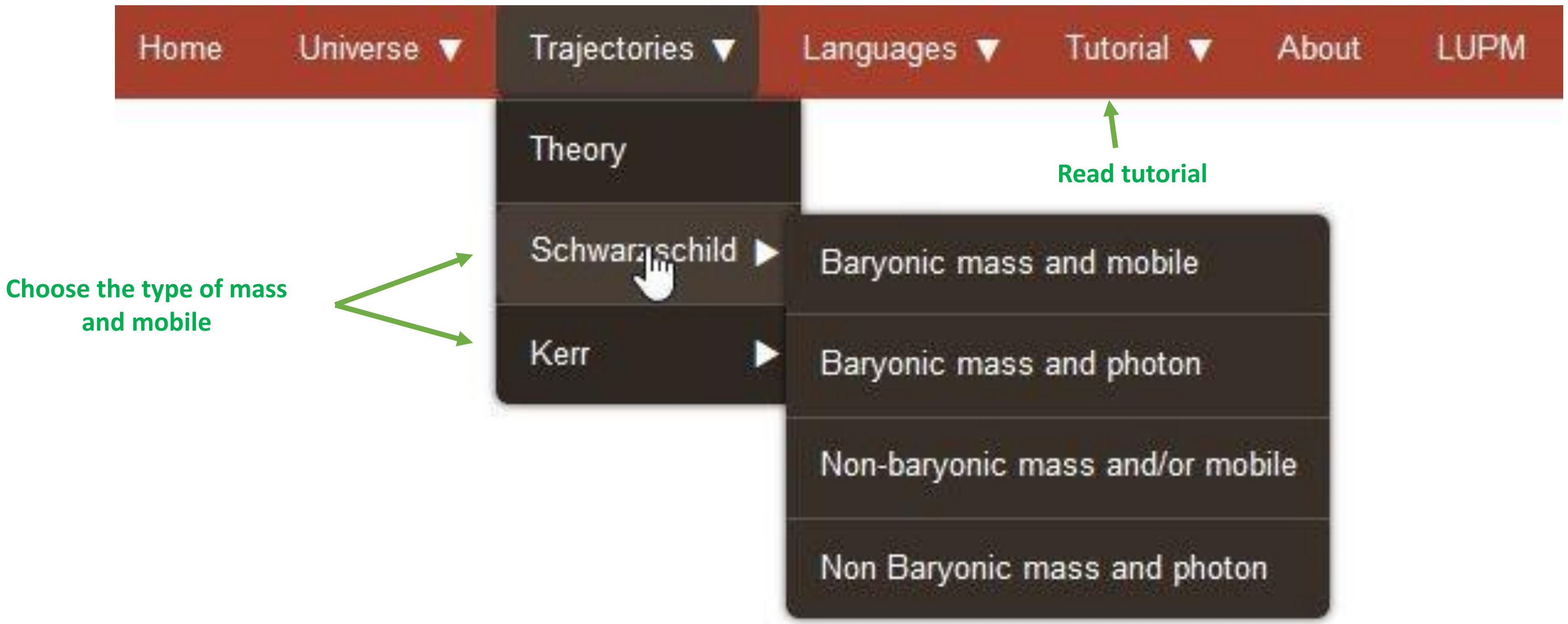
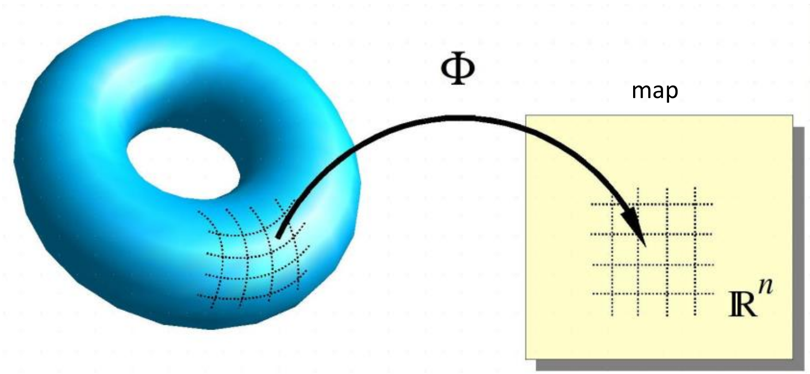
TRAJECTORIES with COSMOGRAVITY TUTORIAL

J.P. CORDONI 2022/08/08



Geometric frame

Relativity has merged space and time, two notions that were completely distinct in Galilean mechanics. Four numbers are needed to determine an event in the space-time continuum: three for its spatial location (e.g. its Cartesian coordinates {x, y, z} or its spherical coordinates $\{r, \theta, \varphi\}$) and one for its date (t). The mathematical structure corresponding to this four-dimensional "continuum" is that of **variety**.

