Cosmological Theory and Cosmogravity

The cosmology summary presented below focuses on the relationships that are used in the simulation code.

1 Friedmann-Lemaître models

Friedmann-Lemaître's models of the universe are based on the cosmological principle and general relativity.

The cosmological principle postulates that the universe is homogeneous and isotropic. Observations of galaxies seem to validate today this principle but at great distances ($d > 10^{24}$ m) and those of the cosmological microwave background radiation show that our universe was remarkably homogeneous and isotropic in density ($\delta\rho/\rho < 10^{-5}$) $\sim$ 380 000 years after the big bang.

General relativity is a relativistic geometric theory of gravitation (RGTG), i.e. a theory which relates the geometry of the space-time container to its matter-energy content and which, locally, joins restricted relativity.

In a RGTG the symmetry of the content must therefore be found in the container. It is shown that the most general space-time metric responding to a RGTG and to the cosmological principle is that of Robertson-Walker (RW) or Friedmann-Lemaître-Robertson-Walker (FLRW). We call «metric» the square ds^2 of the element of space-time length ds. That of RW has for expression :

$$\mathbf{ds^2} = -\mathbf{R^2(t)}\left[\frac{\mathbf{dr^2}}{\mathbf{1-kr^2}} + \mathbf{r^2 d}\theta^\mathbf{2} + \mathbf{r^2}\sin^\mathbf{2}\theta\mathbf{d}\varphi^\mathbf{2}\right] + \mathbf{c^2dt^2} \quad \textbf{avec} \quad \mathbf{k = -1}, \ \ \mathbf{0} \ \ \textbf{ou} \ \ \mathbf{+1} \quad (1)$$

where r is a radial coordinate and where $R(t)$ is the "scale factor", a real, defined, positive function of the variable t that multiplies the distance between fixed points in space. The 3 possibilities for k define the 3 types of monoconnex spatial topology compatible with the hypotheses : S^3 ([hyper]-spherical space), E^3 (Euclidean space), H^3 ([hyper]-hyperbolic space).

Time t is cosmic time. It is orthogonal (independent of spatial coordinates). It is the same for all observers at rest (constant r, θ and ϕ), traditionally referred to as *«comoving»*.

The $H(t)$ expansion rate is defined as the logarithmic derivative of R(t) : $H(t) \overset{def}{=} \dot{R}(t)/R(t)$. Its present value $H_0 = H(t_0)$ is the "Hubble-Lemaître" constant. Its dimension is the inverse of a time. For historical reasons of method of measurement it is often expressed in km s^{-1} Mpc^{-1} (1 pc $\overset{def}{=}$ 3.085677581491 10^{16} m.)

By following the trajectory of the photons (for which $ds = 0$) between emission and reception we deduce that if a source emits two light signals at the times t_e and $t_e + dt_e$, an observer will receive them at the times t_0 and $t_0 + dt_0$ with :

$$\mathbf{\frac{dt_0}{dt_e} = \frac{R(t_0)}{R(t_e)}} \tag{2}$$

Applied to the very small T-periods or wavelengths of light it can be written :

$$\mathbf{\frac{R(t_0)}{R(t_e)} \approx \frac{T_0}{T_e} = \frac{\lambda_0}{\lambda_e} \overset{def}{=} 1 + z} \tag{3}$$

z being thes cosmological [1]redshift.

The complete (1917) differential equation of Einstein's general relativity [2] is written :

$$\mathbf{\frac{1}{2} g_{\mu\nu} \mathcal{R} - \mathcal{R}_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} \mathcal{T}_{\mu\nu}} \tag{4}$$

With general relativity as RGTG and by introducing the coefficients ($g_{\mu\nu}$) of dr^2, $d\theta^2$, dϕ^2 and dt^2 of the RW metric into Einstein's equation it becomes 2 differential equations, the Friedmann-Lemaître equations FL1 and FL2, on the scale factor R(t) in which $p(t)$ and $\rho(t)$ (pressure and density) are the only two physical parameters describing the content : p and ρ are, according to the cosmological principle, spatially homogeneous (therefore not a function of r,θ or ϕ) and isotropic (therefore scalar, even for pressure) .

