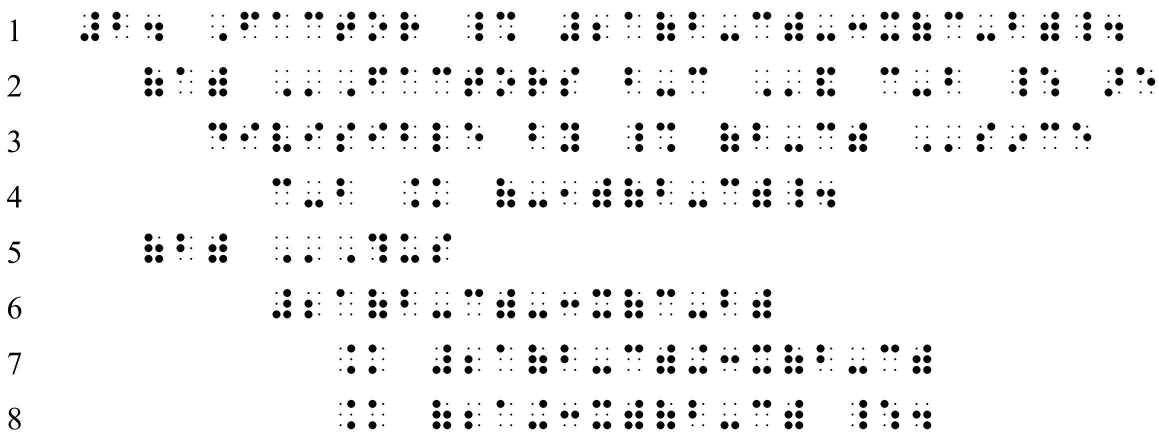
2. Factor $2a(b-c)-3x(c-b)$.

   (a) Factors $b-c$ and $c-b$ are divisible by $(b-c)$ since

   $$c-b=(-1)(b-c)\,.$$

   (b) Thus

   $$\begin{aligned} 2a(b-c)-3x(c-b) &= 2a(b-c)+3x(b-c) \\ &= (2a+3x)(b-c)\,. \end{aligned}$$



Observation: The nested linked expression is displayed to a 3-5 subdivision.
Line 1: Itemized text begins in cell 1.
Lines 2-3: Subdivision (3-5).
Line 4: Displayed math begins in cell 7. This is not a nested linked expression. The link continues on the same line as the anchor.
Line 5: The next subdivision begins in cell 3.
Lines 6-8: Nested linked expression.
Line 6: The anchor is in cell 7 (two cells to the right of the runover cell of the preceding material).
Lines 7-8: Each link begins in cell 9 (two cells to the right of the anchor), regardless of the amount of available space on the preceding line.

Example 8-41

7. Approximating the value of *e*.

    a. Approximate *e* using two terms.

$$\begin{aligned} e &= 1+\frac{1}{1}+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+\frac{1}{1\cdot 2\cdot 3\cdot 4}+\cdots \\ &= ? \end{aligned}$$

```
1  ⠼⠛⠲ ⠠⠁⠏⠏⠗⠕⠭⠊⠍⠁⠞⠬ ⠮ ⠧⠁⠇⠥⠑ ⠷ ⠰⠑⠲
2      ⠁⠲ ⠠⠁⠏⠏⠗⠕⠭⠊⠍⠁⠞⠑ ⠰⠑ ⠥⠎⠬ ⠞⠺⠕ ⠞⠻⠍⠎⠲ ⠸⠩
3            ⠑
4                ⠨⠅ ⠼⠂⠬⠹⠂⠌⠂⠼⠬⠹⠂⠌⠂⠡⠆⠼⠬⠹⠂⠌⠂⠡⠆⠡⠒⠼
5                    ⠬⠹⠂⠌⠂⠡⠆⠡⠒⠡⠲⠼⠬ ⠄⠄⠄
6                ⠨⠅ ⠿ ⠸⠱
```

Observation: The first link has a runover.
Line 1: Itemized text begins in cell 1.
Line 2: Subdivision begins in cell 3.
Lines 3-6: Nested linked expression.
Line 3: The anchor is in cell 7 (two cells to the right of the runover cell of the preceding material).
Lines 4 and 6: Each link begins in cell 9 (two cells to the right of the anchor).
Line 5: The first link requires a runover. The runover begins in cell 11 (four cells to the right of the anchor). The runover begins with a plus sign.

8.22.4 **Itemized Nested Linked Expression with No Narrative.** When a nested linked expression is itemized and immediately follow the identifier, transcribe the anchor on the same line as the identifier. Each link begins a new line, with the comparison sign two cells to the right of the cell in which the identifier begins. If the anchor or any link requires a runover, indent two cells further—that is, four cells to the right of the cell in which the identifier begins.

Example 8-42

Approximate the value of e.

1. $e = 1 + \frac{1}{1} + \frac{1}{12} + \frac{1}{123} + \frac{1}{1234} + \cdots$

   $= ?$

2. …

```
    ⠠⠁⠏⠏⠗⠕⠭⠊⠍⠁⠞⠑ ⠮ ⠧⠁⠇⠥⠑ ⠷ ⠰⠑⠲ ⠸⠩
⠼⠂⠸⠲ ⠑
  ⠨⠅ ⠼⠂⠬⠹⠂⠌⠂⠼⠬⠹⠂⠌⠂⠡⠆⠼⠬⠹⠂⠌⠂⠡⠆⠡⠒⠼
    ⠬⠹⠂⠌⠂⠡⠆⠡⠒⠡⠲⠼⠬ ⠄⠄⠄
  ⠨⠅ ⠿
⠼⠆⠸⠲ ⠄⠄⠄ ⠸⠱
```

Line 1: Instructions begin in cell 5.
Line 2: The identifier begins in cell 1. Because there is no narrative, the anchor begins on this line.
Lines 3 and 5: Each link begins in cell 3 (two cells to the right of the cell in which the identifier begins).
Line 4: The first link requires a runover. The runover begins in cell 5 (four cells to the right of the cell in which the identifier begins). The runover begins with a plus sign.

PRACTICE 8J

Multiplying Mixed Numbers

A. $2\frac{1}{2} \cdot 1\frac{1}{4} = \left(2 + \frac{1}{2}\right) \cdot \left(1 + \frac{1}{4}\right)$

$= 2 + \frac{2}{4} + \frac{1}{2} + \frac{1}{8}$

$= 2 + \frac{1}{2} + \frac{1}{2} + \frac{1}{8}$

$= 2 + 1 + \frac{1}{8} = 3\frac{1}{8}$

B. What will the remainder be in this problem?

$4\frac{1}{3} \times 3\frac{2}{5} \times 9\frac{11}{15} \times 2\frac{3}{4}$

$= \frac{13}{3} \times \frac{17}{5} \times \frac{146}{15} \times \frac{11}{4}$

$= \frac{13 \times 17 \times 146 \times 11}{3 \times 5 \times 15 \times 4}$

$= \frac{354{,}926}{900}$

= 394 with a remainder of ____.

For further practice, see Appendix A—Reading Practice.

EXERCISE 8

Prepare Exercise 8 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 8A

PRACTICE 8B

1 ⠠⠙⠊⠁⠛⠕⠝⠁⠇ ⠠⠎⠊⠍⠏⠇⠑ ⠠⠋⠗⠁⠉⠰⠝ ⠠⠇⠔⠑

2 ⠀⠀

3 ⠠⠁⠐⠜ ⠠⠓⠪ ⠸⠍ ⠸⠩ ⠹⠆⠸⠌⠒⠼⠸⠄⠎ ⠸⠱ ⠜⠑ ⠐⠮ ⠔

4 ⠸⠩ ⠹⠢⠸⠌⠖⠼ ⠸⠱⠦

5 ⠰⠠⠃⠐⠜ ⠠⠢⠻⠛⠽ ⠊⠎ ⠁⠃⠎⠕⠗⠃⠫ ⠁⠞ ⠮ ⠗⠁⠞⠑ ⠷

6 ⠸⠩ ⠼⠦⠦⠴ ⠰⠠⠚⠸⠌⠰⠎ ⠸⠱ ⠿ ⠑⠁⠡ ⠎⠯⠥⠜⠑ ⠍⠑⠞⠻ ⠷

7 ⠮ ⠎⠥⠗⠋⠁⠉⠑⠲

8 ⠰⠠⠉⠐⠜ ⠸⠩ ⠽⠷⠴⠾ ⠨⠅ ⠨⠏⠸⠌⠲

9 ⠰⠠⠙⠾ ⠼⠂ ⠹⠋⠞⠸⠌⠎⠑⠉⠼ ⠈⠱⠈⠱ ⠼⠴⠨⠖⠦⠂⠦ ⠍⠏⠓

10 ⠰⠠⠑⠾ ⠠⠄⠠⠔ ⠽⠹⠂⠸⠌⠢⠼ ⠸⠱⠂ ⠰⠽ ⠊⠎ ⠮ ⠉⠕⠑⠖⠊⠉⠊⠢⠞

11 ⠷ ⠮ ⠋⠗⠁⠉⠰⠝ ⠸⠩ ⠼⠂⠸⠌⠢ ⠸⠱⠲

12 ⠰⠠⠋⠐⠜ ⠠⠞⠗⠥⠑⠸⠌⠠⠋⠁⠇⠎⠑⠒ ⠠⠮ ⠸⠩ ⠗⠊⠎⠑⠸⠌⠗⠥⠝ ⠸⠱

13 ⠗⠁⠞⠊⠕ ⠊⠎ ⠼⠑ ⠔ ⠛⠗⠁⠏⠓ ⠠⠁⠲

14 ⠰⠠⠛⠐⠜ ⠸⠩ ⠹⠁⠸⠌⠃⠼⠡⠹⠉⠸⠌⠙⠼ ⠨⠅ ⠹⠁⠉⠸⠌⠃⠙⠼ ⠸⠱

15 ⠰⠠⠓⠐⠜ ⠠⠁ ⠼⠑⠤⠽⠑⠜ ⠰⠠⠠⠉⠙ ⠺⠢⠞ ⠋ ⠑⠜⠝⠬ ⠔⠞⠻⠑⠌

16 ⠁⠞ ⠮ ⠗⠁⠞⠑ ⠷ ⠸⠩ ⠼⠂⠆⠨⠴⠖⠈⠴⠸⠌⠽⠑⠁⠗ ⠸⠱ ⠔

17 ⠼⠁⠊⠓⠙ ⠞⠕ ⠑⠜⠝⠬ ⠇⠑⠎⠎ ⠹⠁⠝

18 ⠸⠩ ⠼⠴⠨⠦⠶⠈⠴⠸⠌⠽⠑⠁⠗ ⠸⠱ ⠔ ⠼⠃⠚⠁⠑⠲

Line 3: The apostrophe-s is included inside the switch and so a punctuation indicator is needed for the apostrophe. Review 2.7 in Lesson 2.

Line 6: Because the slash means "per" ("Joules per second") a switch to Nemeth Code is required. The value ("880") is included in the switch. An English-letter indicator is required for each single-letter abbreviation.

Line 8: No fraction indicators are used because "pi" and "4" are of normal size and are printed on the baseline.

Line 12: The first slash is in literary context. A switch to Nemeth Code is required for the second slash because it means "over" ("rise over run") in a ratio.

PRACTICE 8C

⠠⠍⠊⠭⠫ ⠠⠝⠥⠍⠃⠻⠎

⠼⠁⠲ ⠠⠋⠔⠙ ⠮ ⠏⠗⠑⠍⠊⠥⠍ ⠿ ⠁
⠸⠩ ⠼⠂⠸⠹⠂⠸⠌⠆⠸⠼⠤⠽⠗⠲ ⠸⠱ ⠏⠕⠇⠊⠉⠽ ⠁⠞ ⠮ ⠽⠑⠜⠇⠽
⠗⠁⠞⠑ ⠷ ⠸⠩ ⠼⠆⠲⠈⠉ ⠠⠄⠏⠻ ⠈⠎⠂⠴⠴⠸⠲
⠼⠆⠸⠲ ⠼⠆⠸⠹⠂⠌⠆⠸⠼ ⠋⠞ ⠬⠦ ⠊⠝ ⠨⠅ ⠿ ⠊⠝⠉⠓⠑⠎
⠼⠒⠸⠲ ⠷⠹⠂⠌⠆⠼⠈⠡⠒⠸⠹⠂⠌⠆⠸⠼⠾⠬⠷⠒⠸⠹⠂⠌⠆⠸⠼⠈⠡⠆⠿
⠼⠲⠸⠲
⠼⠂⠒⠸⠹⠂⠸⠌⠂⠸⠼⠬⠆⠸⠹⠆⠸⠌⠒⠸⠼ ⠨⠅ ⠼⠂⠖⠸⠹⠂⠸⠌⠖⠸⠼
⠼⠢⠸⠲ ⠼⠶⠸⠌⠲ ⠨⠅ ⠼⠂⠸⠹⠿⠸⠌⠲⠸⠼
⠼⠖⠸⠲ ⠹⠔⠌⠲⠼ ⠨⠅ ⠼⠆⠹⠭⠌⠲⠼ ⠸⠱

⠠⠋⠗⠁⠉⠰⠝ ⠠⠗⠑⠧⠊⠑⠺

⠠⠉⠕⠍⠏⠥⠞⠑ ⠑⠁⠡ ⠘⠂⠥⠝⠊⠞ ⠘⠂⠗⠁⠞⠑
⠸⠩ ⠷⠏⠗⠊⠉⠑⠸⠌⠏⠕⠥⠝⠙⠾⠸⠲
⠰⠁⠸⠲ ⠈⠎⠂⠨⠢⠴ ⠠⠄⠿ ⠼⠆⠸⠌⠒ ⠸⠱ ⠏⠨⠙ ⠷ ⠏⠕⠞⠁⠞⠕⠑⠎
⠰⠃⠲ ⠸⠩ ⠈⠎⠲⠨⠆⠴ ⠠⠄⠿ ⠹⠂⠸⠌⠆⠼ ⠸⠱ ⠏⠨⠙ ⠷ ⠠⠫⠁⠍
⠡⠑⠑⠎⠑
⠰⠉⠲ ⠸⠩ ⠈⠎⠖⠨⠴⠴ ⠠⠄⠿ ⠹⠒⠌⠲⠼ ⠸⠱ ⠏⠨⠙ ⠷ ⠙⠑⠇⠊
⠎⠍⠕⠅⠫ ⠞⠥⠗⠅⠑⠽
⠰⠙⠲ ⠸⠩ ⠈⠎⠂⠆⠨⠢⠴ ⠠⠄⠿ ⠼⠂⠸⠹⠂⠌⠆⠸⠼ ⠸⠱ ⠏⠨⠙⠎ ⠷
⠎⠇⠊⠉⠫ ⠓⠁⠍

PRACTICE 8D

⠠⠉⠕⠍⠏⠇⠑⠭ ⠠⠋⠗⠁⠉⠰⠝⠎

⠸⠩ ⠠⠹⠹⠂⠌⠦⠼⠬⠹⠒⠌⠲⠼⠠⠌⠶⠠⠼
⠠⠹⠹⠂⠌⠆⠼⠬⠹⠂⠌⠒⠼⠠⠌⠹⠒⠌⠲⠼⠤⠹⠶⠌⠔⠼⠠⠼
⠠⠹⠂⠸⠌⠒⠬⠂⠸⠌⠲⠠⠌⠲⠸⠌⠢⠤⠂⠸⠌⠆⠠⠼
⠠⠹⠹⠨⠏⠌⠦⠼⠠⠌⠆⠨⠏⠠⠼
⠠⠹⠁⠠⠌⠹⠃⠌⠉⠼⠠⠼
⠠⠹⠒⠒⠸⠹⠂⠌⠒⠸⠼⠠⠌⠂⠴⠴⠠⠼
⠠⠹⠒⠸⠌⠢⠠⠌⠖⠠⠼
⠠⠹⠹⠒⠌⠢⠼⠠⠸⠌⠹⠶⠌⠔⠼⠠⠼ ⠸⠱

PRACTICE 8E

⠠⠘⠮ ⠜⠑ ⠎⠊⠍⠏⠇⠑ ⠋⠗⠁⠉⠰⠝⠎⠒ ⠸⠩
⠹⠂⠌⠆⠼
⠹⠁⠘⠆⠐⠌⠃⠘⠆⠐⠼
⠹⠽⠘⠹⠂⠌⠆⠼⠐⠌⠽⠘⠹⠂⠌⠲⠼⠐⠼ ⠸⠱
⠠⠹ ⠊⠎ ⠸⠂⠝ ⠁ ⠎⠊⠍⠏⠇⠑ ⠋⠗⠁⠉⠰⠝⠒
⠸⠩ ⠠⠹⠂⠸⠌⠒⠠⠌⠆⠸⠌⠒⠠⠼ ⠸⠱
⠠⠗⠑⠧⠊⠑⠺ ⠮ ⠗⠥⠇⠑⠎ ⠔ ⠼⠋⠲⠁⠃⠲⠑ ⠗⠑⠛⠜⠙⠬ ⠁⠝
⠑⠇⠇⠊⠏⠎⠊⠎ ⠕⠝ ⠮ ⠃⠁⠎⠑⠇⠔⠑ ⠷ ⠺⠗⠊⠞⠬ ⠱⠢ ⠭
⠋⠕⠇⠇⠪⠎ ⠁ ⠎⠥⠏⠻⠎⠉⠗⠊⠏⠞⠲ ⠸⠩
⠭⠘⠹⠂⠌⠆⠼⠐⠄⠄⠄ ⠭⠘⠹⠂⠌⠆⠼⠐⠡⠽⠘⠤⠹⠂⠌⠆⠼⠐⠄⠄⠄
⠹⠭⠘⠹⠂⠌⠆⠼⠐⠬⠂⠌⠽⠘⠹⠂⠌⠆⠼⠐⠤⠂⠼ ⠸⠱
⠠⠏⠇⠕⠞ ⠮ ⠏⠕⠔⠞⠎ ⠸⠩ ⠷⠤⠹⠂⠌⠆⠼⠠ ⠲⠾⠠
⠷⠒⠠ ⠲⠸⠹⠂⠌⠲⠸⠼⠾⠠ ⠠⠄⠯ ⠷⠤⠔⠠ ⠹⠒⠌⠲⠼⠾ ⠸⠱⠲ ⠠⠮⠝
⠑⠭⠏⠗⠑⠎⠎ ⠸⠩ ⠹⠁⠘⠒⠸⠌⠲⠐⠌⠁⠘⠢⠸⠌⠲⠐⠼ ⠸⠱ ⠔
⠗⠁⠙⠊⠉⠁⠇ ⠿⠍⠲

PRACTICE 8F

⠠⠗⠁⠙⠊⠉⠁⠇ ⠠⠑⠭⠏⠗⠑⠎⠨⠝⠎

⠀⠀

⠸⠩

⠼⠂⠸⠲ ⠜⠹⠃⠌⠁⠼⠬⠹⠁⠌⠃⠼⠻

⠼⠆⠸⠲ ⠜⠉⠸⠌⠙⠻

⠼⠒⠸⠲ ⠹⠂⠌⠲⠼⠜⠹⠂⠌⠆⠼⠻

⠼⠲⠸⠲ ⠜⠹⠂⠴⠌⠂⠖⠼⠻ ⠨⠅ ⠜⠂⠴⠻⠸⠌⠲

⠼⠢⠸⠲ ⠷⠜⠒⠻⠬⠜⠢⠻⠾⠷⠜⠒⠻⠤⠜⠢⠻⠾

⠼⠖⠸⠲ ⠼⠆⠜⠆⠻⠬⠶⠜⠆⠻ ⠨⠅ ⠷⠆⠬⠶⠾⠜⠆⠻ ⠨⠅ ⠼⠔⠜⠆⠻

⠼⠶⠸⠲ ⠹⠜⠒⠻⠌⠜⠆⠻⠼⠈⠡⠹⠜⠢⠻⠌⠜⠆⠻⠼ ⠨⠅ ⠹⠜⠂⠢⠻⠌⠆⠼

⠼⠦⠸⠲ ⠠⠹⠜⠆⠻⠤⠜⠹⠂⠌⠒⠼⠻⠠⠌⠜⠒⠻⠤⠜⠹⠂⠌⠆⠼⠻⠠⠼

⠼⠔⠸⠲ ⠜⠲⠦⠭⠘⠒⠐⠽⠻

⠼⠂⠴⠸⠲ ⠜⠷⠽⠤⠂⠾⠬⠜⠷⠆⠽⠾ ⠨⠅ ⠼⠂

⠼⠂⠂⠸⠲ ⠣⠝⠜⠙⠻

⠼⠂⠆⠸⠲ ⠣⠁⠬⠃⠜⠵⠤⠽⠻

⠼⠂⠒⠸⠲ ⠣⠲⠜⠶⠆⠔⠻⠬⠣⠖⠜⠆⠶⠻ ⠨⠅ ⠣⠿⠜⠿⠻

⠼⠂⠲⠸⠲ ⠼⠶⠣⠒⠜⠂⠆⠢⠻⠡⠶⠣⠢⠜⠆⠻

⠼⠂⠢⠸⠲ ⠣⠢⠜⠍⠻⠣⠢⠜⠝⠻ ⠨⠅ ⠣⠢⠜⠍⠝⠻ ⠸⠱

PRACTICE 8G

⠠⠝⠑⠌⠫ ⠠⠗⠁⠙⠊⠉⠁⠇ ⠠⠑⠭⠏⠗⠑⠎⠨⠝⠎

⠀⠀

⠸⠩

⠷⠂⠾ ⠜⠤⠹⠂⠌⠆⠼⠤⠊⠹⠨⠜⠒⠨⠻⠌⠆⠼⠻

⠷⠆⠾ ⠜⠨⠜⠂⠒⠨⠻⠬⠨⠜⠂⠢⠨⠻⠬⠨⠜⠂⠂⠶⠨⠻⠻

⠷⠒⠾ ⠜⠂⠤⠨⠜⠁⠤⠃⠨⠻⠻⠈⠡⠎⠂⠬⠨⠜⠁⠤⠃⠨⠻⠻

⠷⠲⠾ ⠜⠃⠨⠣⠒⠜⠃⠨⠨⠜⠃⠨⠨⠻⠨⠻⠻

⠷⠢⠾ ⠣⠁⠜⠨⠣⠃⠜⠨⠨⠣⠉⠜⠁⠃⠉⠨⠨⠻⠨⠻⠻

⠷⠖⠾ ⠜⠁⠘⠆⠐⠻⠜⠃⠸⠲⠐⠻⠜⠉⠻ ⠨⠅ ⠁⠃⠘⠆⠐⠜⠉⠻

⠷⠶⠾ ⠷⠎⠘⠆⠐⠣⠒⠜⠎⠘⠲⠐⠻⠾⠘⠆

⠷⠦⠾ ⠣⠒⠜⠭⠘⠆⠐⠨⠜⠖⠲⠭⠘⠖⠐⠨⠻⠻

⠷⠔⠾ ⠜⠨⠣⠒⠜⠨⠨⠣⠲⠜⠨⠨⠨⠣⠢⠜⠃⠘⠲⠦⠐⠨⠨⠨⠻⠨⠨⠻⠨⠻⠻

⠷⠂⠴⠾ ⠜⠭⠂⠬⠨⠜⠭⠆⠨⠻⠻

⠷⠂⠂⠾ ⠟⠘⠜⠗⠻⠐⠬⠎

⠷⠂⠆⠾ ⠜⠉⠬⠙⠬⠑⠬ ⠄⠄⠄⠻ ⠸⠱

PRACTICE 8H

⠠⠇⠔⠅⠫ ⠠⠑⠭⠏⠗⠑⠎⠨⠝⠎
⠀⠀
⠼⠁⠲ ⠠⠊⠎ ⠮ ⠋⠕⠇⠇⠪⠬ ⠔⠑⠟⠥⠁⠇⠰⠽ ⠞⠗⠥⠑⠦
⠸⠩ ⠹⠒⠌⠢⠼⠷⠹⠆⠌⠒⠼⠭⠤⠹⠂⠌⠆⠼⠾
⠨⠂ ⠹⠆⠌⠢⠼⠷⠹⠂⠌⠲⠼⠭⠬⠹⠂⠌⠒⠼⠾
⠼⠆⠸⠲ ⠼⠒⠒⠸⠹⠂⠌⠒⠸⠼⠈⠴⠬⠲⠴⠈⠴⠬⠖⠂⠸⠹⠆⠌⠒⠸⠼⠈⠴
⠨⠅ ⠼⠂⠒⠲⠸⠹⠒⠌⠒⠸⠼⠈⠴ ⠨⠅ ⠼⠂⠒⠢⠈⠴ ⠸⠱
⠼⠉⠲ ⠠⠔ ⠍⠥⠇⠞⠊⠏⠇⠽⠬ ⠸⠩ ⠼⠢⠸⠹⠒⠌⠲⠸⠼⠈⠡⠖⠖ ⠸⠱⠂
⠗⠑⠉⠁⠇⠇ ⠞ ⠸⠩ ⠼⠢⠸⠹⠒⠌⠲⠸⠼ ⠨⠅ ⠼⠢⠬⠹⠒⠌⠲⠼⠸⠲
⠠⠄⠠⠐⠮⠿⠑⠂ ⠼⠲⠖⠈⠡⠷⠢⠬⠹⠒⠌⠲⠼⠾
⠨⠅ ⠷⠲⠖⠈⠡⠢⠾⠬⠷⠲⠖⠈⠡⠹⠒⠌⠲⠼⠾ ⠨⠅ ⠼⠆⠖⠲⠸⠹⠂⠌⠆⠸⠼
⠸⠱⠲

Lines 10-11: Notice that the print copy divided this equation after the equals sign, but the braille transcription follows Nemeth rules and divides before the comparison sign.

PRACTICE 8I

Lines 7 and 8: Two sets of linked expressions are displayed to the first paragraph. Each linked expression begins in cell 3. Neither linked expression has a runover.

Line 9: The narrative paragraph continues in cell 1.

Line 10: The displayed expression begins in cell 3.

Line 11: The link will not fit on line 11 so it begins in the runover position, cell 3.

Line 12: The Nemeth Code terminator and comma do not fit on line 11. They are placed in the runover position, cell 3.

Lines 13-14: The narrative paragraph continues in cell 1.

Lines 15-18: The print layout makes this a nested linked expression, displayed to a 3-1 paragraph. The anchor begins in cell 3, and each link begins in cell 5,.

Line 19: The final linked expression is not part of the nested expression. It begins in in cell 3 (displayed to a 3-1 paragraph).

PRACTICE 8J

LESSON 9

- ARROWS
  - Construction of Braille Arrows
  - Vertical, Slanted, and Curved Arrow Shafts
  - Boldface and Compounded Arrows
- INTRODUCTION TO SPATIAL ARRANGEMENTS
  - Spatial Arrangements with Addition and Subtraction
  - Introduction to Cancellation
  - Arrangement on the Page
  - Placement of Code Switch Indicators

Answers to Practice Material

LESSON PREVIEW

The construction of braille arrows is demonstrated, showing several styles of arrowheads and arrow shafts. Braille format for spatially arranged addition and subtraction problems is studied.

Note: Due to the amount of vertical space used with spatial arrangements, this lesson consumes more pages than most.

ARROWS

[NC Rule 22]

9.1 Arrows Used in Mathematics

Here are some examples of arrows which bear mathematical meaning.

$A \rightarrow B$	$\overleftrightarrow{PQ}$	$\mathrm{2CO} \rightleftharpoons \mathrm{CO_2} + \mathrm{C}$
$f: a \longmapsto b$	$P \uparrow Q$	$A \longrightarrow B \leftrightarrow C \longleftarrow D$

Note that some chemical arrows are constructed differently from math arrows. In a chemistry transcription, the arrows of *Chemical Notation Using the Nemeth Braille Code* should be used.

9.2 Construction of Braille Arrows

The components of an arrow are transcribed in the following order.

(1) The shape indicator. (See below.)
(2) The arrow direction, if it must be indicated (vertical or slanted). (Sections 9.9.1 and 9.9.2.)
(3) The typeface of the arrow, if it must be indicated. (Section 9.10.)
(4) The left arrowhead or dot, if any. (Sections 9.5, 9.7, and 9.8.)
(5) The arrow shaft, if required. (Sections 9.4 and 9.9.)
(6) The right arrowhead or dot, if any. (Sections 9.4, 9.7, and 9.8.)

An explicit terminator is not necessary.

⠫	Shape Indicator

9.3 Spacing and Punctuation with Arrows

When arrows function as a sign of comparison, the entire construction is spaced according to the rules governing comparison signs. When not functioning as a sign of comparison, an arrow is spaced according to its context. Spacing issues will be illustrated throughout this lesson.

Punctuation associated with an arrow is mathematical.

9.4 Horizontal Arrow Shafts

The length of a shaft is indicated by the number of times the arrow shaft symbol is used. Two braille symbols represent the ordinary shaft length. By comparison, one braille symbol represents a short shaft, and three or more symbols indicate a longer shaft.

Horizontal Arrow Shafts		
⠒⠒	Ordinary length, single	——
⠒	Short, single	–
⠒⠒⠒	Long, single	———
⠶⠶	Ordinary length, double	==
⠶	Short, double	=
⠶⠶⠶	Long, double	===
⠒⠀⠒	Dashed	- -
⠂⠂⠂	Dotted	. . .
⠔⠒⠢	Wavy	∼∼

9.5 Barbed Arrowheads

An arrowhead may be barbed, blunted, straight, or curved, and may occur at the left, at the right, or at both ends of an arrow shaft. An arrowhead may also appear with only its upper or lower barb. First, we will look at barbed arrowheads appearing at the end of a horizontal arrow shaft.

The short line in each image represents the point where the arrow shaft meets the arrowhead.

Barbed Arrowheads		
⠪	Full, left pointing	
⠕	Full, right pointing	
⠠⠪	Lower only, left pointing	
⠠⠕	Lower only, right pointing	
⠈⠪	Upper only, left pointing	
⠈⠕	Upper only, right pointing	

Arrowheads printed as a solid triangle are considered to be barbed arrowheads and are transcribed as such.

Many types of arrows can be constructed using the given shaft and arrowhead symbols. Some examples are shown below.

Note: Code switch indicators are omitted in the isolated examples in this section.

Consider the shaft in the barbed arrows shown below to be the standard length for the book. A two-cell "ordinary length" shaft is transcribed. Notice the similarity between the print and braille symbols.

→	⠫⠒⠒⠕	←	⠫⠪⠒⠒	↔	⠫⠪⠒⠒⠕
⇒	⠫⠶⠶⠕	⇐	⠫⠪⠶⠶	⇔	⠫⠪⠶⠶⠕
⇀	⠫⠒⠒⠈⠕	↼	⠫⠈⠪⠒⠒	⇌	⠫⠠⠪⠒⠒⠈⠕
⇁	⠫⠒⠒⠠⠕	↽	⠫⠠⠪⠒⠒	⇋	⠫⠈⠪⠒⠒⠠⠕
↣	⠫⠕⠒⠒⠕	↢	⠫⠪⠒⠒⠪		
↼⇀	⠫⠈⠪⠒⠒⠈⠕	↽⇁	⠫⠠⠪⠒⠒⠠⠕		

An additional rule may apply to the first arrow shown above. See 9.6.

By comparison, the shafts in the arrows shown below are longer. A three-cell shaft is transcribed.

⟶ ⠫⠒⠒⠒⠕ ⟹ ⠫⠶⠶⠶⠕

⟵ ⠫⠪⠒⠒⠒ ⟸ ⠫⠪⠶⠶⠶

And, by comparison, the shafts in the arrows shown next are shorter. A one-cell shaft is transcribed.

→ ⠫⠒⠕ ← ⠫⠪⠒

Other shaft styles are illustrated below.

Dashed and dotted		Wavy	
--→	⠫⠒ ⠒⠕	↝	⠫⠔⠒⠢⠕
←--	⠫⠪⠒ ⠒	↜	⠫⠪⠔⠒⠢
⋯→	⠫⠂⠂⠂⠕	↭	⠫⠪⠔⠒⠢⠕

a. **Spacing.** Note the spacing before and after these arrows which are functioning as comparison signs.

⫸ $x \rightharpoonup y$ ⠭ ⠫⠒⠒⠈⠕ ⠽

⫸ $X \Rightarrow Y$ ⠠⠭ ⠫⠶⠶⠕ ⠠⠽

b. **Punctuation.** Mathematical punctuation is used with arrows.

⫸ (←, ↔, ⇒)

A dot 6 comma is used in Nemeth.

Example 9-1

Consider the set $\mathcal{P} \longleftarrow 2$.

The script letter P requires an English-letter indicator even though the letter precedes a sign of comparison. (A typeform indicator must always be followed by an alphabetic indicator.)

9.6 Special Case: The Contracted Form of the Right-Pointing Arrow [NC 22.1]

When the following criteria are all true, the right-pointing arrow is represented in its contracted form.

- the shaft is single and is of ordinary length*
- the barb is a full barb and is right pointing
- the arrow is printed in regular typeface

	Contracted Arrow

⫸ $A \longrightarrow B$

⫸ $p \longrightarrow (q \vee r)$

Example 9-2

Is the following equation balanced: $2H_2O + O_2 \longrightarrow 2H_2O$?

*"Ordinary length" is determined by comparing the arrow shafts printed in the book being transcribed.

9.6.1 **Nonuse of the Contracted Form of the Right-Pointing Arrow.** All other right-pointing arrows require the use of the appropriate shaft symbol.

- ⫸ X ⇢ Y ⠠⠭ ⠫⠒ ⠒⠕ ⠠⠽
- ⫸ X ⇝ Y ⠠⠭ ⠫⠔⠒⠢⠕ ⠠⠽
- ⫸ X ⟶ Y ⠠⠭ ⠫⠒⠒⠒⠕ ⠠⠽
- ⫸ X → Y ⠠⠭ ⠫⠒⠕ ⠠⠽

Instructions: Leave one space (one blank cell) between the arrows in A. and B. Assume shafts in items A, C, and D to be of regular length, and in item B to be long. In item D, divide the long expression before the arrow that comes between the double parentheses. Assume all to be in regular type even though they may appear darker.

PRACTICE 9A

Arrows with Barbed Ends

A. ⇔ ⇀ ↔ ⬿ ⇢ ⇢ ⇝ ↳

B. ⟵ ⟷ ⟷ ⟺

C. The principal operator of the left formula in Fig. 7.2 is ↔, while the principal operator of the right formulas is →.

D. Construct a truth table for $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$.

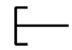
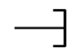
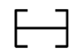
9.7 Blunted, Straight, and Curved Arrowheads

Here are three other types of arrowheads identified in the Nemeth Code. Examples with a single shaft of ordinary length are shown. The longer horizontal line in each image represents the end of the arrow shaft.

Please disregard the differences in size and line weight in these images.

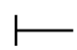
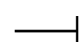
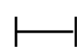
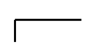
BLUNTED ARROWHEADS

Each blunted arrowhead can be left or right pointing.

⠿	Full	⠫⠿⠒⠒	
		⠫⠒⠒⠿	
		⠫⠿⠒⠒⠿	
⠠⠿	Lower only	⠫⠠⠿⠒⠒	
		⠫⠒⠒⠠⠿	
⠈⠿	Upper only	⠫⠈⠿⠒⠒	
		⠫⠒⠒⠈⠿	

STRAIGHT ARROWHEADS

Each straight arrowhead can be left or right pointing.

⠳	Full	⠫⠳⠒⠒	
		⠫⠒⠒⠳	
		⠫⠳⠒⠒⠳	
⠠⠳	Lower only	⠫⠠⠳⠒⠒	
		⠫⠒⠒⠠⠳	
⠈⠳	Upper only	⠫⠈⠳⠒⠒	
		⠫⠒⠒⠈⠳	

CURVED ARROWHEADS

⠯ Full, left pointing	⠫⠯⠒⠒	
⠽ Full, right pointing	⠫⠒⠒⠽	
	⠫⠯⠒⠒⠽	
⠠⠯ Lower only, left pointing	⠫⠠⠯⠒⠒	
⠠⠽ Lower only, right pointing	⠫⠒⠒⠠⠽	
⠈⠯ Upper only, left pointing	⠫⠈⠯⠒⠒	
⠈⠽ Upper only, right pointing	⠫⠒⠒⠈⠽	

Example 9-3

Use x $\mapsto f(x)$ to denote that an element $x \in X$ is mapped to an element $f(x) \in Y$ by the mapping $f\colon X \to Y$.

  ⠠⠥⠎⠑ ⠸⠩ ⠭ ⠫⠳⠒⠒⠕ ⠋⠷⠭⠾ ⠸⠱ ⠞⠕ ⠙⠢⠕⠞⠑ ⠞ ⠁⠝
⠑⠇⠑⠰⠞ ⠸⠩ ⠭ ⠈⠑ ⠠⠭ ⠸⠱ ⠊⠎ ⠍⠁⠏⠏⠫ ⠞⠕ ⠁⠝ ⠑⠇⠑⠰⠞
⠸⠩ ⠋⠷⠭⠾ ⠈⠑ ⠠⠽ ⠸⠱ ⠃⠽ ⠮ ⠍⠁⠏⠏⠬
⠸⠩ ⠰⠋⠸⠒⠠⠭ ⠫⠕ ⠠⠽ ⠸⠱⠲

Review 5.7 in Lesson 5 regarding transcription of the colon used in mapping notation.

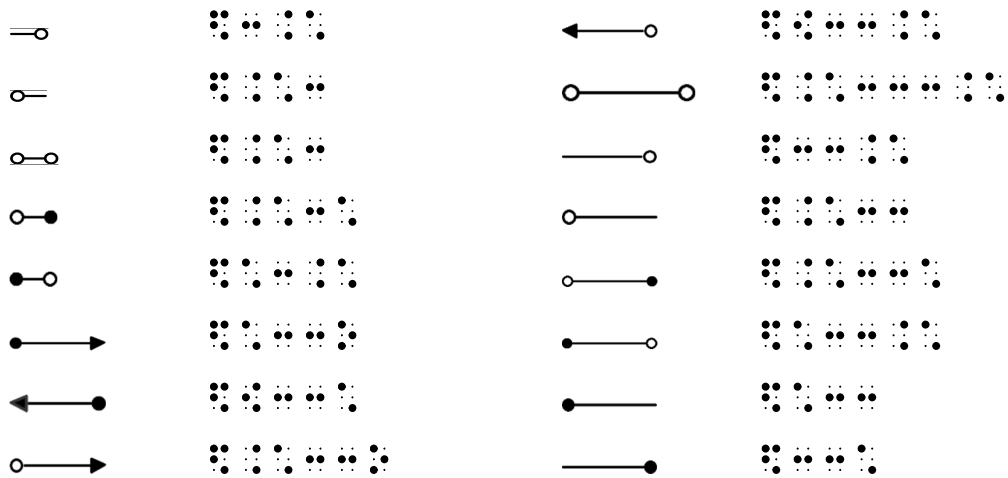
9.8 Arrows with Dotted Ends

Arrows may be represented by a shaft preceded or followed by a solid or hollow dot.

⠡	Solid Dot	•
⠨⠡	Hollow Dot	∘

Dotted ends are most commonly encountered in geometry and will be illustrated in the lesson on modified expressions. *Note*: Arrow symbols used with graphic number lines are not represented by the braille symbols shown above. Refer to the most recent edition of *Guidelines and Standards for Tactile Graphics* for number line techniques.

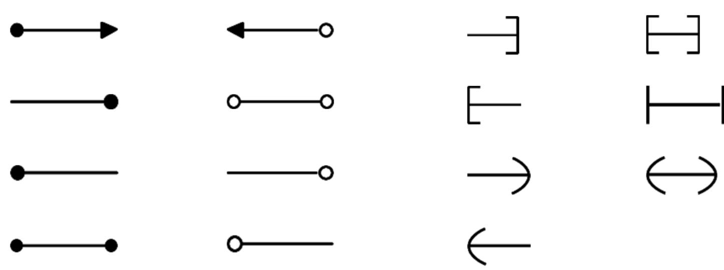
An sample list of arrows with dotted ends is shown on the next page.



Instructions: Transcribe in four columns, as printed. Assume all shafts are of regular length. Arrowheads printed as a solid triangle are considered to be barbed arrowheads.

PRACTICE 9B

Other Types of Arrowheads



Vertical, Slanted, and Curved Arrow Shafts

The arrows shown so far point horizontally – to the left, to the right, or both left and right. When an arrow points in a vertical or slanted direction, arrow direction indicators are used.

9.9 Arrow Direction Indicators [NC 22.4]

An arrow's direction is assumed to be horizontal unless a direction indicator is used. There are two types of direction indicators – one set for vertical arrows and one set for slanted arrows.

9.9.1 **Vertical Arrow Shaft.** A vertical arrow requires either a "directly over" or a "directly under" indicator.

⠣	Pointing Up Indicator
⠩	Pointing Down Indicator

The shape indicator is transcribed first, followed by the direction indicator, the arrow shaft, and the arrowhead.

a. **One-Headed Arrow.** When a vertical arrow has only one arrowhead, the symbol for the right arrowhead is used (dots 135).

⠫⠣⠒⠒⠕	↑	(up-pointing arrow)
⠫⠩⠒⠒⠕	↓	(down-pointing arrow)

b. **Two-Headed Arrow.** To show a two-headed vertical arrow (pointing up and down) the "pointing up" indicator is transcribed before a two-headed arrow.

⠫⠣⠪⠒⠒⠕	↕	(up-and-down-pointing arrow)

Example 9-4

$x \downarrow a$ means x is approaching a "from above", much like $x \rightarrow a^{+}$.

⠸⠩ ⠭ ⠫⠩⠒⠒⠕ ⠁ ⠸⠱ ⠍⠂⠝⠎ ⠰⠭ ⠊⠎ ⠁⠏⠏⠗⠕⠁⠡⠬ ⠁
⠦⠋ ⠁⠃⠧⠴⠂ ⠍⠡ ⠇ ⠸⠩ ⠭ ⠫⠕ ⠁⠘⠬ ⠸⠱⠲

Functioning as a sign of comparison, each arrow is spaced accordingly.

9.9.2 **Slanted Arrow Shaft.** Slanted arrows require either a superscript or a subscript indicator. The superscript indicator elevates the arrowhead by 45 degrees. The subscript indicator depresses the arrowhead by 45 degrees.

⠘	Superscript Indicator
⠰	Subscript Indicator

The shape indicator is transcribed first, followed by the appropriate "slant" indicator, and then the arrow.

a. **One-Headed Arrow.** When a slanted arrow has only one arrowhead, the appropriate left- or right-pointing arrowhead is used. The first two arrows below show a one-headed arrow pointing upward at a slant. The superscript indicator is used to show the upward slant of the arrowhead.

⠫⠘⠪⠒⠒	$\nwarrow$	(arrow points northwest)
⠫⠘⠒⠒⠕	$\nearrow$	(arrow points northeast)

The next two arrows show a one-headed arrow pointing downward at a slant. The subscript indicator is used to show the downward slant of the arrowhead.

⠫⠰⠪⠒⠒	$\swarrow$	(arrow points southwest)
⠫⠰⠒⠒⠕	$\searrow$	(arrow points southeast)

Example 9-5

$x \uparrow a = x \nearrow a = x \rightarrow a^{-}$

⠸⠩ ⠭ ⠫⠣⠒⠒⠕ ⠁ ⠨⠅ ⠭ ⠫⠘⠒⠒⠕ ⠁ ⠨⠅ ⠭ ⠫⠕ ⠁⠘⠤ ⠸⠱

Each arrow is a sign of comparison and is spaced accordingly.

b. **Two-Headed Arrow.** To show a two-headed slanted arrow, the left-pointing arrowhead dictates the indicator used. In the arrow, below, the left arrowhead points upward, therefore the superscript indicator is used.

⠫⠘⠪⠒⠒⠕	⤡	(arrow points northwest to southeast)

The next arrow has the left arrowhead pointing downward, therefore the subscript indicator is used.

⠫⠰⠪⠒⠒⠕	⤢	(arrow points northeast to southwest)

9.9.3 **Curved Arrow Shaft [NC 22.5.1]** Direction indicators are not used with curved arrows. Instead, the curved shaft symbol is used, and the type of arrowhead reveals the direction of curvature.

⠢⠔	Curved Shaft

Rotation direction is indicated as follows. A curved shaft preceded by a *left*-pointing arrowhead represents a clockwise arrow. A curved shaft followed by a *right*-pointing arrowhead represents a counterclockwise arrow.

⠫⠪⠢⠔	Clockwise Curve	↻	⤾	⤸
⠫⠢⠔⠕	Counterclockwise Curve	↺	⤿	⤹

PRACTICE 9C

Other Types of Arrow Shafts

Vertical	Slanted	Curved
↕	↖ ↘	↺
↑	↗ ⤡	↻
↓	↙ ⤢	

Can you figure out how to construct this spear with a northwest, blunted arrowhead?

Boldface and Compounded Arrows

9.10 Boldface Arrow [NC 22.6]

When an arrow is printed in boldface type and the typeform is determined to be significant and therefore retained, the boldface typeform indicator (dots 456) is incorporated into the structure of the arrow. The typeform indicator is placed at the beginning of the arrow symbol, before the arrowhead or arrow shaft. When a direction indicator is required, the direction indicator is transcribed first, followed by the boldface typeform indicator.

⠫⠸⠒⠒⠕	Boldface Pointing Right	$\rightarrow$
⠫⠸⠪⠒⠒	Boldface Pointing Left	$\leftarrow$
⠫⠸⠪⠒⠒⠕	Boldface Pointing Left and Right	$\leftrightarrow$
⠫⠣⠸⠒⠒⠕	Boldface Pointing Up	$\uparrow$
⠫⠩⠸⠒⠒⠕	Boldface Pointing Down	$\downarrow$
⠫⠣⠸⠪⠒⠒⠕	Boldface Pointing Up and Down	$\updownarrow$

Reminder: A right-pointing arrow in nonregular type requires a shaft. The contracted form is not used.

9.11 Arrows Used as Signs of Comparison Compounded Vertically [NC 21.9]

Lesson 5 introduced signs of comparison compounded vertically—two or more simple signs of comparison arranged one under the other in print. When arrows are so arranged, the same principle applies. The combination of symbols becomes a single comparison sign "compounded vertically". The uppermost arrow symbol is transcribed first, immediately followed by and unspaced from the symbol for the lower arrow. Each arrow requires a shape indicator.

When the arrow is part of a sign of comparison compounded vertically, the uncontracted form of the right-pointing arrow is used. The contracted form of the right-pointing arrow is never used in the construction of arrows compounded vertically.

⠫⠒⠒⠕	Uncontracted Right-Pointing Arrow	$\rightarrow$

Arrows compounded vertically are most often encountered in the subject of chemistry. When the rules of *Chemical Notation Using the Nemeth Braille Code* are being followed, use the arrow constructions from that code, which differ from the Nemeth symbols.

When arrows compounded vertically are encountered in mathematics, use the constructions shown below. Mathematical arrows which are not shown in this list should be transcribed in accordance with the principle outlined in this section.

⠫⠒⠒⠕⠫⠪⠒⠒	Pointing Right Over Pointing Left	⇄
⠫⠪⠒⠒⠫⠒⠒⠕	Pointing Left Over Pointing Right	⇆
⠫⠒⠒⠒⠕⠫⠪⠒	Long Pointing Right Over Short Pointing Left	⥂
⠫⠒⠕⠫⠪⠒⠒⠒	Short Pointing Right Over Long Pointing Left	⥄
⠫⠒⠒⠈⠕⠫⠠⠪⠒⠒	Pointing Right, Upper Barb Only Over Pointing Left, Lower Barb Only	⇌
⠫⠒⠒⠕⠫⠸⠪⠒⠒	Pointing Right Over Boldface Pointing Left	$\begin{smallmatrix}\rightarrow\\ \boldsymbol{\leftarrow}\end{smallmatrix}$
⠫⠸⠪⠒⠒⠫⠒⠒⠕	Boldface Pointing Left Over Pointing Right	$\begin{smallmatrix}\boldsymbol{\leftarrow}\\ \rightarrow\end{smallmatrix}$
⠫⠸⠒⠒⠕⠫⠸⠪⠒⠒	Boldface Pointing Right Over Boldface Pointing Left	$\begin{smallmatrix}\boldsymbol{\rightarrow}\\ \boldsymbol{\leftarrow}\end{smallmatrix}$

9.12 Arrows Used as Signs of Comparison Compounded Horizontally [NC 21.11]

Lesson 5 also introduced the topic of signs of comparison compounded horizontally. When arrows are arranged side by side, the same principle applies. The combination of symbols becomes a single comparison sign "compounded horizontally". A multipurpose indicator (dot 5) is inserted between the unspaced symbols to indicate that the symbols are printed side by side. Arrows not shown in this list should be transcribed in accordance with this principle.

⠫⠣⠒⠒⠕⠐⠫⠩⠒⠒⠕	Pointing Up Followed by Pointing Down	↑↓
⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕	Pointing Down Followed by Pointing Up	↓↑
⠫⠣⠒⠒⠕⠐⠫⠩⠸⠒⠒⠕	Pointing Up Followed by Boldface Pointing Down	↑**↓**
⠫⠩⠸⠒⠒⠕⠐⠫⠣⠸⠒⠒⠕	Boldface Pointing Down Followed by Boldface Pointing Up	**↓↑**

Arrows compounded horizontally are most often encountered in the subject of chemistry. When the rules of *Chemical Notation Using the Nemeth Braille Code* are being followed, use the arrow constructions from that code, which differ from the Nemeth symbols.

9.13 Nonmathematical Arrows

Nemeth arrows should not be used for nonmathematical purposes. Rules and guidelines in BANA publications examine flowchart arrows, arrows in graphic organizer diagrams, lead lines in tactile graphics, etc. Refer to *Guidelines and Standards for Tactile Graphics* for techniques regarding arrows in specific applications such as box-and-whisker plots, clock faces, graphs, line plots, measurement tools, number lines, and spinners.

Here are some samples. (Transcriptions are not shown.)

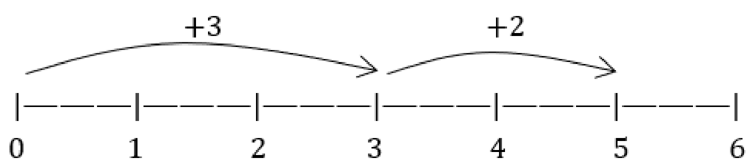
Example 9-6

F O I L

$$(2x+3)(x-5) = 2x^2 - 10x + 3x - 15$$

These "pointing" arrows should be drawn as a raised-line tactile graphic.

Example 9-7



Methods for depicting arrows above number lines are given in Guidelines and Standards for Tactile Graphics.

Example 9-8

$\frac{3}{4}$ ← *fraction line*

The lead line in this diagram could be omitted without a loss of information.

INTRODUCTION TO SPATIAL ARRANGEMENTS

9.14 Background

Up to this point we have looked at mathematical material that is read linearly, from left to right, whether it appears embedded within the narrative or set apart from the text as displayed material. When material is arranged on more than one line in print and there is essential vertical alignment of place values or characters, a spatial arrangement is required in braille. ("Alignment" refers to vertical location in the same column or cell.)

9.14.1 **Numeric Indicator.** The numeric indicator is generally not used in arrangements which are aligned for computation. Use of the numeric indicator in certain situations abides by rules particular to that specific focus. This distinction will be clarified as each topic is presented.

9.14.2 **Format.** A blank line is required above and below a spatial arrangement. However, transition to a new braille page before beginning or after ending a spatial arrangement takes the place of the required blank line. When no running head is used, a spatial arrangement can begin on line 1 unless a code switch indicator is required at the page turn, or unless the print page number interferes with the alignment. Similarly, a spatial arrangement can end on line 25 unless a Nemeth Code terminator is required or unless the braille page number interferes with the alignment. These exceptions are illustrated later in this lesson.

This lesson teaches rules for the transcription of spatially arranged addition and subtraction problems.

Spatial Arrangements with Addition and Subtraction

The Preliminary Lesson introduced linear problems using the symbol + for plus, – for minus, and = for equals.

⫸ $2+3=5$ ⠼⠆⠬⠒ ⠨⠅ ⠼⠢

⫸ $7-6=1$ ⠼⠶⠤⠖ ⠨⠅ ⠼⠂

When addition and subtraction problems are printed in a vertical arrangement, Nemeth rules for spatial arrangements apply. We will use standard terminology to refer to the parts.

2	*addend*	7	*minuend*
+3	*addend*	−6	*subtrahend*
——	*separation line*	——	*separation line*
5	*sum*	1	*difference*

9.15 Separation Line

The line printed between the addends and the sum or between the subtrahend and the difference (signifying "equals") is called the "separation line" and is transcribed as a line of dots 25. In braille, the separation line must be made one cell longer at either end than the overall width of the arrangement.

⠒⠒⠒⠒⠒⠒	Separation Line (varying in length)

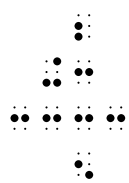
9.16 Alignment with Addition and Subtraction

In spatial arrangements for addition and subtraction, the corresponding digits, commas, and decimal points are vertically aligned one below the other. In other words, digits under digits, commas under commas, decimal points under decimal points.

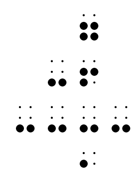
The Examples Placement of code switch indicators with spatial arrangements is discussed later in this lesson. Assume Nemeth context in these examples. Switch indicators and the blank line before and after each example are not shown.

Example 9-9

$$\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array}$$



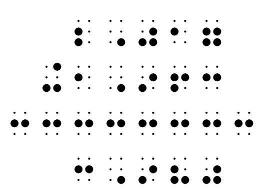
$$\begin{array}{r} 7 \\ -6 \\ \hline 1 \end{array}$$



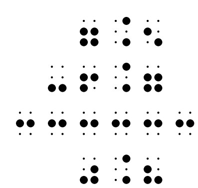
Numeric indicators are not used. Digits are aligned by place value, as printed. The separation line extends one cell to the left and one cell to the right of the overall arrangement.

Example 9-10

$$\begin{array}{r} 2,017 \\ +1,963 \\ \hline 3,980 \end{array}$$



$$\begin{array}{r} 7.5 \\ -6.7 \\ \hline 0.8 \end{array}$$

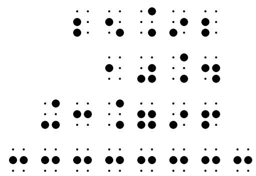


Commas and decimal points are aligned as printed. The separation line extends one cell to the left and one cell to the right of the overall arrangement.

9.16.1 **Intentional Misalignment.** If items have been intentionally misaligned as an exercise for the student, the misalignment is preserved in the transcription. In the next example, the student has been instructed to arrange the digits for proper place value.

Example 9-11

```
 25.92
  10.4
+3.796
------
```

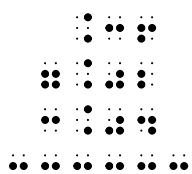


The decimal points are intentionally misaligned. The braille transcription matches the print arrangement.

9.16.2 **Operation Sign is Absent.** The plus sign is not always present in addition problems. If there is no operation sign, examine the surrounding text to determine that this is indeed an addition problem. Then apply alignment rules for addition.

Example 9-12

```
 .36
7.02
3.04
----
```

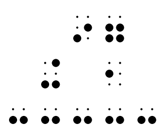


9.17 Placement of Symbols

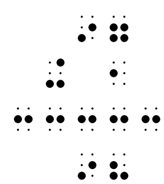
9.17.1 **Operation Symbols.** The plus or minus symbol indicating addition or subtraction is placed one column of cells to the left of the leftmost numeric symbol in the part of the arrangement which lies *above* the separation line, regardless of print layout.

Example 9-13

```
97
+1
--
```



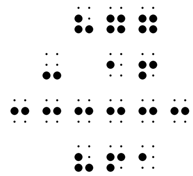
```
97
+1
--
98
```



The plus sign is printed within the column reserved for the tens place. In braille, the plus sign is placed one column to the left of the leftmost numeric symbol that appears above the separation line.

Example 9-14

```
 877
−16
 861
```

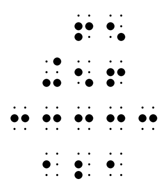


The minus sign is printed under the "8" in the minuend. In braille, the minus sign is placed one column to the left of the leftmost numeric symbol that appears above the separation line.

a. **Considerations Below the Separation Line.** Placement of the operation symbol is in relation to the numeric characters above the separation line. If an answer is printed below the separation line, part of it may fall beneath symbols which appear above the separation line.

Example 9-15

```
  65
+56
 121
```

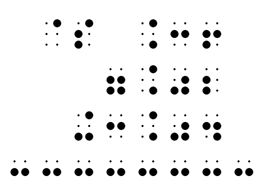


The "ones" and "tens" place values in the sum are aligned with those place values in the addends. Because the plus sign is placed one cell to the left of the "tens" column, the "hundreds" place value in the sum (below the separation line) falls in the same column as the plus sign.

9.17.2 **Currency Symbols.** Currency symbols which appear above the separation line must also be similarly placed—that is, one column of cells to the left of the leftmost numeric symbol in the part of the arrangement which lies *above* the separation line. A plus or minus symbol (one cell) aligned below a currency symbol (two cells) is right adjusted in the column.

Example 9-16

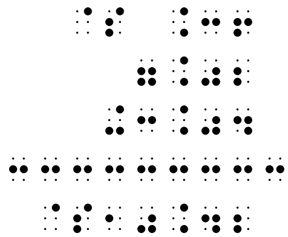
```
$  .36
  7.02
+3.04
```



a. **Considerations Below the Separation Line.** Placement of the currency symbol applies to the numeric characters above the separation line. If an answer is printed below the separation line, part of it may be shown beneath the symbols which appear above it.

Example 9-17

```
$  .36
  7.02
 +3.04
------
$10.42
```

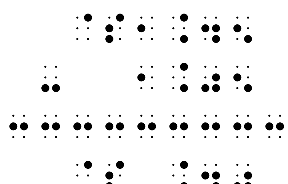


The decimal point and place values in the sum align with the decimal points and place values in the addends. The dollar signs above and below the separation line are not aligned in print and so need not be aligned in braille.

b. **Further Considerations.** A symbol of operation or a currency symbol may be placed in the same position shown in print as long as the symbols above the separation line are placed at least one column of cells to the left of the leftmost column of numeric characters appearing there.

Example 9-18

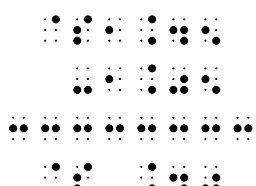
```
  $1.45
–  1.05
-------
  $ .40
```



In print, the minus sign appears to the left of the dollar sign. This alignment is duplicated in braille.

Example 9-19

```
 $1.45
– 1.05
------
 $ .40
```



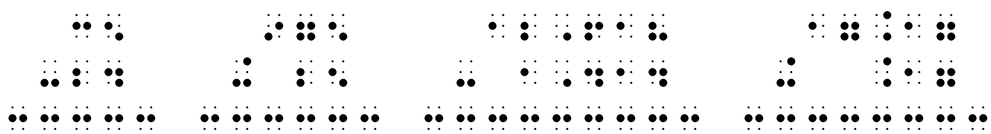
In print, the minus sign appears directly beneath the dollar sign and the two dollar signs are vertically aligned. This alignment is duplicated in braille.

9.18 Side-by-Side Layout

Spatial arrangements may be placed side by side across a page. Care must be taken to leave at least three blank cells between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement. "Any symbol" does not refer to a separation line. This means that one blank space is left between the end of one separation line and the beginning of the next when neighboring arrangements contain separation lines.

Example 9-20

$$\begin{array}{r}35\\ \underline{-24}\end{array}\qquad \begin{array}{r}975\\ \underline{+25}\end{array}\qquad \begin{array}{r}12,618\\ \underline{-1,414}\end{array}\qquad \begin{array}{r}17.17\\ \underline{+\ .17}\end{array}$$

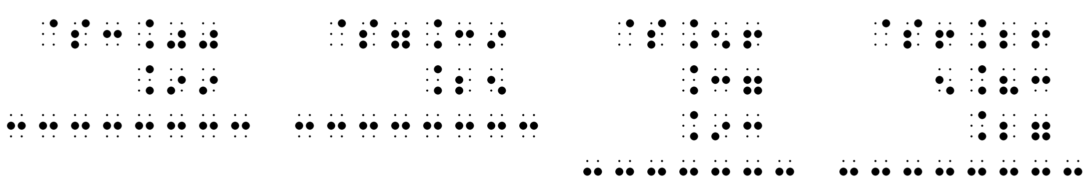


Each separation line extends one cell to the left and one cell to the right of its overall arrangement. One clear column of blank space is between the ends of the separation lines.

9.18.1 **Top Alignment.** The first addend in each problem is transcribed on the same line, regardless of the print arrangement. When the separation lines are not on the same braille line, at least one clear column of blank space (one cell's width) must be maintained between the ends of the separation lines. This assures that at least three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring spatial arrangement (separation lines excluded).

Example 9-21

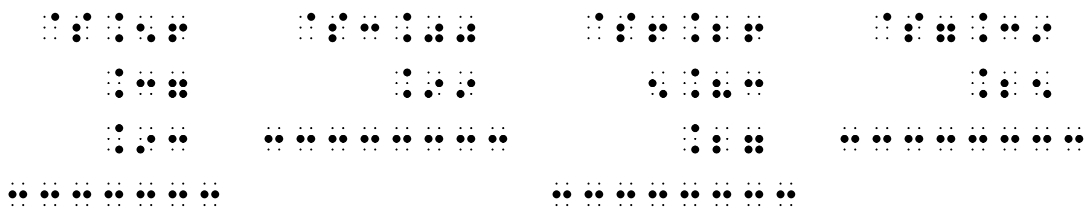
$$\begin{array}{r}\$3.00\\ \underline{.99}\end{array}\qquad \begin{array}{r}\$7.39\\ \underline{.25}\end{array}\qquad \begin{array}{r}\$.56\\ .37\\ \underline{.93}\end{array}\qquad \begin{array}{r}\$6.26\\ 5.83\\ \underline{.27}\end{array}$$



One clear column of blank space is between the ends of the separation lines, even those that do not fall on the same braille line.

Example 9-22

\$.56		\$6.26	
.37	\$3.00	5.83	\$7.39
.93	.99	.27	.25



In print, the problems are arranged so that all separation lines are across the same row. In braille, the addends are top adjusted.

Instructions: Placement of code switches and blank lines has not been discussed yet. Please follow these directives in your transcription of this PRACTICE. Transcribe the opening Nemeth Code indicator on a line by itself, in cell 1. The next line should be blank. Begin the problems on the line following the blank line. Use side-by-side layout, with the first cell of the leftmost separation line in cell 1. Leave only one blank line between the two sets of problems. After completion of the last problem, leave one line blank and transcribe a Nemeth Code terminator in cell 1.

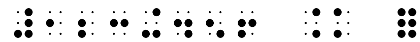
PRACTICE 9D

900	4391	25.763	718	5,025
+ 100	+ 81	− 4.239	− 437	+ 3,999
1000	4472	21.524	281	9,024

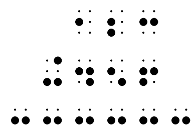
\$ .27	\$3,854	\$14.99
6.13	602	− 6.37
2.15	+ 918	\$ 8.62

9.19 Omissions in Work Arranged Spatially for Computation

Recall that, in a linear problem, the transcriber indicates the omission of an answer by transcribing a general omission symbol. (See Lesson 1.)

⫸ $123 + 456 =$ 

On the other hand, when nothing is printed below the separation line in a spatially arranged problem, the transcriber follows print. Do not insert a general omission symbol to represent blank space below a spatially arranged problem.

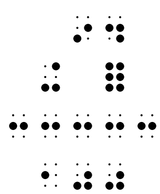
⫸ $\begin{array}{r} 123 \\ \underline{+\ 456} \end{array}$ 

An omission may be printed any number of ways – as a question mark, an ellipsis, an underscore, a box, etc. In braille, only the general omission symbol is used to show an omission within a spatial arrangement. The one-cell symbol allows the transcription to maintain alignment of the place values.

⠿	General Omission Symbol

Example 9-23

$\begin{array}{r} 94 \\ \underline{+\ ?} \\ 100 \end{array}$

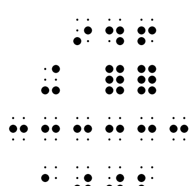


The omission is printed as a questions mark.

9.19.1 The number of general omission symbols transcribed reflects the number of omission signs used in print.

Example 9-24

$\begin{array}{r} 946 \\ \underline{+??} \\ 1002 \end{array}$

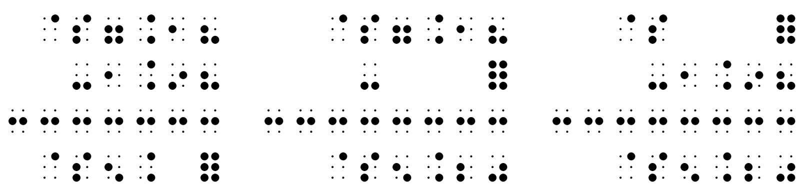


In print, each omission is denoted as a questions mark.

9.19.2 When only one sign of omission is used in print, only one general omission symbol is transcribed. An ellipsis is considered to be one print sign. Align the symbol beneath the rightmost character of the omission.

Example 9-25

$$\begin{array}{r} \$7.18 \\ -1.98 \\ \hline \$5.\underline{\quad} \end{array} \qquad \begin{array}{l} \$7.18 \\ -\ \ldots \\ \hline \$5.20 \end{array} \qquad \begin{array}{r} \$\boxed{\quad} \\ -1.98 \\ \hline \$5.20 \end{array}$$



The first omission is printed as an underscore. The second omission is printed as an ellipsis. The third omission is printed as a box.

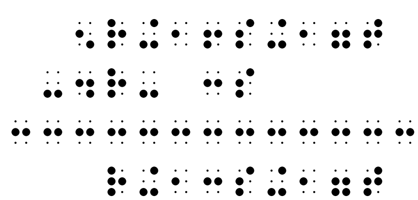
9.20 Spatially Arranged Polynomials

In polynomials arranged spatially for addition or subtraction, vertical alignment is important.

9.20.1 **Alignment.** Terms, symbols, and indicators are aligned.

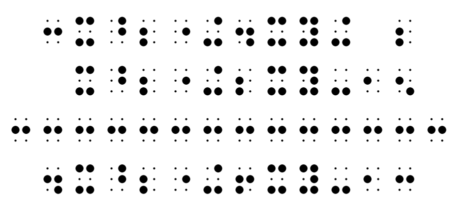
Example 9-26

$$\begin{array}{r} 5r + 16s + 17t \\ -\,4r - \ \ 3s \quad\quad\ \ \\ \hline r + 13s + 17t \end{array}$$



Example 9-27

$$\begin{array}{r} 3x^2 + 4xy + \ \ 2 \\ x^2 + 2xy - 15 \\ \hline 4x^2 + 6xy - 13 \end{array}$$

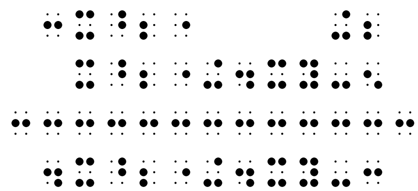


Level indicators align, as well as coefficients and variables.

9.20.2 **Placement of the Baseline Indicator.** A space used for alignment does not cancel the need for a baseline indicator. When the baseline indicator is required, it is placed in the first (leftmost) possible position consistent with the alignment. Any spacing required for alignment is placed following the baseline indicator.

Example 9-28

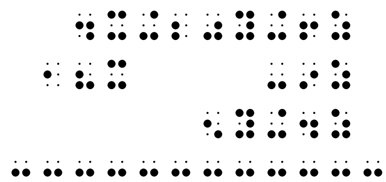
$$\begin{array}{r} 3x^2 \qquad\qquad +2 \\ x^2+4xy-5 \\ \hline 4x^2+4xy-3 \end{array}$$



9.20.3 **Misalignment in Print.** The braille transcription must show proper alignment, even if items are not aligned in print. *Exception*: If the terms are intentionally misaligned in an exercise or illustration, follow print alignment in the braille transcription.

Example 9-29

$$\begin{array}{r} 4x+20y+6z \\ 18x-9z \\ 5y+4z \\ \hline \end{array}$$

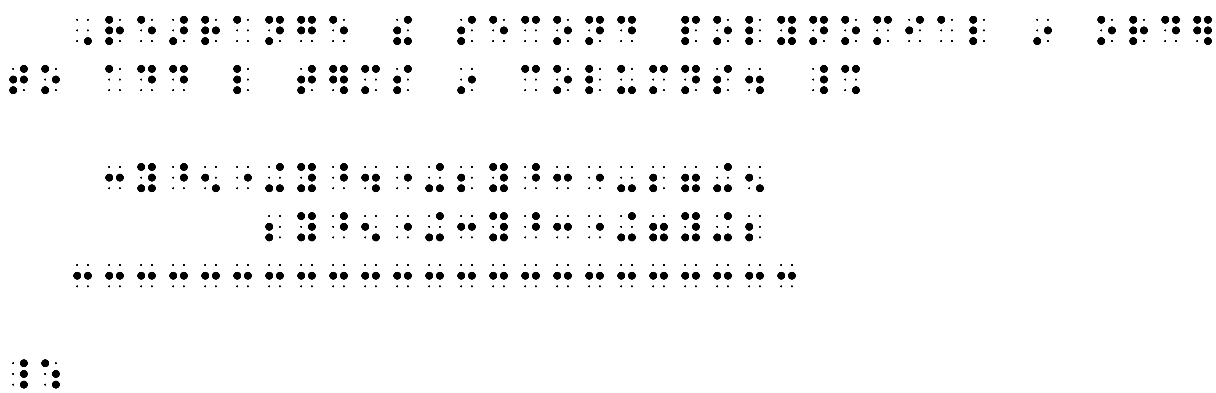


Terms are not aligned in this print example. The braille transcription aligns terms and symbols.

Example 9-30

Rearrange the second polynomial in order to add like terms in columns.

$$\begin{array}{r} 3y^5+y^4+2y^3-27+5 \\ 2y^5+3y^3+7y+2 \\ \hline \end{array}$$



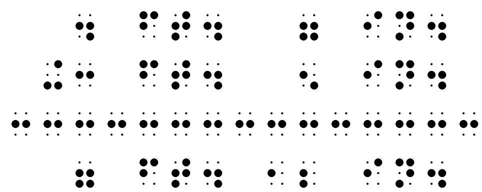
Both polynomials are right justified in print. The student is instructed to align the terms. (This example illustrates the proper placement of switch indicators and margins for displayed spatial material. Both topics are discussed later in this lesson.)

9.21 Abbreviations

When abbreviations occur in a spatially arranged addition or subtraction problem, they are vertically aligned and transcribed according to the rules for abbreviations outlined in Lessons 3 and 4.

Example 9-31

$$\begin{array}{rr} & 4\text{ ft.} \quad 7\text{ in.} \\ + & 3\text{ ft.} \quad 5\text{ in.} \\ \hline & 7\text{ ft.}\ 12\text{ in.} \end{array}$$

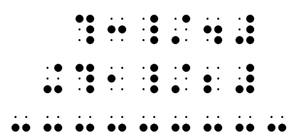


9.22 Fractions

In a spatial addition or subtraction arrangement containing fractions, the fraction lines are vertically aligned. Each numerator and denominator must be in contact with its fraction line. Fraction indicators must be vertically aligned.

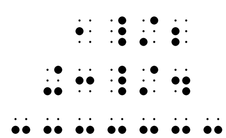
Example 9-32

$$\begin{array}{rr} & {}^{3}\!/_{4} \\ + & {}^{1}\!/_{2} \\ \hline \end{array}$$



Example 9-33

$$\begin{array}{rr} & 1/2 \\ + & 3/4 \\ \hline \end{array}$$

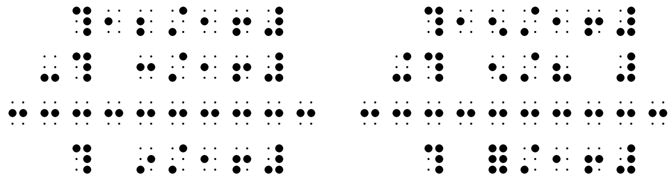


9.22.1 Alignment of Fraction Indicators

a. Blank cells are inserted in order to achieve alignment of fraction indicators when numerators and/or denominators contain a different number of digits.

Example 9-34

$$\begin{array}{rc} & \dfrac{12}{16} \\ - & \dfrac{3}{16} \\ \hline & \dfrac{9}{16} \end{array} \qquad\qquad \begin{array}{rc} & \dfrac{15}{16} \\ + & \dfrac{5}{8} \\ \hline & \dfrac{?}{16} \end{array}$$

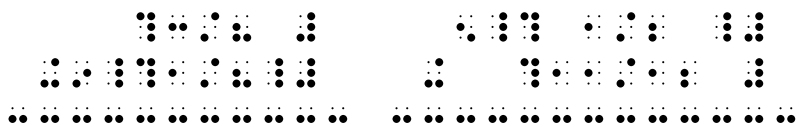


Fraction indicators and fraction lines are aligned.

b. When different kinds of fraction indicators occur in the same problem, align the (1456) and (3456) symbols. Indicators and numbers must not appear in the same column. Blank cells are inserted in order to achieve this alignment.

Example 9-35

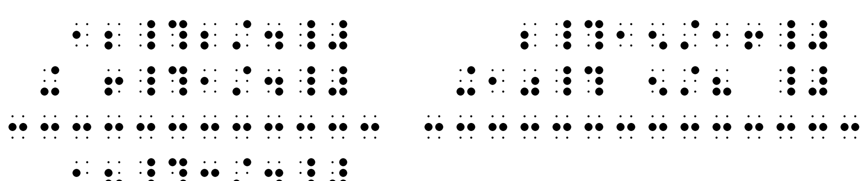
$$\begin{array}{r} \frac{3}{8} \\ +\ 9\,\frac{1}{8} \\ \hline \end{array} \qquad \begin{array}{r} 5\,\frac{1}{2} \\ +\ \frac{11}{12} \\ \hline \end{array}$$



c. The whole number part of a mixed number is vertically aligned by place value.

Example 9-36

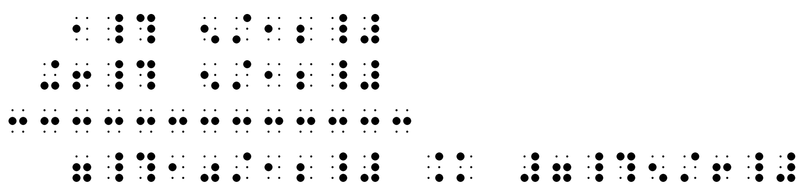
$$\begin{array}{r} 12\,\frac{2}{4} \\ +\ 6\,\frac{1}{4} \\ \hline 18\,\frac{3}{4} \end{array} \qquad \begin{array}{r} 2\,\frac{15}{16} \\ +\ 10\,\frac{5}{8} \\ \hline \end{array}$$



9.22.2 **Linear Portion of a Spatial Arrangement.** An equals sign printed within a spatial arrangement follows linear spacing rules for signs of comparison.

Example 9-37

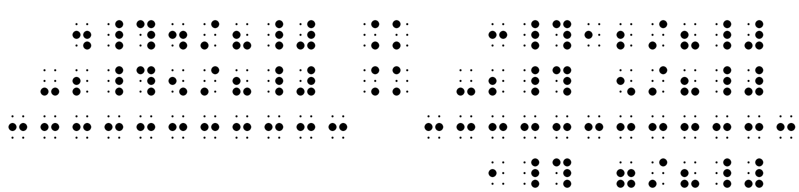
$$\begin{array}{r} 1\,\frac{5}{12} \\ +\,6\,\frac{5}{12} \\ \hline 7\,\frac{10}{12} \end{array} = 7\,\frac{5}{16}$$



A numeric indicator is required for the final answer because it is not part of the vertical computation.

Example 9-38

$$\begin{array}{r} 4\,\frac{4}{8} \\ -\,2\,\frac{5}{8} \\ \hline \end{array} \quad \begin{array}{c} = \\ = \\ \end{array} \quad \begin{array}{r} 3\,\frac{12}{8} \\ -\,2\,\frac{5}{8} \\ \hline 1\,\frac{7}{8} \end{array}$$



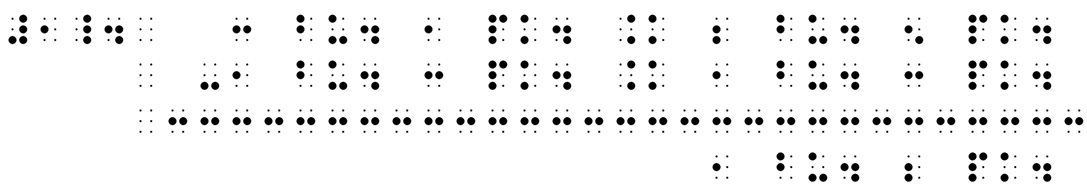
These equals signs show a relationship between the two side-by-side spatial arrangements.

9.23 Placement of Identifiers

The numeric or alphabetic identifier associated with a spatially arranged addition or subtraction problem is positioned at the top line of the problem—that is, with the first addend or with the subtrahend. One clear column of blank space (one cell's width) must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines.

Example 9-39

1. $$\begin{array}{r} 3 \text{ bu. } 1 \text{ pk.} = 2 \text{ bu. } 5 \text{ pk.} \\ -\ 1 \text{ bu. } 3 \text{ pk.} = 1 \text{ bu. } 3 \text{ pk.} \\ \hline 1 \text{ bu. } 2 \text{ pk.} \end{array}$$

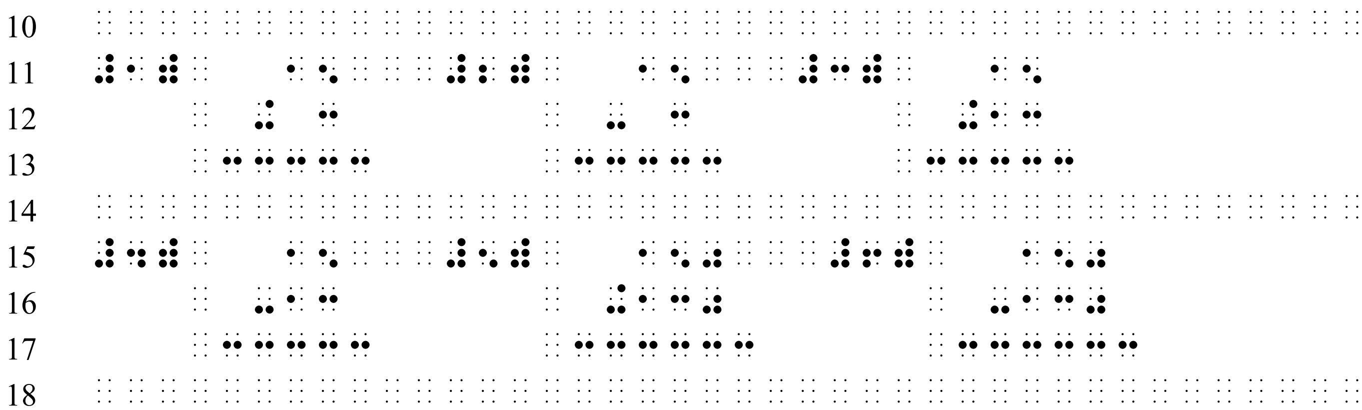


9.23.1 **Side-by-Side Layout.** Spatially arranged itemized material may be arranged side by side. The identifiers are located on the same braille line across the width of the page, in sequential order. Problems are spaced so that no less than three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement (separation lines excluded) including the neighboring identifier.

Note that this is a special rule pertaining to spatially arranged itemized material. As taught in Lesson 1, each main item in *nonspatial* itemized material must start in cell 1, even if the print copy arranges the items side by side across the page.

Example 9-40

1) $\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$ 2) $\begin{array}{r} 15 \\ -3 \\ \hline \end{array}$ 3) $\begin{array}{r} 15 \\ +13 \\ \hline \end{array}$ 4) $\begin{array}{r} 15 \\ -13 \\ \hline \end{array}$ 5) $\begin{array}{r} 150 \\ +130 \\ \hline \end{array}$ 6) $\begin{array}{r} 150 \\ -130 \\ \hline \end{array}$



Lines 11 and 15: Three blank cells come between the rightmost symbol in each problem (the "ones" column) and the numeric indicator of the next identifier.
Lines 11-13 and 15-17: One blank cell comes between the right parenthesis of each identifier and the leftmost cell of the separation line on line 3.
Line 14: A blank line precedes the second group of spatially arranged problems.
Line 15: No attempt is made to align identifiers with those on line 11.

Instructions: Transcribe the opening Nemeth Code indicator on a line by itself, in cell 1. The next line should be blank. Begin the problems on the line following the blank line. Use side-by-side layout. After completion of the last problem, leave one line blank and transcribe a Nemeth Code terminator in cell 1.

PRACTICE 9E

1) $\begin{array}{r} 621 \\ +??? \\ \hline 1096 \end{array}$ 2) $\begin{array}{r} 17x - 8y - z \\ -2x + 17y + 6z \\ \hline 15x + 9y + \_\_\_ \end{array}$ 3) $\begin{array}{r} 3x^2 - 5x + 4 \\ -5x^2 + 12x - 12 \\ \hline -2x^2 + 7x - 8 \end{array}$

4) $\begin{array}{l} \quad 1\frac{2}{3}\text{ yr} \\ +5\frac{5}{12}\text{ yr} \\ \hline \quad 6\frac{13}{12}\text{ yr} = 7\frac{1}{12}\text{ yr} \end{array}$

9.24 Regrouping Numbers in Addition Problems

Small numbers printed above or below digits in a problem are referred to as "regrouping numbers". (Older texts may call them "carried numbers".) The transcription places the regrouping numbers in the same location as printed, inserting the appropriate regrouping indicator between the regrouping numbers and their associated digits. A regrouping line usually does not appear in print, since the smaller font is enough to distinguish them.

The first cell of the regrouping indicator tells the reader to read the numbers above the line or below the line.

⠣⠶⠶⠶⠶⠶	Regrouping Indicator for Numbers Above the Arrangement (varying in length)
⠩⠶⠶⠶⠶⠶	Regrouping Indicator for Numbers Below the Arrangement (varying in length)

The regrouping indicator will be one cell longer on the left than the separation line. It is important to place the regrouping numbers in the same columnar positions as in print.

In the first example, the regrouping numbers are printed above the first addend. In the second example, the regrouping numbers are printed above the separation line.

Example 9-41

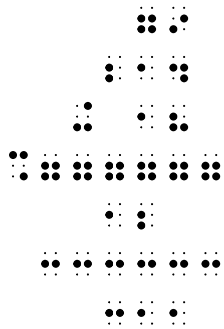
```
 1 1
 213
+ 87
----
 300
```

```
   ⠂⠂
⠣⠶⠶⠶⠶⠶⠶
   ⠆⠂⠒
  ⠬ ⠦⠶
 ⠒⠒⠒⠒⠒⠒
   ⠒⠴⠴
```

In the print copy, the regrouping numbers are in a smaller typeface and a regrouping line is not present.

Example 9-42

$$\begin{array}{r} 7\ 9 \\ 2\ 1\ 4 \\ +\quad 1\ 8 \\ {\scriptstyle 1\ \ 2\ \ } \\ \hline 3\ 1\ 1 \end{array}$$

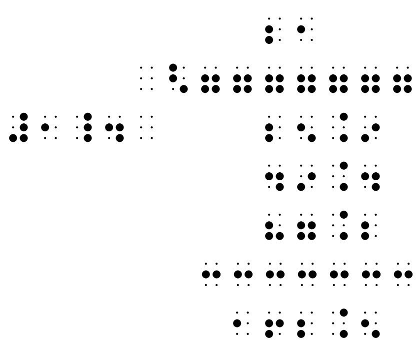


In the print copy, the regrouping numbers are in a smaller typeface and a regrouping line is not present.

9.24.1 **With Identifiers.** If an identifier is present, it is placed on the first line of the addition problem (the first addend) regardless of the presence of regrouping numbers. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines and regrouping indicators.

Example 9-43

1. $$\begin{array}{r} {\scriptstyle 2\ 1\ \ \ \ } \\ 25.9 \\ 49.4 \\ 87.2 \\ \hline 162.5 \end{array}$$



Introduction to Cancellation

[NC Rule 12]

A spatial arrangement is required when numbers, letters, abbreviations, or words in a mathematical expression are canceled in print by any type of stroke through them, and replacement values are shown. The opening and closing cancellation indicators enclose the material being canceled.

⠪	`Opening Cancellation Indicator`
⠻	`Closing Cancellation Indicator`

- ⫸ $\cancel{6}$ ⠪⠖⠻
- ⫸ ~~b~~ ⠪⠃⠻
- ⫸ $\bcancel{\text{ft.}}$ ⠪⠋⠞⠲⠻
- ⫸ $\xcancel{\text{mol}}$ ⠪⠍⠕⠇⠻

9.25 Cancellation in Subtraction Problems

When cancellation is shown in subtraction, a regrouping indicator is not needed. Each regrouping number is aligned with the corresponding canceled digit in the problem.

Digits, decimal points, or other symbols above and below the line with cancellation must be carefully aligned for computation. Spaces are inserted where necessary to achieve alignment. Numerals or other symbols must not appear in the same columns as cancellation indicators.

Example 9-44

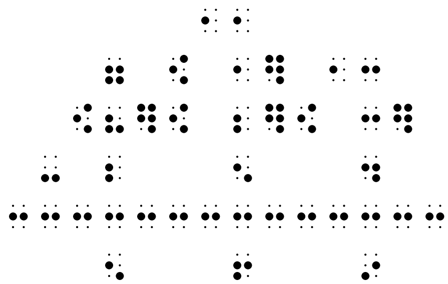
$$\begin{array}{r} {\scriptstyle 4\ \ 10} \\ 1\,\cancel{5}\,\cancel{0} \\ -\ 3\ 9 \\ \hline 1\ 1\ 1 \end{array}$$

```
    ⠲  ⠂⠴
  ⠂⠪⠢⠻⠪ ⠴⠻
 ⠤  ⠒   ⠔
⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
  ⠂ ⠂   ⠂
```

Place values align throughout the arrangement, including the regrouping numbers. To assure a clear column above the cancellation indicators, a space is inserted before the two-digit regrouping number on line 1.

Example 9-45

$$\begin{array}{rccc} & 7 & \overset{11}{\cancel{1}} & 13 \\ & \cancel{8} & \cancel{2} & \cancel{3} \\ - & 2 & 5 & 4 \\ \hline & 5 & 6 & 9 \end{array}$$



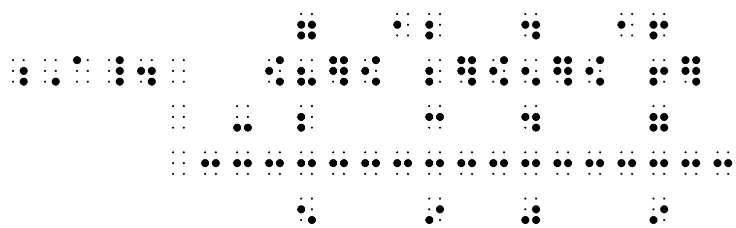
Cancellation indicators align with cancellation indicators. The minus sign on line 4 is placed one column to the left of the opening cancellation indicator on line 3 to assure a clear column below that indicator.

9.25.1 **With Identifiers.** If an identifier is present, it is placed on the first line of the subtraction problem (the minuend) regardless of the presence of canceled items. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 9-46

A.

$$\begin{array}{rcccc} & 7 & 12 & 4 & 16 \\ & \cancel{8} & \cancel{2} & \cancel{5} & \cancel{6} \\ - & 2 & 3 & 4 & 7 \\ \hline & 5 & 9 & 0 & 9 \end{array}$$



PRACTICE 9F

Regrouping Numbers and Cancellation

1.

$$\begin{array}{r} {}^{1\,1} \\ 948 \\ +75 \\ \hline 1023 \end{array}$$

2.

$$\begin{array}{lr} & {}^{12\ \ 2} \\ \$ & 2.52 \\ & 65.89 \\ & 91.99 \\ & 32.03 \\ \hline \$ & 192.43 \end{array}$$

3.

$$\begin{array}{rcccc} & 4 & 16 & 1 & 13 \\ & \cancel{5} & \cancel{6} & \cancel{2} & \cancel{3} \\ - & & 8 & 0 & 4 \\ \hline & 4 & 8 & 1 & 9 \end{array}$$

Arrangement on the Page

A spatial arrangement should be confined to one braille page. Strategies for larger arrangements are beyond the scope of this course.

9.26 Blank Lines

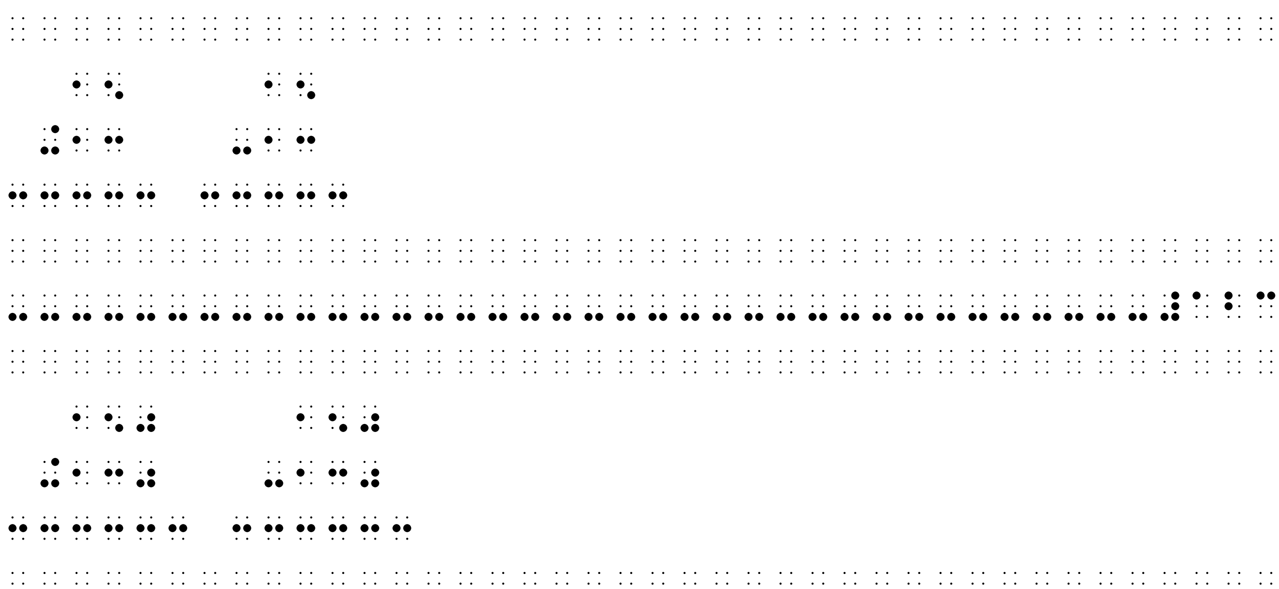
A blank line is required above and below a spatial arrangement in most situations.

9.26.1 **Blank Lines and the Page Change Indicator.** The presence of a page change indicator does not affect the blank line requirement. There may be times when a blank line is needed both before and after the page change indicator, for example, when spatial arrangements occur both before and after the page change.

Example 9-47

```
  15        15
 +13       −13
----      ----
============================== new print page 123

  150       150
 +130      −130
-----     -----
```



Each set of spatial problems requires a blank line before and after it.

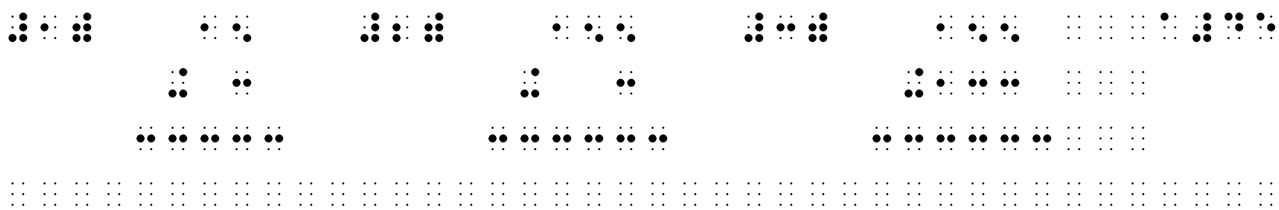
9.26.2 **Blank Lines and Box Lines.** A blank line is left both above and below a spatial arrangement even if the arrangement directly precedes or follows a box line. (No examples are given.)

9.26.3 **Pagination and Blank Lines.** Transition to a new braille page before beginning or after ending the transcription of a spatial arrangement takes the place of the required blank line. *The discussion in this section assumes that code switch indicators are not needed, imagining that Nemeth material precedes and follows each example.*

a. **Starting a Braille Page with a Spatial Arrangement.** When no running head is in use, a spatial arrangement may begin on line 1. There must be room for at least three blank spaces between the symbol furthest to the right of the overall arrangement and the first symbol of the print page number. If a running head is present, line 2 must be blank and the spatial arrangement will begin on line 3.

Example 9-48

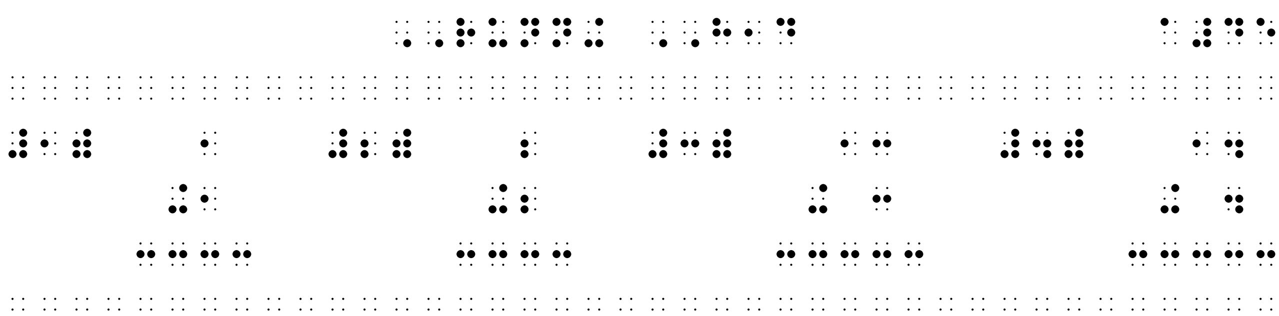
1) $\begin{array}{r} 15 \\ \underline{+3} \end{array}$ 2) $\begin{array}{r} 155 \\ \underline{+3} \end{array}$ 3) $\begin{array}{r} 155 \\ \underline{+133} \end{array}$



This example has no running head. The problems begin on line 1.

Example 9-49

1) $\begin{array}{r} 1 \\ \underline{+1} \end{array}$ 2) $\begin{array}{r} 2 \\ \underline{+2} \end{array}$ 3) $\begin{array}{r} 13 \\ \underline{+3} \end{array}$ 4) $\begin{array}{r} 14 \\ \underline{+4} \end{array}$

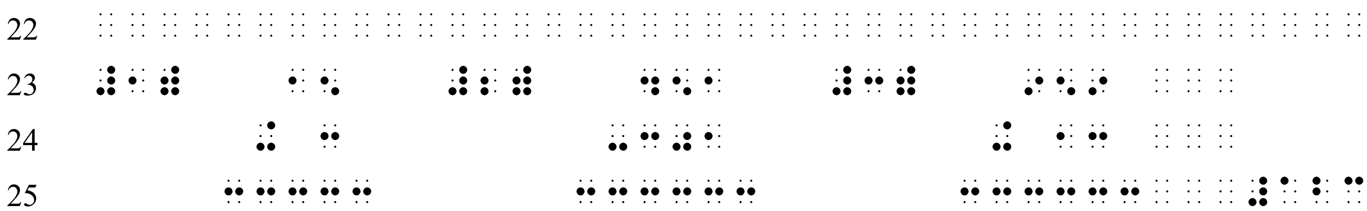


This example has a running head. Line 2 is blank.

b. **Ending a Braille Page with a Spatial Arrangement.** A spatial arrangement may be placed on the last line of the braille page. There must be room for at least three blank spaces between the symbol furthest to the right of the overall arrangement and the first symbol of the braille page number.

Example 9-50

1) $\begin{array}{r} 15 \\ +3 \\ \hline \end{array}$ 2) $\begin{array}{r} 451 \\ -301 \\ \hline \end{array}$ 3) $\begin{array}{r} 959 \\ +\ 13 \\ \hline \end{array}$



9.27 Wide Arrangements [NC 26.6.1]

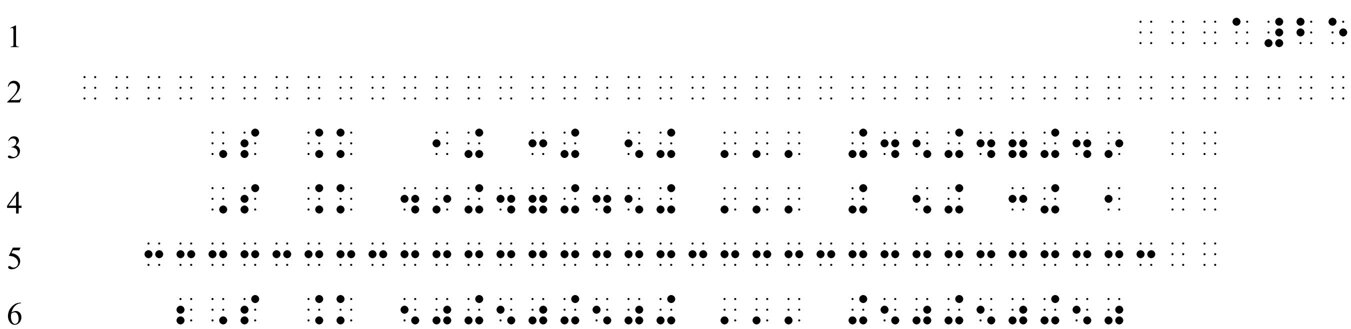
As noted above, if any portion of a spatial arrangement falls on lines 1, 2, 24, or 25 of the braille page, at least three blank spaces must be left between the symbol furthest to the right of the overall arrangement and the first symbol of the page number. This restriction includes the rightmost symbol of a separation line. If this spacing cannot be achieved, the arrangement must begin on line 3 or end on line 23, respectively.

The following example illustrates a wide arrangement that begins in cell 3, assuming it is displayed to narrative.

Example 9-51

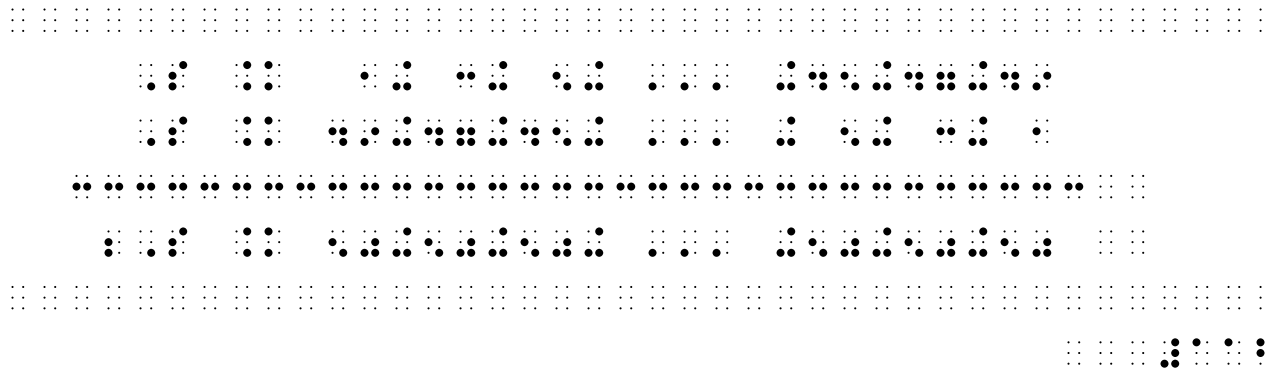
$$\begin{array}{rcl} S &=& 1+\ \ 3+\ \ 5+\dots+45+47+49 \\ S &=& 49+47+45+\dots+\ \ 5+\ \ 3+\ \ 1 \\ \hline 2S &=& 50+50+50+\dots+50+50+50 \end{array}$$

Example 9-51-a (Arranged near the top of a braille page, with no running head.)



The separation line invades the three-cell area before the print page number on line 1, so this problem is not allowed to fall on lines 1 or 2.

Example 9-51-b (Arranged near the bottom of a braille page.)



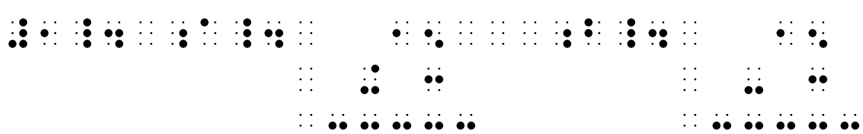
The separation line invades the three-cell area before the braille page number on line 25, so this problem is not allowed to fall on lines 24 or 25.

9.28 Itemized Spatial Problems with Subdivisions

9.28.1 **Without Narrative.** When a set of spatial arrangements contains both main divisions and subdivisions, the following must be observed. If there is no material between the main division identifier and the first subdivision identifier, place the main division at the margin in cell 1, and place the first subdivision on the same braille line after one blank cell. As many subdivisions as possible may be placed across the line. The identifiers are placed as outlined in 9.23, 9.24.1, and 9.25.1, and must be located on the same braille line across the width of the page. If additional subdivisions remain after the width of the page has been met, insert a blank line below the last line of the arrangements. Then begin the next subdivision identifier in cell 3.

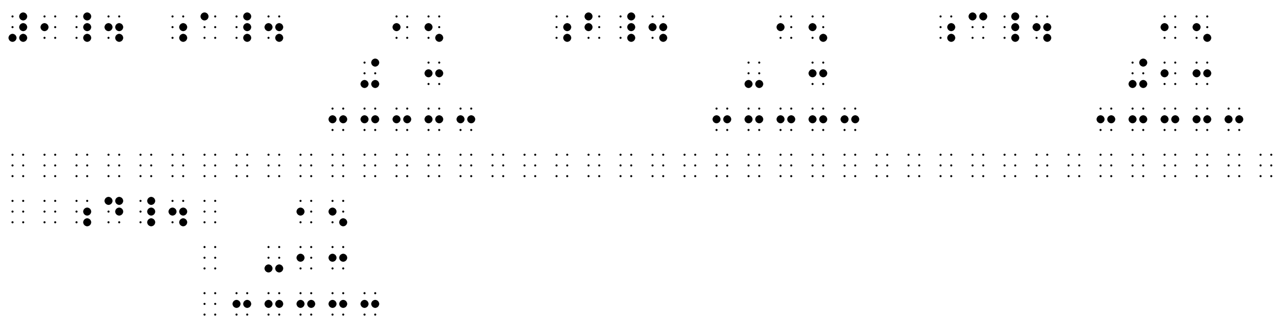
Example 9-52

1. a. $\begin{array}{r} 15 \\ \underline{+3} \end{array}$ b. $\begin{array}{r} 15 \\ \underline{-3} \end{array}$



Example 9-53

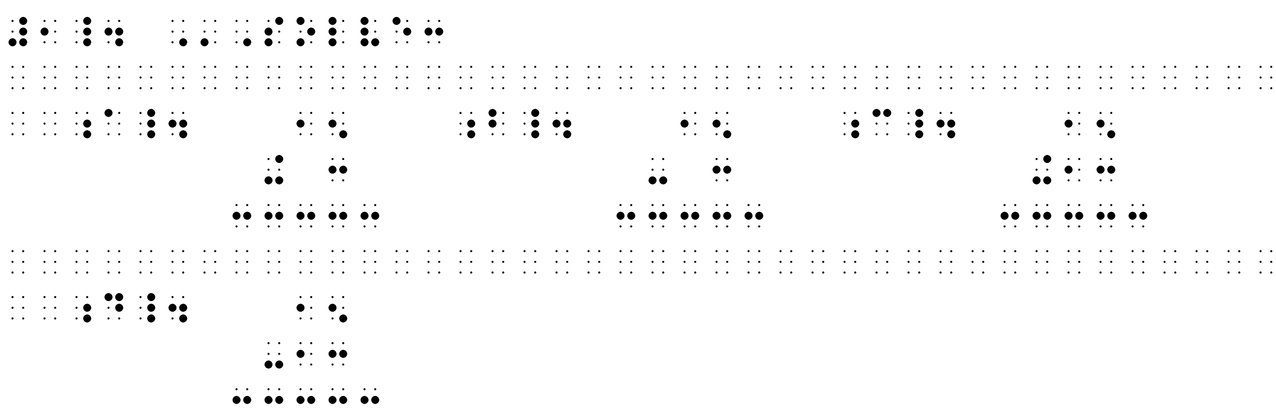
1. a. $\begin{array}{r} 15 \\ \underline{+3} \end{array}$ b. $\begin{array}{r} 15 \\ \underline{-3} \end{array}$ c. $\begin{array}{r} 15 \\ \underline{+13} \end{array}$ d. $\begin{array}{r} 15 \\ \underline{-13} \end{array}$



9.28.2 **With Narrative.** If there is narrative associated with the main division number, the subdivisions begin on the next available line (following the required blank line that precedes spatial material) starting in cell 3.

Example 9-54

1. Solve: a. $\begin{array}{r} 15 \\ \underline{+3} \end{array}$ b. $\begin{array}{r} 15 \\ \underline{-3} \end{array}$ c. $\begin{array}{r} 15 \\ \underline{+13} \end{array}$ d. $\begin{array}{r} 15 \\ \underline{-13} \end{array}$



Placement of Code Switch Indicators

Each code switch indicator should fall on the same braille page as the mathematical material to which it applies. (There are times when this is not possible.)

A code switch indicator does not take the place of a blank line. To keep the code switch indicators from interfering with the layout of the problem, they are placed outside of the arrangement. Position the required blank lines *between* the Nemeth switches as described below.

9.29 Code Switching with Unitemized Spatial Arrangements

i. If the problems are not displayed to text, the opening Nemeth Code indicator is placed in cell 1 on a line by itself. If there is text, the opening Nemeth Code indicator is placed on the same line with the end of the text above the problem unless there is no room, in which case the opening Nemeth Code indicator is placed in cell 1 on a line by itself. This applies to spatial material following instructions and for spatial material following a transcriber's note as well.

ii. The required blank line comes next.

iii. The leftmost symbol of the arrangement is placed in cell 1, or in the appropriate cell for displayed material.

iv. When Nemeth Code is closed after a spatial problem, the required blank line comes first, followed by the Nemeth Code terminator in cell 1 on a line by itself.

v. Resume UEB text on the line following the terminator, unless a format change that requires a leading blank line, such as a heading, follows. See Section 9.31, below.