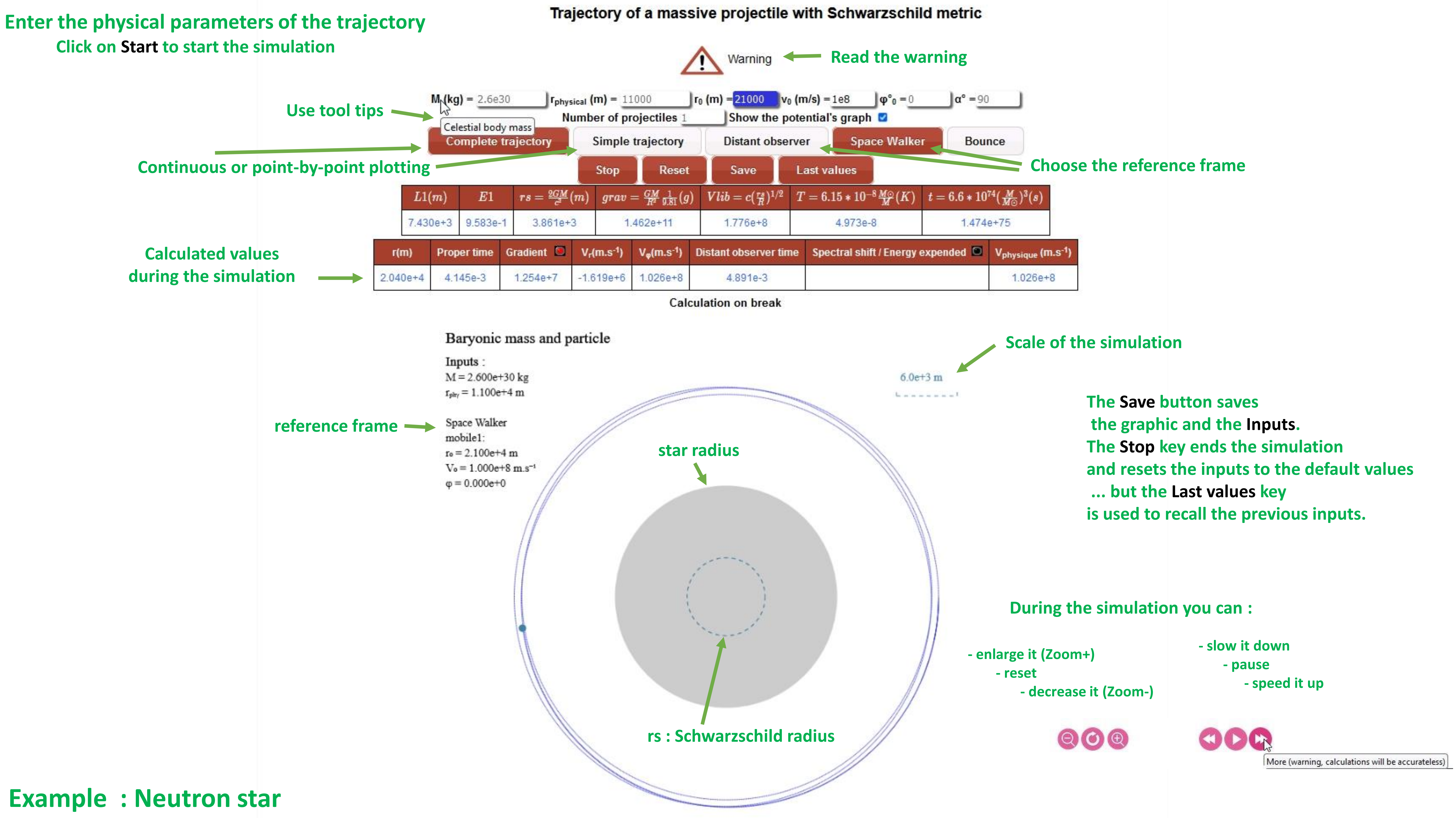


Variety: seen closely, a variety looks like R^n (n = 2 on the figure),
but this is not necessarily true at the global level.

