(Dis)Inflation Targeting

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Abstract

Extending the model proposed by Barro and Gordon (1983b) by introducing an independent central bank, subjective expectations, and announced intermediate inflation targets this paper aims to explain the disinflation process adopted by the Latin American economies. Specifically, given the introduction of a new independent central bank we ask if the policy of the announced intermediate targets were cheap talk or the targets aided the disinflation process. We hypothesise that announcing intermediate targets reduced the cost of disinflation by reducing the inflation surprises the agents face and enabling agents to learn faster. The reduced surprises also help in developing credibility for the central bank further building the basis for faster learning. Finally, we also show that the cost of disinflation reduces when there are announced inflation targets.

Keywords: Disinflation, Credibility, Inflation, Inflation Expectations, Subjective Expectations

JEL Classification: D83, E17, E31, E52, E58

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1 Introduction

The late 1980s and early 1990s saw many of the Latin American economies witness periods of hyperinflation. These periods of hyperinflations were linked to the political cycle where governments would run high deficits for political gain in subsequent elections and the oil price shocks at the end of the 1980s. This political cycle resulted in what is known as the inflation bias. Consequently, the pattern in the economies became one of high price indexation in the labour markets, and financial markets. In an attempt to de-link the monetary and political cycle many of these economies instituted a constitutional amendment, introducing an independent monetary authority tasked with price stabilisation and announcing an annual inflation target.

The aim of the paper is to propose a simple model to rationalise the decision to introduce an independent monetary authority[1] and intermediate inflation targets to disinflate[2]. The new institution is an unknown entity for the agents in these economies. Therefore specifically, we ascertain whether the new policy of intermediate inflation targets is cheap talk or the targets aid the disinflation process? The agents learn about these objectives in two stages. First, agents learn about the introduction of a new institution, the central bank and its difference with the government. Second, the agents learn about the objectives of the bank with a sequence of announced inflation targets.

The introduction of an independent central bank was quickly followed by the introduction of intermediate targets to slowly guide the path of inflation. They key reason to introduce the targets was to reduce the degree of indexation in labour and financial markets in these economies. The belief was that the adoption of these intermediary targets would provide explicit objectives which could diminish the use of indexation mechanisms and thus, subsequently reducing the cost of stabilisation. This paper attempts to model this interaction between the agents and introduction of the independent central bank, and intermediate targets to discern the cost of disinflation.

The paper contributes to the literature on two aspects. First, it internalises the policy decision to introduce an independent central bank, which much of the literature takes for granted. Second, it proposes a mechanism for the disinflation that the economies witnessed. Previous research specifically in the Latin American economies context has focused on explaining the hyper inflationary phases without delving into the disinflation process.

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[1]Brazil: `https://www.bcb.gov.br/en/about/cmnen`
Chile: `https://www.bcentral.cl/en/web/banco-central/the-bank/corporative-goverment/history`
Colombia: `https://www.banrep.gov.co/en/node/22666`

[2]This paper focuses on the experience of the Latin American economies namely, Brazil, Chile, and Colombia.

We do so extending the model by Barro and Gordon (1983b) by introducing a second entity which is the central bank which does not face the same trade-off as the government. That is, the loss function of the central bank only includes minimising the loss from inflation. A further modification is the introduction of non-linear updating of beliefs of agents. That is, agents instead of being rational and understanding the objectives of the central bank immediately, must learn about the objectives using the standard Kalman Filter. Subsequently, we explore the cost of disinflation by including a sequence of announced, and geometrically declining intermediate inflation targets. In this formulation of the model, agents must now evaluate the stance of the central bank based on the information they receive from the public announcements and realised inflation to forecast inflation for the next period.

To motivate our research question, we present time series evidence from three Latin American economies namely, Brazil, Chile and Colombia. We focus on these three economies for two main reasons. First, all three countries adopted similar measures to disinflate and stabilise inflation. Second, all three economies experienced similar shocks during the same period, restricting the feasible set of shocks we need to consider when modelling the disinflationary process.

Figures 1 - 3 delineate the evolution of inflation (blue solid line), inflation expectations[3] (red solid line) and the intermediate inflation targets (upper and median, dashed lines) for the three countries, respectively. The series cover the period ranging from January 1990 - January 2020 with the exception of Brazil[4]. All three countries unanimously, witness a decline in inflation until 1999, when they adopt inflation targeting as the monetary policy. Specifically, the decline was from hyperinflationary states to around 3% over the course of the decade through the use of intermediate inflation targets.

Two aspects of these countries' experiences are worth drawing attention to. First, all three countries after experiencing turbulent inflation in the late 1980s and early 1990s, introduced amendments in the constitution for the central bank. Table 1 lists the dates for the constitutional amendments. The figures present information immediately following the amendments. The amendments introduced a board of governors for the central bank which would have a few members appointed by the incumbent government but any new government would not have influence over. The amendment also led to central banks having full control over monetary, credit and foreign exchange matters[5].

Second, and crucially, the period prior to 1999, is the period where the three countries adopted what is referred to as intermediate inflation targets before assigning a medium to long term target associated with low and stable inflation. The reason to introduce

[3]The figures do not include a measure of inflation expectations prior to 1999, since most central banks only started tracking expectations post the adoption of inflation targeting.