Example 9-55

$$\begin{array}{r} 15 \\ +3 \\ \hline \end{array} \qquad \begin{array}{r} 115 \\ +30 \\ \hline \end{array} \qquad \begin{array}{r} 155 \\ +13 \\ \hline \end{array}$$

```
⠸⠩

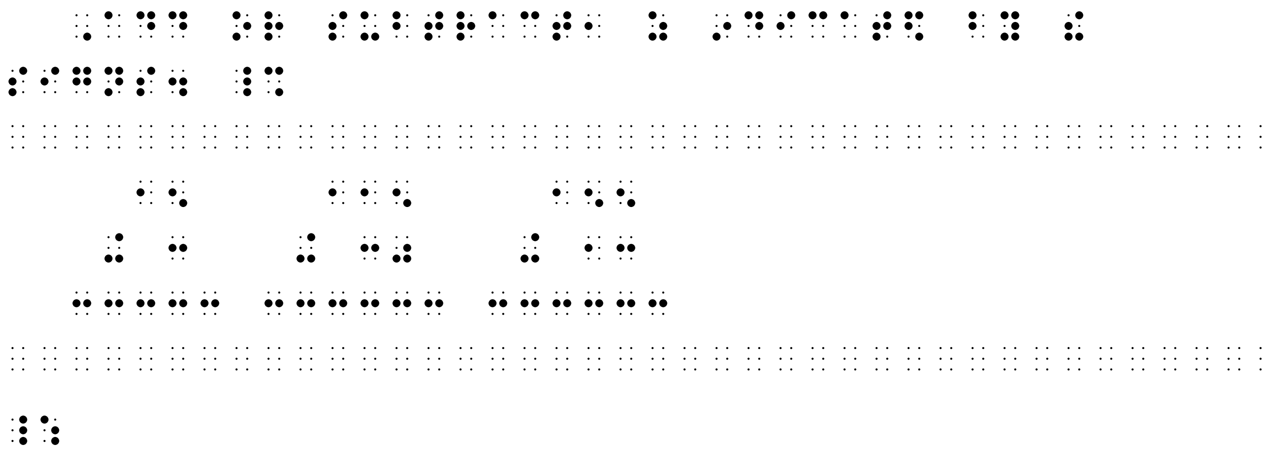
  ⠂⠢    ⠂⠂⠢    ⠂⠢⠢
 ⠬ ⠒   ⠬ ⠒⠴   ⠬ ⠂⠒
⠒⠒⠒⠒⠒ ⠒⠒⠒⠒⠒⠒ ⠒⠒⠒⠒⠒⠒

⠸⠱
```

Example 9-56

Add or subtract, as indicated by the signs.

$$\begin{array}{r} 15 \\ +3 \\ \hline \end{array} \qquad \begin{array}{r} 115 \\ -30 \\ \hline \end{array} \qquad \begin{array}{r} 155 \\ +13 \\ \hline \end{array}$$

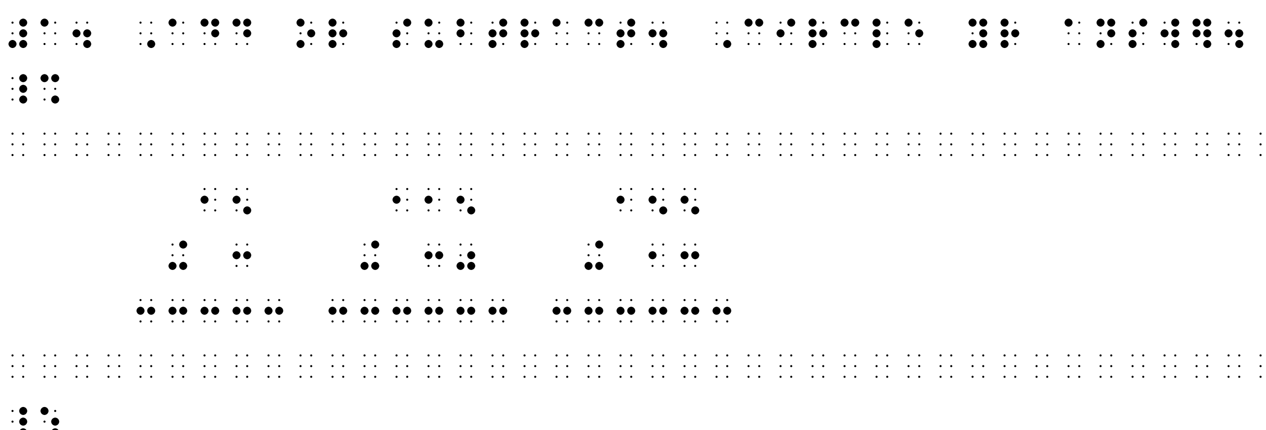


The opening switch fits at the end of the line of text. The arrangements are displayed to narrative and so the leftmost cell (the first separation line) is placed in cell 3. The Nemeth Code terminator is placed outside of the arrangement, after the required blank line, in cell 1.

Example 9-57

1. Add or subtract. Circle your answer.

$$\begin{array}{r} 15 \\ +3 \\ \hline \end{array} \qquad \begin{array}{r} 115 \\ -30 \\ \hline \end{array} \qquad \begin{array}{r} 155 \\ +13 \\ \hline \end{array}$$



The opening switch does not fit at the end of the line of text, so it is placed in cell 1 on the next line. The switch indicator does not take the place of the *required blank line. The arrangements are displayed to itemized text and so the leftmost cell is placed in cell 5. The Nemeth Code terminator is placed outside of the arrangement, after the required blank line, in cell 1.*

9.30 Code Switching with Itemized Spatial Arrangements

i. An opening Nemeth Code indicator which precedes itemized spatial material follows the same directives as with any itemized material. (See Lesson 2.) That is, if there is text, the opening Nemeth Code indicator is placed on the same line with the end of the text above the itemized problem unless there is no room, in which case the opening Nemeth Code indicator is placed in the runover cell.

ii. The required blank line comes next.

iii. The layout of the arrangements follow the directives given in Sections 9.23 and 9.28.

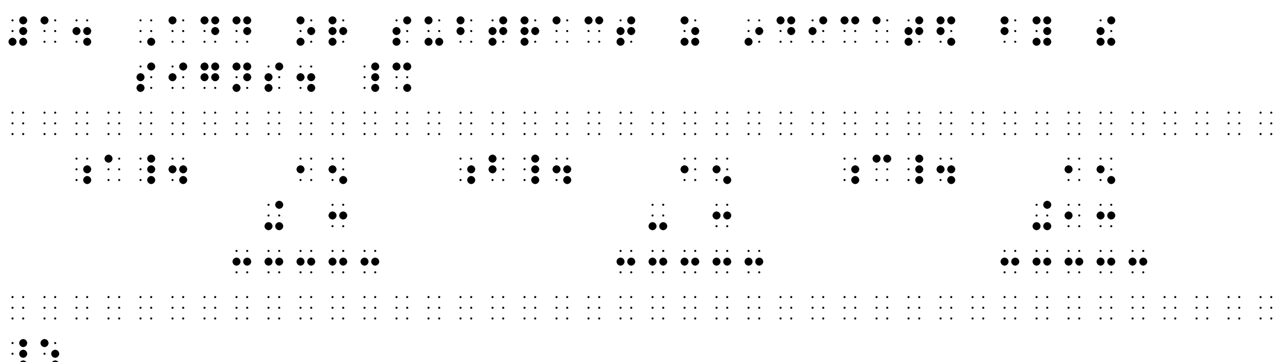
iv. When Nemeth Code is closed after a spatial problem, the required blank line comes first, followed by the Nemeth Code terminator in cell 1 on a line by itself.

v. Resume UEB text on the line following the terminator, unless a format change that requires a leading blank line, such as a heading, follows. See Section 9.31, below.

Example 9-58

1. Add or subtract as indicated by the signs.

a.	$\begin{array}{r} 15 \\ \underline{+3} \end{array}$	b.	$\begin{array}{r} 15 \\ \underline{-3} \end{array}$	c.	$\begin{array}{r} 15 \\ \underline{+13} \end{array}$



The opening switch fits at the end of the line of text. The Nemeth Code terminator is placed outside of the arrangement, after the required blank line, in cell 1.

Example 9-59

1. Solve these problems using counting cubes.

a. 15 +3   b. 15 −3   c. 15 +13

$$\begin{array}{r} 15 \\ +3 \\ \hline \end{array} \qquad \begin{array}{r} 15 \\ -3 \\ \hline \end{array} \qquad \begin{array}{r} 15 \\ +13 \\ \hline \end{array}$$

2. Solve these problems with your pencil.

```
⠼⠁⠲ ⠠⠎⠕⠇⠧⠑ ⠘⠮ ⠏⠗⠕⠃⠇⠑⠍⠎ ⠥⠎⠬ ⠉⠨⠞⠬ ⠉⠥⠃⠑⠎⠲
    ⠸⠩

  ⠰⠁⠸⠲   ⠂⠢   ⠰⠃⠸⠲   ⠂⠢   ⠰⠉⠸⠲   ⠂⠢
        ⠬ ⠒         ⠤ ⠒         ⠬⠂⠒
       ⠒⠒⠒⠒⠒       ⠒⠒⠒⠒⠒       ⠒⠒⠒⠒⠒

⠸⠱
⠼⠃⠲ ⠠⠎⠕⠇⠧⠑ ⠘⠮ ⠏⠗⠕⠃⠇⠑⠍⠎ ⠾ ⠽⠗ ⠏⠢⠉⠊⠇⠲
```

The opening switch does not fit at the end of the line of text, so it is placed in the runover cell (cell 5). The Nemeth Code terminator is placed outside of the arrangement, after the required blank line, in cell 1. Item 2 begins on the next line.

Example 9-60

Solve these problems using counting cubes.

$$1.\ \begin{array}{r} 15 \\ +3 \\ \hline \end{array} \qquad 2.\ \begin{array}{r} 15 \\ -3 \\ \hline \end{array} \qquad 3.\ \begin{array}{r} 15 \\ +13 \\ \hline \end{array}$$

```
    ⠠⠎⠕⠇⠧⠑ ⠘⠮ ⠏⠗⠕⠃⠇⠑⠍⠎ ⠥⠎⠬ ⠉⠨⠞⠬ ⠉⠥⠃⠑⠎⠲
⠸⠩

⠼⠂⠸⠲   ⠂⠢   ⠼⠆⠸⠲   ⠂⠢   ⠼⠒⠸⠲   ⠂⠢
      ⠬ ⠒         ⠤ ⠒         ⠬⠂⠒
     ⠒⠒⠒⠒⠒       ⠒⠒⠒⠒⠒       ⠒⠒⠒⠒⠒

⠸⠱
```

The opening switch does not fit at the end of the line of the instructions, so it is placed in cell 1 because spatially arranged material follows. The switch indicator does not take the place of the required blank line before the spatial *arrangements. The Nemeth Code terminator is placed outside of the arrangement, after the required blank line, in cell 1.*

9.31 Code Switching with Headings and with the Page Change Indicator

Recall from 9.26.1 that a blank line is required before a page change indicator when the print page ends with a spatial arrangement. The next example illustrates blank lines necessary when a heading begins on the new print page and code switching is necessary. Blank lines are required before and after each centered heading, and before and after each problem set. Each code switch indicator is placed in cell 1 on a line by itself.

Example 9-61

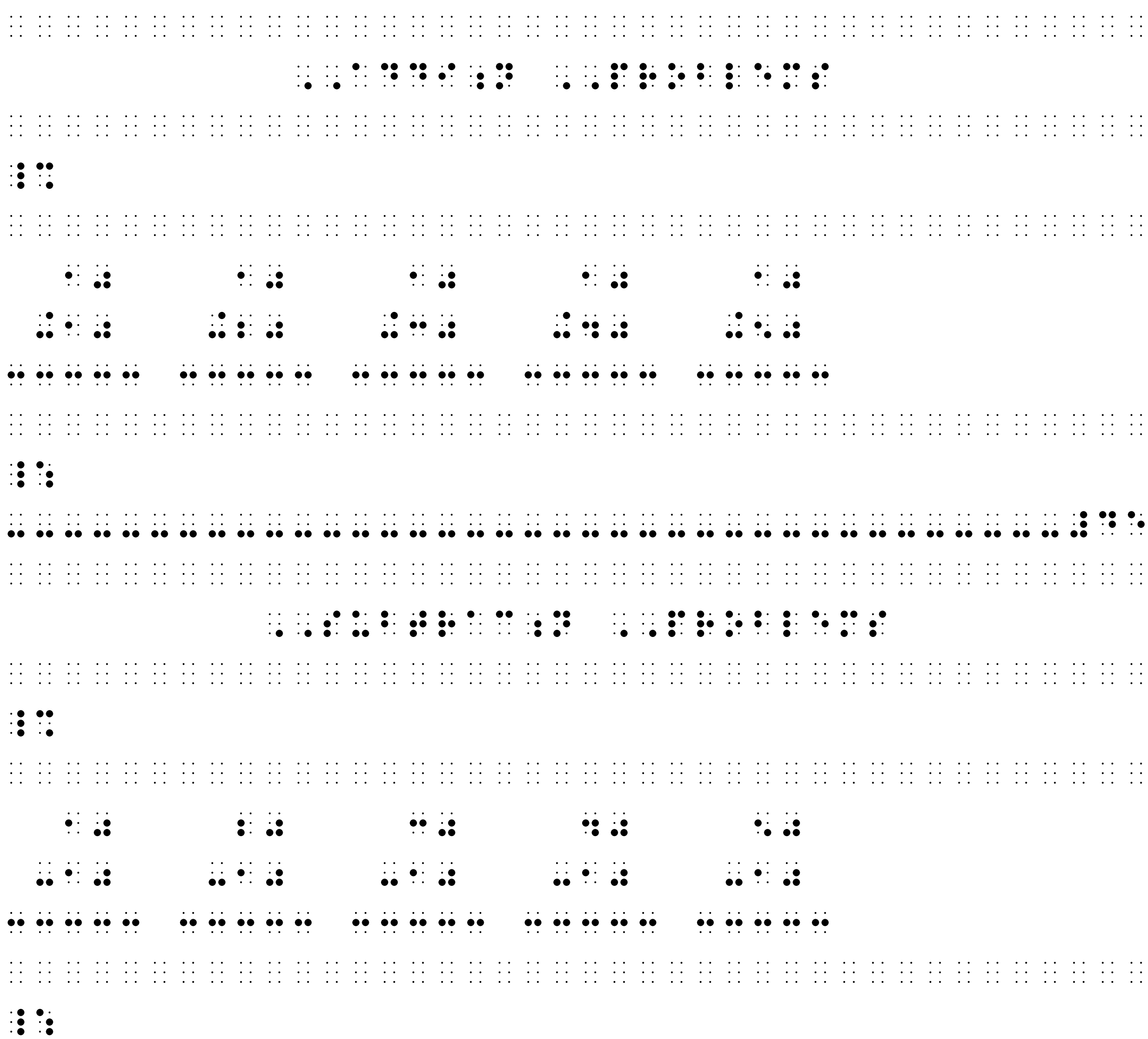
ADDITION PROBLEMS

10	10	10	10	10
+10	+20	+30	+40	+50

=== new print page 45

SUBTRACTION PROBLEMS

10	20	30	40	50
−10	−10	−10	−10	−10



For further practice, see Appendix A—Reading Practice.

EXERCISE 9

Prepare Exercise 9 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 9A

⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠠⠜⠗⠪⠎ ⠿ ⠠⠃⠜⠃⠫ ⠠⠢⠙⠎
⠀⠀
⠸⠩
⠰⠠⠁⠸⠲ ⠫⠪⠶⠶⠕ ⠫⠒⠒⠈⠕ ⠫⠪⠒⠒⠕ ⠫⠪⠒⠒⠠⠕ ⠫⠂⠂⠂⠕
⠀⠀⠫⠒ ⠒⠕ ⠫⠪⠔⠒⠢⠕ ⠫⠈⠪⠒⠒⠠⠕
⠰⠠⠃⠸⠲ ⠫⠪⠒⠒⠒ ⠫⠈⠪⠒⠒⠒⠕ ⠫⠠⠪⠒⠒⠒⠕ ⠫⠪⠶⠶⠶⠕ ⠸⠱
⠰⠠⠉⠲ ⠠⠮ ⠏⠗⠔⠉⠊⠏⠁⠇ ⠕⠏⠻⠁⠞⠕⠗ ⠷ ⠮ ⠇⠑⠋⠞ ⠿⠍⠥⠇⠁
⠀⠀⠔ ⠠⠋⠊⠛⠲ ⠼⠛⠲⠃ ⠊⠎ ⠸⠩ ⠫⠪⠒⠒⠕ ⠸⠱⠂ ⠱⠊⠇⠑ ⠮
⠀⠀⠏⠗⠔⠉⠊⠏⠁⠇ ⠕⠏⠻⠁⠞⠕⠗ ⠷ ⠮ ⠐⠗ ⠿⠍⠥⠇⠁ ⠊⠎
⠀⠀⠸⠩ ⠫⠕ ⠸⠱⠲
⠰⠠⠙⠲ ⠠⠒⠌⠗⠥⠉⠞ ⠁ ⠞⠗⠥⠹ ⠞⠁⠃⠇⠑ ⠿
⠀⠀⠸⠩ ⠷⠏ ⠫⠕ ⠷⠟ ⠫⠕ ⠗⠾⠾
⠀⠀⠫⠕ ⠷⠷⠏ ⠫⠕ ⠟⠾ ⠫⠕ ⠷⠏ ⠫⠕ ⠗⠾⠾ ⠸⠱⠲

NOTES

A. The dashed shaft must not be divided between lines.

D. This long expression will not fit on one line. The arrows are comparison signs, making this is a "linked expression". In order to organize this expression into mathematical units, divide before the main sign of comparison – the third arrow in this case.

PRACTICE 9B

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⠫⠡⠒⠒⠕   ⠫⠪⠒⠒⠨⠡   ⠫⠒⠒⠿   ⠫⠿⠒⠒⠿
⠫⠒⠒⠡   ⠫⠨⠡⠒⠒⠨⠡   ⠫⠿⠒⠒   ⠫⠳⠒⠒⠳
⠫⠡⠒⠒   ⠫⠒⠒⠨⠡   ⠫⠒⠒⠽   ⠫⠯⠒⠒⠽
⠫⠡⠒⠒⠡   ⠫⠨⠡⠒⠒   ⠫⠯⠒⠒

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PRACTICE 9C

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⠫⠣⠒⠒⠕   ⠫⠘⠒⠒⠕ ⠫⠘⠪⠒⠒⠕   ⠫⠪⠢⠔
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Note: It is acceptable to have two blank cells between items in the second column.

PRACTICE 9D

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PRACTICE 9E

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PRACTICE 9F

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     ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
        ⠲   ⠦  ⠂   ⠔

⠸⠱
```

LESSON 10

- SPATIAL ARRANGEMENTS WITH MULTIPLICATION
- DIVISION PROBLEMS
    - Linear (Nonspatial) Representation of Division Problems
    - Spatial Representation of Division Problems
- REVISITING SOME RULES
    - Summary of the Use and Nonuse of the Numeric Indicator
    - Review of Rules for Signs of Grouping

Answers to Practice Material

LESSON PREVIEW

The study of spatial arrangements continues with a look at multiplication and division problems. The lesson ends with a review of the rules for use/nonuse of the numeric indicator and a summary of rules regarding signs of grouping.

NOTE: Code switch indicators are not shown in the isolated Nemeth examples in this lesson.

SPATIAL ARRANGEMENTS, continued

Lesson 9 looked at spatially arranged addition and subtraction problems. The list below summarizes format rules presented in that lesson.

Review of Format for Spatial Arrangements

1. A blank line is required above and below a spatial arrangement. It may begin on line 1 and/or end on line 25.
2. Code switch indicators are placed outside of the arrangement.
3. The numeric indicator is not used. (There are some exceptions.)
4. An identifier associated with a spatial arrangement is positioned according to rules applied to the specific type of problem.
5. Side-by-side arrangement is allowed according to certain spacing rules.
6. If a spatial arrangement falls on lines 1-2, any symbol of the arrangement cannot fall within three blank spaces of the first symbol of the print page number on line 1. If a spatial arrangement falls on lines 24-25, any symbol of the arrangement cannot fall within three blank spaces of the first symbol of the braille page number on line 25.
7. Only the general omission symbol is used to show an omission within a spatial arrangement.

SPATIAL ARRANGEMENT WITH MULTIPLICATION

[NC 25.4]

The parts of a spatial multiplication problem are labeled below.

2	*multiplicand*
$\times 3$	*multiplier*
———	*separation line*
6	*product*

10.1 Alignment

In a spatial arrangement for multiplication, the multiplier and multiplicand must be aligned the same way as in the print copy. Any associated symbols such as dollar signs, commas, and decimal points correspond to the print placement.

10.2 Placement of Multiplication Symbol

In braille, the multiplication symbol must immediately precede the multiplier, regardless of print placement.

The operation sign is not always present. If there is no multiplication sign, examine the surrounding text to determine that this is indeed a multiplication problem. Then apply alignment rules for multiplication.

10.3 Separation Line

In braille, the separation line extends one cell to the left and to the right of the longest entry appearing above or below it. If there is more than one separation line in a given arrangement, each must be the same length regardless of the way it is printed.

⠒⠒⠒⠒⠒⠒	Separation Line (varying in length)

Example 10-1

```
2704
× 12
```

```
 ⠆⠶⠴⠲
 ⠈⠡⠂⠆
⠒⠒⠒⠒⠒⠒
```

Alignment: Digits in the multiplier and multiplicand (lines 1 and 2) are vertically aligned the same way as in the print copy. Placement of Multiplication Symbol: The multiplication cross is unspaced from the multiplier (line 2) even though this symbol is printed further to the left. Separation Line: The separation line extends one cell to the left and to the right of the longest entry appearing above or below it.

Example 10-2

```
132
× 300
39600
```

```
 ⠂⠒⠆
 ⠈⠡⠒⠴⠴
⠒⠒⠒⠒⠒⠒⠒
 ⠒⠔⠖⠴⠴
```

Alignment: Digits in the multiplier and multiplicand (lines 1 and 2) and in the product (line 4) are vertically aligned the same way as in the print copy. Placement of Multiplication Symbol: The multiplication cross is unspaced from the multiplier (line 2) even though this symbol is printed further to the left. Separation Line: The separation line extends one cell to the left and to the right of the longest entry appearing above or below it.

Example 10-3

```
 $421
  × 6
$2526
```

```
  ⠈⠎⠲⠆⠂
    ⠈⠡⠖
⠒⠒⠒⠒⠒⠒⠒⠒
 ⠈⠎⠆⠢⠆⠖
```

Spacing and Alignment: The dollar signs correspond to the print placement.

Example 10-4

```
  1,623
× 5.27
```

```
  ⠂⠠⠖⠆⠒
 ⠈⠡⠢⠨⠆⠶
⠒⠒⠒⠒⠒⠒⠒⠒
```

Spacing and Alignment: The commas and decimal points correspond to the print placement.

Alignment of Partial Products

10.4 Partial Products

When partial products are shown in a sample problem, note that the final answer (the product) is obtained by adding the partial products. Hence, partial products and the final product (the answer) must be aligned for addition.

2704	*multiplicand*	
× 12	*multiplier*	
5408	*partial product*	
2704	*partial product*	*an addition problem*
32448	*product*	

Example 10-5

```
  2704
×   12
  5408
 2704
 32448
```

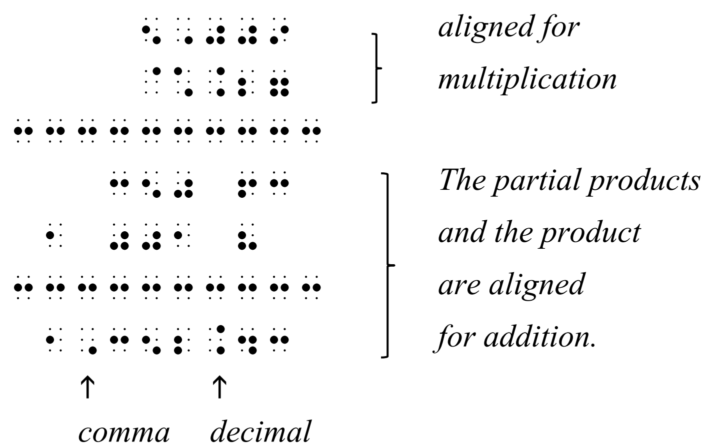
1	⠀⠀⠆⠶⠴⠲	*The multiplicand and the multiplier (lines 1 and 2)*
2	⠀⠀⠈⠡⠂⠆	*are aligned for multiplication, as printed.*
3	⠒⠒⠒⠒⠒⠒⠒	
4	⠀⠀⠢⠲⠴⠦	*The partial products (lines 4 and 5)*
5	⠀⠆⠶⠴⠲	*and the product (line 7) are aligned*
6	⠒⠒⠒⠒⠒⠒⠒	*according to the rules for addition,*
7	⠀⠒⠆⠲⠲⠦	*that is, by place value.*

All of the separation lines in one multiplication problem are the same width and in the same cell column, even though they may not appear this way in print.

10.4.1 **Spacing.** If the product contains a comma or a decimal point, the corresponding cells in the partial products above it are left blank. No blank cells are inserted in the separation lines.

Example 10-6

$$\begin{array}{r} 5,009 \\ \times\ \ .27 \\ \hline 35063 \\ 10018 \\ \hline 1,352.43 \end{array}$$



Instructions: Use side-by-side layout, leaving one blank space between the end of one separation line and the beginning of the next. Include switch indicators in your transcription. Review side-by-side format rules in Lesson 9.

PRACTICE 10A

$$\begin{array}{r} \$98 \\ \times\ 100 \\ \hline \end{array} \qquad \begin{array}{r} \$33 \\ \times\ \ 200 \\ \hline \end{array} \qquad \begin{array}{r} 12.12 \\ \times\ \ 15.3 \\ \hline \end{array} \qquad \begin{array}{r} 7,165 \\ \times\ 85 \\ \hline \end{array}$$

$$\begin{array}{r} \$98 \\ \times\ 100 \\ \hline \$9,800 \end{array} \qquad \begin{array}{r} \$33 \\ \times\ \ 200 \\ \hline \$6,600 \end{array} \qquad \begin{array}{r} 12.12 \\ \times\ \ 15.3 \\ \hline 3636 \\ 6060 \\ 1212 \\ \hline 185.436 \end{array} \qquad \begin{array}{r} 7,165 \\ \times\ 85 \\ \hline 35825 \\ 57320 \\ \hline 609,025 \end{array}$$

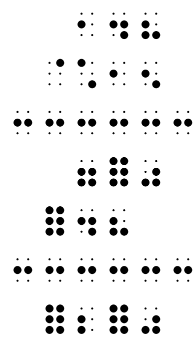
10.5 Omissions in Spatial Multiplication Problems

As noted in Lesson 9, only the general omission symbol is used to show an omission within a spatial arrangement, regardless of the print sign used. The one-cell symbol allows the transcription to maintain alignment.

⠿	`General Omission Symbol`

Example 10-7

$$\begin{array}{r} 148 \\ \times\ \ 15 \\ \hline 7\square 0 \\ \square 48 \\ \hline \square 2\square 0 \end{array}$$

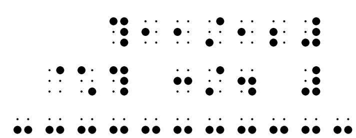


10.6 Fractions and Mixed Numbers

In a multiplication problem which contains fractions and mixed numbers, the various parts of the fractions are aligned vertically, as in an addition problem. You may wish to review Section 9.22, *Fractions*.

Example 10-8

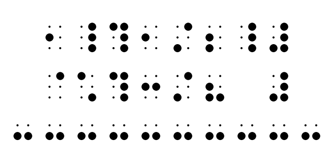
$$\begin{array}{r} \dfrac{11}{12} \\ \times\ \dfrac{3}{4} \\ \hline \end{array}$$



The fraction indicators and the fraction lines are vertically aligned. Numerators and denominators touch the fraction line.

Example 10-9

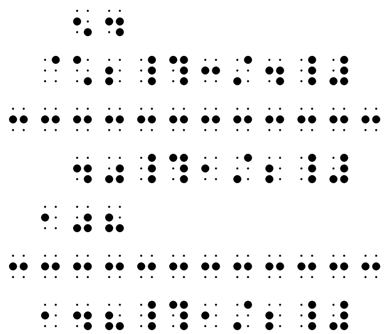
$$\begin{array}{r} 1\dfrac{1}{2} \\ \times\ \dfrac{3}{8} \\ \hline \end{array}$$



The fraction indicators align, including the fractional part of the mixed number. The multiplication cross touches the multiplier.

Example 10-10

$$
\begin{array}{r}
54 \\
\times\ 2\frac{3}{4} \\
\hline
40\frac{1}{2} \\
108\phantom{\frac{1}{2}} \\
\hline
148\ \frac{1}{2}
\end{array}
$$



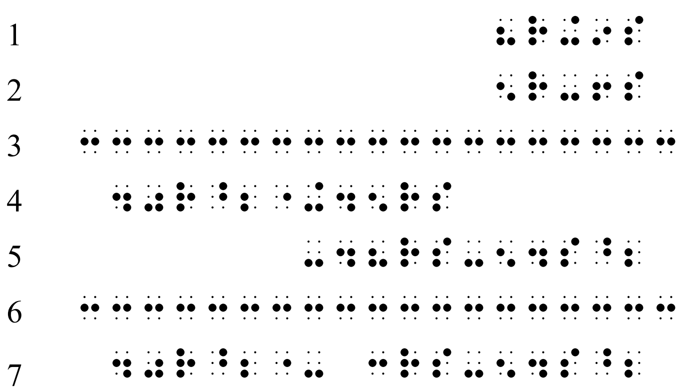
Place values and indicators are aligned throughout.

10.7 Polynomials

In a multiplication problem which contains polynomials, the multiplicand and multiplier (above the first separation line) are aligned as in the print copy. The partial products and final product are aligned following rules for addition.

Example 10-11

$$
\begin{array}{r}
8r + 9s \\
5r - 6s \\
\hline
40r^2 + \ 45rs\phantom{{}-54s^2} \\
-\ 48rs - 54s^2 \\
\hline
40r^2 - \ \ 3rs - 54s^2
\end{array}
$$



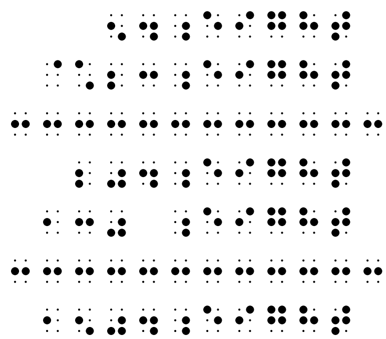
Lines 1-2: The multiplicand and multiplier are aligned as printed.
Lines 4-7: The partial products and the product are aligned for addition.
Lines 4-7: Terms r, rs, and s as well as operation signs, superscript indicators, and baseline indicators are aligned.

10.8 Subscripts Denoting Nondecimal Bases

In arrangements which show multiplication of nondecimal bases, the subscript indicators are vertically aligned. The rightmost partial product sets the location of this alignment. This may differ from the spacing shown in the print copy.

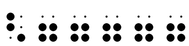
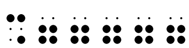
Example 10-12

$$\begin{array}{r} 54_{\text{eight}} \\ \times\ 23_{\text{eight}} \\ \hline 204_{\text{eight}} \\ 130_{\text{eight}} \\ \hline 1504_{\text{eight}} \end{array}$$



To ensure that the subscript indicators align, a spaces is inserted in the second partial product.

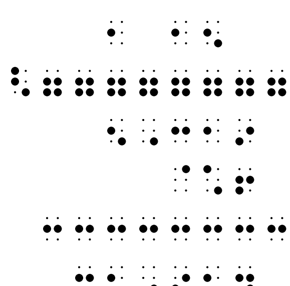
10.9 Regrouping Numbers with Multiplication

	Regrouping Indicator for Numbers Above the Arrangement (varying in length)
	Regrouping Indicator for Numbers Below the Arrangement (varying in length)

If regrouping numbers are shown, follow the rules for regrouping numbers with addition as outlined in Lesson 9. This indicator is transcribed whether or not the line appears in the print copy. It is inserted between the regrouping numbers and the multiplicand, and is one cell wider than the separation line. The regrouping number is placed in the same columnar position as in print.

Example 10-13

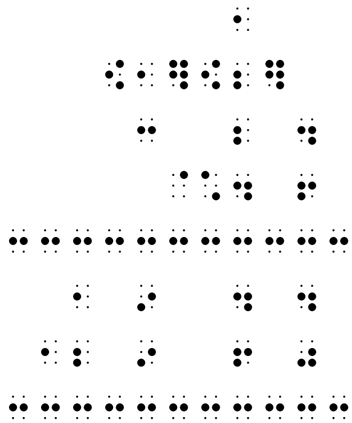
$$\begin{array}{r} {\scriptstyle 1\ 1\ 5} \\ 5{,}319 \\ \times\ 6 \\ \hline 31{,}914 \end{array}$$



a. When the multiplier consists of more than one digit, the regrouping numbers might be canceled and replaced. Recall that regrouping indicators are not needed when numbers are canceled. Review section 9.25.

Example 10-14

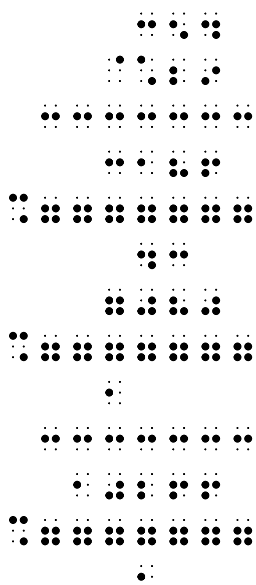
$$\begin{array}{r} 1 \\ \cancel{1}\,\cancel{2} \\ 324 \\ \times\ 46 \\ \hline 1944 \\ 12960 \\ \hline \end{array}$$



b. When the multiplier consists of more than one digit, the regrouping numbers might be written within the partial products and product area. Determine whether the regrouping numbers are written above or below each digit to select the appropriate regrouping indicator – "above" or "below."

Example 10-15

$$\begin{array}{r} 354 \\ \times\ 29 \\ \hline 3186 \\ {\scriptstyle 4\ 3} \\ 7080 \\ {\scriptstyle 1} \\ \hline 10266 \\ {\scriptstyle 1} \end{array}$$



The appropriate regrouping indicator is inserted between the regrouping numbers and the line in the arrangement to which they apply.

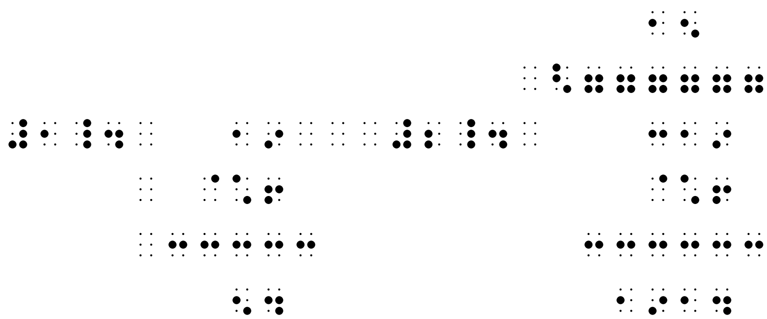
10.10 Placement of Identifiers with Spatial Multiplication

An identifier, if present, is placed on the first line of the multiplication problem (the multiplicand) regardless of its placement in print. If there are regrouping numbers, the identifier is still placed on the line with the multiplicand.

Example 10-16

1. $\begin{array}{r} 19 \\ \times\ 6 \\ \hline 54 \end{array}$

2. $\begin{array}{r} {\scriptstyle 1\,5} \\ 319 \\ \times\ 6 \\ \hline 1914 \end{array}$



Reminders: One blank space comes between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines. No symbol of one spatial arrangement or its identifier may be less than three cells distant from any symbol on any line of a neighboring arrangement or its identifier, except at the ends of separation lines.

Instructions: Use side-by-side layout. Include code switch indicators in your transcription.

PRACTICE 10B

1)
$$\begin{array}{r} \frac{1}{2} \\ \times \frac{15}{16} \\ \hline \end{array}$$

2)
$$\begin{array}{r} 9\frac{3}{4} \\ \times 4\frac{7}{12} \\ \hline \end{array}$$

3)
$$\begin{array}{r} {}^{3}9{}^{3}99 \\ \times\ 4 \\ \hline 3{,}996 \end{array}$$

4)
$$\begin{array}{r} 3p+6q \\ 11p-2q \\ \hline 33p^2 + 66pq \qquad \\ -\ \ 6pq-12q^2 \\ \hline 33p^2-60pq-12q^2 \end{array}$$

DIVISION PROBLEMS

[NC 25.5]

10.11 Notation Devices

It will be helpful to refer to the parts by using the following standard terminology. In this example ("forty divided by ten equals 4"), 4 is the *dividend*, 10 is the *divisor*, and 4 is the *quotient*.

$$\begin{array}{r} \textit{quotient} \\ \textit{divisor}\overline{)\textit{dividend}} \end{array} \qquad\qquad \begin{array}{r} 4 \\ 10\overline{)40} \end{array}$$

Other print styles which can be found in various publications are illustrated below using the same numbers for dividend, divisor, and quotient. Each style includes a division sign (either straight, slanted, or curved) and a separation line. The quotient can be printed above, below, or next to the dividend. Any one publication will likely use only one style throughout a document.

$$\begin{array}{r} 4 \\ 10/\overline{40} \end{array} \qquad 10\overline{)40}(4 \qquad \begin{array}{r} 4 \\ 10|\overline{40} \end{array} \qquad \begin{array}{r} 10|\underline{40} \\ 4 \end{array} \qquad \begin{array}{r} 4 \\ \underline{10}|\overline{40} \end{array}$$

If there is a *remainder,* it may be shown after the quotient, preceded by the letter R or r. In this example ("forty-seven divided by ten"), 7 is the remainder.

$$\begin{array}{r} \textit{quotient}\;\;\textit{remainder} \\ \textit{divisor}\overline{)\textit{dividend}}\phantom{\;\;\textit{remainder}} \end{array} \qquad\qquad \begin{array}{r} 4\text{ R7} \\ 10\overline{)40}\phantom{\text{ R7}} \end{array}$$

10.12 Linear (Nonspatial) Representation of Division Problems

The Preliminary Lesson introduced linear division problems using the symbol ÷ for "divided by", as in 15 ÷ 3 ("fifteen divided by three"). A division problem may also be printed using a curved, straight, or slanted line for the division sign, and a horizontal line printed above or below the divisor.

$$3\overline{)15}$$

When only a divisor and dividend are printed, the problem is formatted as a linear construction as long as the divisor and dividend are composed entirely of numerals. (A "numeral" may include decimal points and/or commas.) The print style is replicated by using one of the following symbols between the divisor and dividend. Note that the straight and slanted division signs are represented with the same braille symbol.

⠕	`Curved Division Sign (curving right)`	)
⠪	`Curved Division Sign (curving left)`	(
⠸	`Straight Division Sign`	|
⠸	`Slanted Division Sign (right or left)`	/ or \

The vertical and slanted division symbol is formed with the same dot configuration as the punctuation indicator (dots 456). In context, it is clear that it is a vertical line, not a punctuation indicator. It is not necessary to explain this in a transcriber's note.

In linear format, the horizontal line printed above or below the dividend is not transcribed. If a horizontal line is printed below the divisor, it, too, is disregarded. The numeric indicator is used where required according to rules governing linear expressions. As with other operation signs, the division symbol is unspaced from the divisor and the dividend, even if the print copy shows a space.

- ⫸ $3\overline{)15}$ ⠼⠒⠕⠂⠢
- ⫸ $2\,|\overline{46}$ ⠼⠆⠸⠲⠖
- ⫸ $3\,/\overline{15.3}$ ⠼⠒⠸⠂⠢⠨⠒
- ⫸ $\overline{1{,}000}(10$ ⠼⠂⠠⠴⠴⠴⠪⠂⠴
- ⫸ $\underline{30}|\overline{90}$ ⠼⠒⠴⠸⠔⠴

The horizontal line printed below this divisor is disregarded.

Example 10-17

Show how to solve $9\overline{)216}$ on your calculator. Is there a remainder?

⠠⠩⠪ ⠓⠪ ⠞⠕ ⠎⠕⠇⠧⠑ ⠸⠩ ⠼⠔⠕⠆⠂⠖ ⠸⠱ ⠕⠝ ⠽⠗
⠉⠁⠇⠉⠥⠇⠁⠞⠕⠗⠲ ⠠⠊⠎ ⠐⠮ ⠁ ⠗⠑⠍⠁⠔⠙⠻⠦

Example 10-18

1. $3\overline{)15}$ 2. $0.3\overline{)30}$ 3. $3\overline{)4.5}$ 4. $30\overline{)9{,}060}$

⠸⠩
⠼⠂⠸⠲ ⠼⠒⠕⠂⠢
⠼⠆⠸⠲ ⠼⠴⠨⠒⠕⠒⠴
⠼⠒⠸⠲ ⠼⠒⠕⠲⠨⠢
⠼⠲⠸⠲ ⠼⠒⠴⠕⠔⠠⠴⠖⠴ ⠸⠱

Reminder: In nonspatial braille, each identifier begins on a new line in cell 1 even though the problems are arranged side by side in print. The opening Nemeth Code indicator is in cell 1 on the line before the first numbered item.

Instructions: Use appropriate margins for embedded, displayed, and itemized problems.

PRACTICE 10C

Converting Fractions to Decimals

To write the fraction 1/2 as a decimal, you can divide "one point zero divided by two", or $2\overline{)1.0}$. Put away your calculator! Using mental math, it's easy! The answer is .5 .

Can you divide *these* fractions in your head?

1/5: $5\overline{)1.0}$

4/5: $5\overline{)4.0}$

9/10: $10\overline{)9.0}$

Practice

1. Solve these division problems.

   a. $8\overline{)128}$ b. $6\overline{)5{,}304}$

   c. $4\overline{)204}$ d. $9\overline{)837}$

10.13 Spatial Representation of Division Problems

A division problem is transcribed as a spatial (vertical) arrangement if it includes a quotient, partial products and differences, or if there are spaces within the dividend. The quotient is aligned with the dividend as shown in the print copy.

⠒⠒⠒⠒⠒	Separation Line (varying in width)	———

10.13.1 **Width of the Separation Line.** The separation line begins in the column containing a division symbol and ends in the column containing the other division symbol if the latter appears in print. When only one division symbol is printed, the separation line ends in the cell which is one column beyond the overall arrangement. *Reminder:* The division symbol is unspaced from the divisor and the dividend. Several styles of division problems are illustrated below.

⫸ $3\overline{)15}$ with quotient 5

```
   ⠢
 ⠒⠒⠒⠒
⠒⠕⠂⠢
```

This separation line begins in the cell directly over the (curved) division sign and ends one cell beyond the overall arrangement.

⫸ $\overline{15}|3$ with quotient 5

```
   ⠢
 ⠒⠒⠒⠒
  ⠂⠢⠸⠒
```

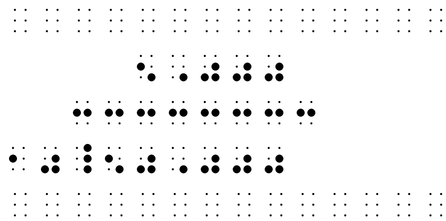
This separation line begins one cell beyond the overall arrangement and ends in the cell directly over the (straight) division sign.

⫸ $3)\overline{15}(5$

```
 ⠒⠒⠒⠒
⠒⠕⠂⠢⠪⠢
```

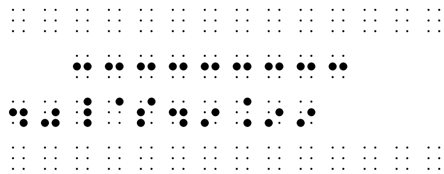
This separation line begins directly over the "curving right" division sign and ends in the cell directly over the "curving left" division sign. Because a quotient is present, it is transcribed as a spatial arrangement, even though the quotient is printed on the same line as the divisor and the dividend.

⫸ $\begin{array}{r} \underline{5,000} \\ 10\big|50,000 \end{array}$



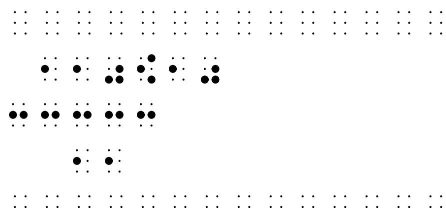
The quotient is aligned with the dividend as shown in the print copy.

⫸ $40\big|\overline{\$49.99}$



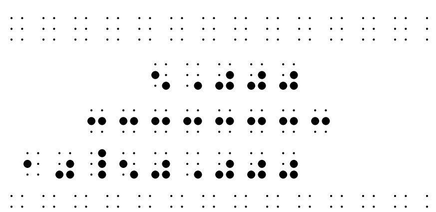
Even though there is no quotient shown, the presence of the dollar sign in the dividend means a spatial arrangement must be constructed.

⫸ $\begin{array}{l} \underline{110}(10 \\ \ \ 11 \end{array}$



This arrangement shows the separation line printed below the dividend, with the quotient printed below that. The separation line begins one cell beyond the overall arrangement and ends in the cell directly under the division sign.

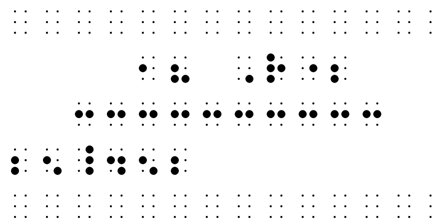
⫸ $\begin{array}{r} \underline{5,000} \\ \underline{10}\big|50,000 \end{array}$



This arrangement shows a horizontal line printed below the divisor. That line is disregarded in braille.

10.13.2 **A Quotient with a Remainder.** When a remainder is printed next to the quotient, the capitalized letter R, or uncapitalized letter r, (indicating "remainder") is preceded by a space. When the quotient is arranged above the separation line, the separation line extends one cell to the right of the overall arrangement, including the remainder, even if the line is not printed in this manner.

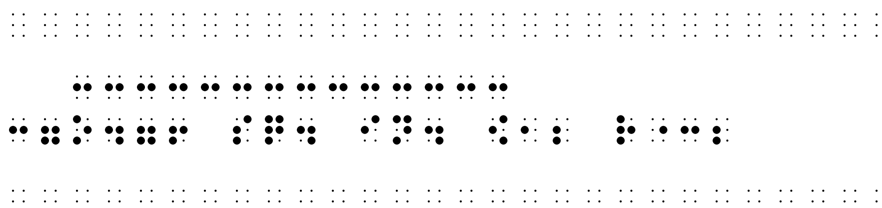
⫸ $\begin{array}{l|l} & \underline{18}\ \text{R2} \\ 25 & 452 \end{array}$



Recall from Lesson 6 that a number printed to the right of a letter needs a multipurpose indicator (dot 5) to show that the number is not a subscript.

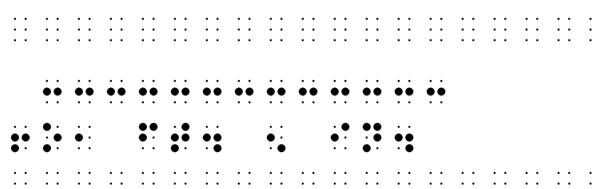
The arrangement in braille reflects the print layout. In the next example, the quotient and remainder are printed to the right of the dividend.

⫸ $37\,\overline{)\,476\,\text{sq. in.}\,(}\;12\,\text{r}32$



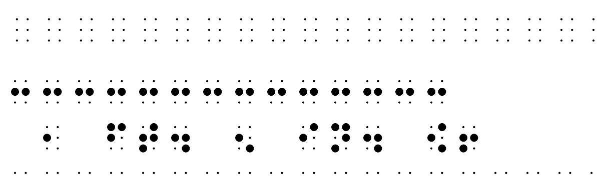
10.13.3 **Abbreviations.** When words or abbreviations appear in the problem, a spatial arrangement is required. When an abbreviation is next to the division symbol, spacing rules for abbreviations next to operation signs are applied. See Lesson 4.

⫸ $6\overline{)1\,\text{ft. } 5\text{ in.}}$



The numeric indicator is not used in a spatially arranged problem.

⫸ $\overline{1\,\text{ft.}\,5\,\text{in.}\,(}\,6$

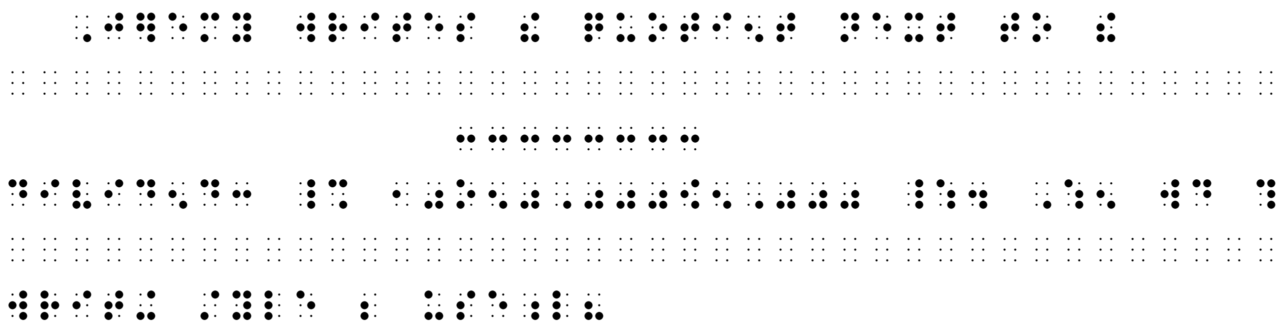


A space is required between an abbreviation and a sign of operation, in this case "in." and the (curved) division sign.

10.13.4 **Embedded Division Problem.** A blank line must precede and follow a spatially arranged problem, even when the problem is embedded within narrative.

Example 10-19

Jeremy writes the quotient next to the dividend: $10\overline{)50,000}(5,000$. When would this writing style be useful?



Notice the alignment of the embedded division problem. The divisor and dividend are placed on the same line as the text.

10.13.5 **Review of Format for Spatial Arrangements**

- A blank line is required above and below a spatial arrangement.
- The numeric indicator is not used. (There are some exceptions.)
- Side-by-side arrangement is allowed.
- An identifier associated with a spatial problem is positioned according to rules applying to the type of arrangement, not necessarily at the top line of the problem.

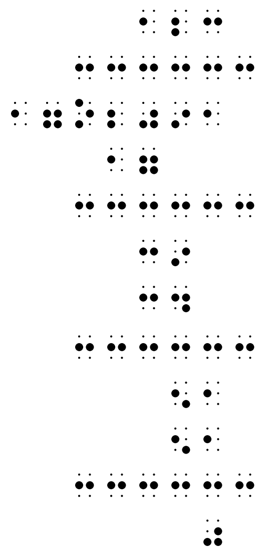
10.13.6 **Long Division.** In a long division problem, multiples are written below the dividend. A series of subtraction problems are performed, giving partial remainders and, finally, a remainder. The long division portion of the problem (bracketed below) is aligned for subtraction.

123		
17)2091	*dividend*	
17	*multiple (17×1)*	subtraction problems
39	*partial remainder*	
34	*multiple (17×2)*	
51	*partial remainder*	
51	*multiple (17×3)*	
0	*remainder*	

a. **Alignment.** The components of the problem are aligned in the same manner as they are aligned in the print copy. All separation lines must be the same length, regardless of their relative lengths in the print copy.

Example 10-20

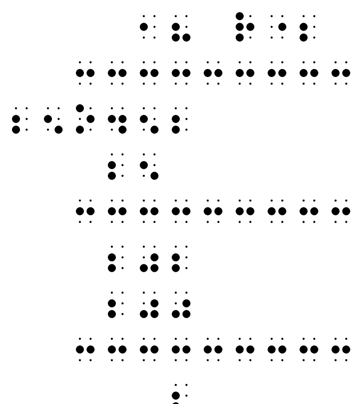
$$\begin{array}{r}123\\17\overline{)2091}\\ \underline{17}\\39\\ \underline{34}\\51\\ \underline{51}\\0\end{array}$$



aligned for subtraction

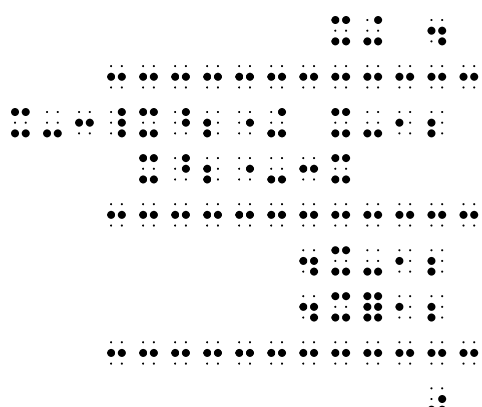
Example 10-21

$$\begin{array}{r}18\,\text{r}2\\25\overline{)452}\\ \underline{25}\\202\\ \underline{200}\\2\end{array}$$



Example 10-22

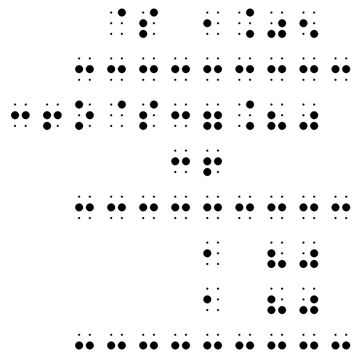
$$\begin{array}{r}x+4\\x-3\overline{)x^2+x-12}\\ \underline{x^2-3x}\\4x-12\\ \underline{4x-12}\\0\end{array}$$



b. **Blank Cells.** When commas or decimal points occur in a dividend, corresponding blank cells should be left throughout the body of the division example, except in the separation lines.

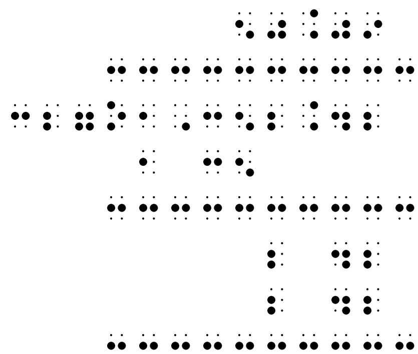
Example 10-23

```
      $1.05
36)$37.80
    36
      1 80
       1 80
```

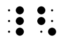


Example 10-24

```
        50.09
27)1,352.42
    1 35
           242
            242
```



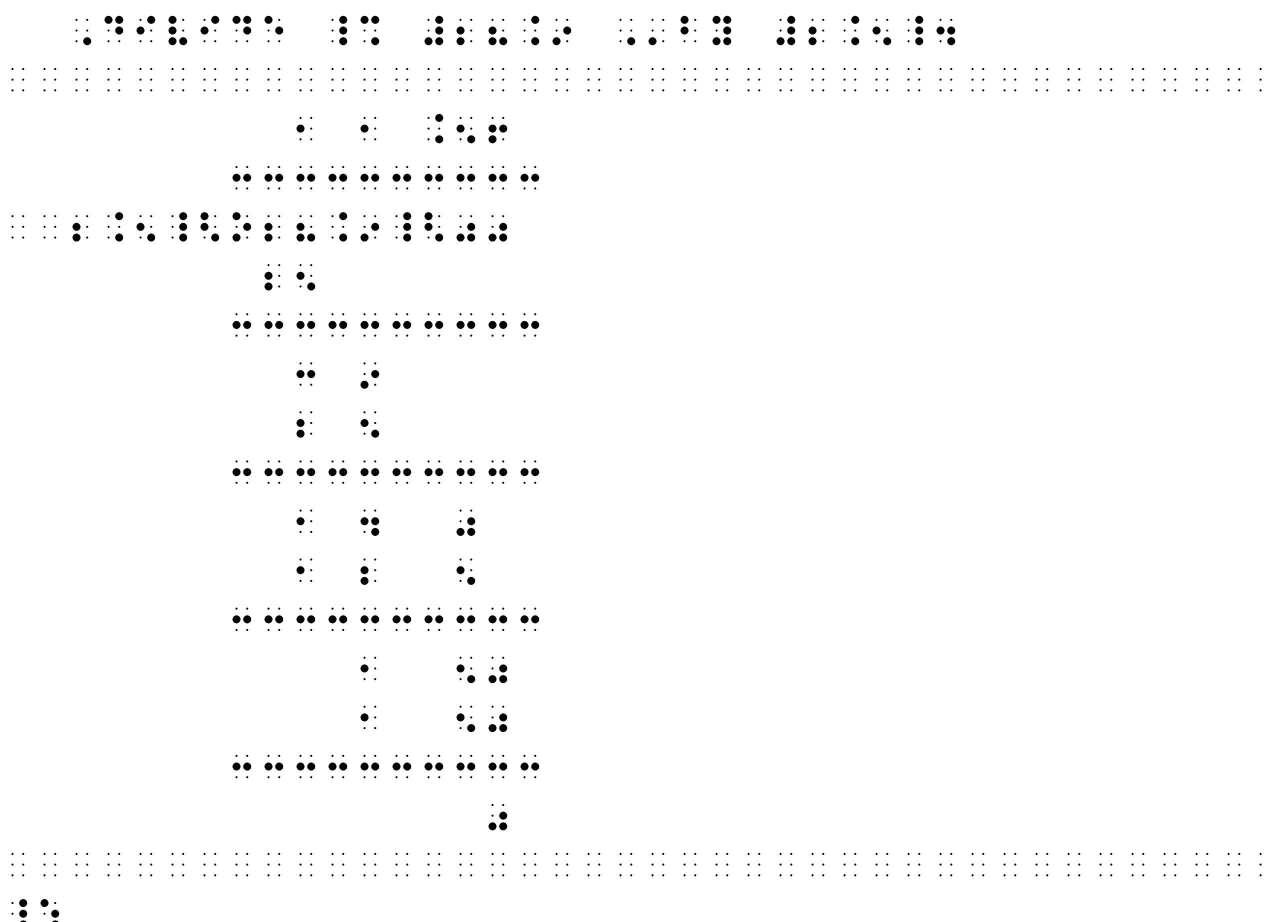
c. **Caret.** If a caret occurs in a dividend, corresponding blank cells are left throughout the body of the division problem, except in the separation lines. Two cells are required for the caret. The decimal point replacing the caret in the quotient is aligned in the right-hand cell.

	Caret	∧

Example 10-25

Divide 28.9 by 2.5.

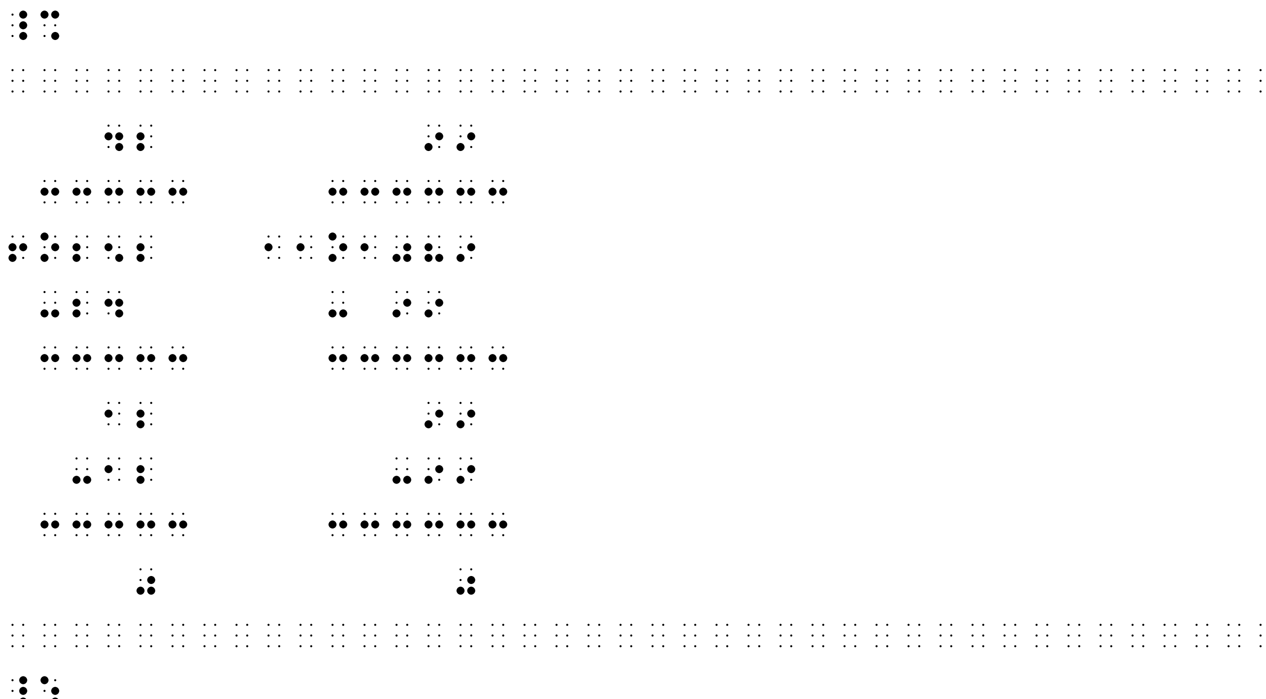
```
         11.56
2.5∧)28.9∧00
     25
      39
      25
       140
       125
         150
         150
           0
```



d. **Minus Sign.** A minus sign appearing in the long division portion of the problem is placed according to the rules of spatially arranged subtraction problems—one cell to the left of the leftmost numeric symbol that appears above its separation line. If a minus sign appears on the first line below the dividend, the symbol will be placed in the same column as the division symbol. In that case, the following separation line will begin in the same column.

Example 10-26

$$\begin{array}{r} 42 \\ 6\overline{)252} \\ -24 \\ \hline 12 \\ -12 \\ \hline 0 \end{array} \qquad \begin{array}{r} 99 \\ 11\,\overline{)\,1089} \\ -99 \\ \hline 99 \\ -99 \\ \hline 0 \end{array}$$

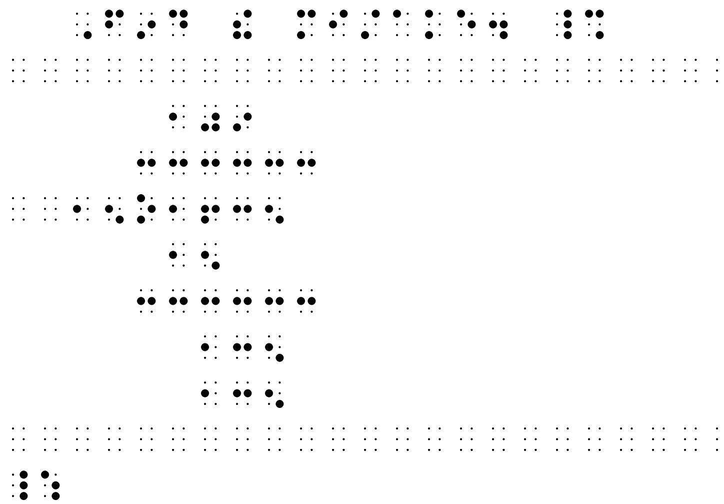


e. **Intentional Misalignment.** If the quotient has been intentionally misaligned as an exercise for the student, the same misalignment is shown in the transcription.

Example 10-27

Find the mistake.

$$
\begin{array}{r}
109 \\
15\overline{)1635} \\
\underline{15} \\
135 \\
\underline{135}
\end{array}
$$



The leftmost character in the spatial arrangement is placed in the appropriate cell for displayed material (cell 3).

Instructions: Although not required, for this practice please keep the last problem on the same page as the paragraph which precedes it.

PRACTICE 10D

Here are seven other ways to write $\begin{array}{r}203\\11\overline{)2233}\end{array}$ or "2233 divided by 11 equals 203".

$$\begin{array}{l}\underline{\;203\;}\\2233(11\end{array} \qquad \begin{array}{r}203\\11/\overline{2233}\end{array} \qquad \begin{array}{r}203\\11|\overline{2233}\end{array}$$

$$\begin{array}{l}\underline{2233}\,|\,11\\\;\;203\end{array} \qquad \begin{array}{l}\underline{2233}(11\\\;\;203\end{array} \qquad \begin{array}{r}11)\,\underline{2233}\\203\end{array}$$

$$11\,)\,\overline{2233}\,(\,203$$

What is $11\overline{)2234}$? Use long division.

$$\begin{array}{r}203\text{ R1}\\11\overline{)2234}\phantom{\text{ R1}}\\\underline{22}\phantom{34\text{ R1}}\\34\phantom{\text{ R1}}\\\underline{33}\phantom{\text{ R1}}\\1\phantom{\text{ R1}}\end{array}$$

10.14 Omissions in Spatial Division Problems

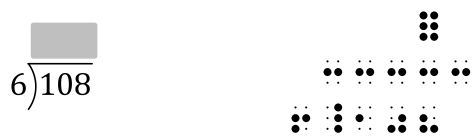
When the arrangement contains omissions, in order to maintain necessary alignment only the general omission symbol is used. When a single general omission symbol represents an unknown number of values, align the symbol above the rightmost character in the dividend.

⠿	General Omission Symbol

Example 10-28

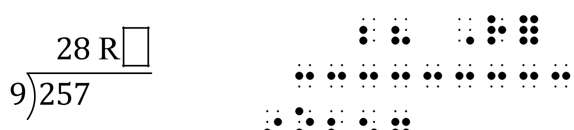
Example 10-29

Example 10-30



a. When a remainder is to be filled in, a multipurpose indicator is not needed following the letter "R" because the general omission symbol is nonnumeric, even though it may represent a numeral.

Example 10-31

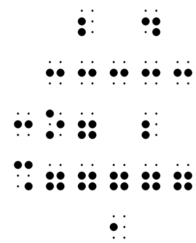


10.15 Regrouping in Division

The regrouping indicator is transcribed above or below the dividend, depending on the position of the separation line. The first cell of each regrouping indicator tells whether read the numbers above or below the line. The regrouping indicator is one cell longer on the left than the separation line. A blank space is left in the dividend and in the quotient where necessary to accommodate a regrouping number. No blank spaces are left in the regrouping line.

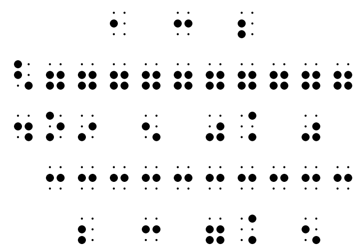
a. When the separation line is printed above the dividend, the regrouping indicator for numbers *below* the arrangement is transcribed below the dividend.

⫸ $$\begin{array}{r} 2\ 4 \\ 3\overline{)7^{1}2} \end{array}$$



b. When the separation line is printed below the dividend, the regrouping indicator for numbers *above* the arrangement is transcribed above the dividend.

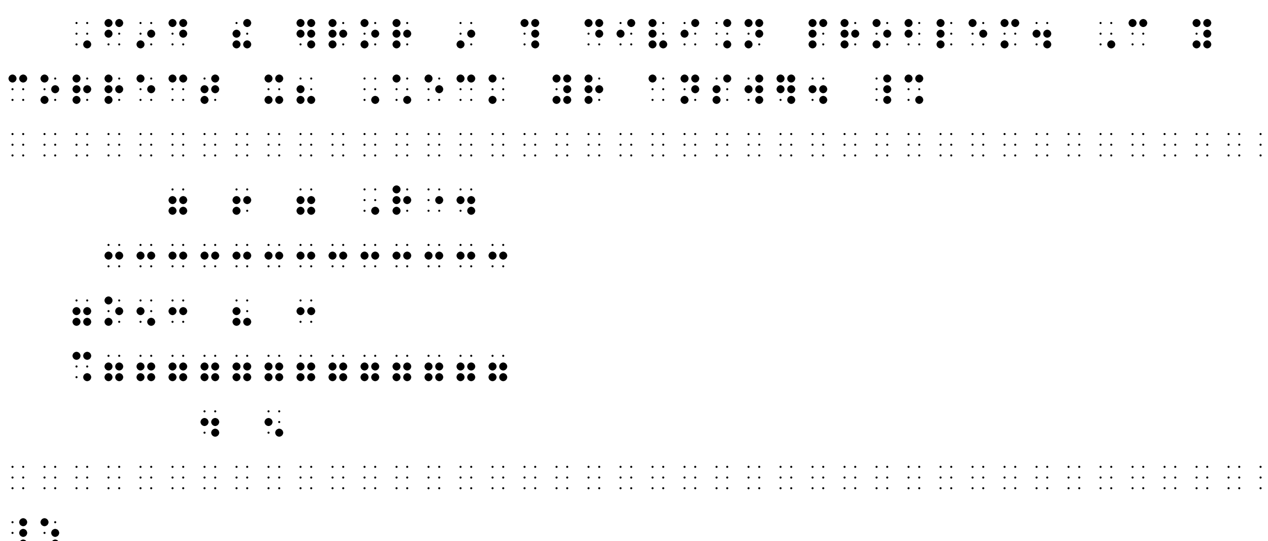
⫸ $$\begin{array}{l} 4)\,9^{1}5^{3}0.^{2}0 \\ \hline \ \ 2\ \ 3\ \ 7.\ \ 5 \end{array}$$



Example 10-32

Find the error in this division problem. Can you correct it? Check your answer.

$$\begin{array}{r} 7\ 6\ 7\ \mathrm{R}4 \\ 7\overline{)53_{4}8_{5}3} \end{array}$$



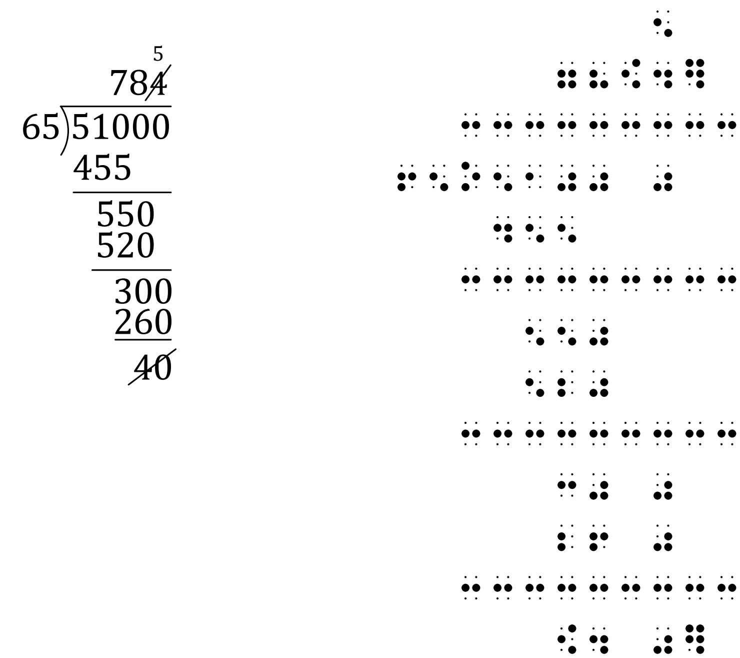
10.16 Cancellation in Long Division

Cancellation was introduced in Lesson 9.

⠪	Opening Cancellation Indicator
⠻	Closing Cancellation Indicator

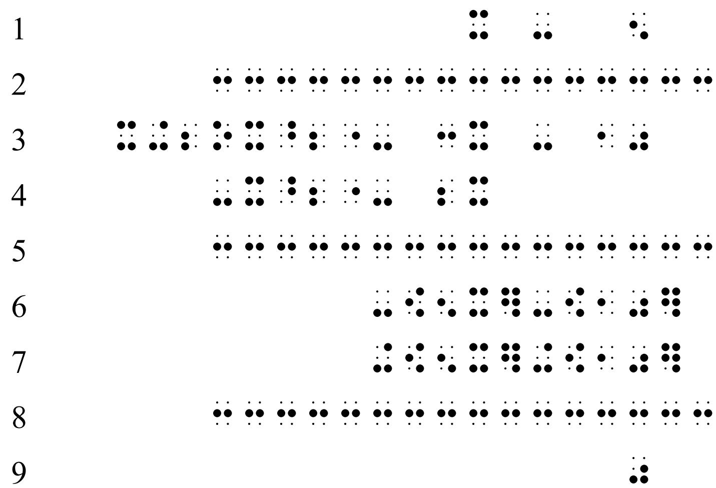
If cancellation is shown in long division, the canceled material is enclosed in braille cancellation indicators. Blank cells are left where necessary for proper alignment. The canceled material must exactly represent what is canceled in print.

Example 10-33



Example 10-34

$$\begin{array}{r} x-\ 5 \\ x+2\,\overline{)\,x^2-3x-10} \\ \underline{-x^2-2x\quad\ } \\ -\cancel{5x}-\cancel{10} \\ \underline{+\cancel{5x}+\cancel{10}} \\ 0 \end{array}$$



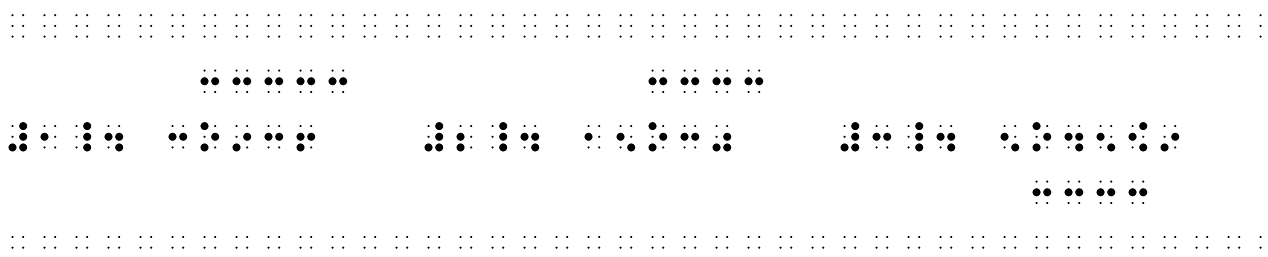
Line 1: "x–5" is aligned as printed.
Line 3: The division sign is unspaced from the divisor.
Line 4: The minus sign is transcribed in the same column (cell) as the division sign.
Line 6: Space inserted above cancellation indicators.
Line 7: Aligned for subtraction.
Line 9: "0" is aligned by place value.

10.17 Placement of Identifiers with Spatial Division

An identifier, if present, is placed on the line with the dividend. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines. When arranged side by side across the width of the page, no symbol of one spatial arrangement or its identifier may be less than three cells distant from any symbol on any line of a neighboring arrangement or its identifier, except at the ends of separation lines.

Example 10-35

1. $\begin{array}{r} 312 \\ 3\overline{)936} \end{array}$ 2. $\begin{array}{r} 2 \\ 15\overline{)30} \end{array}$ 3. $5)\underline{45}(9$



10.18 Other Layouts

Other types of division problems will be taught in a later lesson.

Instructions: Do not use side-by-side layout in your transcription.

PRACTICE 10E

Fill in the boxes with the correct numeral.

(a) $\begin{array}{r} 3\ 9\ 302\ \text{R}\square \\ 6\overline{)23^{5}5^{1}813} \end{array}$

(b) $\begin{array}{r} 3\frac{1}{5} \\ \cancel{3\frac{5}{25}} \\ 25\overline{)80} \\ \underline{75} \\ \square \end{array}$

(c) $\begin{array}{r} \square\ \text{R3} \\ 4\overline{)\ 19} \end{array}$

REVISITING SOME RULES

10.19 Summary of the Use and Nonuse of the Numeric Indicator

It may be helpful to summarize the use and nonuse of the Nemeth numeric indicator studied so far. This is not an all-inclusive list.

The word "numeral" includes a number that begins with a decimal point or a number that begins with a minus sign (a negative number) as well as a simple number consisting of one or more digits. These rules also apply to nondecimal digits such as T and E.

10.19.1 **Use of the Numeric Indicator. [NC Rule 3.3]** A numeric indicator <u>is needed</u> in the following circumstances.

a. when a Nemeth numeral begins a braille line or is preceded by a space.

b. at the beginning of the runover line of a long (divided) numeral.

c. when a Nemeth numeral immediately follows a punctuation mark. (A grouping sign is not a mark of punctuation. See item d. regarding the hyphen, which requires special attention.)

d. after a hyphen which connects a Nemeth numeral to a word, an abbreviation, or a punctuation mark. (A grouping sign is not a punctuation mark.)

e. when a Nemeth numeral follows a mathematical asterisk, crosshatch, paragraph mark, or section mark.

f. when a Nemeth numeral is in nonregular type.

g. when there is change back to regular type from nonregular type within the same Nemeth numeral.

The following examples illustrate points a-g.

(a) .5 −1 t4e

0.333 ... 3 ...

−.7

(b) 0.12345678910 (etc.)

(c) "3.4"

2:30 + 1:15 =

(d) 1.5 million-2.5 billion

4K-5K

6.3?-6.8

(e) 3 ∗ 9 = 27

(f) **183**

(g) 14**625**

10.19.2 **Nonuse of the Numeric Indicator. [NC Rule 3.4]** A numeric indicator <u>is not needed</u> in the following circumstances, as long as the numeral is in regular type.

h. when a numeral is unspaced from and follows a mathematical character or a slash.

i. when a numeral is unspaced from and follows a Nemeth indicator.

j. when a numeral is unspaced from and follows a left grouping sign.

k. when a numeral follows a hyphen which follows a numeral or other mathematical symbol.

l. when a numeral in regular type is part of an "enclosed list".

m. when a numeral is partitioned into segments with spaces.

n. when a numeral is part of a spatially arranged addition, subtraction, multiplication, or division problem.

The following examples illustrate points h-n.

(h) $-7 + 12 - 3 = +2$

$1.5K-$2.5B

2/3

(i) x^2 ⁻2 R5

$\frac{3}{4}$ $\sqrt[3]{8}$

(j) (−4 and −5)

(0 = x)

(7%)

(k) 3.4-3.8 cc

80%-90%

(l) {.5, .7, .9, 1.1}

(m) 987 656 000

(n)

$$\begin{array}{r} 53 \\ +51 \\ \hline 104 \end{array}$$

10.20 Review of Rules for Signs of Grouping

A review of the cited sections is recommended.

a. In mathematical context, grouping symbols are not considered to be punctuation; they are signs of grouping. Grouping symbols inside the switches must be transcribed using the Nemeth symbols. [2.9]

b. Nemeth grouping symbols are punctuated mathematically. [2.13.1]

c. The numeric indicator is not used before a numeral which immediately follows a grouping symbol. [2.10]

d. In mathematical context, non mathematical symbols of enclosure (i.e., grouping signs functioning as punctuation marks) can be transcribed as Nemeth grouping symbols if switching out of Nemeth would be awkward. [2.13] However, paired punctuation marks must be transcribed in the same code. [2.14] When an isolated math symbol is enclosed within parentheses, brackets, or braces, transcribe the paired grouping symbols in Nemeth. [2.13]

e. The English-letter indicator is not used when a single English letter or a Roman numeral is entirely enclosed within signs of grouping. This rule applies to mathematical "single letters" only, not to abbreviations. [3.12.2 and 4.4]

f. The appropriate alphabetic indicator must be used with any letter from the German, Greek, Hebrew, or Russian alphabets. [4.10.1] This rule applies even when non-English letters are enclosed within, or in contact with, signs of grouping.

g. A sequence of mathematical items enclosed in signs of grouping (an "enclosed list") must use the Nemeth grouping symbols. [4.17]

h. When a single English letter or a Roman numeral is in direct contact with only one sign of grouping, the English-letter indicator is or is not used as though the grouping sign was not present. If the grouping sign carries a prime or other modifying symbol, the English-letter indicator is not used. [4.18]

i. When a grouping symbol appears on the baseline level and a level indicator is currently in effect, the baseline indicator is placed before the sign of grouping. If no subscript indicator is used, a return to the baseline is implied. [6.21]

j. When brackets or vertical bars are printed in mathematically significant boldface, dots 456 are used before the grouping symbol. [7.9.3]

For further practice, see Appendix A—Reading Practice.

EXERCISE 10

Prepare Exercise 10 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 10A

PRACTICE 10B

```
⠸⠩

⠼⠂⠾    ⠹ ⠂⠌⠆ ⠼   ⠼⠆⠾    ⠔⠸⠹⠒⠌⠲ ⠸⠼
     ⠈⠡⠹⠂⠢⠌⠂⠖⠼        ⠈⠡⠲⠸⠹⠶⠌⠂⠆⠸⠼
    ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒      ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒

        ⠒⠒
    ⠣⠶⠶⠶⠶⠶⠶⠶
⠼⠒⠾     ⠔⠔⠔   ⠼⠲⠾              ⠒⠏⠬⠖⠟
        ⠈⠡⠲                   ⠂⠂⠏⠤⠆⠟
     ⠒⠒⠒⠒⠒⠒⠒      ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
      ⠒⠠⠔⠔⠖        ⠒⠒⠏⠘⠆⠐⠬⠖⠖⠏⠟
                         ⠤ ⠖⠏⠟⠤⠂⠆⠟⠘⠆
                  ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
                   ⠒⠒⠏⠘⠆⠐⠬⠖⠼⠏⠟⠤⠂⠆⠟⠘⠆

⠸⠱
```

PRACTICE 10C

⠠⠒⠧⠻⠞⠬ ⠠⠋⠗⠁⠉⠰⠝⠎ ⠞⠕ ⠠⠙⠑⠉⠊⠍⠁⠇⠎

⠠⠞⠕ ⠺⠗⠊⠞⠑ ⠮ ⠋⠗⠁⠉⠰⠝ ⠸⠩ ⠼⠂⠸⠌⠆ ⠸⠱ ⠵ ⠁
⠙⠑⠉⠊⠍⠁⠇⠂ ⠽ ⠉ ⠙⠊⠧⠊⠙⠑ ⠦⠐⠕ ⠏⠕⠔⠞ ⠵⠻⠕ ⠙⠊⠧⠊⠙⠫
⠃⠽ ⠞⠺⠕⠴⠂ ⠕⠗ ⠸⠩ ⠼⠆⠕⠂⠨⠴ ⠸⠱ ⠠⠏⠥⠞ ⠁⠺⠁⠽ ⠽⠗
⠉⠁⠇⠉⠥⠇⠁⠞⠕⠗⠖ ⠠⠥⠎⠬ ⠍⠢⠞⠁⠇ ⠍⠁⠹⠂ ⠭⠄⠎ ⠑⠁⠎⠽⠖ ⠠⠮
⠁⠝⠎⠺⠻ ⠊⠎ ⠸⠩ ⠼⠨⠢ ⠸⠱⠲ ⠠⠉ ⠽ ⠙⠊⠧⠊⠙⠑ ⠸⠂⠘⠮
⠋⠗⠁⠉⠰⠝⠎ ⠔ ⠽⠗ ⠓⠂⠙⠦ ⠸⠩
⠼⠂⠸⠌⠢⠸⠒ ⠼⠢⠕⠂⠨⠴
⠼⠲⠸⠌⠢⠸⠒ ⠼⠢⠕⠲⠨⠴
⠼⠔⠸⠌⠂⠴⠸⠒ ⠼⠂⠴⠕⠔⠨⠴ ⠸⠱

⠠⠏⠗⠁⠉⠞⠊⠉⠑

⠼⠁⠲ ⠠⠎⠕⠇⠧⠑ ⠘⠮ ⠙⠊⠧⠊⠨⠝ ⠏⠗⠕⠃⠇⠑⠍⠎⠲ ⠸⠩
⠰⠁⠸⠲ ⠼⠦⠕⠂⠆⠦
⠰⠃⠸⠲ ⠼⠖⠕⠢⠠⠒⠴⠲
⠰⠉⠸⠲ ⠼⠲⠕⠆⠴⠲
⠰⠙⠸⠲ ⠼⠔⠕⠦⠒⠶ ⠸⠱

Lines 14-17: Each subdivision begins on a new line in cell 3 even though the problems are arranged side by side in print.

PRACTICE 10D

Lines 2 and 6 are blank because a spatial arrangement must be preceded and followed by a blank line, even when embedded within a narrative paragraph.

Lines 11 and 13: Three blank cells between symbols in neighboring arrangements is the minimum number allowed (separation lines excluded).

Line 20: The Nemeth Code terminator is placed in cell 1 following the blank line after the completion of the spatially arranged material.

```
  ⠠⠱⠁⠞ ⠊⠎ ⠸⠩ ⠼⠂⠂⠕⠆⠆⠒⠲ ⠸⠱⠦ ⠠⠥⠎⠑ ⠇⠰⠛
⠙⠊⠧⠊⠨⠝⠲ ⠸⠩

      ⠆⠴⠒ ⠠⠗⠐⠂
    ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
  ⠂⠂⠕⠆⠆⠒⠲
     ⠆⠆
    ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
       ⠒⠲
       ⠒⠒
    ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
        ⠂

⠸⠱
```

Line 6: The leftmost character in the problem is placed in cell 3, the margin for material displayed to the 3-1 narrative on lines 1-2.

PRACTICE 10E

```
    ⠠⠋⠊⠇⠇ ⠔ ⠮ ⠃⠕⠭⠑⠎ ⠾ ⠮ ⠉⠕⠗⠗⠑⠉⠞ ⠝⠥⠍⠻⠁⠇⠲
⠸⠩

       ⠒ ⠔ ⠒⠴⠆ ⠠⠗⠿
     ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠷⠁⠾ ⠖⠕⠆⠒ ⠢ ⠦⠂⠒
    ⠩⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶
        ⠢ ⠂

         ⠒⠸⠹⠂⠌⠢ ⠸⠼
        ⠪⠒⠸⠹⠢⠌⠆⠢⠸⠼⠻
      ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠷⠃⠾ ⠆⠢⠕⠦ ⠴
       ⠶ ⠢
      ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
         ⠿

       ⠿ ⠠⠗⠐⠒
     ⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠷⠉⠾ ⠲⠕⠂⠔

⠸⠱
```

Line 1: The instructions begin in cell 5.
Line 2: There is not room for the opening Nemeth Code indicator on line 1. It is placed in cell 1 on line 2 because a spatial arrangement follows.
Line 4: See 10.14.a regarding the transcription of the remainder notation in problem (a).

LESSON 11

LESSON PREVIEW

Signs of shape are studied in depth, including icons and calculator keys. The study of typeform in Nemeth continues with the study of emphasized words in mathematical context. Format guidelines are given for consistent transcription of mathematical statements.

SIGNS OF SHAPE

[NC Rule 17]

11.1 Definition

A sign of shape is a miniature picture of a geometric figure or an object.

△ (triangle)   ∠ (angle)   ○ (circle)

Basic Shapes

[NC 17.1]

A basic shape is represented by the shape indicator followed by a numeral, one or more letters, or a dot combination suggestive of the shape.

⠫	Shape Indicator

11.2 Basic Signs of Shape Represented by Numbers—Regular Polygons

A closed figure that has equal sides and equal angles is called a regular polygon and is represented by the shape indicator followed by a numeral specifying the number of sides in the figure.

⠫⠲	Square (4-sided)	□
⠫⠢	Regular Pentagon (5-sided)	⬠
⠫⠖	Regular Hexagon (6-sided)	⬡

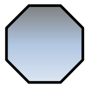
Note that the equilateral triangle, which is a regular polygon, is <u>not</u> represented by the number three. See 11.4.

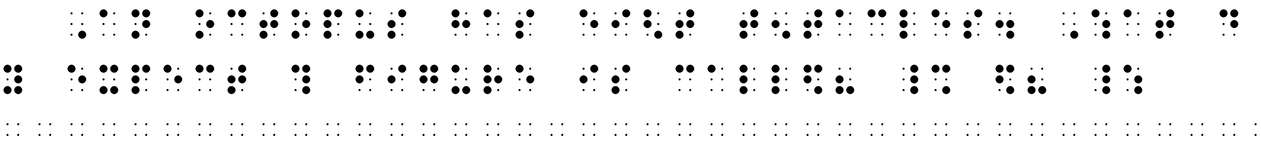
11.2.1 **Unlisted Regular Polygons.** Symbols which represent regular polygons with seven or more sides are not provided for in the Nemeth Code. If the unlisted shape is a *regular polygon*—that is, it is a closed figure with equal sides and equal angles—the transcriber is instructed to devise a symbol in accordance with the principles above, based on the number of sides the shape has. It may be helpful to include a tactile drawing of the shape. Unlisted regular polygon constructions do not require a transcriber's note.

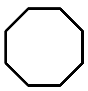
Refer to *Guidelines and Standards for Tactile Graphics* regarding shapes used in kindergarten through third grade materials such as counting symbols, pictographs, etc.

Example 11-1

An octopus has eight tentacles. What do you expect this figure is called?



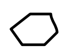
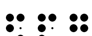
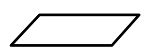
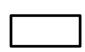
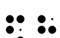
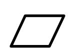
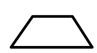




In addition to the transcriber-devised symbol, the shape is presented as a tactile graphic at the first mention of this shape. The graphic's left margin is cell 1. Follow directives in Guidelines and Standards for Tactile Graphics for drawing techniques.

11.3 Basic Signs of Shape Represented by Letters—Irregular Polygons

A closed figure which has at least two unequal sides and/or two unequal angles is called an *irregular polygon* and is represented by the shape indicator followed by a letter or a combination of letters suggestive of the name of the shape. (The derivation of the letter following the shape indicator is underlined in the list below.)

	Diamond	
	Irregular Hexagon	
	Irregular Pentagon	
	Parallelogram	
	Quadrilateral	
	Rectangle	
	Rhombus	
	Trapezoid	

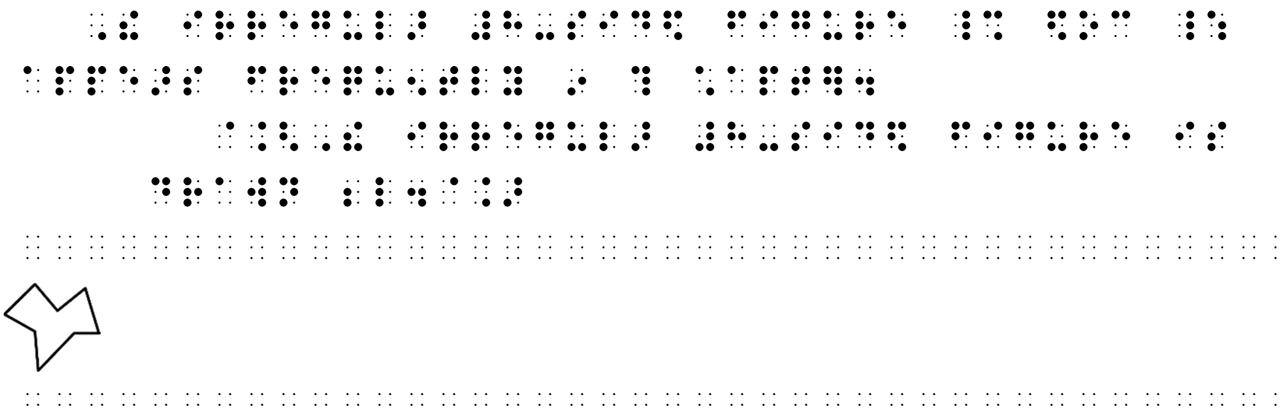
11.3.1 **Unlisted Irregular Polygons.** You may come across a shape which is not provided for in the Nemeth Code. If the unlisted shape is an irregular polygon—that is, it is a closed figure with at least two unequal sides and/or two unequal angles—the transcriber is instructed to devise a symbol if it appears frequently in the transcription. Construct the symbol in accordance with the principles above. Be careful not to choose a letter or letter combination which already has an assigned meaning in the Nemeth Code. Refer to Appendix B of the Nemeth Code for a list of symbols already in use. Find the section for symbols beginning with dots 1246.

A transcriber's note is required to define the figure unless it is described in the narrative. In your note, give the name or description of the symbol used. Include a drawing of the shape when appropriate. Sample transcriber's note:

The irregular 8-sided figure is drawn below.

Example 11-2

The irregular 8-sided figure  appears frequently in this chapter.



A raised outline of the shape is included as a tactile graphic at the first mention of this shape. Follow directives in Guidelines and Standards for Tactile Graphics.

11.4 Other Basic Signs of Shape Represented by Letters

The following shapes are also represented by the shape indicator followed by a letter suggestive of the name. (The derivation of the letter following the shape indicator is underlined.)

⠫⠉	Circle	○
⠫⠑	Ellipse	⬭
⠫⠊	Intersecting Lines	×
⠫⠎	Star	☆
⠫⠞	Triangle	△

Clarification: The triangle shape shown here is an equilateral or equiangular triangle – sides are equal in length, angles are equal in measurement. The symbol ⠫⠞ is used to represent a triangle shape when the print copy uses the drawing to replace the word "triangle" or when it is used to specifically represent an equilateral/equiangular triangle. See 11.8 for other types of triangles.

The following shapes were introduced in Lesson 5 as signs of comparison. They may also be used in print to simply replace the word they represent. Notice that two signs in this category begin with the negation symbol (dots 34) immediately followed by the shape indicator.

⠫⠁	Arc, Concave Upward	⌒
⠫⠇	Is Parallel To	∥
⠌⠫⠇	Is Not Parallel To	∦
⠫⠏	Is Perpendicular To	⊥
⠌⠫⠏	Is Not Perpendicular To	$\not\perp$

Example 11-3

Line AD ∦ Line BC.

⠸⠩ ⠠⠇⠊⠝⠑ ⠠⠁⠠⠙ ⠌⠫⠇ ⠠⠇⠊⠝⠑ ⠠⠃⠠⠉ ⠸⠱⠲

11.4.1 **Other Unlisted Basic Shapes.** Basic shapes not provided for in the Nemeth Code are formed in accordance with the principles above. One must be careful not to choose a symbol which already has an assigned meaning in the Nemeth Code. Refer to Appendix B of the Nemeth Code for a list of symbols already in use. Find the section for symbols beginning with dots 1246 and dots 34.

A transcriber's note is required to define the figure. Give the name or description of the symbol used. Include a drawing of the shape if it is vital to the mathematical topic at hand.

Sample transcriber's note:

In the following problem, ⠫⠓⠗ represents the shape of a heart.

Example 11-4

VALENTINE MATH Replace the heart with the correct math symbol: 14 ♡ 2 = 7.

The transcriber represents the heart shape with since means "rhombus".

11.5 Basic Signs of Shape Represented by Other Dot Combinations

Three additional shapes are identified in the Nemeth Code.

	Angle	∠
	Arc, Concave Downward	⌣
	Inverted Triangle	▽

11.6 Filled-In and Shaded Shapes

A filled-in or shaded closed shape (circle, diamond, square, etc.) is represented as such by the filled-in shape indicator or the shaded shape indicator. The appropriate indicator is placed between the shape indicator and the shape symbol.

	Filled-in shape indicator
	Shaded shape indicator

⋙ ★ (filled-in star)

⋙ ◍ (shaded circle)

⋙ ■ (filled-in square)

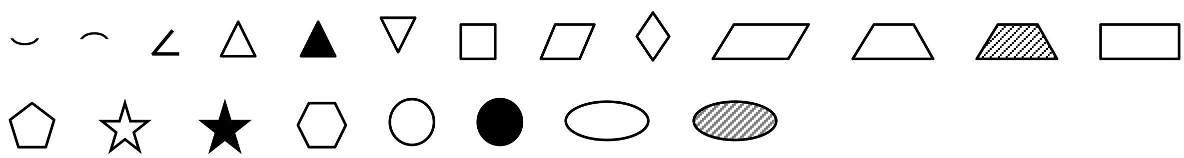
A filled-in square or rectangle used to indicate the end of a proof has its own special symbol. Proofs will be discussed in Lesson 12.

Shapes used as icons are discussed later in this lesson. See 11.26.

Instructions: Format the topic headings as cell-5 headings. Format each series of shapes as one paragraph, placing each opening Nemeth Code indicator at the beginning of each paragraph. Leave one space between each shape. Place as many shapes on one braille line that will fit before beginning a new line. Following the second topic heading, write a transcriber's note defining the flower and chicken shapes. Use "fl" to represent the flower and "ch" to represent the chicken.

PRACTICE 11A

Listed Shapes



Unlisted Shapes

Shapes with Structural Modification

[NC 17.5]

11.7 Definition and Construction

A shape with structural modification is one in which the general print form of a basic shape (such as *triangle*) is changed to show a more specific form (such as *right triangle*).

Basic shape:	△	Triangle
More specific form:		Right Triangle

Composite signs in which two or more signs of shape are combined are also structurally modified shapes, for example, two *angle* shapes in print combine to form the symbol for *adjacent angles*.

Basic shape:	∠	Angle
More specific form:		Adjacent Angles

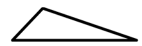
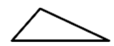
A shape with structural modification is represented by

- the basic shape symbol,
- followed by the structural shape-modification indicator,
- followed by a letter or an uncontracted combination of letters suggestive of the change in the shape,
- ending with the termination indicator which signals the end of the modification.

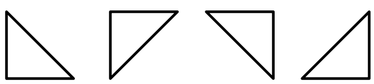
⠫	Shape indicator
⠨	Structural shape-modification indicator
⠻	Termination indicator

11.8 Structurally Modified Triangles

The following five structurally modified triangles are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "triangle" ⠫⠞. The derivation of the letter following the structural shape-modification indicator is underlined in the list of modified triangles below.

⠫⠞⠨⠁⠻	<u>A</u>cute Triangle	
⠫⠞⠨⠊⠻	<u>I</u>sosceles Triangle	
⠫⠞⠨⠕⠻	<u>O</u>btuse Triangle	
⠫⠞⠨⠗⠻	<u>R</u>ight Triangle	
⠫⠞⠨⠎⠻	<u>S</u>calene Triangle	

Know Your Triangles: Triangles are defined by the measure of angles and sides, not by orientation. For example, each of these is a "right triangle" because each contains a 90° angle.

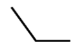


Definitions can be found in Appendix B of this course ("Glossary of Terms").

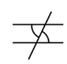
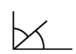
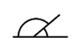
11.9 Structurally Modified Angles

The following twelve structurally modified angles are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "angle" ⠫⠪. The derivation of the letter or letters following the structural shape-modification indicator . is underlined in the list below.

Specific Angles

⠫⠪⠨⠕⠻	Obtuse Angle	
⠫⠪⠨⠗⠻	Right Angle	
⠫⠪⠨⠎⠻	Straight Angle	

Combined Angles

⠫⠪⠨⠚⠻	Adjacent Angles	 or 
⠫⠪⠨⠁⠑⠻	Alternate Exterior Angles	
⠫⠪⠨⠁⠊⠻	Alternate Interior Angles	
⠫⠪⠨⠉⠏⠻	Complementary Angles	
⠫⠪⠨⠉⠻	Corresponding Angles	
⠫⠪⠨⠑⠻	Exterior Angles	
⠫⠪⠨⠊⠻	Interior Angles	
⠫⠪⠨⠎⠏⠻	Supplementary Angles	
⠫⠪⠨⠧⠻	Vertical Angles	

11.10 Unlisted Shapes with Structural Modification

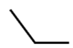
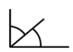
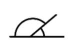
Structurally modified shapes which are not provided for in the Nemeth Code are formed in accordance with the principles above. Review the definition of *structural modification* in 11.7 to properly identify the unlisted shape. Be careful not to choose a symbol which already has an assigned meaning in the Nemeth Code. Refer to Appendix B of the Nemeth Code for a list of symbols already in use. Find the section for symbols beginning with dots 1246.

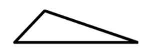
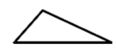
Explain the unlisted shape in a transcriber's note, giving the name or description of the symbol used. Include a drawing of the shape when appropriate.

Instructions: After completing the "Angle/Symbol" table, leave one blank line and then begin the "Triangle/Symbol" table. Do not use box lines. Review simple table format in Lesson 5.

PRACTICE 11B

Structurally Modified Shapes

Angle	Symbol
right	
straight	
obtuse	
complementary	
supplementary	
vertical	

Triangle	Symbol
isosceles	
right	
acute	
obtuse	
scalene	

Shapes with Interior Modification

[NC 17.6]

11.11 Definition and Construction

A shape with interior modification is a basic shape (for example, a *circle*) within which a letter, a numeral, a sign of operation, or other sign appears.

Basic shape:	○	Circle
More specific form:	⑧	Circle with number 8 inside
More specific form:	⊛	Circle with asterisk inside

A shape with interior modification is represented by

- the basic shape symbol,
- followed by the interior shape-modification indicator,
- followed by the symbol corresponding to the interior sign,
- ending with the termination indicator which signals the end of the modification.

⠫	`Shape indicator`
⠸⠫	`Interior shape-modification indicator`
⠻	`Termination indicator`

Symbols, numbers, words, etc. that represent keys on a calculator or a keyboard follow rules for keystrokes. See 11.23-11.25.

11.12 Circles with Interior Modification

Eleven circles with interior modification are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "circle" ⠫⠉ followed by the interior shape-modification indicator. ⠸⠫ Notice that an interior numeral includes a numeric indicator and that the contracted form of the right-pointing arrow is not used.

⠫⠉⠸⠫⠫⠒⠒⠕⠻	Circle with Interior Arrow Pointing Right	
⠫⠉⠸⠫⠫⠪⠒⠒⠻	Circle with Interior Arrow Pointing Left	
⠫⠉⠸⠫⠫⠣⠒⠒⠕⠻	Circle with Interior Arrow Pointing Up	
⠫⠉⠸⠫⠫⠩⠒⠒⠕⠻	Circle with Interior Arrow Pointing Down	
⠫⠉⠸⠫⠠⠺⠻	Circle with Interior Capitalized Letter	Ⓦ
⠫⠉⠸⠫⠼⠢⠻	Circle with Interior Numeral	⑤
⠫⠉⠸⠫⠈⠡⠻	Circle with Interior Cross	⊗
⠫⠉⠸⠫⠡⠻	Circle with Interior Dot	⊙
⠫⠉⠸⠫⠤⠻	Circle with Interior Minus Sign	⊖
⠫⠉⠸⠫⠬⠻	Circle with Interior Plus Sign	⊕
⠫⠉⠸⠫⠳⠻	Circle with Interior Vertical Bar	⦶

11.13 Angles with Interior Modification

Three angles with interior modification are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "angle" ⠫⠪ followed by the interior shape-modification indicator ⠸⠫

⠫⠪⠸⠫⠫⠁⠻	Angle with Interior Arc	
⠫⠪⠸⠫⠫⠪⠢⠔⠻	Angle with Interior Clockwise Arrow	
⠫⠪⠸⠫⠫⠢⠔⠕⠻	Angle with Interior Counterclockwise Arrow	

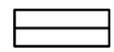
When the print copy uses the "angle with interior arc" symbol throughout the text to simply mean "angle", the two-cell angle symbol may be used: ⠫⠪. A transcriber's note is required to inform the reader of the substitution. Sample note on the Transcriber's Notes page: "In print, the angle shape image includes an interior arc."

When a number with internal measurement is encountered within the text, it is constructed as follows. (See also, 11.18, below.)

⫸  ⠫⠪⠸⠫⠒⠴⠘⠨⠡⠐⠻

11.14 Rectangles and Squares with Interior Modification

One rectangle and seven squares with interior modification are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "rectangle" ⠫⠗ or for "square" ⠫⠲ followed by the interior shape-modification indicator ⠸⠫

⠫⠗⠸⠫⠱⠻	Rectangle with Interior Horizontal Bar	
⠫⠲⠸⠫⠱⠻	Square with Interior Horizontal Bar	
⠫⠲⠸⠫⠳⠻	Square with Interior Vertical Bar	
⠫⠲⠸⠫⠔⠻	Square with Interior Diagonal from Lower Left to Upper Right	
⠫⠲⠸⠫⠢⠻	Square with Interior Diagonal from Upper Left to Lower Right	
⠫⠲⠸⠫⠢⠈⠔⠻	Square with Interior Diagonals	
⠫⠲⠸⠫⠡⠻	Square with Interior Dot	
⠫⠲⠸⠫⠼⠢⠻	Square with Interior Numeral	

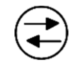
11.15 Words Enclosed in Shapes

Words enclosed in shapes are transcribed according to the methods for shapes with internal modification and must be enclosed within Nemeth switches. *Note*: Words that represent keys on a calculator or a keyboard follow rules for keystrokes. See 11.23-11.25.

11.16 Two or More Vertically Arranged Modifiers

When two or more vertically arranged symbols occur within a basic sign of shape, the basic shape symbol and the interior shape-modification indicator are followed first by the symbol for the upper and then by the symbol for the lower interior sign. The termination indicator is used only after the last symbol.

⠫⠉⠸⠫⠫⠒⠒⠕⠫⠪⠒⠒⠻

Circle with Interior Arrow Pointing Right
Over Interior Arrow Pointing Left 

⠫⠉⠸⠫⠫⠪⠒⠒⠫⠒⠒⠕⠻

Circle with Interior Arrow Pointing Left
Over Interior Arrow Pointing Right 

11.17 Two or More Horizontally Arranged Modifiers

When two or more horizontally arranged symbols occur within a basic sign of shape, a multipurpose indicator (dot 5) is inserted between the interior modifiers to show that they are printed horizontally, not vertically. The termination indicator is used only after the last symbol.

⠫⠉⠸⠫⠫⠣⠒⠒⠕⠐⠫⠩⠒⠒⠕⠻

Circle with Interior Arrow Pointing Up
Followed by Interior Arrow Pointing Down 

⠫⠉⠸⠫⠫⠩⠒⠒⠕⠐⠫⠣⠒⠒⠕⠻

Circle with Interior Arrow Pointing Down
Followed by Interior Arrow Pointing Up 

11.18 Unlisted Shapes with Interior Modification

Shapes with interior modification not provided for in the Nemeth Code are formed in accordance with the principles for the construction of such shapes. Review the definition of interior modification in 11.11 to properly identify the unlisted shape.

⫸ $\boxed{\text{W}}$ ⠫⠲⠸⠫⠠⠺⠻

⫸ $\circledast$ ⠫⠉⠸⠫⠈⠼⠻

⫸ $\angle x^\circ$ ⠫⠪⠸⠫⠭⠘⠨⠡⠐⠻

⫸ ▥ ⠫⠲⠸⠫⠳⠐⠳⠻

A symbol which already has an assigned meaning in the Nemeth Code must not be used for the unlisted sign of shape. If necessary, explain the shape in a transcriber's note giving the name or description of the symbol used. Include a drawing of the shape when appropriate.

Instructions: Transcribe this as a simple list, not as columns.

PRACTICE 11C

Squares with Interior Modification

Square with interior numeral 2 $\boxed{2}$

Square with interior dot ⊡

Square with interior horizontal bar ⊟

Square with interior vertical bar ◫

Square with interior diagonals ⊠

Other Details

11.19 Spacing with Signs of Shape

A sign of shape is spaced in accordance with its assigned meaning. For example, operation signs are unspaced.

⫸ x ⊕ y ⠭⠫⠉⠸⠫⠬⠻⠽

Comparison signs are spaced.

⫸ x $\boxed{>}$ y ⠭ ⠫⠲⠸⠫⠨⠂⠻ ⠽

Exception: Keystroke constructions follow different spacing rules. See 11.24.2.

Example 11-5

Operation signs appear within circles. The comparison sign appears inside a square.

2 ⊕ 3 $\boxed{=}$ 3 ⊕ 2

⠠⠕⠏⠻⠁⠰⠝ ⠎⠊⠛⠝⠎ ⠁⠏⠏⠑⠜ ⠾⠔ ⠉⠊⠗⠉⠇⠑⠎⠲ ⠠⠮
⠉⠕⠍⠏⠜⠊⠎⠕⠝ ⠎⠊⠛⠝ ⠁⠏⠏⠑⠜⠎ ⠔⠎⠊⠙⠑ ⠁ ⠎⠟⠥⠜⠑⠲
⠸⠩ ⠼⠆⠫⠉⠸⠫⠬⠻⠼⠒ ⠫⠲⠸⠫⠨⠅⠻ ⠼⠒⠫⠉⠸⠫⠬⠻⠼⠆ ⠸⠱

11.20 Punctuation with Signs of Shape

Signs of shape are punctuated mathematically when the punctuation falls within the code switches.

⫸ (△, ○, □) ⠷⠫⠞⠠ ⠫⠉⠠ ⠫⠲⠾

11.21 Plurals/Possessives

The uncapitalized letter "s" or the apostrophe-s combination occurring inside or after a sign of shape to show its plural or possessive are placed <u>after</u> the shape symbol. Apply the general rules for the English-letter indicator to the plural or possessive ending.

Example 11-6

∠s and △s.

⠸⠩ ⠫⠪⠎ ⠠⠄⠯ ⠫⠞⠎ ⠸⠱⠲

Each "s" is printed inside the shape.

Example 11-7

∠ s and △ s.

⠸⠩ ⠫⠪⠎ ⠠⠄⠯ ⠫⠞⠎ ⠸⠱⠲

Each "s" follows the printed shape.

Example 11-8

∠'s and △'s.

⠸⠩ ⠫⠪⠸⠄⠎ ⠠⠄⠯ ⠫⠞⠸⠄⠎ ⠸⠱⠲

A punctuation indicator precedes each apostrophe.

Example 11-9

(∠'s and △'s, and ○'s.)

⠸⠩ ⠷⠫⠪⠸⠄⠎⠠ ⠫⠞⠸⠄⠎⠠ ⠠⠄⠯ ⠫⠉⠸⠄⠎⠸⠲⠾ ⠸⠱

Each "apostrophe-s" is punctuated mathematically because each is associated with a mathematical item.

11.22 Further Considerations Regarding Transcriber-Devised Shapes

As previously noted, when encountering a shape not provided for in the Nemeth Code the transcriber may devise a symbol if that shape appears more than occasionally. The print shape should also be drawn as a raised-line diagram the first time the new symbol is introduced. In addition to the guidelines regarding unlisted shapes throughout this lesson, observe the following.

11.22.1 **Usage Rules Regarding Interior Numerals and Arrows.** Transcriber-devised forms should heed the following principles regarding interior numerals and arrows.

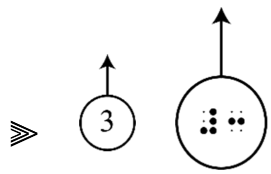
a. The numeric indicator is used before a numeral or before a decimal point and a numeral following the interior shape-modification indicator.

⋙ ⑧ ⠫⠉⠸⠫⠼⠦⠻

b. When a right-pointing arrow is part of a shape symbol, the shaft is transcribed. The contracted form of the right-pointing arrow is not used.

⋙ $\boxed{\rightarrow}$ ⠫⠗⠸⠫⠫⠒⠒⠕⠻

11.22.2 **Shapes Represented by Drawing.** Drawn-in shapes are often more readable than elaborate braille constructions. Since it is not possible to formulate specific rules for the selection of an appropriate form, the decision is left to the experience and judgment of the transcriber. Shapes may also be represented by a combination of drawing and braille symbols. For example, if a modified shape cannot be represented clearly by braille symbols alone, the shape can be drawn and the modification shown in braille.



PRACTICE 11D

1. ▯, ○, △, ∡, ⊙, ∠.
2. (○'s, ∠'s, and △'s.)
3. a ⊕ (b ⊕ c)
4. r ⊗ s ⊗ ____ = rst
5. How many △ can you find in the giant □ ?

Calculators and Keyboards

[NC 17.6.4]

11.23 The Keystroke Indicator

When a print shape with interior modification depicts a labeled calculator or computer key, a contracted form employing a keystroke indicator is used in braille. A keystroke is represented by

- the keystroke indicator,
- followed by the label printed on the calculator key or the computer key,
- ending with the termination indicator which signals the end of the modification.

⠫⠅	Keystroke indicator
⠻	Termination indicator

11.23.1 **Shape in Print.** The keystroke indicator is used regardless of the shape of the key in the print copy.

⫸ $\boxed{+}$ ⠫⠅⠬⠻

⫸ ⊛ ⠫⠅⠈⠼⠻

⫸ $\boxed{\rightarrow}$ ⠫⠅⠫⠒⠒⠕⠻

The actual key shape(s) used in a particular text should be specified in a transcriber's note. Sample transcriber's note:

Calculator keys are depicted in print as square shapes.

11.24 Other Details Concerning Keystrokes

11.24.1 **The Label.** Regarding the label depicted on the key, note the following.

a. Capitalization is duplicated in braille. Words are transcribed without contractions.

⫸ $\boxed{\text{ENTER}}$ ⠫⠅⠠⠠⠑⠝⠞⠑⠗⠻

⫸ $\boxed{\text{Enter}}$ ⠫⠅⠠⠑⠝⠞⠑⠗⠻

b. Follow the usual rules of the Nemeth Code for use of indicators.

⫸ $\boxed{x^2}$ ⠫⠅⠭⠘⠆⠐⠻

A baseline indicator is required before the termination indicator.

Example 11-10

Press £ for help.

A multipurpose indicator (dot 5) is needed to show that the numeral is not a subscript. See Section 6.11.1.c.

c. The numeric indicator is not required within the keystroke construction.

⫸ 8

Compare to a shape with interior modification which <u>does</u> require a numeric indicator. See <u>11.22.1</u>.

d. The contracted form of the right-pointing arrow is not used in a keystroke construction.

⫸ →

11.24.2 **Spacing.** No space is left between keystroke constructions and other similar constructions or mathematical symbols in a sequence of related calculations. Arrows contained in the labels on the keys should not be spaced from the material to which they apply.

⫸ A b/c ↔ d/e

⫸ ENTER ↑

Example 11-11

Solve the division problem (2 × 3 + 4) ÷ 5 on your calculator as follows:

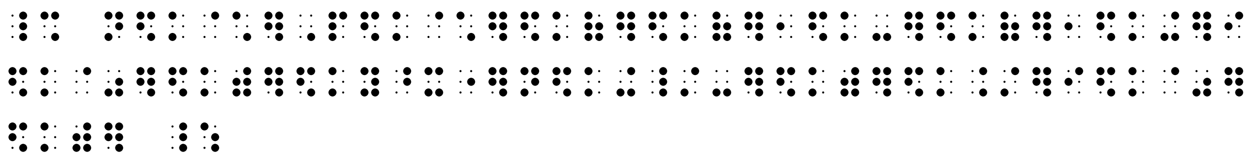
(2 * 3 + 4) ÷ 5 =

11.25 Long Keystroke Constructions

Use the fullest extent of the current braille line, making sure that a single keystroke construction is not divided between braille lines. When the print lines are arranged in a logical sequence, duplicate the arrangement of the print lines if possible.

Example 11-12

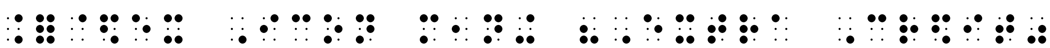
n $\boxed{\times}$ P $\boxed{\times}$ $\boxed{(}$ $\boxed{(}$ 1 $\boxed{-}$ $\boxed{(}$ 1 $\boxed{+}$ i $\boxed{\%}$ $\boxed{)}$ $\boxed{y^x}$ n $\boxed{+/-}$ $\boxed{)}$ $\boxed{\div}$ i $\boxed{\%}$ $\boxed{)}$



Icons

11.26 Consistency in Representation of Icons

When icons appear in mathematical context, the UEB transcriber-defined shape indicator may be used without the insertion of switch indicators. Include the icon on the Special Symbols page. Here is a sample, using the icon in the next example.



Note: An icon used to indicate the end of a proof has its own special symbol. Proofs will be discussed in Lesson 12.

In Nemeth, identifiers do not need to align when certain items are preceded by an icon. The icon is placed in the appropriate cell for the itemized material.

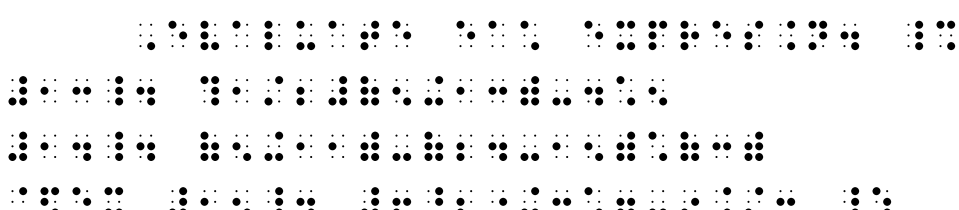
Example 11-13

Evaluate each expression.

13. $\frac{1}{2}(5+13)-4\cdot 5$

14. $(5+11)-(24-15)\cdot(3)$

 15. $6^2+3\cdot 7-9\div 3$



This textbook has stated that a gold ribbon indicates extra credit problems.

Line 1: Instructions begin in cell 5. The opening Nemeth Code indicator is placed at the end of this line of text so that all identifiers may start in the same cell.
Line 4: An icon was created to represent the "extra credit" print symbol. There is no need to switch out of Nemeth in order to transcribe the icon.

Shapes Used as Signs of Omission

11.27 Spacing

When a sign of shape is used as a sign of omission or placeholder to represent a numeral, letter, sign of comparison, sign of operation, abbreviation, or any other item, the sign of shape is spaced in accordance with the rules for the omitted material it represents. A sign of shape is unspaced from any braille indicator which applies to it.

⫸ $12 \div 4 = \square$

(a numeral is omitted—symbol is spaced from comparison sign)

⫸ 40 dimes = $\$\bigcirc$

(a numeral is omitted—symbol is unspaced from monetary symbol)

⫸ $\square$ l = 1000 cc

(a numeral is omitted—symbol is spaced from abbreviation "l")

⫸ $15 \bigcirc 15 = 30$

(an operation sign is omitted—symbol is unspaced)

⫸ 24 hrs. = 1 $\triangle$

(an abbreviation is omitted—symbol is spaced from preceding numeral)

⫸ $24_{\lozenge} + 11_{\lozenge} = 40_5$

(numerals in the subscript position are omitted)

⫸ $y^{\frac{1}{3}} = \sqrt[3]{\blacksquare}$

(the radicand is omitted)

Example 11-14

Fill in the square with the proper sign: $=$ or $\neq$.

$15 \div 3 \square \; 3 \div 15$

11.28 The English-letter Indicator and Comparison Signs

When a sign of shape is used in place of a comparison sign, rules regarding use of the English-letter indicator to items immediately preceding or following the shape apply just as they would for any comparison sign. In these three examples, a "square" symbol is used as a sign of omission representing a comparison sign.

⫸ $x \square y$

An English-letter indicator is not used for a single letter which is immediately preceded or followed by a sign of comparison.

⫸ 1 yr ○ 200 da

An abbreviation whose letters correspond to a shortform will not be misread as a word in Nemeth because contractions are not used inside the switches. An English-letter indicator is not needed. See 4.22.

⫸ ray ab ⬭ ray ac

A mathematical sequence that corresponds to a shortform will not be misread as a word in Nemeth. No English-letter indicator is needed.

⫸ iii □ iv

An English-letter indicator is not used for a Roman numeral which is immediately preceded or followed by a sign of comparison.

11.29 Use of the Multipurpose Indicator

Because a regular polygon with four or more sides is represented by a numeral, when an unspaced numeral follows such a shape, a multipurpose indicator is used before the numeral to prevent it from being misread. The shapes in these two examples represent a sign of operation.

⫸ 15 □ 15 = 30 ⠼⠂⠢⠫⠲⠐⠂⠢ ⠨⠅ ⠼⠒⠴

Without the multipurpose indicator this is read as a 415-sided regular polygon.

The situation stated above occurs only when the shape is represented by a numeral.

⫸ 15 ◯ 15 = 30 ⠼⠂⠢⠫⠉⠂⠢ ⠨⠅ ⠼⠒⠴

The circle shape is represented in braille with a letter ("c"). The following numeral will not be misread.

11.30 Omissions in Spatially Arranged Problems

In a spatial arrangement, omissions are indicated with the general omission symbol regardless of the symbol used in print.

Example 11-15