It should be emphasized that the local similarity with R^4 stops at the labeling of the points and does not extend to the Euclidean space structure of R^4. In particular, the choice of coordinate system is completely free.

These notes are from Gourgoulhon-Relativité Générale

In the **Cosmogravity** software the "trajectories" are the geodesics followed by the different particles (baryonic, non-baryonic, photons) represented by their coordinates (r, φ) in R^2 as a function of the proper time (τ) of the particles or the time of the distant observer (t).
The distance that would be measured (using the scale of the simulation) between two positions of a particle is obviously not equal to the metric distance between these two positions.


Enter the physical parameters of the trajectory
Click on Start to start the simulation
Trajectory of a massive projectile with Schwarzschild metric
Warning
Read the warning
Use tool tips
M (kg) = 2.6e30
r physical (m) = 11000
r0 (m) = 21000
v0 (m/s) = 1e8
φ°0 = 0
α° = 90
Number of projectiles 1
Show the potential's graph
Celestial body mass
Complete trajectory
Simple trajectory
Distant observer
Space Walker
Bounce
Continuous or point-by-point plotting
Choose the reference frame
Stop
Reset
Save
Last values
L1(m)
E1
rs = 2GM/c² (m)
grav = GM/R² 1/9.81 (g)
Vlib = c(rs/R)^1/2
T = 6.15 * 10^-8 M⊙/M (K)
t = 6.6 * 10^74 (M/M⊙)^3 (s)
7.430e+3
9.583e-1
3.861e+3
1.462e+11
1.776e+8
4.973e-8
1.474e+75
Calculated values
during the simulation
r(m)
Proper time
Gradient
Vr(m.s-1)
Vφ(m.s-1)
Distant observer time
Spectral shift / Energy expended
Vphysique (m.s-1)
2.040e+4
4.145e-3
1.254e+7
-1.619e+6
1.026e+8
4.891e-3
1.026e+8
Calculation on break
Baryonic mass and particle
Inputs :
M = 2.600e+30 kg
rphy = 1.100e+4 m
reference frame
Space Walker
mobile1:
r0 = 2.100e+4 m
V0 = 1.000e+8 m.s-1
φ = 0.000e+0
Scale of the simulation
6.0e+3 m
star radius
rs : Schwarzschild radius
The Save button saves
the graphic and the Inputs.
The Stop key ends the simulation
and resets the inputs to the default values
... but the Last values key
is used to recall the previous inputs.
During the simulation you can :
- enlarge it (Zoom+)
- reset
- decrease it (Zoom-)
- slow it down
- pause
- speed it up
More (warning, calculations will be accurateless)
Example : Neutron star

Trajectory of a massive projectile with Schwarzschild metric

2 mobiles around the asteroid

Warning

M (kg) = 2e13 | $r_{physical}$ (m) = 1000 | r_0 (m) = 3000 | 5000 | v_0 (m/s) = 0.4 | 0.5 | $\varphi°_0$ = 0 | 0 | α° = 65 | 230

Number of projectiles 2 | Show the potential's graph ☑

Complete trajectory | Simple trajectory | Distant observer | Space Walker | Bounce

Possibility of bounce

Choose the impact absorption coefficient

Shock absorption : the bounce is limited to an impact speed of 0.3

Stop | Reset | Save | Last values

$L1(m)$	$L2(m)$	$E1$	$E2$	$rs = \frac{2GM}{c^2}(m)$	$grav = \frac{GM}{R^2}\frac{1}{9.81}(g)$	$Vlib = c(\frac{rs}{R})^{1/2}$	$T = 6.15 * 10^{-8}\frac{M\odot}{M}(K)$	$t = 6.6 * 10^{74}(\frac{M}{M\odot})^3(s)$
3.628e-6	-6.388e-6	1.000e+0	1.000e+0	2.970e-14	1.361e-4	1.634e+0	6.464e+9	6.710e+23