$$\mathbf{-\frac{k}{R^2} - \frac{\dot{R}^2}{c^2R^2} - \frac{2\ddot{R}}{Rc^2} + \Lambda = \frac{8\pi G p}{c^4} \quad (FL1) \quad et \quad \frac{k}{R^2} + \frac{\dot{R}^2}{c^2R^2} - \frac{\Lambda}{3} = \frac{8\pi G\rho}{3c^2} \quad (FL2)} \tag{5}$$

FL3 may be deducted from FL1 et FL2 :

$$\mathbf{\frac{d(\rho R^3)}{dR} + 3p\frac{R^2}{c^2} = 0 \quad (FL3)} \tag{6}$$

FL2 can also be written :

$$\mathbf{\dot{R}^2 = H_\circ^2\, R_\circ^2 \left[\Omega_{r\circ}\, \frac{R_\circ^2}{R^2} + \Omega_{m\circ}\, \frac{R_\circ}{R} + \Omega_{\Lambda\circ}\, \frac{R^2}{R_\circ^2} + \Omega_{k\circ}\right]} \tag{7}$$

with

$$\mathbf{\Omega_r(t) \overset{déf}{=} \frac{8\pi G\rho_r(t)}{3H^2(t)}\ ,\ \Omega_m(t) \overset{déf}{=} \frac{8\pi G\rho_m(t)}{3H^2(t)}\ ,\ \Omega_\Lambda(t) \overset{déf}{=} \frac{\Lambda c^2}{3H^2(t)}\ \ \text{et}\ \ \Omega_k(t) \overset{déf}{=} -\frac{kc^2}{R^2(t)H^2(t)}} \tag{8}$$

G is the universal gravitational constant. $\Omega_r(t)$ is the radiation density parameter (light or ultra-relativistic particles that have the same equation of state : $p = \frac{1}{3}\rho c^2$). $\Omega_m(t)$ is the total density parameter of matter (including dark and non-baryonic) and $\Omega_\Lambda(t)$ is the density parameter of Λ. It is also called a reduced cosmological constant (but $\Omega_\Lambda(t)$ is generally not a constant).

$\Omega_k(t)$ is the curvature density parameter or reduced curvature. It depends on k, which represents the spatial (3D) curvature of the universe.

1. There are other causes of redshift, but they're only local since they are related to inhomogeneities or displacements.

2. Λ is the "cosmological constant" introduced in 1917 by Einstein in his equations to make general relativity compatible with his model of a static universe.

— If k = 1, 3D space is hyper-spherical
— If k = 0, 3D-space is Euclidean (in this case $\Omega_k(t) = 0 \;\; \forall t$)
— If k = -1, 3D-space is [hyper]-hyperbolic.

Ω_{r0} is the better known density parameter because the light energy density ρ_{r0} is essentially that of the Cosmic Microwave Background Radiation (CMBR). One can probably add to it that of the primordial neutrinos not yet detected and which in the simplest hypothesis have an energy density equal to 68% of that of the CMBR.

From FL2 : $\mathbf{\Omega_r(t) + \Omega_m(t) + \Omega_\Lambda(t) + \Omega_k(t) = 1} \quad \forall \mathbf{t}$

So, we only need to measure three of the four Ω_i at any given time to determine the fourth. Since today Ω_{r0} is well known and, moreover, less than 10^{-4}, the knowledge of our model of the universe is essentially linked to the measurements of Ω_{m0} and $\Omega_{\Lambda 0}$.

In the simulation part, the values of Ω_{m0} and $\Omega_{\Lambda 0}$ are left to the choice (with by default those of the Λ-CDM model of the 2015 results of the ESA Planck mission). For Ω_{r0}, the RFC having a blackbody thermal spectrum, it is the temperature T_0 which is to be chosen. There is also an option to choose $\Omega_k = 0$.