```
  9 4 6          ⠔⠲⠖
+ □□          ⠬ ⠿⠿
------       ⠒⠒⠒⠒⠒⠒
1 0 0 2       ⠂⠴⠴⠆
```

In print, the omissions are indicated as two squares.

Instructions: Use the letter p for the "pencil" icon. After completing item E, center "SPECIAL SYMBOLS USED IN THIS VOLUME" and show how the icon will be listed on the Special Symbols page.

PRACTICE 11E

Input the equation using the $\boxed{x}$ key. Press $\boxed{\text{SHIFT}}$ and $\boxed{\ln}$ to access $\left(e^{\square}\right)$. Press $\boxed{=}$ for the y value. Now input these values into your calculator.

$\boxed{(}$ 2 $\boxed{*}$ 3 $\boxed{+}$ 9 $\boxed{)}$ $\boxed{\div}$ 5 $\boxed{=}$ and

2 1 2 $\boxed{\circ}$ $\boxed{\to {}^\circ\text{C}}$ $\boxed{\text{End}}$

1. Fill in the box with the correct exponent.

   a. $x^2 \times x^4 = x^{\square}$

   b. $y^3 \times y^{\square} = y^9$

   c. $z^{\square} \times z^5 = z^{15}$

Problems marked with ✎ indicate that you are to show your work.

A. $436 - \square = 102$

B. $5_8 + \bigcirc_8 = 22_8$

✎ C. $5\frac{18}{20} = \square\frac{1}{2}$

D. Name two different operation signs that make this a true statement: $1 \square 1 = 1$.

✎ E. $\frac{15}{20} = \frac{3}{\square}$

Identified Signs of Shape

[NC 17.10.1]

A sign of shape which is followed by a letter, a sequence of letters, or a numeral, is an *identified sign of shape*. The entire unit is mathematical and therefore transcribed in Nemeth.

11.31 Spacing

There must be a space between the shape symbol and its identification. A space often does not appear in the print copy but it must be present in braille.

⋙ △ABC ⠫⠞ ⠠⠁⠠⠃⠠⠉ ("triangle ABC")

⋙ ∠*θ* ⠫⠪ ⠨⠹ ("angle theta")

Example 11-16

⏢5 denotes "trapezoid 5."

⠸⠩ ⠫⠵ ⠼⠢ ⠸⠱ ⠙⠢⠕⠞⠑⠎ ⠦⠞⠗⠁⠏⠑⠵⠕⠊⠙ ⠼⠑⠲⠴

The first 5 is associated with the mathematical shape and is included inside the switches. A switch to Nemeth is not necessary for the second 5 – it is a freestanding numeral in UEB context.

11.31.1 **Keep Together.** A sign of shape and the letter, sequence of letters, or numeral which follows it is regarded as a single mathematical item and therefore should not be divided between braille lines.

Example 11-17

How many different triangles can you draw within the boundaries of these shapes: ⏢ ABCD? △ EFG? ⬡ HIJKLM?

  ⠠⠓⠪ ⠸⠍ ⠙⠊⠖⠻⠢⠞ ⠞⠗⠊⠁⠝⠛⠇⠑⠎ ⠉ ⠽ ⠙⠗⠁⠺ ⠾⠔ ⠮
⠃⠨⠙⠜⠊⠑⠎ ⠷ ⠘⠮ ⠩⠁⠏⠑⠎⠒ ⠸⠩ ⠫⠵ ⠠⠁⠠⠃⠠⠉⠠⠙⠸⠦
⠫⠞ ⠠⠑⠠⠋⠠⠛⠸⠦ ⠫⠖ ⠠⠓⠠⠊⠠⠚⠠⠅⠠⠇⠠⠍ ⠸⠱⠦

11.31.2 **Surrounding Symbols.** The spacing before and after a sign of shape and its identification is subject to the spacing rules for the symbols preceding or following it.

⋙ △PQR ~ △P'Q'R'

⠫⠞ ⠠⠏⠠⠟⠠⠗ ⠈⠱ ⠫⠞ ⠠⠏⠄⠠⠟⠄⠠⠗⠄

A space precedes and follows the tilde, which, in this case, is a sign of comparison meaning "is similar to".

⫸ $\angle 2 + \angle 3 = \angle 4$

⠫⠪ ⠼⠆⠬⠫⠪ ⠼⠒ ⠨⠅ ⠫⠪ ⠼⠲

There is no space before or after the operation sign (plus sign).

11.32 A Shape Within a Superscript or a Subscript

An identified sign of shape is read as a unit, therefore when one appears in a superscript or a subscript the effect of the level indicator extends through the space following the sign of shape. That is, the space preserves the superscript or subscript level where the sign of shape appears.

⫸ $A_{\triangle ABC}$ ⠠⠁⠰⠫⠞ ⠠⠁⠠⠃⠠⠉

Example 11-18

Three-body Geometry. The subscripts define the direction of each vector. $\mathbf{r}_{\oplus\mathrm{sat}} = \mathbf{r}_{\mathrm{sat}} - \mathbf{r}_{\oplus}$

⠘⠂⠠⠹⠗⠑⠑⠤⠃⠕⠙⠽ ⠘⠂⠠⠛⠑⠕⠍⠑⠞⠗⠽⠲ ⠠⠮
⠎⠥⠃⠎⠉⠗⠊⠏⠞⠎ ⠙⠑⠋⠔⠑ ⠮ ⠙⠊⠗⠑⠉⠰⠝ ⠷ ⠑⠁⠡ ⠧⠑⠉⠞⠕⠗⠲
⠸⠩ ⠸⠰⠗⠰⠫⠉⠸⠫⠬⠻ ⠎⠁⠞ ⠨⠅ ⠸⠰⠗⠰⠎⠁⠞⠐⠤⠸⠰⠗⠰⠫⠉⠸⠫⠬⠻
⠸⠱

"⊕sat" *is in the subscript position. Only one subscript indicator is needed because the level continues through the space following the shape. (Reminder: Boldface type used to identify letters as vectors must be preserved.)*

11.33 A Shape Which Carries a Superscript or a Subscript

When a sign of shape carries a superscript or subscript, the level indicator is unspaced from the shape.

⫸ $\triangle^2$ ⠫⠞⠘⠆

⫸ $\square_2$ ⠫⠲⠰⠆

The rule for nonuse of the subscript level indicator does not apply to a shape represented by a letter. A subscript indicator is required.

⫸ $\triangle_2$ ⠫⠞⠰⠆

With an identified sign of shape, the space required between the sign of shape and its identifier follows the superscript or subscript. When the identified shape is on the baseline of writing, the space following the superscript or subscript terminates the effect of the level indicator and reinstates the baseline level.

⫸ $\triangle^{2}$ DEF

Only the numeral "2" is in the superscript position. The shape and letters "DEF" are on the baseline of writing.

11.34 The English-letter Indicator

When an English letter or a Roman numeral identifies a shape, the English-letter indicator is not used. *Exception:* If such letters are in nonregular type, rules regarding typeform are followed and an English-letter indicator is required if the variant typeform is retained. Also, if the sign of shape has a plural or a possessive ending, an English-letter indicator may be required. See 11.21.

⫸ $\angle p$ (*Spoken:* angle p)

⫸ □ ii (*Spoken:* square ii)

Example 11-19

⊙ Q denotes "circle Q."

Example 11-20

Compare ⬭ Z to ⬭ Z.

Example 11-21

Find the sum of ∠s a and b. Find the difference of ∠s acr and adr.

Single letters "a" and "b" require an English-letter indicator in Nemeth. (Review the definition of "single letter" in Lesson 3.)

11.34.1 **The Letter "m" meaning "measure".** Notation regarding angle measurement often uses the letter "m" for "measure". The letter "m" is unspaced from the following symbol regardless of spacing used in the print copy.

⫸ $m\angle\theta$ ⠍⠫⠪ ⠨⠹

(*Spoken:* the measure of angle theta)

⫸ $m°\ \angle\alpha$ ⠍⠘⠨⠡⠐⠫⠪ ⠨⠁

(*Spoken:* the measure in degrees of angle alpha)

11.35 Use of the Numeric Indicator in an Enclosed List

The "enclosed list" was introduced in Lesson 4 where it states that a numeric indicator is not used before a numeral in an enclosed list. More specifically, this rule applies to a numeral that occurs at the <u>beginning</u> of the item. A sign of shape and an identifying numeral which follows it are a single item even though a space occurs between them. In an enclosed list, the numeric indicator is required for the identifying numeral because the numeral is not at the beginning of the item. Look carefully at the use and nonuse of the numeric indicator in each example below, as described in the comments.

⫸ $(\angle 1,\ \angle 2,\ \angle 3)$

⠷⠫⠪ ⠼⠂⠠ ⠫⠪ ⠼⠆⠠ ⠫⠪ ⠼⠒⠾

In this enclosed list, each numeral needs a numeric indicator because it identifies the angle symbol—the numeral does not begin the item.

⫸ $(\angle 1,\ 2\angle 1,\ 3\angle 1)$

⠷⠫⠪ ⠼⠂⠠ ⠆⠫⠪ ⠼⠂⠠ ⠒⠫⠪ ⠼⠂⠾

Only the numerals which begin an item ("2" and "3") are transcribed without a numeric indicator in this enclosed list.

⫸ $(1\angle a,\ 2\angle b)$ ⠷⠂⠫⠪ ⠁⠠ ⠆⠫⠪ ⠃⠾

The numerals ("1" and "2") begin each item in this enclosed list—a numeric indicator is not transcribed. The letters ("a" and "b") are transcribed without a letter indicator according to the rules governing identified signs of shape.

PRACTICE 11F

1. □ABCD is a square. ▱ EFGH is a parallelogram. ▱ JKLM is a rhombus.
2. Compare triangles: △ADM ≅ △A'D'M'. ⊿ BEP ≇ ⊿ CFP.
3. Should △ABV be included in the set {△3, ∠ GHA, ◊2}?
4. $\angle 3 + \angle 4 = 90°$
5. $m\angle p + m\angle q = 180°$
6. $m°\angle\theta = -45$
7. $A_{\Delta DEF} = \frac{1}{2}bh$
8. $\angle ECB = \frac{1}{2}\angle ABC$

Typeform Indicators for Mathematical Words and Phrases

[NC 7.3 and 7.4]

As discussed in Lesson 7, when a variant typeform has mathematical significance or is illustrating a teaching point, the typeform is retained in the braille transcription. Otherwise, the variant typeform is disregarded. In this lesson, we will study how to apply this practice to words and phrases.

11.36 Italic and Boldface Typeform Indicators

The Nemeth Code recognizes two possible typeforms for words: italics and boldface. Different methods are used depending upon the extent of the variant typeform.

11.36.1 **One Word in Italics or Boldface.** When an italicized or boldfaced word is part of a mathematical expression and the typeform is determined to be significant, the following indicators are used. The indicator is unspaced from the following word.

⠨	Single Word Italic Type
⠸	Single Word Boldface Type
⠸⠨	Single Word Bold Italic Type

- *four* ⠨⠋⠕⠥⠗
- **four** ⠸⠋⠕⠥⠗
- ***four*** ⠸⠨⠋⠕⠥⠗

Example 11-22

6 *pieces* of pizza ÷ 2 people = 3 *pieces* per person

⠸⠩ ⠼⠖ ⠨⠏⠊⠑⠉⠑⠎ ⠕⠋ ⠏⠊⠵⠵⠁⠨⠌⠆ ⠏⠑⠕⠏⠇⠑
⠀⠀⠨⠅ ⠼⠒ ⠨⠏⠊⠑⠉⠑⠎ ⠏⠑⠗ ⠏⠑⠗⠎⠕⠝ ⠸⠱

Example 11-23

The subset of *even-number words* is shown in italics.
{one, *two*, three, *four*, five}

In UEB context, the UEB italic word indicator is used. In Nemeth context, the Nemeth italic type indicator is used.

Example 11-24

In this set, the **even-number word** is shown in boldface.
{three, **four**, five}

In UEB context, the UEB boldface word indicator is used. In Nemeth context, the Nemeth boldface type indicator is used.

a. **Probability Statements.** Words encountered in probability statements may be printed in regular type, in italics, in boldface, or fully capitalized. If all such words in the publication are printed in the same typeface, the distinction may be disregarded in braille. (Remember, capitalization is not a typeform, so capitalization is retained.) The word is part of the probability statement and so is transcribed in Nemeth—that is, without the single-word switch indicator and uncontracted. In the examples below, assume the typeface shown for the word is used throughout the document.

- ⫸ $P(A\ and\ B) = 0$

- ⫸ $P(\text{A and B}) = 0$

- ⫸ $\text{P(A OR B)} = ?$

Example 11-25

The rule for the probability of $(A\ and\ B)$ applies when A and B are independent events: $P(A\ and\ B) = P(A) \times P(B)$.

The italicized word "and" is part of the mathematical expression $(A\ and\ B)$ and so is transcribed in Nemeth. The italics are disregarded when all such statements are printed in italics. At the end of line 2, note that the word "and" is part of the sentence structure and is transcribed in UEB.

Example 11-26

Multiplication Rule: $P(\mathrm{A} \cap \mathrm{B}) = P(\mathrm{A}) * P(\mathrm{B} \backslash \mathrm{A})$, or $P(\mathrm{B}) * P(\mathrm{A} \backslash \mathrm{B})$

The word "or" is not part of the rule and so is transcribed in UEB using the single-word switch indicator. The intersection symbol, multiplication asterisk, and back slash are operation signs and so are unspaced.

b. **Hyphenated Expression.** If one or both components of a hyphenated expression are printed in a nonregular typeform, and if it is determined that the typeform must be retained, the appropriate single word typeform indicator is used. The typeform continues through the hyphen. The entire hyphenated expression is transcribed inside the switches.

- ⫸ *4.5-ohm* (both parts are in italics)
- ⫸ **4.5-ohm** (both parts are bold)
- ⫸ 4.5-*ohm* (only the word is in italics)

Example 11-27

Compare: 4.5-*ohm* vs. 4.5-*watt*.

⠠⠉⠕⠍⠏⠜⠑⠒ ⠸⠩ ⠼⠲⠨⠢⠤⠨⠕⠓⠍ ⠠⠄⠧⠎⠲
⠼⠲⠨⠢⠤⠨⠺⠁⠞⠞ ⠸⠱⠲

c. **Typeform Change Following the Hyphen.** Because a hyphen does not terminate typeform, a return to regular type after the hyphen is shown by inserting a typeform terminator before the hyphen.

⠠⠄	Typeform Terminator

⋙ *4.5*-ohm ⠨⠼⠲⠨⠢⠠⠄⠤⠕⠓⠍
(4.5 is in italics, ohm is in regular type)

If there is a change in typeform after the hyphen to a different nonregular type, the first typeform does not need an explicit terminator. The appropriate typeform indicator used after the hyphen terminates the effect of the first indicator.

⋙ *4.5*-***ohm*** ⠨⠼⠲⠨⠢⠤⠸⠨⠕⠓⠍
(4.5 is in italics, ohm is in bold italics)

d. Within narrative, a hyphenated expression may be transcribed in UEB unless the numeral or letter itself requires a switch to Nemeth.

Example 11-28

Compare the *25*-watt bulb to the *60*-watt bulb.

⠠⠉⠕⠍⠏⠜⠑ ⠮ ⠨⠂⠼⠃⠑⠨⠄⠤⠺⠁⠞⠞ ⠃⠥⠇⠃ ⠞⠕ ⠮
⠨⠂⠼⠋⠚⠨⠄⠤⠺⠁⠞⠞ ⠃⠥⠇⠃⠲

11.36.2 **A Phrase in Italics or Boldface.** When an italicized or boldfaced phrase is part of a mathematical expression and the typeform is retained, the following indicators are used.

⠠⠄⠨	Opening Italic Type for two or more words
⠨⠠⠄	Closing Italic Type for two or more words
⠠⠄⠸	Opening Boldface Type for two or more words
⠸⠠⠄	Closing Boldface Type for two or more words

These typeform indicators are preceded and followed by a space and must not stand alone on a line. When both indicators are required for the same word or phrase, they are unspaced from each other and are closed in the opposite order as opened.

⫸ *4.9 sq. ft.*

⠠⠄⠨ ⠼⠲⠨⠔ ⠎⠟⠲ ⠋⠞⠲ ⠨⠠⠄

⫸ *4.9 sq.* ***ft.***

⠠⠄⠨ ⠼⠲⠨⠔ ⠎⠟⠲ ⠸⠋⠞⠲ ⠨⠠⠄

⫸ *4.9* ***sq. ft.***

⠠⠄⠨ ⠼⠲⠨⠔ ⠠⠄⠸ ⠎⠟⠲ ⠋⠞⠲ ⠸⠠⠄⠨⠠⠄

a. When typeform is retained for a fraction, or for a numeral that is associated with a mathematical symbol such as a percent sign or monetary symbol, the appropriate three-cell typeform indicators are used.

$\mathbf{\frac{9}{100}}$ ⠠⠄⠸ ⠹⠔⠌⠂⠴⠴⠼ ⠸⠠⠄

9%   ⠠⠄⠸ ⠼⠔⠈⠴ ⠸⠠⠄

$9   ⠠⠄⠨ ⠈⠎⠔ ⠨⠠⠄

Example 11-29

ValueCo enjoyed a **9%** increase in profits last quarter.

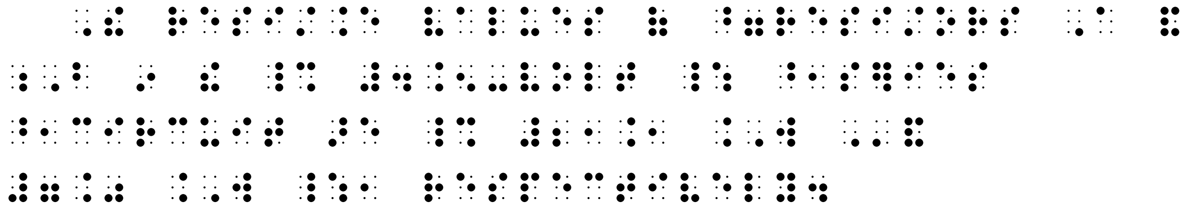
⠠⠧⠁⠇⠥⠑⠠⠉⠕ ⠢⠚⠕⠽⠫ ⠁ ⠸⠩ ⠠⠄⠸ ⠼⠔⠈⠴ ⠸⠠⠄ ⠸⠱
⠔⠉⠗⠂⠎⠑ ⠔ ⠏⠗⠷⠊⠞⠎ ⠇⠁⠌ ⠟⠥⠜⠞⠻⠲

11.37 Code Switching Within an Emphasized UEB Passage

Switching from UEB to Nemeth terminates the effect of a UEB typeform. The appropriate UEB typeform indicator is restated after Nemeth is terminated when emphasis continues. Nemeth typeform indicators are not used in the math portion unless the typeform gives unique meaning to the mathematical word or symbol.

Example 11-30

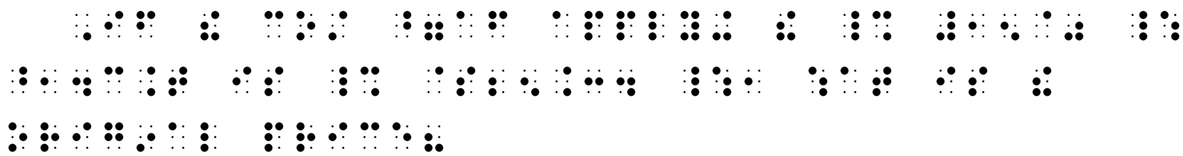
The resistance values of **resistors A and B in the 4.5-volt series circuit** are 21.1 Ω and 7.0 Ω, respectively.



Line 1: The UEB boldface passage indicator applies to "resistors A and B in the".
Line 2: A switch to Nemeth is required for the decimal number. The switch to Nemeth terminates the effect of the UEB typeform indicator without the need for a UEB termination indicator. Boldface is not retained in the Nemeth portion. UEB boldface is reinstated following the Nemeth Code terminator.
Line 4: Although 7.0 Ω fits on line 3, the Nemeth Code terminator does not. All are brought down to line 4.

Example 11-31

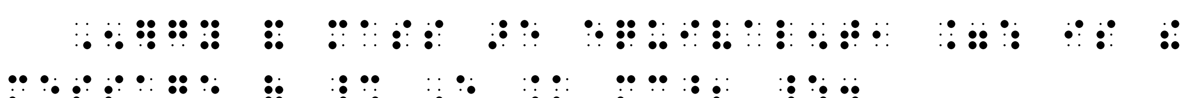
If the cost **after applying the 15% discount** is $25.34, what is the original price?



Line 1: The UEB boldface passage indicator applies "after applying the".
Line 2: A switch to Nemeth is required for the percentage. The switch to Nemeth terminates the effect of the UEB typeform indicator without the need for a UEB termination indicator. Boldface is not retained in the Nemeth portion. UEB boldface is reinstated following the Nemeth Code terminator.

Example 11-32

Energy and mass are equivalent, *which is the message of* $E = mc^2$.



Even though the formula is part of the italicized passage, italics are disregarded inside the switches because the typeform is not mathematically significant.

a. Although switching from UEB to Nemeth terminates typeform, switching from Nemeth to UEB does not. When the three-cell Nemeth typeform indicators are used inside the switches, the closing typeform indicator is required before terminating Nemeth Code.

Example 11-33

Thus, 6 pieces of pizza ÷ 2 people = 3 *pieces per person,* as illustrated in the diagram.

⠨⠹⠥⠎⠂ ⠸⠩ ⠼⠖ ⠏⠊⠑⠉⠑⠎ ⠕⠋ ⠏⠊⠵⠵⠁⠨⠌⠆ ⠏⠑⠕⠏⠇⠑
⠨⠅ ⠼⠒ ⠠⠄⠨ ⠏⠊⠑⠉⠑⠎ ⠏⠑⠗ ⠏⠑⠗⠎⠕⠝ ⠨⠠⠄ ⠸⠱⠂ ⠵
⠊⠇⠇⠥⠌⠗⠁⠞⠫ ⠔ ⠮ ⠙⠊⠁⠛⠗⠁⠍⠲

The italic typeface is retained for the phrase "pieces per person" because it illustrates a teaching point.

Mathematical Statements

[NC 7.4.4 and 26.7]

11.38 Axioms, Corollaries, Definitions, Laws, Lemmas, Propositions, Theorems

A mathematical statement is often printed in a distinctive style to catch the reader's attention. It also may be set off from the main text by different margins or some other means of distinction. The statement is usually introduced by a word such as Axiom, Corollary, Definition, Law, Lemma, Proposition, or Theorem. We will refer to this word as a "label" in this section.

Follow these directives in the braille transcription.

a. To draw attention to the mathematical statement in braille, leave one blank line before the label and one blank line after the statement.

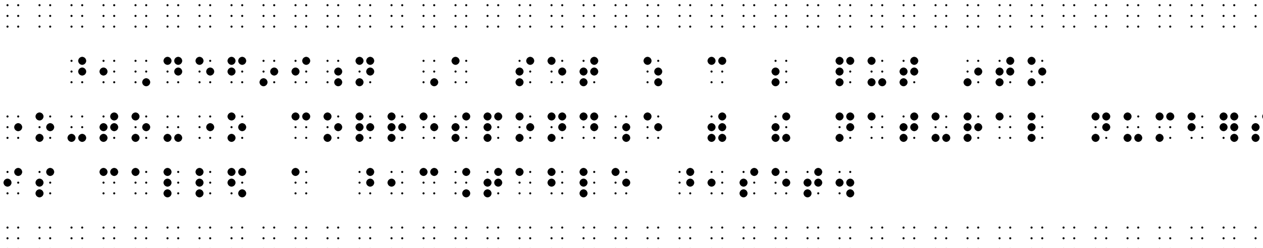
b. The label can be formatted as a paragraph heading or as a cell-5 or cell-7 heading, at the transcriber's discretion. Consistent treatment is important. Follow print for the capitalization style of the label. Typeform is disregarded in a cell-5 or cell-7 heading. Typeform is retained in a paragraph heading unless it is printed as fully capitalized as well as a variant typeform, in which case capitalization is retained but typeform is disregarded.

c. Continue with the text, using normal (3-1) paragraph style. When all statements in the text are printed in the same typeform, the uniform typeform may be disregarded in the transcription.

d. If, in the body of a mathematical statement, a word or phrase is singled out for special attention by using a nonregular typeface, the change in typeface is retained in braille.

Example 11-34

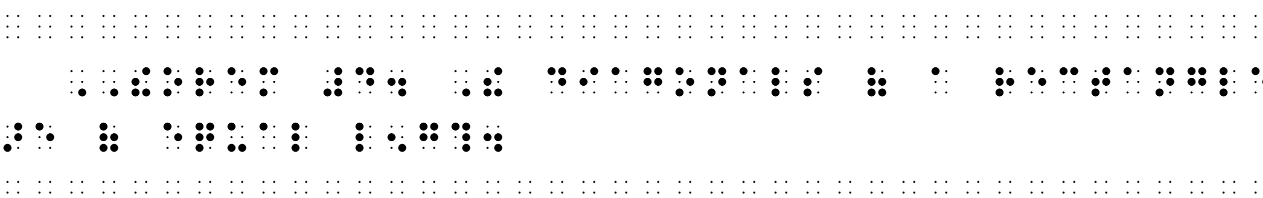
Definition *A set which can be put into one-to-one correspondence with the natural numbers is called a* ***countable set****.*



A blank line precedes and follows the mathematical statement. In print, "Definition" is a marginal heading and is in boldface type. The statement is printed in italics and is set off from the main body text with indented margins. In braille, the label is transcribed as a paragraph heading, but could just as well have been formatted as a cell-5 or cell-7 heading. As a paragraph heading, typeface is retained. We will assume that all Definition statements in this book are printed in italics, therefore the uniform typeface of the statement is disregarded. Boldface is retained for the bold words within the statement.

Example 11-35

THEOREM 4. *The diagonals of a rectangle are of equal length.*

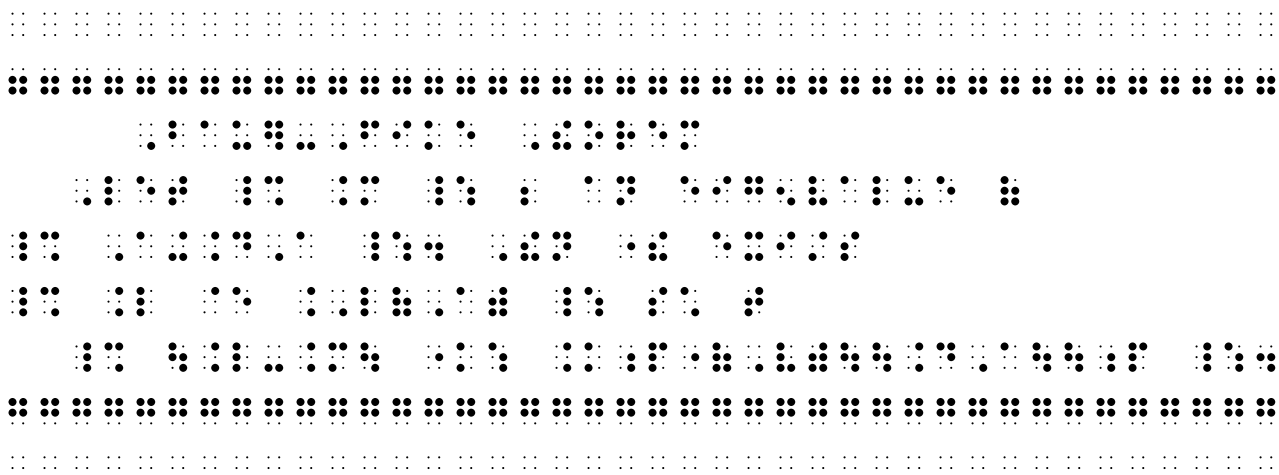


The label is printed in bold type. Typeform of the paragraph heading is disregarded because it is fully capitalized. The statement is printed in italics. We will assume that all Theorem statements in this book are printed in italics, therefore the uniform typeface of the statement is disregarded.

Example 11-36

Bauer-Fike Theorem
Let μ be an eigenvalue of $A + \delta A$. Then there exists $\lambda \in \Lambda(A)$ such that

$$|\lambda - \mu| \leq \kappa_p(V)\|\delta A\|_p.$$



The mathematical statement is boxed. Box lines are retained for distinction. The label, printed in boldface, is formatted as a cell-5 heading. Typeform is disregarded in a cell-5 heading, (The label could just as well have been formatted as a paragraph heading, in which case the bold type would be retained.) The definition is printed in normal typeface, with the exception of the letters in the mathematical expressions which are in italics. In braille, variables are transcribed in normal type unless the typeface has mathematical significance. You may wish to review any unfamiliar symbols in this example. Greek letters mu, delta, lambda (both capital and lowercase), and kappa, as well as the comparison sign for "membership".

Instructions: Format each statement's label as a paragraph heading. Assume all Theorem statements in this book are printed in italics. Retain the box around the Definition.

PRACTICE 11G

Write your answer in the box. Pay close attention to the italicized units.

$$52\ \textit{churros} \div 26\ \textit{children} = \square\ \textit{churros per child}$$

Pythagorean Theorem *In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.*

$$c^2 = a^2 + b^2$$

DEFINITION	A positive number expressed in the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer is said to be written in **scientific notation.**

For further practice, see Appendix A—Reading Practice.

EXERCISE 11

Prepare Exercise 11 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 11A

1 ⠀⠀⠀⠀⠠⠇⠊⠌⠫ ⠠⠩⠁⠏⠑⠎
2 ⠀⠀⠸⠩ ⠫⠄ ⠫⠁ ⠫⠪ ⠫⠞ ⠫⠸⠞ ⠸⠫ ⠫⠲ ⠫⠓ ⠫⠙ ⠫⠛ ⠫⠵
3 ⠫⠨⠵ ⠫⠗ ⠫⠢ ⠫⠎ ⠫⠸⠎ ⠫⠖ ⠫⠉ ⠫⠸⠉ ⠫⠑ ⠫⠨⠑ ⠸⠱
4 ⠀⠀
5 ⠀⠀⠀⠀⠠⠥⠝⠇⠊⠌⠫ ⠠⠩⠁⠏⠑⠎
6 ⠀⠀⠀⠀⠀⠀⠈⠨⠣⠦⠋⠇⠴ ⠌⠯⠎ ⠿ ⠋⠇⠪⠻⠆ ⠦⠉⠓⠴ ⠌⠯⠎ ⠿
7 ⠀⠀⠀⠀⠡⠊⠉⠅⠢⠲⠈⠨⠜
8 ⠀⠀⠸⠩ ⠫⠦ ⠫⠋⠇ ⠫⠉⠓ ⠸⠱

Code Switching Decisions

Line 1: It is acceptable to place the opening Nemeth Code indicator at the end of the cell-5 heading instead of at the beginning of line 2.

Line 8: Because the Nemeth Code terminator fits on the same line as the opening Nemeth Code indicator, the opening switch begins this line.

PRACTICE 11B

⠠⠌⠗⠥⠉⠞⠥⠗⠁⠇⠇⠽ ⠠⠍⠕⠙⠊⠋⠊⠫ ⠠⠩⠁⠏⠑⠎

⠀⠀⠀

⠠⠁⠝⠛⠇⠑ ⠠⠎⠽⠍⠃⠕⠇

⠐⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒ ⠐⠒⠒⠒⠒⠒⠒

⠸⠩

⠗⠊⠛⠓⠞ ⠐⠐⠐⠐⠐⠐⠐ ⠫⠪⠨⠗⠻

⠎⠞⠗⠁⠊⠛⠓⠞ ⠐⠐⠐⠐ ⠫⠪⠨⠎⠻

⠕⠃⠞⠥⠎⠑ ⠐⠐⠐⠐⠐⠐ ⠫⠪⠨⠕⠻

⠉⠕⠍⠏⠇⠑⠍⠑⠝⠞⠁⠗⠽ ⠫⠪⠨⠉⠏⠻

⠎⠥⠏⠏⠇⠑⠍⠑⠝⠞⠁⠗⠽ ⠫⠪⠨⠎⠏⠻

⠧⠑⠗⠞⠊⠉⠁⠇ ⠐⠐⠐⠐ ⠫⠪⠨⠧⠻

⠸⠱

⠀⠀⠀

⠠⠞⠗⠊⠁⠝⠛⠇⠑ ⠠⠎⠽⠍⠃⠕⠇

⠐⠒⠒⠒⠒⠒⠒⠒⠒ ⠐⠒⠒⠒⠒⠒⠒

⠸⠩

⠊⠎⠕⠎⠉⠑⠇⠑⠎ ⠫⠞⠨⠊⠻

⠗⠊⠛⠓⠞ ⠐⠐⠐ ⠫⠞⠨⠗⠻

⠁⠉⠥⠞⠑ ⠐⠐⠐ ⠫⠞⠨⠁⠻

⠕⠃⠞⠥⠎⠑ ⠐⠐ ⠫⠞⠨⠕⠻

⠎⠉⠁⠇⠑⠝⠑ ⠫⠞⠨⠎⠻

⠸⠱

PRACTICE 11C

⠀⠀⠀⠀⠀⠀⠠⠎⠟⠥⠜⠑⠎ ⠾ ⠠⠔⠞⠻⠊⠕⠗ ⠠⠍⠕⠙⠊⠋⠊⠉⠁⠰⠝

⠠⠎⠟⠥⠜⠑ ⠾ ⠔⠞⠻⠊⠕⠗ ⠝⠥⠍⠻⠁⠇ ⠼⠃ ⠸⠩ ⠫⠲⠸⠫⠼⠆⠻ ⠸⠱
⠠⠎⠟⠥⠜⠑ ⠾ ⠔⠞⠻⠊⠕⠗ ⠙⠕⠞ ⠸⠩ ⠫⠲⠸⠫⠡⠻ ⠸⠱
⠠⠎⠟⠥⠜⠑ ⠾ ⠔⠞⠻⠊⠕⠗ ⠓⠕⠗⠊⠵⠕⠝⠞⠁⠇ ⠃⠜
⠀⠀⠸⠩ ⠫⠲⠸⠫⠱⠻ ⠸⠱
⠠⠎⠟⠥⠜⠑ ⠾ ⠔⠞⠻⠊⠕⠗ ⠧⠻⠞⠊⠉⠁⠇ ⠃⠜ ⠸⠩ ⠫⠲⠸⠫⠳⠻ ⠸⠱
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PRACTICE 11D

⠸⠩
⠼⠂⠸⠲ ⠫⠗⠠ ⠫⠉⠠ ⠫⠞⠠ ⠫⠪⠨⠚⠻⠠ ⠫⠉⠸⠫⠨⠱⠻⠠
⠀⠀⠫⠪⠸⠫⠫⠢⠔⠕⠻⠸⠲
⠼⠆⠸⠲ ⠷⠫⠉⠸⠄⠎⠠ ⠫⠪⠸⠄⠎⠠ ⠠⠄⠯ ⠫⠞⠸⠄⠎⠸⠲⠾
⠼⠒⠸⠲ ⠁⠫⠉⠸⠫⠬⠻⠷⠃⠫⠉⠸⠫⠬⠻⠉⠾
⠼⠲⠸⠲ ⠗⠫⠉⠸⠫⠈⠡⠻⠎⠫⠉⠸⠫⠈⠡⠻ ⠤⠤⠤⠤ ⠨⠅ ⠗⠎⠞ ⠸⠱
⠼⠑⠲ ⠠⠓⠪ ⠸⠍ ⠸⠩ ⠫⠞⠎ ⠸⠱ ⠉ ⠽ ⠋⠔⠙ ⠔ ⠮ ⠛⠊⠁⠝⠞
⠀⠀⠸⠩ ⠫⠲ ⠸⠱⠦

PRACTICE 11E

PRACTICE 11F

⠼⠁⠲ ⠸⠩ ⠫⠲ ⠠⠁⠠⠃⠠⠉⠠⠙ ⠸⠱ ⠊⠎ ⠁ ⠎⠟⠥⠜⠑⠲
⠸⠩ ⠫⠛ ⠠⠑⠠⠋⠠⠛⠠⠓ ⠸⠱ ⠊⠎ ⠁ ⠏⠜⠁⠇⠇⠑⠇⠕⠛⠗⠁⠍⠲
⠸⠩ ⠫⠓ ⠠⠚⠠⠅⠠⠇⠠⠍ ⠸⠱ ⠊⠎ ⠁ ⠗⠓⠕⠍⠃⠥⠎⠲
⠼⠃⠲ ⠠⠉⠕⠍⠏⠜⠑ ⠞⠗⠊⠁⠝⠛⠇⠑⠎⠒
⠸⠩ ⠫⠞ ⠠⠁⠠⠙⠠⠍ ⠈⠱⠨⠅ ⠫⠞ ⠠⠁⠄⠠⠙⠄⠠⠍⠄⠸⠲
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⠸⠩ ⠨⠷⠫⠞ ⠼⠒⠠ ⠫⠪ ⠠⠛⠠⠓⠠⠁⠠ ⠫⠙ ⠼⠆⠨⠾⠸⠦
⠼⠲⠸⠲ ⠫⠪ ⠼⠒⠬⠫⠪ ⠼⠲ ⠨⠅ ⠼⠔⠴⠘⠨⠡
⠼⠢⠸⠲ ⠍⠫⠪ ⠏⠬⠍⠫⠪ ⠟ ⠨⠅ ⠼⠂⠦⠴⠘⠨⠡
⠼⠖⠸⠲ ⠍⠘⠨⠡⠐⠫⠪ ⠨⠹ ⠨⠅ ⠤⠼⠲⠢
⠼⠶⠸⠲ ⠠⠁⠰⠫⠞ ⠠⠙⠠⠑⠠⠋ ⠨⠅ ⠹⠂⠌⠆⠾⠃⠓
⠼⠦⠸⠲ ⠫⠪ ⠠⠑⠠⠉⠠⠃ ⠨⠅ ⠹⠂⠌⠆⠼⠫⠪

PRACTICE 11G

Lines 3-4: The displayed expression begins in cell 3. The linked expression will not fit on one line. It is divided before the equals sign. Review the topic of linked expressions in Lesson 8.

LESSON 12

- MODIFIERS AND MODIFIED EXPRESSIONS
    - Common Modifiers
    - Binomial Coefficient
    - Modified Expressions and Superscripts / Subscripts
    - Modified Signs of Comparison
    - Expressions with More Than One Modifier

Format

- Formal Proof

Answers to Practice Material

LESSON PREVIEW

Another type of mathematical notation with vertical components is studied. This lesson also applies the "mathematical statement" format to formal proofs.

MODIFIERS AND MODIFIED EXPRESSIONS

[NC Rule 15]

Some mathematical notation has a vertical aspect that challenges linear braille notation. By using modified expression indicators, the transcriber can relay the material in a compact manner, and the reader can construct the symbols into a meaningful expression.

12.1 Definition

A <u>modifier</u> is a symbol or a combination of symbols occurring *directly over* or *directly under* its related symbol or expression. Here are some typical examples.

$\overset{?}{=}$	a question mark over an equals sign
$3.1\underline{5}$	an underlined digit
$1.\overline{37}$	a line over two numerals (signifying a repeating decimal)
$\overleftrightarrow{AB}$	an arrow over two letters (signifying a line)
$\hat{k}$	a caret over a letter

12.2 Construction of Simple Modified Expressions – The Five-Step Rule

An expression modified using the five-step method is initiated and terminated with special indicators.

⠐	`Multipurpose Indicator`
⠻	`Termination Indicator`

The position of the modifier (above or below the expression) is also shown with the use of an indicator.

⠣	`Directly-Over Indicator`
⠩	`Directly-Under Indicator`

The process of constructing a modified expression is known as "The Five-Step Rule".

(1) The *multipurpose indicator* is placed immediately before the expression to be modified.

(2) The *expression* to be modified is written second.

(3) The *directly-over indicator* or the *directly-under indicator* is written third to show the position of the modifier.

(4) The *modifying symbol* is written fourth.

(5) The *termination indicator* is written last to show the end of the modification.

Notice that the multipurpose indicator, dot 5, signals the beginning of the modified expression and the termination indicator signals the completion of the modified expression.

Prototype for an expression with a modifier printed directly above it: ⠐_____⠣_____⠻

Prototype for an expression with a modifier printed directly below it: ⠐_____⠩_____⠻

All components of an expression modified according to the five-step rule should not be divided between lines. If that is not possible, strategies will be presented in Lesson 15.

Common Modifiers

The most commonly used modifiers are presented in this lesson. You have seen many of these symbols in other contexts in previous lessons. Symbols of the code not shown here may also be used as modifiers.

12.3 Arrows as Modifiers

Any of the arrows of the Nemeth Code may be a modifier. Those used in this section are shown below.

⠫⠕	`Arrow barbed at right, contracted form`	$\rightarrow$
⠫⠪⠒⠒⠕	`Arrow barbed at both ends`	$\longleftrightarrow$
⠫⠒⠒⠨⠡	`Arrow shaft with hollow dot at right`	⊸
⠫⠒⠒⠈⠕	`Upper barb only, right pointing`	$\rightharpoonup$
⠫⠒ ⠒⠕	`Arrow barbed at right, dashed shaft`	$\dashrightarrow$

12.3.1 **Right-Pointing Arrow.** A right-pointing arrow in regular type with a full barb and single shaft of ordinary length is transcribed in its contracted form when used as a modifier above or below a math expression. The shaft length in print is determined by the width of the expression it modifies. If the modified expression is wide, the arrow shaft will be long in print, but this is not considered to be a "longer than ordinary" arrow shaft in this context.

⫸ $\overrightarrow{\mathrm{YZ}}$ ⠐⠠⠽⠠⠵⠣⠫⠕⠻

YZ is the expression being modified. It is modified directly over with a right-pointing arrow.

⫸ $\underrightarrow{\mathrm{T}}$ ⠐⠠⠞⠩⠫⠕⠻

T is the expression being modified. It is modified directly under with a right-pointing arrow.

⫸ $\overrightarrow{M'N'}$ ⠐⠠⠍⠄⠠⠝⠄⠣⠫⠕⠻

The contracted form of the right-pointing arrow is used. The arrow shaft covers a wide expression and so is longer in print.

Show exactly what portion of an expression is being modified. In the next example, only the expressions between the grouping symbols are modified. The grouping symbols are placed outside of the multipurpose and termination indicators.

⫸ $\left|\overrightarrow{\mathrm{O_1P}}\right| + \left|\overrightarrow{\mathrm{P_1P_2}}\right|$

⠳⠐⠠⠕⠂⠠⠏⠣⠫⠕⠻⠳⠬⠳⠐⠠⠏⠂⠠⠏⠆⠣⠫⠕⠻⠳

O_1P and P_1P_2 are the expressions being modified. Each is modified directly over with a right-pointing arrow.

12.3.2 **Other Arrows.** All other types of arrows require the appropriate arrow symbol.

⫸ $\overleftrightarrow{\mathrm{AB}}$ ⠐⠠⠁⠠⠃⠣⠫⠪⠒⠒⠕⠻

AB is the expression being modified. It is modified directly over with an arrow barbed at both ends.

⫸ $\overset{\longrightarrow\!\!\circ}{\mathrm{AB}}$ ⠐⠠⠁⠠⠃⠣⠫⠒⠒⠨⠡⠻

AB is the expression being modified. It is modified directly over with an arrow with a hollow dot at its right end, no barb.

⫸ $\overset{\rightharpoonup}{\mathbf{OT}}$ ⠐⠸⠰⠠⠕⠸⠰⠠⠞⠣⠫⠒⠒⠈⠕⠻

OT (boldface) is the expression being modified. It is modified directly over with an arrow with an upper right-pointing barb.

⫸ $\overset{-->}{\mathrm{OB}}$

OB is the expression being modified. It is modified directly over with a right-pointing arrow with a dashed shaft.

Example 12-1

$\overleftrightarrow{\mathrm{PQ}}$ is a line through points P and Q.

Example 12-2

We now conclude that $\overrightarrow{\mathrm{OP}} = \overset{\rightharpoondown}{\mathrm{OT}} + \overset{\rightharpoondown}{\mathrm{TP}}$

Reminder from Lesson 1: If a math expression will fit on one line but there is not room for one or both of the switch indicators, one or both switch indicators may stand alone on a line. Keeping the mathematical expression intact on one line is the priority.

12.3.3 **When to Omit Arrows.** When identical arrows are used above vectors in boldface type throughout the text, they are omitted from the braille transcription. The boldface font attribute of the vectors is considered to be sufficient identification.

Example 12-3

Vector Addition By adding vectors $\overrightarrow{\mathbf{CD}}$ and $\overrightarrow{\mathbf{EF}}$, the resultant vector, **r**, is found.

$$\overrightarrow{\mathbf{CD}} + \overrightarrow{\mathbf{EF}} = \mathbf{r}$$

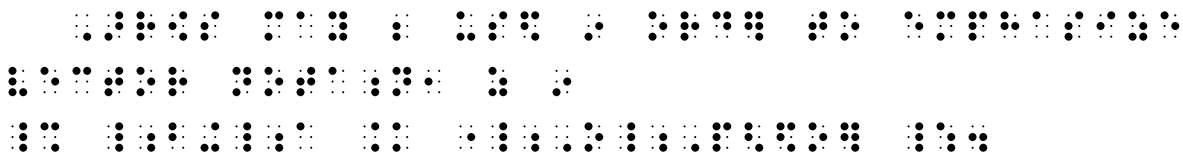
The arrows printed above CD and EF are not transcribed. The bold typeform is retained.

When the transcriber omits the vector arrows, a transcriber's note is required to explain the presence of the arrows in print. Sample transcriber's note: "The right-pointing arrows printed over boldface vectors are omitted."

A notable exception occurs when the author specifically refers to the arrows as a notational device. In that case, the arrow is transcribed in addition to the bold typeform.

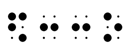
Example 12-4

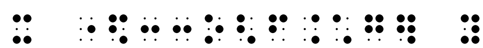
Arrows may be used in order to emphasize vector notation, as in $\mathbf{b} + \mathbf{a} = \overrightarrow{\mathbf{OP}}$.



The arrow is transcribed because it is mentioned in the narrative.

12.3.4 **When the Arrow Itself is Being Modified.** A right-pointing arrow in regular type with a full barb and single shaft of ordinary length is transcribed in its uncontracted form when the arrow itself is being modified.

	`Arrow barbed at right, uncontracted form`	$\longrightarrow$

⋙ $x \xrightarrow{f \circ g} y$ 

A right-pointing arrow is the expression being modified.
It is modified directly over with the operation $f \circ g$
Reminder: In function notation, $\circ$ is a hollow dot, not the letter "o".

12.3.5 **When Other Rules Apply.** Some arrangements that may appear to be modified arrows are not transcribed using the Five-Step Rule.

a. **Signs of Comparison Compounded Vertically.** Horizontal arrows printed one below the other are a sign of comparison compounded vertically. Review this topic in Lesson 9.

b. **Chemistry.** Words or symbols printed above reaction arrows are not treated as modifiers. Rules for these constructions are found in *Chemical Notation Using the Nemeth Braille Code.*

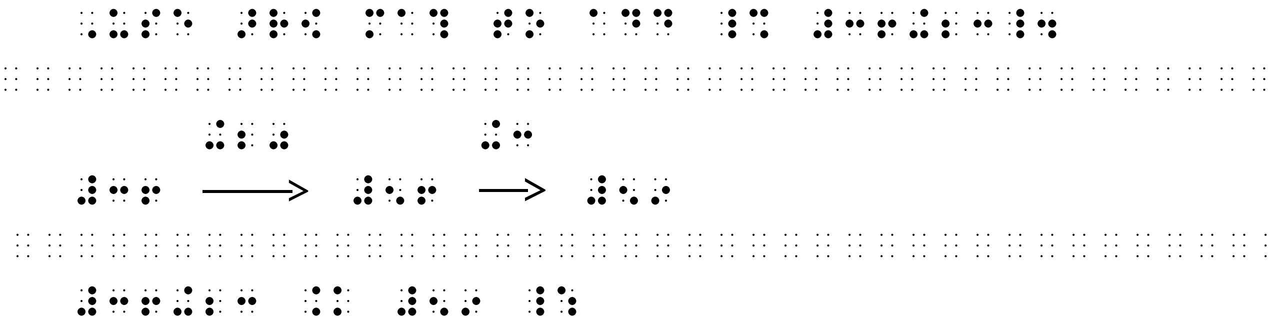
$$Na_2SO_4 \xrightarrow{H_2O} Na^+ + SO_4^{2-}$$

c. **Elementary Mathematics—"ARROW MATH" in Lower Grades.** These arrows are not "modified arrows". They are not being used as a sign of comparison, but rather as a "process". Draw these arrows as a tactile graphic. Refer to BANA's *Guidelines and Standards for Tactile Graphics* for techniques.

Example 12-5

Use arrow math to add 36 + 23.

$$36 \xrightarrow{+20} 56 \xrightarrow{+3} 59 \qquad 36 + 23 = 59$$



Instructions: Review arrow construction in Lesson 9. *Reminder:* In braille, identifiers for nonspatial itemized material must all begin on a new line in cell 1.

In the sentence at the end of this practice, assume that all vectors in the document are shown using that particular arrow notation. Show the proper way to omit the vector arrows in the transcription. Include the required transcriber's note after the topic heading.

PRACTICE 12A

Here are two modified arrows: $x \overset{g}{\rightarrow} y \overset{f}{\rightarrow} z$

Arrows as Modifiers

1. $\overleftharpoonup{\mathrm{F}}$ 2. $\overleftharpoonup{\mathrm{AB}}$ 3. $\overleftrightarrow{\mathrm{CD}}$ 4. $\overset{\mapsto}{\mathrm{OB}} \cup \overset{\mapsto}{\mathrm{OC}}$ 5. $\overset{\circ\!-\!\bullet}{\mathrm{EF}}$

6. $\underset{\leftarrow}{\mathrm{T}}$ 7. $\overleftrightarrow{\mathrm{XZ}} \parallel \overleftrightarrow{\mathrm{RS}}$ 8. $\overleftrightarrow{\mathrm{AB}} + \overleftrightarrow{\mathrm{CD}}$

Vector Addition

$\mathbb{R}$ equals $\overrightarrow{\mathbf{OP}}$ equals $\overrightarrow{\mathbf{OM}}$ plus $\overrightarrow{\mathbf{MC}}$ plus $\overrightarrow{\mathbf{CP}}$.

12.4 Carets as Modifiers

	Caret	∧
	Inverted Caret	∨

Note that the symbol for the inverted caret is the same construction as the opening Nemeth Code indicator. Context will make its meaning clear.

⋙ $\hat{\boldsymbol{k}}$

Bold letter k is modified directly over with a caret.

⋙ $\underset{\vee}{x}$

Letter x is modified directly under with an inverted caret.

Example 12-6

All values of y are located on the regression line $\hat{y} = \alpha + \beta x$.

12.5 Horizontal Bar as a Modifier

Recall that underlining is not considered to be a typeform in the Nemeth Code. When a number or letter with mathematical meaning is underlined, a switch to Nemeth is required and the methods discussed in this section are applied.

12.5.1 **Bar Over or Under More than One Digit or Letter.** When more than one digit or letter is modified by a single horizontal bar, the five-step rule is applied. Only one bar symbol is transcribed.

	Horizontal Bar	—

⋙ $\overline{99}$

The 2-digit number 99 is modified directly over with horizontal bar. Note that a numeric indicator is not needed because the number is not preceded by a space.

⋙ $\underline{419}$

The 3-digit number 419 is modified directly under with horizontal bar.

⫸ $\$11.\underline{25}$

The 2-digit number 25 is modified directly under with horizontal bar. Note that the decimal point is not part of this modified expression. The bar is under the digits only.

⫸ $1.\overline{37}$

The 2-digit number 37 is modified directly over with horizontal bar.

Example 12-7

Prove that $\overline{\mathrm{OP}} + \overline{\mathrm{QR}} = \overline{\mathrm{OR}}$.

Example 12-8

The fraction $\frac{3}{7}$ expressed as a decimal is .428571428571428571 … . Written as a repeating decimal, $\frac{3}{7} = .\overline{428571}$.

The decimal point is not part of the modified expression. The multipurpose indicator is transcribed in the cell immediately preceding the numeral 4.

12.5.2 **Bar Over or Bar Under Only One Digit or Letter.** When a single digit or a single letter in any typeform or any alphabet is modified only by a single horizontal bar either directly over or directly under it, the five-step rule is not applied. Instead, a contracted form is used.

a. **Bar Above a Single Digit or Letter.** When a single digit or a single letter (in any typeform or any alphabet) is modified only by a single horizontal bar directly over it, the symbol for the bar is placed immediately after the digit or letter modified.

⫸ $\overline{x}$

⫸ $\overline{3}$

⫸ $\overline{\mathbf{C}}'$

⫸ $\bar{\lambda}_t$ ⠨⠇⠱⠰⠞

Example 12-9

The $\overline{7}\,'$s are repeated.

⠠⠮ ⠸⠩⠼⠶⠱⠸⠄⠎ ⠸⠱ ⠜⠑ ⠗⠑⠏⠂⠞⠫⠲

The apostrophe-s relates to the numeral, therefore it is transcribed inside the switches. The punctuation indicator prevents the apostrophe from being misread as a prime sign.

Example 12-10

Arabic	Roman
5,000	$\overline{\mathrm{V}}$
10,000	$\overline{\mathrm{X}}$
50,000	$\overline{\mathrm{L}}$
100,000	$\overline{\mathrm{C}}$

⠠⠜⠁⠃⠊⠉   ⠠⠗⠕⠍⠁⠝
⠐⠒⠒⠒⠒⠒⠒⠒   ⠐⠒⠒⠒⠒⠒
⠸⠩
  ⠼⠢⠠⠴⠴⠴   ⠠⠧⠱
 ⠼⠂⠴⠠⠴⠴⠴   ⠠⠭⠱
 ⠼⠢⠴⠠⠴⠴⠴   ⠠⠇⠱
⠼⠂⠴⠴⠠⠴⠴⠴   ⠠⠉⠱
⠸⠱

The Arabic numbers in the first column can be transcribed in either UEB or Nemeth. The Roman numerals in column two require a code switch because they are modified. Smoother reading is accomplished by transcribing both columns in Nemeth. Review simple table format in Lesson 5.

b. **Bar Below a Single Digit or Letter.** When a single digit or a single letter (in any typeform or any alphabet) is modified only by a single horizontal bar directly under it, the directly-under indicator followed by the symbol for the bar is placed immediately after the digit or letter modified.

	Directly-Under Indicator followed by Horizontal Bar

⫸ $\underline{x}$ $\underline{\mathrm{R}}^{+}$

⫸ $\underline{3}$ $5\underline{7}9$

Study the examples below. Notice when the contracted form of the bar can be used. All simulated braille is in Nemeth. Switch indicators are omitted.

Example 12-11

(1) $(\bar{x}, \bar{y})$

(2) $\bar{x}$'s and $\bar{y}$'s

(3) $\overline{x+y}$

(4) $\underline{x+y}$

(5) $A(\underline{s}n)$

(6) $\dfrac{\overline{\mathrm{PR}}}{\overline{\mathrm{OR}}}$

(7) $P(\bar{x}) = \overline{P(x)}$

Instructions: Treat the three examples of unit vectors in problem #2 as displayed mathematical material.

PRACTICE 12B

Carets and Bars as Modifiers

1. Unit vectors can be denoted with normal vector notation, $\boldsymbol{u}$ or $\vec{u}$, or with standard unit vector notation, $\hat{\mathbf{u}}$, spoken "u-hat".
2. Unit vectors in various coordinate systems use Greek and English letters.

$$\text{Cartesian coordinate system: } \hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$$

$$\text{Cylindrical coordinate system: } \hat{\rho}, \hat{\varphi}, \hat{\mathbf{z}}$$

$$\text{Spherical coordinate system: } \hat{\mathbf{r}}, \hat{\theta}, \hat{\varphi}$$

3. $1.142857\overline{142857}$
4. $\frac{7}{15} = .4\overline{6}$
5. $2 \cdot 3 = \overline{2} \cdot 3 = \overline{2 \cdot 3}$
6. $\overline{\mathrm{PQ}}$, $\overline{x'}$, $\overline{\mathrm{R''S''}}$
7. $\overline{\mathbf{s}}, \overline{\alpha}, \overline{m}\,'$
8. $\mathrm{m}\overline{\mathrm{BC}} = a$
9. $\overline{\mathrm{C}} = 100 \times 1000$
10. $F = 2\pi\overline{r}l$
11. $P(\overline{a + bi}) = \overline{0} = 0$
12. $\hat{x}_i = 0.5(\overline{x}_i + \underline{x}_i)$
13. $3.141\underline{59}$

12.6 Other Symbols Used as Modifiers

Apply the five-step rule for the construction of simple modified expressions when the following symbols are used as modifiers.

12.6.1 **Arc**

⠫⠁	Arc Concave Upward	⌒

Example 12-12

Consider $\angle ABD = \frac{1}{2}\overset{\frown}{DC}$.

⠠⠒⠎⠊⠙⠻
⠸⠩ ⠫⠪ ⠠⠁⠠⠃⠠⠙ ⠨⠅ ⠹⠂⠌⠆⠼⠐⠠⠙⠠⠉⠣⠫⠁⠻ ⠸⠱⠲

12.6.2 **Dot**

⠡	Dot	•

In print, the recurrence of one digit in a decimal numeral may be indicated by one dot over the recurring digit.

⋙ $.\dot{6}$ ⠼⠨⠐⠖⠣⠡⠻

"4" is the expression being modified. It is modified directly over with a single dot.

⋙ $\frac{1}{6} = 0.66\dot{6}\ldots$

⠹⠂⠌⠖⠼ ⠨⠅ ⠼⠴⠨⠂⠖⠖⠐⠖⠣⠡⠻ ⠄⠄⠄

The last "6" is the expression being modified. It is modified directly over with a single dot.

The recurrence of one or more digits in a decimal numeral may be indicated by one dot over each recurring digit. In braille, only one dot is used as a modifier. The dot is placed after the last modified digit in the recurring sequence.

⫸ $1.\dot{3}\dot{7}\dot{5}$

In print, a single dot is shown over each of the three digits 3 7 and 5. In braille, only one dot is used as a modifier. The multipurpose indicator and termination indicator clearly show what is included within the modified expression.

Example 12-13

Expressed as a fraction, $0.9090\dot{0}\dot{9} = \frac{1}{11}$.

In print, a single dot is shown over the last digit 0 and a single dot is shown over the last digit 9.

When one or more dots occur over or under a single letter or numeral, the symbol for the dot is used as many times as necessary to conform with the print text.

⫸ $\dddot{x}$

⫸ $\underset{..}{u}$

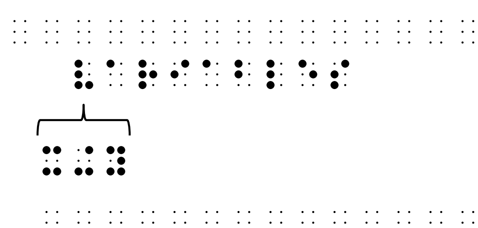
Example 12-14

Prove $\ddot{x} = \frac{d^2x}{dt^2}$

12.6.3 **Horizontal Grouping Sign.** When a horizontal grouping sign occurs over or under a mathematical expression, it is either a part of the expression or is pointing to a label.

a. **As a Pointer.** When the grouping sign points to a label or to explanatory text, it must be drawn as a tactile graphic. Refer to *Guidelines and Standards for Tactile Graphics* for drawing techniques.

⫸ $\overbrace{x+y}^{\textit{variables}}$

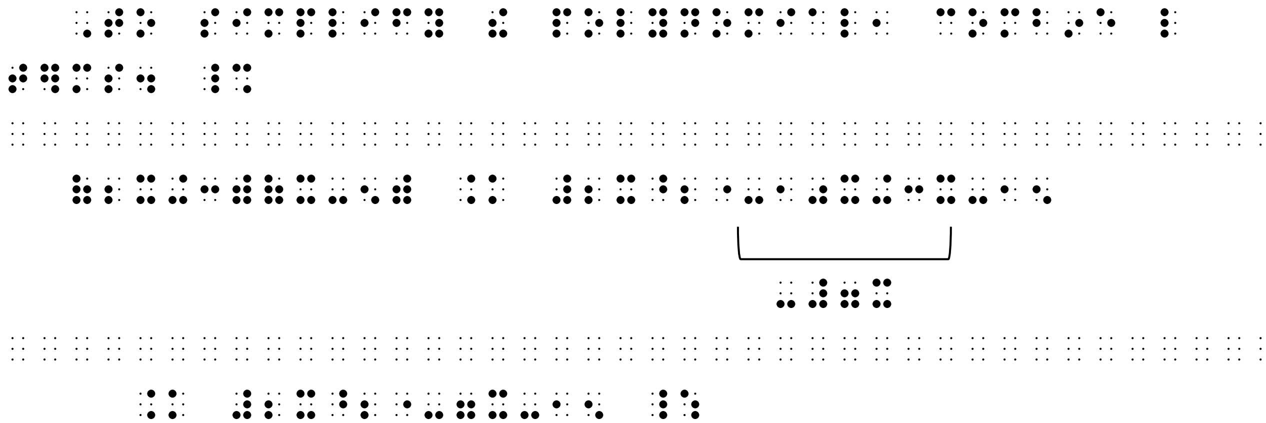


Example 12-15

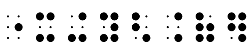
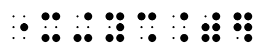
To simplify the polynomial, combine like terms.

$$(2x+3)(x-5) = 2x^2 \underbrace{-10x + 3x} - 15$$

$$= 2x^2 \quad -7x \quad -15$$

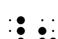


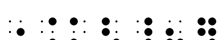
b. **As a Modifier.** When a horizontal grouping sign is part of the math expression, it is regarded as a modifier. Although it is preferable to draw the symbol, the transcriber can represent the construction as shown below, following the five-step rule of modified expressions. The left grouping symbol is transcribed when the modifier is directly over the expression, and the right grouping symbol is transcribed when the modifier is directly under the expression.

⫸ $\overbrace{x+y}$  $\underbrace{x+y}$ 

A brace is printed above $x+y$. *A brace is printed below $x+y$.*

12.6.4 **Question Mark.** When the question mark is not functioning as a sign of omission, the punctuation mark is transcribed. In mathematical context, a punctuation indicator precedes the question mark to prevent it from being misread as the numeral 8.

	Question Mark	?	(not an omission)

⫸ $\stackrel{?}{=}$ 

An equals sign is the expression being modified. It is modified directly ove with a question mark.

Example 12-16

Today our math teacher introduced us to the paradox $0.\overline{9} \stackrel{?}{=} 1$.

12.6.5 Tilde

	Single Tilde	~
	Extended Tilde	$\widetilde{\ \ \ }$

The extended tilde has more than one peak.

⋙ $\tilde{u} = 0.8$

⋙ $\widetilde{A} \cup \widetilde{B}$

⋙ $\widetilde{r+s+t}$

The next example is a single tilde even though it is covering several items. (The single tilde has only one peak.)

⋙ $\widetilde{r+s}$

12.7 Expressions as Modifiers

When a modifier is, itself, a mathematical expression, follow Nemeth Code rules regarding spacing and use of indicators. Each example below contains a comparison sign in the modifier. Nemeth rules applied include the following. A space is required before and after a sign of comparison; an English-letter indicator is not needed when a letter immediately precedes or follows a sign of comparison or when a letter touches an indicator; a numeric indicator is required when a numeral follows a space. The termination indicator marks the end of the modifier.

⋙ $\lim\limits_{x \searrow a}$

lim is modified directly under with the expression $x \searrow a$.

⋙ $\bigcup\limits_{A \subset F}$

The union ("cup") symbol is modified under with the expression $A \subset F$.

Greek capital letters *Sigma* and *Pi* are often encountered in this notation. Sigma is used in summation notation and Pi is used in product notation.

⠨⠠⠎	Sigma	Σ	(capital)
⠨⠠⠏	Pi	Π	(capital)

⫸ $\sum_{i<k}$ ⠐⠨⠠⠎⠩⠊ ⠐⠅ ⠚⠻

Sigma is modified directly under with the inequality $i < j$.

⫸ $\prod_{k=1}$ ⠐⠨⠠⠏⠩⠅ ⠨⠅ ⠼⠂⠻

Pi is modified directly under with the equality $k = 1$.

12.7.1 **Binomial Coefficient.** A binomial coefficient is written as two expressions, one atop the other, enclosed in parentheses. For example, *n choose q* is written like this:

$$\binom{n}{q}$$

The binomial coefficient does not follow the five-step rule for modified expressions. The opening parenthesis is followed by the upper expression; the directly-under indicator is placed next, followed by the lower expression. The closing parenthesis ends the expression.

⠩	Directly-under indicator

⫸ $\binom{n}{q}$ ⠷⠝⠩⠟⠾

⫸ $\binom{n-1}{k-1}$ ⠷⠝⠤⠂⠩⠅⠤⠂⠾

Example 12-17

The **recursive formula** states that $\binom{n}{0} = \binom{n}{n} = 1$ for all integers $n, k : 1 \leq k \leq n - 1$.

⠠⠮ ⠘⠂⠗⠑⠉⠥⠗⠎⠊⠧⠑ ⠘⠂⠿⠍⠥⠇⠁ ⠌⠁⠞⠑⠎ ⠞
⠸⠩ ⠷⠝⠩⠴⠾ ⠨⠅ ⠷⠝⠩⠝⠾ ⠨⠅ ⠼⠂ ⠸⠱ ⠿ ⠁⠇⠇ ⠔⠞⠑⠛⠻⠎
⠸⠩ ⠰⠝⠠ ⠰⠅⠸⠒ ⠼⠂ ⠐⠅⠱ ⠅ ⠐⠅⠱ ⠝⠤⠂ ⠸⠱⠲

Reminder from Lesson 5: A colon that means "such that" is transcribed unspaced from the letter it follows, and is preceded by a punctuation indicator. This spacing is applied regardless of the spacing shown in print. Spacing after the colon follows print.

Example 12-18

Use this formula to calculate the binomial coefficient.

$$\binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$$

⠠⠥⠎⠑ ⠹ ⠿⠍⠥⠇⠁ ⠞⠕ ⠉⠁⠇⠉⠥⠇⠁⠞⠑ ⠮ ⠃⠔⠕⠍⠊⠁⠇
⠉⠕⠑⠖⠊⠉⠊⠢⠞⠲
⠸⠩ ⠷⠝⠩⠅⠾ ⠨⠅ ⠹⠝⠯⠌⠅⠯⠡⠷⠝⠤⠅⠾⠯⠼ ⠸⠱

PRACTICE 12C

1. In Figure 7.3, if $\overset{\frown}{\mathrm{AB}} = \overset{\frown}{\mathrm{CD}}$ in circle 0, then $\angle \mathrm{AOB} = \angle \mathrm{BOC}$.
2. $x \overset{?}{=} y$ means "does x equal y?"
3. $\sum\limits_{d|n}$ (where $d|n$ means "d divides n").
4. $\binom{t}{p} = R_t^p$
5. $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ for all integers $n \geq 0$.
6. Does $(\tilde{x}, \tilde{y})$ mean $\tilde{x}$ and $\tilde{y}$?
7. More modified expressions:

   a. $.249\dot{9}$ b. $2.4313\dot{3}\dot{1}$ c. $\dot{x}\ddot{y} - \dot{y}\ddot{x}$

   d. $\underset{\cdot}{\mathrm{a}} + \underset{\cdots}{\mathrm{a}} = ?$ e. $f \to \tilde{f}$ f. $\widetilde{x+y}$

12.8 Spacing with Modified Expressions

The spacing before and after an entire modified expression is subject to the spacing rules for the symbols preceding or following it.

Example 12-19

Prove that $\overline{\text{OP}} + \overline{\text{QR}} = \overline{\text{OR}}$.

There is no space before or after the operation (plus) sign. There is a space before and after the comparison (equals) sign.

When the modifier is wider than the modified symbol, the print copy will insert extra space to clarify what exactly is being modified. In the next example, only the Sigma is modified, not the letter *a*. The space between the Sigma and the a clarifies the extent of the modifier, i < j. In braille, however, indicators define the extent of a modifier. There is no need for the space in braille.

⫸ $\sum_{i<j} a_{ij}$

Sigma is modified directly under with the inequality $i < j$.
The termination indicator signals the completion of the modifier.

PRACTICE 12D

Spacing with Modified Expressions

A. The probability of the event A, written P(A), is defined as

$$\text{P(A)} = \underset{\wedge}{\sum} f(x)$$

where $\underset{\wedge}{\sum} f(x)$ means sum $f(x)$ over those values x_i that are in A.

B. $\underset{\alpha \in \text{A}}{\times} \text{A}_\alpha$

C. $\prod_{i>j} (x_i - x_j)$

D. $\overline{7} + 2 \overset{?}{=} \overline{7+2}$

Modified Expressions and Superscripts / Subscripts

12.9 Modified Expression on the Baseline of Writing

A modifier and its related expression must be placed on the same level of writing.

12.9.1 **Superscript/Subscript After the Modification.** When a modifier affects only the expression and not the superscript or subscript, the modified expression terminates before the superscript (or subscript) is transcribed.

⫸ $\overline{P_1Q}^{\,2} + \overline{QP_2}^{\,2}$

⠐⠠⠏⠂⠠⠟⠣⠱⠻⠘⠆⠐⠬⠐⠠⠟⠠⠏⠆⠣⠱⠻⠘⠆

A termination indicator precedes each superscript.

⫸ $\bar{x}^2$ ⠭⠱⠘⠆

The contracted form of "bar over" does not have an explicit termination indicator. The superscript follows the "bar" symbol, unspaced.

Note that, when a numeric subscript follows a modified letter, a subscript indicator is required. This rule applies whether the letter is modified by the five-step rule or by the contracted form.

⫸ $\hat{\beta}_3$ ⠐⠨⠃⠣⠸⠣⠻⠰⠒

⫸ $\bar{\alpha}_2$ ⠨⠁⠱⠰⠆

⫸ $\underline{\mathrm{N}}_1$ ⠠⠝⠩⠱⠰⠂

12.9.2 **Superscript/Subscript Within the Modification.** When a modifier affects both an expression on the baseline of writing <u>and</u> its superscript or subscript, the baseline indicator is placed before the directly-over (or the directly-under) indicator. This assures that the expression as a whole appears on the same level of writing.

⫸ $\overline{x^2}$ ⠐⠭⠘⠆⠐⠣⠱⠻

The five-step method is used because the modified expression (x^2) is not a single letter. The first dot 5 begins the modified expression. The second dot 5 is a baseline indicator following the superscript "2".

⋙ $\overline{\mathrm{OD}^2} + \overline{\mathrm{OP}^2}$

⠐⠠⠕⠠⠙⠘⠆⠐⠣⠱⠻⠬⠐⠠⠕⠠⠏⠘⠆⠐⠣⠱⠻

The first dot 5 begins the first modified expression. The second dot 5 is a baseline indicator following the first superscript "2". Similarly, the third dot 5 begins the second modified expression and the fourth dot 5 is a baseline indicator following the second superscript.

⋙ $\overline{x_1}$ ⠐⠭⠂⠣⠱⠻

You may wish to review Section 6.11 regarding nonuse of the subscript indicator.

a. **Binomial Coefficient.** Although a binomial coefficient is not technically a modified expression, notice how this rule applies.

⋙ $\binom{a_x}{b_y}$ ⠷⠁⠰⠭⠐⠩⠃⠰⠽⠐⠾

The baseline indicator precedes the "directly under" indicator. This keeps that indicator on the same level of writing as the letters "a" and "b".

PRACTICE 12E

Superscripts and Subscripts

A) $\overline{\mathrm{AB}}^2 + \overline{\mathrm{BC}}^2$

B) $\overline{\mathrm{A}} = [\,\overline{a}_i]$

C) $\sqrt{\ddot{x}^2 + \ddot{y}^2}$

D) $\underline{\mathrm{Z}}^\circ$

E) If $\overline{a}_1 = 72$, find $\overline{a}_7$.

F) Draw $\overrightarrow{\mathrm{P}_1\mathrm{P}_2}$ if P_1 is the point $(1, 3)$ and P_2 is the point $(2, \text{-}1)$.

G) $\left(\overline{3^{-1}}\right) \in \mathrm{P}$

H) $\overline{x_1} + \overline{y_1}$

12.9.3 **Modified Expression on the Baseline That Follows a Superscript or a Subscript.** When a modified expression written on the baseline of writing immediately follows a superscript or a subscript, several details must be considered in order to determine the necessity of level indicators. Because braille dot 5 has several meanings besides that of baseline indicator (hence the name "multipurpose indicator") mindful use of that symbol is required in order to give the reader the correct information. Several examples will illustrate. Each dot-5 is highlighted in the examples.

a. If a level indicator is needed for the superscript or subscript, the baseline indicator is transcribed before starting the modified expression.

⫸ $x^2\overline{y}$ ⠭⠘⠆⠐⠽⠱

This dot 5 is a baseline indicator, following the superscript 2.

⫸ $\overline{x}_n\overline{z}_m$ ⠭⠱⠰⠝⠐⠵⠱⠰⠍

This dot 5 is a baseline indicator, following the subscript n.

b. If a subscript indicator is not needed, a baseline indicator is not needed before starting the modified expression.

⫸ $x_1\overline{z}_1$ ⠭⠂⠵⠱⠰⠂

Each 1 is printed at the subscript level.There are no dot 5s in this example because a baseline indicator is not needed when a subscript indicator is not used, and the contracted form of the "bar over" does not use a dot 5.

⫸ $x_1\overline{z_1}$ ⠭⠂⠐⠵⠂⠣⠱⠻

Each 1 is printed at the subscript level. A baseline indicator is not needed when a subscript indicator is not used. This dot 5 is the start of the modified expression.

c. Two indicators may be needed. First, a baseline indicator (dot 5) is used to terminate the effect of the superscript or subscript level indicator. Next, a multipurpose indicator (dot 5) is required to begin the five-step modification.

⫸ $3x^2\overline{\Delta x} + 3x\overline{\Delta x}^{\,2}$

⠼⠒⠭⠘⠆⠐⠐⠨⠠⠙⠭⠣⠱⠻⠬⠒⠭⠐⠨⠠⠙⠭⠣⠱⠻⠘⠆

Because x^2 requires a superscript indicator and the five-step rule is used for the modified expression, two dot 5s are needed. The baseline *indicator is immediately followed by the dot 5 which begins the modified expression.*

⪢ $a_k\overline{b_1 \ldots b_p}$

Because a_k requires a subscript indicator and the five-step rule is used for the modified expression, two dot 5s are needed. The baseline indicator is immediately followed by a dot 5 beginning the modified expression. Finally, the directly-over indicator is preceded by a baseline indicator to assure that the entire modified expression is transcribed on the same level.

⪢ $a_i \prod_{j \neq i} (A - r_j I)$

Because a_i requires a subscript indicator and the five-step rule is used for the modified expression, two dot 5s are needed. The baseline indicator is immediately followed by a dot 5 beginning the modified expression. The last dot 5 is a baseline indicator following the subscript "j".

12.10 Modified Expression Within a Superscript or Subscript

Recall that a modifier and its related expression must be placed on the same level of writing. When a modified expression occurs as a superscript or subscript, or as the first part of a superscript or subscript, the level indicator is transcribed first, followed by the multipurpose indicator which begins the modified expression. This assures that the expression as a whole appears on the same level of writing.

⪢ $S^{\tilde{x}}$

This dot 5 begins the modified expression $\tilde{x}$ (which is in the superscript position). It will not be misread as a baseline indicator because nothing comes between it and the superscript indicator.

⪢ $S_{\tilde{x}}$

This dot 5 begins the modified expression $\tilde{x}$ (which is in the subscript position). It will not be misread as a baseline indicator because nothing comes between it and the subscript indicator.

⪢ ${}_{\tilde{x}}A_1$

The first dot 5 begins the modified expression $\tilde{x}$ (which is a left subscript to the letter A). The second dot 5 is a baseline indicator.

If the modified expression occurs in the middle or at the end of the superscript or subscript, the appropriate level indicator must be repeated before the multipurpose indicator to show continuation of the same level of writing. This assures that the multipurpose indicator will not be misread as a baseline indicator.

⫸ $S_{\tilde{x}+\tilde{y}}$ ⠠⠎⠰⠐⠭⠣⠈⠱⠻⠬⠰⠐⠽⠣⠈⠱⠻

The first dot 5 begins the modified expression $\tilde{x}$. The second dot 5 begins the modified expression $\tilde{y}$. To assure that the second dot 5 is not read as a baseline indicator, the subscript level is restated before the dot 5.

Since the multipurpose indicator is absent in the contracted form of "bar over" or "bar under", the level continues with certainty. It is not necessary to repeat the level indicator when the contracted form is used.

⫸ $e^{a\overline{x}}$ ⠑⠘⠁⠭⠱

PRACTICE 12F

More About Superscripts and Subscripts

(1) $a_0\overline{x}^{\,n} + a_1\overline{x}^{\,n-1}$

(2) $W = \frac{2}{3}\pi r^3 \underline{w}\left(h + \frac{3}{8}r\right)$

(3) $\mathrm{S}^{\tilde{\mathrm{x}}}$ and $\mathrm{S}^{\tilde{\mathrm{x}}+\tilde{\mathrm{y}}}$

(4) $\mathrm{D}_{\overline{\mathrm{x}}}$ or $\mathrm{D}_{\overline{\mathrm{x}}+\overline{y}}$

(5) $3_{\overline{x}} - 2_{\overline{x}}$

(6) ${}_{\overline{n}}A_{\,1}$

Modified Signs of Comparison

12.11 Definition

A modified sign of comparison consists of a simple sign of comparison, such as the equals sign or the tilde, modified by a caret, dot, triangle, question mark, vertical bar, or any symbol <u>except</u> another sign of comparison.

> *When a simple sign of comparison occurs above or below another simple sign of comparison the combination is transcribed as a sign of comparison compounded vertically. See Section 5.8 for a review of that construction. Note that many of those signs are printed with a single horizontal line "bar over" or "bar under". For example, "bar over greater than, inclusion with bar under, bar over single tilde, logical sum with bar under," etc. These signs are not to be misinterpreted as a horizontal bar modifying a sign of comparison.*

12.12 Transcription

A modified sign of comparison as defined above is transcribed in accordance with the five-step rule for modified expressions.

In addition to the caret and inverted caret seen earlier in this lesson, you may also encounter a left- or right-pointing caret in a modified sign of comparison. Do not confuse these two symbols with the "less than" and "greater than" comparison signs. Ask an expert if context does not clarify the identity of this symbol.

⠰⠣	Left-Pointing Caret	<
⠰⠩	Right-Pointing Caret	>

The following list contains the modified equals signs most commonly used.

Modified Equals Sign		
⠐⠨⠅⠣⠸⠣⠻	Caret Over Equals Sign	$\overset{\wedge}{=}$
⠐⠨⠅⠩⠸⠣⠻	Caret Under Equals Sign ("is projective to")	$\underset{\wedge}{=}$
⠐⠨⠅⠣⠸⠩⠻	Inverted Caret Over Equals Sign	$\overset{\vee}{=}$
⠐⠨⠅⠣⠰⠣⠻	Left-Pointing Caret Over Equals Sign	$\leqq$
⠐⠨⠅⠣⠰⠩⠻	Right-Pointing Caret Over Equals Sign	$\geqq$

⠐⠨⠅⠣⠡⠻	Dot Over Equals Sign ("is approximately equal to")	$\doteq$
⠐⠨⠅⠩⠡⠣⠡⠻	Dot Over and Dot Under Equals Sign	$\overset{\cdot}{\underset{\cdot}{=}}$
⠐⠨⠅⠩⠡⠡⠣⠡⠡⠻	Two Dots Over and Two Dots Under Equals Sign	$\overset{\cdot\cdot}{\underset{\cdot\cdot}{=}}$
⠐⠨⠅⠣⠨⠡⠻	Hollow Dot Over Equals Sign ("is equal in degrees to")	$\overset{\circ}{=}$
⠐⠨⠅⠣⠫⠞⠻	Equilateral Triangle Over Equals Sign	$\triangleq$
⠐⠨⠅⠣⠸⠦⠻	Question Mark Over Equals Sign	$\overset{?}{=}$
⠐⠨⠅⠣⠳⠻	Vertical Bar Over Equals Sign	$\overset{\vert}{=}$

⫸ $\sqrt{5} \doteq 2.236$

⠜⠢⠻ ⠐⠨⠅⠣⠡⠻ ⠼⠆⠨⠆⠒⠖

⫸ $5(3) + 2(-2) \overset{?}{=} 11$

⠼⠢⠷⠒⠾⠬⠆⠷⠤⠆⠾ ⠐⠨⠅⠣⠸⠦⠻ ⠼⠂⠂

If a symbol is encountered which does not appear on this list, construct a symbol in accordance with the same principles. A transcriber-created symbol should be listed on the Special Symbols page.

⫸ $A \overset{m}{=} B$ ⠠⠁ ⠐⠨⠅⠣⠍⠻ ⠠⠃

The remainder of the list provided in the code contains other modified comparison signs most commonly used.

Modified Tilde		
⠐⠈⠱⠩⠡⠻	Dot Under Tilde	$\underset{\bullet}{\sim}$
⠐⠈⠱⠣⠡⠻	Dot Over Tilde	$\overset{\bullet}{\sim}$
Modified Horizontal Bar		
⠐⠱⠣⠸⠣⠻	Caret Over Bar	$\overset{\wedge}{-}$
⠐⠱⠩⠸⠣⠻	Caret Under Bar ("is perspective to")	$\overline{\wedge}$
⠐⠱⠩⠡⠻	Dot under Bar	$\underset{\bullet}{-}$

If the horizontal bar is modified by a dot *over* it, the combination is a modified sign of operation ("minus with dot over" signifying "proper difference"). The five-step rule is not used for this symbol. See Section 5.4.7.

Instructions: Use the five-step rule to show the horizontal grouping sign in the last item.

PRACTICE 12G

Modified Signs of Comparison and More

1. $A \doteq 3.14r^2$
2. $\angle b \overset{\circ}{=} \frac{1}{2}\overset{\frown}{\mathrm{EB}}$
3. The symbol $\triangleq$ is used to make a definition.
4. $x \dot{\sim} \mathcal{N}(0,1)$
5. $x^n = \underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}$

Expressions with More Than One Modifier

12.13 Modifiers of Higher Order

When two or more modifiers occur one above the other and apply to exactly the same expression, the second, third, etc. modifiers are "modifiers of higher order." In such cases, the directly-over indicator or the directly-under indicator is doubled, tripled, etc., before each modifier to indicate its position. The termination indicator is used only after the last modifier shown.