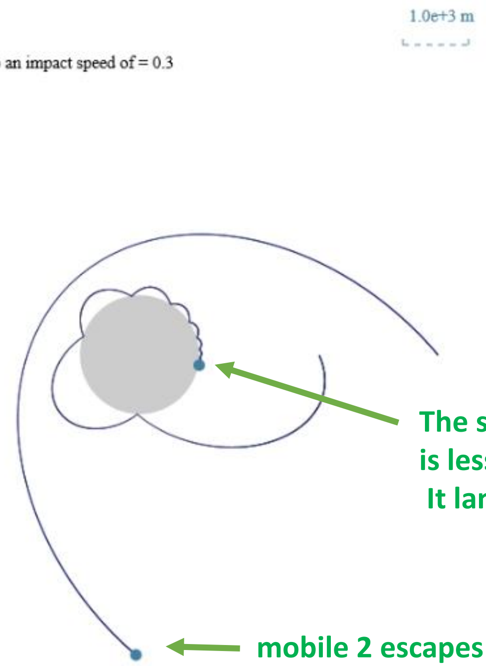
r(m)	Proper time	Gradient	$V_r(m.s^{-1})$	$V_\varphi(m.s^{-1})$	Distant observer time	Spectral shift / Energy expended	$V_{physique}(m.s^{-1})$
1.016e+3	1.829e+4	2.406e-6	0.000e+0	2.150e-1	1.829e+4		2.150e-1
r(m)	Proper time	Gradient	$V_r(m.s^{-1})$	$V_\varphi(m.s^{-1})$	Distant observer time	Spectral shift / Energy expended	$V_{physique}(m.s^{-1})$
5.048e+3	1.829e+4	3.809e-9	0.000e+0	-3.794e-1	1.829e+4		3.794e-1

Calculation on break

Baryonic mass and particle

Inputs :
M = 2.000e+13 kg
r_{phy} = 1.000e+3 m
Shock absorption : the bounce is limited to an impact speed of = 0.3
Space Walker
mobile1:
r_0 = 3.000e+3 m
V_0 = 4.000e-1 m.s^{-1}
φ = 0.000e+0
mobile2:
r_0 = 5.000e+3 m
V_0 = 5.000e-1 m.s^{-1}
φ= 0.000e+0

reference frame



The speed impact of mobile 1 is less than 300 m/s : It lands on the asteroid

mobile 2 escapes

Example : Small asteroid

Example : Photons trajectories

Trajectory of a photon with Schwarzschild metric

Warning

M (kg) = 2e30 | $r_{physical}$ (m) = 0 | r_0 (m) = 10000 | 8000 | $\varphi°_0$ = 50 | 210 | α° = 135 | 135

Number of projectiles 2 | Show the potential's graph ☑

Complete trajectory | Simple trajectory | Distant observer | Photon | Bounce

Stop | Reset | Save | Last values

$L1(m)$	$L2(m)$	$E1$	$E2$	$rs=\frac{2GM}{c^2}(m)$	$grav=\frac{GM}{R^2}\frac{1}{9.81}(g)$	$Vlib=c(\frac{rs}{R})^{1/2}$	$T=6.15*10^{-8}\frac{M\odot}{M}(K)$	$t=6.6*10^{74}(\frac{M}{M\odot})^3(s)$
8.434e+3	7.134e+3	1.000e+0	1.000e+0	2.970e+3			6.464e-8	6.710e+74

r(m)	Proper time	Gradient	$V_r(m.s^{-1})$	$V_\varphi(m.s^{-1})$	Distant observer time	$V_{physique}(m.s^{-1})$
9.912e+3	0.000e+0		2.105e+8	2.135e+8	6.735e-5	2.99792458e+8
r(m)	**Proper time**	**Gradient**	**$V_r(m.s^{-1})$**	**$V_\varphi(m.s^{-1})$**	**Distant observer time**	**$V_{physique}(m.s^{-1})$**
0.000e+0	0.000e+0				Infinity	

Calculation on break

The speed of the photon is identical in all reference frames

The proper time of a photon is always zero.