The mass per unit volume ρ_r ($\rho_r = u_r c^2$ with u_r the density energy) of a black body is related only to its temperature T :

$$\rho_r = \frac{4\sigma T^4}{c^3} \quad \text{avec} \quad \sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} \tag{9}$$

By default the fundamental constants have the values of our universe but they can be modified in the simulations (in the multiverse hypothesis they could be different) :
— Boltzmann k = 1.38064852 10^{-23} $m^2.Kg.s^{-2}.K^{-1}$
— Planck h = 6.62607004 10^{-34} $m^2.Kg.s^{-1}$
— gravitation G = 6.67385 10^{-11} $m^3.Kg^{-1}.s^{-2}$
— speed of light in a vacuum c = 299792458 $m.s^{-1}$.

With the reduced coordinates : $\mathbf{a} \stackrel{\mathbf{def}}{=} \mathbf{R(t)/R(t_0)}$ [3] and $\mathbf{a} \stackrel{\mathbf{def}}{=} \mathbf{H_0(t - t_0)}$ we get the first and second derivatives of $a(\tau)$:

$$\mathbf{\frac{da}{d\tau} = \left[-\frac{\Omega_{r0}}{a^2} + \frac{\Omega_{m0}}{a} + \Omega_{\Lambda 0} \; a^2 + \Omega_{k0} \right]^{\frac{1}{2}}} \tag{10}$$

$$\mathbf{\frac{d^2a}{d\tau^2} = -\frac{\Omega_{r0}}{a^3} - \frac{1}{2}\frac{\Omega_{m0}}{a^2} + \Omega_{\Lambda 0} \; a} \tag{11}$$

with the initial conditions $a(0) = \frac{da}{d\tau}(0) = 1$.

Thus the data H_0, Ω_{r0}, Ω_{m0}, and $\Omega_{\Lambda 0}$ make it possible to solve the differential equation above and thus to know $a(\tau)$ for all τ (and from there $a(t)$ for all t) and thus to draw its graph (which is the first action of the simulation part).

Depending on the values of the selected Ω_i, different patterns with or without singularity(ies) are obtained. If $H_0 > 0$:

3. Then $a = (1+z)^{-1}$ or $z = (1-a)/a$ if z is z is purely cosmological

— Universe with Big-Bang and no Big Crunch.
— Universe with Big-Bang and Big-Crunch
— Universe without singularity

If $H_0 < 0$ universes with a Big Crunch and no Big Bang may be obtained.

The separators of these types of universes in the $\{\Omega_{m0}, \Omega_{\Lambda 0}\}$ field are shown in the right (and interactive) figure of the simulation.

In our present universe the parameter of radiation density is very low ($\Omega_{r0} < 10^{-4}$) whereas Ω_{m0} and $\Omega_{\Lambda 0}$ are close to 1/3 and 2/3. Under these conditions it is essentially Ω_{m0} and $\Omega_{\Lambda 0}$ that determine the type of universe : the separators in the field $\{\Omega_{m0}, \Omega_{\Lambda 0}$ are shown on the right (and interactive) figure of the simulation.

2 Calculation of Duration and Ages

Still following the trajectory of the photons, we demonstrate the relations (between distances, time, spectral shifts, apparent diameters, ...) that are used in Cosmogravity (notably in the toolbox of the "Adjunct computations" window). These expressions often use the $E(x)$ function. This function is deduced from :

$$\frac{H(z)}{H_\circ} \overset{déf}{=} E^{\frac{1}{2}}(z) = \left[\Omega_{r\circ}(1+z)^4 + \Omega_{m\circ}(1+z)^3 + (1-\Omega_{m\circ}-\Omega_{r\circ}-\Omega_{\Lambda\circ})(1+z)^2 + \Omega_{\Lambda\circ}\right]^{\frac{1}{2}} \quad (12)$$

when defining :

$$\mathbf{E(x) \overset{déf}{=} \Omega_{r\circ}(1+x)^4 + \Omega_{m\circ}(1+x)^3 + (1-\Omega_{m\circ}-\Omega_{r\circ}-\Omega_{\Lambda\circ})(1+x)^2 + \Omega_{\Lambda\circ}} \quad (13)$$

This gives simple expressions for the link between dt and dz along a line of sight. From $z \overset{déf}{=} R_0/R \; - 1$ we deduce $dz = -(1+z)\; H(t)dt$ and thus : $dt = -H_0^{-1}(1+z)^{-1}E^{-1/2}(z)dz$.