```
Directly-over indicators
     ⠣⠣       Second order
     ⠣⠣⠣      Third order
Directly-under indicators
     ⠩⠩       Second order
     ⠩⠩⠩      Third order
⠻  Termination indicator
```

⫸ $$\overset{a=3}{\overline{x+y}}$$

⠐⠭⠬⠽⠣⠱⠣⠣⠁ ⠨⠅ ⠼⠒⠻

Analysis: The expression x + y is modified directly over with a horizontal bar which, in turn, is modified directly over with the expression a = 3.

⫸ $$\underset{\substack{a=3\\b=2}}{\underline{x+y}}$$

⠐⠭⠬⠽⠩⠱⠩⠩⠁ ⠨⠅ ⠼⠒⠩⠩⠩⠃ ⠨⠅ ⠼⠆⠻

Analysis: The expression x + y is modified directly under with a horizontal bar which, in turn, is modified directly under with the expression a = 3 which is modified directly under with the expression b = 2.

⫸ $\sum\limits_{\substack{i,j,k \\ i<j<k}}$

Analysis: The Sigma is modified directly under with letters i, j, k. That, in turn, is modified directly under with the expression i < j < k. Review the rules regarding use of the English-letter indicator in Lesson 3. An English-letter indicator is needed for the first letter "j" because it is a "single letter" according to the definition in the Nemeth Code. The other letters are either touching indicators or are preceded or followed by a sign of comparison, so an English-letter indicator is not needed.

12.13.1 **Parallel Horizontal Bars.** When two or more parallel horizontal bars are the same length and apply to <u>exactly</u> the same expression, they are treated as a single modifier. In such cases, the directly-over or the directly-under indicator is used only once, and the symbol for the bar is used as many times as necessary to correspond to the print text.

⫸ $\overline{\overline{9}}$

⫸ $(\overline{\overline{\mathrm{A}}})$

⫸ $\underline{\underline{\underline{x}}}$

Attention: Two parallel horizontal bars must not be misinterpreted as the equals sign, and three parallel horizontal bars must not be misinterpreted as the identity sign. The symbol's meaning can be determined by reading the surrounding text for context. See Section 5.8 ("Signs of Comparison Compounded Vertically") to review equals sign printed above or below a sign of comparison.

12.14 Individual Modifiers

When two or more modifiers do not apply to exactly the same symbols but cover different portions of the same expression, the longer modification encloses the entire modified expression. Within the long expression, each inner expression is modified individually.

⫸ $\overline{\overline{\overline{\overline{x}} \times \overline{\overline{x}}}}$

Analysis: In the simulated braille below, the longer modification indicators and symbols are highlighted; the inner modified expressions are underlined.

The five-step rule and the contracted form of transcription may be used together.

⫸ $\overline{(\bar{\text{a}}\text{A} + \bar{\text{c}}\text{C})}$

Analysis: In the simulated braille below, the longer modification indicators and symbols are highlighted; the inner modified expressions are underlined.

⫸ $\overline{P_{\left(\overline{x^2}\right)}}$

Analysis: In the simulated braille below, the longer modification indicators and symbols are highlighted; the inner modified expression is underlined.

Reminder: The subscript level indicator must be repeated before the modified expression indicator so the dot 5 is not misread as a baseline indicator.

12.15 Simultaneous Modifiers

When an expression is simultaneously modified both above and below, the modifier *below* is transcribed first and the modifier *above* is transcribed second. The termination indicator is used only at the end of the entire modification.

⫸ $\sum\limits_{3}^{7}$

⫸ $\prod\limits_{k=2}^{6} a_k$

Recall from 12.8 that, when the modifier is wider than the modified symbol (in this case, $k = 2$ is wider than Π), the print copy might insert extra space to clarify what exactly is being modified. In braille, the modified Pi is unspaced from the letter "a".

⫸ $\overline{\underline{x}}$

⫸ $\overline{\underline{x + y}}$

As you study the eight constructions in the next example, note the use or nonuse of the English-letter indicator and the use or nonuse of the numeric indicator. This reminder applies to the first item. *Reminder from Lesson 8: When a linked expression will not fit on one braille line, start a new line with the link (the comparison sign).*

Example 12-20

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k \qquad (1)$$

$$\sum_{k=1}^{n} (2k) = n(n+1) \qquad (2)$$

$$\sum_{i,j=1}^{n} a_{ij} x_i x_j \qquad (3)$$

$$\sum_{x_1=0}^{1} \sum_{x_2=0}^{1} \qquad (4)$$

$$\bigcup_{r=1}^{n} A_r = A_1 \cup A_2 \ldots \qquad (5)$$

$$\overline{\overline{\prod_{x\in a} c'x}} \qquad (6)$$

$$\underset{a=3}{\overset{b=2}{\overline{\underline{x+y}}}} \qquad (7)$$

$$\sum_{i=1}^{6} \overline{P_{i-1}P_i} \qquad (8)$$

⠷⠦⠾ ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠂⠣⠖⠻⠐⠠⠏⠰⠊⠤⠂⠐⠠⠏⠰⠊⠐⠣⠱⠻

PRACTICE 12H

Expressions with More Than One Modifier

1. $\overline{\overline{9}} \cdot \overline{\overline{3}} = \overline{\overline{9 \cdot 3}}$

2. $\overline{\overline{A^n}}$

3. $\overline{A \cap \overline{B \cap \overline{C}}}$

4. $\overline{a_{\bar{n}} + b_{\bar{p}}}$

5. $\overline{x_{\bar{a}} + y^{\bar{n}}}$

6. $\overline{\overline{\underline{\mathrm{N}}}}$

7. $\sum_{i=1}^{k} \sum_{j=1}^{k}$

8. $\prod_{\substack{j=1 \\ j \neq k}}^{n}$

12.16 Formal Proof [NC Rule 26.7]

A proof is a valid argument that establishes the truth of a mathematical statement. It is often introduced by a heading such as *Theorem*, *Proposition*, or *Lemma*. A progressive sequence of statements leads to the conclusion. In a formal proof, every step of the argument is shown and each step is supported by a definition or by a previously proven statement.

Lesson 11 explained how to transcribe a mathematical statement. Those guidelines are summarized in steps a-c, below, in the context of a proof.

a. A blank line precedes the beginning of the proof.

b. *Heading:* The heading can be formatted as a paragraph heading or as a cell-5 or cell-7 heading, at the transcriber's discretion. Review Section 11.38.b for details.

c. *Statement:* Continue with the text, using normal (3-1) paragraph style. When the statement is printed in a variant typeform and the proof follows, in regular type, it is recommended that the typeform be preserved for the statement in order to retain distinction.

d. *Auxiliary Captions:* Paragraph headings such as Given, Hypothesis, Prove, or Conclusion begin in cell 3, without a blank line line before the paragraph. Associated material follows the caption. Runovers go in cell 1.

e. *Two-Column Proof:* See 12.16.1, below.

f. *End of Proof Icon:* See 12.16.2, below.

g. When the proof is complete, insert a blank line before continuing with the text.

12.16.1 **Two-Column Proof.** When a formal proof is presented by numbered steps printed in two columns, the layout is changed as follows.

a. If there is a caption such as "Proof", follow the same pattern established in step d, above ("Auxiliary Captions").

b. The column format is changed to a list in braille. A transcriber's note must call attention to the change in format. See step c, below, for a sample transcriber's note.

A blank line is inserted before the list. If there are column headings, such as "Statement" and "Reason", see step c, below. Each step begins in cell 1, starting with the first item from the left column. Runovers are in cell 3. The related item from the right column begins in cell 1 on the next line, with runovers in cell 3.

Each item must be labeled with an identifier as described below.

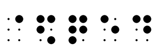
c. *Step Identifiers:* Typically, the print copy includes the column headings "Statements" and "Reasons". In braille, the column headings are replaced with a letter – "S" or "R", respectively – as part of each step number. For example, "1S" is Statement 1, followed on the next line by "1R" which is Reason 1. If other column headings appear in print, the transcriber chooses an appropriate letter to represent each heading. If no column headings appear in print, use "S" for the left-column items and "R" for the right-column items.

If the steps are not numbered in print, identifiers are added in the manner stated above. A transcriber's note must specify the meaning of the letter labels. Any transcriber-added numbers or letters must also be explained.

Sample transcriber's note: "Proofs printed with steps in columns headed *Statements* and *Reasons* are transcribed with an S or R immediately following the step number to show the column in which the step appears. Each step from the *Statements* column is immediately followed by the corresponding step from the *Reasons* column."

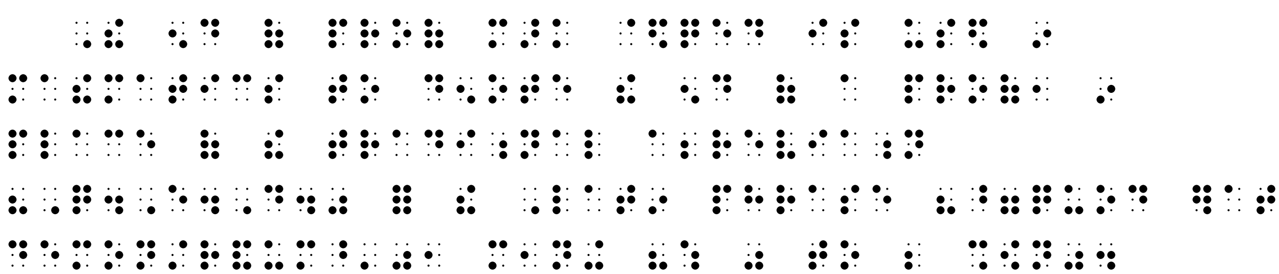
Identifiers are transcribed in either code (UEB or Nemeth) according to code-switching guidelines.

12.16.2 **End of Proof Icon.** Sometimes a shape (often a bold rectangle or square) or the Latin abbreviation "Q.E.D." is used to denote the end of a proof. The shape is often printed at the right margin and may go unnoticed. Watch for it. Regardless of the shape or abbreviation used in print, the icon shown below is used in braille, and is transcribed following the last word or character in the proof, preceded by one blank cell. The icon is created using the UEB transcriber-defined shape indicator and may be used in either Nemeth Code or UEB without switching. The icon must be listed on the Special Symbols page.

	`End of Proof Icon`	*for example,* ▌ *or* ■

Example 12-21

The end of proof mark ▌ is used in mathematics to denote the end of a proof, in place of the traditional abbreviation "Q.E.D." for the Latin phrase "**quod erat demonstrandum**", meaning "which was to be shown".



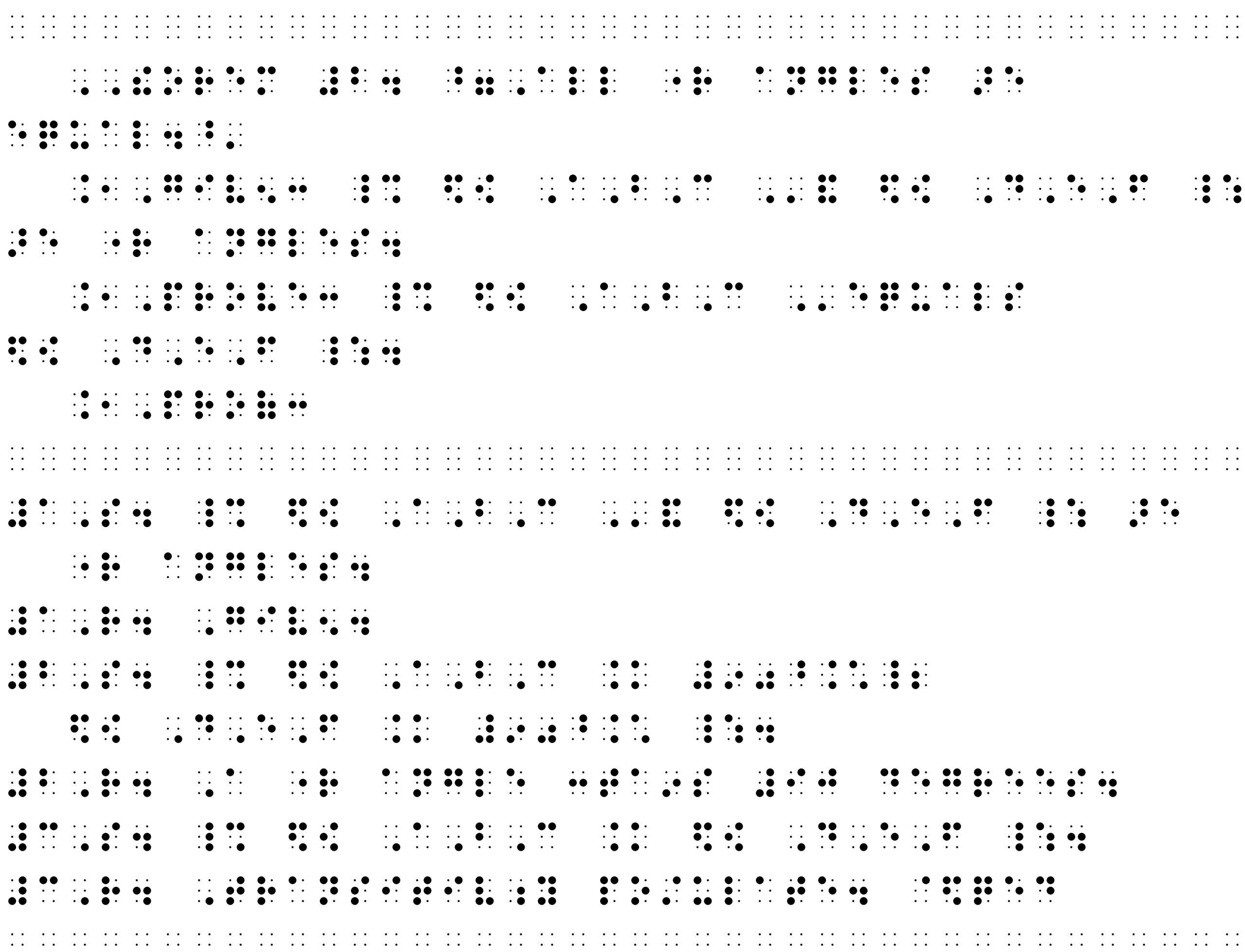
Contractions are used in the Latin words according to Braille Formats.

Example 12-22

(Assume the required transcriber's note regarding the two-column proof appears on the Transcriber's Notes page.)

THEOREM 2. All right angles are equal.
Given: ∠ ABC and ∠ DEF are right angles.
Prove: ∠ABC equals ∠DEF.
Proof:

Statements	Reasons
1. ∠ ABC and ∠ DEF are right angles.	1. Given.
2. $\angle ABC = 90^\circ$, $\angle DEF = 90^\circ$.	2. A right angle contains 90 degrees.
3. $\angle ABC = \angle DEF$.	3. Transitivity postulate. ■



Line 7: A blank line precedes the proof.
Line 8: The heading is printed in capital letters and also in a nonregular typeface (boldface). When a paragraph heading is printed in capital letters, typeform is disregarded.
Lines 8-9: The paragraph style is 3-1. Distinctive typeform (boldface) is retained in the statement.
Lines 10-11, 12-13, 14: Each auxiliary caption follows print regarding typeform (italics, in this example), and uses a 3-1 paragraph style.
Line 15: A blank line precedes the list.
Lines 16-23: Each item in the 2-column proof begins in cell 1, with runovers in cell 3. Identifying letters S and R are combined with each step number.
Line 23: A dark square is printed in the right margin to mark the end of the proof. The "qed" icon is transcribed.
Line 24: A blank line follows the proof.

PRACTICE 12I

Instructions: Create a Transcriber's Notes page that might appear in a volume which contains the proof shown in Practice 12J. A statement citing the code book is required in every volume that uses the Nemeth Code. In the first paragraph, state the title and edition of the Nemeth code book as well as any Updates. Something like this:

> Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 2022.*

In the second paragraph, explain the step-number format as described in 12.16.1.c. Refer to *Braille Formats* for further guidelines regarding the structure of a Transcriber's Notes page.

PRACTICE 12J

Given: $3x = 7 - \frac{1}{2}x$

To Prove: $x = 2$

STEP	REASON
1. $3x - 7 - \frac{1}{2}x$	1. GIVEN
2. $6x = 14 - x$	2. Multiplication Property
3. $7x = 14$	3. Addition Property
4. $x = 2$	4. Division Property ▮

For further practice, see Appendix A—Reading Practice.

EXERCISE 12

Prepare Exercise 12 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 12A

PRACTICE 12B

PRACTICE 12C

⠼⠁⠲ ⠠⠔ ⠠⠋⠊⠛⠥⠗⠑ ⠼⠛⠲⠉⠂ ⠊⠋
  ⠸⠩ ⠐⠠⠁⠠⠃⠣⠫⠁⠻ ⠸⠅ ⠐⠠⠉⠠⠙⠣⠫⠁⠻ ⠸⠱ ⠔ ⠉⠊⠗⠉⠇⠑
  ⠠⠕⠂ ⠮⠝ ⠸⠩ ⠫⠪ ⠠⠁⠠⠕⠠⠃ ⠨⠅ ⠫⠪ ⠠⠃⠠⠕⠠⠉⠸⠲
⠼⠆⠸⠲ ⠭ ⠐⠨⠅⠣⠸⠦⠻ ⠽ ⠸⠱ ⠍⠂⠝⠎ ⠦⠙⠕⠑⠎ ⠰⠭ ⠑⠟⠥⠁⠇
  ⠰⠽⠦⠴
⠼⠉⠲ ⠸⠩ ⠐⠨⠠⠎⠩⠙⠳⠝⠻ ⠠⠄⠐⠣⠐⠱ ⠙⠳⠝ ⠸⠱ ⠍⠂⠝⠎ ⠦⠰⠙
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⠼⠢⠸⠲ ⠷⠝⠩⠅⠾ ⠨⠅ ⠷⠝⠤⠂⠩⠅⠤⠂⠾⠬⠷⠝⠤⠂⠩⠅⠾ ⠸⠱ ⠿ ⠁⠇⠇
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  ⠰⠃⠸⠲ ⠼⠆⠨⠲⠒⠂⠐⠒⠂⠣⠡⠻
  ⠰⠉⠸⠲ ⠐⠭⠣⠡⠻⠐⠽⠣⠡⠡⠻⠤⠐⠽⠣⠡⠻⠐⠭⠣⠡⠡⠻
  ⠰⠙⠸⠲ ⠐⠁⠩⠡⠻⠬⠐⠁⠩⠡⠡⠡⠻ ⠨⠅ ⠿
  ⠰⠑⠸⠲ ⠋ ⠫⠕ ⠐⠋⠣⠈⠱⠻
  ⠰⠋⠸⠲ ⠐⠭⠬⠽⠣⠈⠱⠻ ⠸⠱

PRACTICE 12D

PRACTICE 12E

PRACTICE 12F

⠀⠀⠀⠀⠠⠍ ⠠⠁⠃ ⠠⠎⠥⠏⠻⠎⠉⠗⠊⠏⠞⠎ ⠯ ⠠⠎⠥⠃⠎⠉⠗⠊⠏⠞⠎

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⠷⠲⠾ ⠠⠙⠰⠭⠱ ⠠⠄⠕⠗ ⠠⠙⠰⠭⠱⠼⠽⠱
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⠷⠖⠾ ⠰⠝⠱⠐⠠⠁⠂ ⠸⠱

PRACTICE 12G

⠀⠀⠀⠀⠠⠍⠕⠙⠊⠋⠊⠫ ⠠⠎⠊⠛⠝⠎ ⠯ ⠠⠉⠕⠍⠏⠜⠊⠎⠕⠝ ⠯ ⠠⠍

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⠀⠀⠍⠁⠅⠑ ⠁ ⠙⠑⠋⠔⠊⠰⠝⠲
⠼⠙⠲ ⠸⠩ ⠭ ⠐⠈⠱⠣⠡⠻ ⠈⠰⠠⠝⠷⠴⠠ ⠂⠾
⠼⠢⠸⠲ ⠭⠘⠝ ⠸⠱ ⠂⠭⠡⠭⠡⠭⠡ ⠄⠄⠄ ⠡⠭⠩⠨⠾⠻ ⠸⠱

PRACTICE 12H

⠠⠑⠭⠏⠗⠑⠎⠨⠝⠎ ⠾ ⠠⠍ ⠠⠹⠁⠝ ⠠⠐⠕ ⠠⠍⠕⠙⠊⠋⠊⠻

⠀⠀

⠸⠩

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⠼⠢⠸⠲ ⠐⠭⠰⠁⠱⠐⠬⠽⠘⠝⠱⠐⠣⠱⠻

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⠼⠦⠸⠲ ⠐⠨⠠⠏⠩⠚ ⠨⠅ ⠼⠂⠩⠩⠚ ⠌⠨⠅ ⠅⠣⠝⠻ ⠸⠱

PRACTICE 12I

PRACTICE 12J

LESSON 13

- MISCELLANEOUS SYMBOLS
    - Unspaced Symbols
    - Spaced Symbols
    - Spacing with the Angstrom Unit and Tally Marks
- SUPERPOSED SIGNS
- AMBIGUOUS SIGNS
- MULTIPURPOSE INDICATOR
- REFERENCE SIGNS AND SYMBOLS

Answers to Practice Material

LESSON PREVIEW

Symbols not previously covered are collected in this lesson. Spacing rules differ among the symbols; the spacing rules are grouped accordingly. Signs printed one atop another are examined. Several look-alike print signs are compared. After a review of the multipurpose indicator, four more uses of this indicator are explored. Reference signs in Nemeth context are discussed.

MISCELLANEOUS SYMBOLS

[NC Rule 23]

In this lesson, symbols are grouped according to spacing rules: unspaced, spaced, and those with special rules. Within each section, the symbols are presented in alphabetical order. See Lesson 5 regarding spacing with the crosshatch symbol and the diagonal slash.

Unspaced Symbols

13.1 Spacing Rules for Unspaced Symbols

No space is left between the symbols listed below and any other symbol or quantity to which they apply, regardless of print spacing. However, a space must be left between these symbols and a word, an abbreviation, a sign of comparison, or other symbol which specifically requires a space before or after it.

13.1.1 **Caret**

⠸⠣	`Caret`	∧

The caret was first seen in Lesson 10 in the context of a long division problem. Used as a place indicator, the caret is treated as a numeric symbol.

⋙ $.37_{\wedge}688$ ⠼⠨⠒⠶⠸⠣⠖⠦⠦

The caret was also encountered in Lesson 12, as a modifier.

⋙ $\hat{\mathbf{k}}$ ⠐⠸⠰⠅⠣⠸⠣⠻

Example 13-1

A caret (∧) shows the place to which the decimal is moved: $.37_{\wedge}688$

⠠⠁ ⠉⠜⠑⠞ ⠸⠩ ⠷⠸⠣⠾ ⠸⠱ ⠩⠪⠎ ⠮ ⠏⠇⠁⠉⠑ ⠞⠕ ⠱ ⠮
⠙⠑⠉⠊⠍⠁⠇ ⠊⠎ ⠍⠕⠧⠫⠒ ⠸⠩ ⠼⠨⠒⠶⠸⠣⠖⠦⠦ ⠸⠱

a. **Use of the UEB Caret.** The caret may be transcribed more than one way within a document. The distinction is based on meaning. In mathematical context, use the Nemeth caret. In nontechnical context, use the UEB caret ⠈⠢ to indicate the insertion of literary material, or the UEB circumflex ⠘⠩ to indicate a modified letter or accent. In UEB context, follow UEB rules for spacing of the caret or circumflex symbol.

13.1.2 **Crossed Letters**

Crossed h	ħ
Crossed lambda	ƛ
Crossed R	Ɍ

⫸ $2\pi\hbar v = \phi e$

⫸ $3ƛ \ldots nƛ$

⫸ $n = 3Ɍ$

13.1.3 **Del**

Del (nabla, gradient)	$\bigtriangledown$

When the inverted triangle is used as a sign of omission, it is spaced according to the rules presented in Section 11.27. In all other cases it is spaced in accordance with 13.1.

⫸ $s \bigtriangledown t + t \bigtriangledown s$

⫸ $\bigtriangledown (rn)$

Example 13-2

The symbol "$\bigtriangledown$" is called "del" as in $\mathbf{T} \cdot \bigtriangledown (r_1 + r_2) = 0$.

13.1.4 **Differentials and the Partial Derivative Symbol (Round d).**

Differentials are commonly notated as dx and dy. You may also encounter dt, du, dv, dz, etc. The letter d may be printed in regular or italic type. The italic typeform is disregarded in the braille transcription. In print, it is quite common that dx and dy are spaced within a mathematical expression. In braille, the space is omitted.

⫸ $(1 + 4xy)\mathrm{d}x\ \mathrm{d}y$

The partial derivative sign is represented by the following symbol. Note that this is <u>not</u> the letter "d" in script type.

Partial Derivative	∂

⋙ $\dfrac{\partial f}{\partial x}$

⋙ $\partial/\partial y(\partial b/\partial x)$

⋙ $\dfrac{\partial h}{\partial u} = \dfrac{\partial f}{\partial x}\,\dfrac{\partial x}{\partial u}$

Example 13-3

Geometry The partial derivative of V with respect to r is $\dfrac{\partial V}{\partial r} = \dfrac{2\pi r h}{3}$.

13.1.5 Empty Set (null set, void set)

The print symbols ∅ and Φ used to denote the empty set must not be mistaken for the Greek uncapitalized phi (φ or ϕ) which they resemble. When facing braces are used to denote the empty set, one space is left between the braces.

	Empty Set (null set, void set) Represented by Zero with a Slanted or Vertical Bar through it	∅ or Φ
	Empty Set Represented by Facing Braces	{ }

⋙ $A \cap \emptyset = \emptyset$

⋙ $\emptyset \subseteq A$

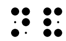
⋙ $S \cup \{\ \}$

Example 13-4

The solution set ∅ is written $R = \emptyset$ or $R = \{\ \}$.

13.1.6 **Factorial Sign**

	Factorial Sign	!

⋙ $n!$ 

⋙ ${}_5C_1 = \frac{5!}{1!4!}n!$ 

Example 13-5

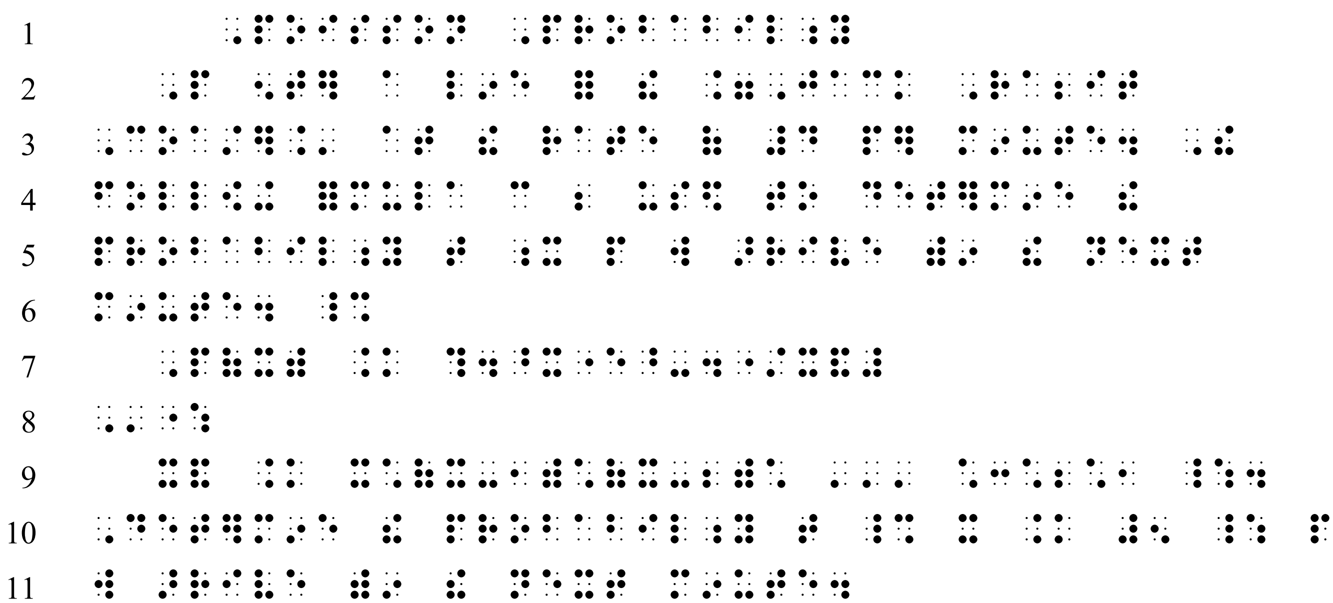
Poisson Probability People enter a line for the *Jack Rabbit Coaster* at the rate of 4 per minute. The following formula can be used to determine the probability that x people will arrive within the next minute.

$$P(x) = \frac{4^x e^{-4}}{x!}$$

where

$$x! = x \cdot (x-1) \cdot (x-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1.$$

Determine the probability that $x = 5$ people will arrive within the next minute.



1

Line 1: The paragraph heading is formatted as a cell-5 heading. Typeform (boldface) is disregarded in a cell-5 heading.
Line 2: The paragraph begins in cell 3, following the rules of the Nemeth Code.
Lines 7 and 9: Each displayed math expression begins in cell 3.
Lines 8 and 10: The narrative continues in the runover cell for paragraphs, cell 1.
Line 9: The last sentence can be interpreted either as a continuation of the same paragraph or as a new paragraph in which case it would begin in cell 3.

13.1.7 **Infinity**

⠠⠿	`Infinity`	∞

- ⫸ $a-(+\infty)=-\infty$ ⠁⠤⠷⠬⠠⠿⠾ ⠨⠅ ⠤⠠⠿
- ⫸ $n \to \infty$ ⠝ ⠫⠕ ⠠⠿
- ⫸ 1^{∞} ⠼⠂⠘⠠⠿
- ⫸ $\sum_{n=-\infty}^{\infty} f(n)$ ⠐⠨⠠⠎⠩⠝ ⠨⠅ ⠤⠠⠿⠣⠠⠿⠻⠋⠷⠝⠾

Example 13-6

Interval	Inequality
(a, +∞)	x > a
[a, +∞)	x > a
(−∞, a)	x < a
(−∞, a]	x < a

⠠⠔⠞⠻⠧⠁⠇ ⠠⠔⠑⠟⠥⠁⠇⠰⠽
⠐⠒⠒⠒⠒⠒⠒⠒⠒ ⠐⠒⠒⠒⠒⠒⠒⠒⠒
⠸⠩
⠷⠁⠠ ⠬⠠⠿⠾ ⠭ ⠨⠂ ⠁
⠈⠷⠁⠠ ⠬⠠⠿⠾ ⠭ ⠨⠂⠱ ⠁
⠷⠤⠠⠿⠠ ⠁⠾ ⠭ ⠐⠅ ⠁
⠷⠤⠠⠿⠠ ⠁⠈⠾ ⠭ ⠐⠅⠱ ⠁
⠸⠱

13.1.8 **Integrals**

⠮	Single Integral	$\int$
⠮⠮	Double Integral	$\iint$
⠮⠮⠮	Triple Integral	$\iiint$

⫸ $\int(\mathrm{d}u + \mathrm{d}v) = \int \mathrm{d}u + \int \mathrm{d}v$

⠮⠷⠙⠥⠬⠙⠧⠾ ⠨⠅ ⠮⠙⠥⠬⠮⠙⠧

⫸ $V = \iiint \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}y$ ⠠⠧ ⠨⠅ ⠮⠮⠮⠙⠭⠙⠽⠙⠽

Reminder: There are no spaces before or after the terms of the expression (dx and dy) even though they are spaced in print.

a. **Superscripts and Subscripts.** Superscripts and subscripts are printed just to the right of the integral symbol. Because the integral symbol is printed at a slant, the superscript and subscript do not exactly align, but they are considered to be simultaneous.The subscript is transcribed first. A subscript indicator is required for numeric subscripts.

⫸ $\int_1^3 \frac{\mathrm{d}x}{x^2-2x+5}$ ⠮⠰⠂⠘⠒⠐⠹⠙⠭⠌⠭⠘⠆⠐⠤⠆⠭⠬⠢⠼

⫸ $\int_1^2 \int_0^{x-1} y\ \mathrm{d}y\ \mathrm{d}x$ ⠮⠰⠂⠘⠆⠐⠮⠰⠴⠘⠭⠤⠂⠐⠽⠙⠽⠙⠭

b. **Modifiers.** Symbols printed directly above and/or directly below the integral symbol are transcribed according to the five-step rule for the transcription of modified expressions.

⫸ $\int\limits_0^\infty f(x)\,\mathrm{d}x$ ⠐⠮⠩⠴⠣⠠⠿⠻⠋⠷⠭⠾⠙⠭

The single integral sign is modified directly below with the numeral zero and directly above with an infinity symbol.

⫸ $\iint\limits_Q f(x, y)\,\mathrm{d}y\,\mathrm{d}x$ ⠐⠮⠮⠩⠠⠟⠻⠋⠷⠭⠠ ⠽⠾⠙⠽⠙⠭

The double integral sign is modified directly below with the letter "Q".

c. **Upper and Lower Integral Signs.** A horizontal bar directly over or under the integral sign is not a modifier—the bar is an essential part of the symbol. The symbols for the upper and lower integral signs are transcribed as follows.

	Upper Integral	$\overline{\int}$
	Lower Integral	$\underline{\int}$

⫸ $\overline{\int}_a^b f(x)\,\mathrm{d}x = 0$

⫸ $\underline{\int}_a^b f(x)\,\mathrm{d}x = 0$

13.1.9 **Quantifiers**

Existential Quantifier		
	There Exists, for Some	∃
	There Exists Uniquely for Exactly One	∃| or ∃!
Universal Quantifier		
	For All, For Each, For Every	∀

⫸ $\exists x,\ x < \frac{1}{n}$

⫸ $(\exists_x \in A)$

⫸ $(\forall_x)$

⫸ $\forall_x \forall_y - \frac{y-x}{x+y}$

Example 13-7

$\exists|v \mid v = -v$ means "there exists exactly one v such that v equals $-v$."

The first vertical bar is part of the Existential Quantifier symbol, and is unspaced. The second vertical bar is a sign of comparison meaning"such that" (understood from the narrative) and is spaced accordingly. Review Section 5.6.16.

Example 13-8

Assuming the domain of variable s is ℝ, is the following statement true?

$$(\forall_s)(s \cdot 0 = 0)$$

13.1.10 **Transcriber-Devised Symbols.** When a symbol is encountered that is not represented in the Nemeth Code, the transcriber may substitute another symbol as long as it is not being used with another meaning within the same subject matter. Another option is to invent a symbol for temporary use. A transcriber-devised symbol should be constructed in keeping with usage and according to the general rules of the Nemeth Code. In either case, the symbol must be explained in a transcriber's note.

a. **"Not-p".** One example of a symbol that is not represented in the Nemeth Code is the negation sign ¬ which is commonly used in the topic of logic. Since this particular print sign is not listed in the Nemeth Code, the transcriber must devise one. A little research reveals that UEB has a braille symbol for this print sign. Although UEB symbols cannot be used inside the Nemeth switches, it can be used here as a transcriber-devised Nemeth symbol. Checking Appendix B of the Nemeth Code, we find that this dot configuration has no other meaning in Nemeth. Sample transcriber's note:

> The symbol ⠈⠹ represents the negation symbol, which is printed as a small horizontal bar with a down-pointing end.

Example 13-9

The negation $\neg p$ is read as "it is not the case that p" or simply as "not p".

PRACTICE 13A

Unspaced Miscellaneous Symbols

$1.2_{\wedge}$

$(v > \phi\hbar)$

℞: 24 grams

$\|\nabla \mathrm{f}(a)\|$

$$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$$

$\{\ \} \cap \{\ \}$

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

$-\infty < x < \infty$

$f'(x) = 0$ or ∞

$$\int_a^b f(x)\,\mathrm{d}x = F(x)\Big]_a^b$$

$$\int\limits_{x=a}^{x=b} f(t)\,\mathrm{d}t$$

$(\exists x)(\exists y)[x + y = 85]$

$\exists|\mathrm{x}$

$\forall_x \ \in \ A$

Spaced Symbols

13.2 Spacing Rules for Spaced Symbols

A space is left *before and after* the symbols listed below. However, no space is left between these symbols and a mark which applies to them such as a sign of grouping, a braille indicator, or a punctuation mark.

13.2.1 "At" Sign in Mathematical Context

⠈⠁	At Sign	@

⫸ (@) ⠷⠈⠁⠾

Example 13-10

15 pencils @45¢ = \$6.75.

⠸⠩ ⠼⠂⠢ ⠏⠑⠝⠉⠊⠇⠎ ⠈⠁ ⠼⠲⠢⠈⠉ ⠨⠅ ⠈⠎⠖⠨⠶⠢ ⠸⠱⠲

In the print copy there is no space following the "at" symbol. In braille, this symbol is spaced. Switching Decision: The problem is not "45¢ = \$6.75" therefore "15 pencils @" is part of the problem and is transcribed in Nemeth.

13.2.2 **Check Mark.** The following symbol may be used within Nemeth context. A space is left before and after a single check mark. A sequence of two or more check marks is written unspaced, but the combination as a whole is preceded and followed by a space.

⠈⠜	Check Mark	✓

⫸ ✓ $1 + 2 = 3$ ⠈⠜ ⠼⠂⠬⠆ ⠨⠅ ⠼⠒

⫸ ✓✓ $2 + 2 = 4$ ⠈⠜⠈⠜ ⠼⠆⠬⠆ ⠨⠅ ⠼⠲

13.2.3 **Ditto Mark.** A ditto mark is centered beneath the material it duplicates. It is separated from any expression which precedes or follows it by at least one space. In Nemeth, the ditto mark shown below is used. For clarity, guide dots are not inserted in the row entries which end with a ditto mark.

⠠⠄	Ditto Mark	"

In UEB context, the UEB ditto mark is used, but Nemeth formatting is applied, centering the ditto under the word or item it duplicates.

Example 13-11

BTB	ACR	pH range
5.55×10^{-5}	2.57×10^{-11}	1.00-6.39
5.30 "	3.24 "	0.99-12.31
4.44 "	2.06 "	1.17-12.01

13.2.4 **Since (because)**

	Since (because)	$\because$

⫸ $\because x = y, x^2 = y^2$

⫸ $(\because)$ RS = RT

No space is left between the symbol and the grouping signs which apply to it.

13.2.5 **Therefore**

Therefore		
⠠⠡	Normal	∴
⠌⠠⠡	Negated (it does not follow that)	/∴

⫸ ∴ CM ⊥ AB ⠠⠡ ⠠⠉⠠⠍ ⠫⠏ ⠠⠁⠠⠃

⫸ /∴ R = S ⠌⠠⠡ ⠠⠗ ⠨⠅ ⠠⠎

Example 13-12

∴ the solution set is {±3}.

⠸⠩ ⠠⠡ ⠸⠱ ⠮ ⠎⠕⠇⠥⠰⠝ ⠎⠑⠞ ⠊⠎ ⠸⠩ ⠨⠷⠬⠤⠒⠨⠾ ⠸⠱⠲

PRACTICE 13B

Spaced Miscellaneous Symbols

1. Su bought 25 boxes of tissue for her classroom. Priced @99¢, can she pay with only one $20 bill?

   $25 \times \$.99 = \$24.75$

   ✓ $\$24.75 > \$20$

   Answer: No. Su needs more than $20 to buy the tissues.

2. $\because\ 8x + 3y = 15$, substituting 0 for x gives $8(0) + 3y = 15$, or $3y = 15$. $\therefore\ y = 5$.

Spacing with the Angstrom Unit and Tally Marks

13.3 Angstrom Unit

The angstrom unit is a mathematical symbol, but it is spaced according to the rules of abbreviations. A switch to Nemeth is required for this symbol.

⠈⠠⠁	Angstrom Unit	Å

⫸ 1Å ⠼⠂ ⠈⠠⠁

a. A space is left before and after the angstrom unit even when it is preceded or followed by a sign of operation.

Example 13-13

1Å + 2Å = 3Å

⠸⠩ ⠼⠂ ⠈⠠⠁ ⠬⠆ ⠈⠠⠁ ⠨⠅ ⠼⠒ ⠈⠠⠁ ⠸⠱

b. A space is <u>not</u> left between the angstrom unit and a sign of grouping, a horizontal or diagonal fraction line, a braille indicator, or a punctuation mark which applies to it.

Example 13-14

An angstrom unit (Å) equals 1×10^{-10} meter.

⠠⠁⠝ ⠁⠝⠛⠌⠗⠕⠍ ⠥⠝⠊⠞ ⠸⠩ ⠷⠈⠠⠁⠾ ⠠⠄⠑⠟⠥⠁⠇⠎
⠼⠂ ⠈⠡⠂⠴⠘⠤⠂⠴ ⠸⠱ ⠍⠑⠞⠻⠲

The angstrom unit is unspaced from the opening and closing parentheses.

Example 13-15

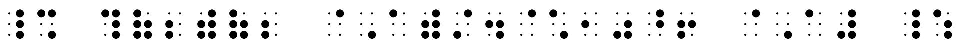
($\lambda > 7{,}000$Å)

⠸⠩ ⠷⠨⠇ ⠨⠂ ⠼⠶⠠⠴⠴⠴ ⠈⠠⠁⠾ ⠸⠱

A space precedes the angstrom unit, but it is unspaced from the closing parenthesis.

Example 13-16

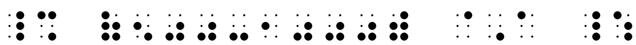
$$\frac{(2)(2\text{Å})}{4\times10^{6}\text{Å}}$$



A space precedes each angstrom unit, but the first angstrom unit is unspaced from the closing parenthesis and the second angstrom unit is unspaced from the closing fraction indicator.

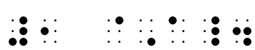
Example 13-17

$(500-1000)\text{Å}$



The closing parenthesis does not apply to the angstrom unit, therefore the symbol is preceded by a space.

c. Although the Angstrom unit is spaced like an abbreviation, it is a mathematical symbol and therefore must be punctuated mathematically when the punctuation falls within the Nemeth switches.

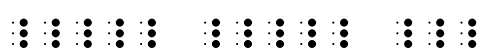
⫸ 1Å. 

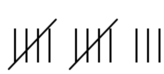
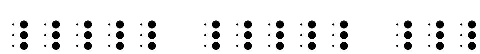
The angstrom unit is unspaced from the punctuation which follows it. (Assume Nemeth continues after this period.)

13.4 Tally Mark

	Tally Mark	|

a. **Grouping.** If tally marks are grouped in print they are similarly grouped in braille, with one blank cell between groupings. When a group of tally marks is crossed through, the cross tally is treated as an additional (456) tally mark which is added to its group. Tally marks belonging to the same grouping should not be divided between braille lines.

⫸ ||||| ||||| ||| 

⫸  

In the print copy, each group of five tally marks appears a long diagonal stroke superposed on four vertical strokes.

b. **Spacing.** One space is left before and after a group of tally marks even when the group is preceded or followed by a sign of operation.

⫸ ||| + ||| = ||||||

⠸⠸⠸ ⠬ ⠸⠸⠸ ⠨⠅ ⠸⠸⠸⠸⠸⠸

No space is left between a group of tallies and a sign of grouping or a braille indicator applying to it.

⫸ (|||||) + (|||) ⠷⠸⠸⠸⠸⠸⠾⠬⠷⠸⠸⠸⠾

⫸ (||| + |||) ⠷⠸⠸⠸ ⠬ ⠸⠸⠸⠾

⫸ $\cancel{||||}^{2}$ ⠸⠸⠸⠸⠸⠘⠆

When the combination of symbols cannot be accommodated on one braille line, transition to another braille line may take the place of a required space.

c. **Punctuation.** No space is left between a group of tallies and a punctuation mark applying to it. When a tally mark is followed by punctuation requiring the punctuation indicator, a multipurpose indicator (dot 5) is placed between the tally mark and the punctuation indicator to avoid misreading the identical symbols. In the next illustration, assume more Nemeth material follows the period.

⫸ $\cancel{||||}$, $\cancel{||||}$. ⠸⠸⠸⠸⠸⠠ ⠸⠸⠸⠸⠸⠐⠸⠲

PRACTICE 13C

I. Tally marks ||||||| equal the number 11_{six}. What numeral does ||||||||| equal in base six?

II. Does |||+||| = $\cancel{||||}$ | ?

III. 5550Å = 555nm or 0.555 micron. (nm is a millimicron.)

SUPERPOSED SIGNS

13.5 Definition and Analysis

Superposed signs are signs which are printed one upon another so that one sign extends beyond the boundary of the other. Contrast this with "shapes with interior modification" presented in Lesson 11, where one symbol is printed inside the boundaries of the other. Here are some examples of superposed signs.

$\oint$ ⋐ ⇸ $\ll$ ∡

In order to transcribe a superposed sign, the basic sign and the superposed sign need to be determined because the basic sign is transcribed first. The following order of preference is used as a guide. A symbol lower on the list is regarded as being superposed upon a symbol higher on the list.

Integral sign
Signs of operation
Horizontal and vertical bars
Signs of shape
Signs of comparison
Signs not listed above

Here is an analysis the first three print examples shown above.

$\oint$ The basic sign is an integral sign; the superposed sign is a sign of shape (circle).

⋐ The basic sign is a sign of operation (dot); the superposed sign is a sign of comparison (inclusion).

⇸ The basic sign is a vertical bar; the superposed sign is a sign of comparison (arrow).

If two signs belong to the same category, the superposition may be represented in either order, provided the same order is followed consistently throughout the transcription. Here is an analysis the last two print examples shown above.

$\ll$ Both signs belong to the same category – signs of comparison (nested "less than" signs).

∡ Both signs belong to the same category – signs of shape (an angle and an arc).

13.6 Transcription of Superposed Signs

The components of a superposed sign are unspaced and are transcribed in the following order:

The symbol for the basic sign is written first.
The superposition indicator is written next.
The symbol for the superposed sign is written third.
The termination indicator is written last.

⠈	Superposition Indicator
⠻	Termination Indicator

13.6.1 **Integral Modified by Superposition.** The most common integrals with superposed symbols are listed below. Unlisted integrals modified by superposition are transcribed in accordance with the rules for superposed signs. Spacing and punctuation follow the same rules as for unmodified integral signs (see 13.1.8).

⠮⠈⠫⠉⠻	Integral with superposed circle	$\oint$
⠮⠈⠠⠿⠻	Integral with superposed infinity	$\int\!\!\!\!\!\infty$
⠮⠈⠫⠗⠻	Integral with superposed rectangle	⨒
⠮⠈⠫⠲⠻	Integral with superposed square	$\int\!\!\!\!\!\square$

⫸ $\oint \frac{\mathrm{d}q}{T} < 0$ ⠮⠈⠫⠉⠻⠹⠙⠟⠌⠠⠞⠼ ⠐⠅ ⠼⠴

⫸ $\oint P\,\mathrm{d}x + Q\,\mathrm{d}y$ ⠮⠈⠫⠉⠻⠠⠏⠙⠭⠬⠠⠟⠙⠽

⫸ $\int\!\!\!\!\!\infty\, f(x, y)\,\mathrm{d}y$ ⠮⠈⠠⠿⠻⠋⠷⠭⠠ ⠽⠾⠙⠽

13.6.2 **Signs of Operation Modified by Superposition.** The most common symbols use the multiplication dot as the basic symbol. Three examples are shown below. (Note that these are symbols modified by superposition, they are not shapes with interior modification. See Lesson 11.) Unlisted signs of operation modified by superposition are transcribed in accordance with the rules for superposed signs.

⠡⠈⠨⠅⠻	Dot between bars of equals sign	$\mathrel{\ooalign{$=$\cr\hfil$\cdot$\hfil}}$
⠡⠈⠸⠐⠅⠻	Dot within inclusion sign	⪽
⠡⠈⠸⠨⠂⠻	Dot within reverse inclusion sign	⪾

⫸ $\pi \mathrel{\ooalign{$=$\cr\hfil$\cdot$\hfil}} 3.14$ ⠨⠏ ⠡⠈⠨⠅⠻ ⠼⠒⠨⠂⠲

13.6.3 **Horizontal and Vertical Bars Modified by Superposition.** The most common symbols are shown below. Unlisted bars modified by superposition are transcribed in accordance with the rules for superposed signs.

⠱⠈⠸⠐⠅⠻	`Horizontal Bar through inclusion sign`	⋲
⠱⠈⠸⠨⠂⠻	`Horizontal Bar through reverse inclusion sign`	⋺
⠳⠈⠫⠒⠒⠕⠻	`Vertical Bar through shaft of right-pointing arrow`	⇸
⠳⠈⠫⠪⠒⠒⠻	`Vertical Bar through shaft of left-pointing arrow`	⇷

⫸ ⊖ ⠱⠈⠫⠉⠻

"Horizontal bar" is higher on the list than a sign of shape (the circle).

13.6.4 **Signs of Shape Modified by Superposition**

⫸ △ ⠫⠞⠈⠫⠏⠻

"Triangle" is a sign of shape, which is higher on the list than "perpendicular to," which is a sign of comparison.

When both signs are signs of shape, the superposition may be represented in either order, provided the same order is followed consistently throughout the transcription.

⫸ ∡ ⠫⠪⠈⠫⠁⠻ ⠕⠗ ⠫⠁⠈⠫⠪⠻

This arc shape extends beyond the boundary of the angle shape, making this a shape modified by superposition. Compare this symbol to the "angle with interior arc" (Lesson 11) which has a different braille form.

When the print copy uses an "angle with interior arc" symbol throughout the text to simply mean "angle", the two-cell angle symbol may be used: ⠫⠪. A transcriber's note is required to inform the reader of the substitution. Sample note on the Transcriber's Notes page: "In print, the angle shape image includes an interior arc."

Signs of shape modified by superposition are spaced and punctuated as other signs of shape. (See Lesson 11).

13.6.5 **Two Signs of Comparison Modified by Superposition.** When both signs are signs of comparison, the superposition may be represented in either order, provided the same order is followed consistently throughout the transcription. Spacing and punctuation follow the same rules as for any other sign of comparison.

```
Equals Sign Through Inclusion Sign

     ⠨⠅⠈⠸⠐⠅⠻                                      ⋹

  or ⠸⠐⠅⠈⠨⠅⠻

Equals Sign Through Reverse Inclusion Sign

     ⠨⠅⠈⠸⠨⠂⠻                                      ∋

  or ⠸⠨⠂⠈⠨⠅⠻

Nested Greater Than Signs
(means "is large compared with")

     ⠨⠂⠈⠨⠂⠻          with straight sides        ≫

     ⠨⠨⠂⠈⠨⠨⠂⠻        with curved sides          ⪼

Nested Less Than Signs
(means "is small compared with")

     ⠐⠅⠈⠐⠅⠻          with straight sides        ≪

     ⠨⠐⠅⠈⠨⠐⠅⠻        with curved sides          ⪻
```

13.6.6 **Negated Symbols and Tally Marks.** Negated symbols are not transcribed as superposed signs. As seen in Lessons 5 and 11, as well as in Section 13.2.5, negated symbols simply include dots 34 in their construction. Also, the print method of showing a group of five tally marks as a long diagonal stroke superposed on four vertical strokes does not follow the Nemeth rules for superposed signs, as shown in Section 13.4 above.

Instructions: See Lesson 11 to review shapes with interior modification, and Lessons 5, 11, and 13 regarding negated symbols.

PRACTICE 13D

Superposed Signs	Interior Modification	Negated Relations
∮	⊙	/∴
ψ	⊡	≠
⦵	⊖	∦
∡C	⊾	≰
5 ≪ y	∠45°	∉
R ⊃̶ s	⑬	≢
Q ⇸ R	✪	∤

AMBIGUOUS SIGNS

13.7 Context

Certain fonts can make it difficult to differentiate between print symbols for letters o, O, and the numeral "zero", or letters l, i, I, and the numeral "one". Additionally, certain print signs look similar to other print signs. The braille symbols may be altogether different. Transcribing the wrong symbol will give the reader false information. In order to assure your transcription is correct, search the surrounding context to determine the meaning of the sign. Magnification may help you identify it. If you are unsure, seek help from someone knowledgeable in the math or science topic who can correctly identify the print sign. Some examples are shown below.

ϕ Greek letter phi	⠨⠋	or	Φ null set ⠸⠴
		or	$\not{0}$ canceled numeral zero ⠪⠴⠻
		or	0 zero in certain fonts ⠼⠴
		or	Φ horizontal bar with superposed circle ⠳⠈⠫⠉⠻
		or	θ Greek letter theta ⠨⠹
α Greek letter alpha	⠨⠁	or	∝ "varies as" ⠸⠿
		or	English letter "a" ⠰⠁
ε Greek letter epsilon	⠨⠑	or	Ɛ "membership" ⠈⠑
ν Greek letter nu	⠨⠝	or	v English letter "vee" ⠰⠧
Δ Greek letter Delta	⠨⠠⠙	or	△ triangle shape ⠫⠞
		or	$\underline{\Delta}$ logical product with underbar ⠈⠩⠱
< "less than"	⠐⠅	or	⟨ opening angle bracket ⠨⠨⠷
		or	< left-pointing caret ⠸⠩
> "greater than"	⠨⠂	or	⟩ closing angle bracket ⠨⠨⠾
		or	> right-pointing caret ⠰⠩
‖ two vertical bars	⠳⠳	or	‖ "is parallel to" ⠫⠇
		or	|| two separate vertical bar symbols ⠸⠐⠸
^ mathematical caret	⠸⠣	or	∧ logical product ⠈⠩
		or	^ literary (UEB) caret ⠈⠣ or circumflex ⠘⠩
√ radical sign	⠜	or	✓ checkmark ⠈⠜

¬ Is it a right-pointing arrow with lower-only, straight arrowhead ⠫⠒⠒⠠⠳ or a long-division structure? or the logic "negation" symbol?

13.7.1 **Vertical Bar and Colon.** The symbols that give transcribers the most trouble due to their ambiguity are the vertical bar and the colon. The vertical bar can be a grouping sign, an operation sign, a comparison sign, or an "end of proof" symbol. The colon can be a ratio symbol or a punctuation mark. You need to recognize the meaning of the sign in order to transcribe the proper symbol.

⠳ Vertical bar used as a sign of grouping, or as a sign of operation meaning "is a factor of", or as a sign of comparison meaning "such that" or "given"

⠐⠂ Colon used as a ratio symbol

⠒ Colon used in digital time, or meaning "is to", or meaning "such that", or used in mapping notation, or used as sentence punctuation. Preceded by a punctuation indicator when unspaced.

⠈⠫⠟⠑⠙ Boldface vertical bar as an "end of proof" icon.

13.7.2 **Spacing.** Some signs use the same braille symbol but have different spacing rules depending on their meaning. You can't depend upon the print copy to show the spacing according to Nemeth rules so you need to recognize the meaning of the sign in order to apply proper spacing. Generally speaking, signs of comparison are spaced; signs of operation are unspaced; punctuation marks are followed by a space but not preceded by a space; signs of grouping are preceded by a space (opening) or followed by a space (closing).

| Is the vertical bar a grouping sign, an operation sign, or a comparison sign?

~ Is the tilde an operation sign ("not") or is it a comparison sign ("is related to" or "is similar to")?

' Is this an apostrophe or single quotation mark (a punctuation mark) or is it a math symbol (prime sign)?

/ Is the slash mathematical (meaning "per", "over", or "divided by") or is it a UEB solidus?

: Are the two vertical dots a ratio symbol (a sign of comparison) or are they a punctuation mark?

13.7.3 **Capital Greek Letters.** Some capital Greek letters are indistinguishable from English letters. Unless the text identifies the letter as Greek, you can safely assume it is an English letter.

13.7.4 **Chemical Notation.** Some signs have yet another meaning in chemical notation. For example, the following symbols can be certain types of chemical bonds.

= ‖ ≡ — | / \ · : ::

Details can be found in *Chemical Notation Using the Nemeth Braille Code*.

⠐	Multipurpose Indicator

13.8 Review

In addition to being a baseline indicator, dot 5 assumes several other functions in the Nemeth Code. Dot 5 is called the *multipurpose indicator* in the following situations which have been discussed previously.

- A multipurpose indicator is used between two unspaced signs to indicate that they are printed horizontally.
  - — side-by-side plus and minus signs. See Section 5.2.
  - — side-by-side tildes. See Section 5.4.9.b.
  - — side-by-side signs of comparison. See Section 5.9.
  - — a number printed on the baseline to the right of a letter. See Section 6.11.1.c.
  - — consecutive superscripts and subscripts. See Section 6.16.
  - — side-by-side arrows. See Section 9.12.
  - — side-by-side modifiers within a sign of shape. See Section 11.17.
- A multipurpose indicator is used between a regular polygon representing a sign of operation and a numeral immediately following it. See Section 11.29.
- A multipurpose indicator begins a modified expression. See Section 12.2.
- A multipurpose indicator is placed between a tally mark and a following punctuation indicator to avoid misreading the similar symbols. See Section 13.4.c.

13.9 Additional Uses of the Multipurpose Indicator

13.9.1 **Letter Followed by a Decimal Point and a Numeral.** When a letter on the baseline of writing is immediately followed by a decimal point and a numeral, a multipurpose indicator is placed between the letter and the decimal point to show that the decimal point and numeral are not subscripts to the letter.

⫸ x.4 ⠭⠐⠨⠲

13.9.2 **Numeric Subscript Followed by a Numeral.** A multipurpose indicator is used after a numeric subscript if the subscript is followed by a numeral on the baseline of writing.

⫸ $x_7 10$ ⠭⠶⠐⠂⠴

13.9.3 **Decimal Point Followed by a Nonnumeric Character.** A multipurpose indicator is used after a decimal point if the symbol following the decimal point is something other than a numeral. When a decimal point is followed by a comma or the punctuation indicator, a multipurpose indicator is not needed.

⫸ $0.a_1a_2a_3 \ldots a_n$

⫸ $\frac{1}{2} = .\underline{?}$

⫸ (5.)

Example 13-18

Can you explain the significance of the decimal point in 1., 10., and 100.? 2., 20., and 200.?

13.9.4 **Side-by-Side Vertical Bars.** A multipurpose indicator is used between two unspaced vertical bars when the first bar is a closing sign of grouping and the second bar is an opening sign of grouping. Similarly, a multipurpose indicator is used between two vertical bars which are nested grouping symbols (one bar is shorter and/or thicker than the other). Examine the surrounding material to be sure the sign is indeed two distinct vertical bars, not a double vertical bar symbol or a parallel sign.

⫸ $|x||y|$

⫸ $\big||x|\big|$

This example shows only one outer closing bar.

⫸ $|x|\big|_{x=0}$

REFERENCE SIGNS AND SYMBOLS

13.10 Reference Signs and Symbols

Reference signs are used in both literary and technical context. Within UEB context, UEB symbols are transcribed. Within Nemeth context, the Nemeth symbols shown below are transcribed.

13.10.1 **Asterisk, Daggers, Star, and Icons.** We have seen these symbols being used elsewhere, in other contexts. The asterisk and the daggers were introduced in Lesson 5 as operation signs; the star was introduced in Lesson 11 as a sign of shape. When these signs are used as reference markers within Nemeth context, the familiar symbols are transcribed.

When a reference sign occurs for which no provision exists in the Nemeth Code such as pictures and icons, the transcriber devises a suitable symbol with an explanatory transcriber's note or a listing in the Special Symbols list. Icons were discussed in Lesson 11.

⠈⠼	Asterisk	*
⠸⠻	Single Dagger	†
⠸⠸⠻	Double Dagger	‡
⠫⠎	Star	☆

13.10.2 **Numerals or Letters.** When reference to a footnote is denoted by a number or a letter, the general reference indicator is used. The number or letter immediately follows the indicator. A numeric indicator or English-letter indicator is required.

⠈⠻	General Reference Indicator

13.10.3 **Layout and Spacing.** Reference signs are often printed in the superscript position, unspaced from the referenced item. In braille, the superscript position is ignored and the reference symbol is spaced away from the word, letter, or number to which it applies. If there is a punctuation mark associated with a reference symbol, no space is left between them. Follow print as to the left-to-right order of reference sign, item being referenced, and punctuation.

Assume that Nemeth continues following each of the examples, below.

Reference sign is printed before the item

⫸ *6.3 ⠈⠼ ⠼⠖⠨⠒

⫸ †2.6 ⠸⠻ ⠼⠆⠨⠖

Reference sign is printed after the item

⫸ 6.3* ⠼⠖⠨⠒ ⠈⠼

⫸ 2.6†

⫸ \$4,265[d]

With punctuation (follow print order)

⫸ 18%‡.

⫸ 18%.‡

⫸ 1.4709×10^8 km[1],

Example 13-19

... distance of 1.4709×10^8 km[1], what is the average distance?

13.10.4 **The Note.** Follow *Braille Formats* for the placement and layout used for the note. The note begins with the same reference indicator used at the point of reference. There may be a mixture of UEB and Nemeth reference symbols in the notes section, depending on the symbol used at the point of reference. The note separation line may be used in either Nemeth Code or UEB, without switching.

Using the models shown above where the reference symbol in the text is transcribed in Nemeth, the next two examples demonstrate the transcription of the note itself. The first transcription of each example shows a page ending in UEB; the second transcription of each example shows a page ending in Nemeth.

Example 13-20

[d] From budget: (\$1,715+\$1,870+\$680) = \$4,265

(a) UEB text ...

(b) Nemeth ...

Example 13-21

[1]Note: Earth's orbit is elliptical, not circular.

(a) UEB text ...

(b) Nemeth ...

Instructions: Explain the function of the multipurpose indicator in each example. Then transcribe the text after the last example. Assume that the equation is the last item on the print page, before the footnote.

PRACTICE 13E

1.	x3, R10	⠭⠐⠒⠠ ⠠⠗⠐⠂⠴
2.	$120° + n320°$	⠼⠂⠆⠴⠘⠨⠡⠐⠬⠝⠐⠒⠆⠴⠘⠨⠡
3.	$C = \pi 2r$	⠠⠉ ⠨⠅ ⠨⠏⠐⠆⠗
4.	$\omega 2 = \omega \neq 2\omega$	⠨⠺⠐⠆ ⠨⠅ ⠨⠺ ⠌⠨⠅ ⠼⠆⠨⠺
5.	$140\mathrm{te}4\mathrm{t}5_{12} + \mathrm{e}5_{12}$	⠼⠂⠲⠴⠞⠑⠲⠞⠢⠰⠂⠆⠐⠬⠑⠢⠰⠂⠆
6.	$\frac{\mathrm{A}_0}{2} = \mathrm{A}_0 2^{-0.05\mathrm{T}}$	⠹⠠⠁⠴⠌⠆⠼ ⠨⠅ ⠠⠁⠴⠐⠆⠘⠤⠴⠨⠴⠢⠠⠞
7.	$x_2 = n_1 5^{-1} - 1n_2 5^{-1}$	⠭⠆ ⠨⠅ ⠝⠂⠐⠢⠘⠤⠂⠐⠤⠂⠝⠆⠐⠢⠘⠤⠂
8.	$0.\alpha_1\alpha_2\alpha_3 \ldots \alpha_n$	⠼⠴⠨⠐⠨⠁⠂⠨⠁⠆⠨⠁⠒ ⠄⠄⠄ ⠨⠁⠰⠝
9.	.%	⠨⠐⠈⠴
10.	4% = .____	⠼⠲⠈⠴ ⠨⠅ ⠨⠐⠤⠤⠤⠤
11.	$5. + .6 = 5.6$	⠼⠢⠨⠐⠬⠨⠖ ⠨⠅ ⠼⠢⠨⠖
12.	$\lVert x \rVert \lVert y \rVert$	⠳⠳⠭⠳⠳⠐⠳⠳⠽⠳⠳
13.	$\lVert \, \lvert x \rvert \, \rVert$	⠳⠳⠐⠳⠭⠳⠐⠳⠳

Finding an Equation for a Sinusoidal Graph

Figure 47 can be viewed as the graph of a sine function with amplitude $A = 5$*, where $T = 4$.

---

* The equation could also be viewed as a cosine function with a horizontal shift.

For further practice, see Appendix A—Reading Practice.

EXERCISE 13

Prepare Exercise 13 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 13A

⠠⠥⠝⠎⠏⠁⠉⠫ ⠠⠍⠊⠎⠉⠑⠇⠇⠁⠝⠑⠳⠎ ⠠⠎⠽⠍⠃⠕⠇⠎

⠀⠀

⠸⠩ ⠼⠂⠨⠆⠸⠣

⠷⠧ ⠨⠂ ⠨⠋⠈⠓⠾

⠈⠠⠗⠸⠒ ⠼⠆⠲ ⠠⠄⠛⠗⠁⠍⠎

⠳⠳⠨⠫⠋⠷⠸⠰⠁⠾⠳⠳

⠹⠈⠙⠠⠏⠌⠈⠙⠭⠼⠬⠹⠈⠙⠠⠟⠌⠈⠙⠽⠼

⠨⠷ ⠨⠾⠨⠩⠨⠷ ⠨⠾

⠷⠝⠩⠗⠾ ⠨⠅ ⠹⠝⠯⠌⠷⠝⠤⠗⠾⠯⠗⠯⠼

⠤⠠⠿ ⠐⠅ ⠭ ⠐⠅ ⠠⠿

⠋⠄⠷⠭⠾ ⠨⠅ ⠼⠴ ⠠⠄⠕⠗ ⠠⠿

⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭ ⠨⠅ ⠠⠋⠷⠭⠾⠈⠾⠰⠁⠘⠃

⠐⠮⠩⠭ ⠨⠅ ⠁⠣⠭ ⠨⠅ ⠃⠻⠋⠷⠺⠾⠙⠺

⠷⠈⠿⠭⠾⠷⠈⠿⠽⠾⠈⠷⠭⠬⠽ ⠨⠅ ⠼⠦⠢⠈⠾

⠈⠿⠳⠰⠭

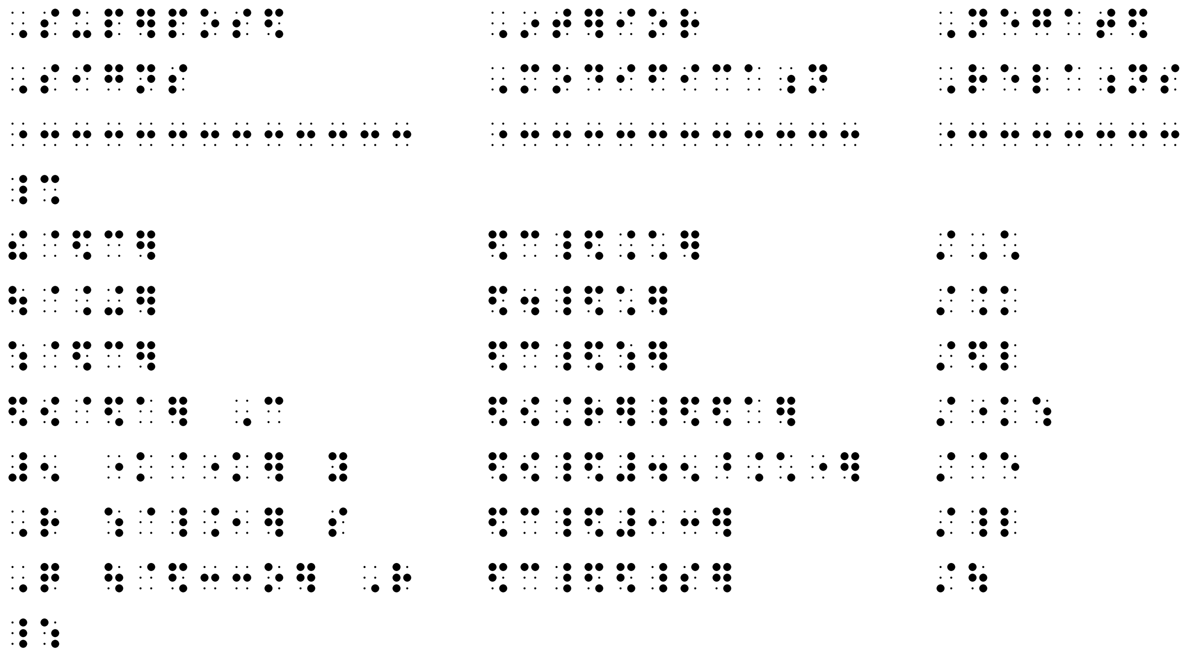
⠈⠯⠰⠭ ⠈⠑ ⠄⠁ ⠸⠱

PRACTICE 13B

PRACTICE 13C

Switch decision: The word "micron" is transcribed in UEB even though it is associated with a Nemeth number.

PRACTICE 13D



Guide dots are not used because the items are not related across the rows.

PRACTICE 13E

1. A multipurpose indicator is used when a letter is followed by a numeral and they are both on the baseline of writing.

2. The first dot 5 is a baseline indicator because the plus sign is on the baseline and it follows a raised hollow dot. The second dot 5 is a multipurpose indicator which is needed to show that the numeral "3" is not a subscript to the letter "n".

3. The same rule applies to letters in any alphabet – a multipurpose indicator is needed to show that the numeral "2" is not a subscript to the Greek letter pi.

4. Same as #3 regarding Greek letter omega followed by numeral "2" in "ω2". Note that a multipurpose indicator is not needed for a letter following a numeral, as in " 2ω".

5. A baseline indicator precedes the plus sign, following the subscript "12". (The multipurpose indicator is not used following the "t" and "e" because they represent numerals in base 12.)

6. A multipurpose indicator is needed after the second numeric subscript "0" because the subscript is followed by a numeral on the baseline of writing ("2").

7. The function of the first and third dot 5 is the same as item 6. The second dot 5 is a baseline indicator which is needed for the minus sign following a superscript.

8. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is the Greek letter "alpha".

9. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is a percent sign.

10. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is a long dash.

11. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is a plus sign.

12. A multipurpose indicator is used between two unspaced vertical bar symbols (in this case, each is a "double vertical bar" symbol) when the first is a closing sign of grouping, and the second bar is an opening sign of grouping.

13. A multipurpose indicator is used between vertical bars which are nested grouping symbols.

⠠⠋⠔⠙⠬ ⠁⠝ ⠠⠑⠟⠥⠁⠰⠝ ⠿ ⠁ ⠠⠎⠔⠥⠎⠕⠊⠙⠁⠇
⠠⠛⠗⠁⠏⠓
⠠⠋⠊⠛⠥⠗⠑ ⠼⠙⠛ ⠉ ⠆ ⠧⠊⠑⠺⠫ ⠵ ⠮ ⠛⠗⠁⠏⠓ ⠷ ⠁
⠎⠔⠑ ⠋⠥⠝⠉⠰⠝ ⠾ ⠁⠍⠏⠇⠊⠞⠥⠙⠑ ⠸⠩ ⠠⠁ ⠨⠅ ⠼⠢ ⠈⠼⠠
⠠⠄⠐⠱ ⠠⠞ ⠨⠅ ⠼⠲⠸⠲
⠐⠒⠒⠒⠒⠒⠒
⠈⠼ ⠸⠱ ⠠⠮ ⠑⠟⠥⠁⠰⠝ ⠉⠙ ⠁⠇ ⠆ ⠧⠊⠑⠺⠫ ⠵ ⠁ ⠉⠕⠎⠔⠑
⠋⠥⠝⠉⠰⠝ ⠾ ⠁ ⠓⠕⠗⠊⠵⠕⠝⠞⠁⠇ ⠩⠊⠋⠞⠲

LESSON 14

- FUNCTION NAMES AND THEIR ABBREVIATED FORMS

Spatial Arrangements, continued

- SQUARE ROOT DIVISION
- OTHER PRINT LAYOUTS SHOWING DIVISION

Answers to Practice Material

LESSON PREVIEW

Rules regarding function names and their abbreviated forms are presented. Many examples are shown. The study of spatial arrangements continues with a other forms of division problems: square root division, partial quotient layout, synthetic division, and others.

FUNCTION NAMES AND THEIR ABBREVIATED FORMS

[NC Rule 18]

14.1 List of Common Function Names and Their Abbreviated Forms

The most common function names and their abbreviated forms are listed below. Function names that do not appear in this list are subject to the same rules taught in this lesson. Note that abbreviated function names are printed in regular type.

Function Name	Abbreviated Function Name
amplitude	amp
antilogarithm	antilog
arc	arc
argument	arg
cologarithm	colog
cosine	cos
hyperbolic cosine	cosh
cotangent	cot
hyperbolic cotangent	coth
coversine	covers
cosecant	csc
hyperbolic cosecant	csch
cotangent	ctn
hyperbolic cotangent	ctnh
determinant	det
error function	erf
exponential	exp
exsecant	exsec
gradient	grad
haversine	hav
imaginary part	im
infimum	inf
limit	lim
upper limit	$\overline{\mathrm{lim}}$ or $\overline{\mathrm{limit}}$
lower limit	$\underline{\mathrm{lim}}$ or $\underline{\mathrm{limit}}$
natural logarithm	ln
logarithm	log
maximum	max
minimum	min
modulo	mod
real part	re
secant	sec

hyperbolic secant	sech
sine	sin
hyperbolic sine	sinh
supremum	sup
tangent	tan
hyperbolic tangent	tanh
versine	vers

14.2 Code Switching and Punctuation

14.2.1 **Function Names in UEB.** A function name which is not associated with a math term is transcribed in UEB and appropriate contractions and punctuation are used.

Example 14-1

Consider the Law of Sines.

"Sines" is not associated with a mathematical item and so is transcribed in UEB, using appropriate contractions.

Example 14-2

Some trigonometric functions are sine, cosine, and tangent.

14.2.2 **Function Names in Nemeth.** A function name which occurs in mathematical context is transcribed in Nemeth, without contractions. A function name used in conjunction with an abbreviated function name is a mathematical term and a switch to Nemeth is required.

- sine α

  Because the Greek letter requires Nemeth Code, the associated function name is also transcribed in Nemeth.

- logsine

  "log" is an abbreviated function name, therefore "logsine" requires a switch to Nemeth.

Example 14-3

What is the meaning of logsine?

14.2.3 **Abbreviated Function Names.** The abbreviated forms of function names are mathematical items and are transcribed following the rules of the Nemeth Code. An abbreviated function name is punctuated mathematically inside the switches.

Example 14-4

Some trigonometric functions are sin, cos, and tan.

The abbreviated function names are punctuated in mathematical mode.

Example 14-5

The abbreviated form of "logarithm" is "log".

This function name is in UEB. Its abbreviated form is in Nemeth. A punctuation indicator is required before the closing quotation mark because an abbreviated function name is a mathematical term.

Example 14-6

The inverse sine function is written $\sin^{-1}$.

14.2.4 **"Arc" in Context.** "Arc" can be a function name, an abbreviated function name, or a word referring to a curve.

Example 14-7

What is the arc sine function?

The function name "sine" is in UEB, so "arc" is also transcribed in UEB.

Example 14-8

What is the arc sin function?

⠠⠱⠁⠞ ⠊⠎ ⠮ ⠸⠩ ⠁⠗⠉ ⠎⠊⠝ ⠸⠱ ⠋⠥⠝⠉⠰⠝⠦

The abbreviated function name "sin" is Nemeth, so "arc" is also transcribed in Nemeth..

Example 14-9

Arc ACB is a major arc in Circle O.

⠠⠜⠉ ⠸⠩ ⠠⠁⠠⠉⠠⠃ ⠸⠱ ⠊⠎ ⠁ ⠍⠁⠚⠕⠗ ⠜⠉ ⠔
⠠⠉⠊⠗⠉⠇⠑ ⠠⠕⠲

"Arc" is a word referring to the curved line ACB.

PRACTICE 14A

1. "sin θ" is pronounced "sine theta".
2. "Arcsin" is the "inverse sine".
3. sin 30° cos 45°
4. The logsine function is related to the logcosine function by $S_n = 2C_n$.

14.3 Spacing of Abbreviated Function Names

Within a mathematical expression, the following spacing rules are observed. These rules apply regardless of the spacing used in the print copy.

a. No space comes before an abbreviated function name unless it follows a sign of comparison or other symbol that requires spacing.

b. A space is required after an abbreviated function name or its inverse (the space follows the superscript). There is one exception – see 14.3.2, below.

⫸ cos 20°	⠉⠕⠎ ⠼⠆⠴⠘⠨⠡
⫸ 3 cos 20°	⠼⠒⠉⠕⠎ ⠼⠆⠴⠘⠨⠡
⫸ sin θ	⠎⠊⠝ ⠨⠹
⫸ i sin θ	⠊⠎⠊⠝ ⠨⠹
⫸ tan (x)	⠞⠁⠝ ⠷⠭⠾
⫸ $\tan^{-1}(x)$	⠞⠁⠝⠘⠤⠂ ⠷⠭⠾
⫸ $f(x) = \sin\,(x)$	⠋⠷⠭⠾ ⠨⠅ ⠎⠊⠝ ⠷⠭⠾

Example 14-10

For any angle θ, $\sin(\theta + 360°) = \sin\theta$ and $\cos(\theta + 360°) = \cos\theta$.

⠠⠿ ⠁⠝⠽ ⠁⠝⠛⠇⠑ ⠸⠩ ⠨⠹⠠
⠎⠊⠝ ⠷⠨⠹⠬⠒⠖⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠎⠊⠝ ⠨⠹ ⠠⠄⠯
⠉⠕⠎ ⠷⠨⠹⠬⠒⠖⠴⠘⠨⠡⠐⠾ ⠨⠅ ⠉⠕⠎ ⠨⠹ ⠸⠱⠲

In print, there is no space before each opening parenthesis. In braille, a space is required following each abbreviated function name.

Example 14-11

sin(35°) = Opposite/Hypotenuse.

⠸⠩
⠎⠊⠝ ⠷⠒⠢⠘⠨⠡⠐⠾ ⠨⠅ ⠠⠕⠏⠏⠕⠎⠊⠞⠑⠸⠌⠠⠓⠽⠏⠕⠞⠑⠝⠥⠎⠑
⠸⠱⠲

In print, there is no space between sin and (35°). In braille, a space is required following the abbreviated function name.

14.3.1 **Spacing with Operation Symbols.** In braille, an operation symbol is usually unspaced from the symbols which precede and follow it. However, when an abbreviated function name is followed by an operation symbol, a space is required.

⋙ $\tan \cdot \sin$

"tan" is followed by a space. A space is not required before "sin".

⋙ $1/\cos - \cos = \tan \cdot \sin$

Each abbreviated function name is followed by a space.

14.3.2 **Spacing with Indicators.** A space is not inserted between an abbreviated function name and an indicator which applies to it.

⋙ $\frac{1}{\cos}$

Example 14-12

Reciprocal Functions: $\frac{1}{\cos} - \cos = \tan \cdot \sin$

The abbreviated function name in the denominator is unspaced from the closing fraction indicator. The expression continues, following other spacing rules of the Nemeth Code.

14.3.3 **Examples.** Examine the spacing in the following examples.

Example 14-13

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $\cos\theta = \frac{1}{\sin\theta}$

(2) $\sin(\theta + 90°) = \cos\theta$

(3) $y = 3\tan 2x$

(4) $a^2 = 2ac\cos\beta + c^2$

(5) $\sin(\alpha+\beta) + \sin(\alpha-\beta)$

(6) $y' = x\cos\varphi - y\sin\varphi$

(7) $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} = C$

(8) $6\sin 2A\cos 4A$

(9) $\cos 203° \csc 203°$

(10) $\dfrac{2\sin\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}}$

(11) $\ln\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right| + C$

"ln" is the abbreviated form of "natural logarithm". Refer to the table at the beginning of this lesson.

PRACTICE 14B

(1) $\sin\theta/\cos\theta$

(2) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

(3) $\dfrac{\tan 90°}{\cot 90°}$

(4) $r[3\cos\theta + 4\sin\theta] = 5$

(5) $7(\cos 20° + i\sin 20°)$

(6) $\frac{1}{2}\ln|\sec 2t + \tan 2t| + C$

(7) $a\sin\frac{x}{a}\cdot\frac{1}{a} = \sin\frac{x}{a}$

14.3.4 **Spacing with Consecutive Abbreviated Function Names.** A space is required between two or more consecutive abbreviated function names unless they are clearly unspaced in the print text. When there is doubt concerning the presence of a space between abbreviated function names, a space should be inserted.

⫸ $y = \text{arc}\ \sin x$

Example 14-14

What is cos arctan(-1)?

14.3.5 **Examples.** Study the following examples.

Example 14-15

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $n = \log \sin 50°27'$

(2) $\cos\left[2\ \text{Arc}\ \csc\left(-\frac{7}{5}\right)\right]$

(3) $\cos\left(\text{arc Tan}\ x + \frac{\pi}{3}\right)$

(4) $\text{Arctan}\ x + \text{Arccot}\ x = \frac{1}{2}\pi$

14.4 Nonuse of the English-letter Indicator

The English-letter indicator is not used with an English letter or a Roman numeral following an abbreviated function name.

⫸ $\sin x$

⫸ cot I

14.4.1 **Examples.** Examine the English letters and the spacing in the following examples.

Example 14-16

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $\sin x + y$

⠎⠊⠝ ⠭⠬⠽

(2) $\text{ctn} - \text{A} = -\text{ctn A}$

⠉⠞⠝ ⠤⠠⠁ ⠨⠅ ⠤⠉⠞⠝ ⠠⠁

(3) $y = 2\sin x + \sin 2x$

⠽ ⠨⠅ ⠼⠆⠎⠊⠝ ⠭⠬⠎⠊⠝ ⠼⠆⠭

(4) $y = \sqrt{\cot x}$

⠽ ⠨⠅ ⠜⠉⠕⠞ ⠭⠻

(5) $\{\sin x \mid \sin x + 2 \leq +1\}$

⠨⠷⠎⠊⠝ ⠭ ⠳ ⠎⠊⠝ ⠭⠬⠆ ⠐⠅⠱ ⠬⠂⠨⠾

(6) $y = \ln |\tan x|$

⠽ ⠨⠅ ⠇⠝ ⠳⠞⠁⠝ ⠭⠳

14.5 Keep Together

A function name or its abbreviated form and the sign which follows it (known as the "argument") is regarded as a single mathematical item and therefore should not be divided between braille lines. Also, a two-part function name should not be divided between braille lines. These rules also applies in UEB context.

Example 14-17

If θ = 51° is the angle between vectors, determine sin θ and cos θ.

⠠⠊⠋ ⠸⠩ ⠨⠹ ⠨⠅ ⠼⠢⠂⠘⠨⠡ ⠸⠱ ⠊⠎ ⠮ ⠁⠝⠛⠇⠑ ⠆⠞
⠧⠑⠉⠞⠕⠗⠎⠂ ⠙⠑⠞⠻⠍⠔⠑ ⠸⠩ ⠎⠊⠝ ⠨⠹ ⠠⠄⠯ ⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠉⠕⠎ ⠨⠹ ⠸⠱⠲

"cos θ" is not divided between lines even though "cos" fits on the previous line.

Example 14-18

Inverse Functions The inverse function $\tan^{-1} x$ may also be called the arc tangent of x, or arctan x.

"arc tangent" is not divided between lines even though "arc" fits on the previous line.

14.6 Clarification— Abbreviated Function Names in an Enclosed List

An abbreviated function name and the item which follows it are regarded as a single item. Although the numeric indicator is not used at the beginning of an item in an enclosed list, it must be used before a numeral (or decimal point and a numeral) following an abbreviated function name.

⫸ (2 sin 30°, 3 cos 60°)

PRACTICE 14C

(A) sine $x -$ sine y

(B) $2 \sin x + 3 \cos y$

(C) $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x}$

(D) The logarithm of sine 18° is written log sin 18°.

(E) $\cos 225° = -\sqrt{\frac{1+\cos 450°}{2}}$

(F) ArcTan[x, y] gives the arc tangent of $\frac{y}{x}$, taking into account in which quadrant the point (x, y) lies.

(G) The arc tangent of the complex number q is written "ArcTan[q]".

(H) Consider the ordered pair (cos .8000, 2 cos .8000).

14.7 Superscripts and Subscripts

When a function name or an abbreviated function name carries a superscript or a subscript, the required space follows the superscript or subscript. A letter, numeral, or other mathematical expression following this space assumes the same level as the function name or abbreviated function name.

In the following three examples, letters "θ" and "x" are printed on the baseline of writing and are unspaced.

⫸ $\text{sin}^2\theta$

⫸ sin^2x

⫸ sine^2x

Example 14-19

The coordinates are the cosine and sine, so we conclude $\sin^2\theta + \cos^2\theta = 1$.

The required space follows the superscript in sin² and cos². There is no space following the plus sign.

Example 14-20

Verify that $1 - \cos\frac{2\pi}{3} = 2\sin^2\frac{4\pi}{3}$.

The required space follows "cos" and "sin²". There is no space following the minus sign.

14.7.1 **Examples.** Examine the spacing in the following examples.

Example 14-21

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $\sin^2 A + \cos^2(B + A)$

(2) $(1 - \sin^2 x)^2\cos^2 x$

(3) $\sin^2\theta \times \dfrac{\cos^2\theta}{\sin^2\theta} - 1$

(4) $\dfrac{1 - \dfrac{\sin^2 x}{\cos^2 x}}{\sec^2 x}$

14.7.2 **Use/Nonuse of the Subscript Indicator.** The subscript indicator is not used when an abbreviated function name carries a numeric subscript on the first level below the baseline of writing. A subscript indicator is required in all other circumstances, including a function name which carries a numeric subscript.

⫸ $\log_3 81 = 4$

The numeral "3" is printed at the subscript level.

⫸ $\text{logarithm}_3\ 81 = 4$

A subscript indicator is required because the subscript applies to a word (a function name is a word).

⫸ $\log_b N \cdot \log_a b$

A subscript indicator is required because each subscript is a letter.

⫸ $\log_{2e} x = -1.4$

A subscript indicator is required because the subscript contains a letter.

14.7.3 **Abbreviated Function Names Within a Superscript or a Subscript.** When an abbreviated function name occurs within a superscript or subscript, the required space following it maintains the level at which the abbreviated function name appears. A restatement of the level indicator is not needed.

⫸ $e^{\sin x}$

"sin x" is in the superscript position.

⫸ $y = e^{\cos^2 x}$

"$\cos^2 x$" is in the superscript position.

14.7.4 **Examples.** Study the following examples.

Example 14-22

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $y = e^{\sin x}$

(2) $y = e^{\text{sine}\, x}$

(3) $y = (\sin x)^{\tan x}$

(4) $e^{\ln x - 2\ln y}$

(5) $\mathrm{a}^{\mathrm{g(x)log_a f(x)}}$

(6) $3^{\log_3 9}$

Recall from Lesson 6 that a subscript indicator is required in superscript and subscript combinations. The super/sub indicator shows a numeric subscript in the superscript position.

(7) $3^{\log_3 7} + 2^{\log_2 5}$

Same note as (6), above.

(8) $a^{\log_a x} = x$

Recall from Lesson 6 that the space before a comparison sign returns the reader to the baseline.

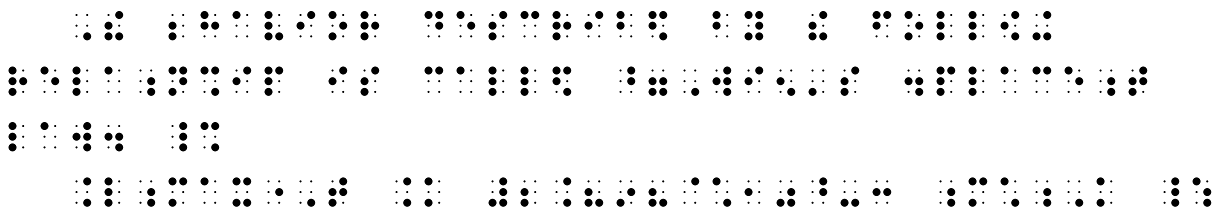
(9) $e^{\sin x = a} > y$

Recall from Lesson 6 that when a comparison sign occurs within a superscript, the level is restated before the comparison sign.

Example 14-23

The behavior described by the following relationship is called **Wien's displacement law.**

$$\lambda_{\text{max}}T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$$



Line 3: The switch to Nemeth terminates the UEB bold passage.
Line 4: Since both Nemeth switches do not fit on this line, the opening switch is placed on line 3. The multiplication dot which comes between the two-part abbreviation is unspaced. Review 4.20.1 in Lesson 4.

PRACTICE 14D

1. $\log_n .125 = -.6$
2. $\text{antilog}_\text{a}\, \text{x} = \text{N}$
3. $\log_{.0543}\text{x} = -.8$
4. $\cot^{-1} x + \frac{\pi}{2} - \tan^{-1} x$
5. $\sin^2 90° + \cos^2 90° = 1$
6. $e^{x+\ln x}$
7. $e^{\sin x} + e^{\sin y}$
8. $2^{\sec x} = y$

14.8 Modifiers

Modified abbreviated function names are transcribed according to the five-step rule for the transcription of modified expressions introduced in Lesson 12. When an abbreviated function name carries a modifier, the required space after the abbreviated function name follows the termination of the modifier.

⫸ $\lim\limits_{x \to a}$ ⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠁⠻

⫸ $\lim\limits_{x \to a} f(x) = 1$

⠐⠇⠊⠍⠩⠭ ⠫⠕ ⠁⠻ ⠋⠷⠭⠾ ⠨⠅ ⠼⠂

14.8.1 **Examples.** Study these additional examples.

Example 14-24

Examples are in Nemeth. The code switch indicators are omitted from the simbraille.

(1) $\lim\limits_{x \uparrow 4} (x - 4)^{-1}$

⠐⠇⠊⠍⠩⠭ ⠫⠣⠒⠒⠕ ⠼⠲⠻ ⠷⠭⠤⠲⠾⠘⠤⠂

(2) $\lim\limits_{\theta \to \theta_0} (\tan \theta)$

⠐⠇⠊⠍⠩⠨⠹ ⠫⠕ ⠨⠹⠴⠻ ⠷⠞⠁⠝ ⠨⠹⠾

14.8.2 **Special Case—Upper Limit and Lower Limit.** The symbols below denote "upper limit" or "lower limit". The horizontal bar directly over or under "lim" or `limit" is <u>not</u> treated as a modifier.

⠣⠇⠊⠍	upper limit	$\overline{\text{lim}}$
⠣⠇⠊⠍⠊⠞	upper limit	$\overline{\text{limit}}$

⫸ $\overline{\lim\limits_{n \to \infty}} f_n(x)$

⠐⠣⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

⫸ $\overline{\text{limit}}_{n \to \infty} f_n(x)$

⠐⠣⠇⠊⠍⠊⠞⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

⠩⠇⠊⠍	`lower limit`	$\underline{\lim}$
⠩⠇⠊⠍⠊⠞	`lower limit`	$\underline{\text{limit}}$

≫ $\underline{\lim}_{n\to\infty} f_n(x)$

⠐⠩⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

≫ $\underline{\text{limit}}_{n\to\infty} f_n(x)$

⠐⠩⠇⠊⠍⠊⠞⠩⠝ ⠫⠕ ⠠⠿⠻ ⠋⠰⠝⠐⠷⠭⠾

PRACTICE 14E

1. Find $\lim\limits_{x\to 0.6} 2^{25x^2-10x-1}$.
2. Formulate a precise definition for $\lim\limits_{x\downarrow -\infty} f(x) = L$.
3. If $\overline{\lim}_{n\to\infty} a_n = A$ and $\overline{\lim}_{n\to\infty} b_n = B$, must it be true that $\overline{\lim}_{n\to\infty} (a_n + b_n) = A + B$?
4. Find $\overline{\text{limit}}_{n\to\infty} a_n$ when $a_n = (-1)_n$.
5. $\lim\limits_{x\to 0} \csc x \ln(1+x)$

You may wish to revisit the Review of Format for Spatial Arrangements in Lesson 10. NOTE: Code switch indicators are omitted and blank lines are implied in the examples that do not contain narrative.

SQUARE ROOT DIVISION

[NC Rule 25.6]

14.9 Review of Terminology

Radical expressions were presented in Lesson 8. When an answer is shown, a spatial arrangement is required. Here are the names of the parts of a radical expression. The line above the radicand is the vinculum. √ is the radical sign.

$$\begin{array}{rl} 12 & \textit{root} \\ \sqrt{144} & \textit{radicand} \end{array}$$

14.10 Spatial Arrangement for Square Root Problems

In the spatially arranged radical expression, the first cell of the vinculum is placed directly above the radical symbol. The last cell of the vinculum extends one cell beyond the radicand.

```
⠒⠒⠒
⠜    Radical Sign (with Vinculum)    √‾‾
```

Example 14-25

The square root of 144 is 12, and is written as follows.

$$\begin{array}{r} 12 \\ \sqrt{144} \end{array}$$

```
  ⠠⠮ ⠎⠟⠥⠜⠑ ⠗⠕⠕⠞ ⠷ ⠼⠁⠙⠙ ⠊⠎ ⠺⠗⠊⠞⠞⠢ ⠵
⠋⠕⠇⠇⠪⠎⠲ ⠸⠩
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
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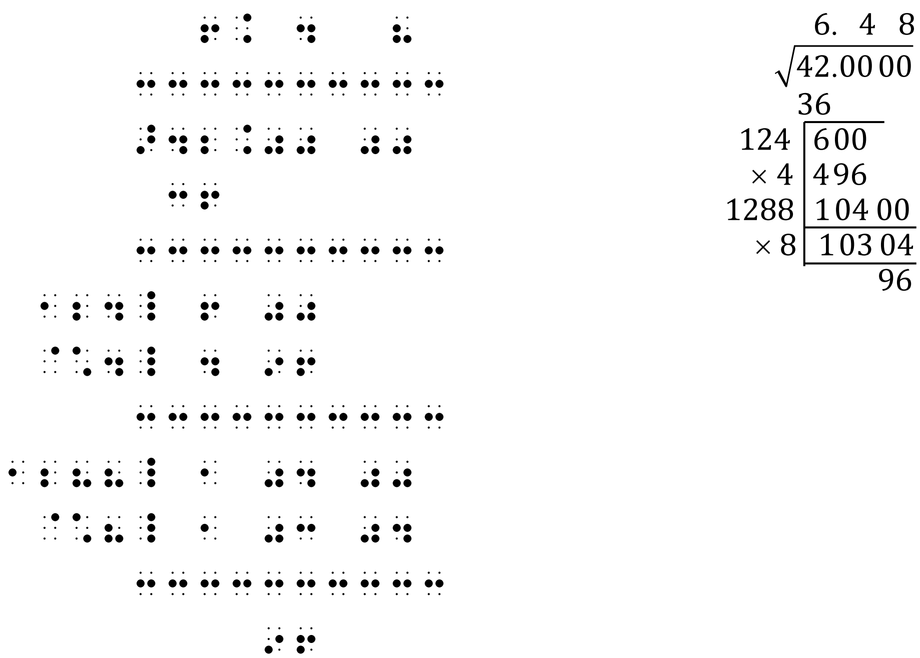
  ⠜⠂⠲⠲
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠸⠱
```

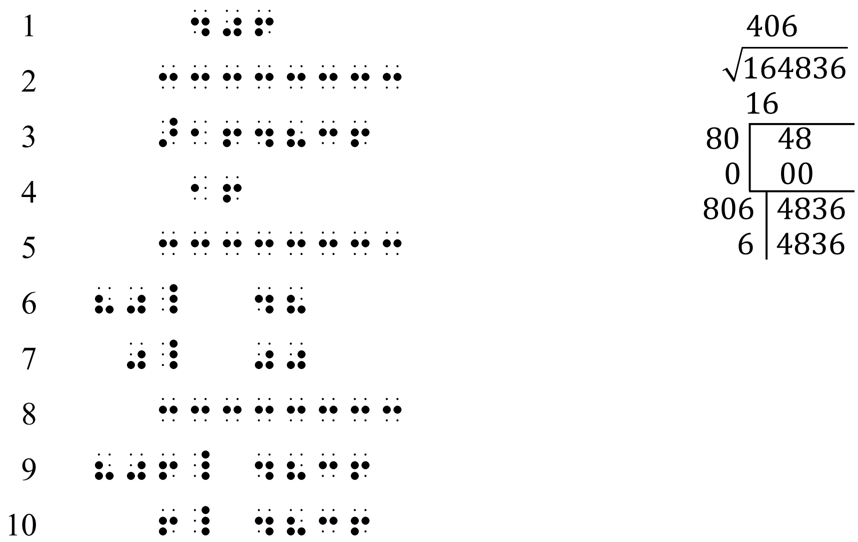
a. **Solving a Square Root Problem.** The procedures used with long division arrangements are applied to a spatially arranged square root problem. Review alignment and spacing rules for long division in Lesson 10. The vertical line that separates the parts of the problem is represented by dots 456. Spacing between digits replicates spacing in print. Follow print regarding the alignment of the vertical lines.

Example 14-26



All lines: Spacing between digits matches print in order to attain proper vertical alignment.
Line 2: The vinculum begins in the cell above the radical sign and ends one cell beyond the rightmost character in the entire arrangement.
Lines 2, 5, 8, 11: Separation lines are all the same width.
Lines 6, 7, 9, 10: These vertical lines align below the radical sign.
Lines 7, 10: The multiplication cross is unspaced from the multiplier.

Example 14-27

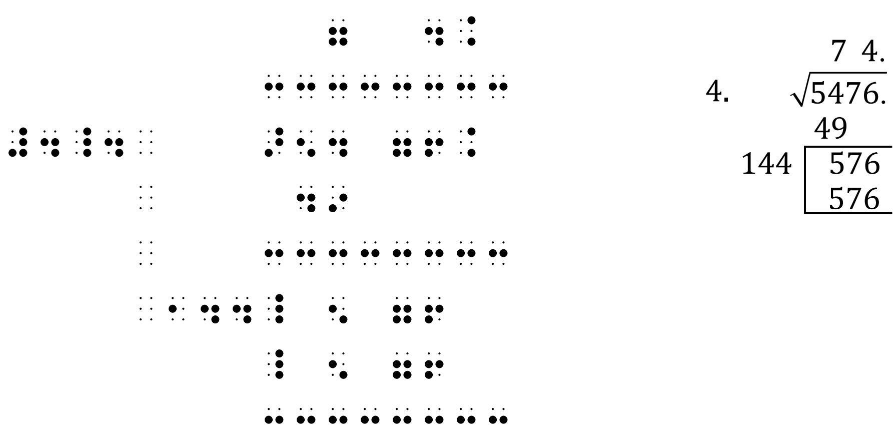


Lines 9-10: These vertical lines are aligned to the right of the vertical lines on lines 6-7, as printed.

14.11 Placement of Identifiers with Spatial Radical Expressions

An identifier, if present, is placed on the line with the radicand. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 14-28



PRACTICE 14F

(A)

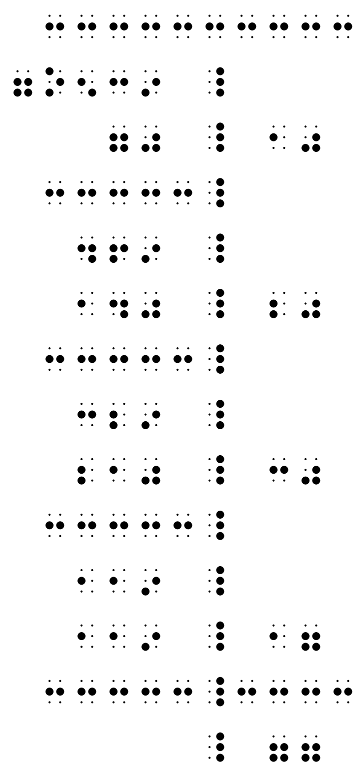
$$
\begin{array}{r|l}
 & 5.\ \ 7\ \ \ 4 \\
 & \sqrt{33.00\,00} \\
 & 25 \\
\hline
107 & 800 \\
\times 7 & 749 \\
\hline
1144 & 5100 \\
\times 4 & 4576 \\
\hline
 & 524
\end{array}
$$

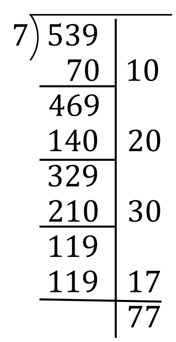
OTHER PRINT LAYOUTS SHOWING DIVISION

14.12 Partial Quotients [NC Rule 25.5.8]

This layout shows partial quotients printed to the right of the division problem. A vertical line separates the partial quotients from the rest of the problem. The vertical line may be either drawn as a tactile graphic or it may be represented by dots 456. One space (one blank cell) is left between the vertical line and any digit preceding or following it.

Example 14-29





Notice the comparative lengths of the separation lines as well as their vertical alignment.

Instructions: Review Section 10.13.6.d regarding alignment of the minus signs.

PRACTICE 14G

$$
\begin{array}{r|l}
132 & \\
6\overline{)792} & \\
-600 & 100 \\
\hline
192 & \\
-\ 60 & 10 \\
\hline
132 & \\
-\ 60 & 10 \\
\hline
72 & \\
-\ 60 & 10 \\
\hline
12 & \\
-\ 12 & 2 \\
\hline
0 &
\end{array}
$$

14.13 Synthetic Division [NC Rule 25.7]

Synthetic division is a method of showing division of polynomials. There is not a standard print layout. The transcription replicates the print design, following alignment rules discussed below, and using the standard separation line and vertical line of the Nemeth Code. Here is an example of one possible layout of a synthetic division problem.

$$\begin{array}{r|rrrr} +2 & 1 & -3 & +4 & +5 \\ & & +2 & -2 & +4 \\ \hline & 1 & -1 & +2 & \multicolumn{1}{|r}{+9} \end{array}$$

The parts to this problem are labeled as follows.

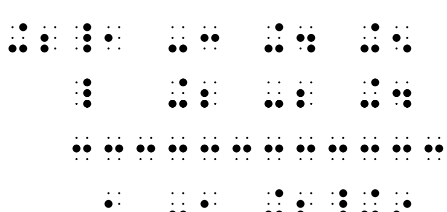
$$\begin{array}{lr|rrrrl} \textit{divisor} & +2 & 1 & -3 & +4 & +5 & \textit{dividend} \\ & & & +2 & -2 & +4 & \textit{product} \\ \cline{3-6} \textit{quotient} & \multicolumn{1}{r}{} & 1 & -1 & +2 & \multicolumn{1}{|r}{+9} & \textit{remainder} \end{array}$$

14.13.1 **Alignment and Spacing.** In the examples which follow, look carefully at the vertical alignment. The numerals in the dividend, product, and quotient are aligned in vertical columns as in the print copy. Signs of operation, if any, are also vertically aligned. At least one blank cell is left between adjacent columns.

14.13.2 **Vertical Line.** Dots 456 represent the vertical line in the print copy. The braille symbol is shown between the divisor and the division arrangement, beginning on the line with the dividend and ending on the line with the product. No space is left between the vertical line and the dividend or divisor. The separation line (dots 25) extends from the vertical line to one cell beyond the entire arrangement. Another unspaced vertical line is transcribed between the quotient and the remainder.

Example 14-30

$$\begin{array}{r|rrrr} +2 & 1 & -3 & +4 & +5 \\ & & +2 & -2 & +4 \\ \hline & 1 & -1 & +2 & \multicolumn{1}{|r}{+9} \end{array}$$

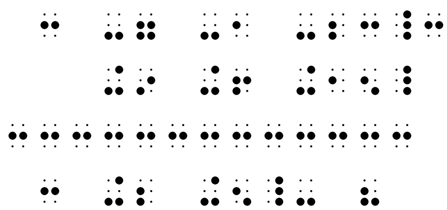


Note the vertical alignment of the numerals and the operation signs.
In this problems, the divisor is +2, the dividend is 1 -3 +4 +5, the product is +2 -2 +4, the quotient is 1 -1 +2, and the remainder is +9.

14.13.3 Another Print Style—Divisor on the Right. If the divisor is printed to the right of the overall problem, the same layout is followed in braille. Follow the alignment and spacing rules outlined above, particularly noting that at least one blank cell must be left between adjacent columns. The vertical lines are unspaced from the dividend and the divisor, as well as from the quotient and the remainder.

Example 14-31

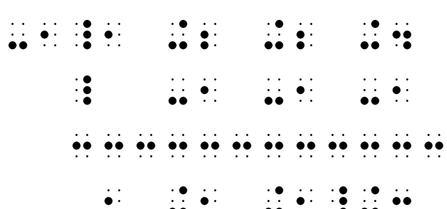
```
3   −7    −1    −23 | 3
    +9    +6    +15 |
--------------------
3   +2    +5 |  −8
```



14.13.4 Another Print Style—Boxed Divisor. If the divisor appears boxed in on two sides, the boxing is omitted. A vertical line between the divisor and the dividend is inserted in order to differentiate the divisor from the rest of the arrangement, even though this vertical line does not appear in print. Follow the same alignment and spacing rules outlined above. The first example shows the divisor at the left; the second shows the divisor at the right.

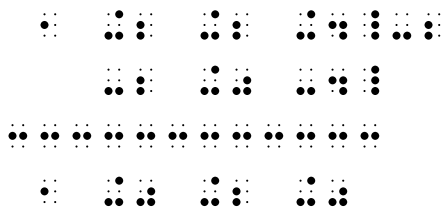
Example 14-32