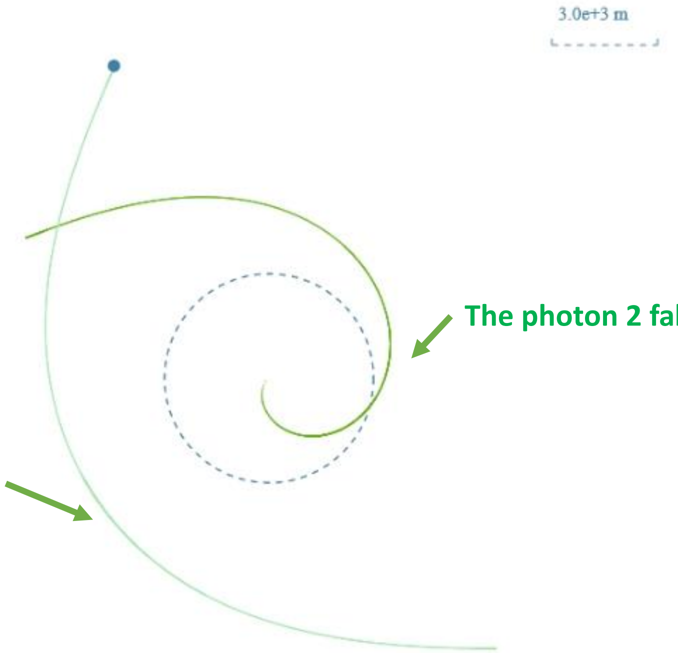
For the distant observer the mobile takes an infinite time to reach rs

The speed of the photon is meaningless inside the black hole horizon

Baryonic mass and photon

Inputs :
M = 2.000e+30 kg
r_{phy} = 0.000e+0 m

Photon
mobile1:
r_0 = 1.000e+4 m
V_0 = 2.998e+8 m.s^{-1}
φ = 8.727e-1
mobile2:
r_0 = 8.000e+3 m
V_0 = 2.998e+8 m.s^{-1}
φ= 3.665e+0



The photon 2 falls into the black hole

The trajectory of photon 1 is deviated in the gravitational field of the black hole

Trajectory of a massive projectile with Schwarzschild metric
(non baryonic case)

Warning

M (kg) = 2e30 | $r_{physical}$ (m) = 7e8 | r_0 (m) = 9e8 | 4e8 | 6e8 | $v_0(m.s^{-1})$ = 3e5 | 3e5 | 3e5 | φ $°_0$ = 0 | 90 | 180 | α° = 90 | 90 | 90

Number of projectiles 3 | Show the potential's graph ☑

Complete trajectory | Simple trajectory | Distant observer | Space Walker

Stop | Reset | Save | Last values

$L1(m)$	$L2(m)$	$L3(m)$	$E1$	$E2$	$E3$	$rs = \frac{2GM}{c^2}(m)$	$grav = \frac{GM}{R^2}\frac{1}{9.81}(g)$	$Vlib = c(\frac{rs}{R})^{1/2}$	$T = 6.15 * 10^{-8}\frac{M\odot}{M}(K)$	$t = 6.6 * 10^{74}(\frac{M}{M\odot})^3(s)$
9.006e+5	4.003e+5	6.004e+5	1.000e+0	1.000e+0	1.000e+0	2.970e+3	2.777e+1	6.176e+5	6.464e-8	6.710e+74

r(m)	Proper time	Gradient	$V_r(m.s^{-1})$	$V_\varphi(m.s^{-1})$	Distant observer time	Spectral shift / Energy expended	$V_{physique}$ $(m.s^{-1})$
8.546e+8	2.176e+4	1.773e-8	7.706e+4	3.159e+5	2.176e+4	2.326e-6	3.252e+5

r(m)	Proper time	Gradient	$V_r(m.s^{-1})$	$V_\varphi(m.s^{-1})$	Distant observer time	Spectral shift / Energy expended	$V_{physique}$ $(m.s^{-1})$
4.593e+8	2.176e+4	1.360e-6	4.372e+4	2.613e+5	2.176e+4	3.116e-6	2.649e+5

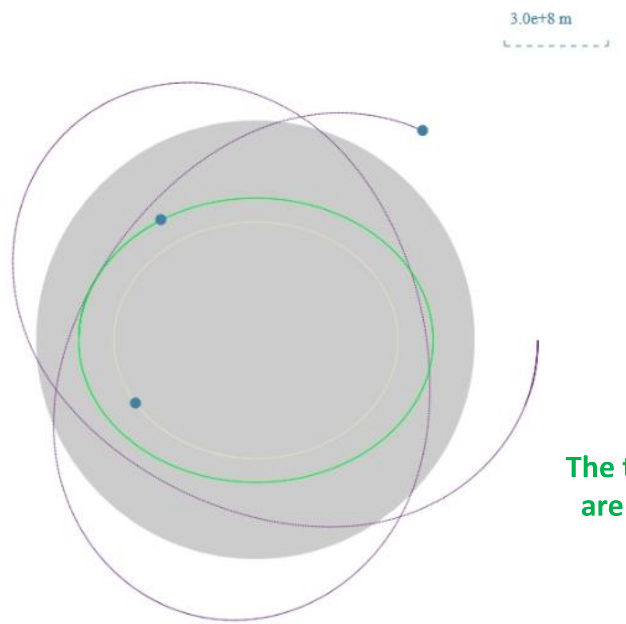
r(m)	Proper time	Gradient	$V_r(m.s^{-1})$	$V_\varphi(m.s^{-1})$	Distant observer time	Spectral shift / Energy expended	$V_{physique}$ $(m.s^{-1})$
5.177e+8	2.176e+4	1.742e-6	-7.006e+4	3.477e+5	2.176e+4	3.302e-6	3.547e+5