By integrating this relation we obtain the durations according to the redshifts :

$$\mathbf{t_2 - t_1 = \frac{1}{H_\circ}\int_{z_2}^{z_1}(1+x)^{-1}E^{-\frac{1}{2}}(x)\; dx} \quad (14)$$

We deduce the expression of the âge ($t_\circ$) of an FL Big Bang universe according to its parameters $H_\circ$ and $\Omega_{i\circ}$ and that of the age of this universe when light received today with a redshift z was emitted :

$$\mathbf{t_\circ = \frac{1}{H_\circ}\int_0^\infty(1+x)^{-1}E^{-\frac{1}{2}}(x)\; dx \quad t_e = \frac{1}{H_\circ}\int_z^\infty(1+x)^{-1}E^{-\frac{1}{2}}(x)\; dx} \quad (15)$$

Cosmogravity also allows the inverse calculation : z as a function of t

3 Distance Calculation

Expression of the "metric distance" d_m is obtained by integrating the trajectory of a photon from its emission at time t_e and coordinate r until it is received at time $t_\circ$ in $r = 0$

with a cosmological redshift z. Depending on whether the spatial curvature is negative (d_{m-}), null ($d_{m\circ}$), or positive (d_{m+}) [4] :

$$\mathbf{d_{m-}} = \frac{\mathbf{c}}{\mathbf{H_\circ} \mid \mathbf{\Omega_{k\circ}} \mid^{\frac{1}{2}}} \sinh \left\{ \mid \mathbf{\Omega_{k\circ}} \mid^{\frac{1}{2}} \int_{\mathbf{0}}^{\mathbf{z_c}} \mathbf{E}^{-\frac{1}{2}}(\mathbf{x})\mathbf{dx} \right\} \tag{16}$$

$$\mathbf{d_{m\circ}} = \frac{\mathbf{c}}{\mathbf{H_\circ}} \int_{\mathbf{0}}^{\mathbf{z_c}} \mathbf{E}^{-\frac{1}{2}}(\mathbf{x})\mathbf{dx} \tag{17}$$

$$\mathbf{d_{m+}} = \frac{\mathbf{c}}{\mathbf{H_\circ} \mid \mathbf{\Omega_{k\circ}} \mid^{\frac{1}{2}}} \sin \left\{ \mid \mathbf{\Omega_{k\circ}} \mid^{\frac{1}{2}} \int_{\mathbf{0}}^{\mathbf{z_c}} \mathbf{E}^{-\frac{1}{2}}(\mathbf{x})\mathbf{dx} \right\} \tag{18}$$

Cosmogravity also allows the reverse calculation : z according to d.

4 Calculation of angular diameters

Due to the radial trajectory of the photons received in an isotropic space-time the apparent diameter of an object (or the angular difference between two sources of the same z is defined at the time of emission, i.e. at the time when the distance from the source was $(1+z)$ times smaller than at the time of observation. More mathematically the metric of the surface $r = r$ and $t = t_e$ is that of an Euclidean 2-sphere of radius $R_e r = R_0 r/(1+z)$ and the Euclidean relation between linear diameter D_e (at time t_e), metric distance d_m and apparent diameter ϕ_0 is :

$$\phi_{\mathbf{0}} = \frac{\mathbf{D_e(1+z)}}{\mathbf{d_m}} \tag{19}$$

Cosmogravity also allows the reverse calculation : z according to ϕ.