```
|−1   1   +2    +2    +4
          −1    −1    −1
      ------------------
      1   +1    +1 |  +3
```



Example 14-33

$$\begin{array}{rrrr|l} 1 & +2 & +2 & +4 & \underline{-2} \\ & -2 & +0 & -4 & \\ \hline 1 & +0 & +2 & +0 & \end{array}$$

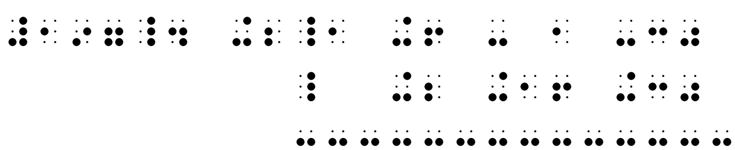


Note that this example has no remainder.

14.13.5 **Placement of Identifiers with Synthetic Division.** An identifier, if present, is placed on the line with the dividend (the top line of the arrangement, in this case). One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 14-34

$$\begin{array}{ll rrrr} 197. & \underline{+2} & 1 & +6 & -1 & -30 \\ & & & +2 & +16 & +30 \\ \cline{3-6} \end{array}$$



Notice the vertical alignment of the operation signs. The numerals are aligned by place value, with the "1" directly above the "6" of "16".

PRACTICE 14H

Dividing Polynomials: Divide $(3x^4 + 12x^3 - 5x^2 - 18x + 8) \div (x + 4)$

–4	3	12	–5	–18	8
		–12	0	20	–8
	3	0	–5	2	0

Answer: $3x^2 - 5x - 2$

For further practice, see Appendix A—Reading Practice.

EXERCISE 14

Prepare Exercise 14 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 14A

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⠮⠞⠁⠴⠲

⠼⠃⠲ ⠸⠩ ⠦⠠⠁⠗⠉⠎⠊⠝⠸⠴ ⠸⠱ ⠊⠎ ⠮ ⠦⠔⠧⠻⠎⠑ ⠎⠔⠑⠴⠲

⠼⠉⠲ ⠸⠩ ⠎⠊⠝ ⠼⠒⠴⠘⠨⠡ ⠉⠕⠎ ⠼⠲⠢⠘⠨⠡

⠼⠲⠸⠲ ⠠⠄⠠⠮ ⠇⠕⠛⠎⠊⠝⠑ ⠸⠱ ⠋⠥⠝⠉⠰⠝ ⠊⠎ ⠗⠑⠇⠁⠞⠫ ⠞⠕

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PRACTICE 14B

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⠷⠆⠾ ⠎⠊⠝ ⠨⠹⠸⠌⠉⠕⠎ ⠨⠹

⠷⠒⠾ ⠎⠊⠝ ⠼⠆⠨⠁ ⠨⠅ ⠼⠆⠎⠊⠝ ⠨⠁⠉⠕⠎ ⠨⠁

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PRACTICE 14C

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PRACTICE 14D

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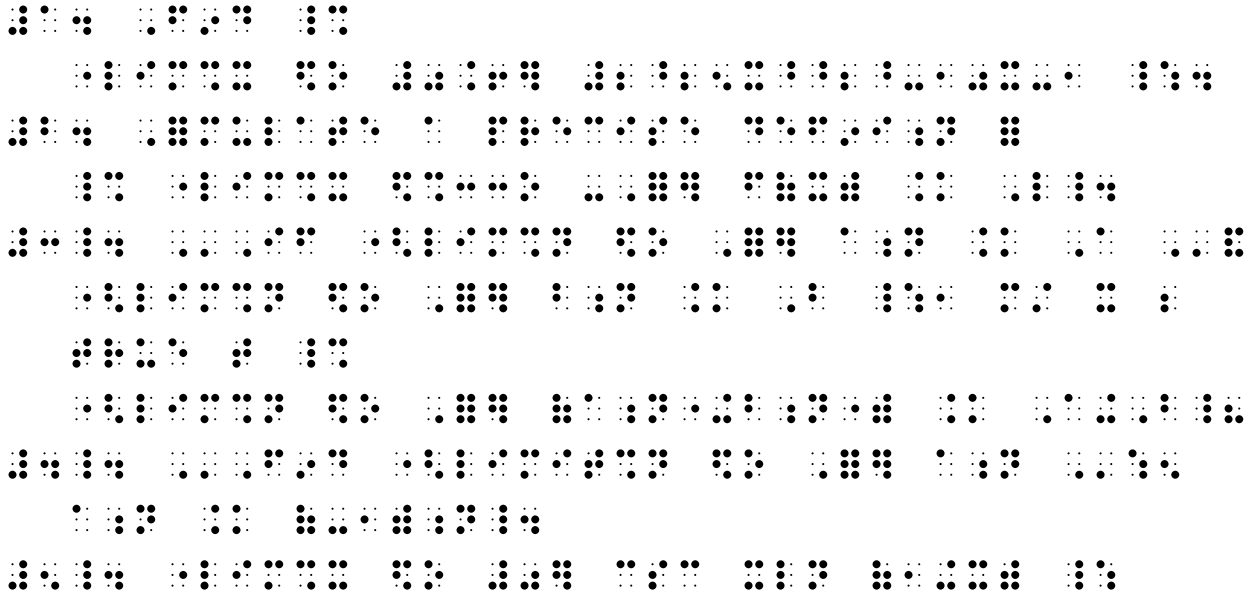
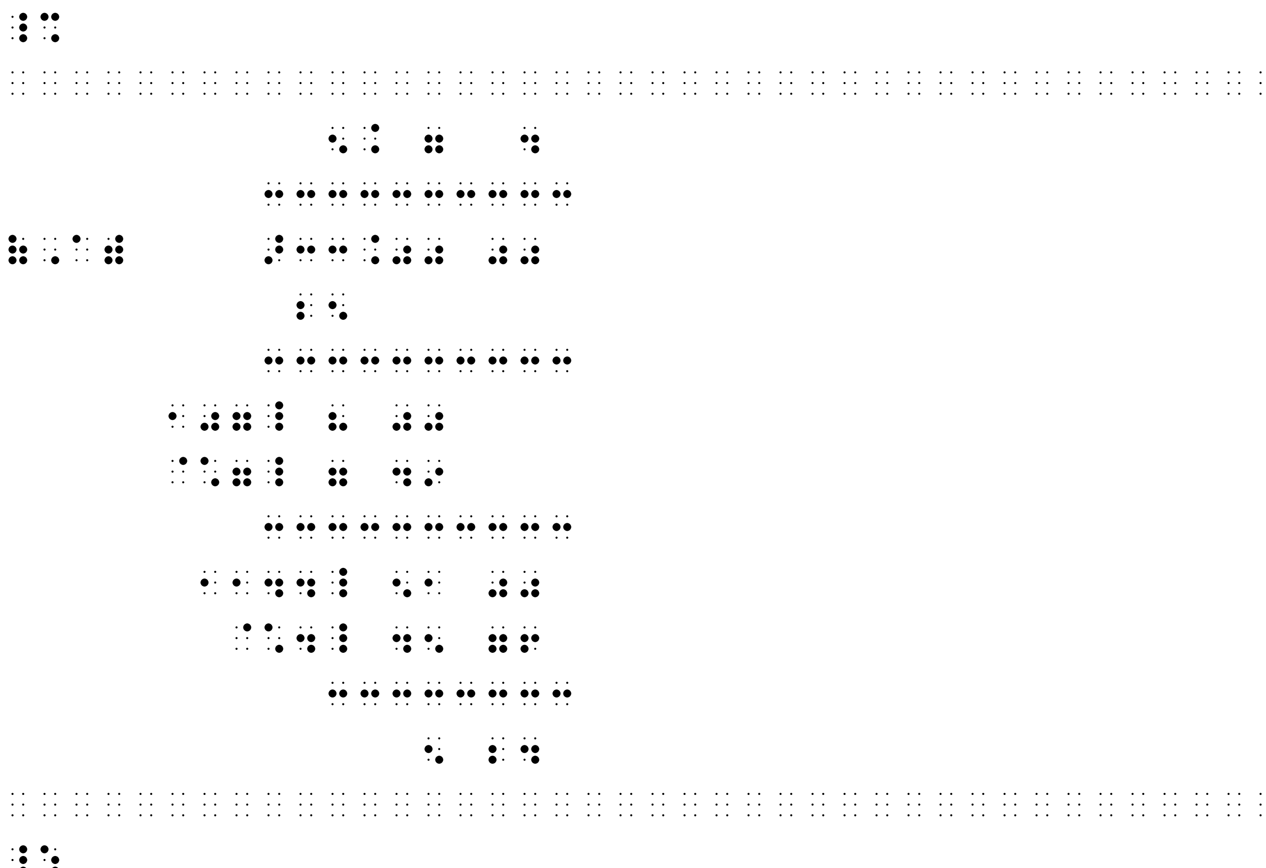
PRACTICE 14E



PRACTICE 14F



PRACTICE 14G

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PRACTICE 14H

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⠸⠩ ⠷⠒⠭⠘⠲⠐⠬⠂⠆⠭⠘⠒⠐⠤⠢⠭⠘⠆⠐⠤⠂⠦⠭⠬⠦⠾⠨⠌⠷⠭⠬⠲⠾

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    ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
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LESSON 15

Format

- MATHEMATICAL EXPRESSIONS REQUIRING RUNOVERS

Answers to Practice Material

LESSON PREVIEW

In this lesson, we look at the methods for transcribing a mathematical expression that is too long to fit on the current line. This often occurs when line length is restricted due to the indented margins in displayed mathematical material. However, even a 40-cell line may not provide enough room for a particularly long expression. The examples in this lesson serve as a good review of other aspects of the Nemeth Code.

MATHEMATICAL EXPRESSIONS REQUIRING RUNOVERS

[NC 26.2]

15.1 Review

A mathematical expression must not be divided between lines if it will fit on one braille line within the current margins. When there is insufficient space on the remainder of a line, the entire expression is brought down to the next line. One or both switch indicators can be placed on a separate line if, by doing so, the math will fit, undivided, on one line. Likewise, an identifier can be placed alone on the line if, by doing so, the math will fit, undivided, on the next line. Keeping the mathematical expression intact on one line is the priority.

In this lesson we discuss what to do when a mathematical expression is too long to be contained within the current margins – that is, when a division is unavoidable. First, here is a summary of items that must not be divided, and a review of runover rules already covered.

15.1.1 **Symbols to Keep Together.** The components of the following symbols must not be divided between braille lines.

a. A symbol of operation using plus and minus (Lesson 5)

b. A symbol of comparison compounded vertically or horizontally (Lesson 5)

c. A shape symbol with structural or interior modification (Lesson 11)

d. A character within a keystroke construction (Lesson 11)

e. Superposed symbols (Lesson 13)

f. Tally marks belonging to the same group (Lesson 13)

15.1.2 **Expressions to Keep Together.** The following expressions must not be divided between braille lines, even if divided in print.

a. A hyphenated expression of which one component is mathematical. (Lesson 2)

b. An abbreviation and its related numeral or letter. (Lessons 3 and 4)

c. An enclosed list. (Lesson 4)

d. A fraction, a mixed number. (Lesson 8)

e. A shape symbol and its name (numeral, letter, or sequence of letters). (Lesson 11)

f. The components of an expression modified according to the five-step rule. (Lesson 12)

g. A function name (or its abbreviated form) and its argument. (Lesson 14)

h. A two-part function name. (Lesson 14)

15.1.3 **Runover Rules Already Studied.** When a long expression won't fit on the braille line within current margins, the following rules apply. The new line begins in the runover cell of the current format.

a. **Long Numeral** (Lesson 1) Divide a long numeral after a comma if a comma is present. A hyphen is inserted at the point of division. If the numeral does not contain a comma, the hyphen may be inserted after any digit. The numeric indicator is restated before the first digit of the continuation of the numeral on the next line.

b. **Enclosed List** (Lesson 4) Divide an enclosed list after a comma used to separate the items.

c. **Linked Expression** (Lesson 8) Divide a linked expression before a comparison sign. It is not necessary to divide at every comparison sign unless it is a nested linked expression.

d. **Keystroke** (Lesson 11) Division may be made after any item in the keystroke string, but not within the keystroke.

15.1.4 **Nemeth Code Switch Indicators.** (Lessons 1 and 3) If both switch indicators will not fit on the same line as the math expression, the opening Nemeth Code indicator may fall on the previous line. The Nemeth Code terminator and any related punctuation may be placed on the following line.

15.2 Mathematical Units

The braille transcriber has only 40 cells available on a line, at most – perhaps as few as 30 cells for the runover to a nested link displayed to a subdivision. We often encounter a mathematical expression that will not fit on the current line. The margins in place at the time should not be changed in order to accommodate a long expression.

Runover sites should be chosen carefully. Every attempt should be made to keep the following mathematical units intact.

- a fraction
- a numerator
- a denominator
- a mixed number
- a base and its exponent; a subscript and its related item
- a grouped expression
- a radical expression
- a modified expression

A long or complicated mathematical expression can be organized into a series of mathematical units by following the procedures presented in this section. When the transcriber applies these principles, the reader is able to mentally reassemble the expression. On the other hand, a poorly divided expression will hinder the reader's understanding of the mathematics. Shrewd application of these guidelines can be properly rendered even if the transcriber is unfamiliar with the particular mathematics.

15.2.1 Follow the list below when choosing division sites, starting with Step i, keeping in mind that items enclosed within grouping symbols should not be divided.

i. Before a comparison sign on the baseline. (See 15.3)

ii. Before an operation sign on the baseline. (See 15.4)

iii. Before a mathematical unit. (See 15.5)

iv. After a termination indicator. (See 15.6)

Special considerations affect division of function notation, integral notation, and Sigma and Pi notation. (See 15.7)

In order to focus on the layout, the isolated examples with no narrative do not include code switch indicators. Unless otherwise noted, embedded material is assumed to be within a 3-1 paragraph and displayed material begins in cell 3 with runovers in cell 5.

15.3 Step i: Divide Before a Comparison Sign on the Baseline

Linked Expressions: As you learned in Lesson 8, when a linked expression will not fit on one braille line, a division is made at the link, before the sign of comparison. A few examples are shown here, as a review. You may wish to revisit the topic of linked expressions and nested linked expressions in Lesson 8. Key points are noted below.

- The comparison sign at which the division is made must be on the baseline of writing. A comparison sign within a modifier, superscript, subscript, fraction, radical expression, etc. is not a suitable division site. (Examples 15-7 and 15-8)
- The print copy may divide after a comparison sign, but the braille transcription follows Nemeth rules and divides before the comparison sign. (Example 15-35)
- The link begins in the appropriate cell according to the current format. (Examples 15-1 and 15-2)
- If the expression contains more than one link, it is not necessary to divide at every link unless it is a nested linked expression. (Example 15-2)
- Even if the anchor consists of only one letter or number, if the link will not fit on the line with the anchor, the line is divided after the anchor. (Example 15-3)
- When a line begins with a sign of comparison, the transition to a new braille line terminates the effect of any level indicator used on the line above, just as it would if it were not divided between lines. (Example 15-4)
- In itemized formats, if an anchor will not fit on the line with its identifier but it fits on the next line starting in the runover cell, put it there in order to keep the anchor intact. The identifier will then be the only item on the first line. (Examples 15-5 and 15-6)
- Material within mathematical grouping signs is a unit and should not be divided. (Example 15-8)

Example 15-1

(two layouts)

$1{,}778 + 1{,}294 + 865 + 905 + 2574 + 485 + 100 \; > \; 8{,}000$

Embedded:

As an embedded expression, the link continues on the next line in the runover cell of the current format.

Displayed:

As a displayed expression, the link begins on the next line, indented two cells from the anchor.

Example 15-2

(displayed)

To factor the expression $-2ab + a^2 + b^2$,
Jared wrote: $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b - a)^2$
Dom wrote: $-2ab + a^2 + b^2 = a^2 - 2ab + b^2 = (a - b)^2$
Both solutions are correct. Explain.

Lines 1-2: Paragraph begins in cell 3 with runovers in cell 1.
Lines 3-6: Two displayed expressions – each begins in cell 3, with runover in cell 5.
Lines 4 and 6: It is not necessary to divide the linked expression at every comparison sign.
Line 7: Paragraph continues in the runover cell (cell 1).

Example 15-3

(embedded)

The degree of the polynomial $y = 6x^3 - 2x^2y^1 + x^2y^2 - 3x^2y^2 + z$ is 4.

⠀⠀⠠⠮ ⠙⠑⠛⠗⠑⠑ ⠷ ⠮ ⠏⠕⠇⠽⠝⠕⠍⠊⠁⠇⠒ ⠸⠩ ⠽
⠨⠅ ⠼⠖⠭⠘⠒⠐⠤⠆⠭⠘⠆⠐⠽⠘⠂⠐⠬⠭⠘⠆⠐⠽⠘⠆⠐⠤⠒⠭⠘⠆⠐⠽⠘⠆⠐⠬⠵
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This anchor consists of the letter y and sits alone at the end of line 1 because it does not fit on the same line as its long link.

Example 15-4

(as a subdivision starting in cell 3)

13.
a. $16x^4 + 81y^4 = (4x^2)^2 + (9y^2)^2$

⠼⠂⠒⠸⠲
⠀⠀⠰⠁⠸⠲ ⠼⠂⠖⠭⠘⠲⠐⠬⠦⠂⠽⠘⠲
⠀⠀⠀⠀⠨⠅ ⠷⠲⠭⠘⠆⠐⠾⠘⠆⠐⠬⠷⠔⠽⠘⠆⠐⠾⠘⠆

The link begins in the current runover cell—cell 5, in this case. Transition to a new line following a sign of comparison returns the reader to the baseline.

Example 15-5

(a main division starting in cell 1)

13. $16x^4 + 81y^4 = (4x^2)^2 + (9y^2)^2$

⠼⠂⠒⠸⠲
⠀⠀⠼⠂⠖⠭⠘⠲⠐⠬⠦⠂⠽⠘⠲ ⠨⠅ ⠷⠲⠭⠘⠆⠐⠾⠘⠆⠐⠬⠷⠔⠽⠘⠆⠐⠾⠘⠆

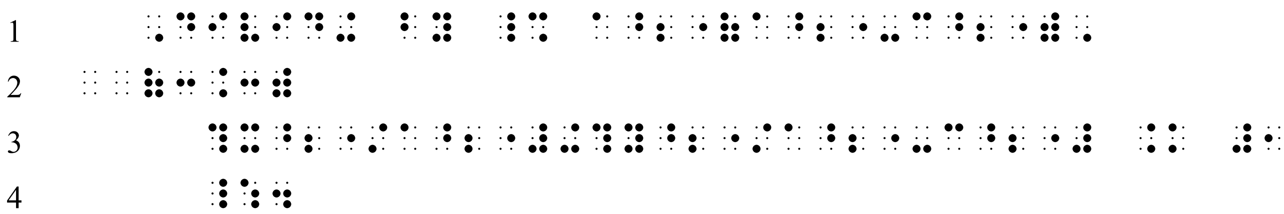
The expression begins in the current runover cell—cell 3, in this case—because it does not fit on the line above with its idenifier.

Example 15-6

(displayed)

Dividing by $a^2(a^2 - c^2)$,

$$\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1. \qquad (3.3)$$

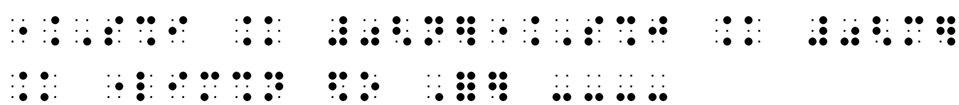


Line 1: The paragraph begins in cell 3.
Lines 2-4: Displayed margins to narrative are 3-5.
Line 2: The label stands alone because the math will not fit on this line. (Review Lesson 7 regarding displayed material with labels printed to the right.)
Line 3: The displayed expression will fit on one line in the runover position—cell 5, in this case.
Line 4: The Nemeth Code terminator and punctuation stand alone because they do not fit on the line above.

Example 15-7

(embedded)

$\sum_{i=0}^{n}\sum_{j=0}^{m} = \lim_{n\to\infty} \underline{\hspace{2em}}$

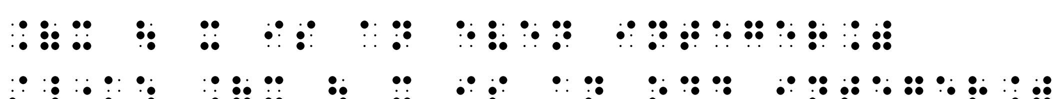


Only the comparison sign on the baseline is a suitable division site.

Example 15-8

(embedded)

{x | x is an even integer} $\not\subseteq$ {x | x is an odd integer}



Although $\not\subseteq$ {x *will fit at the end of the first line, only the comparison sign outside of the grouped expressions is a suitable division site.*

Instructions: Keep the following points in mind as you select runover sites. (1) Keep mathematical units intact on one line, if possible; (2) before dividing an expression, try placing the switch indicators on a different line; (3) before dividing an expression, try placing the identifier on a different line; (4) if the entire expression will not fit on the line, divide before a comparison sign on the baseline.

PRACTICE 15A

A. Solve the linear inqualities.

i. $(x+\frac{10}{3})(x+\frac{19}{3}) > (3x+\frac{46}{3})(\frac{x}{3}+1)$

ii. $\frac{2x}{3} - 3 > \frac{16x}{21} - \frac{13}{3} - \frac{2x}{15}$

iii. $(a-1)^2 - (a-7)(a-3) < 2a + 0.8$

B. $2 \times 423 = (2 \times 400) + (2 \times 20) + (2 \times 3) = 800 + 40 + 6 = 846$

Binomial coefficients get their name because they are the *coefficients* in the expansion of a *binomial:*

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

15.4 Step ii: Divide Before an Operation Sign on the Baseline

As foreshadowed in Lesson 8 in the context of a nested linked expression, a new line may begin before a sign of operation on the baseline of writing. It is not necessary to divide at every operation sign.

Keep these rules in mind as you study the following examples.

- Use as much of the line as possible before dividing the expression.
- The operation sign at which the division is made must be on the baseline of writing. An operation sign within a modifier, superscript, subscript, fraction, radical expression, etc. is not a suitable division site. (Example 15-14)
- In a linked expression, if either anchor or link is divided, a each link must begin on a new line. (Examples 15-15, 15-16, and 15-18)
- The print copy may divide after an operation sign, but the braille transcription follows Nemeth rules and divides before the operation sign. (Example 15-18)
- Transition to a runover line does not take the place of a necessary baseline indicator. If an operation sign follows a superscript, begin the new line with the baseline indicator. (Example 15-11) Similarly, if an operation sign follows a subscript that has a subscript indicator, begin the new line with the baseline indicator. If the subscript does not use a subscript indicator, there will be no subsequent baseline indicator. Review Lesson 6 regarding nonuse of the subscript indicator. (Example 15-17)
- If the transition to a new line is made before a minus sign and a numeral, or before a minus sign and a decimal point and a numeral, a numeric indicator is required. (Example 15-10)
- Material within mathematical grouping signs is a unit and should not be divided. (Example 15-13)

Example 15-9

(embedded)

$$1\frac{5}{6}+9\frac{1}{2}+3\frac{1}{12}+2\frac{1}{4}+3\frac{1}{2}$$

⠼⠂⠸⠹⠢⠸⠌⠖⠸⠼⠬⠔⠸⠹⠂⠸⠌⠆⠸⠼⠬⠒⠸⠹⠂⠸⠌⠂⠆⠸⠼⠬⠆⠸⠹⠂⠸⠌⠲⠸⠼
⠬⠒⠸⠹⠂⠸⠌⠆⠸⠼

Runover line (line 2): A numeral preceded by a plus sign does not require a numeric indicator even when the plus sign begins a braille line.

Example 15-10

(displayed)

$$2x^4 + 14x^3 - 15y^3 + 5x^2 - 4y^2 + 16x - 10y + 31$$

Runover line (line 2): A numeral preceded by a minus sign requires a numeric indicator when the minus sign begins a braille line.

Example 15-11

(displayed)

$$49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2$$

The baseline indicator is the first symbol in the runover line of this divided expression.

15.4.1 A Sign of Operation within a Mathematical Unit

a. **Keep Together: Grouped Expressions.** Unless unavoidable, items enclosed within grouping signs should not be divided between lines. An operation sign within a grouped expression is not a suitable division site.

Example 15-12

(embedded)

Solve, using PEMDAS: $(2^2 + 3^3) - 2^5 \div 4 + (22 \div 2 - 2 \cdot 5) + (14 - 6 \div 6)$

The first grouped expression will fit on the current line of this embedded expression.

Example 15-13

(embedded)

$(2^2 + 3^3) - 2^5 \div 4 + (22 \div 2 - 2 \cdot 5) + (14 - 6 \div 6)$

Division is made before an operation sign that is outside of the grouping symbols.

b. **Keep Together: Fractions and Other Mathematical Units.** An operation sign within a fraction, modifier, superscript, subscript, radical expression, etc. is not a suitable division site.

Example 15-14

(displayed)

$$\frac{1}{y^2 - 6y + 8} + \frac{1}{y^2 - 16} - \frac{5}{y^2 + 2y - 8}$$

Only the operation signs on the baseline are suitable division sites, not those in the denominators of these fractions.

15.4.2 **Linked Expressions.** If an anchor or a link must be divided, further rules apply. ("Anchor" and "link" were defined in Lesson 8.) If a division occurs within the anchor or its link, a division must always be made before the link (Step i, divide before a comparison sign on the baseline). Furthermore, in a linked expression with more than one link, division must occur before <u>each</u> link if <u>any</u> link requires division. The resulting transcription maintains an orderly representation of mathematical units.

Example 15-15

(displayed)

$$20\frac{1}{6} = 1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2}$$

Step i: The linked expression is divided before the equals sign. Step ii: The long link is divided before a plus sign.

Example 15-16

(itemized)

5. $144\ \text{ft}^2 + 17\ \text{ft}^2 + 112\ \text{ft}^2 + 15\ \text{ft}^2 - 131\ \text{ft}^2 = N$

Working backwards, the linked expression is divided before the equals sign (Step i). The long anchor is divided before a plus sign (Step ii). The link must not be placed on line 2, even though it will fit, because Step ii has been applied.

Example 15-17

(itemized)

1. Consider the following chemical equations.

   a. $CH_3CH_2CH_2COONa + CH_3OH \rightarrow CH_3CH_2CH_2COOCH_3 + NaOH$

   b. …

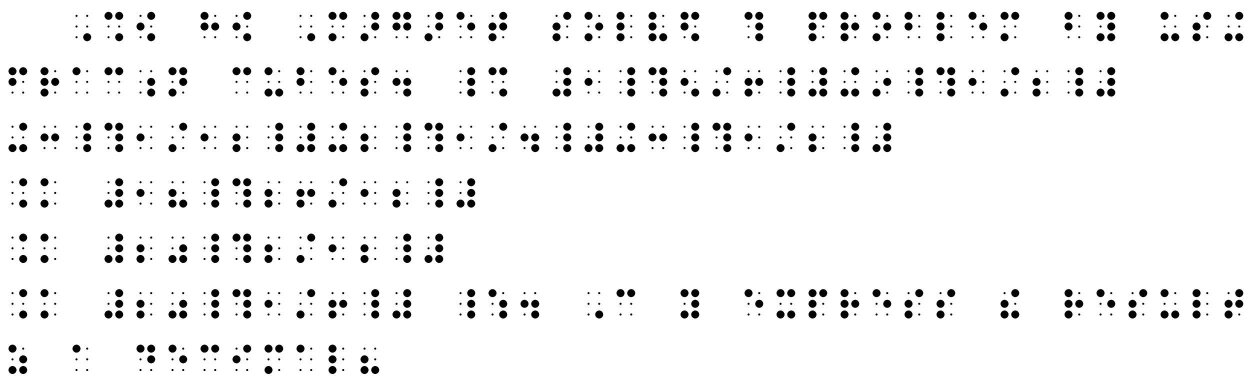
Lines 2-3: The anchor does not fit on the line with the identifier, but it will will fit entirely on the next line. It begins in the runover position for subdivisions, which is cell 5.

Lines 4-5: Line 4 begins with the comparison sign (right-pointing arrow). The link will not fit on one line. It is divided before the plus sign. Because the subscript 3 does not require a subscript indicator, there is no baseline indicator needed before the plus sign on line 5.

Example 15-18

(embedded)

Show how Margaret solved this problem by using fraction cubes. $1\frac{5}{6}+9\frac{1}{2}+3\frac{1}{12}+2\frac{1}{4}+3\frac{1}{2}=18\frac{26}{12}=20\frac{2}{12}=20\frac{1}{6}$. Can you express the result as a decimal?



Because the anchor is divided, the expression is also divided before <u>*each*</u> *link. The paragraph continues following the completion of the embedded expression.*

Instructions: In addition to the tactics outlined with PRACTICE 15A, keep the following point in mind as you select runover sites. Apply Step i (divide before a comparison sign) before applying Step ii (divide before an operation sign).

PRACTICE 15B

A. $\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7=490°$

B. $2\frac{3}{4}$ yd $+\ 1\frac{3}{4}$ yd $+\ \frac{3}{4}$ yd $=5\frac{1}{4}$ yd

C. $\sqrt{(x+a^2)+(y+a^2)}-\sqrt{(x-a^2)+(y-a^2)}=\pm 2a$

Sommer's Routine: Sommer's routine can be represented by the following expression.

Routine S: $5+3+4(1+(-1))+(-3)+(-5)+3\big(5+(-2)\big)+1$

Draw a simple diagram to represent *Routine S*.

15.5 Step iii: Divide Before a Mathematical Unit

The need to apply this rule occurs most often when the line length is restricted due to indented margins applied to displayed material. As mentioned at the beginning of this lesson, by seeing an expression as a series of mathematical units, the transcriber can make wise decisions when a long expression must be divided. Prudent division sites allow the reader to mentally reassemble the expression in an orderly fashion.

15.5.1 **A Fraction is a Unit.** A fraction, as a whole, is a unit. Within a fraction, the numerator and the denominator are each a unit. If an expression requires division when a fraction is encountered, follow these guidelines.

- If the entire fraction will fit on one line, divide before the opening fraction indicator. If a baseline indicator is required before the fraction indicator, divide the expression before the baseline indicator.
- If the entire fraction will not fit on one line, divide before the fraction line. (Examples 15-19 and 15-20) If a baseline indicator is required before the fraction line, divide the expression before the baseline indicator. (Example 15-21)
- If the numerator or denominator requires division, division must also be made before the fraction line. (Example 15-21)
- If the fraction is part of a mixed number, see 15.5.2.

Example 15-19

(displayed)

$$\frac{93{,}000 \times 0.0006 \times 2.0 \times 10.56}{32 \times 1.257 \times 10}$$

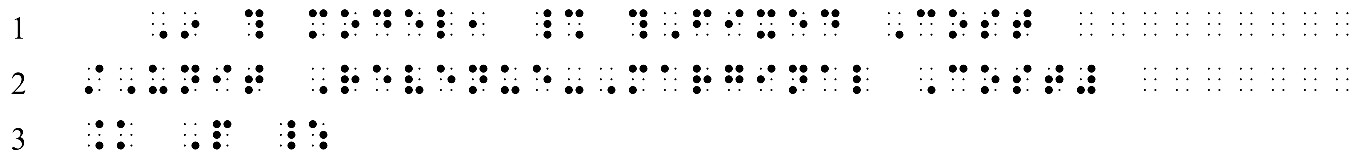
⠀⠀⠹⠔⠒⠠⠴⠴⠴⠈⠡⠴⠨⠴⠴⠴⠖⠈⠡⠆⠨⠴⠈⠡⠂⠴⠨⠢⠖
⠌⠒⠆⠈⠡⠂⠨⠆⠢⠶⠈⠡⠂⠴⠼

A division is made before the fraction line.

Example 15-20

(embedded)

In this model, $\frac{\text{Fixed Cost}}{\text{Unit Revenue}-\text{Marginal Cost}} = P$



Line 1: Since the entire fraction will not fit on one line, the numerator falls on the same line as the narrative. A division is made before the fraction line.

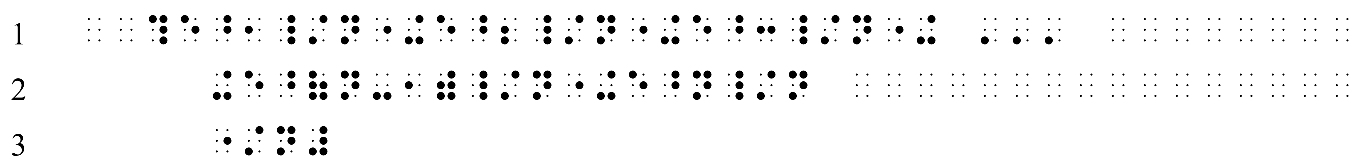
Line 2: The fraction line and the denominator, with the closing fraction indicator, are on line 2.

Line 3: Even though the link (= P) will fit on line 2, a division before the comparison sign is required. (Step i: divide before a comparison sign on the baseline.)

Example 15-21

(displayed)

$$\frac{e^{1/n} + e^{2/n} + e^{3/n} + \ \ldots + e^{(n-1)/n} + e^{n/n}}{n}$$



Lines 1-2: The numerator will not fit on one line. It is divided before a sign of operation that falls on the baseline within the numerator.

Line 3: Because the numerator is divided, the denominator must be on a new line, beginning with the fraction line. A baseline indicator is needed before this fraction line because the last item in the numerator is a superscript. The baseline indicator begins the second runover line.

15.5.2 **A Mixed Number is a Unit.** Do not separate the fractional part of a mixed number from its whole number. Review the topic of mixed numbers in Lesson 8.

Example 15-22

(displayed)

$$7\frac{4}{9}+13\frac{7}{12}+9\frac{5}{18}=7\frac{16}{36}+13\frac{21}{36}+18\frac{10}{36}=38\frac{47}{36}=38+1+\frac{11}{36}=39\frac{11}{36}$$

```
1   ⠼⠶⠸⠹⠲⠌⠔⠸⠼⠬⠂⠒⠸⠹⠶⠌⠂⠆⠸⠼⠬⠔⠸⠹⠢⠌⠂⠦⠸⠼
2     ⠨⠅ ⠼⠶⠸⠹⠂⠖⠌⠒⠖⠸⠼⠬⠂⠒⠸⠹⠆⠂⠌⠒⠖⠸⠼ ⠀⠀⠀⠀⠀⠀⠀⠀⠀
3     ⠬⠂⠦⠸⠹⠂⠴⠌⠒⠖⠸⠼
4     ⠨⠅ ⠼⠒⠦⠸⠹⠲⠶⠌⠒⠖⠸⠼
5     ⠨⠅ ⠼⠒⠦⠬⠂⠬⠹⠂⠂⠌⠒⠖⠼
6     ⠨⠅ ⠼⠒⠔⠹⠂⠂⠌⠒⠖⠼
7     ⠨⠅ ⠼⠲⠆⠸⠹⠢⠌⠂⠦⠸⠼
```

Line 1: The anchor fits on line 1, starting in cell 3.
Lines 2-3: The first link requires division. Each mixed number is a mathematical unit. Division is made before the second plus sign.
Lines 4-7: Each link must begin on a new line because one of the links is divided.

15.5.3 **A Base and its Exponent are a Unit.** The general rule is to keep a base and its exponent together on the same line, but if the unit must be divided, begin the new line with the superscript indicator.

If the exponent (superscript) does not fit in its entirety on the new line, apply further division rules within the superscript. If transition to a new braille line must be made within a superscript, the level in effect continues just as it would if the expression were not divided. (<u>Example 15-25</u>)

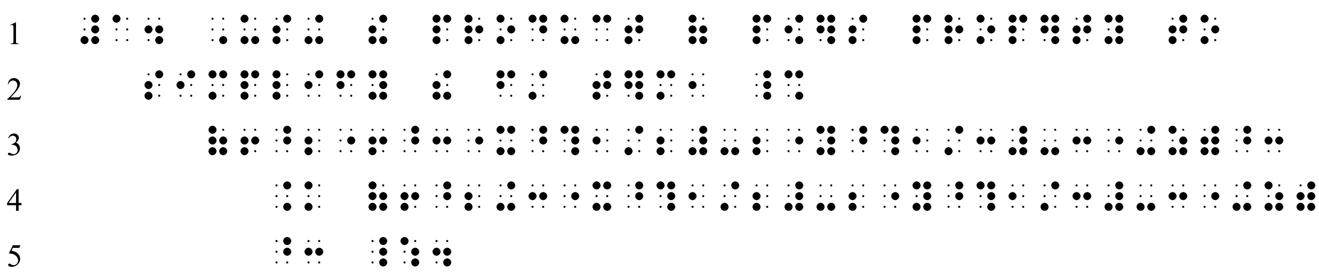
Similar rules apply to a subscript and its related item.

Example 15-23

(displayed)

1. Using the product of powers property to simplify the first term,

$$(6^2 6^3 x^{\frac{1}{2}-2} y^{\frac{1}{3}-3} + z)^3 = (6^{2+3} x^{\frac{1}{2}-2} y^{\frac{1}{3}-3} + z)^3.$$

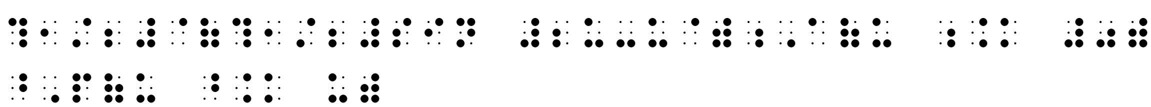


Line 3: The anchor fits on one line.
Line 4: The link must be divided. The expression between the parentheses (the base) fits on the line but the superscript (the exponent) does not.
Line 5: The superscript begins the runover line, starting with the superscript indicator.

Example 15-24

(embedded)

$$\frac{1}{2}\left[\frac{1}{2}\sin 2u - u\right]_{A(u=0)}^{P(u=u)}$$

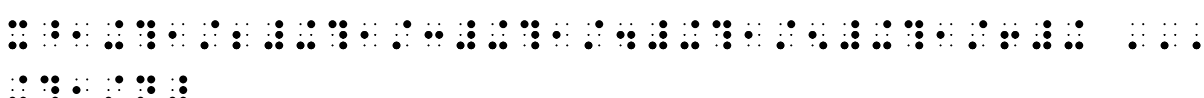


The expression enclosed between brackets is a unit and so is not divided. The subscript is transcribed first, according to Nemeth rules for simultaneous superscripts and subscripts (Lesson 6). The subscript fits on the line but the superscript does not. The superscript begins the runover line, starting with the superscript indicator.

Example 15-25

(embedded)

$$x^{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\dots+\frac{1}{n}}$$



The superscript level initiated on line 1 continues on line 2 without need for restatement, as it would were division not necessary. (Review Section 6.12.4 in Lesson 6 regarding an ellipsis in the superscript position.)

Instructions: Apply the tactics outlined with PRACTICE 15A and PRACTICE 15B as you select runover sites. Note that the ellipsis in item c. is on the baseline of writing.

PRACTICE 15C

1. Compute and/or simplify.

a. $$\frac{\left(\frac{3}{2}\right) \times \left(\frac{1}{2}\right) \times \left(-\frac{1}{2}\right)}{1 \times 2 \times 3}$$

b. $$\frac{\frac{\mathrm{d}x}{\mathrm{d}t}\frac{\mathrm{d}^2y}{dt^2} - \frac{\mathrm{d}^2x}{\mathrm{d}t^2}\frac{\mathrm{d}y}{\mathrm{d}t}}{(\mathrm{d}x/\mathrm{d}t)^3}$$

c. $(\pm)a_{1i_1}a_{2i_2}a_{3i_3}a_{4i_4} \ldots a_{ni_n}$

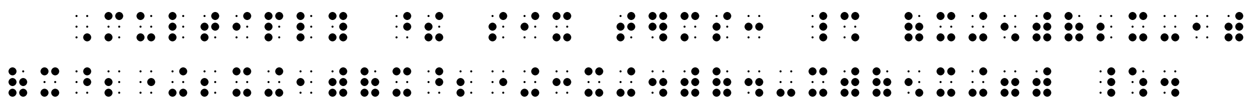
15.5.4 A Grouped Expression is a Unit

a. **A Series of Unspaced Grouped Expressions.** If consecutive groupings do not fit on the line, division may be made between groupings, beginning a new line with the left grouping symbol of the next factor.

Example 15-26

(embedded)

Multiply these six terms: $(x+5)(2x-1)(x^2+2x+1)$ $(x^2+3x+4)(4-x)(5x+7)$.



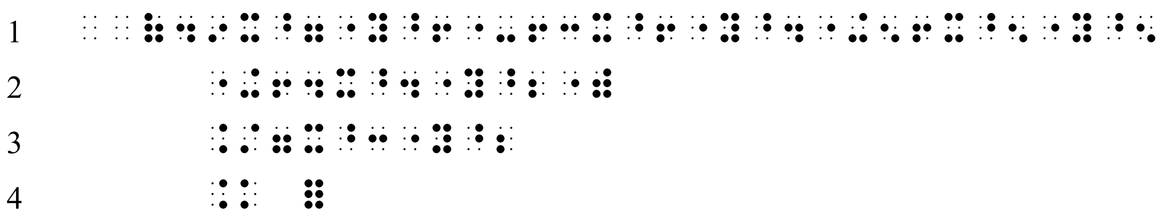
Division is made between terms that are enclosed between parentheses.

b. **A Grouped Expression Will Not Fit on the Line.** If a grouped expression will not fit on one line, follow the guidelines in this lesson to determine the best place to divide it. If the grouped expression is an enclosed list, divide after a comma.

Example 15-27

(displayed)

$$(49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2) \div 7x^3y^2 = ?$$



Line 1: The entire grouped expression will not fit on one line. Division is made before the baseline indicator associated with a plus sign.
Line 3: Although the rest of the anchor will fit on line 2, division is made before the operation sign (division symbol) according to Step ii.
Line 4: A new line must begin with the equals sign because Step i (dividing before a comparison sign) must be applied when the anchor is divided.

Example 15-28

(displayed)

$$y = \left(-8, -4, \frac{1}{2}, 3, 8, 10, 20\frac{3}{4}, 100\right)$$

⠀⠀⠽
⠀⠀⠀⠀⠨⠅ ⠷⠤⠦⠠ ⠤⠲⠠ ⠹⠂⠌⠆⠼⠠ ⠒⠠ ⠦⠠ ⠂⠴⠠
⠀⠀⠀⠀⠆⠴⠸⠹⠒⠌⠲⠸⠼⠠ ⠂⠴⠴⠾

The first division is made before the comparison sign (Step i). The link is an enclosed list and will not fit on one line. Division is made after a comma. See Lesson 4 for further rules regarding enclosed lists.

PRACTICE 15D

1. Multiply these four polynomials: $(4x^9y)(7x^8y^2)(5x^7y^3)(6x^6y^4)$.

2. $\pm\left(\frac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}}{3}\right)^{\frac{3}{2}}$

3. $(16x^4 + 8x^3y + 4x^2y^2 + 2xy^3 + y^4)(2x - y) = 32x^5 - y^5$

15.6 Step iv: Divide After a Termination Indicator

If no suitable division site can be found within a long string of expressions, and if a termination indicator is present, begin a new line after the termination indicator.

15.6.1 **A Radical Expression is a Unit.** Division may be made after the termination indicator that ends a radical expression. If the entire radical expression will not fit on the line, apply division strategies to the radicand.

Example 15-29

(displayed)

$$\sqrt[5]{32a^5b^5c^5d^5}\ \sqrt[5]{2abc^2d^3e^2}$$

⠀⠀⠣⠢⠜⠒⠆⠁⠘⠢⠐⠃⠘⠢⠐⠉⠘⠢⠐⠙⠘⠢⠐⠻
⠀⠀⠀⠀⠣⠢⠜⠆⠁⠃⠉⠘⠆⠐⠙⠘⠒⠐⠑⠘⠆⠐⠻

Division is made after the termination indicator of the first radical expression.

Example 15-30

(itemized)

2. $\sqrt[3]{x^1}\ \sqrt[6]{x^2}\ \sqrt[12]{x^3}\ \sqrt[24]{x^4}\ \sqrt[48]{x^5}\ \ldots$

⠼⠆⠸⠲⠀⠣⠒⠜⠭⠘⠂⠐⠻⠣⠖⠜⠭⠘⠆⠐⠻⠣⠂⠆⠜⠭⠘⠒⠐⠻⠣⠆⠲⠜⠭⠘⠲⠐⠻
⠀⠀⠣⠲⠦⠜⠭⠘⠢⠐⠻⠀⠄⠄⠄

Division is made after the termination indicator of the fourth radical expression.

15.6.2 **A Modified Expression is a Unit.** Division may be made after the termination indicator that ends an expression which is modified according to the five-step rule of modification. (Lesson 12). If the entire modified expression will not fit on the current line, it is acceptable to divide before the directly-over or the directly-under indicator. The next section discusses strategies for dividing longer modified expressions.

Note: In item 2, the expression is displayed.

PRACTICE 15E

1. $\sqrt[3]{x^1}\ \sqrt[6]{x^2}\ \sqrt[12]{x^3}\ \sqrt[24]{x^4}\ \sqrt[48]{x^5}\ \ldots$
2. The general solution for one root of the cubic equation is

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}\ .$$

15.7 Function Notation, Integral Notation, Sigma Notation, and Pi Notation

We hope you will bear with us as we conclude this lesson with some rather complicated-looking notation which is commonly encountered in the study of calculus. The Nemeth Code gives basic guidelines for choosing runover sites in a long mathematical expression. In this lesson manual, we look more specifically at the topic as it applies to functions, integral, Sigma, and Pi notation. Frequently, the transcriber needs to choose between which mathematical unit to keep intact and which unit to divide in order to present such notation clearly.

There often is more than one valid layout. In this section, we have collected advice from mathematicians to help in your decision making. Please note that these strategies are not explicitly discussed in the Nemeth Code.

15.7.1 **A Function and its Argument are a Unit.** The space between a function and the following expression (the "argument") is not a suitable division site. If the entire expression will not fit on the line, apply division strategies to the argument. Sometimes, this will mean dividing within a mathematical unit.

Example 15-31

In the logarithmic number system, LNS $= -\log\left[2\sin\frac{\theta}{2}\cdot e^{-i\left(\frac{\pi}{2}-\frac{\theta}{2}\right)}\right]$.

⠀⠀⠠⠔ ⠮ ⠇⠕⠛⠜⠊⠹⠍⠊⠉ ⠝⠥⠍⠃⠻ ⠎⠽⠌⠑⠍⠂ ⠸⠩ ⠠⠠⠇⠝⠎
⠨⠅ ⠤⠇⠕⠛ ⠈⠷⠆⠎⠊⠝ ⠹⠨⠹⠌⠆⠼
⠡⠑⠘⠤⠊⠷⠹⠨⠏⠌⠆⠼⠤⠹⠨⠹⠌⠆⠼⠾⠐⠈⠾ ⠸⠱⠲

LNS is the abbreviation for "logarithmic number system" and so uses the double capitalization indicator. Even though the grouped expression will fit on one line, division is made within the argument, not at the space after the abbreviated function name.

PRACTICE 15F

1. Daylight ...

   a. The number of minutes of daylight for any location at 60° N latitude is modeled by the function below, where $d =$ day of the year.

$$m(d) = 390\sin\left[\frac{2\pi}{365.25}(d-80)\right] + 738$$

15.7.2 **Integral Notation.** It is helpful to understand that this mathematical unit is comprised of the entire expression from the integral symbol to the dx, dy, etc. (the "differential"). To a mathematician, it is important to keep the integral and its associated modifiers or superscript/subscript intact, and also to begin the associated expression which follows the integral on the same line, if possible. Additionally, it is desirable not to leave the differential dangling alone on a line.

When a choice must be made, it is advisable to apply division strategies to the expression between the integral and the differential. This may mean dividing within a mathematical unit such as a fraction or a grouped expression. This is illustrated showing the same example using two types of integral notation—printing the limits as a superscript and subscript, and printing the limits as modifiers.

Example 15-32

$$\int_{-\infty}^{\infty} \frac{\dfrac{a(x)}{b(x)}}{\dfrac{c(x)}{f(x)}}\, dx$$

Embedded: The 40-cell expression will fit on one line.

⠮⠰⠤⠠⠿⠘⠿⠐⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭

Using displayed margins starting in cell 5, the expression will no longer fit on one line. Two options are illustrated, below.

Displayed: Option 1, dividing before dx (not recommended).

⠀⠀⠀⠀⠮⠰⠤⠠⠿⠘⠿⠐⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼
⠙⠭

Although this interpretation keeps the fraction intact, dx is now in an undesirable location, sitting alone on the next line.

Displayed: Option 2, dividing the fraction (recommended).

⠀⠀⠀⠀⠮⠰⠤⠠⠿⠘⠿⠐⠠⠹⠹⠁⠷⠭⠾⠌⠃⠷⠭⠾⠼
⠠⠌⠹⠉⠷⠭⠾⠌⠋⠷⠭⠾⠼⠠⠼⠙⠭

By dividing before the complex fraction line, the integral notation is on the same line as the beginning of its related expression.

Example 15-33

$$\int_{-\infty}^{\infty} \frac{\frac{a(x)}{b(x)}}{\frac{c(x)}{f(x)}}\, dx$$

Embedded: The 40-cell expression will fit on one line.

Using displayed margins, the expression will no longer fit on one line. Three options are illustrated, below.

Displayed: Option 1, dividing after the termination indicator (less desirable).

The Nemeth Code recommends dividing after the termination indicator of the modified integral. However, this separates the integral from the related expression which follows it. The mathematicians we consulted recommend Option 2, below.

Displayed: Option 2, dividing the fraction (recommended).

By dividing before the complex fraction line, the integral symbol is on the same line as the beginning of its related expression.

Displayed: Option 3, dividing before dx (not recommended).

The entire fraction will fit on the first line, but dx is now in an undesirable location, sitting alone on the next line.

Instructions: Make your division decision based on the mathematician's preference. (The unusual letter in the last numerator is a lowercase Greek zeta.)

PRACTICE 15G

The solutions involving the direct functions

$$\int \frac{\log^2(z+1)}{z}\,\mathrm{d}z = \log(-z)\log^2(z+1) + 2\mathrm{Li}_2(z+1)\log(z+1) - 2\mathrm{Li}_3(z+1)$$

and

$$\int_0^\infty \frac{\log(t+1)\log\left(1+\frac{1}{t^2}\right)}{t}\,\mathrm{d}t = C\pi - \frac{3\zeta(3)}{8}$$

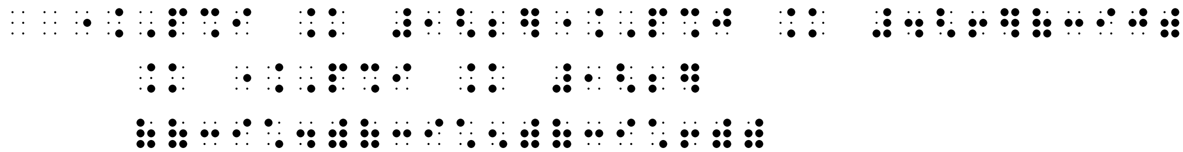
can be found in Chapter 7.

15.7.3 **Sigma and Pi Notation.** To a mathematician, it is important to keep the Sigma or Pi and its associated modifiers or superscript/subscript intact, and also to begin the associated expression which follows (the "argument") on the same line, if possible. When a choice must be made, it is advisable to apply division strategies to the argument. This may mean dividing within a mathematical unit such as a fraction or a grouped expression.

Example 15-34

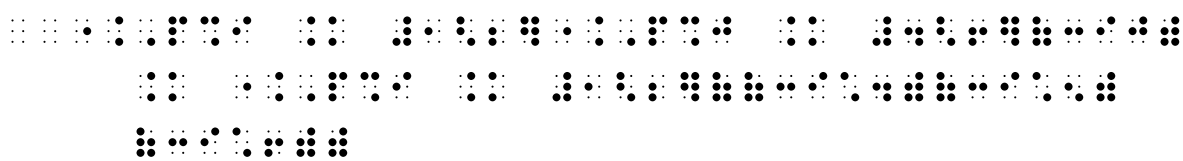
$$\prod_{i=1}^{2}\prod_{j=4}^{6}(3ij) = \prod_{i=1}^{2}\left((3i\cdot4)(3i\cdot5)(3i\cdot6)\right)$$

Displayed: Option 1, dividing after the termination indicator.



The link must be divided. The Nemeth Code recommends dividing after the termination indicator of the modification. Although this keeps the grouped expression intact, it separates the Pi notation from its related expression. The mathematicians we consulted recommend Option 2, below.

Displayed: Option 2, dividing between grouped factors.



The link must be divided. To keep the argument starting on the same line as the Pi notation, division is made between factors inside the larger grouping signs.

Example 15-35

$$d(\vec{x},\vec{y}) =$$

$$\sum_{\substack{Z_{xy}\in\vec{Z}_{xy}\\ \forall x\in\vec{x}\\ \forall y\in\vec{y}}} f\big(Z_{xy}\big)$$

Displayed: Option 1, dividing after the termination indicator.

```
1 ⠀⠀⠙⠷⠐⠭⠣⠫⠕⠻⠠⠀⠐⠽⠣⠫⠕⠻⠾
2 ⠀⠀⠀⠀⠨⠅⠀⠐⠨⠠⠎⠩⠠⠵⠰⠭⠽⠀⠈⠑⠀⠐⠠⠵⠣⠫⠕⠻⠰⠭⠽
3 ⠀⠀⠀⠀⠐⠩⠩⠈⠯⠭⠀⠈⠑⠀⠐⠭⠣⠫⠕⠻⠩⠩⠩⠈⠯⠽⠀⠈⠑⠀⠐⠽⠣⠫⠕⠻
4 ⠀⠀⠀⠀⠋⠷⠠⠵⠰⠭⠽⠐⠾
```

Line 1: The anchor fits on one line, in cell 3.
Line 2: The link begins in cell 5. The entire modified expression will not fit on the line. Division is made before the second order directly-under indicator, including the baseline indicator associated with it.
Line 3: There is not enough room on this line to complete the link. The Nemeth Code recommends dividing after the termination indicator of the modification.
Line 4: The argument is alone on this line. The mathematicians we consulted recommend Option 2, below.

Displayed: Option 2, dividing before a directly-under indicator.

```
1 ⠀⠀⠙⠷⠐⠭⠣⠫⠕⠻⠠⠀⠐⠽⠣⠫⠕⠻⠾
2 ⠀⠀⠀⠀⠨⠅⠀⠐⠨⠠⠎⠩⠠⠵⠰⠭⠽⠀⠈⠑⠀⠐⠠⠵⠣⠫⠕⠻⠰⠭⠽
3 ⠀⠀⠀⠀⠐⠩⠩⠈⠯⠭⠀⠈⠑⠀⠐⠭⠣⠫⠕⠻
4 ⠀⠀⠀⠀⠩⠩⠩⠈⠯⠽⠀⠈⠑⠀⠐⠽⠣⠫⠕⠻⠋⠷⠠⠵⠰⠭⠽⠐⠾
```

Line 1: The anchor fits on one line, in cell 3.
Line 2: The link begins in cell 5. The entire modified expression will not fit on the line. Division is made before the second order directly-under indicator, including the baseline indicator associated with it.
Line 3: There is not enough room on this line to complete the link. It is undesirable to leave the argument alone on the next line, so division is made before the third order directly-under indicator.

Instructions: Transcribe this example as if it were embedded within narrative, beginning with an opening Nemeth Code switch indicator in cell 1. Make your division decision based on the mathematician's preference.

PRACTICE 15H

$$\sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}$$

15.7.4 **Notation Combinations.** As with any formatting choice, consistency in your treatment of runovers in complicated mathematical expressions will give your reader the advantage of knowing what to expect. By keeping in mind the strategies learned in this lesson, your transcription will be well done.

Example 15-36

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n}\Delta x\cdot f(x_i)$$

Displayed: Option 1, dividing before the Sigma notation.

```
1  ⠀⠀⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭
2      ⠨⠅ ⠐⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻
3      ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠂⠣⠝⠻⠨⠠⠙⠭⠡⠋⠷⠭⠰⠊⠐⠾
```

Line 1: The anchor fits on one line.
Line 2: Division is made before the Sigma notation, although this puts the function's argument on a different line.
Line 3: The entire Sigma notation fits on one line.

Displayed: Option 2, dividing before the multiplication dot.

```
1  ⠀⠀⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭
2      ⠨⠅ ⠐⠇⠊⠍⠩⠝ ⠫⠕ ⠠⠿⠻ ⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠂⠣⠝⠻⠨⠠⠙⠭
3      ⠡⠋⠷⠭⠰⠊⠐⠾
```

Line 1: The anchor fits on one line.
Line 2: The function's argument begins on this line. In the Sigma notation, the argument is divided before the multiplication dot.
Line 2: Recall from Lesson 14 that, when a function name carries a modifier, the required space after the function name follows the termination of the modifier.

Instructions: Make your division decision based on the mathematician's preference.

PRACTICE 15I

1. The rectangle rule for numerical integration (The Midpoint Rule) is inspired by the use of Riemann sums.

$$\frac{b-a}{n}\sum_{i=0}^{n-1} f(a+i\frac{b-a}{n}) \approx \int_a^b f(x)dx$$

SUMMARY

If a mathematical expression must be divided, the following strategies and rules were explored in this lesson.

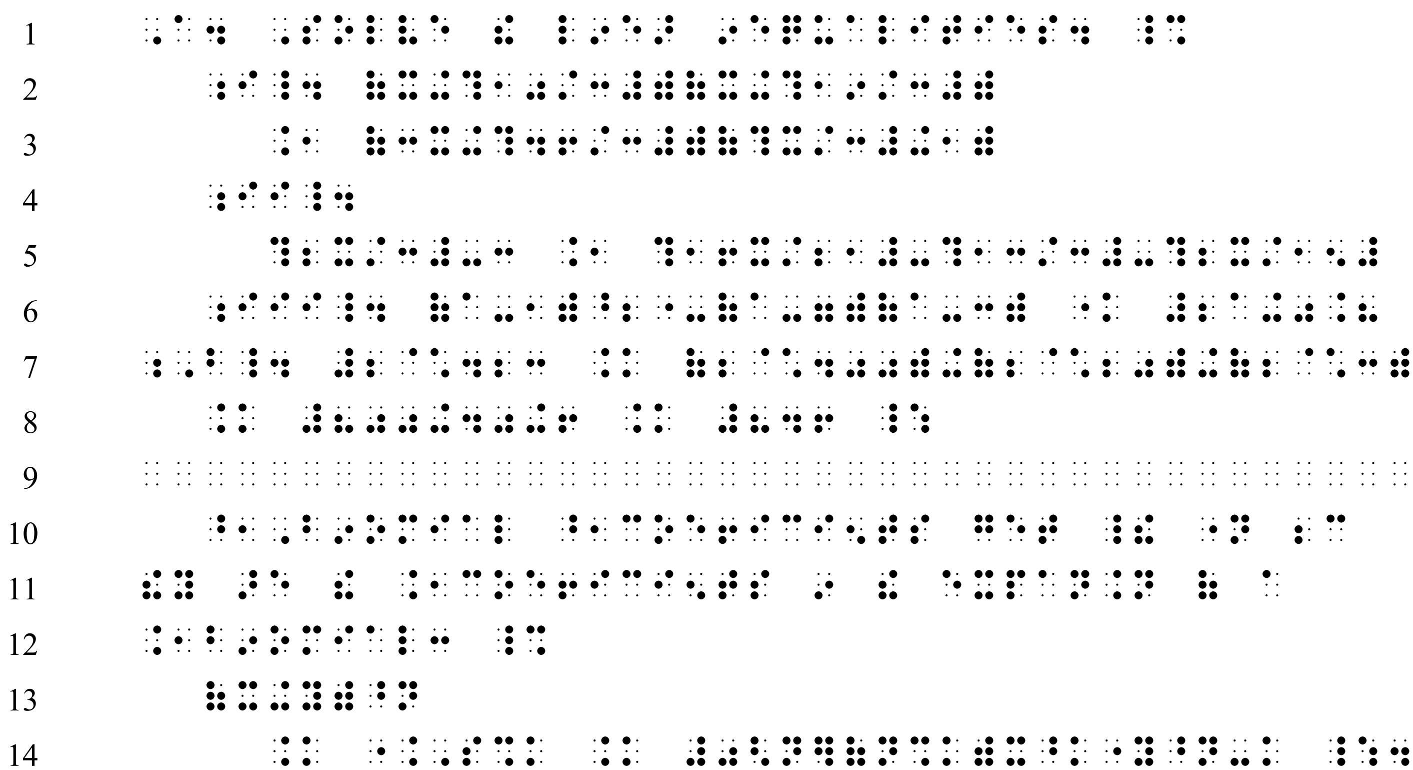
- When a linked expression will not fit on one line, division is made before the comparison sign. A comparison sign within a grouped expression, fraction, modifier, superscript, subscript, radical expression, etc. is not a suitable division site.
- When a link will not fit on one line, division is made before an operation sign. An operation sign within a grouped expression, fraction, modifier, superscript, subscript, radical expression, etc. is not a suitable division site.
- In a linked expression, if either the anchor or any link must be divided, a division must also be made before each link.
- Fractions are kept intact by dividing before an opening fraction indicator. If a fraction must be divided, division is made before the fraction line. A mixed number should not be divided.
- Transition to a runover line does not take the place of a necessary baseline indicator. The baseline indicator will be the first symbol on the new line.
- When an item and its exponent or subscript are too long to fit on a single braille line, division is made before a change-of-level indicator.
- The space between a function and its argument is not a suitable division site. If the entire expression will not fit on the line, division strategies are applied to the argument.
- Items within grouping symbols should not be divided. If consecutive groupings do not fit on the line, a division may be made between groupings.
- If a grouped expression will not fit on one line, division strategies are applied within the grouping symbols. If the grouped expression is an enclosed list, division is made after a comma.
- Division may be made after a mathematical termination indicator such as termination of a radical expression or termination of a modified expression.
- When an embedded math expression must be divided, it may begin on the current line provided division is made in accordance with the principles defined in this section.
- When a displayed math expression must be divided, all runovers begin two cells to the right of the original display cell unless it is a nested linked expression in which case a second indent level is applied.

EXERCISE 15

Prepare Exercise 15 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 15A



Line 2: The anchor fits on the line with the identifier.
Line 3: The link begins in the runover cell (cell 5).
Line 5: This expression will fit on one line if it begins on the line below its identifier.
Line 7: The anchor and the first link fit on the line with the identifier.
Line 8: The remaning two links fit on one line, beginning in the runover cell (cell 3).
Line 9: A blank line precedes the change in format from itemized material to a narrative paragraph.
Line 14: The runover line begins with the equals sign that begins the link. The equals sign in the modifier is not a suitable division site because it would disrupt the modifier.

PRACTICE 15B

```
1   ⠸⠩
2   ⠰⠠⠁⠸⠲ ⠫⠪ ⠼⠂⠬⠫⠪ ⠼⠆⠬⠫⠪ ⠼⠒⠬⠫⠪ ⠼⠲⠬⠫⠪ ⠼⠢
3     ⠬⠫⠪ ⠼⠖⠬⠫⠪ ⠼⠶
4     ⠨⠅ ⠼⠲⠔⠴⠘⠨⠡
5   ⠰⠠⠃⠸⠲
6     ⠼⠆⠸⠹⠒⠌⠲⠸⠼ ⠽⠙ ⠬⠂⠸⠹⠒⠌⠲⠸⠼ ⠽⠙ ⠬⠹⠒⠌⠲⠼ ⠽⠙
7     ⠨⠅ ⠼⠢⠸⠹⠂⠌⠲⠸⠼ ⠽⠙
8   ⠰⠠⠉⠸⠲ ⠜⠷⠭⠬⠁⠘⠆⠐⠾⠬⠷⠽⠬⠁⠘⠆⠐⠾⠻
9     ⠤⠜⠷⠭⠤⠁⠘⠆⠐⠾⠬⠷⠽⠤⠁⠘⠆⠐⠾⠻
10    ⠨⠅ ⠬⠤⠆⠁ ⠸⠱
11
12    ⠘⠂⠠⠎⠕⠍⠍⠻⠄⠎ ⠘⠂⠠⠗⠳⠞⠔⠑⠒ ⠠⠎⠕⠍⠍⠻⠄⠎ ⠗⠳⠞⠔⠑ ⠉
13  ⠆ ⠗⠑⠏⠗⠑⠎⠢⠞⠫ ⠃⠽ ⠮ ⠋⠕⠇⠇⠪⠬ ⠑⠭⠏⠗⠑⠎⠨⠝⠲
14    ⠠⠗⠳⠞⠔⠑ ⠰⠠⠎⠒ ⠸⠩ ⠼⠢⠬⠒⠬⠲⠷⠂⠬⠐⠷⠤⠂⠾⠾⠬⠷⠤⠒⠾
15      ⠬⠷⠤⠢⠾⠬⠒⠷⠢⠬⠷⠤⠆⠾⠾⠬⠂ ⠸⠱
16  ⠠⠙⠗⠁⠺ ⠁ ⠎⠊⠍⠏⠇⠑ ⠙⠊⠁⠛⠗⠁⠍ ⠞⠕ ⠗⠑⠏⠗⠑⠎⠢⠞
17  ⠨⠂⠠⠗⠳⠞⠔⠑ ⠨⠆⠰⠠⠎⠲
```

Line 4: Since a division is made in the anchor, a division must also be made before the link.
Line 6: The anchor will fit on one line if it starts in the runover position of this itemized problem (cell 3). The only necessary division is made before the link.
Line 8: Since the anchor will not fit on line 9, it begins on this line (8) until an appropriate division site is encountered.
Line 9: Division is made before the operation sign between the radical expressions.
Line 10: Since a division is made in the anchor, a division must also be made before the link.
Line 11: A blank line precedes the change in format from itemized material to a narrative paragraph.
Line 14: Since the expression will not fit, undivided, on line 15, it begins on this line (14) until an appropriate division site is encountered..
Line 15: The runover line begins with an operation sign that is not part of an expression enclosed between parentheses.

PRACTICE 15C

1 ⠼⠁⠲ ⠠⠉⠕⠍⠏⠥⠞⠑ ⠯⠸⠌⠕⠗ ⠎⠊⠍⠏⠇⠊⠋⠽⠲ ⠸⠩
2 ⠰⠁⠸⠲ ⠠⠹⠷⠹⠒⠌⠆⠼⠾⠈⠡⠷⠹⠂⠌⠆⠼⠾⠈⠡⠷⠤⠹⠂⠌⠆⠼⠾
3 ⠠⠌⠂⠈⠡⠆⠈⠡⠒⠠⠼
4 ⠰⠃⠸⠲ ⠠⠹⠹⠙⠭⠌⠙⠞⠼⠹⠙⠘⠆⠐⠽⠌⠙⠞⠘⠆⠐⠼
5 ⠤⠹⠙⠘⠆⠐⠭⠌⠙⠞⠘⠆⠐⠼⠹⠙⠽⠌⠙⠞⠼
6 ⠠⠌⠷⠙⠭⠸⠌⠙⠞⠾⠘⠒⠐⠠⠼
7 ⠰⠉⠸⠲ ⠷⠬⠤⠾⠁⠰⠂⠊⠰⠰⠂⠐⠁⠰⠆⠊⠰⠰⠆⠐⠁⠰⠒⠊⠰⠰⠒
8 ⠐⠁⠰⠒⠊⠰⠰⠲⠐⠄⠄⠄ ⠁⠰⠝⠊⠰⠰⠝ ⠸⠱

Line 4: The numerator will not fit on one braille line. It begins on this line.
Line 5: A division is made before the minus sign.
Line 6: Since the numerator is divided, a division is also required before the fraction line. (Review complex fractions in Lesson 8.)
Lines 7-8: Each base is on the same braille line as its subscript.
Line 8: A baseline indicator begins the runover line.
Line 8: Review the rules regarding an ellipsis on the baseline following a subscript in Lesson 6.

PRACTICE 15D

1 ⠼⠁⠲ ⠠⠍⠥⠇⠞⠊⠏⠇⠽ ⠘⠮ ⠋⠳⠗ ⠏⠕⠇⠽⠝⠕⠍⠊⠁⠇⠎⠒
2 ⠸⠩ ⠷⠲⠭⠘⠔⠐⠽⠾⠷⠶⠭⠘⠦⠐⠽⠘⠆⠐⠾⠷⠢⠭⠘⠶⠐⠽⠘⠒⠐⠾
3 ⠷⠖⠭⠘⠖⠐⠽⠘⠲⠐⠾⠸⠲
4 ⠼⠆⠸⠲
5 ⠬⠤⠷⠹⠁⠘⠹⠆⠌⠒⠼⠐⠬⠁⠘⠹⠂⠌⠒⠼⠐⠃⠘⠹⠂⠌⠒⠼⠐⠬⠃⠘⠹⠆⠌⠒⠼
6 ⠐⠌⠒⠼⠾⠘⠹⠒⠌⠆⠼
7 ⠼⠒⠸⠲
8 ⠷⠂⠖⠭⠘⠲⠐⠬⠦⠭⠘⠒⠐⠽⠬⠲⠭⠘⠆⠐⠽⠘⠆⠐⠬⠆⠭⠽⠘⠒⠐⠬⠽⠘⠲⠐⠾
9 ⠷⠆⠭⠤⠽⠾
10 ⠨⠅ ⠼⠒⠆⠭⠘⠢⠐⠤⠽⠘⠢

Lines 4-5: The grouped expression (the fraction) must be divided. The numerator will fit on one line when you start the expression on the line after the identifier, starting in the runover cell (cell 5).

Line 6: The simple fraction is divided at the fraction line. The line begins with its associated baseline indicator.

Line 8: The first grouping will fit on one line if it starts on the line following the identifier.

Line 10: Because the anchor is divided, division must also be made before the link.

PRACTICE 15E

1 ⠼⠁⠲ ⠸⠩ ⠣⠒⠜⠭⠘⠂⠐⠻⠣⠖⠜⠭⠘⠆⠐⠻⠣⠂⠆⠜⠭⠘⠒⠐⠻
2 ⠀⠀⠣⠆⠲⠜⠭⠘⠲⠐⠻⠣⠲⠦⠜⠭⠘⠢⠐⠻ ⠄⠄⠄ ⠸⠱
3 ⠼⠃⠲ ⠠⠮ ⠛⠢⠻⠁⠇ ⠎⠕⠇⠥⠰⠝ ⠿ ⠐⠕ ⠗⠕⠕⠞ ⠯ ⠮ ⠉⠥⠃⠊⠉
4 ⠀⠀⠑⠟⠥⠁⠰⠝ ⠊⠎
5 ⠀⠀⠀⠀⠭
6 ⠀⠀⠀⠀⠀⠀⠨⠅ ⠣⠒⠜⠤⠹⠟⠌⠆⠼
7 ⠀⠀⠀⠀⠀⠀⠬⠨⠜⠹⠟⠘⠆⠐⠌⠲⠼⠬⠹⠏⠘⠒⠐⠌⠆⠶⠨⠻⠻
8 ⠀⠀⠀⠀⠀⠀⠬⠣⠒⠜⠤⠹⠟⠌⠆⠼⠤⠨⠜⠹⠟⠘⠆⠐⠌⠲⠬⠹⠏⠘⠒⠐⠌⠆⠶⠼⠨⠻⠻
9 ⠀⠀⠀⠀⠀⠀⠸⠱⠲

Line 1: Division is made after the termination indicator, before the fourth radical expression.
Line 2: The runover line begins with the index-of-radical indicator.
Lines 3-4: The second itemized narrative begins in cell 1 with runovers in cell 3.
Line 5: The anchor consists only of the letter x (cell 5).
Line 6: The link begins with an equals sign in cell 7. Although the first radical expression will fit entirely on the next line, the equals sign cannot be the only symbol on line 6, so a division must be made within the radical expression. Division is made before the first plus sign in the radicand. The second plus sign is not a suitable division site because the nested radicand is a mathematical unit.
Line 7: The nested radical fits on this line. Division is made after the two termination indicators.
Line 8: This line begins with the plus sign before the second radical expression (Step ii). This radical expression will fit on one line, but the Nemeth Code terminator and the final period do not, so they fall line 9.

PRACTICE 15F

1 ⠼⠁⠲ ⠠⠐⠙⠇⠊⠣⠞ ⠲⠲⠲

2 ⠁⠲ ⠠⠮ ⠝⠥⠍⠃⠻ ⠷ ⠍⠔⠥⠞⠑⠎ ⠷ ⠐⠙⠇⠊⠣⠞ ⠿ ⠁⠝⠽

3 ⠇⠕⠉⠁⠰⠝ ⠁⠞ ⠸⠩ ⠼⠖⠴⠘⠨⠡ ⠰⠠⠝ ⠸⠱ ⠇⠁⠞⠊⠞⠥⠙⠑

4 ⠊⠎ ⠍⠕⠙⠑⠇⠫ ⠃⠽ ⠮ ⠋⠥⠝⠉⠰⠝ ⠆⠇⠂ ⠐⠱

5 ⠸⠩ ⠙ ⠨⠅ ⠙⠁⠽ ⠕⠋ ⠞⠓⠑ ⠽⠑⠁⠗⠲

6 ⠍⠷⠙⠾

7 ⠨⠅ ⠼⠒⠔⠴⠎⠊⠝ ⠈⠷⠹⠆⠨⠏⠌⠒⠖⠢⠨⠆⠢⠼

8 ⠷⠙⠤⠦⠴⠾⠈⠾

9 ⠬⠶⠒⠦ ⠸⠱

Line 5: The entire statement "d = day of the year" is mathematical. The words are uncontracted and are punctuated without the use of a punctuation indicator.
Lines 6-9: The displayed expression begins in cell 7, with runovers in cell 9.
Lines 7-8: The space between the function and its argument is not a suitable division site. The bracketed expression must be divided. Division is made before the left parenthesis in order to keep the expression grouped within the inner parentheses intact.
Line 9: Division is made before the plus sign, according to Step ii.

PRACTICE 15G

1 ⠀⠀⠠⠮ ⠎⠕⠇⠥⠰⠝⠎ ⠔⠧⠕⠇⠧⠬ ⠮ ⠙⠊⠗⠑⠉⠞ ⠋⠥⠝⠉⠰⠝⠎ ⠸⠩
2 ⠀⠀⠮⠹⠇⠕⠛⠘⠆ ⠷⠵⠬⠂⠾⠌⠵⠼⠙⠵
3 ⠀⠀⠀⠀⠨⠅ ⠇⠕⠛ ⠷⠤⠵⠾⠇⠕⠛⠘⠆ ⠷⠵⠬⠂⠾
4 ⠀⠀⠀⠀⠬⠆⠠⠇⠊⠆⠷⠵⠬⠂⠾⠇⠕⠛ ⠷⠵⠬⠂⠾⠤⠆⠠⠇⠊⠒⠷⠵⠬⠂⠾
5 ⠠⠄⠯
6 ⠀⠀⠮⠰⠴⠘⠠⠿⠐⠠⠹⠇⠕⠛ ⠷⠞⠬⠂⠾⠇⠕⠛ ⠷⠂⠬⠹⠂⠌⠞⠘⠆⠐⠼⠾
7 ⠀⠀⠀⠀⠠⠌⠞⠠⠼⠙⠞
8 ⠀⠀⠀⠀⠨⠅ ⠠⠉⠨⠏⠤⠹⠒⠨⠵⠷⠒⠾⠌⠦⠼ ⠸⠱
9 ⠉ ⠆ ⠋⠨⠙ ⠔ ⠠⠡⠁⠏⠞⠻ ⠼⠛⠢

1

This is one paragraph interrupted twice by displayed material.
Line 1: The paragraph begins in cell 3.
Line 2: The displayed expressions begin in cell 3.
Line 3: The link begins in the runover cell for displayed material (cell 5). Division is made before the plus sign (Step ii).
Line 4: It is not necessary to divide before the second operation sign (the minus sign).
Line 5: The paragraph continues in the runover cell for narrative (cell 1).
Line 6: The displayed expression begins in cell 3. The notation from the integral symbol to dt will not fit on one line. A decision needs to be made whether to (a) divide before the opening fraction indicator in order to keep the fraction unit together or (b) begin the fraction on this line, dividing at the complex fraction line. Choice (b) was transcribed, in order to prevent dt from being left alone on line 7.
Line 7: This line begins with the complex fraction line and ends with the dt that finishes the integral notation.
Line 8: A new line begins with the final link (Step i), placed in the runover cell, cell 5.
Line 9: The paragraph concludes.

PRACTICE 15H

1 ⠸⠩ ⠐⠨⠠⠎⠩⠝ ⠸⠇ ⠼⠴⠣⠠⠿⠻⠹⠷⠁⠾⠰⠝⠂⠷⠃⠾⠰⠝⠌⠷⠉⠾⠰⠝⠼
2 ⠹⠵⠘⠝⠐⠎⠝⠯⠼ ⠸⠱

The expression is divided before the second fraction. This keeps the Sigma notation intact with the beginning of its argument.

PRACTICE 15I

1 ⠼⠁⠲ ⠠⠮ ⠗⠑⠉⠞⠁⠝⠛⠇⠑ ⠗⠥⠇⠑ ⠿ ⠝⠥⠍⠻⠊⠉⠁⠇
2 ⠔⠞⠑⠛⠗⠁⠰⠝ ⠐⠣⠠⠮ ⠠⠍⠊⠙⠏⠕⠔⠞ ⠠⠗⠥⠇⠑⠐⠜ ⠊⠎
3 ⠔⠎⠏⠊⠗⠫ ⠃⠽ ⠮ ⠥⠎⠑ ⠷ ⠠⠗⠊⠑⠍⠁⠝⠝ ⠎⠥⠍⠎⠲ ⠸⠩
4 ⠹⠃⠤⠁⠌⠝⠼
5 ⠐⠨⠠⠎⠩⠊ ⠨⠅ ⠼⠴⠣⠝⠤⠂⠻⠋⠷⠁⠬⠊⠹⠃⠤⠁⠌⠝⠼⠾
6 ⠈⠱⠈⠱ ⠮⠰⠁⠘⠃⠐⠋⠷⠭⠾⠙⠭ ⠸⠱

Line 4: The anchor will not fit on one line. Division is made after the first closing fraction indicator.
Line 5: The Sigma notation and its argument fit on one line. No further division decisions are necessary.
Line 6: The link is placed on a new line, starting with the double tilde.

LESSON 16

- SPATIAL ARRANGEMENTS OF FRACTIONS
- HYPERCOMPLEX FRACTIONS
- CONTINUED FRACTIONS
- INSTRUCTIONAL COMMENTARY
- STEM-AND-LEAF PLOTS

Answers to Practice Material

LESSON PREVIEW

The remaining lessons take a look at a variety of formatting topics. In this lesson, spatially arranged fractions are studied, a braille format for instructional commentary is given, and layouts for stem-and-leaf plots are introduced.

SPATIAL ARRANGEMENTS OF FRACTIONS

[NC 13.10]

Fractions are usually transcribed linearly, as discussed in Lesson 8. However, certain situations suggest or require a spatial arrangement.

Recall the terminology used for the parts of a fraction.

$$\frac{3}{4}$$

numerator
fraction line
denominator

16.1 Spatial Fraction Line

The spatial fraction line is comprised of a series of dots 25. An opening fraction indicator (1456) marks the beginning of the fraction line; a closing fraction indicator (3456) signals the end.

⠹⠒⠒⠒⠼	`Spatial Fraction Line with opening and closing fraction indicators (varying length)`

In a spatially arranged fraction the numerator is placed above the spatial fraction line and the denominator is placed below it. The fraction line is comprised of the same number of cells as the widest item above (numerator) or below (denominator), with the fraction indicators placed one cell beyond the width on either end, as illustrated in the next section.

Note: Code switch indicators are absent in many of the isolated examples in this lesson.

16.2 Numerator and Denominator

The numerator is centered above the fraction line; the denominator is centered below the fraction line. If exact centering is not possible, the item is moved to the left one cell. Since fractions do not contain material aligned for computation, the numeric indicator is used in the numerator and/or denominator according to the rules of the Nemeth Code. For the same reason, the English-letter indicator is used where necessary in the numerator and/or denominator.

Example 16-1

(a)

$$\frac{3}{4}$$

```
 ⠼⠒
⠹⠒⠒⠼
 ⠼⠲
```

(b)

$$\frac{x}{y}$$

```
 ⠰⠭
⠹⠒⠒⠼
 ⠰⠽
```

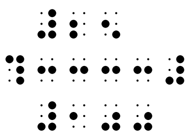
(c)

$$\frac{3}{100}$$

```
  ⠼⠒
⠹⠒⠒⠒⠒⠼
 ⠼⠂⠴⠴
```

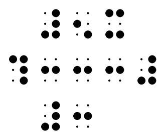
(d)

$$\frac{25}{100}$$



(e)

$$\frac{5x}{3}$$

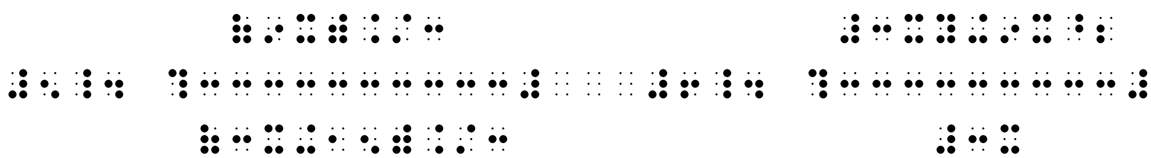


16.3 Placement of Identifiers with Spatially Arranged Fractions

An identifier, if present, is placed on the same braille line as the fraction line. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement.

Example 16-2

5. $\dfrac{(9x)\div 3}{(3x+15)\div 3}$ 6. $\dfrac{3xy+9x^2}{3x}$



Recall from Lesson 9 that three blank cells come between the rightmost symbol of an arrangement and the next identifier.

Situations Requiring Spatial Presentation of Simple Fractions

16.4 Fractions Arranged Spatially for Illustration

When the parts of a simple fraction are labeled, use a spatial arrangement. The labels are uncontracted because they are within the code switches.

Example 16-3

$\frac{3}{4}$	*numerator* *fraction line* *denominator*

```
⠸⠩

 ⠼⠒   ⠝⠥⠍⠑⠗⠁⠞⠕⠗
⠹⠒⠒⠼ ⠋⠗⠁⠉⠞⠊⠕⠝  ⠇⠊⠝⠑
 ⠼⠲   ⠙⠑⠝⠕⠍⠊⠝⠁⠞⠕⠗

⠸⠱
```

Example 16-4

Four children share a dozen cookies equally. Expressed as a fraction, how many cookies does each child receive?

12	number of cookies
4	number of children

```
  ⠠⠋⠳⠗ ⠡⠝ ⠩⠜⠑ ⠁ ⠙⠕⠵⠢ ⠉⠕⠕⠅⠊⠑⠎ ⠑⠟⠥⠁⠇⠇⠽⠲
⠠⠑⠭⠏⠗⠑⠎⠎⠫ ⠵ ⠁ ⠋⠗⠁⠉⠰⠝⠂ ⠓⠪ ⠸⠍ ⠉⠕⠕⠅⠊⠑⠎ ⠙⠕⠑⠎
⠑⠁⠡ ⠡ ⠗⠉⠧⠦ ⠸⠩

   ⠼⠂⠆  ⠝⠥⠍⠃⠑⠗ ⠕⠋ ⠉⠕⠕⠅⠊⠑⠎
  ⠹⠒⠒⠒⠼
   ⠼⠲   ⠝⠥⠍⠃⠑⠗ ⠕⠋ ⠉⠓⠊⠇⠙⠗⠑⠝

⠸⠱
```

PRACTICE 16A

1. Shandra invited seven friends to a pizza party. Two pizzas were ordered. Each pizza had eight slices. Which fraction shows how many slices of pizza each child can have if they share equally?

   a. $\frac{2}{8}$ $\frac{\text{number of pizzas}}{\text{number of children}}$

   b. $\frac{8}{8}$ $\frac{\text{number of slices}}{\text{number of children}}$

   c. $\frac{16}{8}$ $\frac{\text{number of slices}}{\text{number of children}}$

16.5 Cancellation with Replacement Values [NC 12.1]

Recall that a spatial arrangement is required when numbers, letters, or abbreviations are canceled in print and replacement values are shown. *Reminder:* The print copy may show any type of stroke to indicate cancellation.

⠪	Opening Cancellation Indicator
⠻	Closing Cancellation Indicator

When cancellation occurs within a fraction, replacement items are placed above canceled items in the numerator and below canceled items in the denominator. Each replacement item is centered with respect to the canceled term. If exact centering is not possible, the term is moved to the left one cell.

Example 16-5

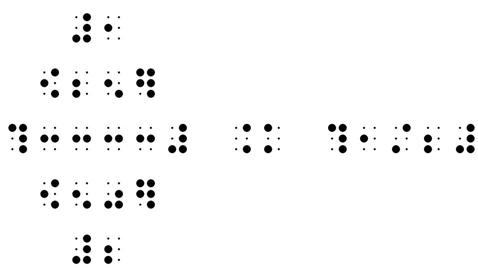
$$\frac{\overset{1}{\cancel{25}}}{\underset{2}{\cancel{50}}}$$

```
1    ⠼⠂
2   ⠪⠆⠢⠻
3  ⠹⠒⠒⠒⠒⠼
4   ⠪⠢⠴⠻
5    ⠼⠆
```

Line 1: Replacement "1" is centered above the canceled number. A numeric indicator is required.
Line 2: A numeric indicator is not required because the numeral is not preceded by a space.
Line 3: The fraction line is as long as the widest item in the fraction. In this case, both numerator and denominator have a 4-cell item.
Line 4: A numeric indicator is not required because the numeral is not preceded by a space.
Line 5: Replacement "2" is centered below the canceled number. A numeric indicator is required.

Example 16-6

$$\frac{\overset{1}{\cancel{25}}}{\underset{2}{\cancel{50}}} = \frac{1}{2}$$

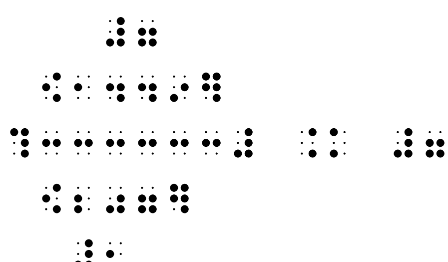


Note that the second fraction ("one-half") is transcribed as a linear fraction because it doesn't contain cancellation. The equals sign and the linear fraction are placed on the same line as the spatial fraction line.

16.5.1 **Alignment Considerations with Cancellation.** Vertical alignment of place value is not a consideration in spatially arranged fractions when the material is not aligned for computation. Replacement items are centered above or below their canceled item. Placement of the replacement item is relative to the width of the canceled item, without respect to place value. Note the location of the replacement items in the next example.

Example 16-7

$$\frac{\overset{7}{\cancel{1449}}}{\underset{1}{\cancel{207}}} = 7$$

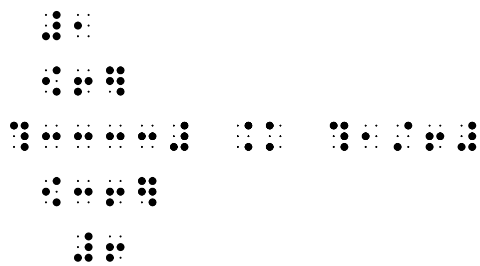


a. **Alignment of Indicators.** When the canceled number and its replacement contain the same number of digits, the numeric indicator and the opening cancellation indicator will align.

Example 16-8

$$\frac{\overset{1}{\cancel{6}}}{\underset{6}{\cancel{36}}} = \frac{1}{6}$$

1
2
3
4
5



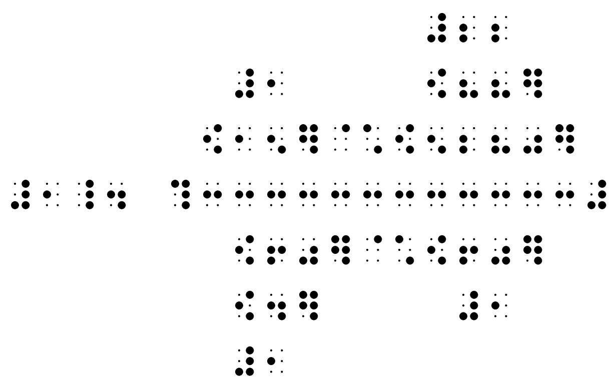
1

Line 1:Replacement "1" is centered above the canceled number on line 2. In this case, the numeric indicator lies directly above the opening cancellation indicator below it.
Line 2:Canceled "6" is centered above the fraction line. Exact centering is not possible so it is moved left one cell.
Line 3: The fraction line is as long as the widest item in the fraction (the denominator).
Line 4: Canceled "36" is centered below the fraction line.
Line 5: Replacement "6" is centered below the canceled number above it.

16.5.2 **Stacked Cancellation.** More than one level of cancellation may be encountered.

Example 16-9

1. $$\frac{\overset{1}{\cancel{15}} \times \overset{\overset{22}{\cancel{88}}}{\cancel{5280}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{60}} \times \underset{1}{\cancel{60}}}$$



PRACTICE 16B

Simplify using cross canceling. Then multiply the numerators and multiply the denominators.

1. $\frac{\overset{1}{\cancel{3}}}{\underset{3}{\cancel{15}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} = \frac{1}{6}$

2. $\overset{1}{\cancel{2}} \times \frac{8}{\underset{1}{\cancel{2}}} = \frac{1 \times 8}{1} = 8$

16.5.3 **Cancellation and Level Indicators.** Care must be taken to place the opening cancellation indicator and its paired closing cancellation indicator on the same level.

⫸ $\cancel{b^2}$ ⠪⠃⠘⠆⠐⠻

Both the base and its superscript are canceled with one stroke. A baseline indicator places the cancellation terminator on the same level as the paired opening cancellation indicator.

⫸ $b^{\cancel{3}}$ ⠃⠘⠪⠒⠻

Only the superscript is canceled. The opening and closing cancellation indicators are at the superscript level.

Example 16-10

$\frac{\overset{a}{\cancel{a^2}}}{\underset{1}{\cancel{a}}} = a$

⠰⠁
⠪⠁⠘⠆⠐⠻
⠹⠒⠒⠒⠒⠒⠒⠼ ⠨⠅ ⠁
⠪⠁⠻
⠼⠂

The denominator is centered under the fraction line.

Example 16-11

$$\frac{4x^2q}{p^{\cancel{3}\,2}}$$

The replacement numeral "2" is placed below the canceled superscript in the denominator, even though it is printed beside the canceled superscript.

PRACTICE 16C

1. $$\frac{2b+a}{b} \div \frac{4b^2-a^2}{b^2} - \frac{\overset{1}{\cancel{(2b+a)}}}{\underset{1}{\cancel{b}}} \cdot \frac{\overset{b}{\cancel{b^2}}}{\underset{1}{\cancel{(2b+a)}}(2b-a)} = \frac{b}{2b-a}$$

16.6 Cancellation Without Replacement Values [NC 12.1]

Material containing cancellation with no replacement may be transcribed linearly.

16.6.1 **Linear Presentation of Simple Fractions with Canceled Items.** When replacement items are not give for canceled material in the numerator and denominator, a linear fraction may be transcribed. The cancellation indicators must exactly represent what is canceled in print.

Example 16-12

$$\frac{\cancel{r}\;\cancel{s}\;\cancel{t}}{\cancel{r}\;\cancel{s}\;\cancel{t}\;\mathrm{v}}$$

Each letter "r" "s" and "t" is printed with a slash through it.

Example 16-13

$$\frac{\cancel{(\mathrm{rst})}}{\cancel{(\mathrm{rst})}\mathrm{v}}$$

Each letter combination "rst" as well as the parentheses are printed with a line through them.

Example 16-14

Use factorization and cross cancellation to simplify $\frac{x^2-5x+6}{-x+2}$.

$$\begin{aligned}\frac{x^2-5x+6}{-x+2} &= \frac{(x-2)(x-3)}{-(x-2)}\\ &= \frac{\cancel{(x-2)}\,(x-3)}{-\cancel{(x-2)}}\\ &= -x+3\end{aligned}$$

16.6.2 **Canceled Abbreviations.** When canceled abbreviations show no replacements, linear transcription may be applied. The abbreviation is unspaced from the cancellation indicators. As a whole, however, the abbreviation must follow the spacing rules for abbreviations (Lesson 3). If a space is required, the space comes before and/or after the related indicator or indicators.

⫸ $\cancel{\text{in.}}$

⫸ $\cancel{\text{in.}^3}$

⫸ $7888\ \cancel{\text{in}^3} + n$

A space precedes the opening cancellation indicator. A space follows the the closing cancellation, before the plus sign.

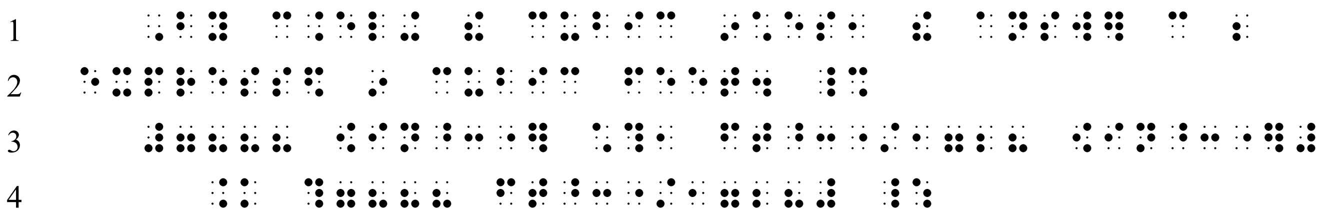
⫸ $\frac{1\ \text{ft}^3}{1728\ \cancel{\text{in}^3}}$

A space precedes the opening cancellation indicator. There is no space between the closing cancellation indicator and the closing fraction indicator.

Example 16-15

By canceling the cubic inches, the answer can be expressed in cubic feet.

$$7888\ \cancel{in^3} \cdot \frac{1\ ft^3}{1728\ \cancel{in^3}} = \frac{7888\ \ ft^3}{1728}$$



Lines 3-4: The fractions are transcribed linearly because the canceled portions do not show replacements.
Line 3: A space is required between each abbreviation and its related numeral, even when the abbreviation is canceled. A space is required between the abbreviation and the operation symbol (multiplication dot) even when the abbreviation is canceled.
Line 4: A space is required between the numeral and its associated abbreviation.

PRACTICE 16D

How many meters are in 100 yards? By multiplying 100 yd by several expressions of "1" and canceling units of measure, a solution is found. *Answer:* There are 91.44 m in 100 yd.

$$\begin{aligned} ?\ m &= 100\ \cancel{yd} \times \frac{3\ \cancel{ft}}{1\ \cancel{yd}} \times \frac{12\ \cancel{in}}{1\ \cancel{ft}} \times \frac{2.54\ \cancel{cm}}{1\ \cancel{in}} \times \frac{1\ \cancel{m}}{100\ \cancel{cm}} \\ &= \frac{100 \times 3 \times 12 \times 2.54}{100}\ m \\ &= 91.44\ m \end{aligned}$$

HYPERCOMPLEX FRACTIONS

[NC 13.7 and 13.8]

16.7 Definition and Recognition

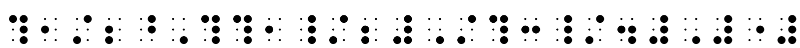
The term "hypercomplex" fraction is used only in the context of the Nemeth Code—it is unlikely that you will encounter this term in a math book. First, we will review the definition of a complex fraction.

Recall from Lesson 8 that a *complex* fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers. In other words, a complex fraction is a fraction within a fraction.

⫸ $\dfrac{1/2}{5}$ This is a complex fraction.

If a simple fraction contains a complex fraction at the superscript or subscript level, it is still a simple fraction. In this example, the numerator is "1" and the denominator is "2 raised to the (one-half over three-fourths) power."

⫸ $\dfrac{1}{2^{\frac{1/2}{3/4}}}$ This is a simple fraction.



The complex fraction is at the superscript level.

A hypercomplex fraction is one whose numerator or denominator, or both, contain at least one complex fraction. Here is a print example of a hypercomplex fraction. The denominator is "10" and the numerator contains the complex fraction shown above, "one-half over five."

⫸ $\dfrac{\frac{1/2}{5}}{10}$ This is a hypercomplex fraction.

The numerator contains a complex fraction.

16.8 Transcription of Hypercomplex Fractions

Although a linear transcription is possible, using a linear arrangement for the numerator and a linear arrangement for the denominator, within a spatial arrangement overall, gives a clearer representation of the hypercomplex fraction.

16.8.1 **Hypercomplex Fraction Indicators.** Opening and closing hypercomplex fraction indicators mark the beginning and end of a spatial hypercomplex fraction line.

⠠⠠⠹⠒⠒⠒⠠⠠⠼	`Spatial Hypercomplex Fraction Line with opening and closing fraction indicators (varying length)`

Here is the transcription of the hypercomplex fraction shown on the previous page. The complex fraction in the numerator is transcribed as a linear arrangement within the overall spatial arrangement.

Example 16-16

$$\frac{\;\frac{1}{2}\;}{\frac{5}{10}}$$

```
       ⠠⠹⠹⠂⠸⠌⠆⠼⠠⠌⠢⠠⠼
⠠⠠⠹⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠠⠠⠼
           ⠼⠂⠴
```

Terms of the fraction are centered above and below their fraction lines.

If the numerator and denominator are too long to fit on one line, follow the rules regarding division of mathematical expressions between braille lines. (Lesson 15) The next example shows an expression which will fit in 40 cells as first transcribed, but if it is displayed, division of the denominator is necessary.

Example 16-17

1. Show that $\dfrac{\dfrac{3\frac{1}{3}}{7\frac{1}{5}} - \dfrac{1}{2\frac{2}{7}}}{4\frac{1}{5} \times \dfrac{2\frac{2}{3}}{6\frac{3}{8}}}$ of $\dfrac{11\frac{1}{5}}{17} \times 52\frac{4}{11} = \dfrac{1}{2}$.

Numerator and denominator are divided before an operation sign. The numerator is divided before the minus sign; the denominator is divided before the multiplication cross. Terms of the fraction are centered above and below the spatial fraction line. Note that the word "of" is part of the equation, so it is transcribed in mathematical context, without contractions.

16.8.2 **Other Layouts.** The combined transcription method illustrated above is the preferred way of transcribing a hypercomplex fraction. However, it is permissible to use an entirely spatial arrangement or an entirely linear arrangement. In particular, linear hypercomplex fractions are difficult to assemble mentally. Example 16-16 is illustrated below using these two alternate methods. Symbols and indicators used in the linear method are as follows.

	Opening Hypercomplex Fraction Indicator
	Horizontal Hypercomplex Fraction Line
	Diagonal Hypercomplex Fraction Line
	Closing Hypercomplex Fraction Indicator

Example 16-18

$$\frac{\frac{1/2}{5}}{10}$$

(entirely spatial)

```
      ⠹⠂⠸⠌⠆⠼
    ⠠⠹⠒⠒⠒⠒⠒⠒⠠⠼
         ⠼⠢
⠠⠠⠹⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠠⠠⠼
        ⠼⠂⠴
```

(entirely linear)

⠠⠠⠹⠠⠹⠹⠂⠸⠌⠆⠼⠠⠌⠢⠠⠼⠠⠠⠌⠂⠴⠠⠠⠼

16.9 Higher Orders of Complexity

Hypercomplex fractions of higher order may be transcribed in the manner described above, with dot 6 added the appropriate number of times before the fraction indicators and the matching fraction lines. No examples are shown.

Instructions: Transcribe this hypercomplex fraction using the combined method—that is, transcribe each complex fraction as a linear arrangement within the overall spatial arrangement. Numerators and denominators can be determined by noting the length of each fraction line in print.

PRACTICE 16E

1. Solve for r in terms of s. $$\frac{\dfrac{\dfrac{r^2-4s^2}{s^2}}{\dfrac{r+2s}{s}}}{\dfrac{\dfrac{4r-2s^2}{3s}}{\dfrac{2s^2-3r}{4r}}}$$

CONTINUED FRACTIONS

[NC 13.9]

16.10 Definition and Recognition

In essence, a continued fraction is a fraction within a fraction. Each denominator is the sum of a whole number and a fraction. The digit in the denominator of one fraction becomes the digit in the numerator of the next fraction. You may find them in the study of irrational numbers such as pi and certain square roots.

The continued fraction shown below can be described as follows.The fraction begins with $1 - \frac{7}{3}$. The first denominator is $3 + \frac{3}{2}$. The next denominator is $2 + \frac{2}{2}$. The final denominator is $2 - \frac{2}{3}$.

$$n = 1 - \cfrac{7}{3 + \cfrac{3}{2 + \cfrac{2}{2 - \cfrac{2}{3}}}}$$

16.10.1 **Transcription Rules.** A continued fraction is transcribed as a spatial arrangement. The layout essentially mimics the print arrangement. Each fraction line begins in the cell after the plus or minus sign (unspaced). Opening and closing fraction indicators are not used. All fraction lines end in the same cell, aligned with the rightmost symbol in the arrangement.

Each new denominator's whole number is aligned directly below the first cell of the separation line above it. Its numerator is centered with respect to the fraction line below it. If that number cannot be exactly centered, it is placed one cell to the left of center.

The rules will make sense when you study the example below, which is the transcription of the continued fraction shown above.

Example 16-19

$$n = 1 - \cfrac{7}{3 + \cfrac{3}{2 + \cfrac{2}{2 - \cfrac{2}{3}}}}$$

