

SOEN 6481- Software Requirements and Specification

Champernowne Constant- C_{10}

In mathematics, Champernowne constant is constructed by concatenating the positive integers and interpreting them as the digits to the right of the decimal point. It is named after the economist and mathematician D.G. Champernowne who published the constant in 1933.

The constant is a **transcendental real constant** whose decimal expansion has important properties.

Essentially, Champernowne constant for base 10 can be constructed by juxtaposing successive integers:

$$C_{10} = 0.12345678910111213141516\dots$$

Champernowne constant can also be created in other bases such as,

$$C_2 = 0.11011100101110\dots$$

$$C_3 = 0.12101112202122\dots$$

Mathematically, for any base m , as an infinite series, Champernowne constant can be expressed as,

$$C_m = \sum_{n=1}^{\infty} \frac{n}{m^{(n + \sum_{k=1}^{n-1} \text{floor}(\log_m(k+1)))}}$$

One of the most defining qualities about the Champernowne constant, **C_b is normal in any base b** (base 10 proved by D.G. Champernowne and the generic proof given by Nakai and Shiokawa). The normality of a real number would be defined as it's digits in any base following a uniform distribution. Champernowne normality says that every finite sequence in the constant occurs infinitely (for example, 123 occurs as part of 123, 1230, 12300, 123000..) and also that there are an infinite number of finite sequences occurring. This property is also defined as the **disjunctiveness of the constant**.

Additionally, the **constant is also transcendental**. This means that the constant cannot be expressed as the root of a polynomial equation. Since the number is transcendental, its continued fraction does not terminate which also signifies that it is an irrational number. Although, what's perceived of an irrational number is that there exists no pattern in it's continued expansion and determining the next digit in the decimal expansion is not possible. But with Champernowne constant, the next integer in the expansion can be determined easily as it is the juxtaposition of successive integers. Therefore, the constant can be called as rationally irrational.