5 Calculating photometry

With a calculation similar to the previous one but on the "wave surfaces" emitted by a source and arriving on the observer, we show that the brightness E_0 (orthogonal illuminance) of a source of intensity I_e (flux emitted per unit of solid angle) in the direction of the observer and of cosmological redshift z presents (in the absence of absorption on the path) an observed brightness

$$\mathbf{E_0} = \frac{\mathbf{I_e}}{\mathbf{d_m^2(1+z)^2}} \tag{20}$$

Assuming the source intensity is isotropic, its luminosity (emitted power) : $L_e = 4\ \pi\ I_e$ et :

$$\mathbf{E_0} = \frac{\mathbf{L_e}}{\mathbf{4\ \pi\ d_m^2(1+z)^2}} \tag{21}$$

4. The three expressions can be summed up into one by defining a function $S_k(x)$ (or *sinn (x)*), $S_k(x) \overset{def}{=} \sinh x,\ x$ or $\sin x$ depending on $k = -1,\ 0,$ or$+1$: $\mathbf{d_m} = \frac{\mathbf{c}}{\mathbf{H_\circ}|\mathbf{\Omega_{k\circ}}|^{\frac{1}{2}}}\ \mathbf{S_k}\left\{ \mid \mathbf{\Omega_{k\circ}} \mid^{\frac{1}{2}} \int_{\mathbf{0}}^{\mathbf{z_c}} \mathbf{E}^{-\frac{1}{2}}(\mathbf{x})\mathbf{dx} \right\}$

6 Single-fluid models

Analytical solutions exist for some of Friedmann-Lemaître's universe models. This is notably (but not only) the case for those for which a single density parameter Ω_i is non-zero. These particular models are sometimes approximations at certain times of a multi-fluid FL universe.

Since $\mathbf{\Omega_m(t) + \Omega_r(t) + \Omega_\Lambda(t) + \Omega_k(t) = 1} \;\; \forall \mathbf{t}$, if only one of the Ω_i is non-zero then it's always equal to one. Four cases present themselves : $\Omega_m(t)$ =1, $\Omega_r(t)$ =1, $\Omega_\Lambda(t)$ =1 and, why not, $\Omega_k(t)$ =1.

6.1 Matter, $\Omega_m = 1 \;\; \forall t$

For this universe "of dust" (non-relativistic matter) by Einstein-de Sitter (1932) $p \approx 0$ and : $\rho_m R^3 = cte = \rho_{m0} R_0^3$

The Friedmann-Lemaître equation FL2 then leads (by taking as origin (t=0) of time that of the singularity : $R(t=0) = 0$) to :

$$\mathbf{R(t) = (6\pi G\rho_{m0}R_0^3)^{\frac{1}{3}} \;\; t^{\frac{2}{3}}} \tag{22}$$

As :

$$\Omega_m(t) \overset{déf}{=} \frac{8\pi G\rho_m(t)}{3H^2(t)}, \qquad \rho_{m0} = \frac{3H_0^2}{8\pi G} \tag{23}$$

$$H \overset{déf}{=} \frac{\dot{R}}{R}, \qquad H(t) = \frac{2}{3\;t} \qquad \Rightarrow \qquad \mathbf{H_0 = \frac{2}{3\;t_0}} \tag{24}$$

Then

$$\mathbf{a(t) \overset{déf}{=} \frac{R(t)}{R_0} = \left[\frac{3H_0}{2}\right]^{2/3} t^{\frac{2}{3}}} \tag{25}$$

6.2 Radiation, $\Omega_r = 1 \;\; \forall t$

Ror this entity : $p_r = \frac{1}{3}\rho_r c^2$ et FL3 : $\rho_r R^4 = cte = \rho_{r0} R_0^4$