```
1   ⠸⠩
2
3                 ⠼⠶
4     ⠝ ⠨⠅ ⠼⠂⠤⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
5                   ⠼⠒
6             ⠼⠒⠬⠒⠒⠒⠒⠒⠒⠒⠒
7                    ⠼⠆
8                ⠼⠆⠬⠒⠒⠒⠒⠒
9                      ⠼⠆
10                  ⠼⠆⠤⠒⠒
11                     ⠼⠒
12
13  ⠸⠱
```

Line 1: The opening Nemeth Code indicator is transcribed on the line preceding the required blank line.
Line 3: The numerator is centered over the fraction line below it.
Line 4: The length of this fraction line extends to the rightmost character in the arrangement, which is on lines 9 and 11.
Line 5: This numerator "3" is centered over the fraction line below it. Likewise for the numerators on lines 7 and 9.
Line 6: The numeric indicator is directly aligned with the first cell of the fraction line two lines above. Likewise for lines 8 and 10.
Line 13: The Nemeth Code terminator is in cell 1 on the line following the required blank line.

PRACTICE 16F

The square root of 3 can be expressed as a continued fraction.

$$\sqrt{3} = 1 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \ldots}}}$$

INSTRUCTIONAL COMMENTARY

[NC 26.4.5]

16.11 Format for Instructional Commentary

When instructional commentary appears alternated with math problems, place the comment on the line following the related expression, blocked 4 cells to the right of the runover position for the expression. Explain this format in a transcriber's note.

Sample transcriber's note: "Comments printed beside related math problems are placed on the line following the expression, blocked four cells to the right of the runover position of the expression."

When switching into or out of Nemeth before a change of margins, place the switch indicators after the last item of the line rather than at the beginning of the next line. This maintains clarity in the indented margin pattern. The single-word switch indicator is used, as needed.

Comments are often printed in a different color or typeface. (In the examples in this section, instructional commentary is printed in italics.) The variant typeform is disregarded in the braille transcription.

Notice in the examples how the comments are clearly set off by indentation. Generous line-by-line notes are provided with each example.

Example 16-20

If y varies inversely as x, and $y = 3$ when $x = 4$, find y when $x = 18$.

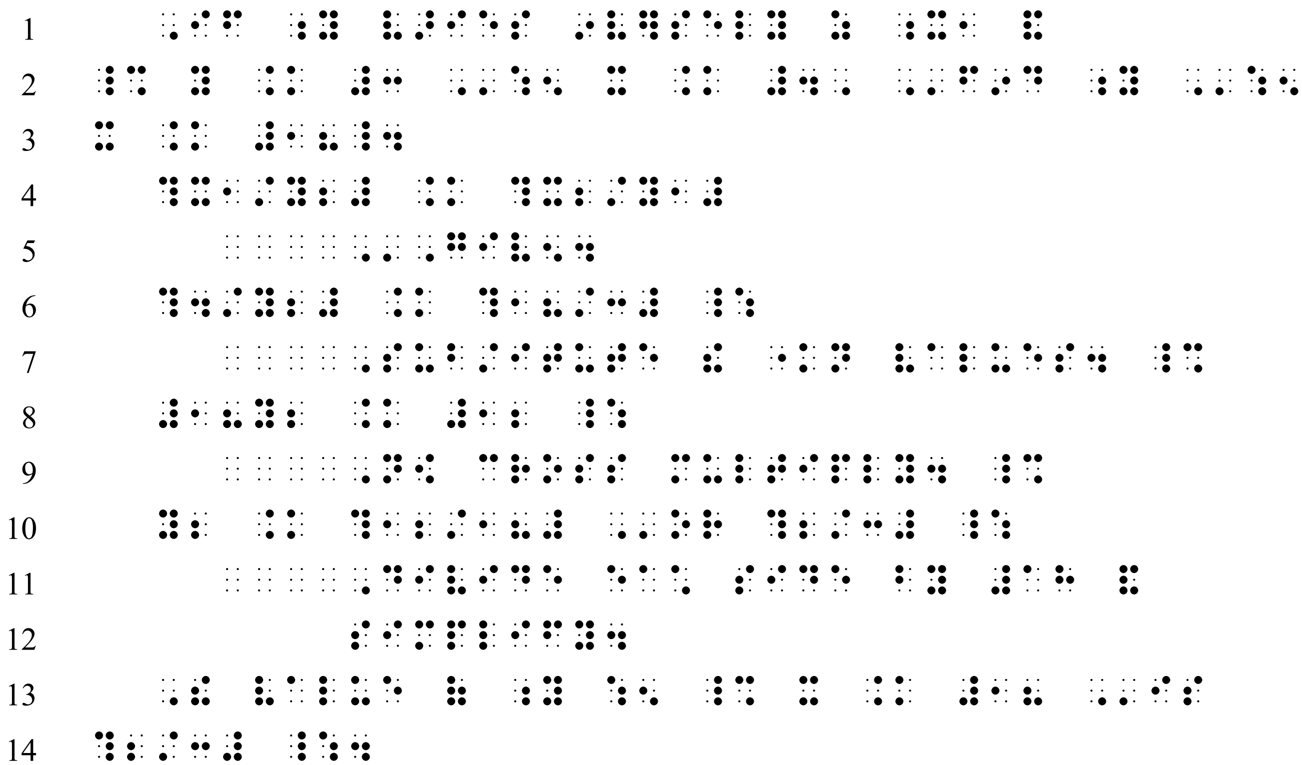
$\frac{x_1}{y_2} = \frac{x_2}{y_1}$ *Given.*

$\frac{4}{y_2} = \frac{18}{3}$ *Substitute the known values.*

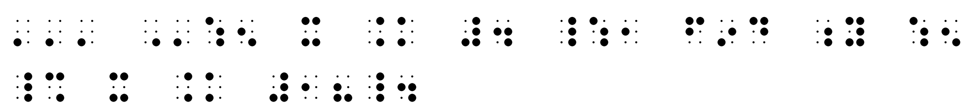
$18y_2 = 12$ *Now cross multiply.*

$y_2 = \frac{12}{18}$ or $\frac{2}{3}$ *Divide each side by 18 and simplify.*

The value of y when $x = 18$ is $\frac{2}{3}$.



Lines 1-3: The narrative begins in cell 3, with runovers in cell 1.
Lines 2-3: This phrase can also be transcribed as follows:



Lines 4, 6, 8, 10: Each displayed math expression begins in cell 3.
Line 5: The comment begins in cell 9, which is four cells to the right of the runover cell of the displayed material even though there is no runover present. A single-word switch is used for this comment. The italic typeform is disregarded.
Lines 6, 8, 10: A Nemeth Code terminator ends each line, preparing the reader for the following comment.
Lines 7, 9, 11: Each comment begins in cell 9.
Line 12: The runover is blocked (cell 9).
Lines 7 and 9: The opening Nemeth Code indicator is placed at the end of the line of text preceding the Nemeth material to assure that all of the displayed math expressions begin in the same cell.
Lines 13-14: The narrative begins in cell 3, with its runover in cell 1. No blank line is needed.

16.11.1 **Alignment of Equals Signs.** If the print copy aligns equals signs of separate displayed equations, the layout is disregarded in the braille transcription. This assures that each step begins in the same cell, as the first example below illustrates. On the other hand, if the print layout meets the definition of a nested list expression, braille format for nested list expressions is applied. (Lesson 8) The second example below illustrates this layout.

Example 16-21

We write the equation in slope-intercept form by solving for y.

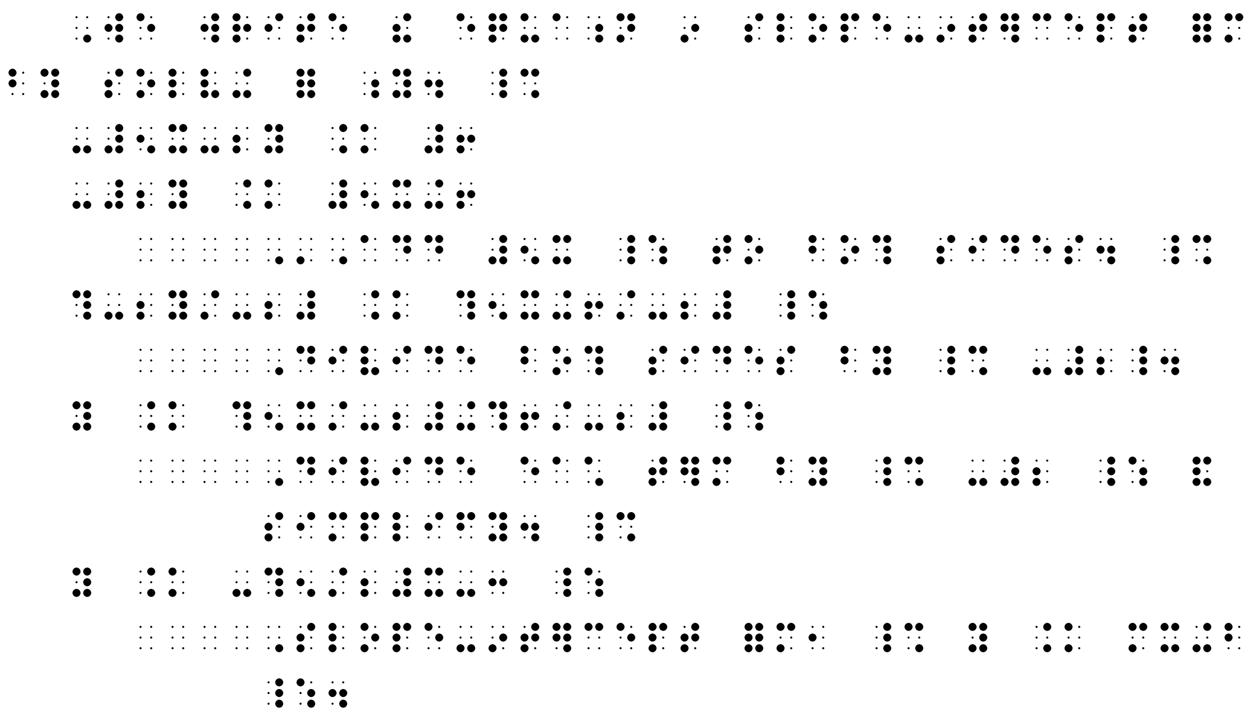
$$-5x - 2y = 6$$

$$-2y = 5x + 6 \qquad \textit{Add } 5x \textit{ to both sides.}$$

$$\frac{-2y}{-2} = \frac{5x+6}{-2} \qquad \textit{Divide both sides by -2.}$$

$$\mathrm{y} = \frac{5\mathrm{x}}{-2} + \frac{6}{-2} \qquad \textit{Divide each term by -2 and simplify.}$$

$$\mathrm{y} = -\frac{5}{2}\mathrm{x} - 3 \qquad \textit{Slope-intercept form, } \mathrm{y} = \mathrm{mx} + \mathrm{b}.$$



Overall: Although the equals signs are aligned in print, these are five separate equations, not a nested linked expression. Alignment of the equals signs is disregarded. Each expression will start in the same cell (cell 3, in this case—displayed to a narrative paragraph). Study the placement of the code switch indicators to see how they do not interfere with the starting cell of each displayed expression.

Lines 1-2: Narrative paragraph in (1-3).

Lines 3, 4, 6, 8, 11: Each displayed equation is placed in cell 3. There are no runovers, but if there were, the runover would start in cell 5.

Lines 5, 7, 9, 12: Each comment begins in cell 9, which is four cells to the right of the runover position of the previous line, even though no runovers are present.

Lines 10 and 13: Runovers to comments are blocked (cell 9, in this case).

Example 16-22

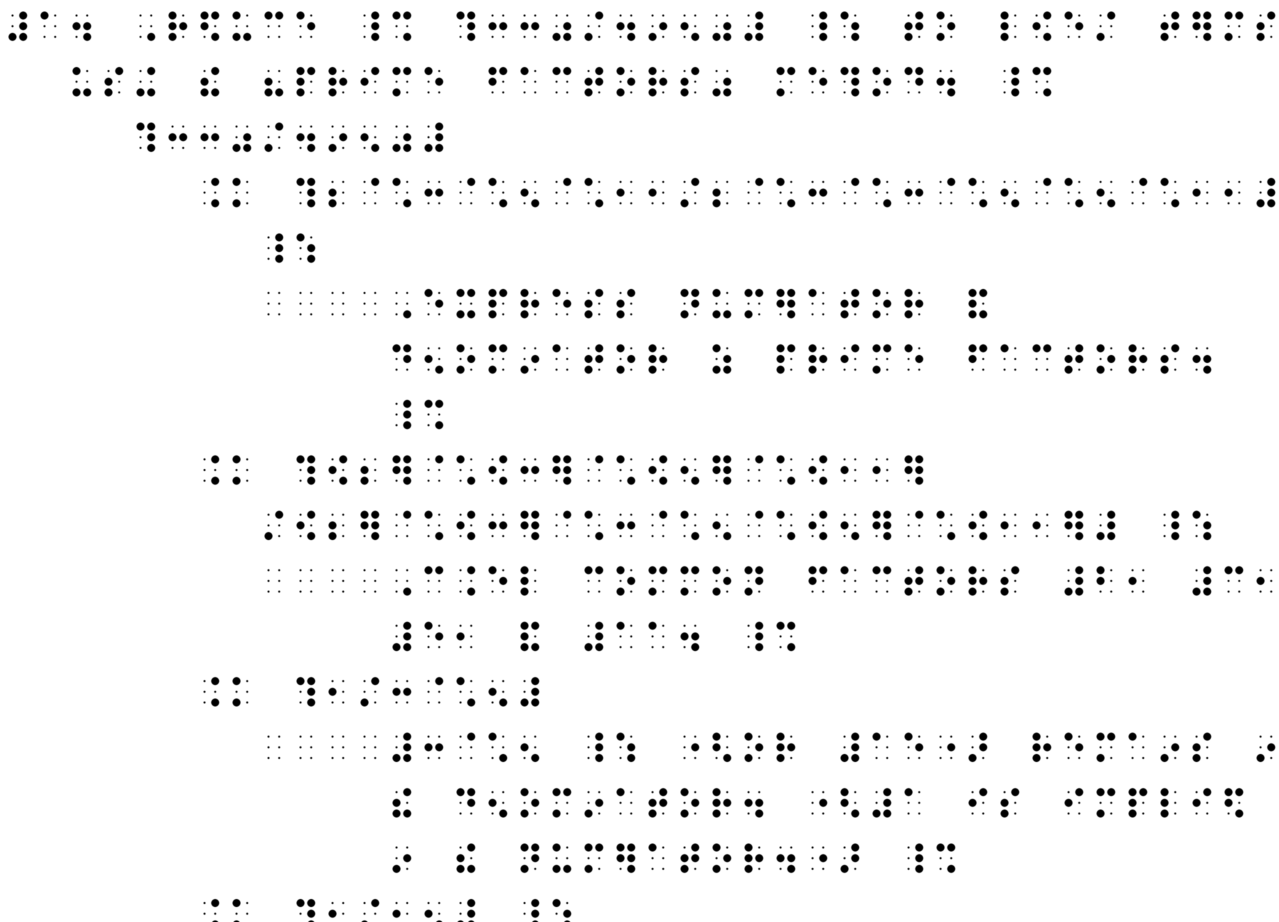
1. Reduce $\frac{330}{4950}$ to lowest terms using the "prime factors" method.

$$\frac{330}{4950} = \frac{2\times3\times5\times11}{2\times3\times3\times5\times5\times11}$$ *Express numerator and denominator as prime factors.*

$$= \frac{\cancel{2}\times\cancel{3}\times\cancel{5}\times\cancel{11}}{\cancel{2}\times\cancel{3}\times3\times5\times\cancel{5}\times\cancel{11}}$$ *Cancel common factors 2, 3, 5, and 11.*

$$= \frac{1}{3\times5}$$ *3×5 remains in the denominator.*

$$= \frac{1}{15}$$



Overall: The displayed math is a nested linked expression. The pattern for this nested linked expression is 5-9, 7-9 (displayed to 1-3 itemized text). The comments are blocked in cell 13. Study the placement of the code switch indicators to see how they do not interfere with the starting cell of each pattern (anchor in cell 5, links in cell 7, comments in cell 13).

Line 3: The anchor begins in cell 5
Lines 4, 9, 13, 17: Each link begins in cell 7. Runovers to the links are in cell 9 (lines 5 and 10).
Lines 6, 11, 14: Each comment begins in cell 13, which is four cells to the right of the runover position of the previous line, even if no runover is present.
Lines 7-8, 12, 15-16: Runovers to comments are blocked (cell 13, in this case).
Lines 9-10: Because the canceled numbers do not have replacements, the fraction is transcribed linearly.

PRACTICE 16G

What percent of 48 is 54?

$n \cdot 48$	$= 54$	*Write an equation.*
$\frac{48n}{48}$	$= \frac{54}{48}$	*Divide each side by 48.*
n	$= 1.125$	*Simplify.*
	$= 112.5\%$	*Change the decimal to a percent.*

Answer: 54 is 112.5% of 48.

STEM-AND-LEAF PLOTS

16.12 Recognition

A stem-and-leaf plot is a method of showing data distribution in columns and rows. A vertical line partitions the information into data on the left, called the stem, and data on the right, called the leaf. Stem and leaf data may consist of numbers, letters, and/or blank entries and may be spaced or unspaced in the print copy. Stem data are right justified to the vertical line; leaf data are left justified to the vertical line. The columns may include a heading. Here is an example.

Stem	Leaf
10	0 4
9	3 4 5 7 8 9
8	0 2 2 9
7	5 8 9
6	3

A distinctive feature of a stem-and-leaf plot is the vertical line separating the stems from the leaves.

16.13 The Table

A stem-and-leaf plot is transcribed in Nemeth notation using the guidelines for tables outlined in *Braille Formats*. The entire table is transcribed in Nemeth, including the column headings (if present), even if the numbers or letters are unmodified. Column headings begin in cell 1 and no contractions are used. Column separation lines follow on the next line below the headings, covering the same number of cells as the widest data entry in each column. Column headings are not transcribed if they do not appear in print.

The vertical line between the stem column and the leaf column is transcribed as dots 456, preceded and followed by one blank cell. This symbol is transcribed in every row, including between column headings and between column separation lines.

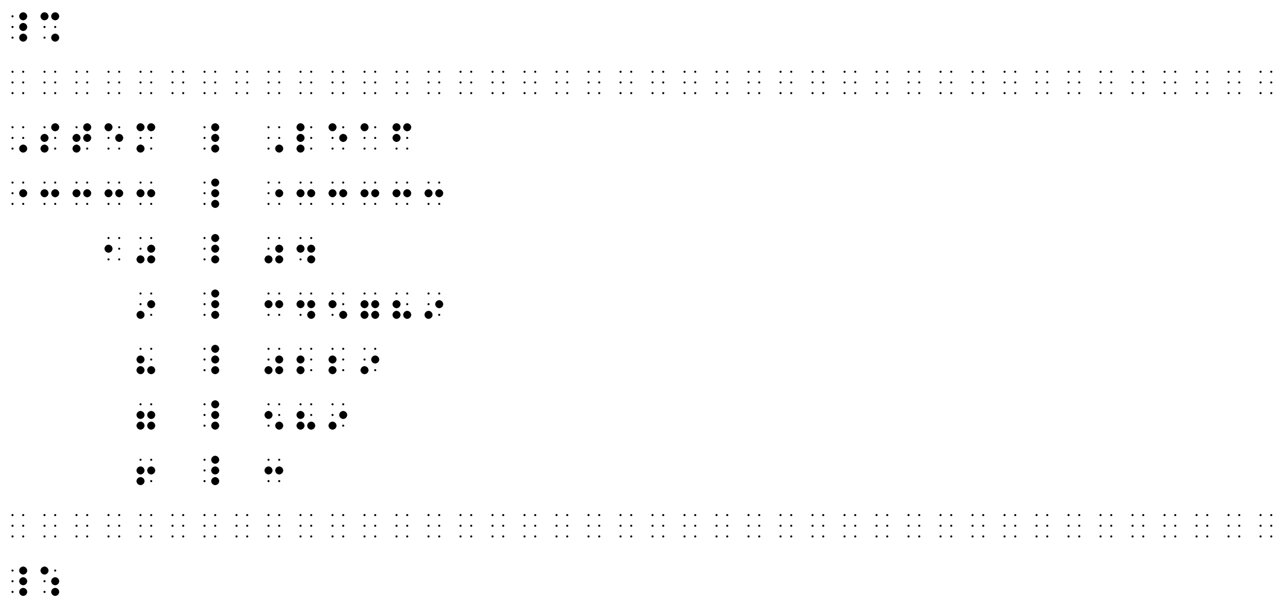
⠸	`Vertical Line (preceded and followed by a space)`

The following Nemeth rules apply to this example.

- The numeric indicator is omitted in the body of the plot.
- Spaces are not inserted between single-digit entries.
- The data are justified to the vertical line as shown in print.

Example 16-23

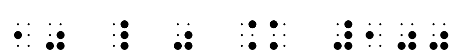
Stem	Leaf
10	0 4
9	3 4 5 7 8 9
8	0 2 2 9
7	5 8 9
6	3



The table is preceded and followed by a blank line. Code switch indicators do not replace required blank lines.

16.14 The Key

A key is often provided in the print copy and may be located above, below, next to, or within the table. The key is transcribed first, regardless of its placement in the print copy. Numeric indicators are not used for the portion of the key that replicates a numeric entry in the plot. (In this illustration, 10|0 is the portion of the key replicating the stem "10" and the leaf "0".) The value assigned to the key is transcribed using the numeric indicator as required by the Nemeth Code. (In this illustration, a numeric indicator is used for the value "100".)

⫸ $10 \mid 0 = 100$ 

If the key is printed in a variant typeform, the typeform is disregarded in braillle. A key is not constructed by the transcriber if one does not appear in print.

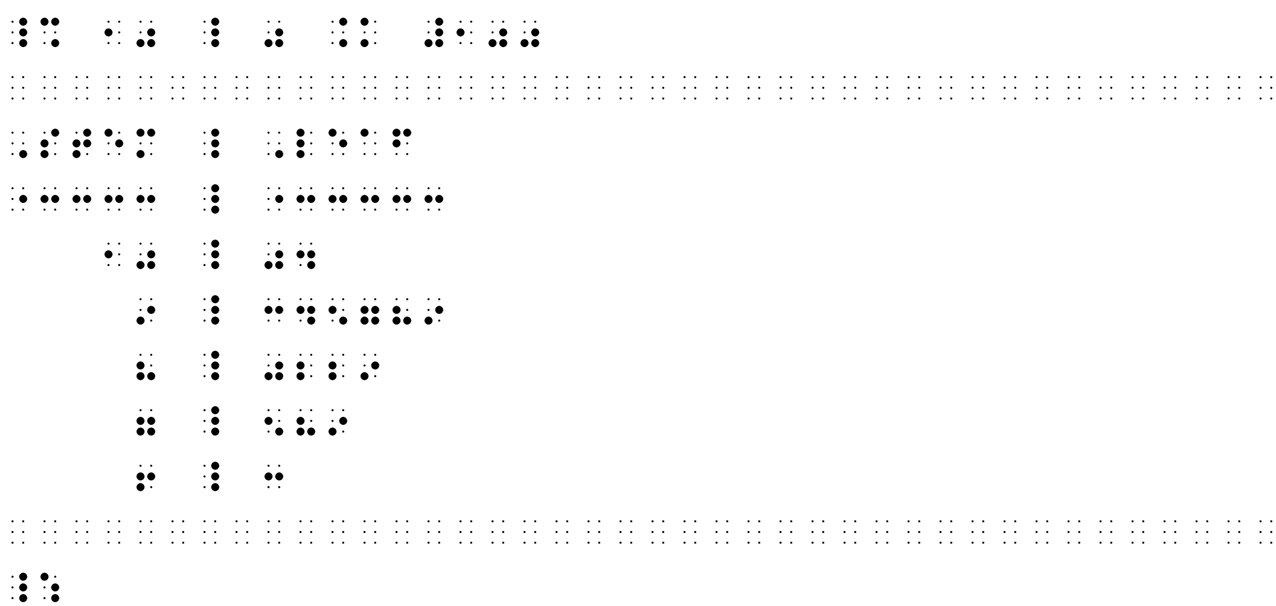
The following Nemeth rules apply to this example.

- The key is transcribed first, beginning in cell 1.
- The numeric indicator is omitted in the body of the plot.
- Spaces are not inserted between single-digit entries.

Example 16-24

Stem	Leaf
10	0 4
9	3 4 5 7 8 9
8	0 2 2 9
7	5 8 9
6	3

$10 \mid 0 = 100$



16.14.1 **Runovers in the Key.** If the key requires a runover line, the runover begins in cell 3. When there are words in the key, they may be transcribed in UEB, switching to Nemeth for the portion that reflects the stem and leaf.

The key in the next example includes a descriptive label ("Key:"). The label is transcribed only if it appears in print. If the key is set off by shading or in a box, box lines are not transcribed.

Example 16-25

Key: 2 | 2 represents
2,150,000 to 2,249,999
copies sold.

16.15 Data Consisting of More Than One Character; Punctuation Between Entries

The next example shows leaf entries with a decimal point and more than one digit. It also shows commas between leaf entries. The key is printed within the body of the table. The following rules are applied:

- The key is transcribed first.
- The numeric indicator is omitted in the body of the plot.
- Data consisting of groups of two or more digits require one blank cell between entries.
- Punctuation printed between units of data is omitted in the transcription.

Example 16-26

Stem	Leaf	
5	8.3	
6	4.3, 5.1, 5.5, 6.7, 7.0, 8.7, 9.3	
7	0.0, 2.8, 3.2, 7.4, 7.4	5 | 8.3 = 58.3

16.16 Alphabetic Data

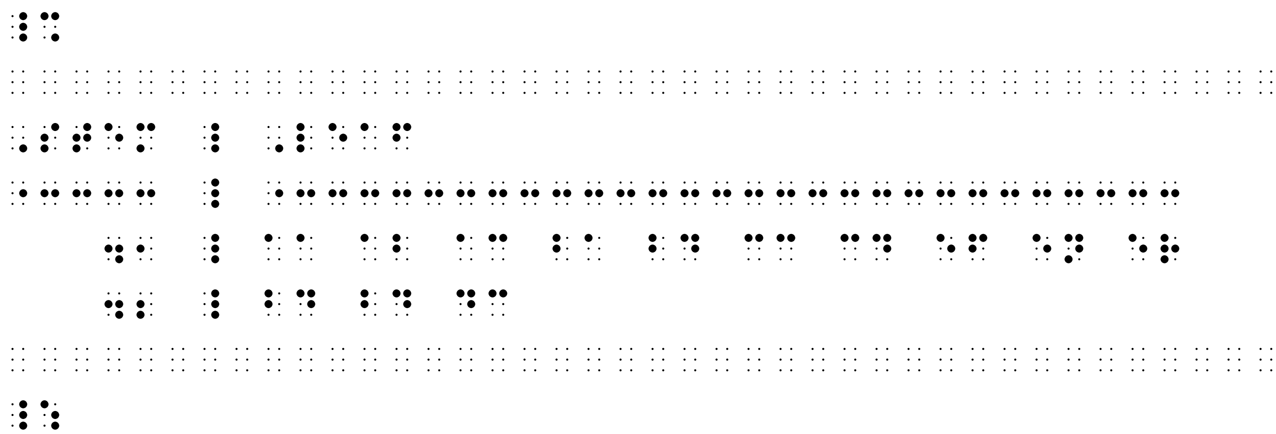
For alphabetic data, the following rules apply.

- The English-letter indicator is omitted in the body of the plot.
- Spaces are not inserted between single-letter data entries.
- Data consisting of groups of two or more letters require one blank cell between entries.

The leaves in the next example consist of pairs of lowercase letters. There is no key.

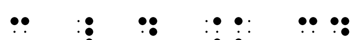
Example 16-27

Stem	Leaf
41	aa ab ac ba bd cc cd ef en er
42	bd bd dc



Reminder: The numeric indicator is omitted in the body of the plot (Stem column).

16.16.1 **Alphabetic Key.** In an alphabetic key, the English-letter indicator is not used for the portion of the key that replicates an entry in the plot. The value assigned to the key is transcribed using the English-letter indicator as required by the Nemeth Code.

⫸ c | d = cd 

16.17 Blank Entries

A blank entry in a stem-and-leaf plot is shown as blank space in braille. Do not transcribe a general omission symbol. Do not fill the width of the column with dot 5s.

The following rules also apply to the next example.

- The numeric indicator and the English-letter indicator are omitted in the body of the plot.
- Spaces are not inserted between single-letter data entries.
- Capital letters are capitalized individually.

Example 16-28

95	M
96	
97	M
98	M
99	M M A M

⠸⠩

⠔⠢ ⠸ ⠠⠍
⠔⠖ ⠸
⠔⠶ ⠸ ⠠⠍
⠔⠦ ⠸ ⠠⠍
⠔⠔ ⠸ ⠠⠍⠠⠍⠠⠁⠠⠍

⠸⠱

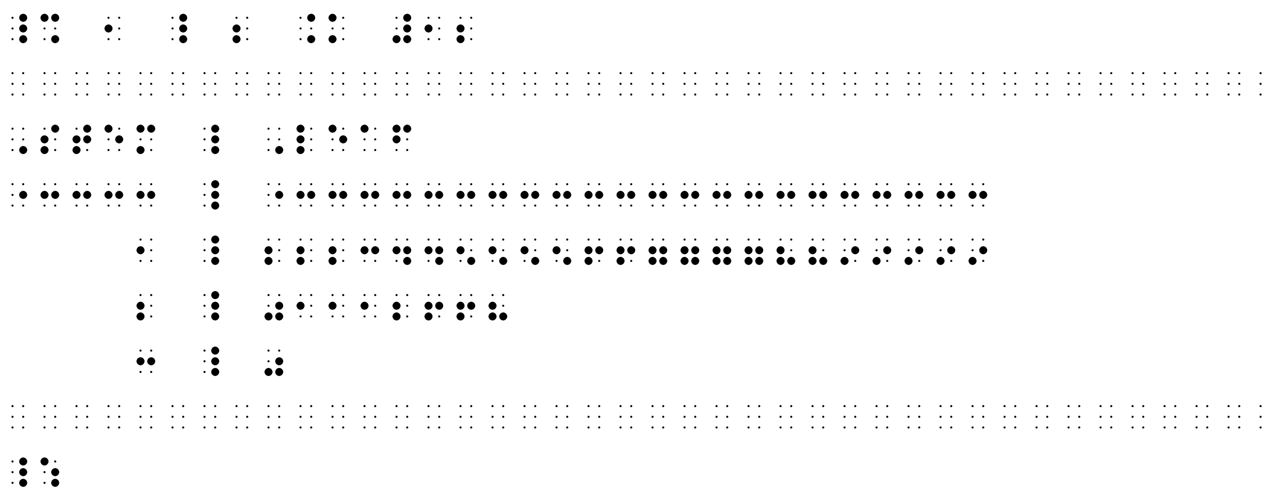
This plot has no key and no column headings—none are added in braille. The blank space in the second row indicates a blank entry in the leaf column. Single letters are unspaced and are capitalized individually.

16.18 Runovers Within the Table

16.18.1 **Print Runovers.** A runover of leaves shown in the print copy should be ignored if all leaves will fit on one row in braille. Use the full width of the available cells in the row.

Example 16-29

Stem	Leaf
1	2 2 2 3 4 4 5 5 5 5 6 6 7
	7 7 7 8 8 9 9 9 9 9
2	0 1 1 1 2 6 6 8
3	0
	1 | 2 = 12

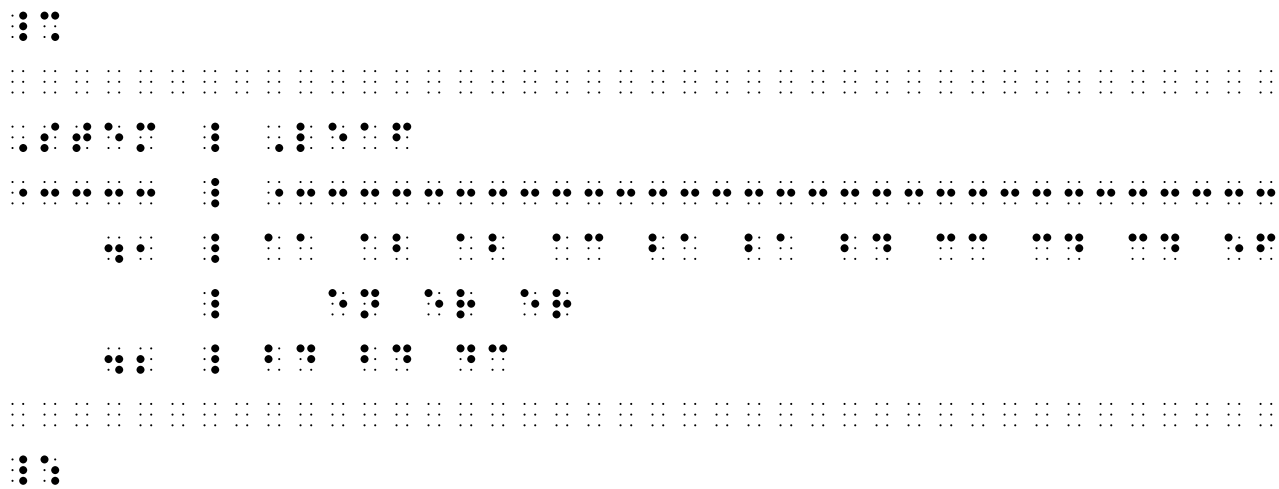


The first row of leaf data requires two lines in print, but not in braille.

16.18.2 **Braille Runovers.** If leaf data will not fit on one line in the braille transcription, indent the runover line two cells to the right after using as much of the braille line as possible. Do not repeat the stem row heading. The next example illustrates, expanding the two-letter data example shown previously.

Example 16-30

Stem	Leaf
41	aa ab ab ac ba ba bd cc cd cd ef en er er
42	bd bd dc



The vertical line is transcribed on every line in the table.

16.19 Back-To-Back Plot

A back-to-back stem-and-leaf plot is used when two sets of data being compared. There are three columns. The stem is the middle column. Data is read from the stem outward, which means that data in the left leaf is read from right to left. There will be two keys. Here are the rules as they apply to the next example.

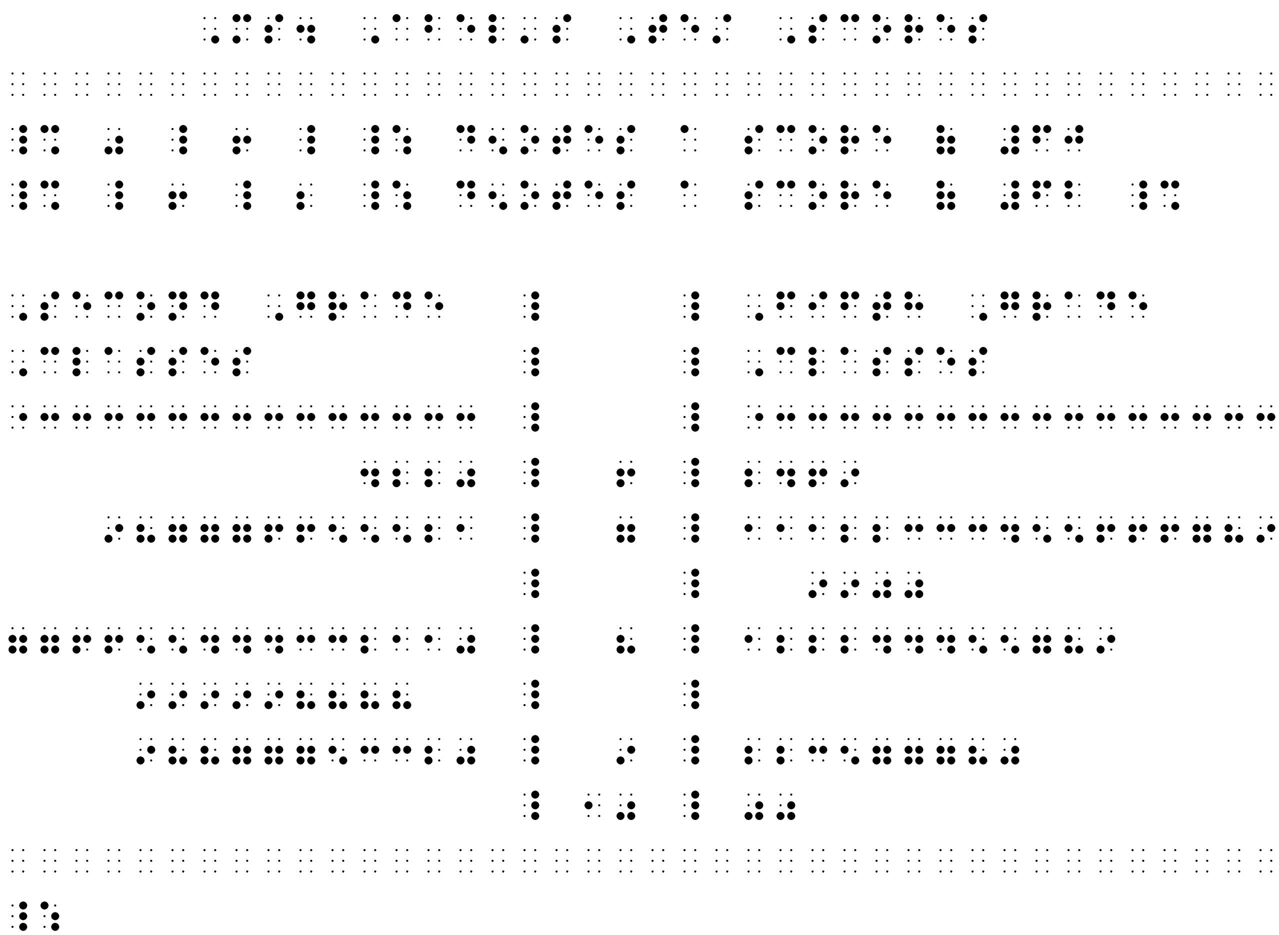
- *The Keys*: The left column's key is transcribed first, followed on the next line by the right column's key.
- *The Columns*: Column separation lines are transcribed below only the lines that contain column headings.
- *The Runovers*: Right leaf data are read from left to right; runovers to the right leaf data column are shifted two cells to the right. Left leaf data are read from right to left; runovers to the left leaf data column are shifted two cells to the left.

Example 16-31

Ms. Abel's Test Scores

Second Grade Classes		Fifth Grade Classes
4220	6	2469
987776655521	7	111223334556667899900
999998888776655444332110	8	122244455789
98877753320	9	223577780
	10	00

0 | 6 | *denotes a score of 60* | 6 | 2 *denotes a score of 62*



Line 3: Key to left column. Italic typeform is disregarded.
Line 4: Key to right column. Italic typeform is disregarded.
Line 5: A blank line precedes the plot.
Lines 6-7: Column headings are uncontracted in a stem-and-leaf plot. These column headings require two lines. Runovers to column headings are blocked.
Line 8: The stem column (middle column) has no heading, so no column separation line is transcribed.
Line 11: Reading left to right, runovers to the right leaf data column are shifted two cells to the right.

Line 13: Reading right to left, runovers to the left leaf data column are shifed two cells to the left.
Line 15: The blank left-leaf entry in the last row is a blank space in braille.
Line 16: The blank line is required following the table.
Line 17: The Nemeth Code terminator is alone on this line, outside of the table arrangement.

PRACTICE 16H

A pet store owner constructed the following stem-and-leaf plot showing the number of guinea pigs at each of her seventeen stores.

Stem	Leaf
0	7 8
1	
2	0 6 8 8 8
3	0 2 6 6 7 8
4	1 2 6 6
5	
	Key: 2 | 0 represents 20 guinea pigs

How many stores have fewer than 36 guinea pigs?

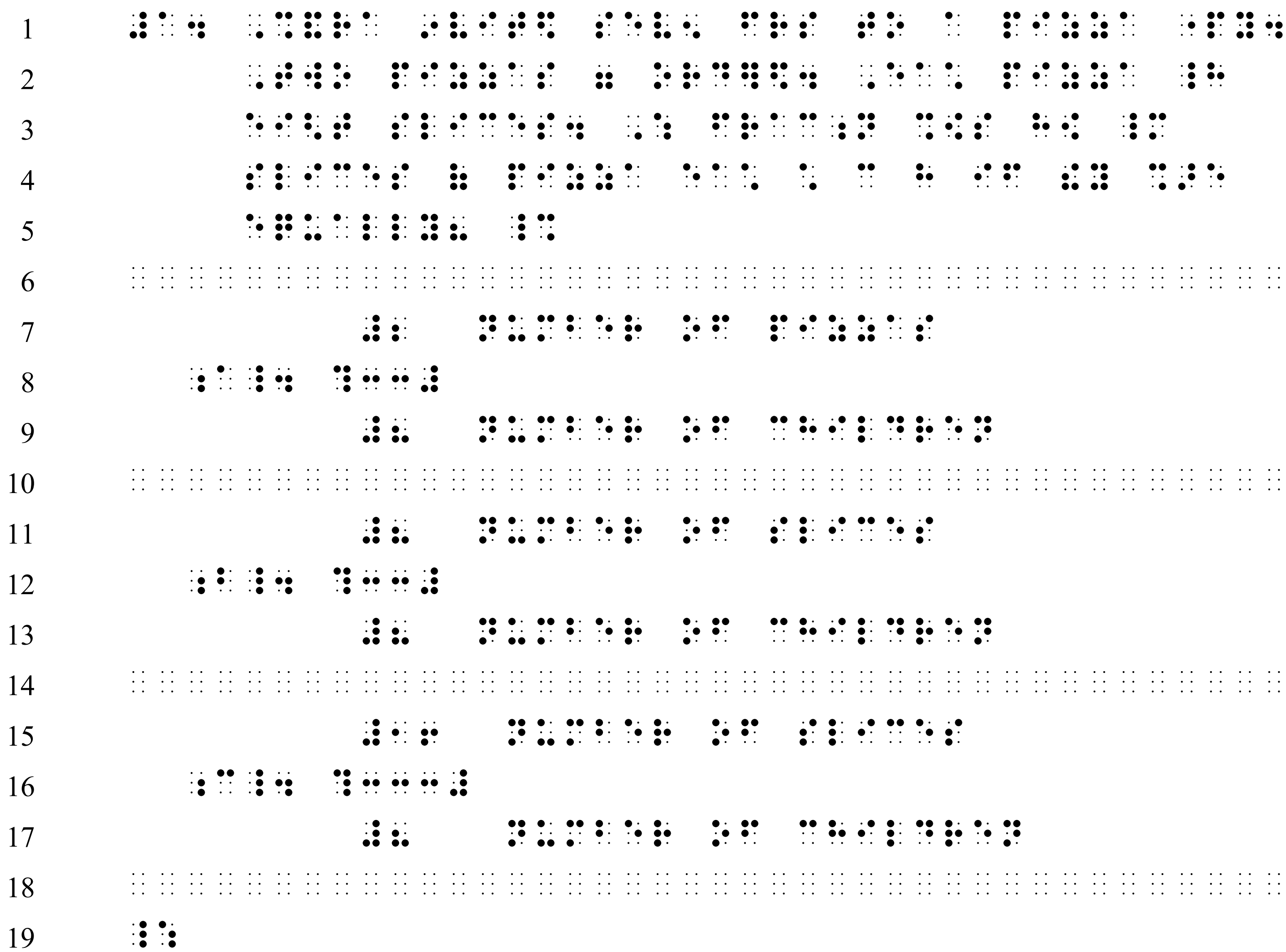
For further practice, see Appendix A—Reading Practice.

EXERCISE 16

Prepare Exercise 16 for your grader.

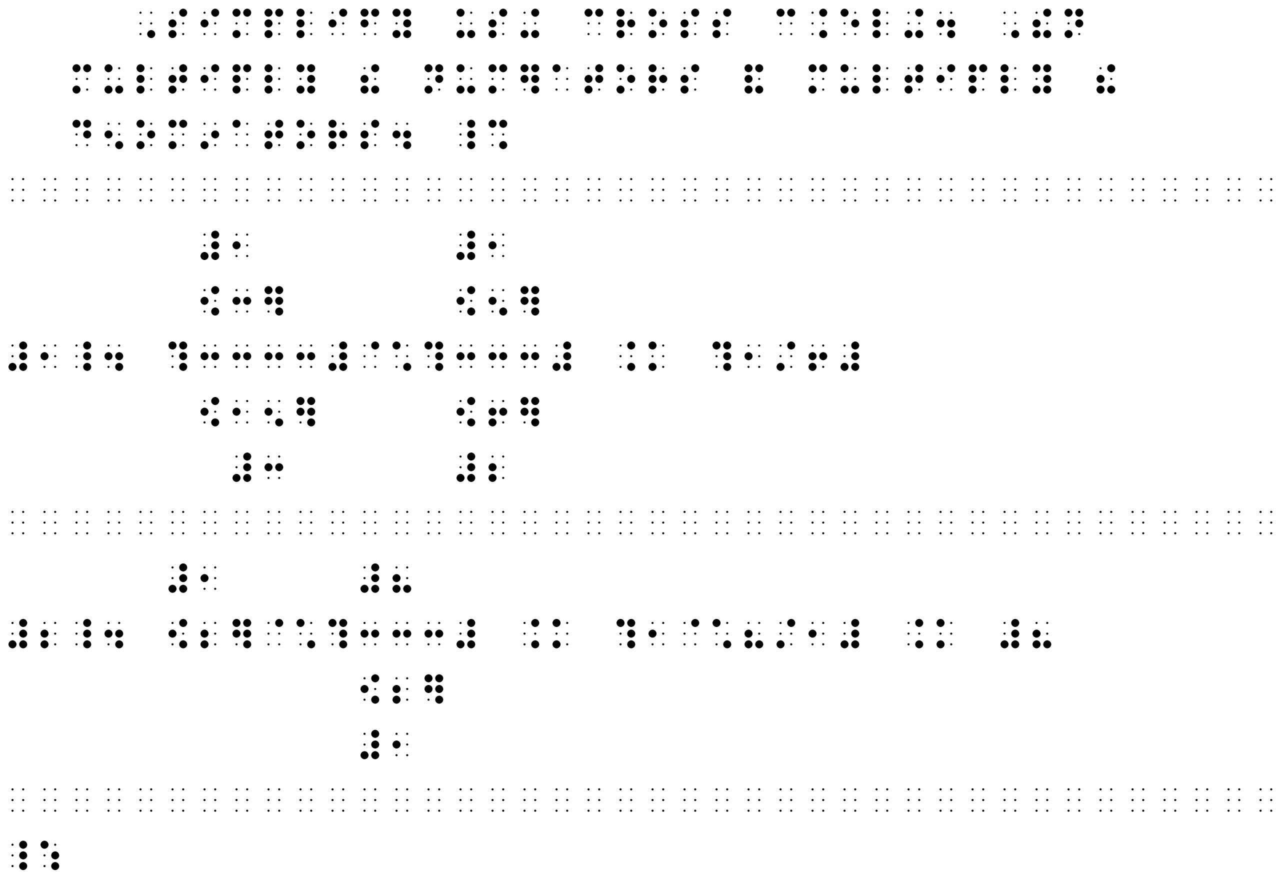
ANSWERS TO PRACTICE MATERIAL

PRACTICE 16A



Lines 18-19: Recall from Lesson 9 that, following a spatial arrangement, the required blank line comes first, followed by the Nemeth Code terminator on the next line in cell 1.

PRACTICE 16B



Lines 1-3: Nemeth "Instructions" format is applied. Margins are 5-3 and the bold typeform is disregarded.
Line 4: A blank line must precede the spatial material which follows.

PRACTICE 16C

PRACTICE 16D

Line 5: Excessive code switching is avoided by staying in Nemeth Code to transcribe "100 yd."
Lines 6-12: This is a nested list format because the equals signs are aligned in the print copy.
Runovers to the first link are in cell 7 (lines 8-10).

PRACTICE 16E

⠼⠁⠲ ⠠⠎⠕⠇⠧⠑ ⠿ ⠰⠗ ⠔ ⠞⠻⠍⠎ ⠷ ⠰⠎⠲ ⠸⠩
⠀⠀
⠠⠹⠹⠗⠘⠆⠐⠤⠲⠎⠘⠆⠐⠌⠎⠘⠆⠐⠼
⠠⠌⠹⠗⠬⠆⠎⠌⠎⠼⠠⠼
⠠⠠⠹⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠠⠠⠼
⠠⠹⠹⠲⠗⠤⠆⠎⠘⠆⠐⠌⠒⠎⠼
⠠⠌⠹⠆⠎⠘⠆⠐⠤⠒⠗⠌⠲⠗⠼⠠⠼
⠀⠀
⠸⠱

PRACTICE 16F

⠠⠮ ⠎⠟⠥⠜⠑ ⠗⠕⠕⠞ ⠷ ⠼⠉ ⠉ ⠆ ⠑⠭⠏⠗⠑⠎⠎⠫ ⠵ ⠁
⠒⠞⠔⠥⠫ ⠋⠗⠁⠉⠰⠝⠲ ⠸⠩
⠀⠀
⠼⠂
⠜⠒⠻ ⠨⠅ ⠼⠂⠬⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠼⠂
⠼⠒⠬⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠼⠂
⠼⠒⠬⠒⠒⠒⠒⠒⠒⠒
⠼⠒⠬ ⠄⠄⠄
⠀⠀
⠸⠱

PRACTICE 16G

```
1   ⠀⠀⠠⠱⠁⠞ ⠏⠻⠉⠢⠞ ⠷ ⠼⠙⠓ ⠊⠎ ⠼⠑⠙⠦ ⠸⠩
2     ⠝⠡⠲⠦ ⠨⠅ ⠼⠢⠲ ⠸⠱
3       ⠀⠀⠀⠀⠠⠺⠗⠊⠞⠑ ⠁⠝ ⠑⠟⠥⠁⠰⠝⠲ ⠸⠩
4     ⠹⠲⠦⠝⠌⠲⠦⠼ ⠨⠅ ⠹⠢⠲⠌⠲⠦⠼ ⠸⠱
5       ⠀⠀⠀⠀⠠⠙⠊⠧⠊⠙⠑ ⠑⠁⠡ ⠎⠊⠙⠑ ⠃⠽ ⠼⠙⠓⠲ ⠸⠩
6     ⠝
7       ⠨⠅ ⠼⠂⠨⠂⠆⠢
8         ⠀⠀⠀⠀⠠⠄⠠⠎⠊⠍⠏⠇⠊⠋⠽⠲
9       ⠨⠅ ⠼⠂⠂⠆⠨⠢⠈⠴ ⠸⠱
10        ⠀⠀⠀⠀⠠⠡⠁⠝⠛⠑ ⠞⠕ ⠁ ⠏⠻⠉⠢⠞⠲
11  ⠨⠂⠠⠁⠝⠎⠺⠻⠒ ⠼⠑⠙ ⠊⠎ ⠸⠩ ⠼⠂⠂⠆⠨⠢⠈⠴ ⠸⠱ ⠷ ⠼⠙⠓⠲
```

Line 1: The narrative paragraph begins in cell 3. There is no runover.
Line 2, 4, 6: Each displayed expression begins in cell 3.
Lines 2 and 4: Alignment of the equals signs in print is disregarded.
Lines 3, and 5: Each comment is blocked in cell 9, which is four cells to the right of the runover cell (cell 5). The italic typeform is disregarded.
Lines 6-9: This is a nested linked expression. Each link begins in the runover cell (cell 5) according to the rules for nested linked expressions.
Lines 8 and 10: Each comment is blocked in cell 11, which is four cells to the right of the runover cell in this nested list format (cell 7).
Line 8: The single-word switch indicator is used even though there are no contractions in the word. Nemeth continues on the next line.
Line 11: The narrative paragraph continues from line 1, in the runover cell (cell 1).

PRACTICE 16H

⠀⠀⠠⠁ ⠏⠑⠞ ⠌⠕⠗⠑ ⠪⠝⠻ ⠒⠌⠗⠥⠉⠞⠫ ⠮ ⠋⠕⠇⠇⠪⠬
⠌⠑⠍⠤⠯⠤⠇⠂⠋ ⠏⠇⠕⠞ ⠩⠪⠬ ⠮ ⠝⠥⠍⠃⠻ ⠷ ⠛⠥⠔⠑⠁ ⠏⠊⠛⠎
⠁⠞ ⠑⠁⠡ ⠷ ⠓⠻ ⠎⠑⠧⠢⠞⠑⠢ ⠌⠕⠗⠑⠎⠲

⠠⠅⠑⠽⠒ ⠸⠩ ⠆ ⠸ ⠴ ⠸⠱ ⠗⠑⠏⠗⠑⠎⠢⠞⠎ ⠼⠃⠚ ⠛⠥⠔⠑⠁
⠀⠀⠏⠊⠛⠎ ⠸⠩

⠠⠎⠞⠑⠍ ⠸ ⠠⠇⠑⠁⠋
⠐⠒⠒⠒⠒ ⠸ ⠐⠒⠒⠒⠒⠒
⠀⠀⠀⠀ ⠴ ⠸ ⠶⠦
⠀⠀⠀⠀ ⠂ ⠸
⠀⠀⠀⠀ ⠆ ⠸ ⠴⠖⠦⠦⠦
⠀⠀⠀⠀ ⠒ ⠸ ⠴⠆⠖⠖⠶⠦
⠀⠀⠀⠀ ⠲ ⠸ ⠂⠆⠖⠖
⠀⠀⠀⠀ ⠢ ⠸

⠸⠱
⠠⠓⠪ ⠸⠍ ⠌⠕⠗⠑⠎ ⠓ ⠋⠑⠺⠻ ⠹⠁⠝ ⠼⠉⠋ ⠛⠥⠔⠑⠁ ⠏⠊⠛⠎⠦

LESSON 17

- SYSTEM OF EQUATIONS
    - Enlarged Signs of Grouping
- DETERMINANTS AND MATRICES

Answers to Practice Material

LESSON PREVIEW

Enlarged grouping signs are often encountered in the topics studied in this lesson: systems of equations and arrays (matrices and determinants). Format rules are given for these spatial arrangements, including further considerations regarding commentary.

SYSTEM OF EQUATIONS

[NC 25.9]

17.1 Definition and Recognition

A system of equations, sometimes called "simultaneous equations," is a collection of two or more equations which share variables. The student identifies the value of each variable by "solving the system." You can recognize a system by noticing the arrangement of equations on two or more lines. If the system consists of two equations, there will be two variables to solve (typically x and y); if the system consists of three equations, there will usually be three variables to solve (typically x, y, and z). The equations may or may not be joined by an enlarged grouping sign.

Here is an example of a system of two equations using variables x and y.

$$4x - y = 10$$
$$2x = 12 - 3y$$

17.2 Transcription Rules for Systems of Equations

A system of equations is a spatial arrangement and is transcribed as follows.

(a) One blank line is left above and below the system. Placement of code switch indicators follows the general rules for spatial arrangements.

(b) Use and nonuse of the numeric indicator follows Nemeth rules for nonspatial material, even though this is a spatial arrangement. (See also, rule (d), below.)

(c) Alignment is maintained only if terms and symbols are aligned in print. If terms are aligned on one side of the equals sign but not on the other side, follow print. Alignment is disregarded if only the equals signs are aligned.

(d) When a space is inserted <u>within</u> the equation for the purpose of maintaining alignment, a numeric indicator is not used.

The same rules apply to a system of inequalities. (See Example 17-4).

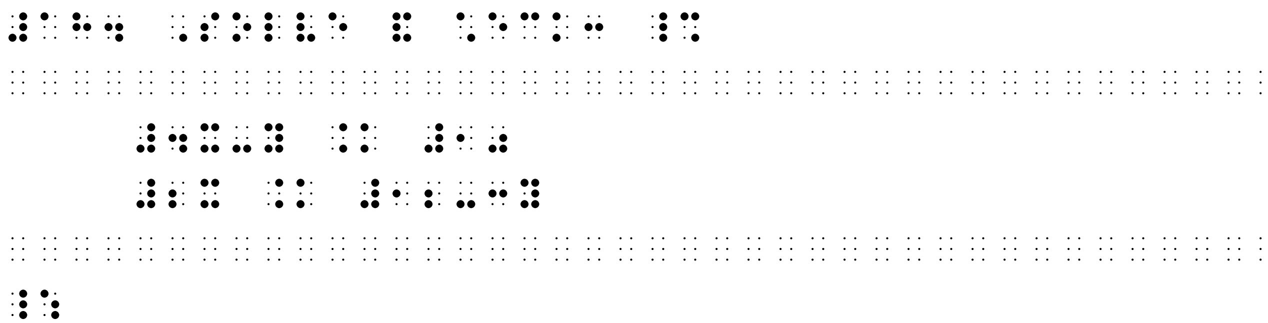
Note that, if a separation line is present, it is a spatially-arranged addition problem and Nemeth rules for spatial addition are followed. (See Lesson 9.)

Example 17-1

18. Solve and check.

$$4x - y = 10$$
$$2x = 12 - 3y$$

Print observation: This system has two variables (x, y) – two equations. The terms are not vertically aligned.



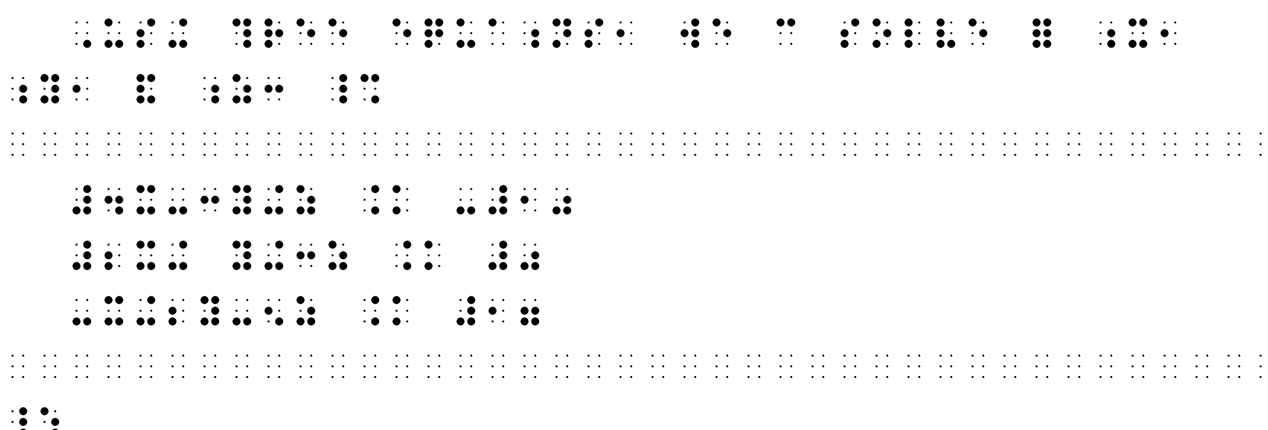
A blank line is inserted before and after the system (a). Each equation begins in cell 5, displayed to itemized material. Numeric indicators are used as needed according to Nemeth rules for nonspatial material (b).

Example 17-2

Using three equations, we can solve for x, y, and z.

$$\begin{aligned} 4x - 3y + z &= -10 \\ 2x + y + 3z &= 0 \\ -x + 2y - 5z &= 17 \end{aligned}$$

Print observation: The terms on both sides of the equals signs are not vertically aligned. Only the equals signs are aligned.

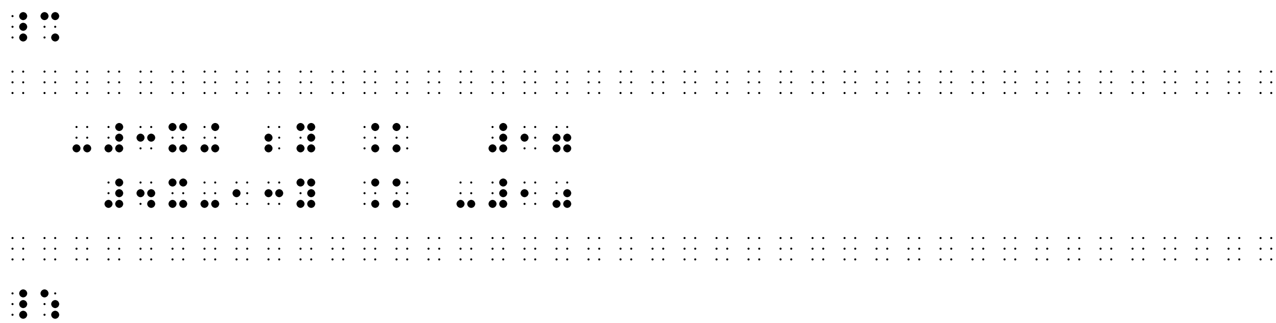


A blank line is inserted before and after the system (a). Each equation begins in cell 3, displayed to 3-1 narrative. Alignment of the equals signs is disregarded (c).

Example 17-3

$$\begin{array}{rcrcr} -3x & + & 2y & = & 17 \\ 4x & - & 13y & = & -10 \end{array}$$

Print observation: The terms on both sides of the equals signs are vertically aligned.

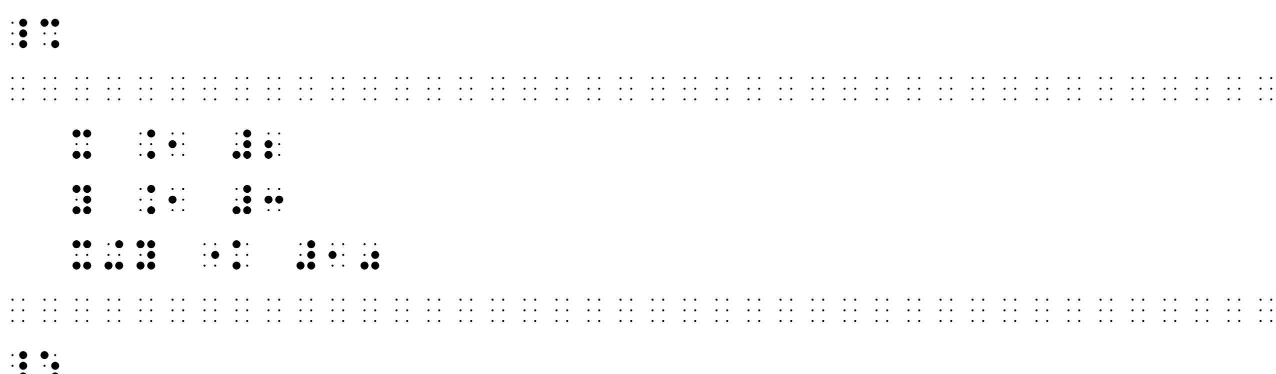


The leftmost symbol is placed in the appropriate cell for displayed material. Spaces are inserted in order to maintain alignment as printed (c). A numeric indicator is not used before the numeral "2" even though it is preceded by a space because when a space is inserted in order to achieve alignment, no numeric indicator is inserted (d). This rule does not apply to the first numeral to the right of the equals sign.

Example 17-4

$$x > 2$$
$$y > 3$$
$$x + y < 10$$

Print observation: Three inequalities are printed on three lines. They are centered to each other—terms are not aligned.



A blank line precedes and follows the arrangement. Each inequality begins in the same cell.

PRACTICE 17A

1. Solve and check:

$$\begin{array}{l} 2x+3y=2 \\ 8x-4z=3 \\ 3y-8z=-1 \end{array}$$

2. Solve:

$$\begin{array}{lr} 2x-5y+6z & =11 \\ 3x-2y+3z & =9 \\ 2x+4y-9z & =-3 \end{array}$$

3. Add:

$$\begin{array}{r} 3x-y=\ 7 \\ 2x+y=\ 8 \\ \hline 5x+0=15 \end{array}$$

Enlarged Signs of Grouping

[NC 19.6]

17.3 A Unified Expression

The Nemeth Code calls an arrangement a "unified expression" when equations are grouped together using an enlarged grouping symbol. Here is a print example a unified system of equations, using a left enlarged brace.

$$\begin{cases} x=y \\ 5x-y=4 \end{cases}$$

17.4 Transcription Rules for Enlarged Signs of Grouping

(a) If only the left or right grouping sign is shown in print, only that symbol is shown in braille.

(b) Enlarged grouping symbols are transcribed on each line of the unified expression and are vertically aligned.

(c) When terms are not aligned in print, each line of the unified expression begins in the same cell. When there is a left enlarged grouping symbol, each expression will begin in the cell which immediately follows the left enlarged grouping symbol.

(d) When the expressions require vertical alignment, at least one item must begin in the cell which immediately follows the left enlarged grouping symbol. The numeric indicator is not required before the first numeric character following a left grouping symbol.

(e) At least one item ends in the cell which immediately precedes the right enlarged grouping symbol.

17.4.1 **Left Enlarged Brace.** The left enlarged brace curves and points to the left in print. Notice that the enlarged braille symbol is formed by inserting a dot 6 before the second cell of the normal brace symbol.

⠨⠷	Left Brace (normal size)	$\{$
⠨⠠⠷	Left Enlarged Brace (covering two or more lines)	

Example 17-5

$$\begin{cases} x = y \\ 5x - y = 4 \end{cases}$$

Print observation: A left enlarged brace groups the unified system of equations. Terms are not vertically aligned.

```
⠸⠩

  ⠨⠠⠷⠭ ⠨⠅ ⠽
  ⠨⠠⠷⠢⠭⠤⠽ ⠨⠅ ⠼⠲

⠸⠱
```

The enlarged grouping symbols are aligned. The terms are not aligned in print; each equation begins in the cell following the left enlarged brace. A numeric indicator is not needed for the numeral 5 because it is not preceded by a space. A numeric indicator is required for the numeral 4.

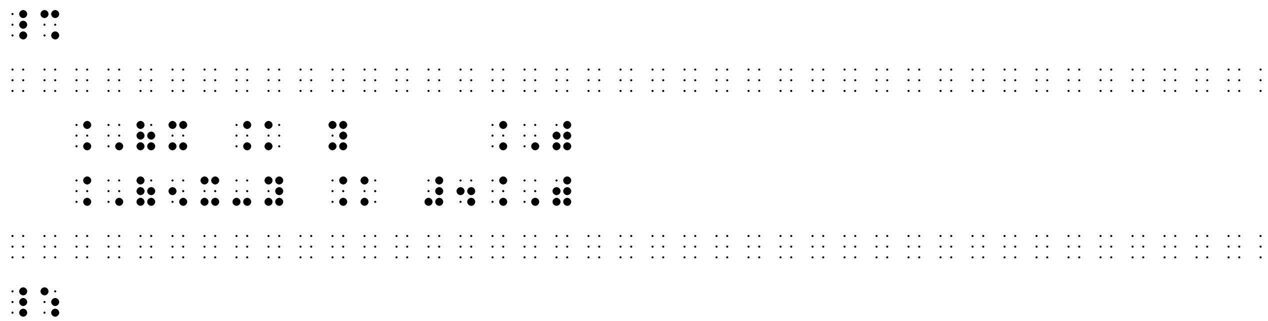
17.4.2 **Right Enlarged Brace.** The right enlarged brace curves and points to the right in print. Notice that the enlarged braille symbol is formed by inserting a dot 6 before the second cell of the normal brace symbol.

⠨⠾	Right Brace (normal size)	$\}$
⠨⠠⠾	Right Enlarged Brace (covering two or more lines)	

The same system is shown below, with a right enlarged brace added and with each expression centered within the braces.

Example 17-6

$$\left\{\begin{array}{c} x = y \\ 5x - y = 4 \end{array}\right\}$$



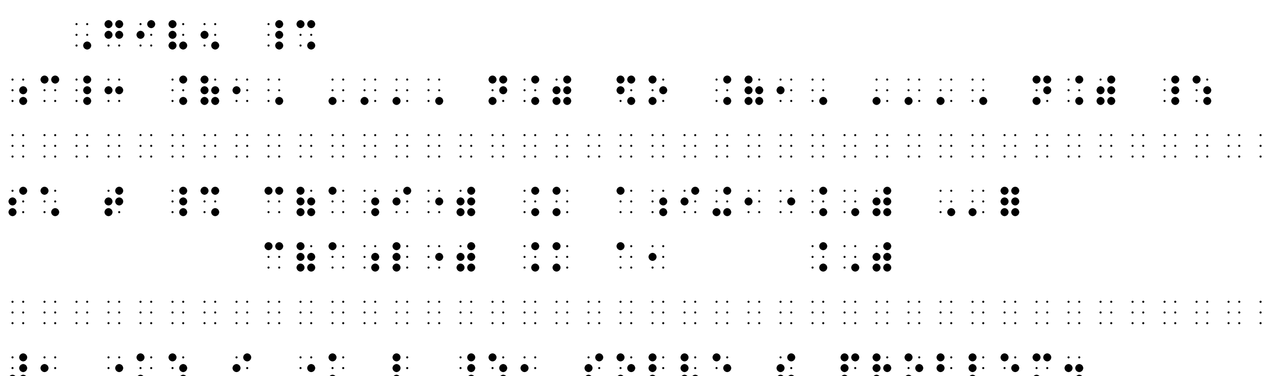
Each expression begins in the cell next to the left grouping symbol. The right grouping symbols are vertically aligned, starting in the cell next to the widest equation.

17.5 Embedded Vertical Groupings

Enlarged grouping signs are used in other structures as well, not only with systems of equations. If the grouping is embedded within narrative, the required blank lines before and after the arrangement become part of the paragraph. The surrounding text is placed on the top line of the arrangement only – before and/or after the math as it fits in context of the narrative. Text continues in the runover cell of the paragraph after the second required blank line. The opening Nemeth Code indicator and the Nemeth Code terminator are placed only on the top line of the embedded arrangement. The switch indicators apply to the whole arrangement.

Example 17-7

Given $c\colon \{1, \ldots, n\} \to \{1, \ldots, n\}$ such that $\left.\begin{array}{l} c(a_i) = a_{i+1} \\ c(a_l) = a_1 \end{array}\right\}$ for $1 \le i < l$, solve the problem.



Lines 1-2: The paragraph begins in cell 3. The runover is in cell 1.
Line 3: A blank precedes the spatial arrangement that is embedded in the paragraph.
Line 4: The narrative continues,in the runover cell of the paragraph. The opening switch is placed on this line only. The narrative continues after the top right enlarged grouping symbol.
Line 5: The second line of the embedded spatial arrangement is aligned with the line above, starting in the same cell and with the right grouping symbols aligned.
Line 6: A blank follows the spatial arrangement that lies within the paragraph.
Line 7: The paragraph continues.

PRACTICE 17B

After solving for x and y, we determine that the system $\left\{\begin{matrix} 2x-3y=17 \\ 3x+2y=6 \end{matrix}\right\}$ has the solution set $\{(4,-3)\}$. Name the solution set for this system:

$$\left\{\begin{matrix} x+2y=6 \\ 2x-y=7 \end{matrix}\right\}.$$

17.6 Enlarged Parentheses

Notice that the enlarged braille parentheses are formed by inserting a dot 6 before the normal parenthesis symbol.

⠷	Left Parenthesis (normal)	(
⠠⠷	Left Enlarged Parenthesis (covering two or more lines)	
⠾	Right Parenthesis (normal)	)
⠠⠾	Right Enlarged Parenthesis (covering two or more lines)	

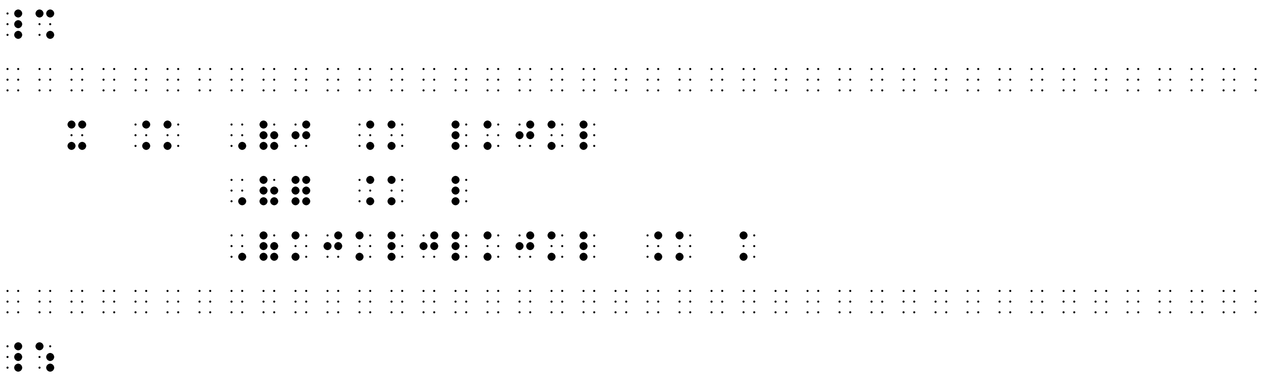
17.7 Placement of Symbols

Symbols which appear outside of the enlarged grouping symbol and which apply to the arrangement are placed on the top line of the arrangement, even if the items are centered in print.

Example 17-8

$$x = \left(\begin{array}{r} j = lkjkl \\ = l \\ kjkljlkjkl = k \end{array} \right.$$

Print observation: A three-line system is grouped on the left with an enlarged parenthesis. The anchor, x, is centered with respect to the enlarged grouping symbol to its right. The equals signs in the grouped arrangement are aligned, but the terms are not.



The anchor is aligned with the top line of the arrangement. Each enlarged grouping symbol begins in the same cell. The grouped items are left adjusted because the terms are not aligned. A general omission symbol denotes the blank space before the middle equals sign. English-letter indicators are not needed for the single letters s, j, l, and k because they are next to a sign of comparison. Code switch indicators do not interfere with the spatial arrangement.

17.8 Placement of Identifiers and Punctuation

Identifiers and punctuation which appear outside of the enlarged grouping symbol and which apply to the arrangement are placed on the top line of the arrangement, even if the items are centered in print. Here is the same example, now identified with an item number "15."

Example 17-9

$$15. \quad x = \left(\begin{array}{r} j = lkjkl \\ = l \\ kjkljlkjkl = k \end{array} \right.$$

Print observation: The item number and the anchor are centered with respect to the associated enlarged grouping symbol to the right.

```
⠸⠩

⠼⠂⠢⠸⠲  ⠭ ⠨⠅ ⠠⠷⠚ ⠨⠅ ⠇⠅⠚⠅⠇
           ⠠⠷⠿ ⠨⠅ ⠇
           ⠠⠷⠅⠚⠅⠇⠚⠇⠅⠚⠅⠇ ⠨⠅ ⠅

⠸⠱
```

Item number and anchor are aligned with the top line of the arrangement.

Example 17-10

... such that $\left.\begin{array}{l} \frac{1}{2}x + y = 7 \\ 5x - y = 4 \end{array}\right)$. First, solve for y in terms of x.

Print observation: A vertical arrangement is embedded within the narrative. It is grouped on the right with a right enlarged parenthesis. A period follows the enlarged parenthesis, centered to the arrangement. The narrative continues on the same line.

```
1
2 ⠲⠲⠲ ⠎⠡ ⠞ ⠸⠩ ⠹⠂⠌⠆⠼⠭⠬⠽ ⠨⠅ ⠼⠶ ⠠⠾ ⠸⠱⠲ ⠠⠋⠌⠂
3             ⠼⠢⠭⠤⠽ ⠨⠅ ⠼⠲     ⠠⠾
4
5 ⠎⠕⠇⠧⠑ ⠿ ⠰⠽ ⠔ ⠞⠻⠍⠎ ⠷ ⠰⠭⠲
```

Line 1: A blank line precedes the line with the embedded spatial arrangement.
Line 2: The top line of the arrangement begins on the main line of the text. The code switch indicators also are placed on this line. The paragraph continues after the period.
Line 3: The second line of the arrangement is placed here. The right enlarged parenthesis is aligned with the identical symbol on the line above. No code switch indicators are placed in this line.
Line 4: A blank line follows the embedded spatial arrangement.
Line 5: The paragraph text continues.

17.9 Nested Grouping Symbols

Recall from Lesson 2 that nested grouping symbols may be printed in different sizes in order to visually distinguish the nested symbols, but they are not transcribed as enlarged symbols if they apply to only one line of characters.

⫸ $\mathrm{p}\Bigg(q\ \bigg(f\ \bigg(g\ \big(h\,(\,x\,)\big)\bigg)\bigg)\Bigg)$

⠏⠷⠟⠷⠋⠷⠛⠷⠓⠷⠭⠾⠾⠾⠾⠾

Similarly, grouping signs which enclose taller print constructions such as fractions, modified expressions, integrals, or binomial coefficients (12.7.1) are transcribed as normal grouping symbols because they apply to only one line of braille characters.

⫸ $\left(\frac{3}{5}\right)^3 = \frac{27}{125} = 0.216$

⠷⠹⠒⠌⠢⠼⠾⠘⠒ ⠨⠅ ⠹⠆⠶⠌⠂⠆⠢⠼ ⠨⠅ ⠼⠴⠨⠆⠂⠖

⫸ $\left(\sum_{k} \binom{n}{k}\right)$ ⠷⠐⠨⠠⠎⠩⠅⠻⠷⠝⠩⠅⠾⠾

Review: Notice the two types of question marks found in this Practice. In item A, each question mark is a modifier, transcribed as a punctuation mark placed directly above the equals sign. (See Lesson 12.) Each question mark in item B denotes an omitted item in an enclosed list, and so is transcribed as an omission symbol. (See Lesson 1).

PRACTICE 17C

A) Verify (YES or NO):

$$2\left(-\frac{4}{3}\right) - 1 \overset{?}{=} 4\left(-\frac{4}{3}\right) + 9$$

$$\left(-\frac{8}{3}\right) - 1 \overset{?}{=} \left(-\frac{16}{3}\right) + 9$$

$$-8 - 3 \overset{?}{=} -16 + 27$$

$$-11 = 11. \qquad \text{NO}$$

B) $\begin{pmatrix} \frac{x}{3} - \frac{y}{2} = 2 \\ 5x + 3y = 51 \end{pmatrix} = (?, ?)$

17.10 Conditions or Commentary Printed Next to a Spatial Arrangement [NC 25.10.1]

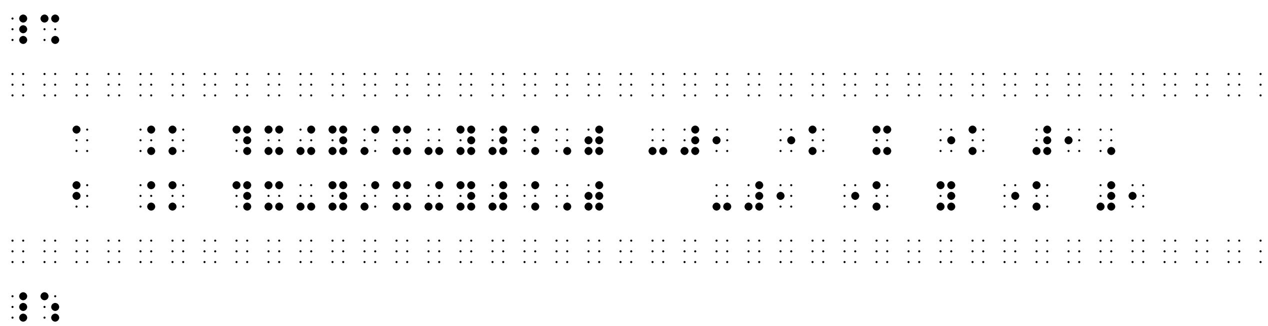
The format for instructional commentary presented in Lesson 16 apply to comments that alternate with linear math problems. Now we will look at a condition or comment which applies to a spatial arrangement.

17.10.1 **Commentary Printed Next to Enlarged Grouping Symbols.** The comment begins on the first line of the arrangement, even if it is centered to the arrangement in print. Runovers are indented two cells from the cell in which the comment begins.

Example 17-11

$$\left.\begin{array}{l} a=\dfrac{x+y}{x-y} \\[2ex] b=\dfrac{x-y}{x+y} \end{array}\right\} \quad -1<x<1, -1<y<1$$

Print observation: A right enlarged brace groups two equations. Conditions for x and y are printed to the right of the brace.



Each equation begins in cell 3. The comment begins to the right of the first (top) enlarged brace. The runover is indented two cells from the beginning of the comment.

a. **Code Switching when Comments are in UEB.** Code switching is necessary if the comment contains narrative. Switching to UEB in the comment does not affect the reading of the math in the spatial arrangement. The code in effect at the end of the comment will apply to the material which follows the completion of the spatial arrangement, even if the comment ends on a different line.

If the comment begins with an opening Nemeth Code indicator, runovers are indented two cells from the cell in which the comment begins, not from the beginning of the Nemeth Code terminator.

Example 17-12

Now we will look at a system with three variables: x, y, and z.

$$\left.\begin{array}{r} y+3z=10 \\ x+y+z=6 \\ 3y-z=13 \end{array}\right\} \quad \textit{Solve by isolating each term.}$$

Write your answer below.

Print Observation: Three equations are grouped with a right enlarged brace. A comment is printed to the right of the middle equation. The comment is narrative text.

⠀⠀⠠⠝⠪ ⠺⠑ ⠺ ⠇⠕⠕⠅ ⠁⠞ ⠁ ⠎⠽⠌⠑⠍ ⠾ ⠹⠗⠑⠑
⠧⠜⠊⠁⠃⠇⠑⠎⠒ ⠰⠭⠂ ⠰⠽⠂ ⠯ ⠰⠵⠲ ⠸⠩

⠀⠀⠽⠬⠒⠵ ⠨⠅ ⠼⠂⠴ ⠨⠠⠾ ⠸⠱ ⠠⠎⠕⠇⠧⠑ ⠃⠽ ⠊⠎⠕⠇⠁⠞⠬
⠀⠀⠭⠬⠽⠬⠵ ⠨⠅ ⠼⠖ ⠨⠠⠾⠀⠀⠀⠀⠀⠀ ⠑⠁⠡ ⠞⠻⠍⠲
⠀⠀⠼⠒⠽⠤⠵ ⠨⠅ ⠼⠂⠒⠨⠠⠾

⠠⠺⠗⠊⠞⠑ ⠽⠗ ⠁⠝⠎⠺⠻⠎ ⠆⠇⠲

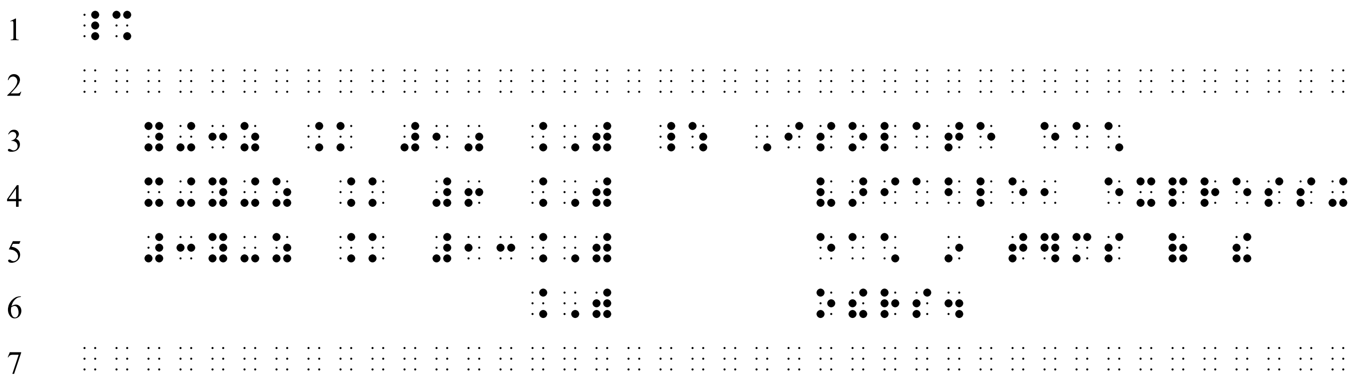
The comment begins on the top line of the arrangement. A Nemeth Code terminator precedes the comment, which is transcribed in UEB. The runover is indented two cells from the first cell of the comment. UEB narrative continues after the spatial arrangement.

b. **A Long Comment.** If the comment requires more lines than the spatial arrangement does, the enlarged grouping symbol is extended to cover the explanation.

Example 17-13

$$\left.\begin{aligned} y+3z&=10\\ x+y+z&=6\\ 3y-z&=13 \end{aligned}\right\}$$ *Isolate each variable, expressing each in terms of the others.*

Print Observation: This is similar to the previous example, but the comment is longer.



Line 3: The comment begins on the top line of the arrangement. A Nemeth Code terminator precedes the comment, which is transcribed in UEB.
Lines 4-6: The runovers are indented two cells from the first cell of the comment text.
Line 6: Four lines are required to complete the comment, so another right enlarged grouping symbol is transcribed.
Line 7: A blank line is required following the spatial arrangement, before narrative resumes.

c. **An Alternate Option.** If there is little room beside the math for the comment, it may be placed before or after the math arrangement, at the transcriber's discretion. A transcriber's note must explain that the remark applies to the spatial arrangement.

17.10.2 **Transcriber-Inserted Grouping Symbol [NC 19.8 and 25.10].** When a comment refers to more than one print line but no enlarged grouping sign is printed, the implied grouping is indicated by using the transcriber-inserted grouping symbol. Such a comment will be transcribed to right of the arrangement, regardless of its position in print. This symbol begins with dots (6, 3) but, in context, will not be misread as a single-word switch indicator due to the vertical aspect.

⠠⠄⠿	Transcriber-Inserted Grouping Symbol (on two or more lines)

Apply the same formatting rules as you would for printed enlarged grouping symbols, discussed in 17.10.1. If the comment takes fewer lines than the arrangement to which it refers, a transcriber-inserted grouping symbol is still placed on every line of the arrangement.

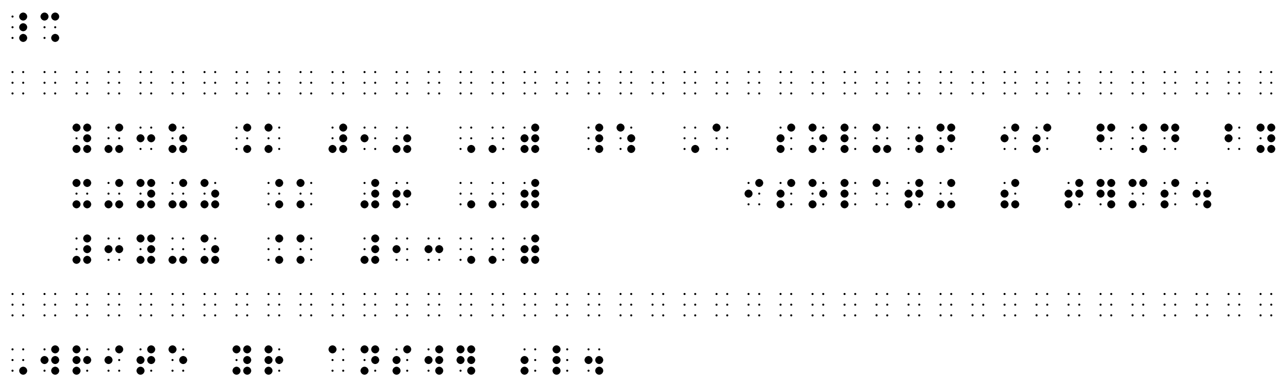
Example 17-14

$$\begin{aligned} y + 3z &= 10 \\ x + y + z &= 6 \\ 3y - z &= 13 \end{aligned}$$

A solution is found by isolating the terms.

Write your answer below.

Print observation: A comment is printed to the right of a 3-line system. The comment applies to all three lines of the system.



Assuming the system is displayed to narrative, each line begins in cell 3. The transcriber-inserted grouping symbol is placed on all three lines of the system, even though the comment takes only two lines. UEB is in effect at the end of the comment, and continues with the following text.

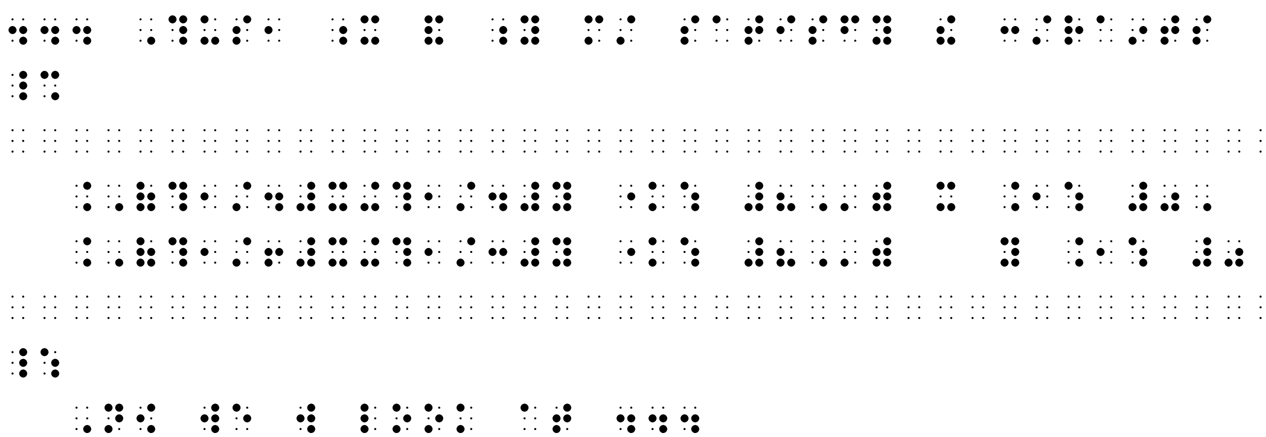
Example 17-15

... Thus, x and y must satisfy the constraints

$$\begin{cases} \frac{1}{4}x + \frac{1}{4}y \leq 8 \\ \frac{1}{6}x + \frac{1}{3}y \leq 8 \end{cases} \qquad x \geq 0,\ y \geq 0$$

Now we will look at ...

Print observation: The arrangement is grouped on the left using a left enlarged brace. Conditions are printed to the right, separated from the arrangement by spacing. The conditions consist of mathematical symbols.



The enlarged left brace begins in cell 3 (displayed to narrative). The transcriber-inserted grouping symbol is placed between the arrangement and the comments. Nemeth Code continues following the arrangement because the comment ends in Nemeth. The Nemeth Code terminator is placed in cell 1 after the blank line. The new paragraph is UEB narrative text.

PRACTICE 17D

Find x and y in terms of a and b.

1. $\begin{cases} x + y = 0 \\ x + ay = 1 \end{cases} \quad (a \neq 1)$

2. $\begin{cases} ax + by = 0 \\ x + y = 1 \end{cases} \quad (a \neq b)$

3. $\begin{cases} ax + by = 0 \\ a^2x + b^2y = 1 \end{cases} \quad (a \neq 0,\ b \neq 0,\ a \neq b)$

4. $\left.\begin{aligned} a &= \frac{x+y}{x-y} \\ b &= \frac{x-y}{x+y} \end{aligned}\right) \quad -1 < x < 1,\ -1 < y < 1$

17.11 More Enlarged Signs of Grouping [NC Rule 19]

In addition to the enlarged braces and parentheses, the Nemeth Code provides symbols for six other enlarged signs of grouping. Notice that each enlarged braille symbol is formed by inserting a dot 6 before the ⠷, ⠾, or ⠳ symbol of the normal-sized grouping symbol.

Vertical Bar		
⠳	Single, Normal	|
⠳⠳	Double, Normal	||
⠠⠳	Single, Enlarged	
⠠⠳⠠⠳	Double, Enlarged	
Barred Brace		
⠨⠸⠷	Left, Normal	⦃
⠨⠸⠾	Right, Normal	⦄
⠨⠸⠠⠷	Left, Enlarged	
⠨⠸⠠⠾	Right, Enlarged	
Bracket		
⠈⠷	Left, Normal	[
⠈⠾	Right, Normal	]
⠈⠠⠷	Left, Enlarged	
⠈⠠⠾	Right, Enlarged	
Angle Bracket		
⠨⠨⠷	Left, Normal	<
⠨⠨⠾	Right, Normal	>
⠨⠨⠠⠷	Left, Enlarged	
⠨⠨⠠⠾	Right, Enlarged	

Barred Bracket		
⠈⠸⠷	Left, Normal	⟦
⠈⠸⠾	Right, Normal	⟧
⠈⠸⠠⠷	Left, Enlarged	
⠈⠸⠠⠾	Right, Enlarged	
Half Bracket		
⠈⠘⠷	Upper Left, Normal	⸢
⠈⠘⠾	Upper Right, Normal	⸣
⠈⠘⠠⠷	Upper Left, Enlarged	
⠈⠘⠠⠾	Upper Right, Enlarged	
⠈⠰⠷	Lower Left, Normal	⸤
⠈⠰⠾	Lower Right, Normal	⸥
⠈⠰⠠⠷	Lower Left, Enlarged	
⠈⠰⠠⠾	Lower Right, Enlarged	

Example 17-16

$$\left\langle \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \right\rangle$$

⠸⠩
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⠀⠀⠨⠨⠠⠷⠆⠀⠀⠨⠨⠠⠾
⠀⠀⠨⠨⠠⠷⠒⠀⠀⠨⠨⠠⠾
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Instructions: Follow "top alignment" rules for these side-by-side arrays.

PRACTICE 17E

$$\left\langle x \right\rangle \left\langle \begin{matrix} x \\ y \\ z \end{matrix} \right\rangle \left\langle \begin{matrix} u \\ v \\ w \\ x \\ y \\ z \end{matrix} \right\rangle$$

DETERMINANTS AND MATRICES

[NC 25.8]

17.12 Definition and Recognition

Determinants and matrices (singular: matrix) are arrangements of items in rows and columns which are enclosed between left and right grouping symbols. Items can be numbers, symbols, or mathematical expressions.

Here is a 2 × 2 ("two by two") determinant enclosed between enlarged vertical bars.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Here is a 2 × 3 ("two by three") matrix enclosed between enlarged brackets.

$$\begin{bmatrix} 1 & 9 & -13 \\ 20 & 5 & -6 \end{bmatrix}$$

A matrix can also be composed of only one column or only one row. Row matrices will be studied at the end of this section.

Determinants and matrices may also be referred to as "arrays."

17.13 Transcription Rules for Determinants and Matrices

Observe the following rules regarding the transcription of the two examples shown above.

17.13.1 **Blank Lines.** Determinants and matrices are spatial arrangements. Thus, a blank line is to be left above and below each array. If the arrangement begins at the top of a braille page, it may begin on line 1 provided no running head is in use; if the arrangement ends at the bottom of a braille page, it may end on line 25. In either case, the rightmost symbol of the arrangement must not fall within three cells of the page number. (See Lesson 9 for details regarding layout of a spatial arrangement at the top or bottom of a page.)

17.13.2 **Grouping Symbols.** For an array consisting of two or more rows, enlarged grouping symbols are used on each braille line. Grouping symbols are vertically aligned.

17.13.3 **Alignment and Spacing of Items.** Each entry is moved as far left as possible in its column. Consequently, each left grouping symbol will be in direct contact with the first entry of each row in the array.

One column of blank cells is left between the columns of the arrangement. That is, one blank cell separates the widest entry in a column from the beginning of the next column. Even if entries contain a space, only one space is left between columns. See Example 17-19.

At least one right grouping symbol must be in direct contact with an entry in the array. The widest entry in the rightmost column determines the placement of the right enlarged grouping symbols.

17.13.4 **Numeric and Letter Indicators.** The numeric indicator is used with numeric entries in an array, even when such entries are in direct contact with a grouping symbol. The English-letter indicator is not used with any English letter in an array.

Example 17-17

$$\begin{bmatrix} 1 & 9 & -13 \\ 20 & 5 & -6 \end{bmatrix}$$

Print observation: Enlarged brackets enclose this two-row array. The numerical entries are centered in their columns.

```
⠸⠩

⠀⠀⠈⠠⠷⠼⠂⠀⠀⠼⠔⠀⠤⠼⠂⠒⠈⠠⠾
⠀⠀⠈⠠⠷⠼⠆⠴⠀⠼⠢⠀⠤⠼⠖⠀⠈⠠⠾

⠸⠱
```

The numeric indicator is used for each number. Items begin in the same cell in each column (entries are left aligned within each column). One blank cell separates the widest entry in a column from the beginning of the next column. The widest entry in the last column determines the placement of the right enlarged grouping symbols.

Example 17-18

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

```
⠸⠩

⠀⠀⠠⠳⠁⠀⠃⠠⠳
⠀⠀⠠⠳⠉⠀⠙⠠⠳

⠸⠱
```

Enlarged vertical bars enclose this two-row array. Each English letter is transcribed without a letter indicator.

Example 17-19

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
  ⠈⠠⠷⠉⠕⠎ ⠨⠹  ⠎⠊⠝ ⠨⠹ ⠼⠴⠈⠠⠾
  ⠈⠠⠷⠤⠎⠊⠝ ⠨⠹ ⠉⠕⠎ ⠨⠹ ⠼⠴⠈⠠⠾
  ⠈⠠⠷⠼⠴      ⠼⠴     ⠼⠂⠈⠠⠾
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠸⠱
```

One space separates each column, even when entries contain a space.
Columns are left aligned, regardless of print layout.

17.13.5 **Placement of Identifiers, Symbols, and Punctuation.** Material outside of an array (such as identifiers, punctuation, operation or comparison signs) are placed on the top line of the arrangement even though the material may be centered in print.

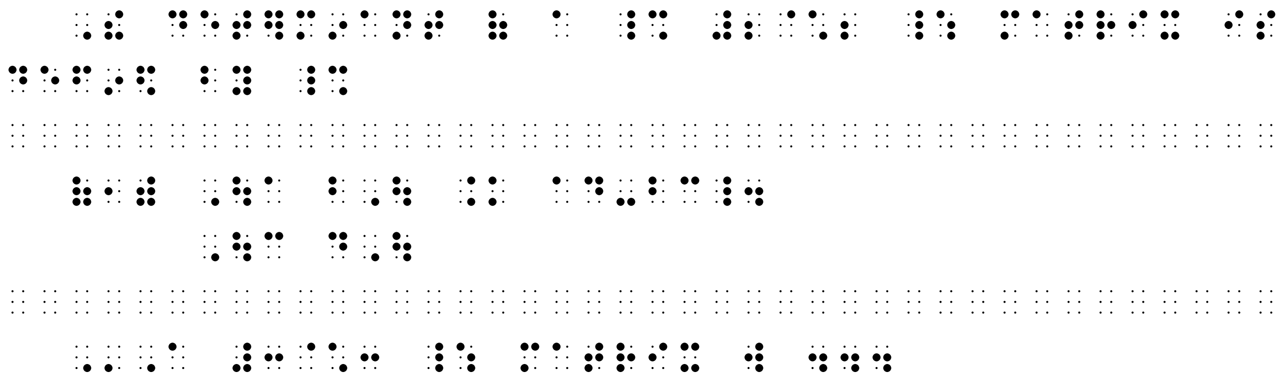
Example 17-20

The determinant of a 2×2 matrix is defined by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc. \qquad (1)$$

A 3×3 matrix will ...

Print observation: An equals sign and expression are centered to the array. A period ends the statement. The expression is identified to the right with a numeral in parentheses.

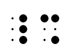


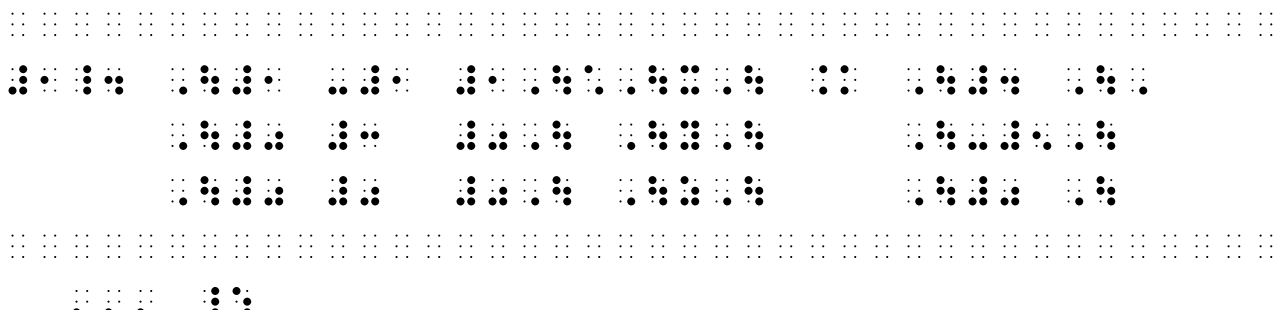
The identifier is moved to the left of the displayed expression according to the rules of the Nemeth Code. (Lesson 7) The identifier, the comparison symbol and the link, and the period are placed on the top line of the arrangement. A punctuation indicator precedes the period.

Example 17-21

$$1.\ \begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 4 \\ -5 \\ 0 \end{vmatrix},\ \ldots$$

Print observation: Three 3-row arrays are printed side-by-side. Aligned with row 2 are symbols before, between, and after the arrays: an identifier, a multiplication dot, an equals sign, a comma, and an ellipsis that implies that more math follows. The numbers are right adjusted within their columns.





The material outside of the array is placed on the top line of the arrangement—the identifier, the operation symbol, the comparison symbol, and the comma.. The multiplication dot is unspaced from the items being multiplied, following spacing rules for operation signs. The ellipsis is placed in the runover position (cell 3) to show where further math would be placed.

PRACTICE 17F

1. $\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$

2. $\begin{pmatrix} 1 & -\frac{3}{4} & \frac{5}{3} \\ 2 & 5 & 12 \end{pmatrix}$

3. $\begin{bmatrix} a & b & c \\ 0 & 0 & 0 \end{bmatrix}$

4. $\begin{vmatrix} ab & cd \\ ac & ce \end{vmatrix}$

5. Explain why points $(a_1,\ b_1), (a_2,\ b_2)$, and $(a_3,\ b_3)$ are collinear if and only if

$$\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0$$

6. The unit vectors of a three dimensional Cartesian coordinate system are

$$\hat{\mathbf{i}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ \hat{\mathbf{j}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ \text{and } \hat{\mathbf{k}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Further Considerations with Determinants and Matrices

17.14 Multiplying Arrays

Recall that a multiplication problem can be printed without a multiplication symbol. When each factor is enclosed in grouping signs, it is understood that the side-by-side factors are to be multiplied. For example, $3 \cdot 2$ and $(3)(2)$ both mean "three times two". Similarly, when arrays are being multiplied, the multiplication symbol is often not printed. This array

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \bullet \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

can also be printed without the multiplication dot, like this:

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

The spacing between the arrays shown (in print) is there only to distinguish between the two factors. The space is not inserted in braille. When vertical bars are used, a multipurpose indicator is transcribed between the two symbols, otherwise it will be misread as a double vertical bar.

Example 17-22

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

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⠠⠳⠼⠂ ⠤⠼⠂ ⠼⠂⠠⠳⠐⠠⠳⠭⠠⠳
⠠⠳⠼⠴ ⠼⠒ ⠼⠴⠠⠳⠐⠠⠳⠽⠠⠳
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The multipurpose indicator is transcribed between each side-by-side enlarged vertical bar, on every line of the arrangement.

17.15 Ellipses, Single Dots, and Blank Entries

The following symbols represent ellipses, a single dot, or a blank entry occur in a matrix or a determinant.

⠤⠤	Blank Entry	
⠘⠄⠄	Ellipsis, Diagonal, lower left to upper right	$\iddots$
⠰⠄⠄	Ellipsis, Diagonal, upper left to lower right	$\ddots$
⠄⠄⠄	Ellipsis, Horizontal	$\cdots$
⠩⠄⠄	Ellipsis, Vertical	$\vdots$
⠡	Single Dot	$\cdot$

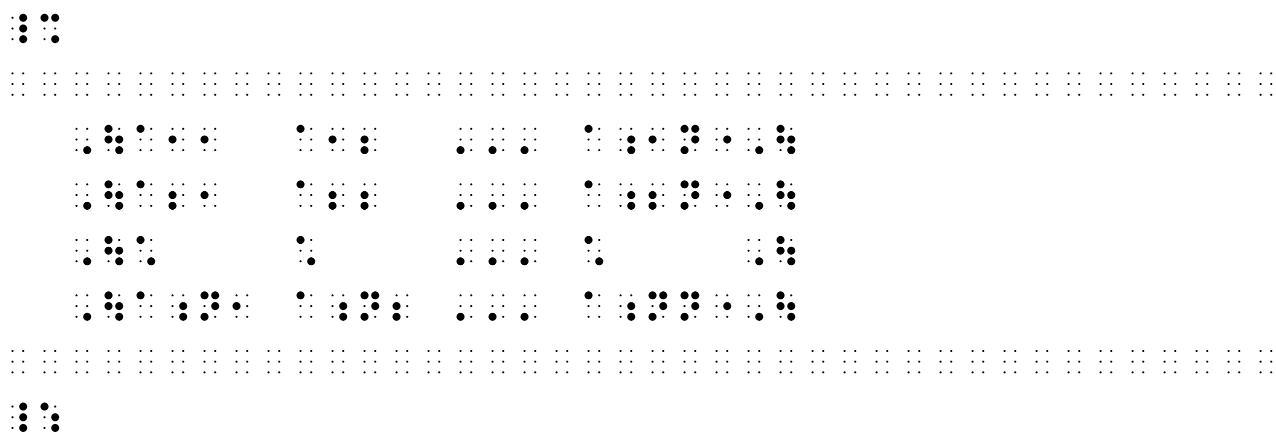
Each symbol is positioned as far left as possible in its column. A transcriber's note must explain the use of the short dash to represent the blank space. Sample transcriber's note:

A short dash represents a blank entry.

Example 17-23

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

Print observation: A horizontal ellipsis is in each entry in column 3. A single dot is printed in row 3, columns 1, 2, and 4.



A baseline indicator is needed to assure the right grouping symbols on rows 1, 2, and 4 are on the same level as the left grouping symbols.

Example 17-24

$$\det(\mathrm{A}) = \begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \vdots & & & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{vmatrix}$$

Print observation: The leftmost and rightmost columns show a vertical ellipsis between the first and the last entry in the column. In columns 2 and 3, the same area is blank. The first, second, and last row show a horizontal ellipsis printed in the third column.

A transcriber's note explains the use of the short dash. In rows 1, 2, and 4, a baseline indicator is needed to assure each right grouping symbol is on the same level as the related left grouping symbol.

Example 17-25

$$\begin{pmatrix} a & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

Print observation: Horizontal, vertical, and diagonal ellipses occur in this matrix.

17.15.1 **A Row of Dots Printed Without Spaces Between Columns.** When dots are strung completely across the omitted row and the dots occupy space between the columns, a row of unspaced dot 3's is transcribed across the full width of the array, beginning in the first cell of the first column and extending to the end of the longest entry in the last column.

Example 17-26

$$\det(\mathrm{A}) = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ . & . & . & . \;\; . \;\; . \;\; . \;\; . \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Print observation: The third row is printed as a series of dots across the width of the array.

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              ⠈⠠⠳⠁⠆⠂  ⠁⠆⠆  ⠄⠄⠄ ⠁⠰⠆⠝⠐⠈⠠⠳
              ⠈⠠⠳⠄⠄⠄⠄⠄⠄⠄⠄⠄⠄⠄⠄⠄⠄⠄⠄⠄⠄⠄⠈⠠⠳
              ⠈⠠⠳⠁⠰⠝⠂ ⠁⠰⠝⠆ ⠄⠄⠄ ⠁⠰⠝⠝⠐⠈⠠⠳
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On rows 1, 2, and 4 a baseline indicator is needed to assure each right grouping symbol is on the same level as the related left grouping symbol.

PRACTICE 17G

In the next equation, matrix Y is expressed as the product of matrices B and X.

$$\begin{vmatrix} y_1 \\ y_2 \\ y_3 \\ \cdot \\ y_r \end{vmatrix} = \begin{vmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ b_{31} & b_{32} & \cdots & b_{3n} \\ \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot & \cdot\ \cdot \\ b_{r1} & b_{r2} & \cdots & b_{rn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ x_n \end{vmatrix}$$

1. $A = \begin{bmatrix} a_{11} & \cdots & a_{1M} \\ \vdots & \ddots & \vdots \\ a_{K1} & \cdots & a_{KM} \end{bmatrix}$

2. $B = \begin{bmatrix} b_{11} & b_{12} & \cdots \\ \vdots & \ddots & \\ b_{K1} & & b_{KK} \end{bmatrix}$

17.16 Augmented Matrix

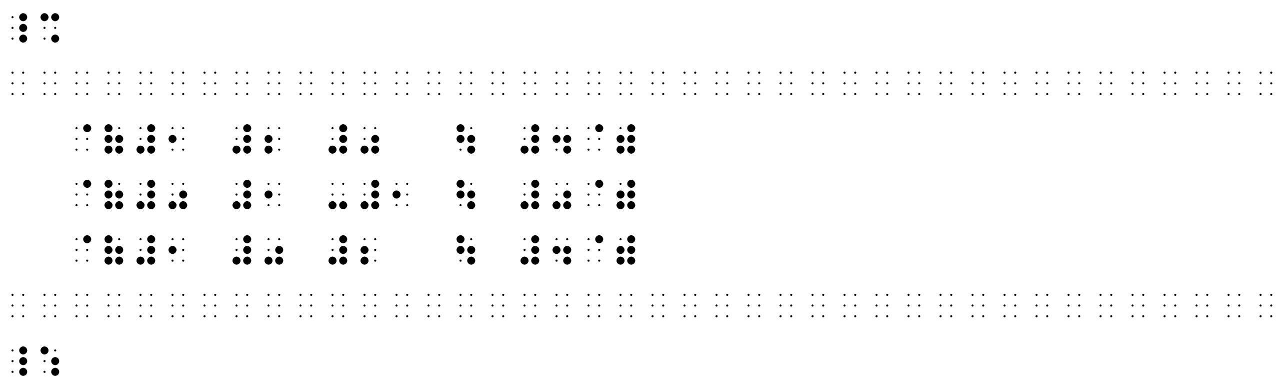
The vertical line in an augmented matrix (the "augmentation line") is represented in braille with the vertical bar symbol, preceded and followed by at least one space, in each row of the matrix. Print may use a solid, dashed, or gray line. In all cases, a vertical bar is transcribed.

	Vertical Bar	|

If space is an issue, the vertical line may be represented as a tactile drawing. See *Guidelines and Standards for Tactile Graphics* for drawing techniques.

Example 17-27

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 2 & 4 \end{array}\right)$$



17.17 Runovers in Arrays

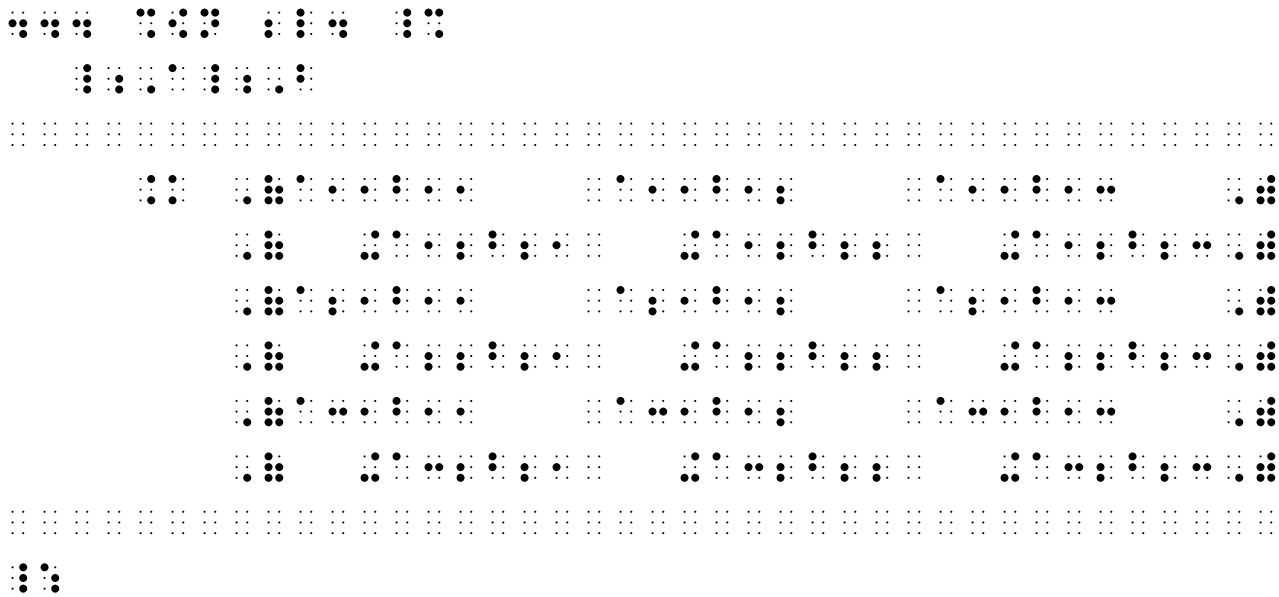
When row entries are too wide to fit on one braille line within the current margins, the arrangement may begin in cell 1 if this will allow each row to be contained on one line. If that strategy fails, apply one of the following techniques. Another option is to draw the enlarged grouping signs as a tactile graphic instead of using the braille symbols. See *Guidelines and Standards for Tactile Graphics* for drawing techniques.

17.17.1 **Runovers With Indentation.** This method is the preferred runover technique. Entries may be run over to a new line, indented two cells from the first cell of the first line of the entry. Attempt to apply the priority list regarding division of long expressions, but those rules can be disregarded if there is no other way around the issue. There is no need to explain the runover format to the reader.

Example 17-28

... shown below.

$$\mathbf{AB} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{pmatrix}$$



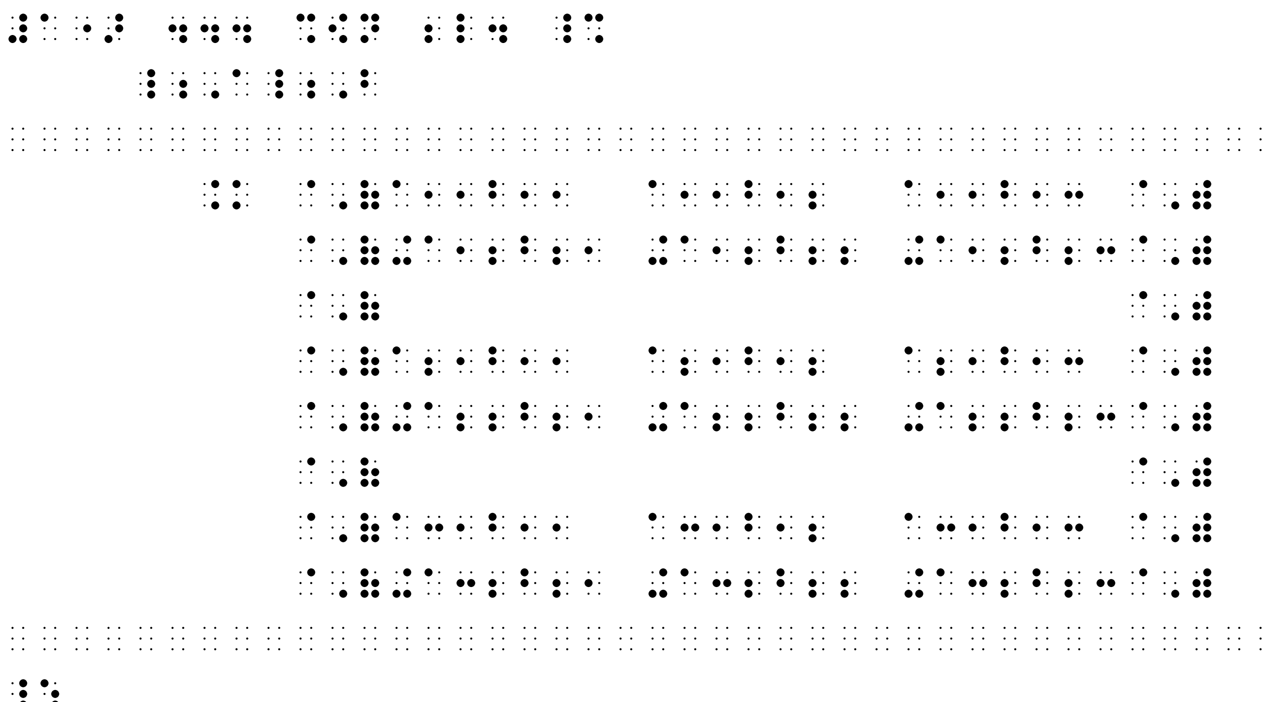
One blank cell separates the widest entry in a column, including any runovers, from the beginning of the next column.

17.17.2 **Runovers Without Indentation.** If the technique above is not feasible, entries may be run over to new lines without indentation. Preference rules for runovers of mathematical expressions need not be observed if space would be saved. In order to distinguish each row, a blank line is inserted between them. Enlarged grouping symbols are transcribed on those blank lines within the arrangement.

Example 17-29

1) ... shown below.

$$\mathbf{AB}=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} & a_{11}b_{13}+a_{12}b_{23} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} & a_{21}b_{13}+a_{22}b_{23} \\ a_{31}b_{11}+a_{32}b_{21} & a_{31}b_{12}+a_{32}b_{22} & a_{31}b_{13}+a_{32}b_{23} \end{pmatrix}$$

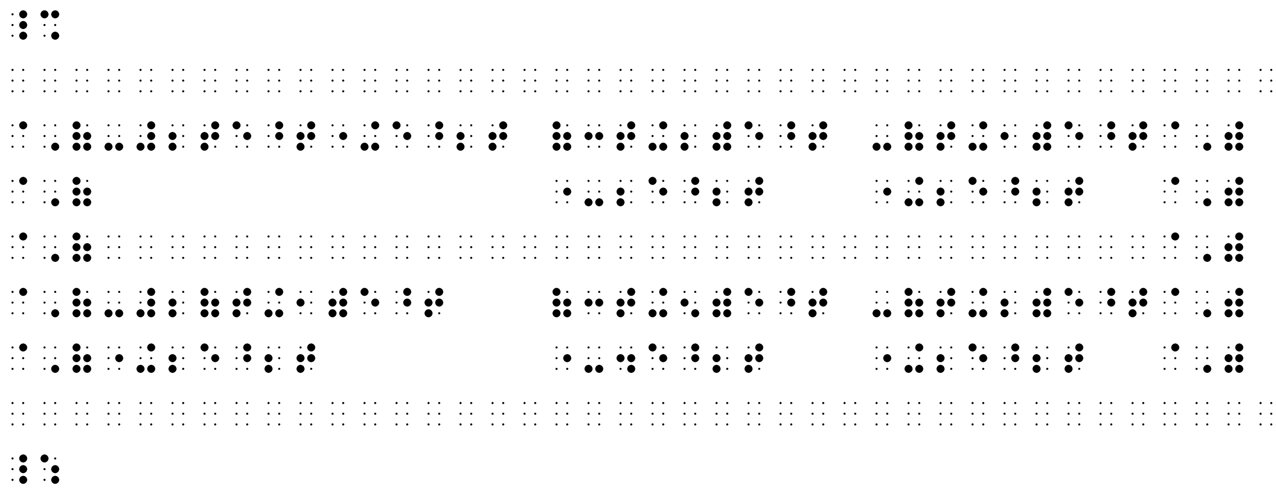


17.17.3 **Runovers and Level Indicators.** As noted in examples throughout this section, a baseline indicator needed when a row entry in the rightmost column ends at a level other than the baseline, and it touches the right grouping symbol (subscripts which don't require a subscript indicator excepted). When a row entry requires a runover, this rule applies only to the last symbol of the entry. A baseline indicator is not used before a right enlarged grouping symbol if the expression which touches the right grouping symbol is continued on the next line.

This rule is illustrated in the next example, which is transcribed first using the "runovers without indentation" technique, and then again using the "runovers with indentation" technique. Focus your attention on the third column.

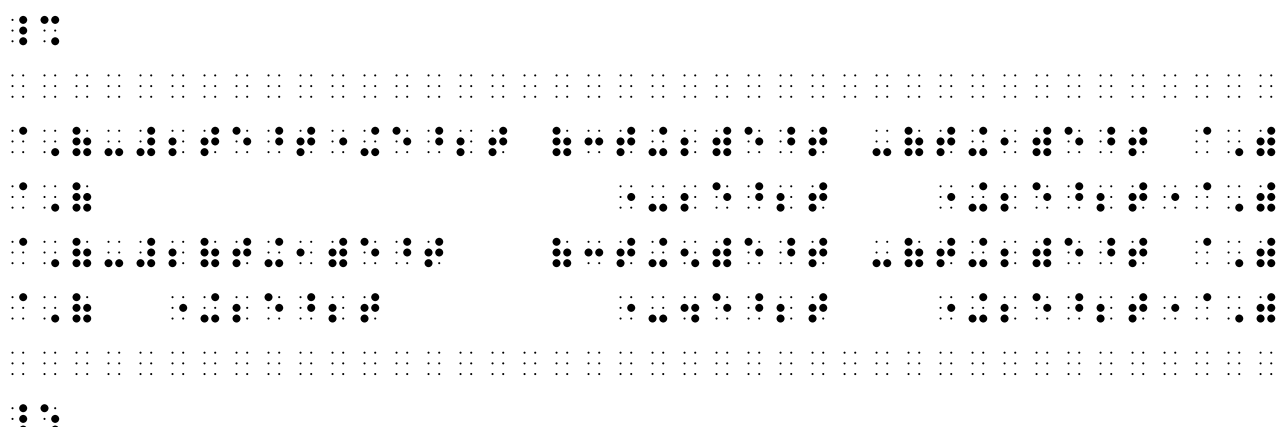
Example 17-30

$$\begin{bmatrix} -2te^{t}+e^{2t} & (3t+2)\,e^{t}-2e^{2t} & -(t+1)\,e^{t}+2e^{2t} \\ -2\,(t+1)\,e^{t}+2\,e^{2t} & (3t+5)\,e^{t}-4e^{2t} & -(t+2)\,e^{t}+2e^{2t} \end{bmatrix}$$



The first line of each row entry ends with the letter t in the superscript position. There is no baseline indicator inserted before the right grouping sign which follows because the expression continues on the next line. The baseline indicator before the operation sign is the first symbol of each runover line.

Note the difference when the same array is transcribed using the preferred runover layout.

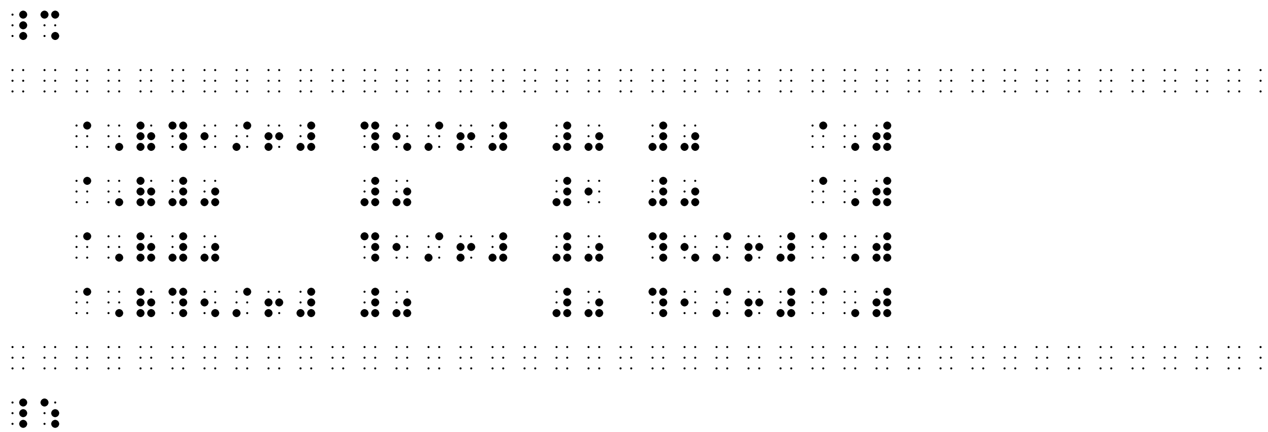


The runover line of each row entry ends with the letter t in the superscript position. A baseline indicator is required before transcribing the right grouping symbol which follows, unspaced.

17.17.4 **Fractions in Arrays.** Fractions in an array may be transcribed linearly, but may be arranged spatially if linear fractions take up too much space.

Example 17-31

$$\begin{bmatrix} \frac{1}{6} & \frac{5}{6} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{6} & 0 & \frac{5}{6} \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} \end{bmatrix}$$

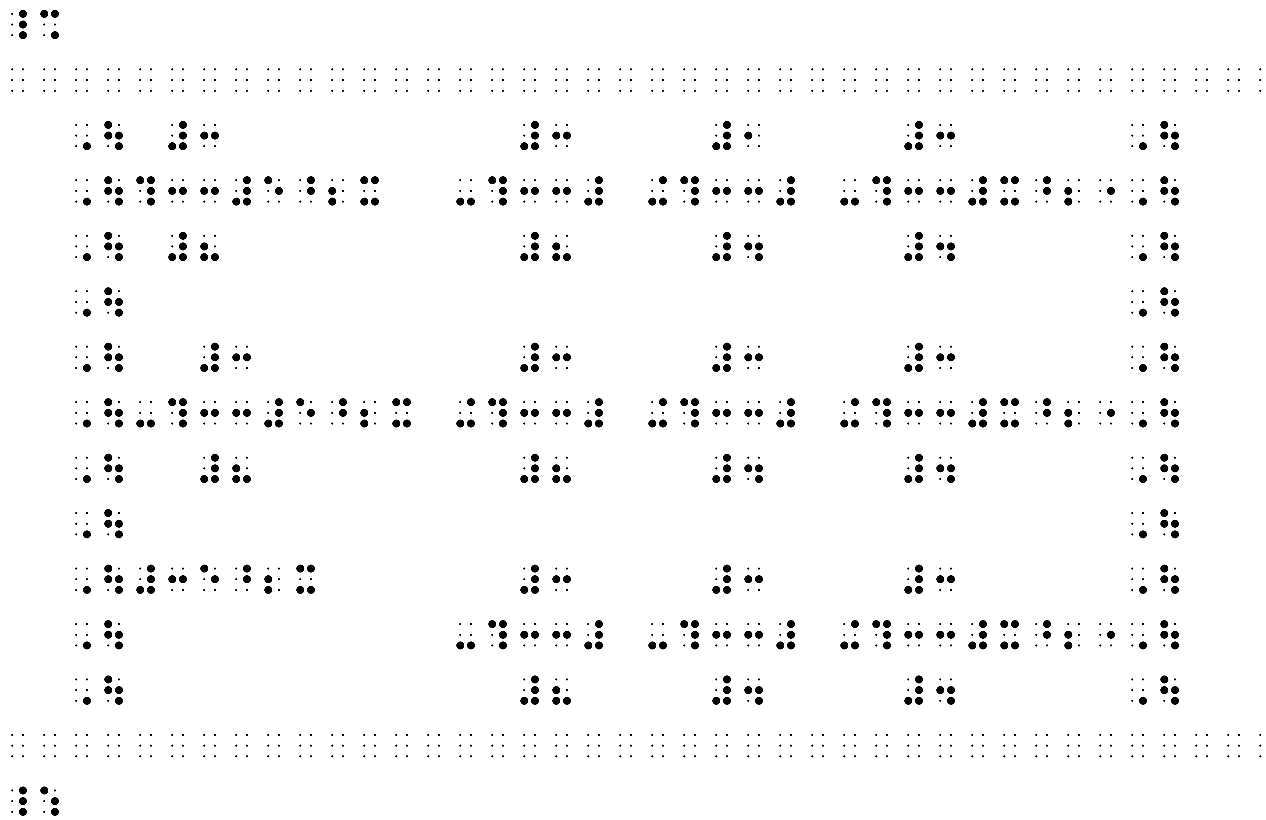


Observe the following details when transcribing the fractions spatially.

- A line is skipped between the rows containing the spatial fraction. Enlarged grouping symbols are transcribed on those blank lines within the arrangement.
- Operation signs and variables are placed on the same line as the spatial fraction line.
- Each entry is moved as far up as possible in its row. This includes entries that are not fractions.
- If the last item in a row is a superscript, or a subscript that uses a subscript indicator, a return to the baseline must occur before the right grouping symbol is transcribed. If the row extends fully to the grouping symbol, a baseline indicator is required to return to the baseline. If the row does not extend to the right grouping symbol, the space returns the reader to the baseline.

Example 17-32

$$\begin{vmatrix} \frac{3}{8}e^{2x} & -\frac{3}{8} & +\frac{1}{4} & -\frac{3}{4}x^2 \\ -\frac{3}{8}e^{2x} & +\frac{3}{8} & +\frac{3}{4} & +\frac{3}{4}x^2 \\ 3e^{2x} & -\frac{3}{8} & -\frac{3}{4} & +\frac{3}{4}x^2 \end{vmatrix}$$



The single-line entry in row 3, column 1, is transcribed on the top line of the row even though it is centered in its row in print.

17.17.5 **Keying.** When the array will not fit on the page even after trying the methods described above, a key may be devised. The technique of keying will be discussed in the next lesson.

Instructions: In the first array, use spatial fractions. In the second array, use runovers with indentation. Review spacing rules and "keep together" rules for abbreviated function names in Lesson 14.