Calculation on break

Nonbaryonic mass and or particle

Inputs :
M = 2.000e+30 kg
r_{phy} = 7.000e+8 m
Distant observer
mobile1:
r_0 = 9.000e+8 m
V_0 = 3.000e+5 $m.s^{-1}$
φ = 0.000e+0
mobile2:
r_0 = 4.000e+8 m

V_0 = 3.000e+5 $m.s^{-1}$
φ= 1.571e+0



For the calculation of the spectral shift the observer is assumed to be very far (non-cosmological distance) in a direction perpendicular to the trajectory plane

The three particles of baryonic matter are subject only to the gravitational field of the central mass

Example : Matter of the Sun supposed to be non-baryonic with constant mass density

Example :

photon and massive rotating black hole

Trajectory of a photon with Kerr metric

Warning

M (kg) = 2e39 | r_0 (m) = 5e12 | J (kg.m^2.s^{-1})= 8.4e59 | φ_0 (°) = 0 | φ_D (°) = 138 | nzoom = -5 | Show the potential's graph ☑

Complete trajectory | Simple trajectory | Distant observer | Photon

Stop | Reset | Save | Last values | Pre-zoom

Choose a decrease - or an increase + of scale before the plot

$L(m)$	E	$rs=\frac{2GM}{c^2}(m)$	$a=\frac{J}{cM}(m)$	$Rh+(m)$	$Rh-(m)$	$g=\frac{c^2}{2Rh+}\frac{(Rh+^2-a^2)}{(Rh+^2+a^2)}(m.s^{-2})$
3.686e+12	1.000e+0	2.970e+12	1.401e+12	1.978e+12	9.921e+11	1.492e+16

r(m)	Proper time	Acceleration gradient	V_r (m.s^{-1})	V_φ (m.s^{-1})	$V_{physique}$ (m.s^{-1})	Distant observer time
1.97829545e+12	0.00000000e+0					2.11961102e+5

Calculation on break

The proper time of a photon is always zero.

Trajectory of a photon with Kerr metric

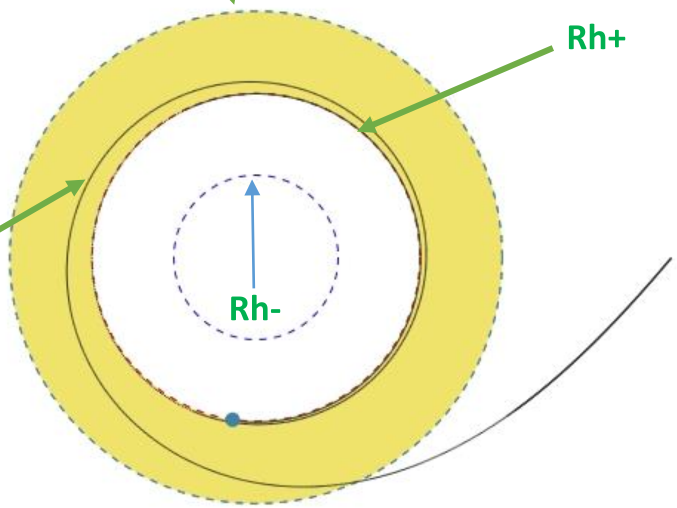
Inputs :
M = 2.000e+39 kg
r₀ = 5.000e+12 m
a = 1.401e+12 m
φ = 1.380e+2 °
Distant observer

Reference frame

1e+12 m

rs

Rh+



In the reference frame of the distant observer, the photon wraps itself indefinitely around the event horizon Rh+

Potential graph (see **Theory**)