In this Weinberg universe, the Friedmann-Lemaître equation FL2 becomes

$$R^2\dot{R}^2 = (8\pi\;G/3)\rho_{r0}R_0^4 \quad \Rightarrow \quad \mathbf{R(t) = \left[\frac{32\pi G\rho_{r0}R_0^4}{3}\right]^{\frac{1}{4}} t^{\frac{1}{2}}} \tag{26}$$

$$\Omega_r(t) \overset{déf}{=} \frac{8\pi G\rho_r(t)}{3H^2(t)}, \quad \Rightarrow \quad \rho_{r0} = \frac{3H_0^2}{8\pi G} \tag{27}$$

$$H \overset{déf}{=} \frac{\dot{R}}{R}, \quad \Rightarrow \quad H = \frac{1}{2\;t} \qquad \Rightarrow \qquad \mathbf{H_0 = \frac{1}{2\;t_0}} \tag{28}$$

Then

$$\mathbf{a(t)} \overset{\text{déf}}{=} \frac{\mathbf{R(t)}}{\mathbf{R_0}} = [\mathbf{2H_0}]^{\frac{1}{2}} \mathbf{t}^{\frac{1}{2}} \tag{29}$$

Note : Since the radiation energy density of a black body is $u_r = \rho_r c^2 = \frac{4\sigma T^4}{c}$ and $\Omega_r \overset{déf}{=} \frac{8\pi G\rho_r}{3H^2} = 1$, the temperature T_r depends only on H :

$$T_r = \left[\frac{3H^2c^3}{32\pi G\sigma}\right]^{\frac{1}{4}} \quad \text{with} \quad \sigma = \frac{2\pi^5 k^4}{15c^2h^3} \quad \text{et} \quad \mathbf{T_r} = \left[\frac{\mathbf{45c^5h^3}}{\mathbf{64\pi^6Gk^4}}\right]^{\frac{1}{4}} \mathbf{H}^{\frac{1}{2}} \tag{30}$$

.

6.3 Cosmological constant, $\Omega_\Lambda = 1 \quad \forall t$

For this de-Sitter universe the Friedmann-Lemaître equation FL2 leads to :

$$\frac{\dot{R}^2}{R^2} = \frac{\Lambda c^2}{3} = cte \tag{31}$$

Then $\Lambda \geq 0$ and

$$\mathbf{H} = \pm\, \mathbf{c} \left[\frac{\mathbf{\Lambda}}{\mathbf{3}}\right]^{\mathbf{1/2}} = \mathbf{cte} = \mathbf{H_0} \tag{32}$$

and

$$\mathbf{a(t)} \overset{\text{déf}}{=} \frac{\mathbf{R(t)}}{\mathbf{R_0}} = \mathbf{e}^{\mathbf{H_0\cdot(t-t_0)}} \tag{33}$$

6.4 Curvature, $\Omega_k = 1 \quad \forall t$

The sum of Friedmann-Lemaître equations FL1+FL2 results for this Milne's universe to : $\ddot{R}$ =0. Therefore :

$$R(t) = \alpha t + \beta \quad , \quad \dot{R} = \alpha = cte \quad \text{et} \quad H_0 \overset{déf}{=} \frac{\dot{R}_0}{R_0} = \frac{\alpha}{R_0} \tag{34}$$

As a result

$$\mathbf{a(t)} \overset{\text{déf}}{=} \frac{\mathbf{R(t)}}{\mathbf{R_0}} = \mathbf{H_0 t} + \frac{\beta}{\mathbf{R_0}} = \mathbf{H_0\cdot t + cte} \tag{35}$$

Taking as the origin of time that of a=0 :

$$\mathbf{a(t) = H_0 \cdot t} \quad \text{et} \quad \mathbf{t_0 = (H_0)^{-1}} \tag{36}$$

7 Dark Energy

7.1 Relativistic equations of state

The matter, the radiation, does intervene only through their *equation of state* $p = p(\rho)$ in the energy-momentum tensor of a perfect fluid. We can thus, while remaining within the framework of Friedmann-Lemaître's equations, take into account different known or hypothetical i, characterized by a state equation like $p_i = w_i\rho_i c^2$. The total density and pressure of a universe at n fluids... are then :

$\rho = \sum_{i=1}^{n} \rho_i \quad et \quad p = c^2 \sum_{i=1}^{n} w_i \rho_i$.