PRACTICE 17H

The derivative of T can be expressed as follows.

$$dT = \begin{bmatrix} \dfrac{\partial y_1}{\partial x_1} & \dfrac{\partial y_1}{\partial x_2} & \cdots & \dfrac{\partial y_1}{\partial x_n} \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot \\ \dfrac{\partial y_m}{\partial x_1} & \dfrac{\partial y_m}{\partial x_2} & \cdots & \dfrac{\partial y_m}{\partial x_n} \end{bmatrix}$$

In <u>this</u> Jacobian determinant,

$$\begin{vmatrix} 0 & 5 & 0 \\ 8x_1 & -2x_3\cos(x_2x_3) & -2x_2\cos(x_2x_3) \\ 0 & x_3 & x_2 \end{vmatrix} = -8x_1 \begin{vmatrix} 5 & 0 \\ x_3 & x_2 \end{vmatrix} = -40x_1x_2$$

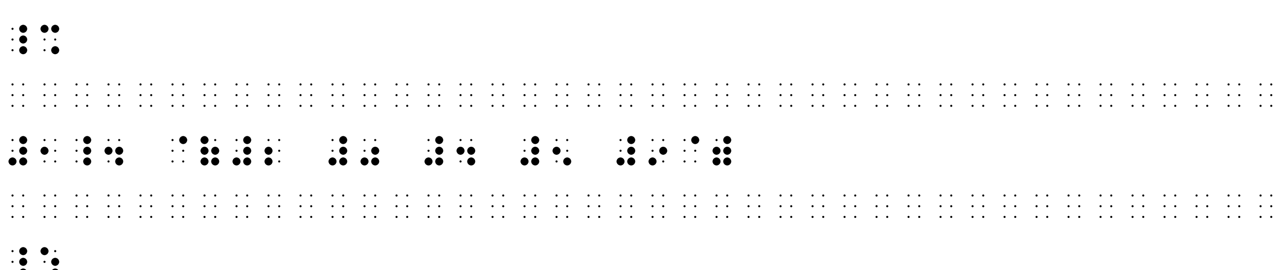
the orientation of the resulting object is reversed.

17.18 Row Matrix

A row matrix has only one row. Because it is a matrix, it is transcribed as a spatial arrangement—that is, a line is left above and below the arrangement. Regular grouping symbols are used, not enlarged symbols, despite the larger appearance of the grouping symbols in print. Here is an example of a 1 × 5 row matrix.

Example 17-33

1. ˆ2 0 4 5 9 ˋ#



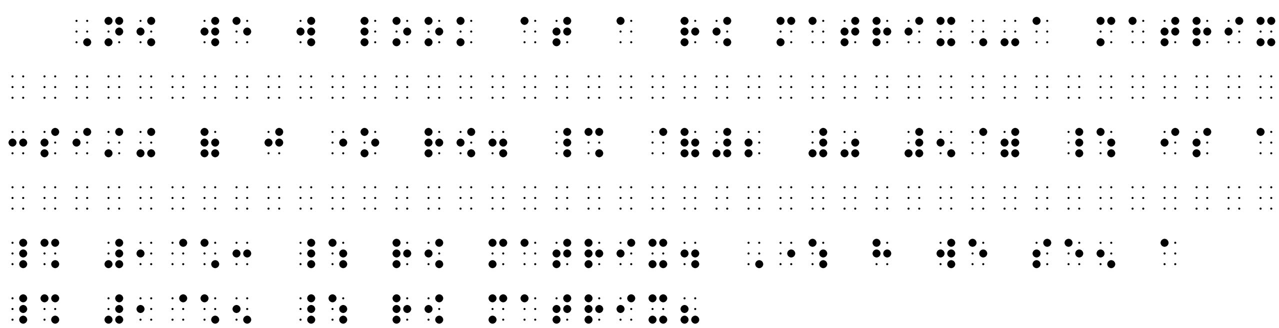
Observation: This is not an enclosed list because there are no commas between items. Context will make it clear that this is a row matrix.

17.19 Embedded Arrays

If the array is embedded within narrative text, the required blank line is inserted on the line before and the line after the line where the arrangement lies. Text is placed on the same line as the array if it fits.

Example 17-34

Now we will look at a row matrix—a matrix consisting of just one row. $\begin{bmatrix} 2 & 0 & 5 \end{bmatrix}$ is a 1×3 row matrix. Where have we seen a 1×5 row matrix?



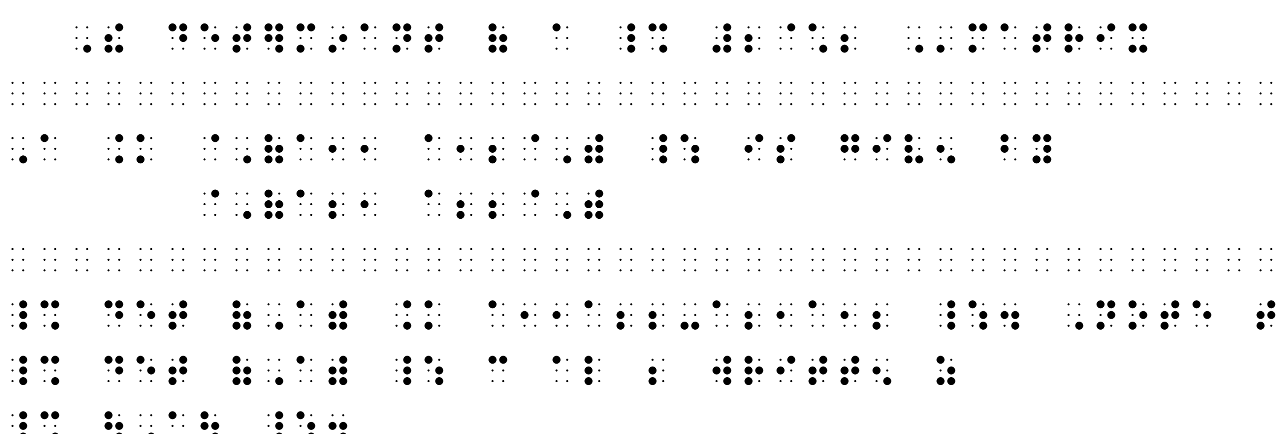
a. **Top Alignment.** If the embedded array consists of more than one line, the surrounding text is placed on the top line of the arrangement only – preceding and/or following the top row of the array, as it fits in context of the narrative. The opening Nemeth Code indicator and the Nemeth Code terminator are placed only on the top line of the embedded arrangement. The switch indicators apply to the whole arrangement.

Example 17-35

The determinant of a 2×2 matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by

$\det(A) = a_{11}a_{22} - a_{21}a_{12}$. Note that $\det(A)$ can also be written as $|A|$.

Print observation: The two rows of the embedded matrix are vertically centered to the equals sign in print.



The first row of the embedded matrix is transcribed on the same line as "A =". The second row is aligned beneath the first row; no text is transcribed on that line. The Nemeth Code terminator is on the main line of text. Note that "det" is an abbreviated function name and therefore is followed by a space in the transcription.

PRACTICE 17I

1) If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$, find AB.

Answer: $AB = \begin{bmatrix} 1 & -25 & 10 \\ 29 & 1 & -18 \end{bmatrix}$.

2) Here are three examples of matrix operations.

a) $\begin{bmatrix} 5 & 4 \end{bmatrix} + \begin{bmatrix} 20 & 30 \end{bmatrix} = \begin{bmatrix} 25 & 34 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 1 & 5 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 7 \\ 1 & 9 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 2 \end{bmatrix}$

17.20 Use of Tactile Graphics for Enlarged Grouping Signs

Enlarged grouping symbols may be drawn in place of the braille equivalents, especially when space saving is a factor. Refer to BANA's *Guidelines and Standards for Tactile Graphics* for drawing techniques. See Section 12.6.3 in Lesson 12 for horizontal grouping signs.

For further practice, see Appendix A—Reading Practice.

EXERCISE 17

Prepare Exercise 17 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 17A

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    ⠼⠦⠭⠤⠲⠵ ⠨⠅ ⠼⠒
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⠼⠆⠸⠲ ⠠⠄⠠⠎⠕⠇⠧⠑⠒

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    ⠼⠆⠭⠬⠲⠽⠤⠔⠵ ⠨⠅ ⠤⠼⠒

⠼⠒⠸⠲ ⠠⠄⠠⠁⠙⠙⠒

     ⠒⠭⠤⠽ ⠨⠅  ⠶
     ⠆⠭⠬⠽ ⠨⠅  ⠦
    ⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
     ⠢⠭⠬⠴ ⠨⠅ ⠂⠢

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Lines 3-5, 9-11, and line 17: The left margin for material displayed to itemized text is cell 5, even if the text has no runovers.

PRACTICE 17B

1 ⠀⠀⠠⠁⠋ ⠎⠕⠇⠧⠬ ⠿ ⠰⠭ ⠯ ⠰⠽⠂ ⠺⠑ ⠙⠑⠞⠻⠍⠔⠑ ⠞ ⠮
2 ⠀⠀
3 ⠎⠽⠌⠑⠍ ⠸⠩ ⠨⠠⠷⠆⠭⠤⠒⠽ ⠨⠅ ⠼⠂⠶⠨⠠⠾ ⠸⠱ ⠓⠁⠎ ⠮
4 ⠀⠀⠀⠀⠀⠀⠀⠀⠀⠨⠠⠷⠒⠭⠬⠆⠽ ⠨⠅ ⠼⠖⠨⠠⠾
5 ⠀⠀
6 ⠎⠕⠇⠥⠰⠝ ⠎⠑⠞ ⠸⠩ ⠨⠷⠷⠲⠠ ⠤⠒⠾⠨⠾ ⠸⠱⠲ ⠠⠐⠝ ⠮
7 ⠎⠕⠇⠥⠰⠝ ⠎⠑⠞ ⠿ ⠹ ⠎⠽⠌⠑⠍⠒ ⠸⠩
8 ⠀⠀
9 ⠀⠀⠨⠠⠷ ⠭⠬⠆⠽ ⠨⠅ ⠼⠖⠨⠠⠾⠸⠲
10 ⠀⠀⠨⠠⠷⠆⠭⠤ ⠽ ⠨⠅ ⠼⠶⠨⠠⠾
11 ⠀⠀
12 ⠸⠱

Line 9: The final period could also be transcribed after the Nemeth Code terminator on line 12.

PRACTICE 17C

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2 ⠀⠀⠀⠀⠼⠆⠷⠤⠹⠲⠌⠒⠼⠾⠤⠂ ⠐⠨⠅⠣⠸⠦⠻ ⠼⠲⠷⠤⠹⠲⠌⠒⠼⠾⠬⠔
3 ⠀⠀⠀⠀⠷⠤⠹⠦⠌⠒⠼⠾⠤⠂ ⠐⠨⠅⠣⠸⠦⠻ ⠷⠤⠹⠂⠖⠌⠒⠼⠾⠬⠔
4 ⠀⠀⠀⠀⠤⠼⠦⠤⠒ ⠐⠨⠅⠣⠸⠦⠻ ⠤⠼⠂⠖⠬⠆⠶
5 ⠀⠀⠀⠀⠤⠼⠂⠂ ⠨⠅ ⠼⠂⠂⠸⠲ ⠠⠄⠠⠠⠝⠕
6 ⠀⠀
7 ⠰⠠⠃⠾ ⠠⠷⠹⠭⠌⠒⠼⠤⠹⠽⠌⠆⠼ ⠨⠅ ⠼⠆⠠⠾ ⠨⠅ ⠷⠿⠠ ⠿⠾
8 ⠀⠀⠀⠀⠀⠠⠷⠢⠭⠬⠒⠽ ⠨⠅ ⠼⠢⠂⠀⠀⠀⠀⠀⠠⠾
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10 ⠸⠱

PRACTICE 17D

PRACTICE 17E

PRACTICE 17F

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⠠⠉⠜⠞⠑⠎⠊⠁⠝ ⠉⠕⠕⠗⠙⠔⠁⠞⠑ ⠎⠽⠌⠑⠍ ⠜⠑ ⠸⠩

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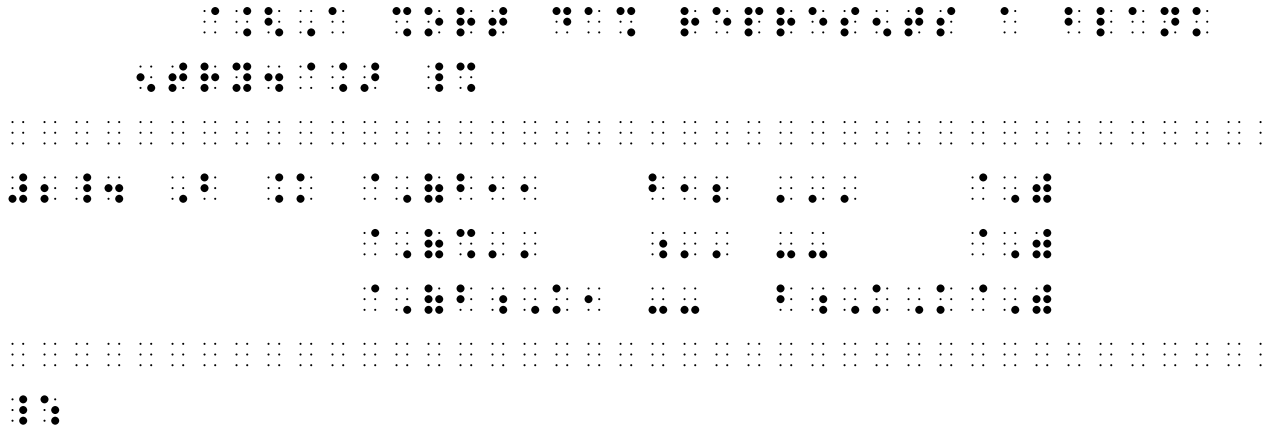
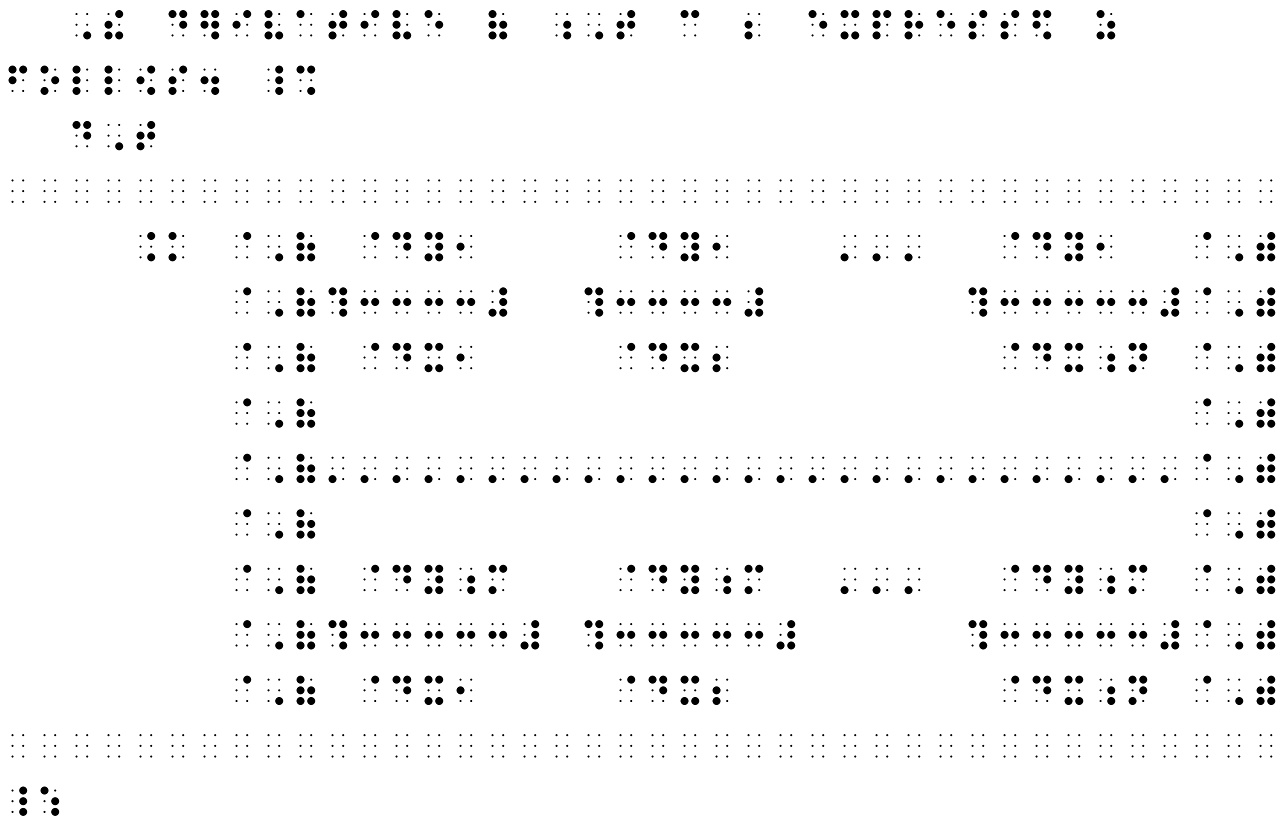
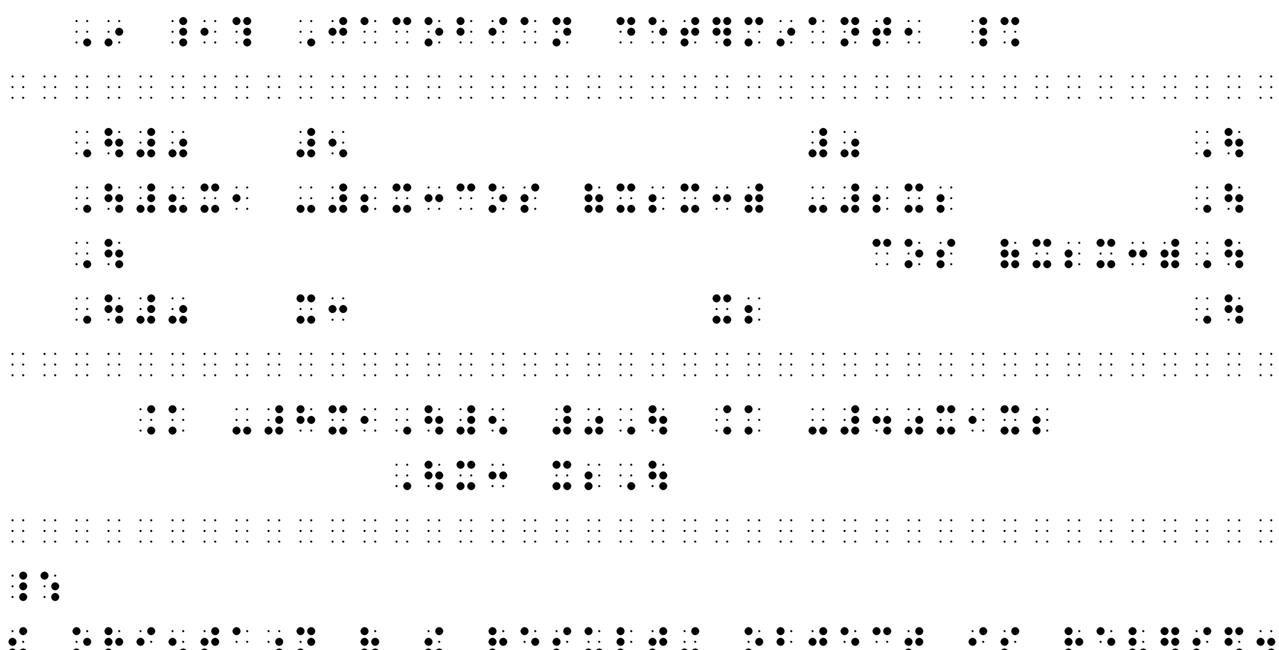
PRACTICE 17G

PRACTICE 17G, cont.



PRACTICE 17H





PRACTICE 17I

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LESSON 18

- TABLES
- FIGURES AND DIAGRAMS
- KEYING TECHNIQUE

Answers to Practice Material

LESSON PREVIEW

Code switching in tables is examined, including considerations regarding box lines and transcriber's notes. Some rules about technical diagrams are introduced. The technique of keying long labels and table entries is explored.

TABLES

An introduction to table format was given in Lesson 5. Stem-and-leaf plots were studied in Lesson 16. Further rules which apply to tables are illustrated in this lesson.

18.1 Structure of Tables

Tables consisting entirely of words are transcribed in UEB. When mathematical data occur in the table, code switching decisions depend upon the content of the entire table and the spacing restrictions encountered on the braille page. If you are unfamiliar with table formats, please read the Tables section in *Braille Formats, Principles of Print-to-Braille Transcription*. There you will find details concerning the layout of the columns and rows, how to handle omissions and blanks within the table, and strategies to consider when a table is too wide to fit within the margins of the braille page.

18.2 Table Label and Title

Follow appropriate rules according to the Nemeth Code or UEB in table labels and titles, switching to Nemeth when necessary.

Example 18-1

(Table label and title only)

Table 2-3. MINIMUM TOLERANCE LEVELS

⠠⠞⠁⠃⠇⠑ ⠼⠃⠤⠼⠉⠲
⠠⠠⠠⠍⠔⠊⠍⠥⠍ ⠞⠕⠇⠻⠨⠑ ⠇⠑⠧⠑⠇⠎⠠⠄

The table label and the table title are not mathematical. UEB is used.

Example 18-2

(Table title only)

DIVISION (÷) TABLE

⠠⠠⠙⠊⠧⠊⠨⠝ ⠸⠩ ⠷⠨⠌⠾ ⠸⠱ ⠠⠠⠞⠁⠃⠇⠑

Only the math symbol requires a switch to Nemeth.

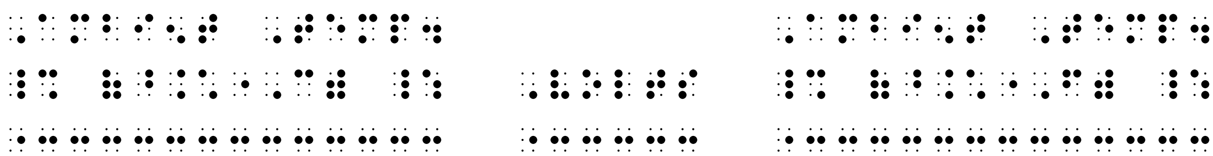
18.3 Column Headings

Follow appropriate rules according to the Nemeth Code or UEB for column headings, switching to Nemeth when necessary.

Example 18-3

(Column headings only)

Ambient Temperature (°C)	**Volts**	**Ambient Temperature (°F)**



Only the degree designations are mathematical. (Column headings may be abbreviated in order to fit in the limited space. A transcriber's note is not required when the abbreviation is easily identifiable. See Braille Formats for further details.)

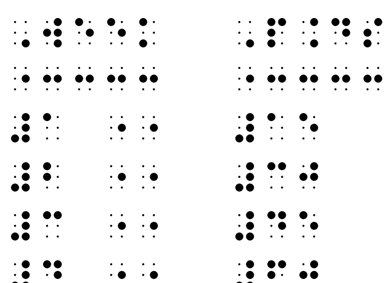
18.4 Table Entries

Table entries may not require a switch to Nemeth, or they may be entirely mathematical, or they may contain a mixture of UEB and Nemeth.

18.4.1 **Tables in UEB.** When entries do not require a switch to Nemeth, the table may be transcribed entirely in UEB.

Example 18-4

Week	Pounds
1	15
2	30
3	45
4	60



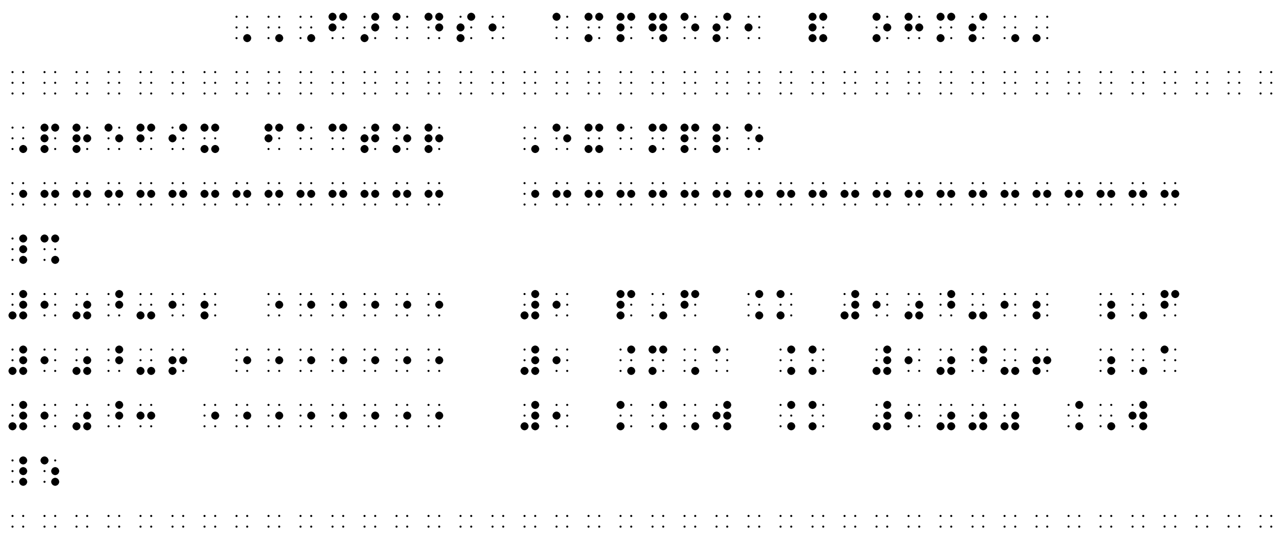
Nothing in this table requires a switch to Nemeth Code.

18.4.2 **Tables in Nemeth Code.** When entries are entirely mathematical, the opening Nemeth Code indicator is placed at the margin (cell 1) of the line following the column separation line. The entries begin on the next line. The Nemeth Code terminator follows the last line of entries, placed at the margin (cell 1).

Example 18-5

FARADS, AMPERES, AND OHMS

Prefix factor	Example
10^{-12}	$1\ \text{pF} = 10^{-12}\ \text{F}$
10^{-6}	$1\ \mu\text{A} = 10^{-6}\ \text{A}$
10^{3}	$1\ \text{k}\Omega = 1000\ \Omega$

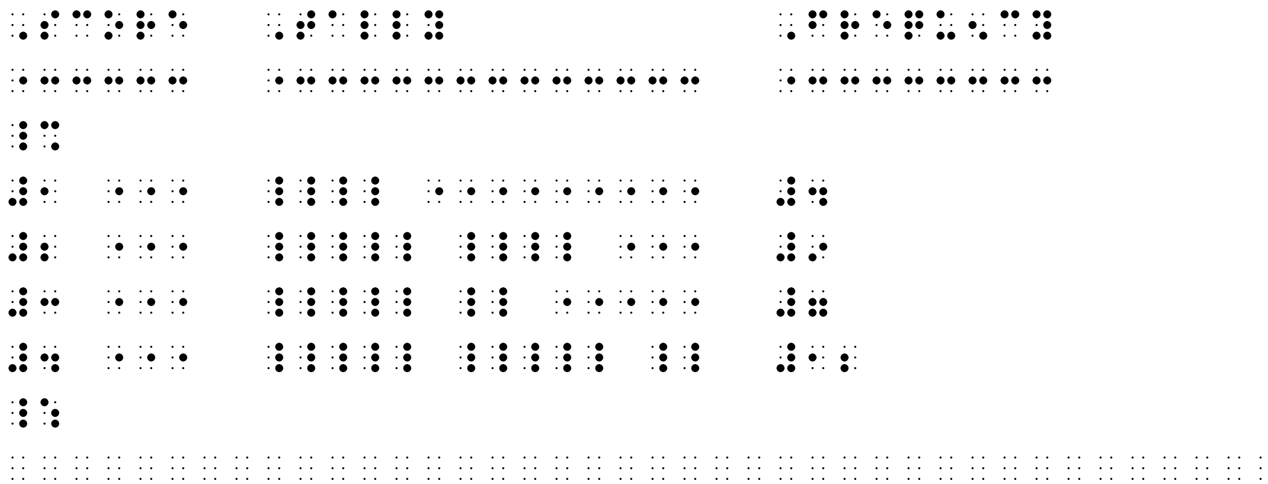


18.4.3 **Code Switching Decisions.** When a mixture of narrative entries and mathematical data occur in a table, the transcriber may switch to Nemeth only where needed, or may transcribe the table entirely in Nemeth. Each table must be individually assessed in order to determine the clearest representation in braille. Keep in mind that a table is read both vertically and horizontally. It is best if a minimum of code switching is encountered within the body of the table.

a. **Column Headings.** When the column headings consist entirely of words, the preferred method is to transcribe them in UEB.

Example 18-6

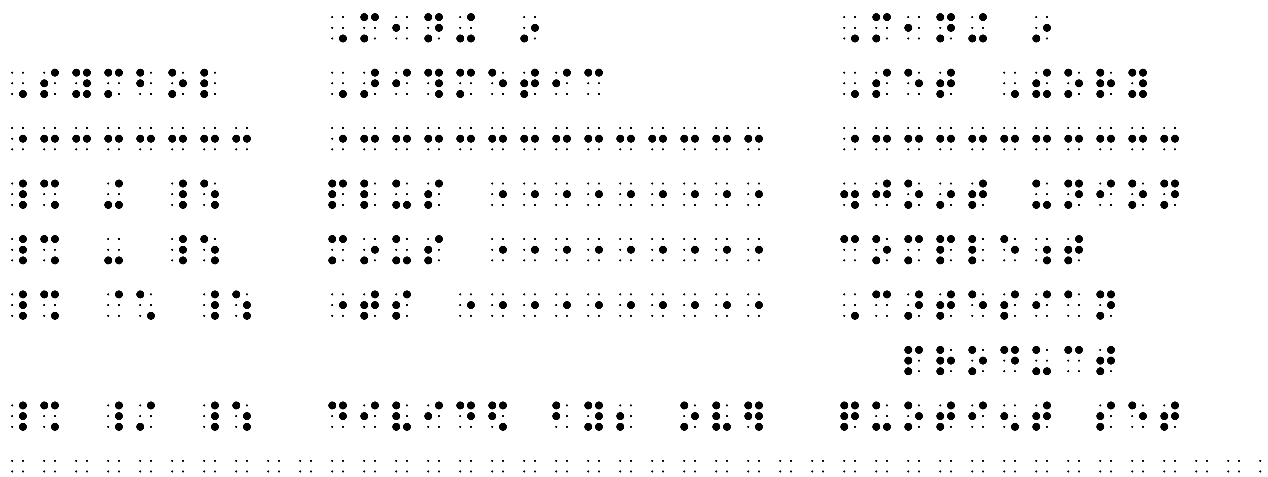
Score	Tally	Frequency
1	$\vert\vert\vert\vert$	4
2	$\cancel{\vert\vert\vert\vert}$ $\vert\vert\vert\vert$	9
3	$\cancel{\vert\vert\vert\vert}$ $\vert\vert$	7
4	$\cancel{\vert\vert\vert\vert}$ $\cancel{\vert\vert\vert\vert}$ $\vert\vert$	12



All of these table entries are transcribed in Nemeth, including the unmodified numbers in columns one and three. Digits in column three are left adjusted in the print table; the same alignment is followed in braille.

Example 18-7

Symbol	**Meaning in Arithmetic**	**Meaning in Set Theory**
$+$	plus	disjoint union
$-$	minus	complement
$\times$	times	Cartesian product
$/$	divided by; over	quotient set



Only the math symbols in column one require Nemeth Code. The words in the rest of the table are more easily read in UEB.

18.4.4 **A Table of Values.** A table of values showing a set of ordered pairs is best presented entirely in Nemeth, including the column headings. (When printed horizontally, a tables of values will not have column headings.)

Example 18-8a

x	y
–1	–2
0	–3
1	–2
2	1
3	6

```
⠸⠩

⠰⠭   ⠰⠽
⠐⠒⠒  ⠐⠒⠒
⠤⠼⠂  ⠤⠼⠆
 ⠼⠴  ⠤⠼⠒
 ⠼⠂  ⠤⠼⠆
 ⠼⠆   ⠼⠂
 ⠼⠒   ⠼⠖

⠸⠱
```

The digits in the table entries are aligned as printed.

Example 18-8b

x	−1	0	1	2	3
y	−2	−3	−2	1	6

```
⠸⠩

⠰⠭ ⠤⠼⠂ ⠼⠴  ⠼⠂  ⠼⠆ ⠼⠒
⠰⠽ ⠤⠼⠆ ⠤⠼⠒ ⠤⠼⠆ ⠼⠂ ⠼⠖

⠸⠱
```

The digits in the print table are centered in their columns. In braille, they are left aligned.

Tables of values are discussed further in the section about boxed tables.

Instructions: If the body of the table can be transcribed entirely in UEB, do so. The first table has a table label and a caption. *Braille Formats* tells us to center sequentially numbered table headings. Center the caption on the next line, disregarding the typeform. Show two ways to transcribe the second table —first, with the column headings in UEB; then, repeat the table heading and transcribe the column headings in Nemeth.

PRACTICE 18A

Table 18.1-5 Values and iterations of e.

e	e^2	S
1	1	6
2	4	24
3	9	54
4	16	96

RTD TABLE

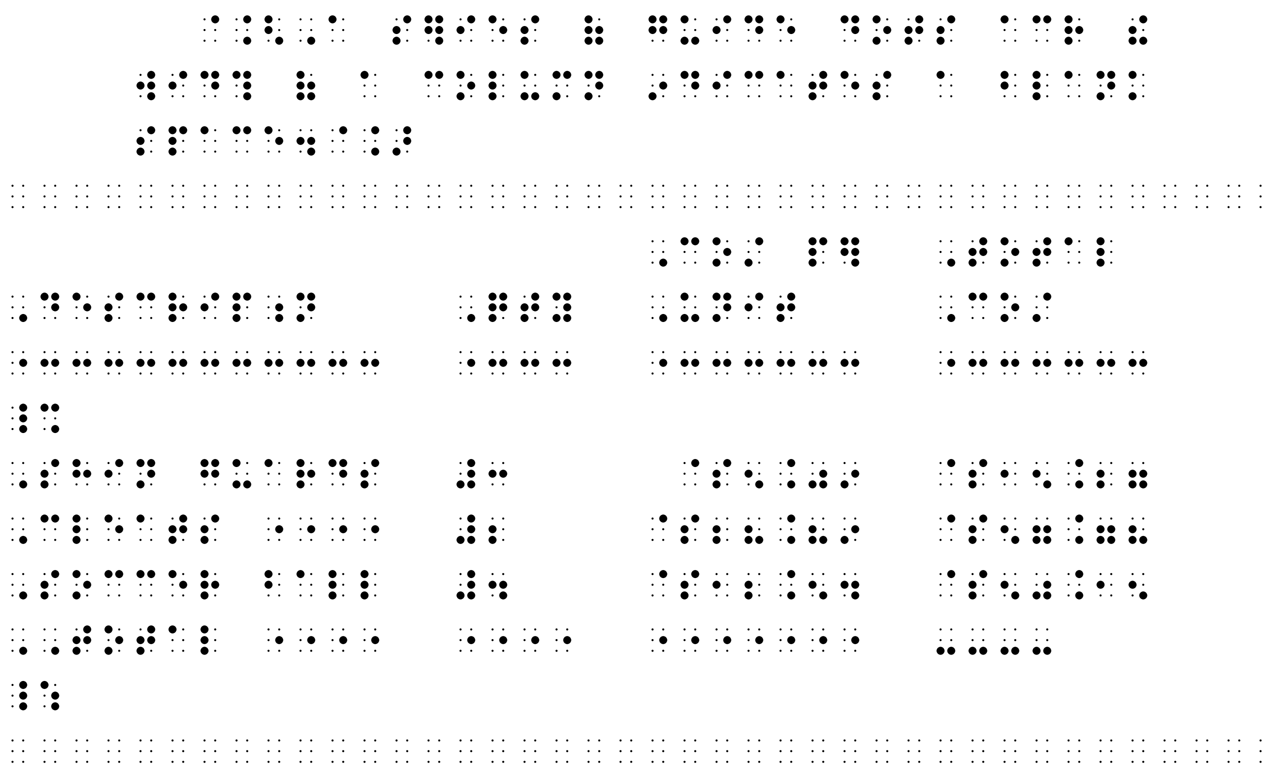
R	T	D
30	$t+2$	$30(t+2)$
45	t	$45t$

18.5 When Row Headings are Words

When table entries consist of technical material but the row headings are words, to minimize the use of code switch indicators the entire table (excluding the table title and column headings) is considered to be technical material. Words within the table are transcribed without contractions. The single-word switch indicator is not used.

Example 18-9

Description	**Qty**	**Cost per Unit**	**Total Cost**
Shin guards	3	\$5.09	\$15.27
Cleats	2	\$28.89	\$57.78
Soccer ball	4	\$12.54	\$50.15
TOTAL			_______



Other rules of note in this table: The transcriber's note regarding blank entries is required (see Braille Formats). The table is preceded and followed by a blank line. Dollar amounts are aligned by place value in print; the same alignment is followed in braille. The long dash of the Nemeth Code represents the omission that is printed as a low line.

PRACTICE 18B

Table A.7

Group	Light	5 days	10 days
Control	12	70.3 ± 2	90 ± 10.5
Test	12	60.4 ± 1.5	78 ± 7.9
Control	16	75.7 ± 8	100 ± 3
Test	16	52.2 ± 2	81 ± 6.7

Boxed Tables

18.6 Code Switching and Box Lines

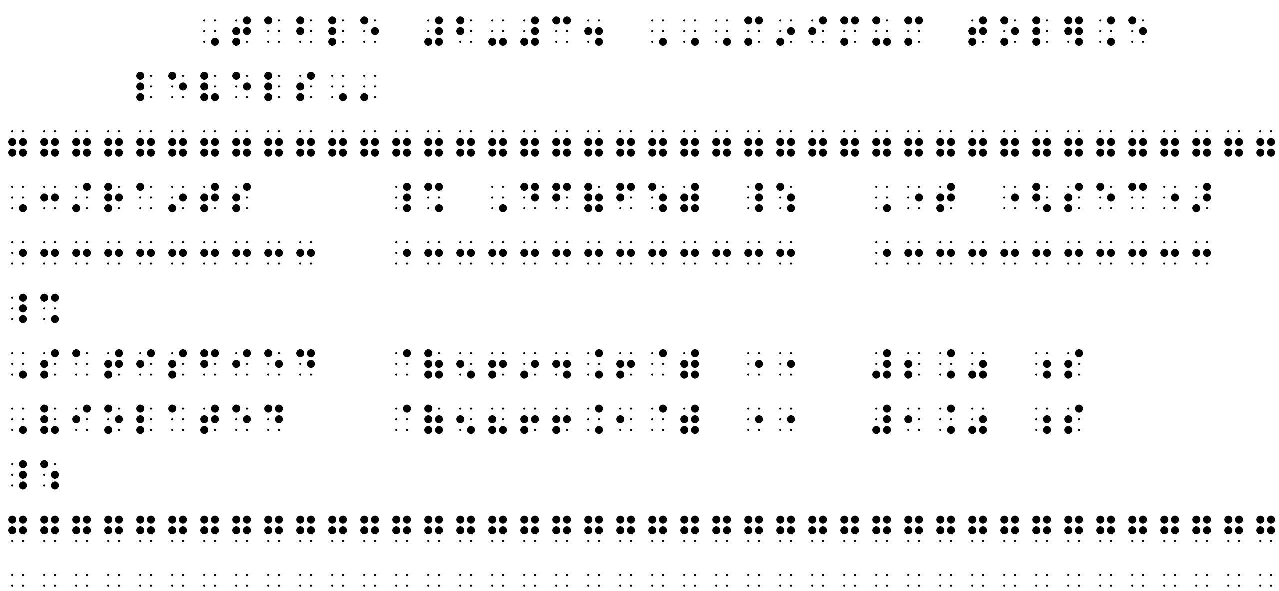
Box lines may be transcribed in either code. Rules regarding box lines are given in *Braille Formats*. Code switching considerations are discussed below.

18.6.1 **Switching Within the Table.** When only the body of the table is in Nemeth, the bottom box line is transcribed on the line following the Nemeth Code terminator.

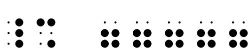
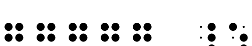
Example 18-10

Table 2-3. MINIMUM TOLERANCE LEVELS

<u>Constraints</u>	<u>$Df(\bar{f})$</u>	<u>Time (sec)</u>
Satisfied	[5694.6]	2.0 s
Violated	[5866.1]	1.0 s



18.6.2 **Switching Within the Box Lines.** For boxed material that is transcribed entirely in Nemeth, the opening Nemeth Code indicator may be included at the beginning of the top box line, followed by a blank space. The Nemeth Code terminator may be included at the end of the bottom box line, preceded by a space.

	`Top box line with opening Nemeth indicator`
	`Bottom box line with Nemeth terminator`

Example 18-11

x	**y**
–2	–3
1	3
3	–3
5	3

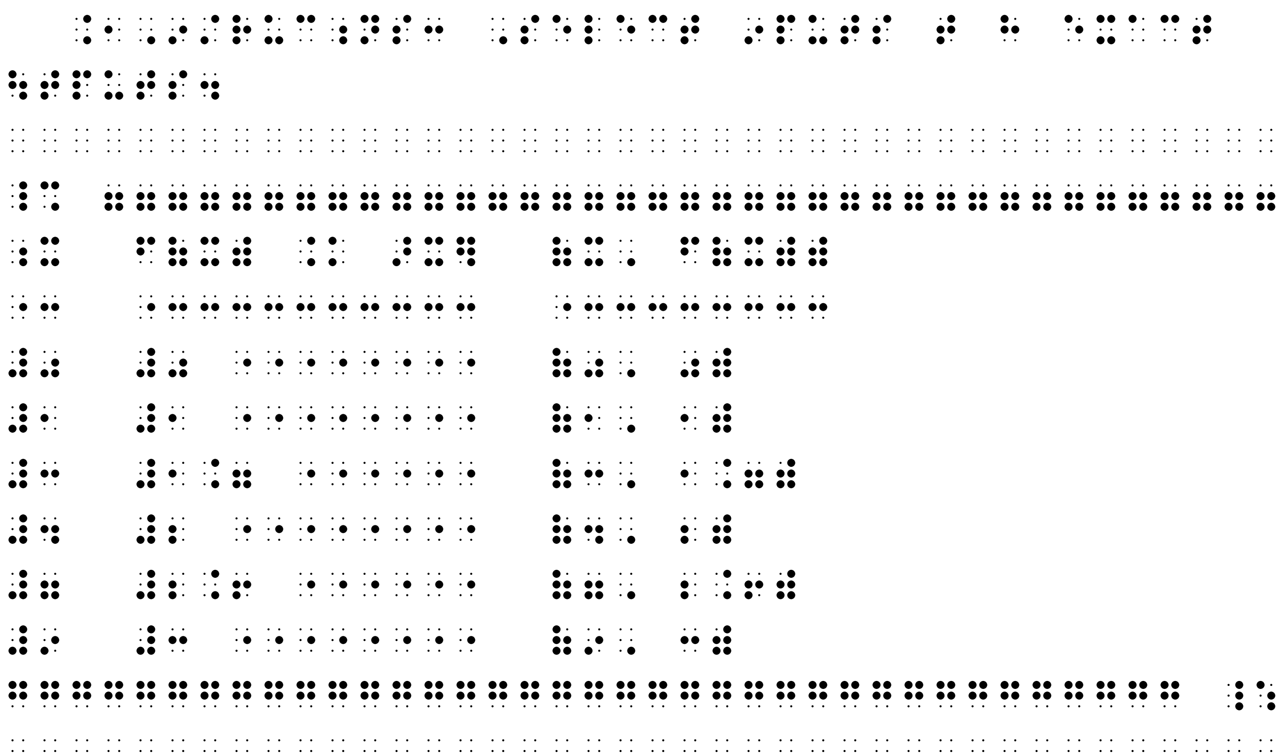
```
⠸⠩ ⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶⠶
⠰⠭   ⠰⠽
⠐⠒⠒  ⠐⠒⠒
⠤⠼⠆  ⠤⠼⠒
 ⠼⠂   ⠼⠒
 ⠼⠒  ⠤⠼⠒
 ⠼⠢   ⠼⠒
⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛ ⠸⠱
```

The negative sign dictates use of Nemeth in this table of values. The values are aligned as printed, by digit. The vertical line between columns is not transcribed.

Example 18-12

Instructions: Select inputs that have exact outputs.

x	$f(x) = \sqrt{x}$	$\big(x, f(x)\big)$
0	0	(0, 0)
1	1	(1, 1)
3	1.7	(3, 1.7)
4	2	(4, 2)
7	2.6	(7, 2.6)
9	3	(9, 3)



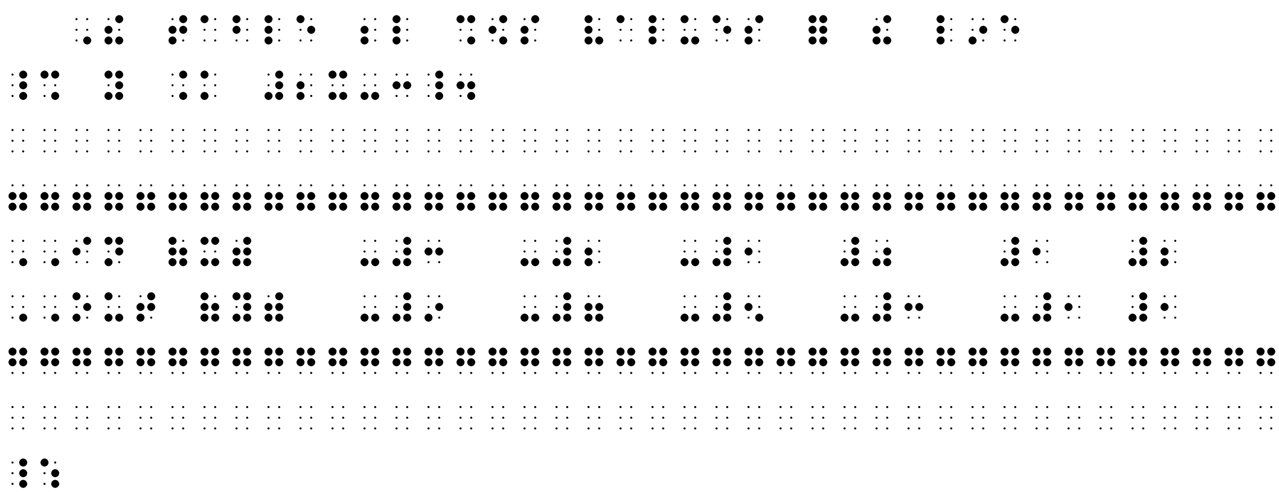
Recall that Nemeth "Instructions" format (5-3) is applied only when followed by itemized material. These instructions are formatted as a narrative paragaph (3-1). A blank line is required before the top box line as well as after the bottom box line unless it falls on line 25 of the braille page.

18.6.3 **Technical Material Before or After a Box.** If technical material immediately precedes or follows the box, the code currently in use continues through the box line.

Example 18-13

The table below shows values for the line $y = 2x - 3$.

IN (x)	–3	–2	–1	0	1	2
OUT (y)	–9	–7	–5	–3	–1	1

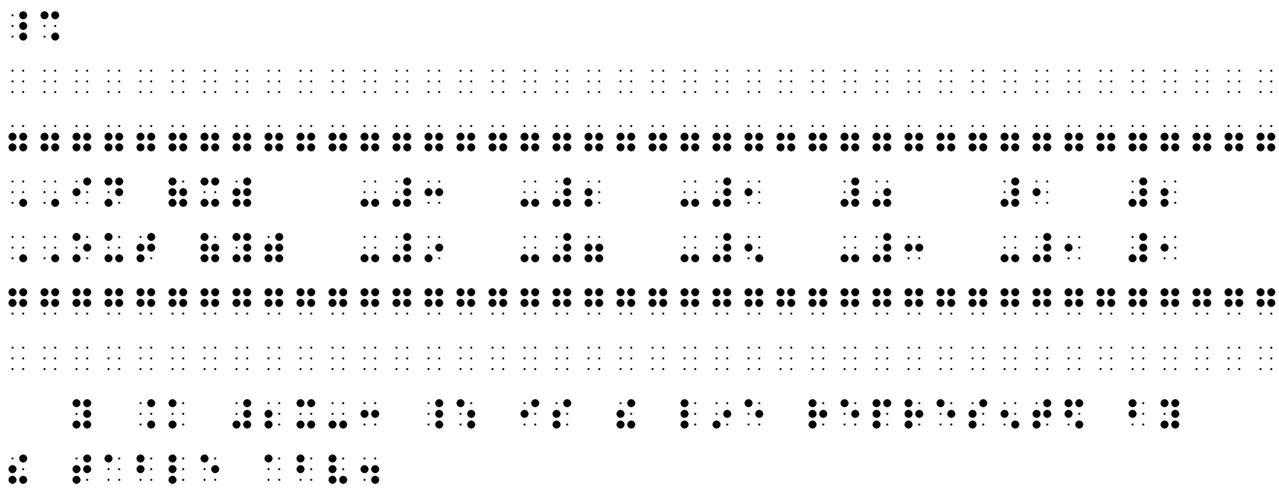


Nemeth begins before the technical material preceding the box and is terminated after the completion of the table. For box lines to match, the Nemeth Code terminator is not included in the bottom box line. The Nemeth Code terminator follows the blank line required following the bottom box line. Note that, in print, the numerals are centered in their columns. In braille, they are left aligned. Also note that internal table lines are disregarded.

Example 18-14

IN (x)	–3	–2	–1	0	1	2
OUT (y)	–9	–7	–5	–3	–1	1

$y = 2x - 3$ is the line represented by the table above.



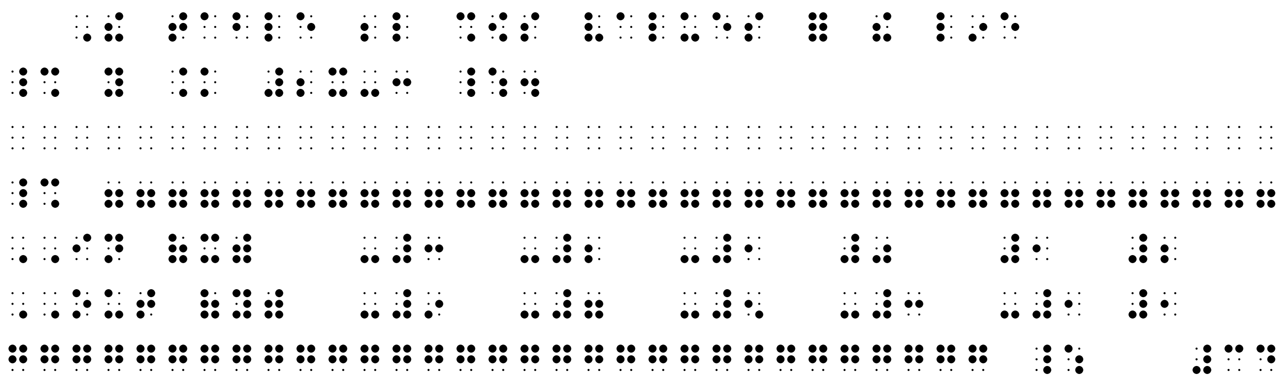
Nemeth continues after the boxed table. UEB resumes after the equation. For box lines to match, Nemeth must be opened before the box line is transcribed. The required blank lines precede and follow the box lines. Note that, in print, the numerals are centered in their columns. In braille, they are left aligned. Also note that internal table lines are disregarded.

a. **Space-Saving Alternative.** If space on the braille page is limited, the code switch indicators can be included in the box lines, independent of the preceding or following technical material. The previous two examples are shown on the next page, illustrating this option.

Example 18-13, revisited

The table below shows values for the line $y = 2x - 3$.

IN (x)	–3	–2	–1	0	1	2
OUT (y)	–9	–7	–5	–3	–1	1

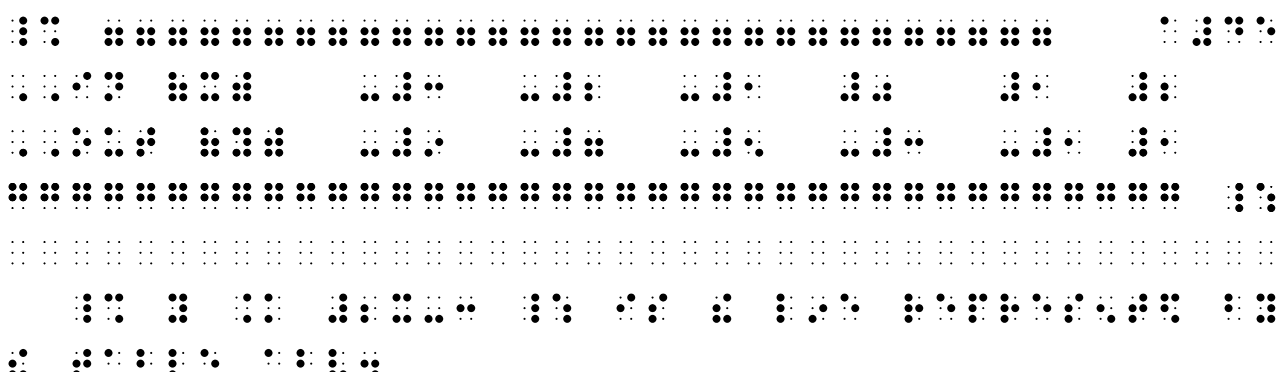


The bottom box line falls on line 25 of the braille page. The box line is shortened in order to accommodate the braille page number.

Example 18-14, revisited

IN (x)	–3	–2	–1	0	1	2
OUT (y)	–9	–7	–5	–3	–1	1

$y = 2x - 3$ is the line represented by the table above.



The top box line can be placed on line 1 when there is no running head. The box line is shortened in order to accommodate the print page number. With a running head, the box would start on line 3.

18.6.4 **Placement of Transcriber's Note.** A transcriber's note that refers to boxed material is generally placed inside the box. However, since the transcriber's note indicators are a UEB symbol, an exception is allowed for boxed material that is entirely in Nemeth. The note may be transcribed above the top box line in order to allow the insertion of switch indicators in the box lines themselves. Two versions of the next example illustrate these options.

Example 18-15

Notice the pattern formed in the table of equivalent fractions in the box below.

$\frac{0}{12}$	$\frac{0}{6}$	$\frac{0}{4}$	$\frac{0}{3}$	$\frac{0}{2}$
$\frac{2}{12}$	$\frac{1}{6}$			
$\frac{4}{12}$	$\frac{2}{6}$		$\frac{1}{3}$	
$\frac{6}{12}$	$\frac{3}{6}$	$\frac{2}{4}$		$\frac{1}{2}$
$\frac{8}{12}$	$\frac{4}{6}$		$\frac{2}{3}$	
$\frac{10}{12}$	$\frac{5}{6}$			
$\frac{12}{12}$	$\frac{6}{6}$	$\frac{4}{4}$	$\frac{3}{3}$	$\frac{2}{2}$

Because the note refers to the information in the box, the blank line required before the top box line precedes the transcriber's note.

Example 18-15, alternate transcription

This transcription requires two additional lines.

Format Instructions: A table must begin in cell 1, even when following an identifier. Use top and bottom box lines, and column separation lines. Disregard typeform in the column headings.

PRACTICE 18C

1) Given exponent x, compute the value of y by completing each table.

a)

$\boldsymbol{x}$	$\boldsymbol{2^x = y}$	$\boldsymbol{y}$
-1	$2^{-1} = y$	?
2	$2^2 = y$	?

b)

$\boldsymbol{x}$	$\boldsymbol{2^{x+1} = y}$	$\boldsymbol{y}$
3	$2^4 = y$	?
5	$2^6 = y$	?

More Table Rules Specific to the Nemeth Code

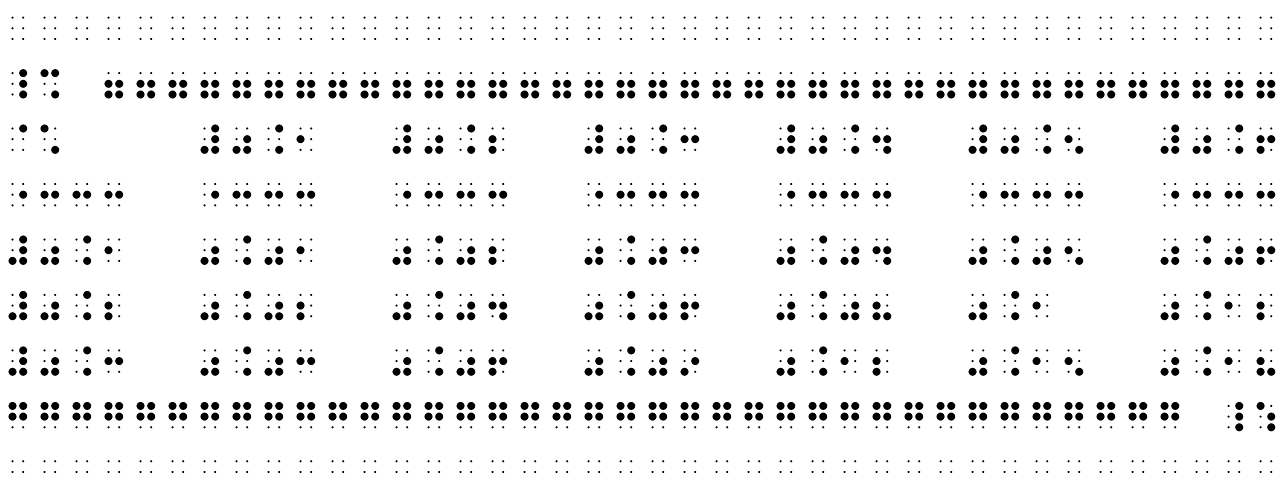
18.7 A Table of Numbers

18.7.1 **Numeric Indicator May Be Omitted.** When row headings and entries in a table consist entirely of numerals, the numeric indicator may be omitted. The numerals can contain commas or decimal points but may not contain any other symbol. This rule applies only to the body of a table and not to the headings.

This technique is used only as a space saving option. The table must be transcribed in Nemeth when the numeric indicator is omitted. (The UEB numeric passage indicator is not used in a Nemeth transcription.) A transcriber's note is not required to explain the omitted numeric indicator.

Example 18-16

×	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
0.1	0.01	0.02	0.03	0.04	0.05	0.06
0.2	0.02	0.04	0.06	0.08	0.1	0.12
0.3	0.03	0.06	0.09	0.12	0.15	0.18



In order to retain print layout, the numeric indicator is omitted in the body of the table. The column headings and the row headings use a numeric indicator.

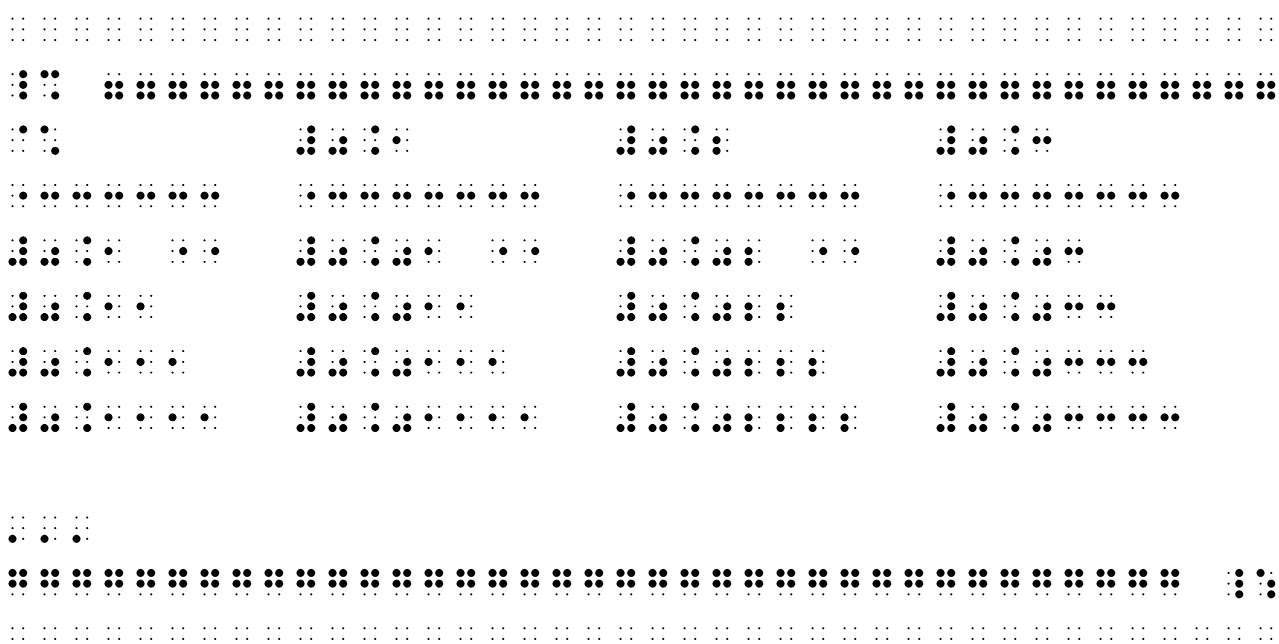
Note: *Braille Formats* guidelines allow for one blank column of cells between columns when the table consists entirely of numbers, as long as the column headings are no wider than the longest entry in the column. If this option allows you to include the numeric indicator, it may be a preferable option.

18.7.2 **Numeric Indicator Required.** This rule does not apply to tables whose entries include any of the following items, all of which are considered to be nonnumeric symbols. In such tables, the numeric indicator must be used throughout the table.

- words
- letters
- mathematical signs such as the dollar sign, percent sign, prime, fraction line, etc.
- a minus symbol
- a general omission symbol
- an ellipsis or a long dash
- guide dots within any column

Example 18-17

×	**0.1**	**0.2**	**0.3**	**0.4**
0.1	0.01	0.02	0.03	0.04
0.11	0.011	0.022	0.033	0.044
0.111	0.0111	0.0222	0.0333	0.0444
0.1111	0.01111	0.02222	0.03333	0.04444



Because guide dots are needed in this table, numeric indicators required throughout. (The ellipsis indicates that this table will need to be divided vertically into two sections according to Braille Formats guidelines.)

Instructions: Do not transcribe tables side by side even though they are printed in this manner. Each table should be preceded and followed by a top and bottom box line, with a blank line between boxes. Treat each table individually regarding code switching. If the body of the table can be transcribed entirely in UEB, do so.

PRACTICE 18D

Create one table which combines data from the three tables shown below.

Age	**Height**
24	5'3"
26	5'9"
30	6'1"
34	5'10"
35	5'4"

Age	**Blood Pressure**
24	108
26	104
30	122
34	119
35	128

Age	**BMI**
24	18.4
26	33.5
30	23.8
34	19.6
35	25.0

FIGURES AND DIAGRAMS

Teaching drawing techniques is beyond the scope of these lessons. Instructions for producing figures, diagrams, and number lines are given in the BANA publication *Guidelines and Standards for Tactile Graphics*. The Nemeth transcriber should obtain a current copy of that publication. UEB methods for drawing lines in line mode are not to be used in a Nemeth transcription.

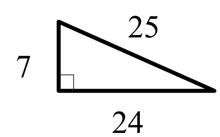
The examples in this section illustrate a few types of diagrams you may encounter in a typical math curriculum.

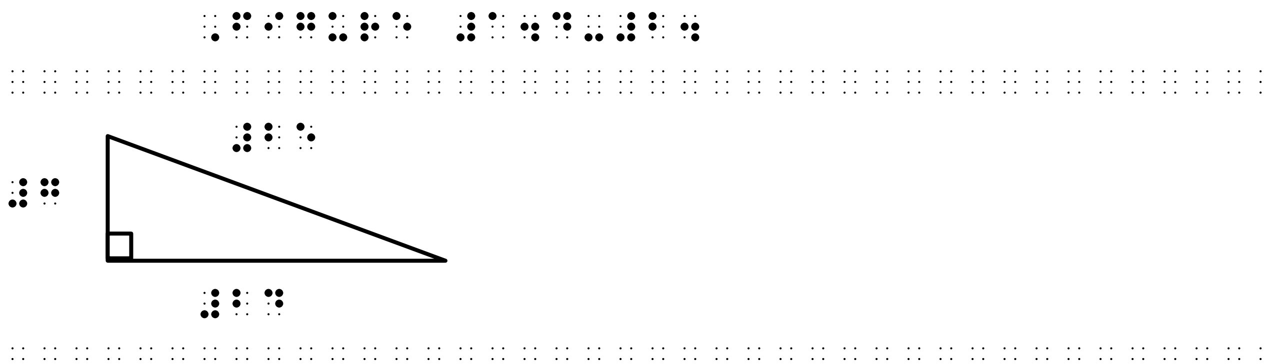
18.8 Which Code?

Numbered titles for figures and diagrams are transcribed in UEB. Diagram labels may not require a switch to Nemeth.

Example 18-18

Figure 1.4-2





These figure labels are transcribed in UEB because they are freestanding, unmodified numbers: 25, 7, and 24. A blank line precedes and follows the diagram.

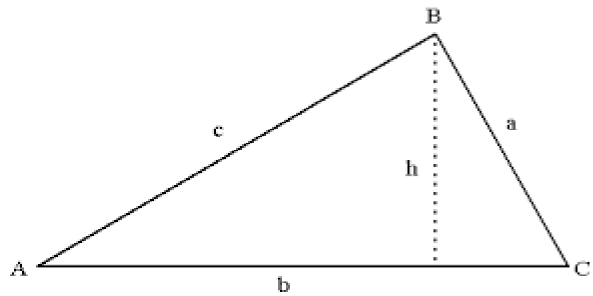
18.8.1 **Letters Used as Diagram Labels.** *Guidelines and Standards for Tactile Graphics* dictates the rules regarding diagram labels, for both UEB and Nemeth.

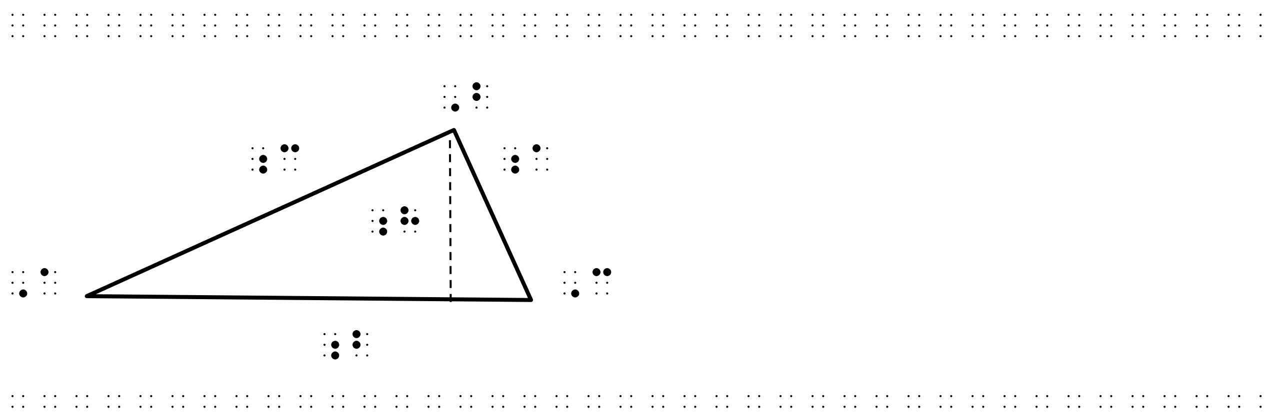
a. **Single Letters.** A single English letter used as a label requires requires a UEB grade 1 indicator or a Nemeth English-letter indicator when the letter is in lowercase. This includes letters a, i, and o in either code. The grade 1/English-letter indicator is omitted if the letter is capitalized.

b. **More Than One Letter.** A The rules differ for more than one letter, depending on the code in use with the diagram. In UEB, a grade 1 indicator is required when an uncapitalized combination of letters corresponds to a shortform (e.g., ab, cd). In Nemeth, the

English-letter indicator is not used for any letter combination in regular type. (See Section 3.16 in Lesson 3.)

Example 18-19





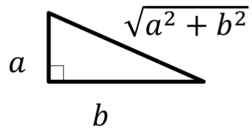
These figure labels are transcribed in UEB because they are freestanding, unmodified letters. The leftmost item in a diagram is placed in cell 1, regardless of the surrounding format.

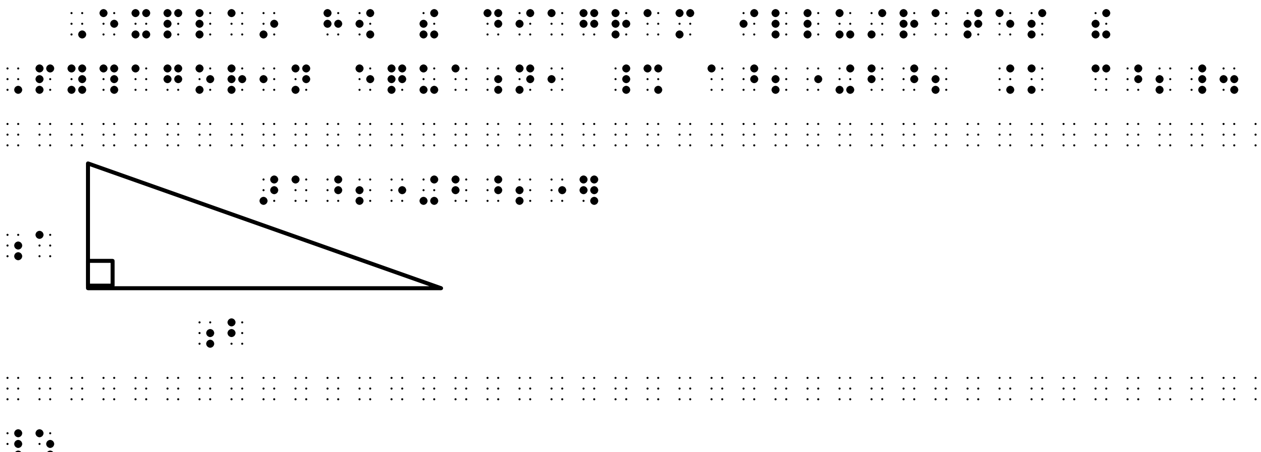
18.9 Switch Indicators and Tactile Graphics

When a tactile graphic contains material that requires Nemeth, and when the preceding text is already in Nemeth, Nemeth Code continues to be in effect for the graphic. If the preceding text is in UEB and if a switch to Nemeth must be made for the tactile graphic, the opening switch indicator is placed at the end of the preceding text or in cell 1 on the line before the required blank line.

Example 18-20

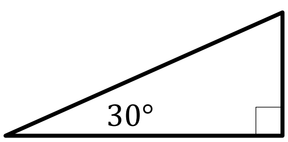
Explain how the diagram illustrates the Pythagorean equation, $a^2 + b^2 = c^2$.

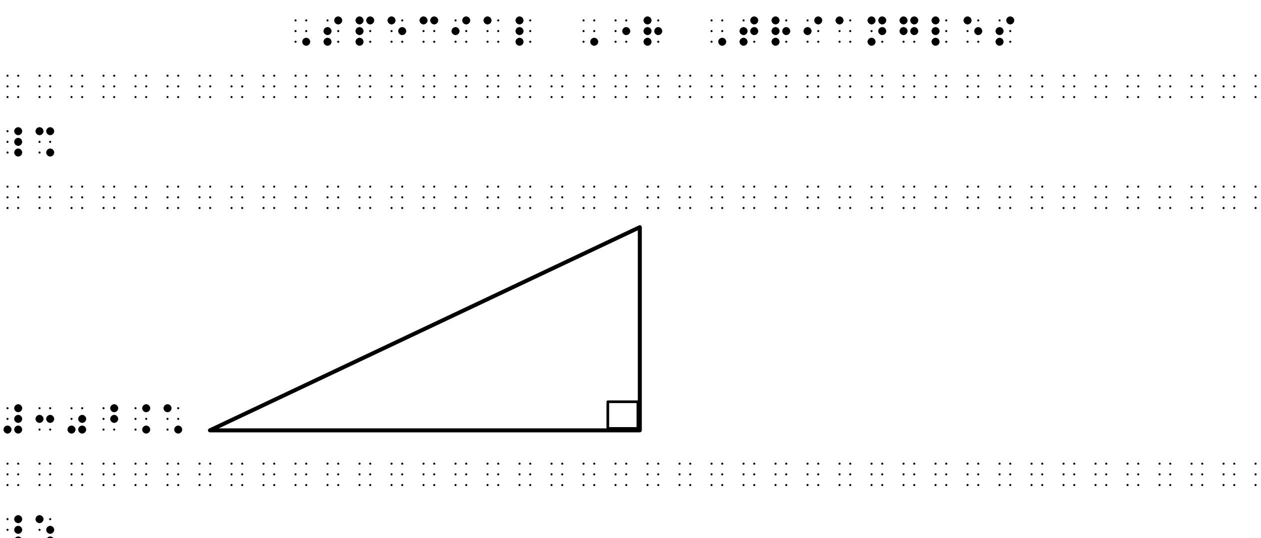




Example 18-21

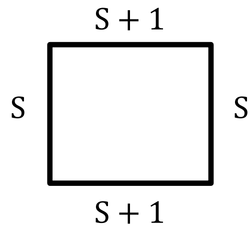
Special Right Triangles

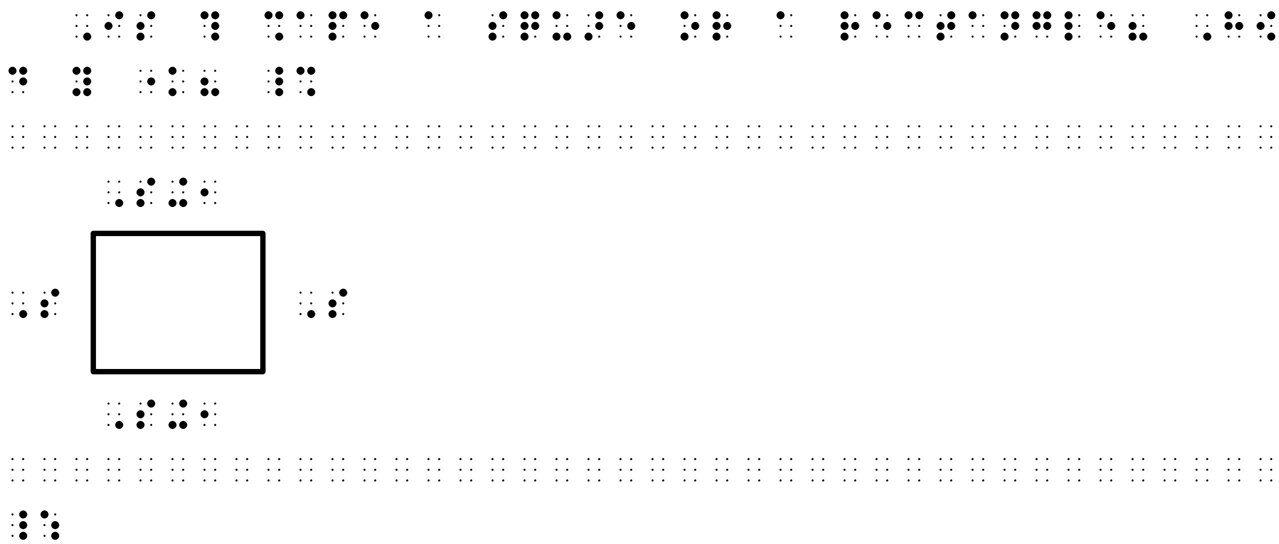




Example 18-22

Is this shape a square or a rectangle? How do you know?





18.10 Number Lines

Clear instructions for producing number lines are given in the BANA publication *Guidelines and Standards for Tactile Graphics*. Symbols used in a graphic number line are required to be listed on the Special Symbols page.

18.11 Diagrams in Exercise Material

If a diagram, number line, or other graphic is placed between instructions and the itemized exercise material which follows, apply the spacing and margin rules for the graphic as outlined in *Guidelines and Standards for Tactile Graphics*. Then continue Nemeth formatting for the exercise material.

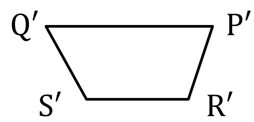
18.12 Molecular Diagrams

Transcribing chemical notation requires further study and is beyond the scope of this lesson manual. Refer to *Chemical Notation Using the Nemeth Braille Code* for rules and guidance.

Instructions: Leave blank space for tooling the lines.

PRACTICE 18E

Raj transformed quadrilateral PQRS to form quadrilateral P′Q′R′S′.



KEYING TECHNIQUE

[NC 26.9]

18.13 Keying

When space does not permit the inclusion of labels, column or row headings, entries, etc., in a figure, in a table, or in an array, one or more of the labels, headings, entries, etc., may be keyed. A keyed item consists of two or three cells made up of letters, numbers, or a combination of letters and numbers. The key items are placed in the same position as the material which they replace. Two items which are identical will have the same key assigned to them.

Keep in mind that keyed items add an extra step for the reader. The technique of keying should not be relied upon as a catch-all technique when other methods may be available. Judicious use of keying can be a good solution after other strategies fail to give a clear presentation.

In addition to the keying guidelines outlined in *Braille Formats* and in *Guidelines and Standards for Tactile Graphics*, the following rules apply in Nemeth.

18.13.1 **Alphabetic Key.** An alphabetic key consists of two or three lowercase English letters. At least one cell of a two- or three-letter key must contain a dot 3 or dot 6. The letter combination should be suggestive of the item it represents, if possible. Quoting *Braille Formats*, "Keys work best when they are related to the terms used in the text to help the reader remember what they are. Typically a letter key will be more memorable for the reader."

An alphabetic key cannot be used if any items remaining in the figure, determinant, matrix, or table are made up of two or three lowercase letters. In that case, a numeric key is used.

18.13.2 **Numeric Key.** A numeric key consists of one or two digits transcribed in the upper part of the braille cell, preceded by the numeric indicator. There must not be punctuation associated with a key number.

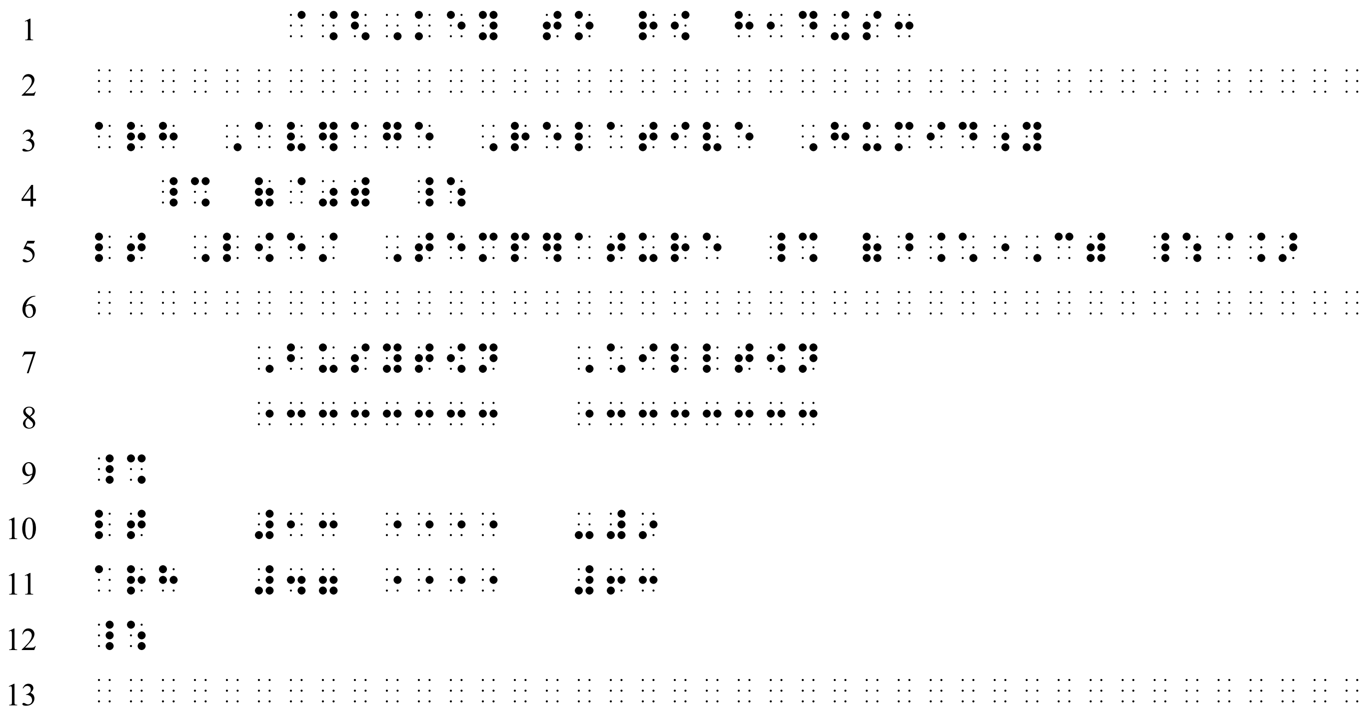
18.13.3 **Combination Key.** The combination of letters and numbers must not exceed three cells. One of the symbols must contain a dot 3 or dot 6.

18.13.4 **The Key List.** A list of numeric and/or alphabetic keys and their meanings is given in a transcriber's note. Letter keys are generally listed in alphabetic order, but may, if appropriate, be listed in order of appearance (see *Braille Formats*). In a circle graph, the keyed items are listed in clockwise order, starting at the top (see *Guidelines and Standards for Tactile Graphics).* Number keys are listed in numeric order.

If the last item in the key listing is in Nemeth, Nemeth Code must be terminated before closing the transcriber's note.

Example 18-23

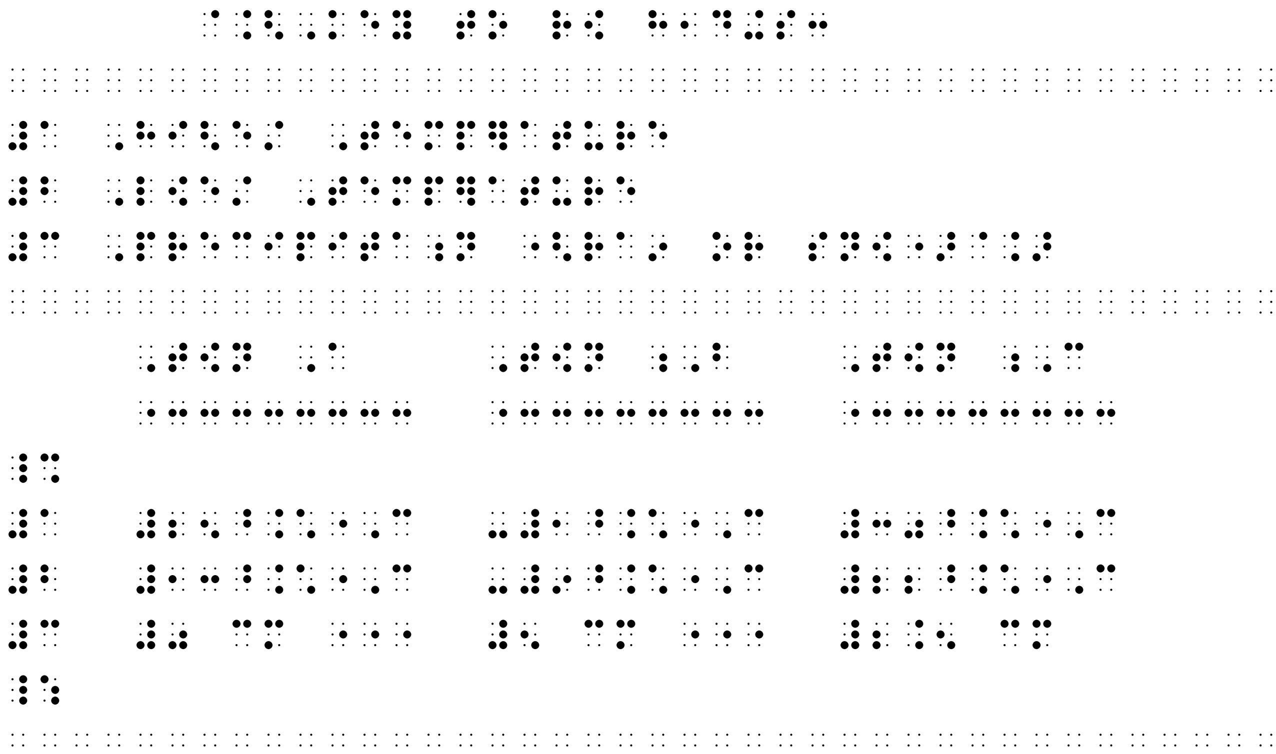
	Busytown	Chilltown
Lowest Temperature (°C)	13	–9
Average Relative Humidity (%)	47	63



Line 1: The transcriber's note "Key to row headings:" begins in cell 7.
Line 2: A blank line precedes the key list.
Lines 3-5: An alphabetic key provides the reader with clues regarding each item's meaning. The key is listed in alphabetical order. Each letter combination is followed by one space and then the words it replaces.
Line 6: A blank line follows the key list.
Lines 7-8: The column headings and column separation lines with two blank cells between them.
Line 9: The opening Nemeth Code indicator is in cell 1.
Lines 10-11: A negative number in the body of the table requires Nemeth Code. All of the entries are in Nemeth.
Line 12: The Nemeth Code terminator is in cell 1.
Line 13: A blank line follows the table.

Example 18-24

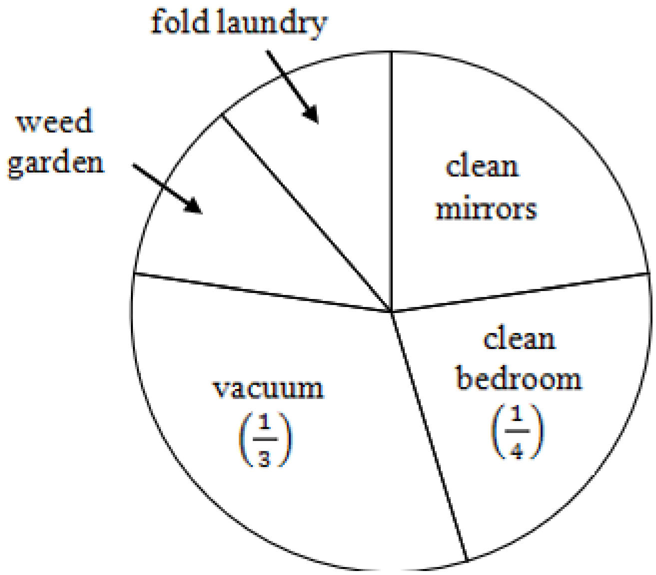
	Town A	Town B	Town C
Highest Temperature	25°C	–1°C	30°C
Lowest Temperature	13°C	–9°C	22°C
Precipitation (rain or snow)	0 cm	5 cm	2.5 cm



The row headings are keyed. Because there are entries in the table consisting of two lowercase letters ("cm"), a numeric key must be used.

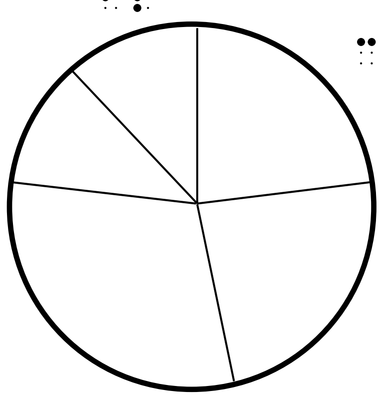
Example 18-25

ADYLYN'S CHORE SPINNER



Adylyn hopes she will spin either "vacuum" or "weed garden" today. What is the probability that she will spin one of these chores?

- What is P(vacuum)?
- What is P(weed garden)?
- What is P(vacuum) OR P(weed garden)?



[next page]

(Commentary is on the next page)

Page 1

Lines 1-2: Centered heading and blank line following.

Lines 3-4: The transcriber's note "Key to labels:" begins in cell 7. A blank line precedes the key list.

Lines 3-5: An alphabetic key provides the reader with clues regarding each item's meaning. The key is listed in clockwise order as stipulated in Guidelines and Standards for Tactile Graphics.

Line 7: "clean bedroom" cannot use the key letters "cb" because there is no dot 3 or dot 6 in that letter combination. "cbr" is chosen to represent "clean bedroom".

Lines 10-19: The graphic is drawn and labeled as outlined in Guidelines and Standards for Tactile Graphics.

Page 2

Lines 5-8: Bulleted items follow guidelines given in Braille Formats. The probability notation is mathematical. Nemeth switch indicators are used and words are not contracted.

PRACTICE 18F

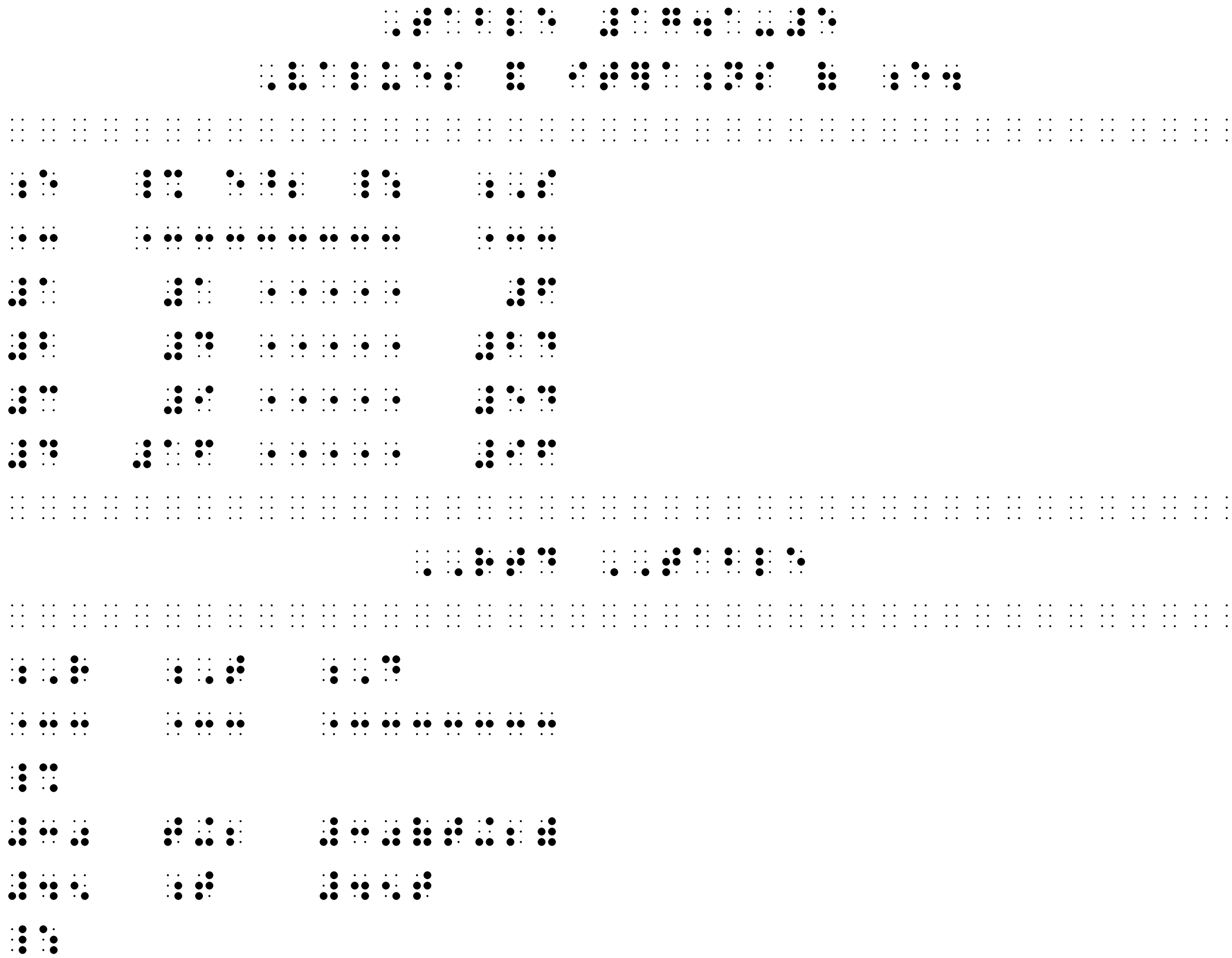
Substance	**Melting Point** (°C)	**Boiling Point** (°C)	**Heat of Fusion** (kJ/kg)	**Heat of Vaporization** (kJ/kg)
Aluminum	660	2467	396	10500
Ammonia	−78	−33	332	1370
Lead	328	1740	25	866

EXERCISE 18

Prepare Exercise 18 for your grader.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 18A



In the first table, only the heading for column two requires Nemeth. Although the first table could just as well be transcribed entirely in Nemeth, the instructions to the practice said to transcribe the body in UEB if possible.

(See the alternate transcription of the RTD table on the next page.)

PRACTICE 18A, RTD TABLE, alternate transcription

⠠⠠⠗⠞⠙ ⠠⠠⠞⠁⠃⠇⠑

⠸⠩

⠰⠠⠗   ⠰⠠⠞   ⠰⠠⠙
⠐⠒⠒   ⠐⠒⠒   ⠐⠒⠒⠒⠒⠒⠒⠒⠒
⠼⠒⠴   ⠞⠬⠆   ⠼⠒⠴⠷⠞⠬⠆⠾
⠼⠲⠢   ⠰⠞   ⠼⠲⠢⠞

⠸⠱

PRACTICE 18B

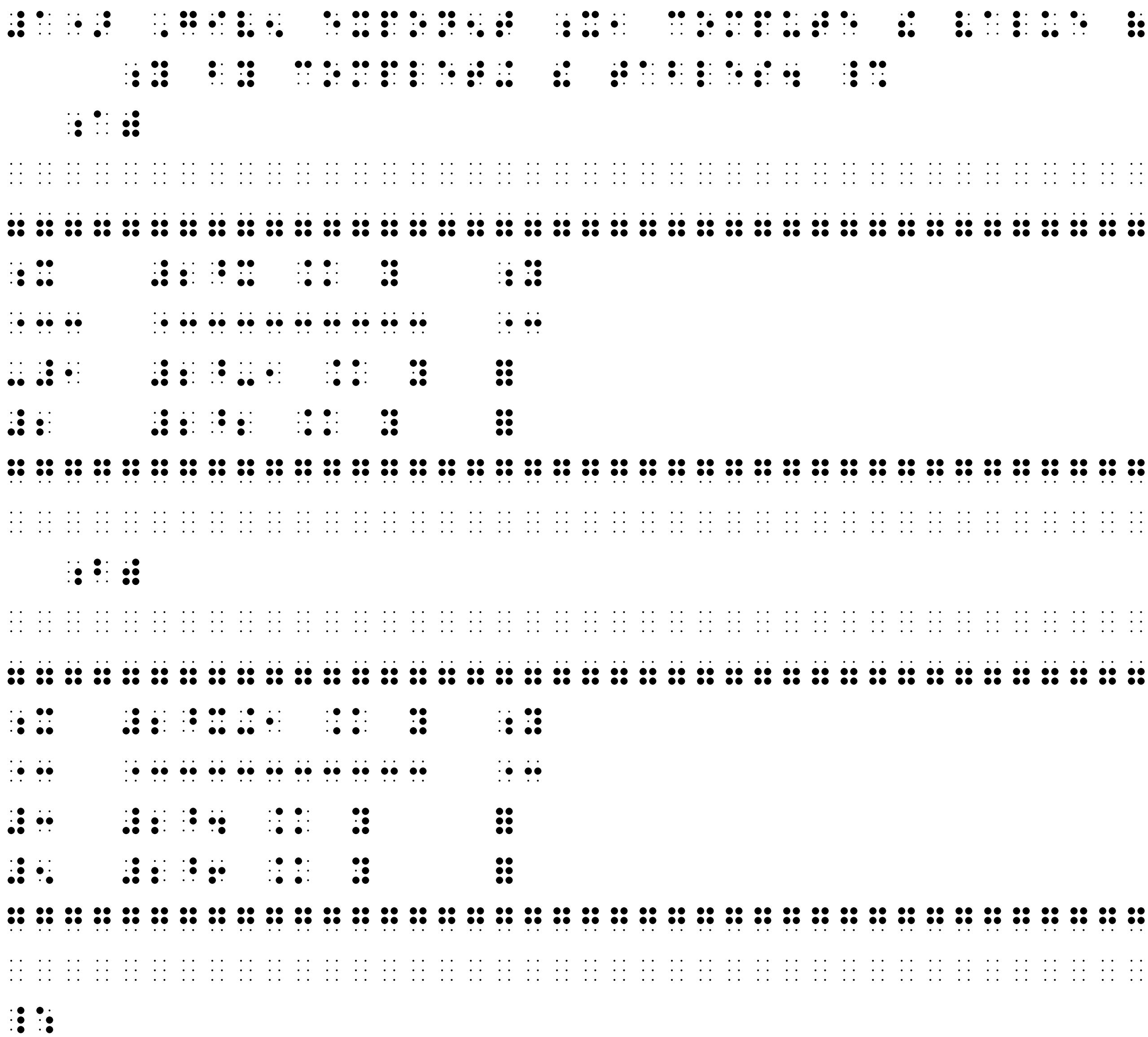
⠠⠞⠁⠃⠇⠑ ⠠⠁⠲⠼⠛

⠠⠛⠗⠳⠏   ⠠⠇⠊⠣⠞   ⠼⠑ ⠐⠙⠎   ⠼⠁⠚ ⠐⠙⠎
⠐⠒⠒⠒⠒⠒⠒⠒   ⠐⠒⠒⠒⠒   ⠐⠒⠒⠒⠒⠒⠒⠒⠒⠒   ⠐⠒⠒⠒⠒⠒⠒⠒⠒
⠸⠩
⠠⠉⠕⠝⠞⠗⠕⠇   ⠼⠂⠆   ⠼⠶⠴⠨⠒⠬⠤⠆   ⠼⠔⠴⠬⠤⠂⠴⠨⠢
⠠⠞⠑⠎⠞ ⠐⠐   ⠼⠂⠆   ⠼⠖⠴⠨⠲⠬⠤⠂⠨⠢   ⠼⠶⠦⠬⠤⠶⠨⠔
⠠⠉⠕⠝⠞⠗⠕⠇   ⠼⠂⠖   ⠼⠶⠢⠨⠶⠬⠤⠦   ⠼⠂⠴⠴⠬⠤⠒
⠠⠞⠑⠎⠞ ⠐⠐   ⠼⠂⠖   ⠼⠢⠆⠨⠆⠬⠤⠆   ⠼⠦⠂⠬⠤⠖⠨⠶
⠸⠱

The table label and column headings are transcribed in UEB. The row headings are part of the technical material and are uncontracted. The single-word switch indicator is not used.

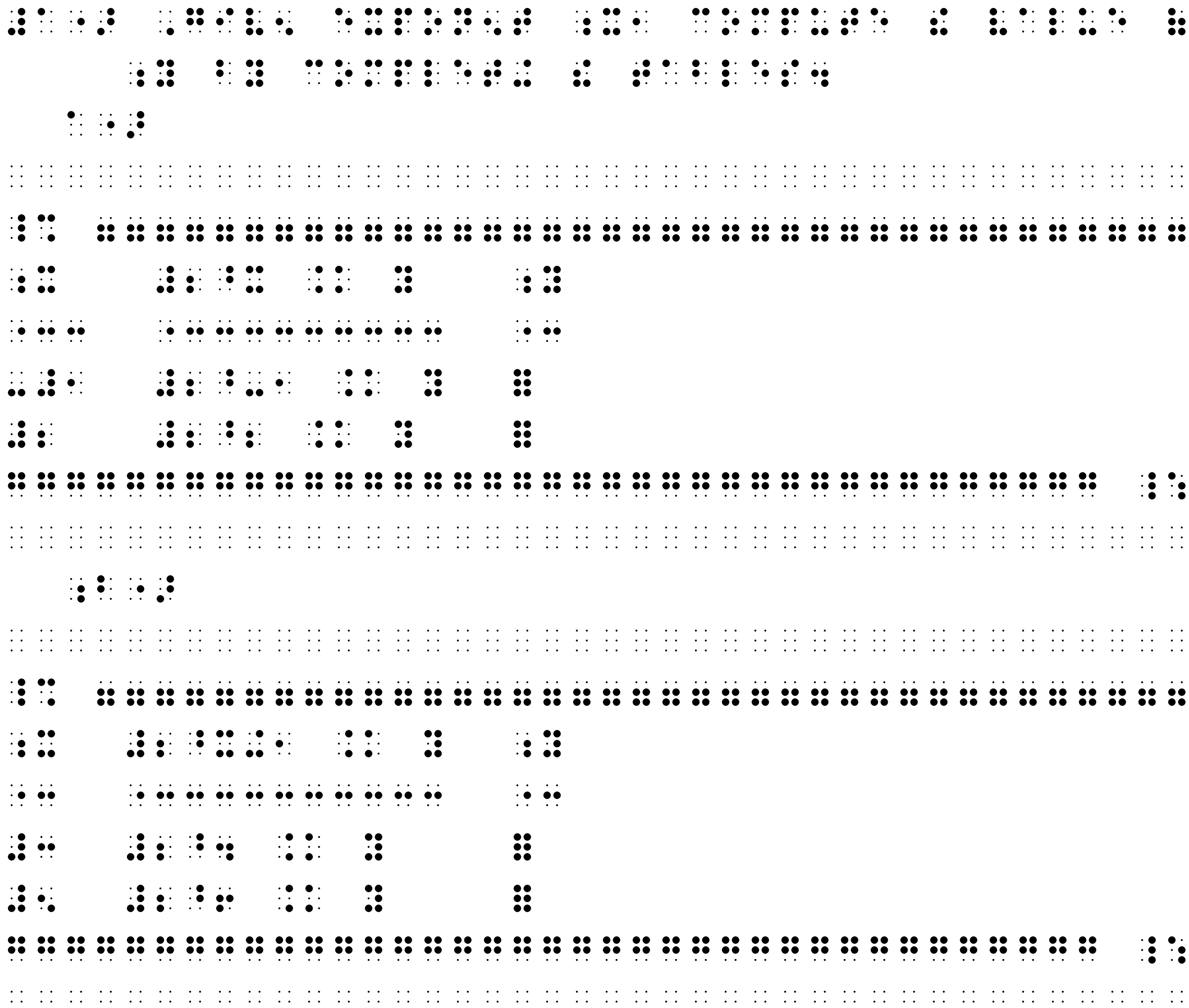
PRACTICE 18C

The next page shows another way this practice could be transcribed. Either transcription is viable.



Formatting Notes: *Subdivision identifiers begin in cell 3. Column 1 begins in cell 1. Box lines use the full width of the page. Boxes are preceded and followed by a blank line. In column 3, the general omission symbol represents the question mark. The opening Nemeth Code indicator precedes the identifier "a)". Nemeth is terminated after the completion of the table. Box lines match —since the top box line does not incorporate a switch indicator, the Nemeth Code terminator is not part of the bottom box line.*

PRACTICE 18C, alternate transcription



Formatting Notes: *Subdivision identifiers begin in cell 3. Column 1 begins in cell 1. Box lines use the full width of the page. Boxes are preceded and followed by a blank line. In column 3, the general omission symbol represents the question mark. The opening Nemeth Code indicator and Nemeth Code terminator are incorporated into each of the top and bottom box lines. Each subdivision identifier is in UEB.*

PRACTICE 18D

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⠼⠆⠖   ⠼⠢⠄⠔⠄⠄
⠼⠒⠴   ⠼⠖⠄⠂⠄⠄
⠼⠒⠲   ⠼⠢⠄⠂⠴⠄⠄
⠼⠒⠢   ⠼⠢⠄⠲⠄⠄
⠸⠱
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⠐⠒⠒⠒  ⠐⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
⠼⠃⠙   ⠼⠁⠚⠓
⠼⠃⠋   ⠼⠁⠚⠙
⠼⠉⠚   ⠼⠁⠃⠃
⠼⠉⠙   ⠼⠁⠁⠊
⠼⠉⠑   ⠼⠁⠃⠓
⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛

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Lines 1-2: Although the text is "instructional", it is formatted as a narrative paragraph (3-1) because it is not followed by itemized material.
Line 3: A blank line precedes the top box line.
Lines 7-13: The presence of prime signs in the first table dictates Nemeth. Nemeth Code is opened on line 7 and terminated on line 13.
Lines 19-23: The numbers in the second table can be transcribed in UEB.

PRACTICE 18D, continued

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⠐⠒⠒⠒  ⠐⠒⠒⠒⠒
⠸⠩
⠃⠲    ⠂⠦⠨⠲
⠆⠖    ⠒⠒⠨⠢
⠒⠴    ⠆⠒⠨⠦
⠒⠲    ⠂⠔⠨⠖
⠒⠢    ⠆⠢⠨⠴
⠸⠱
⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛⠛
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Lines 4-10: Nemeth Code is required because the entries contain decimal points. Since the data is composed entirely of numbers, numeric indicators are not used. The Nemeth Code terminator is placed alone on the line following the last entry, before the bottom box line.

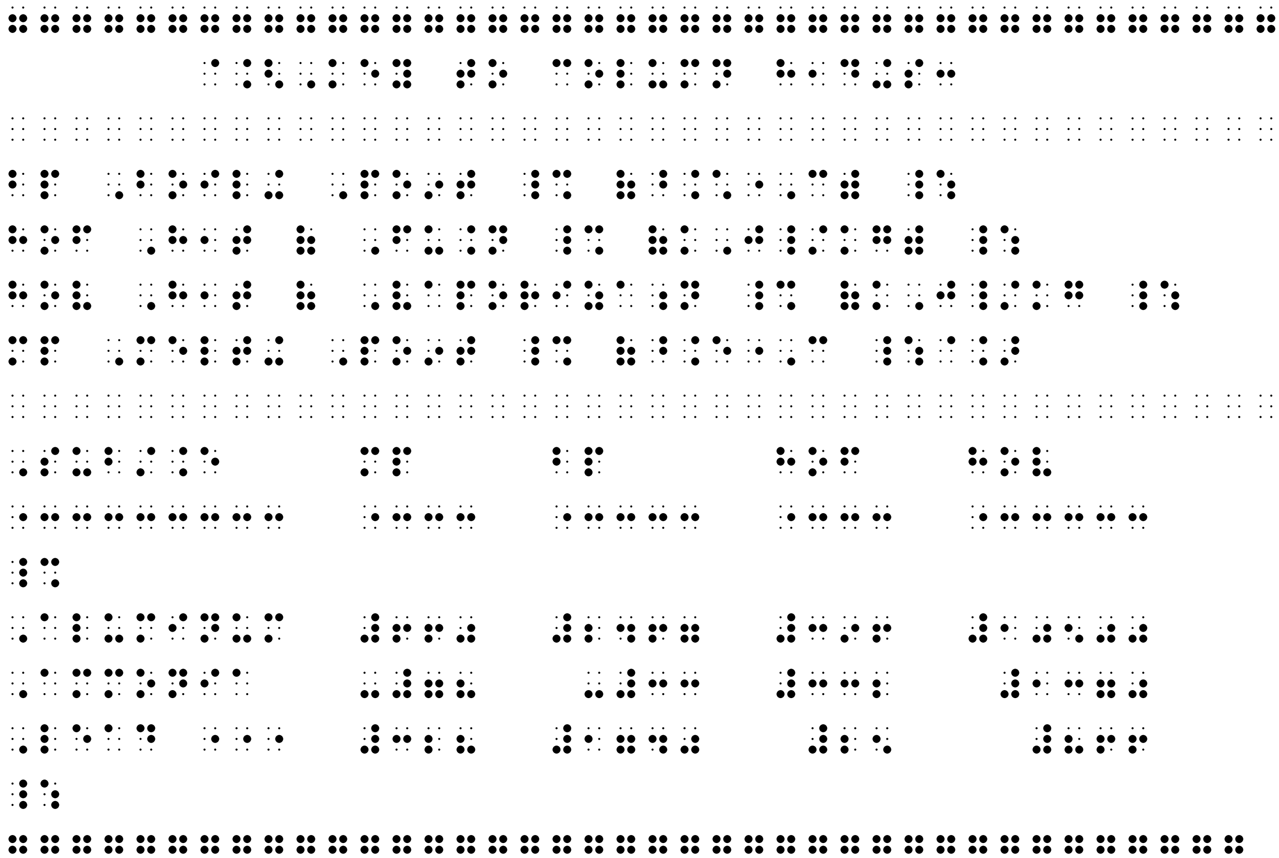
PRACTICE 18E

```
  ⠠⠗⠁⠚ ⠞⠗⠁⠝⠎⠿⠍⠫ ⠟⠥⠁⠙⠗⠊⠇⠁⠞⠻⠁⠇
⠸⠩ ⠠⠏⠠⠟⠠⠗⠠⠎ ⠸⠱ ⠞⠕ ⠿⠍ ⠟⠥⠁⠙⠗⠊⠇⠁⠞⠻⠁⠇
⠸⠩ ⠠⠏⠄⠠⠟⠄⠠⠗⠄⠠⠎⠄⠸⠲
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠠⠟⠄         ⠠⠏⠄

 ⠠⠎⠄      ⠠⠗⠄
⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀⠀
⠸⠱
```

The presence of prime symbols in the labels requires Nemeth. Nemeth Code is terminated after the required blank line following the graphic.