For the *dust* (non-relativistic matter) $p \ll \rho c^2$ and $w_d = w_{nr} \approx 0$. For radiation (light or ultra-relativistic particles) $w_r = 1/3$.

The evolution of the scale factor a and that of other cosmological parameters can be generalized to a mixture of fluid n and i. Restricting ourselves, for example, to fluids of constant w_i we obtain for the evolution of the rate of expansion $H = \dot{a}/a$:

$$\frac{H}{H_\circ} = \left[\sum_{i=1}^{n} \Omega_i a^{-3(1+w_i)} \right]^{\frac{1}{2}} = \left[\sum_{i=1}^{n} \Omega_i (1+z)^{3(1+w_i)} \right]^{\frac{1}{2}} \tag{37}$$

The introduction of the terms of the other fluids in the expressions of the metric distance d_m opens the way to observational tests and to the constraint of the new parameters, by inverting the relation {parameters} $\longrightarrow$ {observable}, since the latter, such as brightness or apparent diameter, depend on d_m [5].

7.2 From Λ to dark energy

The cosmological constant Λ is equivalent to (and can be interpreted as) [6] a fluid of vacuum, Lorentz's invariant of parameter $w_\Lambda = w_v = -1$ and density $\rho_{DE} = \frac{\Lambda c^2}{8\Pi G}$. This substitution leaves Einstein's equations mathematically unchanged (and consequently those of Friedmann-Lemaître if we keep the cosmological principle).

On the other hand, the geometrical cosmological constant Λ can pose (by its constancy) a severe initial adjustment problem [7] and one can try to replace it by a physical fluid. If this new entity, for the moment speculative, is described by a parameter state equation w this parameter must be spatially constant (cosmological principle) but different from -1, or even variable with time t (i.e. with z and a).

It is sometimes distinguished between dark energy ($w_{DE}(z) \neq -1$), the quintessence ($w_Q \neq \; cte$), the phantom energy ($w_{PE} < -1$) ...but it has become customary to generalize the name *« dark energy»* to all possibilities and to use the CPL [8] with two parameters w_0 and w_1 (or $w_a = w_1$) :

$$w(z) = w_\circ + w_1 \frac{z}{1+z} \qquad \text{or} \qquad w(a) = w_0 + w_a(1-a) \tag{38}$$

In this representation Λ would appear as the special case of a dark energy of parameters $w_0 = -1$ and $w_1 = 0$.

Observations allow us to constrain the area of our universe in a plane (w_0, w_1) as we do for the field of $(\Omega_{m0}, \Omega_{\Lambda 0})$.

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5. Evidently multiplying the number of free parameters in the model with the same data expands the uncertainties on each one when inverting

6. Lemaître, 1934, Proc.National. Acad. Sciences USA, vol 20, pp12-17

7. its very small but constant value makes it insignificant in the the primordial universe, but if its value were larger, the early acceleration of the expansion would have prevented the formation of the structures

8. Chevalier & Polarski 2001,Int. J. Mod. Phys. D10,213 ; Linder 2003 Phys. Rev. Lett. 90.091301

At more than 100 years the geometric cosmological constant Λ [9] or its physical equivalent ($w_0 = -1$ and $w_1 = 0$) remains compatible at 1σ with the observational constraints.

7.3 Calculation

As with $E(x)$ for the model with the constant Λ, functions simplify the writing of relations, $Y(x)$ and $F(x)$:

$$\mathbf{Y(x)} \overset{\text{déf}}{=} \exp\{-\mathbf{3(1+w_0+w_1)}\log\mathbf{x} - \mathbf{3w_1(1-x)}\} \tag{39}$$

$$\mathbf{F(x)} \overset{\text{déf}}{=} \left[\frac{\mathbf{H(x)}}{\mathbf{H_0}}\right]^{\mathbf{2}} = \mathbf{(1+x)^2\Omega_{k0} + (1+x)^3\Omega_{m0} + (1+x)^4\Omega_{r0} + Y((1+x)^{-1})\Omega_{DE0}} \tag{40}$$

Thus the first and second derivatives of $a(\tau)$ become :

$$\frac{\mathbf{da}}{\mathbf{d}\tau} = \left[-\frac{\mathbf{\Omega_{r0}}}{\mathbf{a^2}} + \frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{\Omega_{m0}}}{\mathbf{a}} + \mathbf{\Omega_{DE0}\ a^2Y(a) + \Omega_{k0}}\right]^{\frac{1}{2}} \tag{41}$$

$$\frac{\mathbf{d^2a}}{\mathbf{d}\tau^{\mathbf{2}}} = -\frac{\mathbf{\Omega_{r0}}}{\mathbf{a^3}} - \frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{\Omega_{m0}}}{\mathbf{a^2}} + \mathbf{\Omega_{DE0}}\left[\mathbf{aY(a)} + \frac{\mathbf{a^2}}{\mathbf{2}}\frac{\mathbf{dY}}{\mathbf{da}}\right] \tag{42}$$

and that of metric distance :

$$\mathbf{d_m} = \frac{\mathbf{c}}{\mathbf{H_\circ} \mid \mathbf{\Omega_{k\circ}} \mid^{\frac{1}{2}}}\ \ \mathbf{S_k}\left\{\mid \mathbf{\Omega_{k\circ}} \mid^{\frac{1}{2}} \int_{\mathbf{0}}^{\mathbf{z_c}} \mathbf{F}^{-\frac{1}{2}}\mathbf{(x)dx}\right\} \tag{43}$$

The sequence of expressions used in the calculations of E, d_L, d_A, θ, d_{LT}, l . . .is identical to those of the model with Λ replacing $E(x)$ by $F(x)$.

7.4 Big Rip

Substituting the cosmological constant Λ by a fluid whose parameter w ($p \overset{def}{=} w\rho c^2$) can become less than -1 opens the door to a fourth possibility of accident for the universe. In addition to the Big Bang, the Big Crinch and the Big Chill it is possible to get a "Big Rip", i.e. an infinite expansion of $a(t)$ in a finite time. Unlike the Big Chill of the standard model where the expansion rate H tends to a constant (smaller than its current value) which preserves the not currently expanding structures, the expansion rate can itself tend to infinity in a finite time and then disintegrate to the smallest material structures. It is thus an existentially extreme singularity for matter, just as extreme as the $a = 0$ of the "Big Bang". The present (2021) observational constraints of our universe in the plane (w_0, w_1) are compatible with $w_0 = -1$ and $w_1 = 0$ (i.e. the geometric cosmologic constant Λ) but also with slightly different values of w_0 and w_1 that predict a Big Rip.

For the dark energy option (with the CPL parametrisation), Cosmogravity first looks for a Big Cunch. If (and only if) the answer is negative then there are 4 possibilities :

9. Einstein, 1917, Sitzungsberichte der Koniglich PreuBischen Akademie der Wissenschaften (Berlin), Seite 142-152

1. If $w_1 < 0$ then $w \rightarrow +\infty$ with $z \rightarrow -1$ no Big Rip.
2. If $w_1 > 0$ then $w \rightarrow -\infty$ with $z \rightarrow -1$ Big Rip
3. If $w_1 = 0$ and $w_0 < -1$ then $w(\text{constant}) < -1$ Big Rip
4. If $w_1 = 0$ and $w_0 > -1$ then $w(\text{constant}) > -1$ no Big Rip

In the above Big Rip cases (2 and 3), the time $t_{BR} - t_0$ remaining before the Big Rip is then [10]

$$\mathbf{t_{BR} - t_0 = \frac{1}{H_\circ} \int_{-1}^{0} (1+x)^{-1} F^{-\frac{1}{2}}(x)\, dx} \tag{44}$$

The "duration of the universe" is obviously $t_0 + t_{BR}$.

10. in fact we sum from -0.999999 to 0 to avoid the singularity of $(1+z)^{-1}$