PRACTICE 18F



Other key choices are possible. "hof" was chosen here because "hf" cannot be used (it lacks lower dots); "hov" was chosen to mirror "hof" for easier recognition. The data are all transcribed in Nemeth. The row headings are uncontracted because they occur within the Nemeth switches.

FINAL LESSON

- Preparing for the Certification Exam
- Structuring a Textbook
- Four Practices

Answers to Practice Material

This is the final lesson but in no way is it the end of your training. As this book's title states, this is an introduction to the Nemeth braille code. In your work you will encounter symbols and usage that will require creative and consistent application of the rules. Each new assignment will present challenges. As you research answers, your understanding of the rules and guidelines will develop.

F1 Preparing for the Certification Exam

The topics addressed in this lesson are offered to help the transcriber who is preparing for the certification exam. A combination of textbook format guidelines from *Braille Formats* as well as Nemeth formats are to be applied to the practice material. The student can gauge their readiness for the exam by checking their work in the Answers section. There is no exercise to turn in for this lesson.

F2 The Nemeth Code Book

The Nemeth Braille Code For Mathematics and Science Notation should become your primary source for transcribing technical materials. Subject matter from several of the lessons in this training manual may be grouped into one section in the code book, giving new perspective and understanding of a topic or rule.

To get started, read the following "use/nonuse" sections in the Nemeth code book. Review the rules, study the examples, and follow the cross references.

	Use of	Nonuse of
Capitalization indicator	5.1	5.2
English-letter indicator	6.3, 10.3	6.4
Enlarged grouping symbol	19.6	19.7
Level indicator	14.9	14.10

	Use of	Nonuse of
Multipurpose indicator	24.1	
Nemeth Code	4.3	
Numeric indicator	3.3	3.4
Punctuation indicator	8.2	8.3
Simple fraction indicator	13.2	13.3
Typeform indicator	7.2, 7.3	7.4

Review situations when we do not follow print layout

Here is a sampling of situations for which the Nemeth Code has specific rules regarding spacing and arrangement on the braille line. *Centering*: cancellation, spatially arranged fractions. *Spacing*: abbreviations, factors in math expressions, math symbols, functions, alignment of items in spatial arrangements. *Linage*: "keep together" rules (an abbreviation and its associated numeral or letter, abbreviated function names, hyphenated expressions, signs of shape), division of long math expressions, side-by-side arrangements (e.g., itemized, unitemized, displayed, spatial). *Margins*: formal proofs, instructions, itemized material, mathematical statements, linked expressions, paragraphing.

Revisit rules which you find to be troublesome

For example, review the various forms of fractions, the many uses of the multipurpose indicator, correct assessment of ambiguous mathematical signs, and the use of literary punctuation for words and abbreviations in mathematical context.

Back matter

Familiarize yourself with Appendix B of the Nemeth Code which offers a useful index of braille symbols. Skimming the General Index is a good way to reinforce the vocabulary and terminology used in the Nemeth braille code.

Review code switching guidelines

The only time Nemeth symbols and indicators may be used is within the switch indicators. Review guidelines regarding when code switching is optional and when it is required, as well as considerations concerning placement of the switch indicators in embedded material, displayed material, spatial material, and at page turns. A summary appears in Appendix C of this course.

F3 Beyond the Nemeth Code

Every aspect of the Nemeth Code has been introduced in this course. Certification in this code implies that you are prepared to transcribe a textbook which contains mathematical notation. This requires knowledge of the structure of a braille textbook as well as how the Nemeth Code cooperates with textbook formatting and the rules of *Unified English Braille*. Format guidelines which apply to the structuring of a textbook are in the BANA publication *Braille Formats: Principles of Print-to-Braille Transcription.* The transcriber should be thoroughly familiar with that resource as well as the other sources listed below. These documents are available online at www.brailleauthority.org and https://iceb.org/ueb.html. Keep up to date as newer editions or updates are posted.

The editions listed below are current at the time of this writing. The Practice answers and commentary in this lesson may become outdated when future editions of these resources become available. Please contact us at transcribers@nfb.org if you find this lesson is not up to date.

The Rules of Unified English Braille, Second Edition, 2013 and the 2019 Updates

Braille Formats: Principles of Print-to-Braille Transcription, 2016 and the 2016 Addendum

The Nemeth Braille Code for Mathematics and Science Notation, 2022

This course does not address topics regarding creating a tactile graphic. A thorough reading of *Guidelines and Standards for Tactile Graphics* is recommended before undertaking a technical transcription that contains diagrams. Strategies presented in this resource include 2-D and 3-D drawings, clocks (analog and digital), complex geometric shapes, counting symbols, graphs (circle graphs, bar graphs, line graphs, histograms, Cartesian graphs, pictographs, pie charts, scatter plots, line or dot plots, box-and-whisker plots), measurement tools, money, nets, number lines, orthographic drawings, spinners, tessellations, thermometers, and Venn diagrams.

This course does not address topics pertaining to chemistry notation and chemical diagrams. Refer to *Chemical Notation Using the Nemeth Braille Code* when transcribing a chemistry assignment.

Symbols, arrangements, and structures not covered in the courses and code books are frequently encountered when preparing assignments. Many excellent materials have been shared and distributed by transcriber organizations. Professional development resources and online forums can be an excellent way to familiarize yourself with the finer points of code switching, formatting, division of expressions, and other topics for which experience is the best teacher. However, be aware that workshop materials may be outdated or may contain errors. They may contain strategies that are not supported by rules of the braille codes. For these reasons workshop materials should not be used or referenced as sources of information when preparing your certification exam.

F4 Transcriber-Generated Pages and Front Matter

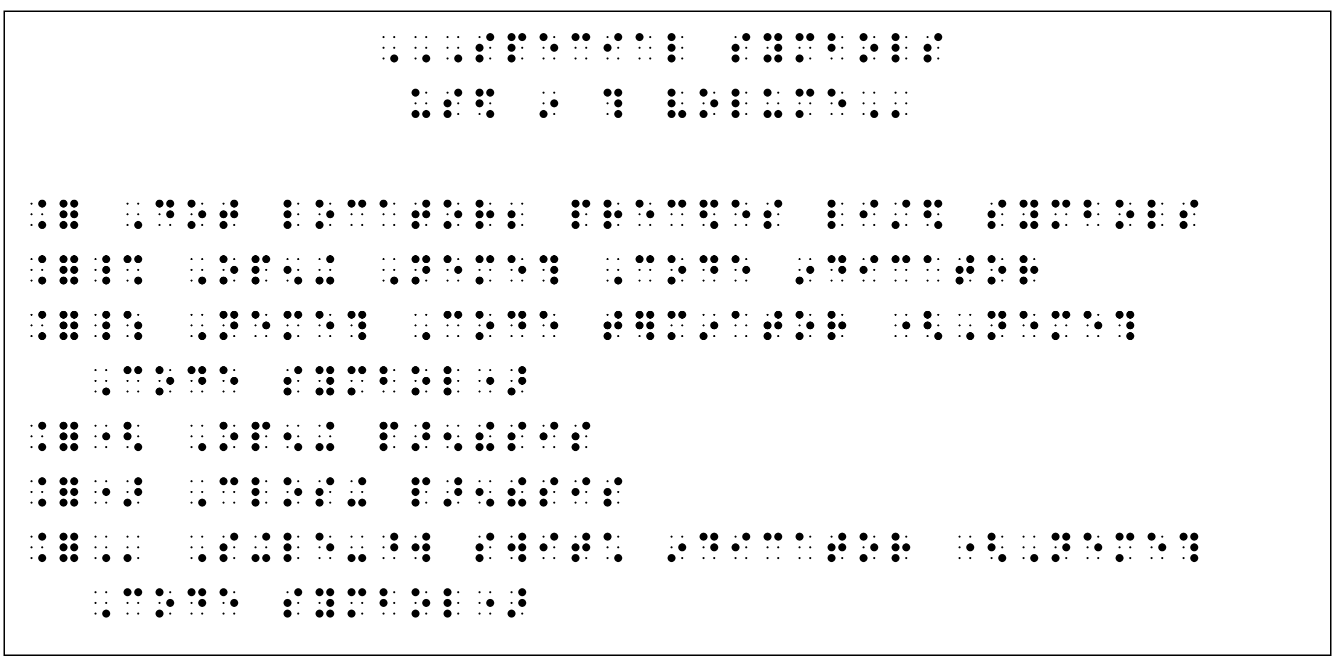
Before transcribing the first page of text, certain transcriber-generated and front matter pages are required. Review the guidelines for the transcriber-generated pages (title pages, special symbols page, and transcriber's notes page) in Section 2 of *Braille Formats 2016*. You may also wish to review the lesson on "Braille Book Format" in the most recent edition of the *Instruction Manual for Braille Transcribing – UEB Edition.*

F4.1 **Title Page.** Use the following statement in the Transcriber/Transcription segment: "Transcribed ___(year) into Unified English Braille with Nemeth by ___(Your Name)"

F4.2 **Special Symbols Page.** The heading for this "t" page is SPECIAL SYMBOLS USED IN THIS VOLUME. Follow the guidelines in Appendix G of *Braille Formats 2016* to select UEB symbols that should be included. Disregard the UEB math symbols given in that list since UEB math symbols are not used in a Nemeth transcription. List the UEB symbols that appear throughout the volume, including any that may occur on the title page.

The Nemeth code switch indicators are to be included in the list, following braille order, as explained in Section 1.1.2 of *The Rules of Unified English Braille*. The phrase “(Nemeth Code symbol)” should follow the definition of both the Nemeth Code terminator and the single-word switch indicator. Other Nemeth symbols are not listed on the Special Symbols page unless they are symbols devised by the transcriber.

Here is a model, showing a few UEB symbols as well as the three switch indicators. Additional symbols will be inserted into the model as needed in each braille volume.



F4.3 **Transcriber's Notes Page.** The heading for this "t" page is TRANSCRIBER'S NOTES. The purpose of this page is to identify special formats or usage found throughout a braille volume, and to cite sources. When Nemeth is used in a transcription, the Transcriber's Notes page should include the following note: "Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 2022.* You can find items that require explanation listed throughout *Braille Formats*. In addition to those requirements, the Nemeth Code identifies the following as items which require explanation either at the point in the transcription where they appear or on the Transcriber's Notes page:

- Use of capitalized letters in the print copy for digits in nondecimal bases
- Use of alternative forms of Greek letters in the print copy
- Omission of vector arrows which appear in the print copy
- Description of the shapes used in print to depict calculator or computer keys
- When identifiers to displayed expression are moved to the left in the braille edition
- Changing the column format of a formal proof to an itemized list in the braille edition

PRACTICE A

Instructions: Prepare transcriber-generated pages and one contents page according to the guidelines in Section 2 of *Braille Formats 2016*. Do not use a running head.

Title Page: Use the following information to prepare a title page.

- Book title: *ADVENTURES IN Y2K MATHEMATICS*
- Book subtitle: *Math in the New Millennium*
- Authors: Monica and Matías Cruz
- Publisher information: Math4You Publications, Inc., Antelope Valley, CA, www.M4YPub.edu
- Copyright information: ©2018 by M&M Publishers
- ISBN: 9-6230-99228x
- Transcriber segment: Use your name, your city, and your state
- Volume Information segment: Assume this is the second volume of a three-volume transcription. The braille page designation is "t1-t3, p1, and 1-120" and the print page designation is "v and 87-a123"

Special Symbols Page: A list of UEB symbols to be included on the Special Symbols page can be found in Appendix G of *Braille Formats 2016.* Follow your agency's decision regarding the "may be included" list, but in Practice A do not include them. List the symbols which appear in the all of the Practices in this lesson, as well as symbols from your title page: acute accent (on the title page); end of proof icon; Nemeth code switch indicators; opening and closing parentheses; typeform indicators for boldface passage, boldface terminator, boldface word, italic passage, italic terminator, italic word, underlined passage, and underlined terminator.

Arrange the symbols in braille order as explained in Section 1.1.2 of *The Rules of Unified English Braille.* To get started with your Special Symbols page, use the model on page F–4 of this lesson.

Transcriber's Notes Page: In addition to the required statement citing the use of the Nemeth Code, write a description of the change made to the column format of the formal proof which appears in Practice E.

Front Matter: Include a transcription of the contents page shown below.

ADVENTURES IN Y2K MATHEMATICS v

CONTENTS

F5 Body of Text

The entire transcription follows a collaborative pattern between *Braille Formats* and the formats provided for in the Nemeth braille code. When a format is specified in the Nemeth Code, those rules are applied not only to the technical material but also to the UEB material. Some examples are given below.

F5.1 **Follow Nemeth Formatting Rules.** The following matters are governed by Nemeth format and apply both to the technical material and to UEB portions of text.

- Runover margins for itemized material are determined individually for each item. That is, a problem with no subdivisions will be (1-3); the next problem in the same exercise set may have subdivisions and so will be (1-5; 3-5), etc. (Review 6.1.4 in Lesson 6.)
- Instructions preceding itemized material begin in cell 5 and runover in cell 3.
- "Keep together" rules for hyphenated expressions and for abbreviations and a numeral or letter associated with it.
- Mathematical statements—Nemeth rules apply regarding paragraphing, blank lines, and typeface.
- Margins applied to itemized material and their subparagraphs.
- Paragraphs begin in cell 3 and runover in cell 1. (Blocked paragraphing is not allowed.)

F5.2 **Follow Braille Formats Guidelines.** Items which are not addressed in the Nemeth code rely on *Braille Formats* for positioning. Some items governed by *Braille Formats* include:

- Blank lines
- Box lines
- Content and structure of the transcriber-generated pages and front matter
- Displayed literary text
- Exercise questions with answer choices
- Headings
- Indented list format for nontechnical text (e.g., table of contents)
- Lists
- Margins for captions, notes, sidebars, and transcriber's notes
- Page numbering
- Table layout

F5.3 Context-Dependent Formats

These formats occur only in Nemeth context.

- Blank lines with spatial arrangements
- Division of mathematical expressions
- Margins for displayed mathematical expressions

In tables, these items follow Nemeth formatting between the switches, and follow *Braille Formats* outside of the switches.

- Omissions in table entries
- Keying
- Tables consisting only of numbers

Final Word

Thank you for making the extra effort to learn the Nemeth braille code. We hope you have noticed your transcribing and proofreading skills improve over the course of the lessons. As you take on assignments, check the NFB website periodically for changes to this lesson material and check the BANA website for updates to the braille codes. We also encourage you to take advantage of opportunities to stay informed and connected to other transcribers. The National Braille Association (NBA) publishes a quarterly Bulletin and hosts several learning opportunities—an online forum "Ask An Expert", monthly webinars, and professional development conferences. Local groups offer similar support, for example, the California Transcribers and Educators for the Blind and Visually Impaired (CTEBVI), and the Visual Aid Volunteers of Florida (VAVF).

We wish you success and satisfaction providing much-needed mathematics and science materials for braille readers.

This course concludes with four practices which are characteristic of K-12 grade level topics.

Four Practices

Instructions: Use a 40-cell line and a 25-line page. Do not use a running head. Show print and braille page numbers on every page. Use the number shown in the upper right corner as the print page number. A dashed line in print shows where a new print page begins.

Begin numbering with braille page 1, showing the book title at the top of the page as required according to *Braille Formats 2016*.

Begin each Practice on a new braille page, but within each Practice do not force a new braille page unless a rule supports doing so. Do not include the PRACTICE headings in your transcription.

Even though the print page numbers of the Practices are not consecutive, number the braille pages consecutively throughout the four Practices.

If a transcriber's note is required, write it in context—in other words, do not create a Transcriber's Notes page.

Please revisit section F3 of this lesson regarding the editions of UEB and *Braille Formats* used in the transcription of these Practice answers.

PRACTICE B

ADVENTURES IN Y2K MATHEMATICS 1

ADDING AND SUBTRACTING

$$\begin{array}{r}53\\+36\\\hline\end{array}\quad\begin{array}{r}14\\+44\\\hline\end{array}\quad\begin{array}{r}33\\+21\\\hline\end{array}\quad\begin{array}{r}98\\+1\\\hline\end{array}\quad\begin{array}{r}2\\+2\\\hline\end{array}\quad\begin{array}{r}456\\+13\\\hline\end{array}$$

$$\begin{array}{r}4\\+84\\\hline\end{array}\quad\begin{array}{r}23\\+32\\\hline\end{array}\quad\begin{array}{r}71\\+1\\\hline\end{array}\quad\begin{array}{r}33\\-22\\\hline\end{array}\quad\begin{array}{r}150\\-40\\\hline\end{array}\quad\begin{array}{r}9\\-6\\\hline\end{array}$$

$$\begin{array}{r}826\\-16\\\hline\end{array}\quad\begin{array}{r}55\\-2\\\hline\end{array}\quad\begin{array}{r}77\\-27\\\hline\end{array}\quad\begin{array}{r}420\\+519\\\hline\end{array}\quad\begin{array}{r}95\\-62\\\hline\end{array}\quad\begin{array}{r}93\\+6\\\hline\end{array}$$

ADVENTURES IN Y2K MATHEMATICS 2

COMPARING FRACTIONS

Directions: Write the correct comparison symbol (>, <, or =) in each box.

1) $\frac{1}{6}\ \square\ \frac{3}{4}$ 2) $\frac{1}{2}\ \square\ \frac{3}{6}$ 3) $\frac{2}{4}\ \square\ \frac{1}{3}$

4) $\frac{2}{5}\ \square\ \frac{2}{3}$ 5) $\frac{9}{10}\ \square\ \frac{4}{5}$ 6) $\frac{1}{6}\ \square\ \frac{2}{12}$

PRACTICE C

— —

Exercise Set 18-6

1. **Home Economics** The cost of using a 60-watt light bulb is given by the function $y = 0.0036x$. The cost is in dollars, and x represents the number of hours the bulb is lit.
   a. How much does it cost to use a 60-watt light bulb 8 hours a day for a week?
   b. If the total cost of using a 60-watt bulb is $1.98, for how many hours can it be used?
2. **What's the Question?** The following set of points defines a function: $\{(3, 6), (-4, 1), (5, -5), (9, -6), (10, -2), (-2, 10)\}$. If the answer is 6, 1, –5, –6, –2, and 10, what is the question?
3. **Physics** Ohm's law can be described by the simple formula

   $$I = \frac{V}{R}$$

   where I = current (in amps, A), V = voltage (in volts, V), and R = resistance (in ohms, Ω). Which equation would you use to solve for voltage?

   a. V = I/R
   b. V = IR
   c. V = R/I

— —

ADDING, SUBTRACTING, MULTIPLYING, AND DIVIDING INTEGERS

Find the sum, product, or quotient as indicated by the signs $+, \times, \div$.

1) $-6 + -5 =$ ____ 2) $-2 \times -1 =$ ____

3) $35 \div -5 =$ ____ 4) $5 + -19 =$ ____

5) $-24 \div 4 =$ ____ 6) $-132 \div -11 =$ ____

7) $9 \times 9 \times -5 =$ ____

UNIT 6 REVIEW

Fill in the correct answers.

1. If $7n = 0$, then $n =$ ____.
2. Replace the ☐ with a numeral to make a true sentence: $(4 + 8) + \square = 4 + (8 + 3)$.
3. $\left(\frac{1}{3} + \frac{5}{6}\right) - \frac{5}{12} = ?$
4. $5 \times 5 \times 5 = 5^{?}$

Study the equivalencies.

5. To find the mixed-number name for $\frac{154}{9}$, divide 9 into 154.

$$\begin{array}{r} 17 \text{ R}1 \\ 9\overline{)154} \\ \underline{9} \\ 64 \\ \underline{63} \\ 1 \end{array}$$

$$\frac{154}{9} = 17\frac{1}{7}$$

6. Multiply $4\frac{2}{3}$ by $\frac{3}{14}$. $4\frac{2}{3} = \frac{12+2}{3} = \frac{14}{3}$.

$$\frac{14}{3} \times \frac{3}{14} = 1!$$

— —

Review various topics.

7. If $R = \{a, b, c, d\}$ and $S = \{a, c, e, g, h\}$, then the intersection of sets R and S is $\{a, c\}$.
8. Only the following pairs of symbols may occur out of the natural order: I before V or X, X before L or C, C before D or M. For example, XL = L–X = 50–10 = 40.
9. Use the distributive property to multiply 14 by 3.

$$\begin{aligned} 3 \cdot 14 &= 3\,(10 + 4) \\ &= 3 \cdot 10 + 3 \cdot 4 \\ &= 30 + 12 = 42 \end{aligned}$$

10. "$24_{(\text{five})}$" is read "two four, base five."
11. A gain of 5 yd followed by a loss of 2 yd gives a total gain of 3 yd. Expressed mathematically, $(^{+}5) + (^{-}2) = {}^{+}3$.

Explain the relationships.

12. $\overline{AB}$ is congruent to $\overline{DE}$.
13. $\overline{RS} \approx \overline{EF}$; $\overline{RS} \neq \overline{LK}$.
14. $\overrightarrow{DA} \cup \overrightarrow{DB} = \overleftrightarrow{AB}$
15. $\dfrac{7}{\underset{6}{\cancel{12}}} \times \dfrac{\overset{11}{\cancel{22}}}{5} = \dfrac{77}{30}$

Simplify each radical expression.

16. $\sqrt[4]{x} \cdot \sqrt{x}$ 17. $\sqrt[5]{\sqrt[3]{a^2}}$ 18. $\dfrac{\sqrt{36x}}{\sqrt[3]{8x^2}}$

PRACTICE E

THEOREM The sum of the angles in a triangle is 180 degrees.

Given: $\triangle ABC$

Prove: $\angle a + \angle b + \angle c = 180°$

STATEMENT	REASON
1. Let BD be a line through B parallel to AC.	1. Parallel postulate.
2. $\angle a = \angle d$.	2. Corresponding angles are equal.
3. $\angle b = \angle b$.	3. Identity.
4. $\angle c = \angle e$.	4. Alternate-interior angles are equal.
5. $\angle a + \angle b + \angle c = \angle d + \angle b + \angle e$.	5. Sum of equal quantities are equal.
6. $\angle d + \angle b + \angle c = 180°$.	6. A straight angle equals 180°.
7. $\therefore \angle a + \angle b + \angle c = 180°$.	7. Quantities equal to the same quantity are equal to each other. ■

Use π/12 = π/3 – π/4 and the identity for the tangent of a difference to solve **tan** $\left(\frac{\pi}{12}\right)$.

$$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \frac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3-2\sqrt{3}+1}{2} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

The answer is $\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$.

ANSWERS TO PRACTICE MATERIAL

Answers observe guidelines in the 2016 edition of *Braille Formats*. See the resource list on page F–3.

PRACTICE A

Title Page

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Special Symbols Page

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PRACTICE A, cont.

Transcriber's Notes Page

Contents Page

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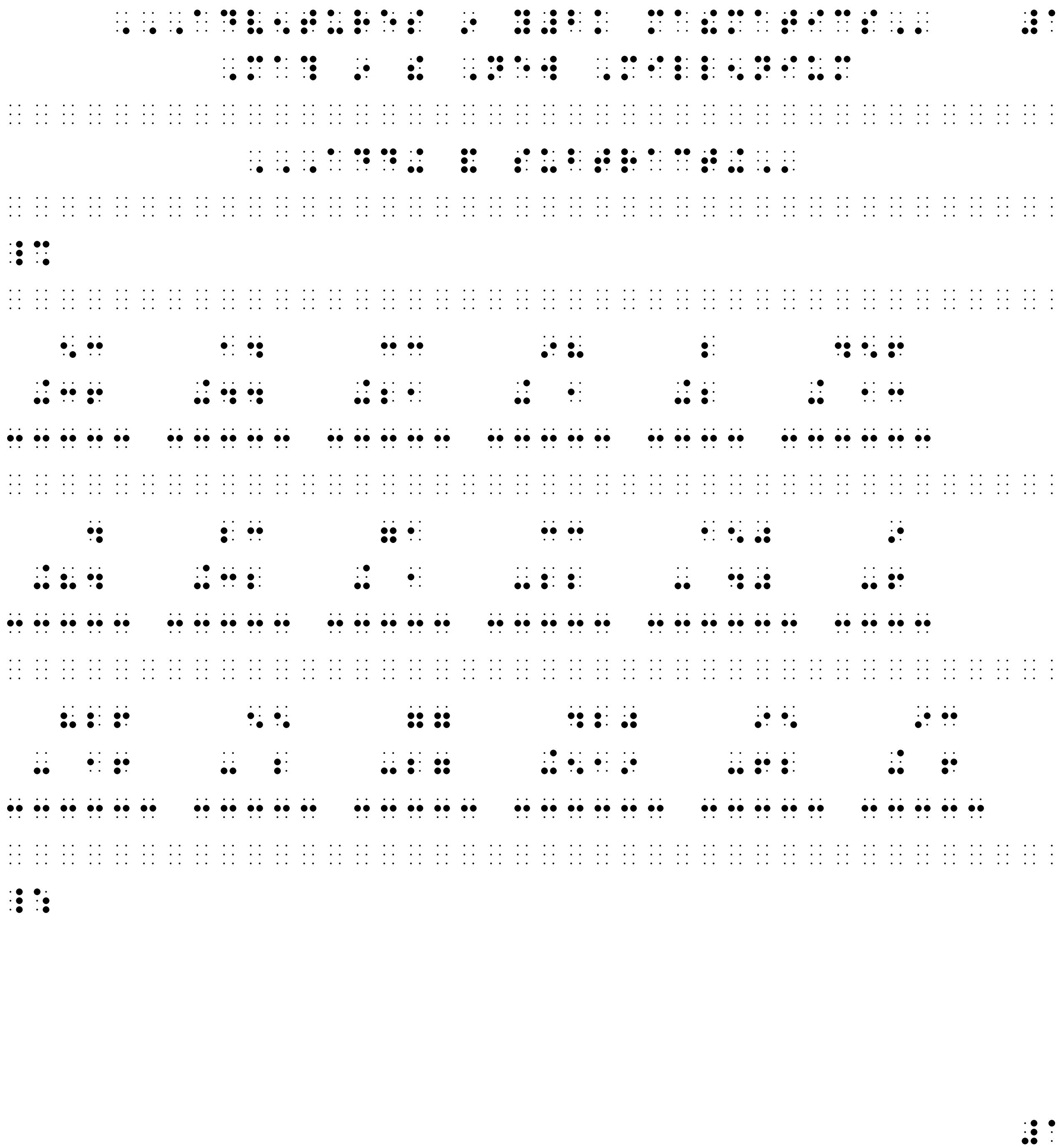
Line 1: Instructions for this page say to assume this is the second volume of a transcription. Implied page numbers i-iv are transcribed in Volume 1 only. See Section 2.7.1 of Braille Formats 2016.

PRACTICE B

Braille Page 1



Format Comment: The switch indicators do not take the place of required blank lines. A blank line follows the centered heading; a blank line precedes and follows the spatial material.

PRACTICE B, cont.

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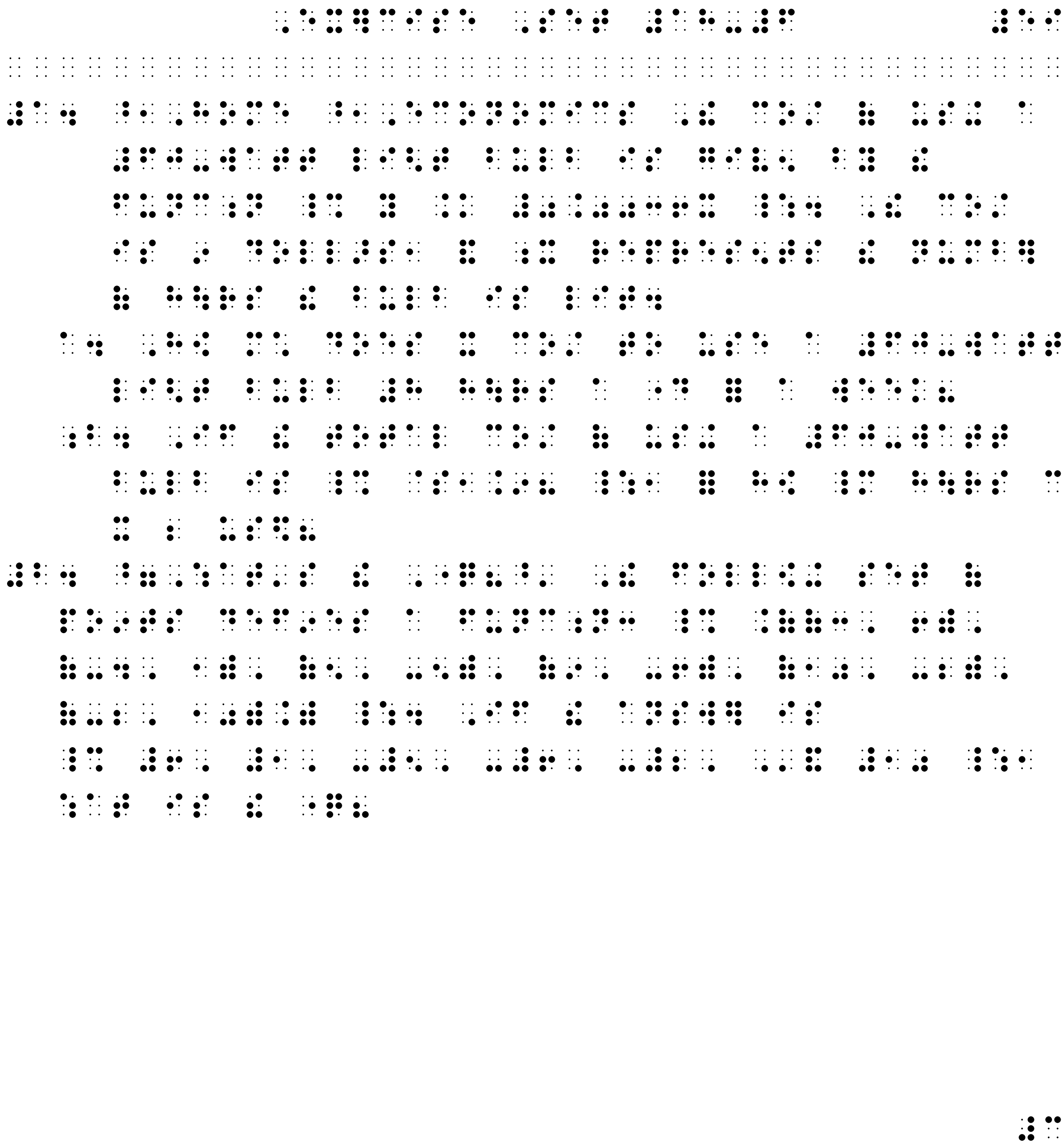
Lines 4-5: The embedded comparison symbols need not all fall on the same line because this is not a mathematical "enclosed list".
Lines 5-10: Side-by-side arrangement of nonspatial itemized material is not allowed in the braille transcription. Each identifier starts in cell 1. (Nemeth format rule.)

PRACTICE C



Line 1: The heading could be formatted as a cell-5 heading, in which case line 2 would not be blank. (Braille Formats guideline.)

Lines 14-18: Runover margins for itemized material are determined individually for each question. Problem #2 has no subdivisions, so its runovers are in cell 3. (Nemeth format rule.)

Lines 17-18: Alternate transcription: The freestanding numbers without negative signs may be transcribed in UEB, switching to Nemeth only for the three negative numbers.

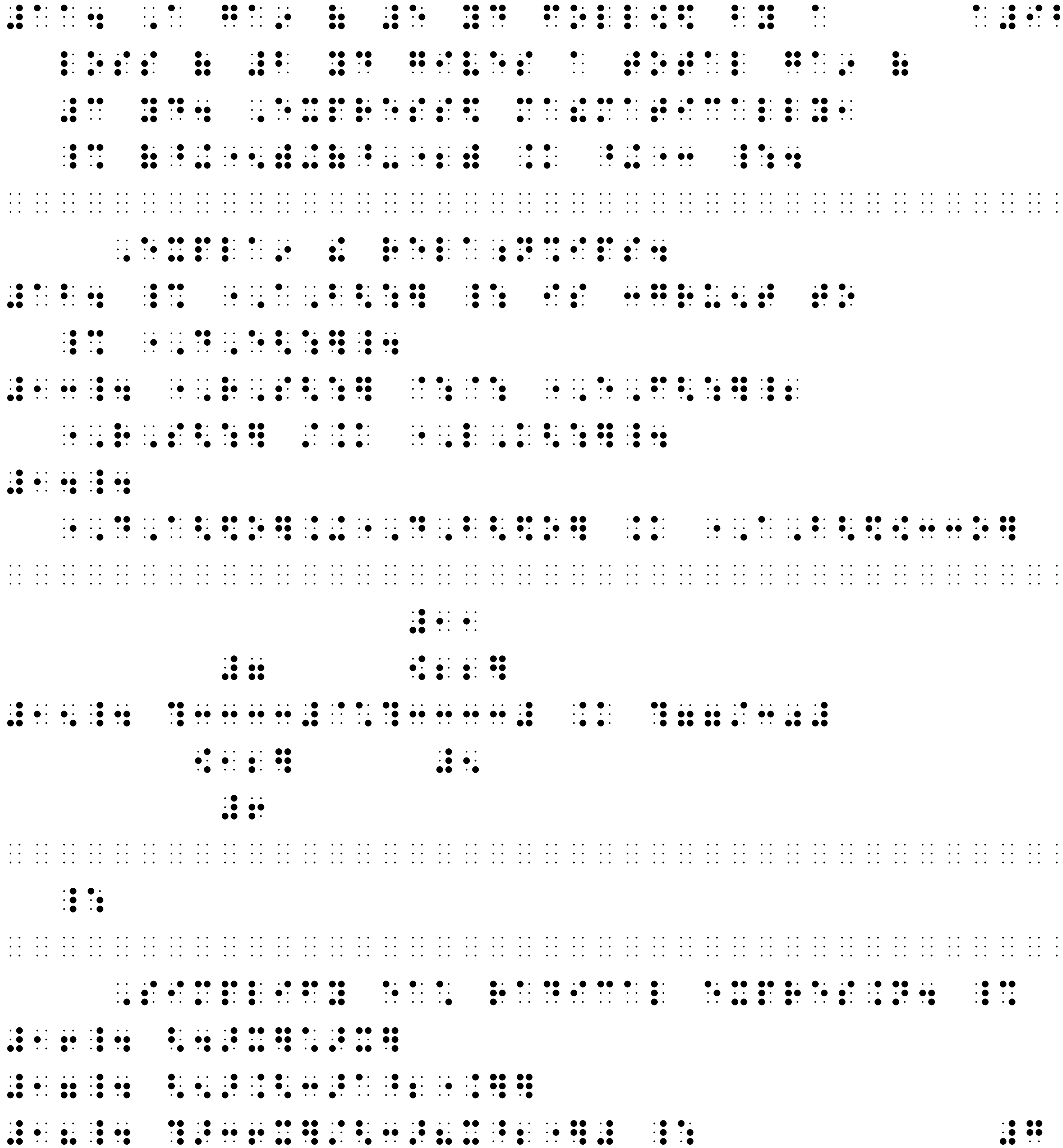
PRACTICE C, cont.

Lines 1-11: This problem set starts on a new braille page because question and answer choices should appear on a single page when space permits, regardless of the amount of blank space resulting on the preceding page. [Braille Formats 2016, Section 10.1.7]

Lines 19-25: Side-by-side arrangement of nonspatial itemized material is not allowed in the braille transcription. Each identifier starts in cell 1. (Nemeth format rule.)

PRACTICE D

PRACTICE D, cont.



PRACTICE E

Lines 1-8: Your transcriber's note may say anything similar, describing the change in format of the two-column proof.

Line 9: The line before the mathematical statement (THEOREM) is blank.

PRACTICE E, cont.

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Line 10: The line following the end of the proof is blank. (See Section 8.3.2 of Braille Formats regarding the blank line after a list when a page change indicator follows.)
Line 15: The displayed expression begins in cell 3 with runovers in cell 5.
Lines 16-23: Because one of the links requires division, each link begins on a new line. (Nemeth format rule.)
Line 18: The divided link begins with the main fraction line, and is not indented further because, in print, the signs of comparison are not vertically aligned.

Revision Log for Uploads of
AN INTRODUCTION TO BRAILLE MATHEMATICS
Using UEB and the Nemeth Code
Provisional Online Edition 2017

Updated: 8-28-2018

This document contains information about revisions to the uploaded PDF and BRF files of *An Introduction to Braille Mathematics Using UEB and the Nemeth Code, Provisional Online Edition 2017.* Individuals who downloaded these files prior to the current upload dates should note this information about what has been changed or should download the most recent copy and discard the old one. Upload dates are reflected in each filename as well as in the footer of the print document and the title page of the braille document.

Please report errors by sending an email to: transcribers@nfb.org.

FRONT MATTER

February 17, 2017: First upload

April 25, 2017: First revision

Added the revision date (footer in PDF; on title page in BRF); changed prerequisite title to Braille Formats 2016 (Pre-Publication Release); edited/added text to the first paragraph under the heading ABOUT THE PROVISIONAL ONLINE EDITION

BRF: Added a braille page number to even-numbered pages.

July 24, 2017: Second revision

BRF: title page: Lines 17-18 changed to say "Transcribed 2017 into Unified English Braille and Nemeth Braille by Lindy Walton"; inserted numeric indicator on line 24; Special Symbols page: added symbols according to Braille Formats Appendix G.

February 1, 2018: *Third revision*

"ABOUT" page: "completed" changed to "completely".

"PREREQUISITES" page: Corrected title of "Guidance"; deleted "(Pre-Publication Release)" from title of *Braille Formats*; deleted citation of "*Application of the Formats Guidelines 2011 to Nemeth Transcriptions*"; corrected date for Nemeth Code Updates to 2007-2015.

May 15, 2018: Fourth revision

Foreword: Sentence added regarding Appendix A; name added to thank yous.

Complete Table of Contents merged with Front Matter file.

June 25, 2018: Fifth revision

Table of Contents:

14.12.2 moved to become new "14.21 Margins for Embedded Expressions".

14.21-14.26 renumbered as 14.22-14.27

14.23-14.25: Heading changed to "Square Root Division"

17.8-17.11 updated.

18.5.3 title changed to "Context-Dependent Formats"

LESSONS 1-4

April 25, 2017

Added the revision date (footer in PDF; on title page in BRF).

BRF: Added a braille page number to even-numbered pages; added a running head; deleted "brief contents".

More changes to Lesson 1-4 are listed by lesson number, below.

LESSONS 1-6

July 24, 2017

PDF: page 1: Added statement to recheck the NFB website periodically for updated file; brief contents: moved FORMAT list to be a right column (if applicable).

BRF: title page: Lines 17-18 changed to say "Transcribed 2017 into Unified English Braille and Nemeth Braille by Lindy Walton"; inserted numeric indicator on line 24; Special Symbols page: added symbols according to Braille Formats Appendix G; Transcriber's Notes page: inserted capitals indicator for letter S in TRANSCRIBER'S; added a paragraph about the dash being omitted from page numbers; page 1: inserted full book title on lines 1-4; page 1: added statement to recheck the NFB website periodically for updated file.

More changes to Lessons 1-6 are listed by lesson number, below.

LESSONS 1-8

February 1, 2018

Braille examples changed to a 3-1 paragraph style in order to avoid misunderstanding. Sometimes this required slight changes in wording.

PDF: "Answers to Practice Material" section forced to start on a page front.

More changes to Lessons 1-8 are listed by lesson number, below.

LESSONS 1-10

May 15, 2018

Throughout: The name of the BANA Mathematics Committee changed to "Nemeth Code Technical Committee".

Throughout: The name of the BANA "Guidance" document changed to "Guidance for Transcription Using the Nemeth Code within UEB Contexts".

More changes to Lessons 1-10 are listed by lesson number, below.

LESSONS 2-10

May 15, 2018

BRF: page t4 (Transcriber's Notes page): Added paragraph about use of the Nemeth Code; changed the last paragraph to have a paragraph heading.

More changes to Lessons 2-10 are listed by lesson number, below.

ALL BRAILLE FILES

August 28, 2018

Changes made to reflect new guidelines:

Special Symbols page: Nemeth Code symbols included in the main symbols list, with (Nemeth Code symbol) added to the end of the description

Transcriber’s Notes page: Changes made to the statement about the Nemeth Codebook and the Guidance document.

LESSON 1

February 17, 2017: First upload

In addition to previous notes, the following changes have been made.

April 25, 2017: First revision

1.10: Clarified statement about Special Symbols page.

July 24, 2017: Second revision

PDF: Brief contents: "General Rules" changed to "General Principles"; 1.7: "(blank lines)" inserted to clarify the term "line spacing".

BRF: 1.7: "(blank lines)" inserted to clarify the term "line spacing".

February 1, 2018: *Third revision*

1.3.1, 1.4.1, 1.5, 1.6: Language clarified.

Example 1.3-1: Line numbers added to print copy.

Instructions before Practice 1A: First paragraph: "he" changed to "the"; second paragraph revised.

Practice 1B: "0.37" corrected in answer section.

May 15, 2018: Fourth revision

BRF: page t3, Deleted all three switch indicators on Special Symbols page.

BRF: Practice 1B: Added transcriber's note referring to notes at end of lesson.

August 28, 2018: Fifth revision

1.10: "is" changed to "are" in the BANA decision box.

LESSON 2

February 17, 2017: First upload

In addition to previous notes, the following changes have been made.

April 25, 2017: First revision

Practice 2E: Braille corrected to use a 40-cell line.

July 24, 2017: Second revision

PDF: 2.14: Deleted the word "All" in "All paragraphs".

BRF: Example 2.7-7: Dot 6 deleted from minus sign in commentary; 2.14: deleted the word "All" in "All paragraphs"; Answer section: lesson number deleted from combined page numbers.

February 1, 2018: *Third revision*

2.1: Language clarified regarding the term "switch".

2.3: Two additional paragraphs and examples added (Examples 2.3-5 and 2.3-6).

Example 2.4-1: Citation added to comment.

Example 2.5-5: Changed "12s" to "twelves".

Examples in 2.5.3 (print copy only): Added line numbers.

Practice 2C: Second paragraph changed.

Practice 2D: Added a period at the end of the second paragraph.

Example 2.7-2: Clarification added to the comment.

Example 2.7-3: Comment added.

Example 2.7-5: "Dan" changed to "Danny".

Text preceding Example 2.10-2: "UEB dash" changed to "UEB underscore".

2.14: Word added to note.

May 15, 2018: Fourth revision

BRF: page t3: Deleted single-word switch indicator on Special Symbols page.

2.1, note: "See Appendix A" changed to "See Lesson 18".

BRF: Practice 2E, F: Added transcriber's note referring to notes at end of lesson.

August 28, 2018: Fifth revision

Example 2.7-5 (braille and simbraille): letter “p” changed to capitalized letter “P” in the word “Phew!”

LESSON 3

February 17, 2017: First upload

In addition to previous notes, the following changes have been made.

April 25, 2017: First revision

3.5.3 Clock Time: Entire section rewritten; Example 3.18-2: replaced; Example 3.19-1: shadow dots added to PDF copy.

July 24, 2017: Second revision

3.14: "math term" changed to "math expression".

BRF: 3.14: Two displayed chevron items moved to cell 3 (from cell 5); answer section: lesson number deleted from combined page numbers.

February 1, 2018: *Third revision*

Example 3.5-1: Sentence added to comment.

Example 3.5-3: Sentence added to comment.

Practice 3B: Second sentence of instructions changed.

Example 3.8-1: Comma inserted after "14.6" in the (simulated) braille.

Example 3.8-4: Text added; period added at end of sentence.

Sample braille before Example 3.8-5: corrected the second cell of the equals symbol.

Example 3.8-5: Text added.

3.10: Corrected narrative in second sentence: "immediate" changed to "immediately".

3.13: Narrative changed to explain recommended practice; further clarification added before the second isolated example.

Example 3.13-2: Comment deleted.

3.13.1: New subsection added, with new Example 13.3-3.

3.14: Removed "it makes more sense to" and changed ellipsis to period; text after Example 3.14-2 changed.

Example 3.18-2: Comment removed.

Example 3.20-1: Comment added.

May 15, 2018: Fourth revision

BRF: page t3: Deleted single-word switch indicator on Special Symbols page.

LESSON 4

February 17, 2017: First upload

In addition to previous notes, the following changes have been made.

April 25, 2017: First revision

Many changes made in the first three sections and the following sections are renumbered: paragraph after Practice 4B: "in a later lesson" changed to "in Lesson 5"; answers to Practice 4C corrected to use a 40-cell braille line; FORMAT SUMMARY #2, last item: removed "or letter".

July 24, 2017: Second revision

BRF: A displayed chevron item moved to cell 3 (from cell 5); inserted a blank line before Example 4.6-4; inserted a blank line before section 4.6.4; removed the box lines around the full-page Practice 4E; FORMAT SUMMARY #2, last item: removed "or letter".

February 1, 2018: *Third revision*

Example 4.2-1 (line 2): Contracted "sh" in "Shape".

Example 4.2-5 Removed; 4.2-6 renumbered.

4.3: First sentence clarified.

Example 4.3-2: Remark reworded.

Practice 4A:

--Sentence "B" (braille): period deleted from the end.

--Sentence "C" (braille): the word "named" moved to fall at the end the first line.

--Sentence "D" added.

4.4: Text edited before Example 4.4-3; isolated example removed.

4.4.2: Heading changed to "Punctuation with Abbreviations"; first sentence clarified.
4.4.2.a: Language clarified.
4.5: Text added.
Practice 4B, item 2 (braille): The grave accent in "Gruyère" corrected.
Practice 4C, braille edition: Nemeth Code terminator corrected in second sentence.
4.9: New Example 4.9-2 with accompanying text; old Example 4.9-2 renumbered as 4.9-3; exception moved.
4.11.3: First sentence changed to "In mapping notation, the symbol : is brailled as an unspaced colon."

May 15, 2018: Fourth revision

BRF: Practice 4A, B, D, F: Added transcriber's note referring to notes at end of lesson.
Practice 4A, Item C: Opening switch indicator moved to the beginning of the next line.
4.6: "See Appendix A" changed to "See Lesson 18".
Example 4.16-1 changed.
Practice 4F (braille) single-word switch indicator inserted before word "does".

LESSON 5

April 25, 2017: First upload

In addition to previous notes, the following changes have been made.

July 24, 2017: First revision

PDF: Added new section 5.1.1; old 5.1.1 renumbered as 5.1.2.
BRF: Added new section 5.1.1; old 5.1.1 renumbered as 5.1.2; several displayed chevron items moved to cell 3 (from cell 5); Greek Alphabet Table: information added to the transcriber's note regarding blank entries, unused column headings deleted from the third page of the table; pagination readjustments for a couple of pages after the Greek Alphabet Table.

February 1, 2018: *Second revision*

Practice 5A, braille edition: Item #1, the opening Nemeth Code indicator moved to begin the runover line.
Example 5.3-1, first line of braille: "Formulas" contracted and "to" moved up to that line.
5.3.1: Last sentence deleted.
5.3.2: Removed redundancy; changed "is" to "are"; last sentence deleted.
Example 5.3-2: Sentence rearranged to include a comma; comment reworded.
5.5: Clarification added regarding required switch to Nemeth Code; cross-reference added.
Example 5.5-1: Wording changed; comment added.
Example 5.5-2: Comment added.
Example 5.9-4, Braille edition: Transcriber's Note indicators removed.
Practice 5F:
--Third sentence: opening quotation mark moved; note added regarding placement of the opening quotation mark before the switch indicator.
--Fourth sentence: a second sentence has been added to the note.
--Last sentence: comment rewritten.
Example 5.15-6: P(3 and 7) changed to P(3 AND 7); explanation of logical operators added to the comment.
New example 5.15-15 and accompanying text.
Practice 5G (braille), 16th item removed; opening parenthesis inserted before "San Antonio"; runover margins corrected.
Example 5.16-3: P(A and B) changed to P(A AND B).
Practice 5H:
--Item C: comma inserted after "nth".
--Item E: not and or changed to NOT and OR.
Example 5.23-1: Clarification added to the comment.

5.24: Entirely revised, including new subsections 5.24.1 and 5.24.2.
Practice 5I: Deleted first two sentences in the instructions.

May 15, 2018: Third revision
BRF: Practice 5F: Added transcriber's note referring to notes at end of lesson.

June 25, 2018: Fourth revision
Practice 5C: Letter a changed to letter A in the second sentence.

August 28, 2018: Fifth revision
Example 5.9-5: (Nemeth Code) changed to (Nemeth Code symbol)

LESSON 6

April 25, 2017: First upload
In addition to previous notes, the following changes have been made.

July 24, 2017: First revision
6.7: Wwording change (clarification).

February 1, 2018: *Second revision*
6.2: New paragraph added to the end.
6.3: Language clarified (after the table).
6.3.1-6.3.6: Each symbol is restated before the examples.
6.4.1-6.4.12: Each symbol is restated before the examples.
6.5: Text clarified; print example inserted before commentary; another line added to sample; commentary adjusted accordingly. BRF: ellipsis corrected to Nemeth Code symbol.
Example 6.7-2: Deleted a space within the word “Relational” in the braille.
6.10: Added "Negated Perpendicular To" symbol; added to the second example.
Practice 6F: Instructions added (to treat the headings as cell-5 headings) and in the braille copy the headings were adjusted.

May 15, 2018: Third revision
6.5: Symbols box added to show column separation line and guide dots.
6.5 (print): Commentary moved to follow the braille example.
BRF: Practice 6C: Added transcriber's note referring to notes at end of lesson.
Practice 6F: The first three item numbers 1. 2. 3. (braille) changed to (1) (2) (3) to match print.

LESSON 7

July 24, 2017: First upload
In addition to previous notes, the following changes have been made.

February 1, 2018: *First revision*
7.10: Changed fifth sentence slightly.
Example 7.11-3, note: The specific section number in the *Braille Formats* citation was removed.
Example 7.18-1: Nemeth Code terminator deleted from the end of item b; single-word switch indicator applied to the word "If" at the beginning of item c; punctuation indicators added before the two periods which now fall inside the switches.
Practice 7C: Nemeth Code terminator removed from the end of item 2 as Nemeth Code continues in item 3; punctuation indicators adjusted and identifier "3" changed to Nemeth Code.

May 15, 2018: Second revision
Insignificant capitalization changes.
Practice 7A: Reorganized for clarity. One more row added to the beginning. Commas deleted.
Example 7.5-7: Math moved to begin on the second line and continue on the third.
7.6: Comment added.
BRF: Practice 7D: Added transcriber's note referring to notes at end of lesson.

June 25, 2018: Third revision
Practice 7B (print): Period deleted from end of sentence ii.

LESSON 8

July 24, 2017: First upload

In addition to previous notes, the following changes have been made.

February 1, 2018: *First revision*

Practice 8B, item 4 (print): Quotation mark deleted before the word "is".

8.6.1: Third mini-example removed.

Example 8.7-3: Comment added.

8.7.3: (print copy): Apostrophes changed to look like punctuation, not like prime signs.

8.10.4: Sentence moved from "is not used" paragraph to "must be used" paragraph; order of paragraphs switched.

Example 8.11-1 (braille): The word “and” added.

Example 8.11-2 (braille): Corrected spelling of the word “shown”.

8.11.2, second mini-example (braille): Corrected spelling of the word "obtuse".

8.11.3 (print): Code switch indicators deleted from the two isolated examples.

Example 8.16-1 (braille): Opening Nemeth Code indicator added following the word “anion”.

Practice 8D, item 6 (braille): Ssingle-word switch indicator added to the word “or”.

Practice 8I (braille): Opening Nemeth Code indicators moved to follow the last word in each cell-5 heading; several extraneous punctuation marks and one extraneous code switch indicator deleted.

May 15, 2018: Second revision

Example 8.3-7 Changed; comment removed.

LESSON 9

February 1, 2018: *First upload*

May 15, 2018: First revision

Practice 9A: Note added to "Instructions" regarding end-of-sentence punctuation.

BRF: Practice 9B: Added transcriber's note referring to notes at end of lesson.

9.7.1: Description of last mini-example clarified.

LESSON 10

February 1, 2018: *First upload*

May 15, 2018: First revision

BRF: Practices 10A, C: Added transcriber's note referring to notes at end of lesson.

10.2: Second sentence deleted.

BRF: 10.5: Opening switch indicator corrected. Examples changed to two columns to match print copy. Pagination changes throughout.

10.15: Second sentence repaired.

Third mini-example before 10.29: Stroke direction changed to be upper left to lower right. BRF: Transcriber's note added to explain the three print cancellation strokes.

Example 10.29-3: Misplaced carried numbers corrected in the print example.

10.34: Question box inserted regarding BANA ruling about opening switch indicator.

Example 10.37-1: Punctuation indicator inserted before period in identifier 2.

LESSON 11

May 15, 2018: First upload

August 28, 2018: First revision

11.23.1: "as a square shape" changed to "as square shapes".

Practice 11G, Instructions: Wording of standard statement changed to "Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision,* 2007-2015 Updates including the *Guidance for Transcription Using the Nemeth Code within UEB Contexts."*

Practice 11G, Braille: same as above.

LESSON 12

May 15, 2018: First upload

August 28, 2018: First revision

Practice 12I, Instructions: Wording of standard statement changed to "Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision,* 2007-2015 Updates including the *Guidance for Transcription Using the Nemeth Code within UEB Contexts."*

Practice 12I, Braille: same as above.

LESSON 13

May 15, 2018: First upload

August 28, 2018: First revision

13.1.2 Deleted the sentence "List Nemeth Code symbols under a separate cell-5 heading." Sample Special Symbols page changed to comply with new guidelines.

LESSONS 14-15: *June 25, 2018: First upload*

LESSONS 16-17: *July 17, 2018*: *First upload*

LESSON 18

July 17, 2018: *First upload*

August 28, 2018: First revision

18.4.1 Special Symbols Page: Revised to comply with newest guidelines.

18.4.2 Transcriber's Notes Page: Revised to comply with newest guidelines.

Practice 18A: Special symbols page and transcriber's notes page revised to comply with newest guidelines.

APPENDIX A

May 15, 2018: First upload (partial)

June 25, 2018: Second upload (complete)

Corrections made to readings 3, 5, 8, and 11 (print) and readings 8, 9, 11, 12, and 13 (braille).

APPENDIX B: *May 15, 2018: First upload*

APPENDIX C

July 24, 2017: First upload (through Lesson 8)

February 1, 2018: *Second upload (through Lesson 10)*

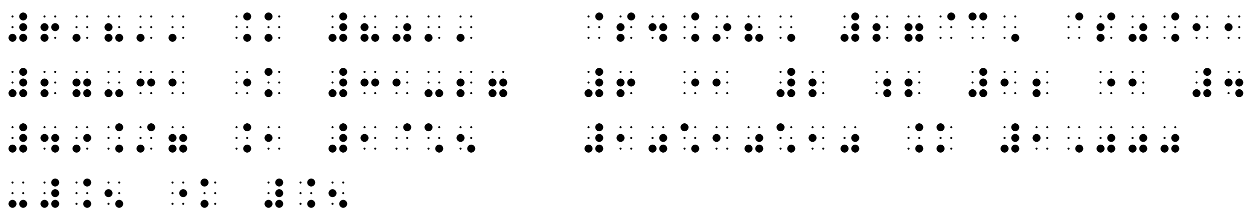
May 15, 2018: Final upload (all lessons)

APPENDIX A — READING PRACTICE

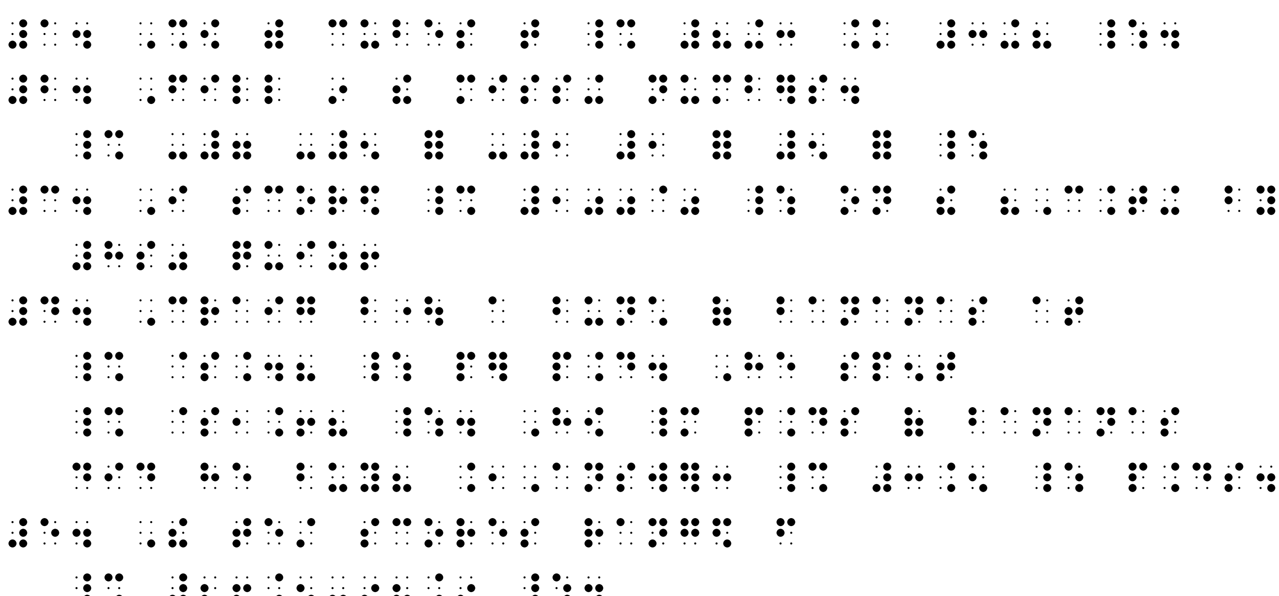
Proofreading your work will improve as you become more proficient at recognizing the braille symbols. The exercises in this appendix will develop your braille reading skills. After studying each lesson, write the back translation of the simulated braille. Check your interpretation by comparing to the print version found in the second half of this appendix.

These readings contain only a few of the topics covered in the course. You can gain more reading practice by back translating the answers to practice material at the end of each lesson.

Introductory Lesson



Lesson 1



Lesson 2

Lesson 3

Lesson 4

Lesson 5

Lesson 6

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Lesson 7

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Lesson 8

Lesson 9

Lesson 10

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Lesson 11

Lesson 12

Lesson 13

Lesson 14

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There is no reading practice for Lesson 15.

Lesson 16

Lesson 17

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There are no reading practices for the remaining lessons.

ANSWERS

Introductory Lesson

$6'8'' = 80''$ $\$4.98,\ 27¢,\ \$0.11$

$27 - 31 < 31 - 27$ $6{:}\,2 :: 12{:}\,4$

$49 \div 7 > 1 \times 5$ $10 \cdot 10 \cdot 10 = 1{,}000$

$-.5 < .5$

Lesson 1

1. Show with cubes that $8 + 3 = 3 + 8$.
2. Fill in the missing numbers. $-7 \quad -5 \quad \underline{\ \ ?\ \ } \quad -1 \quad 1 \quad \underline{\ \ ?\ \ } \quad 5 \quad \underline{\ \ ?\ \ }$
3. I scored 100% on the "Counting By 8s" quiz!
4. Craig bought a bunch of bananas at \$.48 per pound. He spent \$1.68. How many pounds of bananas did he buy? *Answer:* 3.5 pounds.
5. The test scores ranged from 26.5-98.9.

Lesson 2

1) On a number line, the <u>distance</u> from 0 to –3 is its <u>absolute</u> <u>value</u>—that is, $|-3| = 3$.
2) What is $|-13|$? $-(-13) = +13$ because two $-$'s make a $+$.
3) $[(3 + 2) \times (6 - 4) + 2] \times 4 = [(5 \times 2) + 2] \times 4 = 48$
4) Complete the number series: .25, .5, .75, ___, ___.

Lesson 3

Since 1 yr = 52 weeks, how many weeks are there in 2 yrs?

This is the set of children wearing red socks today: {Chloe, Oliver, Charlie}.

Ava counted 7 drops in 1.5 sec. How many drops will fall in 1.5 min?

Here are the commutative properties of addition and multiplication expressed using a and b: $a + b = b + a$ and $a \cdot b = b$ or $ab = ba$.

The area of rectangle PQRS is 4.5 sq. m. If side PQ is 3 cm, how long is side QR?

Lesson 4

i) The area of an ellipse, expressed as "A": $A = \pi ab$.

ii) Point (5, 7) is on ray ST.

iii) Power set notation may use the "Weierstrass P" as in P(S). If $S = \{\ \}$ then P(S)= {{ }} is returned.

iv) The hexadecimal system uses symbols 0-9 and A-F. For example, 45,997 in base 10 is B3AD in base 16.

TN Letters representing hexadecimal digits are capitalized in print. TN

Lesson 5

$\sim p \vee q$ is spoken: "not p or q (or both)".

Integer division is sometimes denoted \, as illustrated here: $10/3 = 3 + 1/3$, so $10\backslash 3 = 3$. The remainder is not noted.

Graph this inequality. $y \leq x + 2$

Use a number line to explain why $-6 - -6 = 0$. Is this the same as $-6 - +6$?

1 # (2 & 3) = (1 # 2) & (1 # 3)

Lesson 6

Isotopes

Hydrogen		${}^{1}_{1}H$
Uranium		${}^{238}_{92}U$

Cations

Sodium	Na^{+}	${}^{23}_{11}Na^{+}$
Aluminum	Al^{3+}	${}^{27}_{13}Al^{3+}$

Anions

Iodine	I^{-}	${}^{127}_{53}I^{-}$
Oxygen	O^{2-}	${}^{16}_{8}O^{2-}$

Lesson 7

1. Several parallel β-sheets form a left-handed β-helix.

2. In the study of logic, use of the boldface equality sign $\boldsymbol{=}$ avoids unintended mixups with the standard equals sign.

3. Which symbol denotes the set of natural numbers?

   a. $\mathbb{Z}$

   b. $\mathbb{N}$

   c. $\mathbb{R}$

4. Is there a vector $\mathbf{s}$ such that $\mathbf{r}+\mathbf{s}=\mathbf{t}$?

Lesson 8

To simplify a radical expression, use the product and quotient properties of radicals,

$$\sqrt{xy} = \sqrt{x}\cdot\sqrt{y} \text{ and } \sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

as demonstrated below.

$$\sqrt{16x} = \sqrt{16}\cdot\sqrt{x} = \sqrt{4^2}\cdot\sqrt{x} = 4\sqrt{x}$$

$$\sqrt{\frac{25}{16}\,x^2} = \frac{\sqrt{25}}{\sqrt{16}}\cdot\sqrt{x^2} = \frac{5}{4}x$$

$$\sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{\sqrt{16}} = \frac{\sqrt{15}}{4}$$

Lesson 9

Prove that S is a subspace of V if and only if:

(1) $\mathrm{v},\mathrm{w} \in \mathrm{S} \Rightarrow \mathrm{v}+\mathrm{w} \in \mathrm{S}$

(2) $\lambda \in \mathrm{K}, \mathrm{v} \in \mathrm{S} \rightarrow \lambda\cdot\mathrm{v} \in \mathrm{S}$

Find the mistake:

$$\begin{array}{r} \overset{4}{\cancel{5}}\,\overset{13}{\cancel{3}}35 \\ -4\,925 \\ \hline 1\,4\,1\,0 \end{array}$$

Lesson 10

The division problem $4.2\overline{)3313.8}$ is demonstrated below.

$$\begin{array}{r} 789.0 \\ 4.2_{\wedge}\overline{)3313.8_{\wedge}0} \\ \underline{294} \\ 373 \\ \underline{336} \\ 378 \\ \underline{378} \\ 0 \end{array}$$

Fill in the missing digits.

$$\begin{array}{r} 197 \\ \underline{\times 76} \\ 11\square 2 \\ \underline{1\square 79} \\ 1\square 9\square 2 \end{array}$$

Note: Since the braille general omission symbol doesn't specify what is used in print, your translation may show a different omission sign.

Lesson 11

1. Calculate the total cost of an item selling for $8.79 with 5.5% tax added.

   8 $\boxed{.}$ 7 9 $\boxed{+}$ 5 $\boxed{.}$ 5 $\boxed{\%}$ $\boxed{=}$

2. $\angle ABD + \angle DBE = ?$
3. $[a, b] \ominus [c, d]$
4. $\{\triangle, \Diamond, \bigcirc\} \cup \{\square\}$
5. Add $\angle \underline{30°}$ and $\angle \underline{20°}$.

Lesson 12

(i) Rewrite as a simplified fraction.

$0.\overline{3}$

$3.1\overline{6}$

$2.\overline{18}$

(ii) Versor $\hat{\mathbf{u}}$ of a nonzero vector **u** is $\hat{\mathbf{u}} = \dfrac{\mathbf{u}}{|\mathbf{u}|}$ where $|\mathbf{u}|$ is the length of **u**.

Lesson 13

Compute the numbers indicated by the factorial notation.

1. $\dfrac{6!}{3!2!}$

2. $7!$

3. $\dfrac{10!}{5!5!}$

Here is a partial integration solution, using ψ[1]. $f(x,y) = \int M(x,y)\partial x$ $= \int (2xy^2 + x^2 - y)\partial x$, therefore $f(x,y) = x^2y^2 + \frac{1}{3}x^3 - xy + \psi(y)$.

[1] Greek letter psi.

Lesson 14

Find the square root of 484.

$$
\begin{array}{r|l}
 & \ \ 22 \\
 & \sqrt{484} \\
 & \ 4 \\
\cline{2-2}
42 & 084 \\
\times 2 & \ \ 84 \\
\cline{2-2}
 & \ \ \ \ 0
\end{array}
$$

Since the remainder is 0, $\sqrt{484} = 22$.

Provide your answers to the following problems on a separate sheet of paper.

1. Find the value of $\sin\frac{5\pi}{12}$.
2. What is Arc csc $\left(-\sqrt{2}\right)$?
3. Simplify: $e^{\log_e e}$
4. Prove this reduction formula : $\tan(90° + \theta) = -\cot\theta$.
5. Is the following identity true? $\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}$

Lesson 16

8) $$\begin{aligned} 9\frac{1}{2} &= \cancel{9\frac{4}{8}}^{\,8\frac{12}{8}} \\ -4\frac{7}{8} &= 4\frac{7}{8} \\ \hline & \quad 4\frac{5}{8} \end{aligned}$$

9) If $\frac{1}{2} \div \frac{1}{3} = \frac{1/2}{1/3}$, express $\frac{\frac{1/2}{1/3}}{11}$ using ÷ symbols.

9) If $\frac{1}{2} \div \frac{1}{3} = \frac{1/2}{1/3}$, express $\frac{\frac{1/2}{1/3}}{11}$ using ÷ symbols.

Lesson 17

Scalar Multiplication Multiply the matrix by the scalar "2", like this:

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}.$$

The second matrix above is the result of four calculations: $2 \times 4 = \mathbf{8}$, $2 \times 0 = \mathbf{0}$, $2 \times 1 = \mathbf{2}$, and $2 \times -9 = \mathbf{-18}$.

The system $\begin{pmatrix} x + 2y = 8 \\ 2x - 3y = 2 \end{pmatrix}$ has the solution set $\{(4,\ 2)\}$.

APPENDIX B — GLOSSARY OF TERMS

This glossary contains a sampling of terminology encountered in K-12 mathematics which transcribers may find useful when communicating in workshops, forums, and blogs. There are many dictionaries of mathematical terms for those interested in a more comprehensive glossary.

Mathematical expression: A combination of mathematical symbols grouped together to designate the value of something.

Numerical expression: A mathematical expression containing only numbers and operation symbols. EXAMPLE $14 \div 2$

Algebraic expression: A mathematical expression containing variables, numbers, and operation symbols. EXAMPLE $3x + 2y - 7$

The Four Basic Operations

Addition

2	addend
+3	addend
5	sum

Subtraction

7	minuend
−1	subtrahend
6	difference

Multiplication

2	multiplicand
×3	multiplier
6	product

5.009	multiplicand
× .27	multiplier
35063	partial product
10018	partial product
1,352.43	product

Division dividend ÷ divisor = quotient

$$10\overline{)40}^{\;4}$$

$$\text{divisor}\overline{)\text{dividend}}^{\;\text{quotient}}$$

$$9\overline{)40}^{\;4\text{ r}4}$$
the "r" number is the remainder

36	partial quotient
4	difference

Many Ways to Denote Multiplication

Print has several ways of denoting multiplication. Nemeth Code provides symbols for all of them, which enables us to follow print. Print may space between items for clarity, but factors are not spaced in braille unless abbreviations, function names, or certain symbols are contained in the expression.

All of the following mean "two times y".

$2 \times y$	⠼⠆⠈⠡⠽	(a)
$2 \cdot y$	⠼⠆⠡⠽	(b)
$2 * y$	⠼⠆⠈⠼⠽	(c)
$2y$	⠼⠆⠽	(d)
$(2)(y)$	⠷⠆⠾⠷⠽⠾	(e)

(a) 2×3 The multiplication cross is used the most in the lower grades.

(b) $2 \cdot 3$ The multiplication dot is used when the "x" variable is introduced, since the x and the multiplication cross can be confused in print.

(c) $2 * 3$ The multiplication asterisk is used most often in calculator operations since, on calculator keys, the multiplication cross may be confused with the variable x and the multiplication dot may be confused with the decimal.

(d) $2y$ This is the form you will see most often when one of the factors is a variable, a constant, or a radical expression. $2xy$ means "two time x times y". 2π means "two times pi". $2\sqrt{5}$ means "two times the square root of 5".

(e) (2)(y) Grouping symbols are used to group factors when the multipliers require calcuations.

EXAMPLE $(x+y)(x-y)$ means "multiply $x+y$ times $x-y$".

If a factor uses parentheses, brackets group the outer factor.

EXAMPLE $5[(x+y)(x-y)]$ means "five times the product of $x+y$ times $x-y$".

The expression $(3\sqrt{2})(5\sqrt{3}) + (3\sqrt{2})(2\sqrt{5})$ can be analyzed as follows.

- The overall expression is the sum (addition) of two terms. Addition is indicated with a plus sign. __ + __
- Each addend is the product (multiplication) of two factors, indicated with side-by-side parentheses. (__)(__)
- Each multiplier is the product of a numeral and a square root. No multiplication sign is shown but multiplication is implied. $\_\sqrt{\_}$

$(3\sqrt{2})(5\sqrt{3}) + (3\sqrt{2})(2\sqrt{5})$

Spoken: "(Three times the square root of two TIMES five times the square root of three) PLUS (three times the square root of two TIMES two times the square root of five)."

Transcription: The braille transcription contains no spaces.

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Fractions

A fraction consists of a *numerator* and a *denominator* separated by a *fraction line*.

Simple Fraction

This fraction has a horizontal fraction line.

$\frac{3}{4}$ numerator / denominator

This fraction has a diagonal fraction line. ${}^{3}/_{4}$

Mixed Number

A mixed number is one quantity made up of a whole number and a simple fraction.

$7\frac{1}{2}$

Complex Fraction

A complex fraction's numerator and/or denominator is also a fraction.

$\frac{{}^{1}/_{2}}{4}$ numerator / denominator $\frac{4}{{}^{1}/_{2}}$

Geometry

Irregular polygon: A closed figure with at least two unequal sides and two unequal angles

Regular polygon: A closed figure with equal sides and equal angles

Triangles

Acute Triangle: All angles are less than 90°

Equilateral Triangle: Three equal sides; three equal angles (always 60°)

Isosceles Triangle: Two equal sides; two equal angles

Obtuse Triangle: Has one angle greater than 90°

Right Triangle: Contains one right angle (90°)

Scalene Triangle: No equal sides; no equal angles

Radical Expressions

$$\overset{12}{\sqrt{144}} \qquad \overset{\text{root}}{\sqrt{\text{radicand}}}$$

Radical sign: √

Vinculum: The line which extends above the radicand.

Index: A small number that may appear next to the radical sign is the index of the radical.

$\sqrt[3]{9}$ 3 is the index of this radical.

Statements and Functions

Exponential statement:	$b^e = x$	b is the base; e is the exponent.
Logarithmic statement:	$\log_b x = y$	x is the argument of the log.
Trigonometric function:	$\sin\theta = 1$	θ (theta) is the argument of the function.

Algebra

The following algebraic expression is used as a sample in the definitions below. $3x + 2y - 7$

Coefficient: When a term is made up of a constant placed before a variable, the constant is called a coefficient. In the sample expression, the coefficient of *x* is 3. The coefficient of *y* is 2.

Constant: A constant is a number that is fixed and known, or a letter which stands for a fixed number, such as π. The constants in the sample expression are 3, 2, and 7.

Term: An algebraic expression is made up of terms. Each term is separated by a + or a – sign. The terms in the sample expression are 3*x*, 2*y*, and 7.

Variable: A variable is a symbol for an unknown number, usually a letter. The variables in the sample expression are *x* and *y*.

Other Terminology Used With Algebraic Expressions

Binomial: An algebraic expression consisting of the sum or the difference of two terms is a binomial. EXAMPLE 3x + 9

Degree: The degree of a term is the sum of the term's exponents. For example, the degree of the term $2y^3$ is 3. The degree of the term $16x^2y^3$ is 5.

Equality: An equality consists of two expressions which have the same value. EXAMPLE A = B

Equation: An equation is an equality which contains at least one variable. EXAMPLE $x + 1 = 4$

The process of finding out the variable value that makes the equation true is called *solving* the equation.

Equivalent equations: Two equations that have the same solution are called equivalent equations. EXAMPLE $x = 1$ and $2x = 2$.

Exponent: In x^2, the exponent is 2.

Factor: Factors are numbers which, when multiplied together, result in another number. A number can have many factors. For example, 3 and 4 are factors of 12 because $3 \times 4 = 12$. 2 and 6 are also factors of 12 because $2 \times 6 = 12$.

Inequality: An inequality consists of two expressions, one on each side of a comparison sign that is not an equals sign. EXAMPLES $3x < 10$ and $a \leq b$.

Like terms: Like terms are terms which have the same variable raised to the same exponent. For example, $3x^2$ and $9x^2$ are like terms.

Monomial: An algebraic expression consisting of one term is a monomial. Monomials include numbers, whole numbers and variables that are multiplied together, and variables that are multiplied together. EXAMPLE $3xy$

Polynomial: An algebraic expression consisting of more than two terms is a polynomial. The sample expression $3x + 2y - 7$ is a polynomial. A polynomial may also include a term with exponents. EXAMPLE $x^2 - 4x + 7$.

APPENDIX C
NEMETH CODE FORMAT SUMMARIES

- General Principles
- "Keep Together"
- Margins
- Itemized Material
- Displayed Mathematical Material
- Mathematical Statements
- Formal Proofs
- Division of Mathematical Expressions Between Braille Lines
- Placement of Code Switch Indicators
- Spatial Arrangements

"Formatting" refers to layout on the page, such as indentations (margins), line spacing (blank lines), centering, and pagination. The Nemeth Code specifies certain formats which are summarized in this appendix. For illustrative examples, go to the sections in this lesson material which are cited as bold numbers in brackets.

General Principles

When an item in a UEB transcription requires the use of Nemeth Code symbols, format rules of *The Nemeth Braille Code for Mathematics and Science Notation* are to be applied to the entire transcription including those portions transcribed in UEB. When a format is not specifically addressed in the Nemeth Code, the principles provided in *Braille Formats Principles of Print-to-Braille Transcription* should be followed. [**Preliminary Lesson, Section P7**]

"Keep Together"

A mathematical expression that will fit on one braille line within the current margins must not be divided between lines. The entire expression is brought down to the next line. [**1.5**] If a page number on line 25 or line 1 does not allow the entire expression to fit on the line, the expression is brought down to the next line that has enough usable cells. [**3.7**]

Within a paragraph, a code switch indicator and/or terminator should appear on the same line as the expression to which it applies, if there is room on the line. [**1.5.1**]

A hyphenated expression containing one or more mathematical components must not be divided between braille lines. [**2.19**]

An abbreviation and a preceding or following numeral or letter to which it applies must not be divided between braille lines. [**3.4.1, 4.1.1**]

Items in an enclosed list must not be divided between braille lines if the entire list will fit on a single braille line. If the enclosed list will not fit on a single braille line, use as much of the current line as possible and begin a runover line after a comma. [**4.17.3**]

The components of the following symbols must not be divided between braille lines: a symbol of operation using plus and minus (**Lesson 5**); a symbol of comparison compounded vertically or horizontally (**Lesson 5**); a shape symbol with structural or interior modification (**Lesson 11**); the the components of an expression modified according to the five-step rule. (**Lesson 12**); superposed symbols (**Lesson 13**); a two-part function name (**Lesson 14**).

A fraction must not be divided unless the entire fraction will not fit on the line. [**8.3.b**]

A mixed number must not be divided from its fractional part. [**8.5.1.b**]

A single keystroke construction must not be divided between braille lines. [**11.25**]

A sign of shape and the letter, sequence of letters, or numeral which follows it is regarded as a single mathematical item and therefore should not be divided between braille lines. [**11.31.1**]

Tally marks belonging to the same group must not be divided between braille lines. [**13.4.a**]

A function name or an abbreviated function name and the sign which follows it must not be divided between braille lines. [**14.5**]

Unless unavoidable, items enclosed within grouping signs should not be divided between lines. [**15.2**]

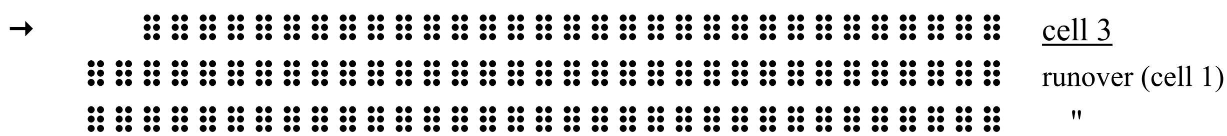
Margins

For margins with spatial material, see the section on Spatial Arrangements at the end of this Appendix.

Nemeth formatting can be broken down into two major styles: indented and hanging. In the indented styles, the first line is indented two cells to the right of the runover margin. In the hanging styles, the first line begins two cells or four cells to the left of the runover margin.

Narrative

3-to-1 Narrative paragraphs in a document governed by Nemeth formatting begin in cell 3 with runovers in cell 1 (indented style). Blocked paragraphing is not used in a Nemeth transcription. [**1.14**]



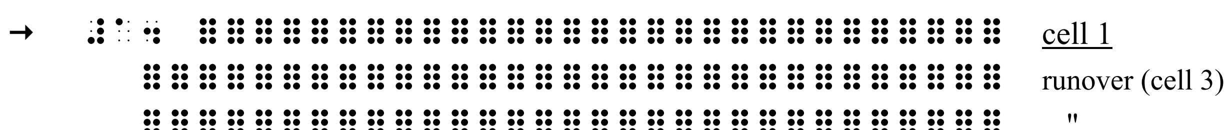
Directions

3-to-1 If instructional text is not followed by lettered or numbered exercise material, the text is considered to be a narrative paragraph and begins in cell 3 with runovers beginning in cell 1 (indented style). The (5-5) style of “directions” given in *Braille Formats* does not apply in a transcription that contains Nemeth. [**5.12.1**]

Margins: Itemized Material

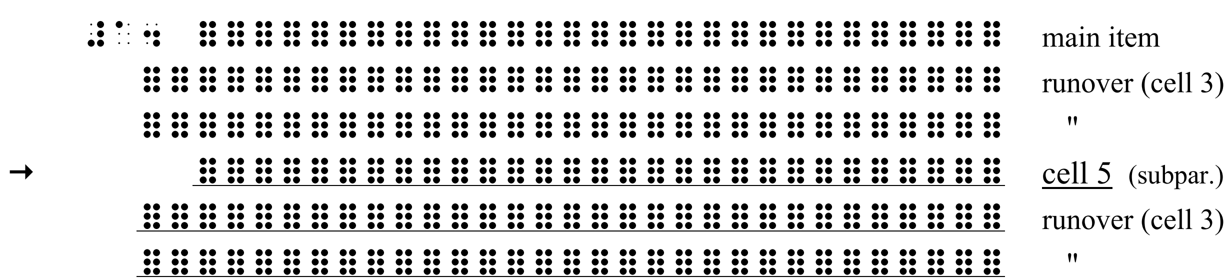
Main Item (no subdivisions)

1-to-3 The identifier begins in cell 1; runovers begin in cell 3 (hanging style). [**1.16**]



Subparagraph of a Main Item (no subdivisions)

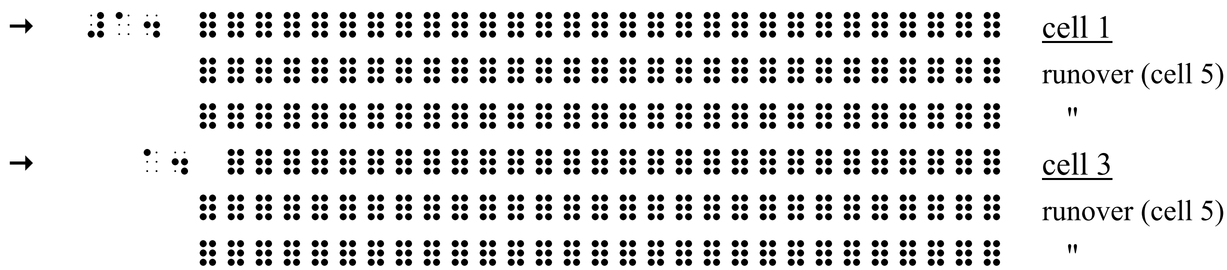
5-to-3 If a main item contains more than one paragraph, each subparagraph begins in cell 5 with runovers in cell 3 (indented style). [**1.16**]



When the print copy arranges nonspatial itemized material side by side across the page, the braille format is changed so that all identifiers start in the same cell. [**2.20, 6.1.2**]

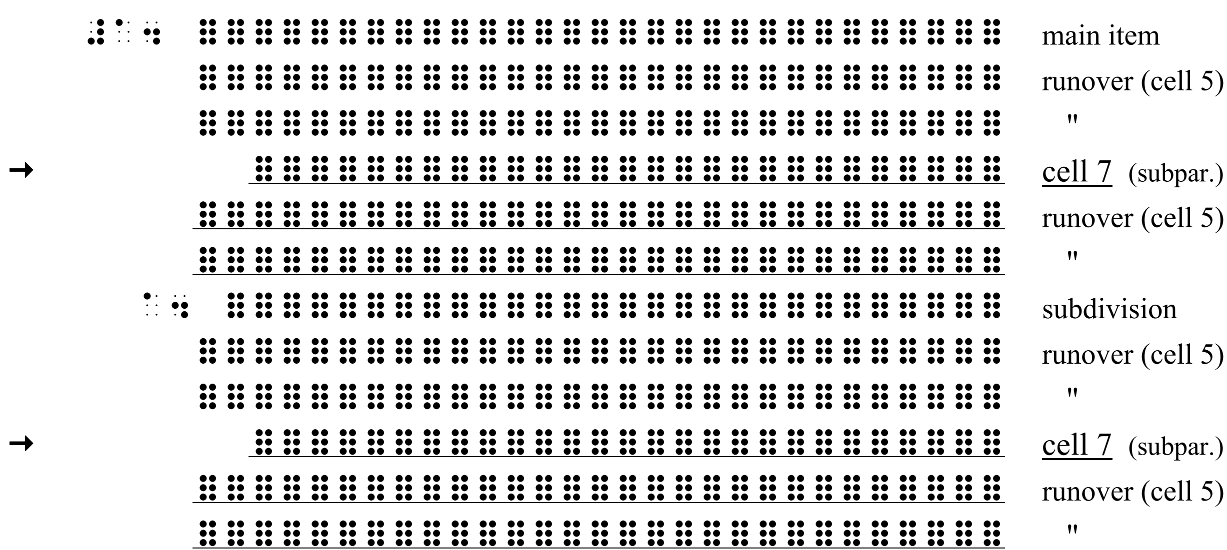
Main Item with Subdivision

1-to-5, 3-to-5 The main item designation begins in cell 1, and its runovers begin in cell 5. Each lettered or numbered subdivision—regardless of print indentation—begin in cell 3, with any runovers in cell 5 (hanging styles). [**6.1**]



Subparagraph of Items with Subdivision

7-to-5 If a main item or a subdivision has more than one paragraph, each new paragraph begins in cell 7, and its runovers begin in cell 5 (indented style). [**6.1.1**]



When itemized material is arranged in tabular form so that items are numbered at the margin and subdivisions are aligned beneath lettered column headings, the material is transcribed in one of the following ways: [**6.1.3**]

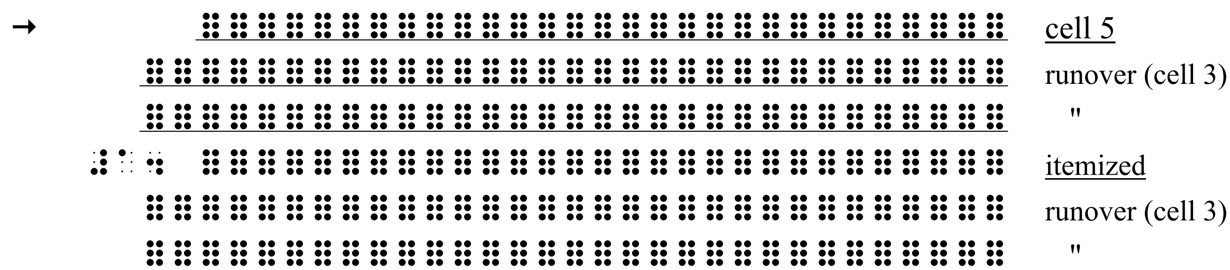
- If all the columns can be accommodated across the braille page, the print columnar arrangement is followed. Each problem number begins in cell 1. The letter identifying each column is aligned with the first cell of the related column. A blank line is left above and below the lettered column headings. Two blank cells separate the columns.
- If all the columns cannot be accommodated across the braille page, each subdivision in each problem must be lettered individually, and the (1-5; 3-5) format is followed.

Exercise Set

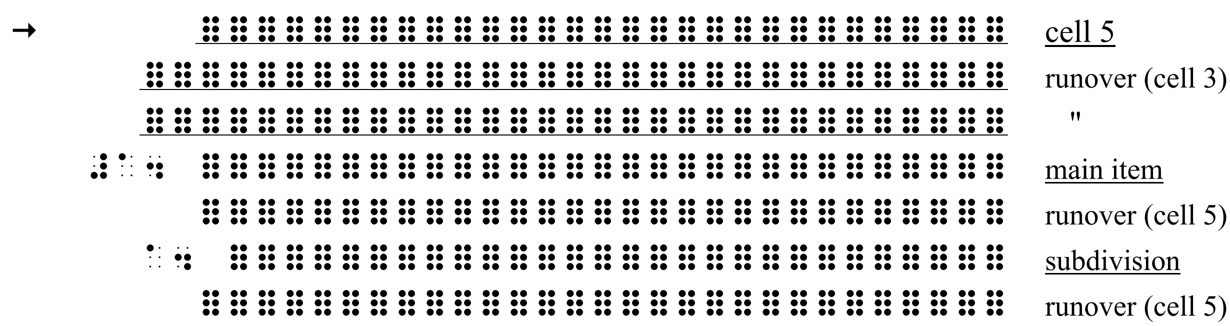
Runover margins for itemized material are determined individually for each question. A numbered or lettered problem with no subdivisions will be (1-3); the next numbered or lettered problem may have subdivisions and so will be (1-5; 3-5), etc. [**6.1.4**]

Instructions

5-to-3 When a group of numbered or lettered problems is preceded by instructions, the instructions begin in cell 5, with runovers in cell 3 (indented style). One line is left blank above instructions unless the instructions follow a cell-5 or a cell-7 heading. Instructions may begin on line 1 of the braille page if no running head is in use. The related itemized material follows on the next line unless the material itself requires a blank line before it. It is preferable to keep instructions on the same braille page with the exercise. To accomplish this, instructions may need to be moved to the next braille page. However, when there is not sufficient space on that page for the instructions and part of the exercise, instructions may be placed on the preceding page. [**5.11**]



OR



Margins: Displayed Mathematical Material

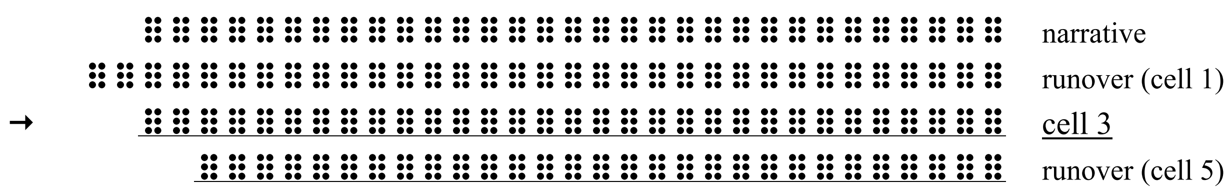
When mathematical material is set apart from the body of the text in the print copy, it is referred to as a displayed expression. Displayed math uses a hanging style. The margins depend upon the layout of the surrounding text. The first cell of the displayed material is indented two cells to the right of the runover cell of the preceding material, whether or not a runover is actually present. Runovers of the displayed math are usually indented two cells further. [**7.1**]

A line is not skipped above or below displayed mathematical material unless the preceding or following material requires a blank line. [**7.1**]

When a number or letter is used to label a displayed mathematical expression it is placed at the left of the expression in braille regardless of the location of the label in the print copy. The label begins in the appropriate cell for displayed material. [**7.2.1**]

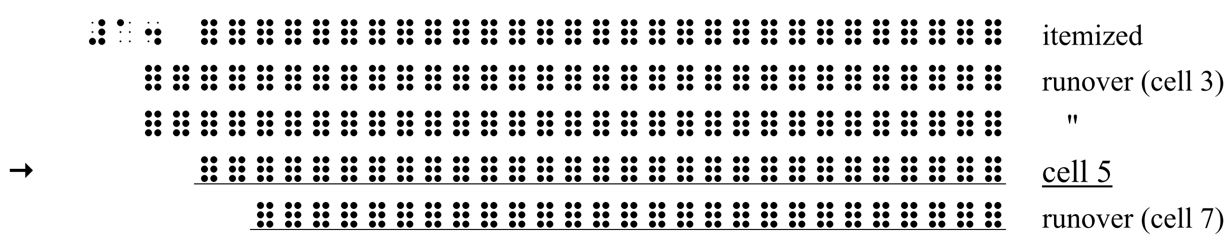
Math Displayed to Narrative

3-to-5 In unitemized explanatory portions of the text, displayed mathematical material begins in cell 3. Runovers begin in cell 5 (hanging style). [Text (3-1); displayed material (3-5)]



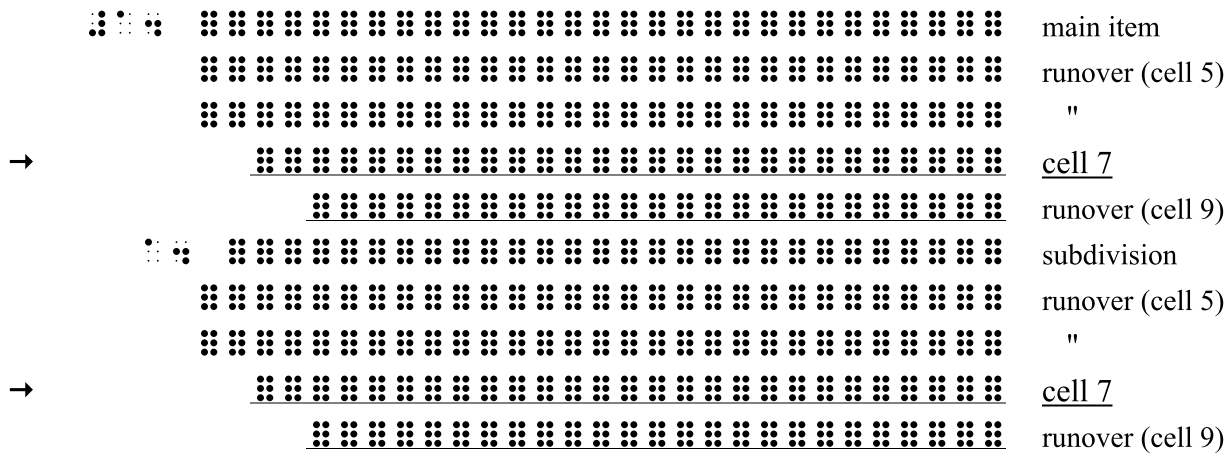
Math Displayed to Itemized Text

5-to-7 In itemized text without subdivisions, displayed mathematical material begins in cell 5. Runovers begin in cell 7 (hanging style). [Text (1-3); displayed material (5-7)]



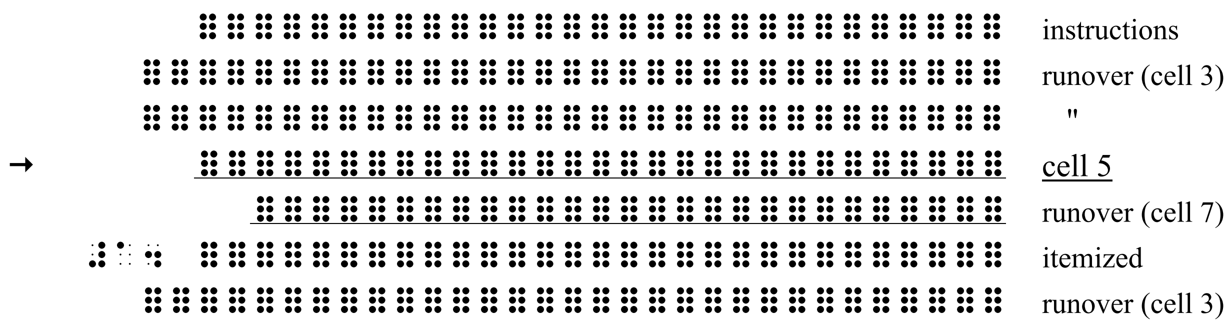
Math Displayed to Itemized Text with Subdivisions

7-to-9 In itemized text with subdivisions, displayed mathematical material begins in cell 7. Runovers begin in cell 9 (hanging style). [Main division text (1-5); displayed material (7-9). Subdivision text (3-5); displayed material (7-9)]



Math Displayed to Instructions

5-to-7 Within or following instructions, displayed mathematical material begins in cell 5. Runovers begin in cell 7 (hanging style). [Instructions (5-3); displayed material (5-7)] [**7.1.5**]



Nested Linked Expressions

A nested linked expression, defined in **8.22**, can occur in one of the following displayed layouts. Note that the first cell of the anchor is indented two cells to the right of the runover cell of the preceding material.

- **In Narrative.** When a nested linked expression is displayed to (3-1) unitemized explanatory portions of text, the anchor begins in cell 3. If the anchor has runovers, they begin in cell 7. Each link begins in cell 5. If a link has runovers, they also begin in cell 7. [**8.22.2**]
- **In Itemized Text Without Subdivisions.** When a nested linked expression is displayed to (1-3) itemized text containing no subdivisions, the anchor begins in cell 5. If the anchor has runovers, they begin in cell 9. Each link begins in cell 7. If a link has runovers, they also begin in cell 9. [**8.22.3.a**]

- **In Itemized Text With Subdivisions.** When a nested linked expression is displayed to itemized text containing subdivisions (1-5; 3-5), the anchor begins in cell 7. If the anchor has runovers, they begin in cell 11. Each link begins in cell 9. If a link has runovers, they also begin in cell 11. [**8.22.3.b**]
- **In Itemized Text With No Narrative.** When a nested linked expression follows an identifier with no intervening narrative, the anchor is placed on the same line as the identifier. Each link begins a new line, two cells to the right of the cell in which the identifier begins. Runovers are indented two cells further—that is, four cells to the right of the cell in which the identifier begins. [**8.22.4**]

Mathematical Statements and Proofs

A line is left blank before the beginning and after the end of a mathematical statement or a proof. Normal paragraphing (3-1) is applied. The label is treated as a paragraph heading. [**11.38**]

If a mathematical statement or a proof contains auxiliary captions such as *Given*, *Prove*, or *Conclusion*, etc., such captions begin a new paragraph in cell 3 with runovers in cell 1. A line is not skipped above a caption. Capitalization and typeform follows print, but if fully capitalized and also in a nonregular typeform, capitalization is retained and typeform is disregarded. [**12.16.d**]

Formal Proof in Two Columns

When a formal proof is printed in step-number form and divided into two columns, follow the format described in **12.16.1**.

Division of Mathematical Expressions Between Braille Lines

When a mathematical expression is too long to fit on one braille line within the current margins the expression is divided between braille lines according to the rules of the Nemeth Code. A new line need not be forced if there is room on the line to begin the expression, provided the division is made in accordance with the principles defined below. Runovers conform to the margin requirements currently in effect – be it itemized, instructional, explanatory, labeled, subdivided, or displayed material.

Long Numeral

A long numeral is divided after a comma if a comma is present, and a hyphen is inserted. If the numeral does not contain a comma, the hyphen may be inserted after any digit. When a numeral is divided between braille lines, the numeric indicator is used before the first digit of the numeral on the next line. [**1.7.1**]

Enclosed List

If an enclosed list will not fit on a single braille line, use as much of the current line as possible and begin a runover line after a comma. [**4.17.3.a**]

Linked Expressions

If a linked expression is too long to fit on one line, the expression continues on the next line, beginning with a sign of comparison. If the expression contains more than one link and the anchor with all links will fit on one line, do not divide it. If all links will not fit, use as much as the line as possible before dividing the expression. The new line begins with a link, placed in the runover cell of the current format. [**8.21**]

The comparison sign at which the new line begins must be on the baseline. An expression should not be divided before a comparison sign that is part of an item enclosed in grouping symbols, between fraction indicators, or within radical signs. [**8.21.1**]

A transition to a new braille line made before a sign of comparison terminates the effect of any level indicator used on the line above. [**8.21.2**]

Mathematical Expressions

Mathematical expressions which will not fit on one braille line within the boundaries of the current margins can be organized into a series of mathematical units in order to choose runover sites. The strategies are studied in Lesson 15, roughly outlined as follows. (i) Divide before a comparison sign on the baseline [**15.3**]; (ii) divide before an operation sign on the baseline [**15.4**]; (iii) divide before a mathematical unit [**15.5**]; (iv) divide after a termination indicator [**15.6**].

Placement of Code Switch Indicators

When mathematical content occurs anywhere in a UEB transcription, the nontechnical notation follows the rules of *Unified English Braille* and the technical notation follows the rules of the *Nemeth Braille Code*. Readers will assume they are reading UEB unless signaled otherwise by the use of a UEB code switch indicator, in this case, the opening Nemeth Code indicator. Between the opening Nemeth Code indicator and the Nemeth Code terminator are Nemeth symbols, following Nemeth rules. UEB symbols are not used within the Nemeth Code switch indicators. [**1.1**] The objective within a paragraph is to keep the switch indicators on the same line as the mathematics to which they apply. Displayed material, spatial arrangements, and tables have other considerations. There are guidelines to follow when the switch indicators do not fall neatly on a line or on a page.

General Principle in Narrative Context

The opening Nemeth Code indicator is followed by a space (unless it ends a line). The Nemeth Code terminator is preceded by a space (unless it begins a line). These spaces do not represent spaces in print. [**1.1**] Within a paragraph, a switch indicator should not stand alone on a line if there is room for it to fall on the line with the math expression to which it applies. [**1.5.1.b**] If two or more math expressions are transcribed between the same code switch indicators, the line may wrap at the space between the expressions even if the entire Nemeth portion could fit on one line. [**1.5.1.a**]

Switch Indicators with Itemized Material

Identifiers are transcribed according to the rules for the code in use at the time. All identifiers in a section do not need to be transcribed in the same code. [**2.18.2**] When at least the first two items require Nemeth, the opening Nemeth Code indicator is placed at the end of the line of text that precedes the itemized material. If there is no room on that line, the opening Nemeth Code indicator is placed in the runover position of the narrative. [**2.18.1**] This placement may be applied to a heading that precedes the identified Nemeth material, centered heading excepted. [**4.25**] A code switch indicator does not take the place of the blank line that may be required preceding the itemized material. For further details regarding placement of switch indicators in a list of mixed items, see **2.18.2**.

Switch Indicators with Displayed Mathematical Material

When displayed mathematical material is both preceded and followed by UEB text, the expression and its two switch indicators may be placed all together on one line if they will fit within current margins. If more than one line is required for the expression, the opening Nemeth Code indicator is placed at the end of the text line preceding the displayed material and the Nemeth Code terminator is placed at the completion of the displayed expression. If either indicator will not fit on the current line, it is placed on the following line in the runover position. [**7.1.1**]

Switch Indicators with Spatial Arrangements

Code switch indicators are placed outside of the spatial material in order not to interfere with alignment. The blank line required before and after the arrangement are part of the spatial problem and so must be inside the Nemeth switches. The opening Nemeth Code indicator and the Nemeth Code terminator do not take the place of that required blank line. If there is not room for the opening Nemeth Code indicator at the end of the line with the preceding text, it is placed on the next line in cell 1. The required blank line is on the line following the opening switch. To close Nemeth after a spatial problem, first insert the required blank line, then place the Nemeth Code terminator in cell 1 by itself on the following line. [**9.29, 9.30**]

Switch Indicators May Stand Alone on a Line

If a math expression will fit on one line but there is not room for one or both of the switch indicators, one or both switch indicators may stand alone on a line. Keeping the mathematical expression intact on one line is the priority. [**1.5.2**]

Switch Indicators and Punctuation

Punctuation that relates to the main text is placed outside of the switch indicators when the surrounding text is in UEB. There is no space between the terminator and the following punctuation. [**1.4]** To avoid excessive code switching between mathematical items, punctuation which belongs to the sentence structure may be transcribed inside the switches. **[2.3**] Paired punctuation (parentheses, brackets, braces, quotation marks) are transcribed inside the code switches when they enclose technical material. **[2.13**]

Switch Indicators after a Heading

An opening Nemeth Code indicator may be placed at the end of a cell-5 or cell-7 heading. [**4.25.2**] An opening Nemeth Code indicator cannot be placed at the end of a centered heading. [**4.25.1**]

Switch Indicators and Transcriber's Notes

Transcriber's note indicators are UEB symbols and therefore must be transcribed outside of the Nemeth switches. When the note itself contains mathematical material, code switching occurs within the note. Nemeth Code must be terminated before the closing transcriber's note indicator is transcribed. When itemized or spatial mathematical material follows the transcriber's note, the opening Nemeth Code indicator may be placed following the closing transcriber's note indicator only if it fits on the same line. [**4.26**]

Switch Indicators at Page Turns

When Nemeth is in effect, Nemeth Code is not terminated by transition to a new braille page or across a page turn line. When code switching occurs at a braille page turn, the opening Nemeth Code indicator and the Nemeth Code terminator should appear on the same braille page as the expression to which they apply. The opening Nemeth Code indicator should not stand alone at the bottom of a braille page, nor should the Nemeth Code terminator stand alone at the top of a braille page. [**1.5.3, 3.7, 3.8**]

Switch Indicators with Boxed Material

Box lines may be transcribed in either code. When literary content is followed by boxed mathematical material, if all of the material in the box is in Nemeth, the opening Nemeth Code indicator may be placed at the beginning of the top box line, followed by a blank space and the Nemeth Code terminator may be placed at the end of the bottom box line, preceded by a space. [**18.6**]

Switch Indicators with Instructional Commentary

When instructional commentary alternates with math problems, switch indicators are used in order to transcribe the comments in contracted braille. When switching into or out of Nemeth before a change of margins, the switch indicators are placed after the last item of the line rather than at the beginning of the next line to maintain clarity in the indented margin pattern. [**16.11**] When comments or conditions occur outside of enlarged grouping symbols, code switching within the comment or condition is independent of the grouped spatial arrangement. [**17.10.1.b**]

Switch Indicators with Instructions

If instructions end with an expression in Nemeth and the subsequent math problem starts with Nemeth, Nemeth Code remains in effect between the end of the instructions and the start of the problem. [**5.11.1**]

Switch Indicators with Tables

When mathematical data occur in the table, code switching decisions depend upon the content of the entire table and the spacing restrictions encountered on the braille page. Each table must be individually assessed in order to determine the clearest representation in braille. [**Lesson 18**]

Column headings which contain words are transcribed in UEB. There may be items within the column headings that require switching to Nemeth. [**18.3**]

It is best if a minimum of code switching is encountered within the body of the table. When a mixture of narrative entries and mathematical data occur in a table, a switch to Nemeth may be applied only where needed. However, a table may be more clearly presented by transcribing it entirely in Nemeth, even when some entries do not require a switch. For example, when only one column requires Nemeth, the opening switch and the Nemeth terminator must be applied to each entry, however, spacing restrictions may make that option unmanageable. Instead, it may be better to transcribe the entire body of the table in Nemeth, including any words.

When the entire body of the table is transcribed in Nemeth, the opening switch indicator is placed in cell 1 of the line following the column separation line (if present), and the entries begin on the next line. The Nemeth Code terminator follows the last line of entries, placed in cell 1. [**18.4.2**] Words within the table, including row headings, are transcribed without contractions. If a row heading consists of one word, the single-word switch indicator is not used. **[18.5]**

Switch Indicators with Tactile Graphics

Nemeth remains in effect for a tactile graphic if the graphic intervenes between two items in Nemeth. If the preceding text is in UEB and if a switch to Nemeth must be made for the tactile graphic, the opening switch indicator is placed at the end of the preceding text or in cell 1 on the line before the required blank line. [**18.9**]

Spatial Arrangements

Details regarding various spatial arrangements are in the following lesson sections.