[4]Brazil experienced very high inflation in the early 1990s, order of magnitude in the thousands. Thus, we exclude the data from this presentation.

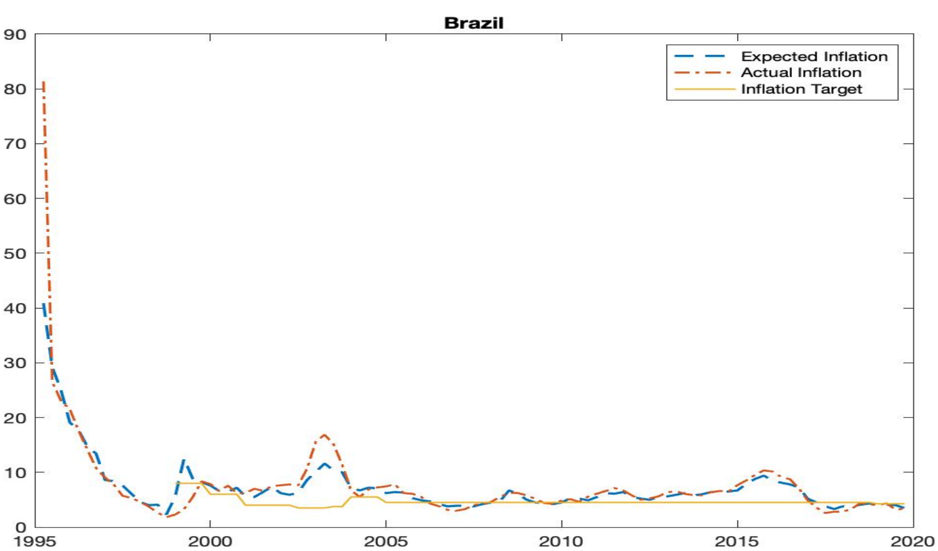
[5]For example, there was a constitutional amendment in Colombia in 1991

an intermediate inflation target is to build credibility for the central bank in order to meet the ultimate objective of price stability. Moreover, as Svensson (1999) notes, targets allows the monetary authority degrees of constrained discretion through a target horizon, escape clause, price index and range.

Concretely, focus on Figure 2, the experience of Chile. It announced an annual inflation target of 20% in September 1990 which was close to the average inflation rate during the 1980s[6]. The adoption of the target coincided with the independence of the central bank. From 1991-1999, the inflation target was linked to the current annual inflation forecast[7] of the central bank. This is known as the period where Chile was a soft inflation targeter. Colombia shares its experience with Chile in the process of disinflation. Colombia[8] also introduced a sequence of intermediate inflation targets in 1991 with a constitutional redesign of central bank which involved the central bank to be responsible for monetary, exchange, and credit policies. However, during the period of 1992-1999, there was significant deviation of inflation from the target. Therefore, during this period the central bank had low credibility.

Brazil is the most distinct case amongst the three countries in its process of disinflation. The process for stabilisation started in 1994. Brazil introduced an independent Monetary Policy Committee (Copom), whose members are the Governor and Deputy Governors. The Compon made decisions to raise the short term interest over the period between 1996 and 1999. Brazil did not have explicitly announced intermediate targets but rather focused on using monetary policy instruments to achieve disinflation.

Figure 1: Inflation Target, Inflation and Inflation Expectations: Brazil



[6]Based on Morandé (2002)

[7]The targets prior to 1999 are approximated based on Céspedes and Soto (2006)

[8]See also Gómez et al. (2002), Echavarría et al. (2013)

Figure 2: Inflation Target, Inflation and Inflation Expectations: Chile

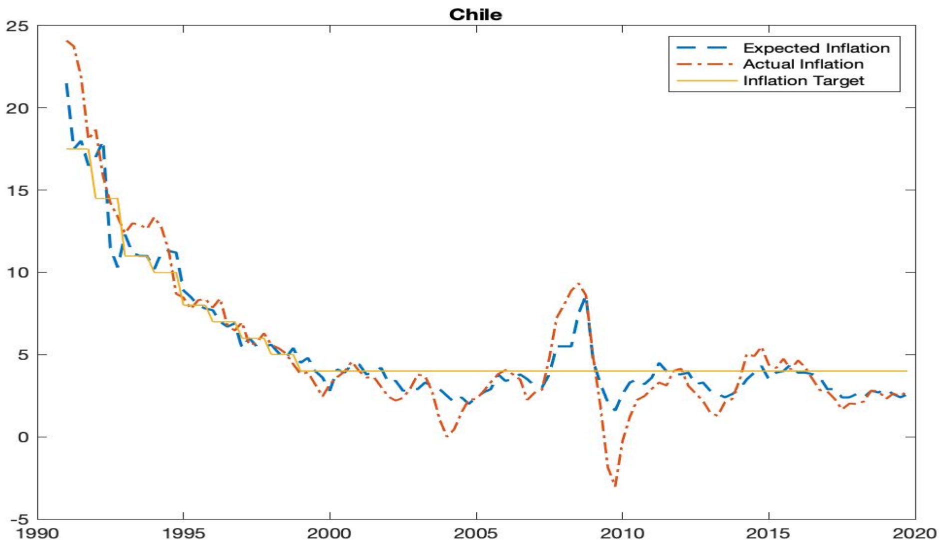
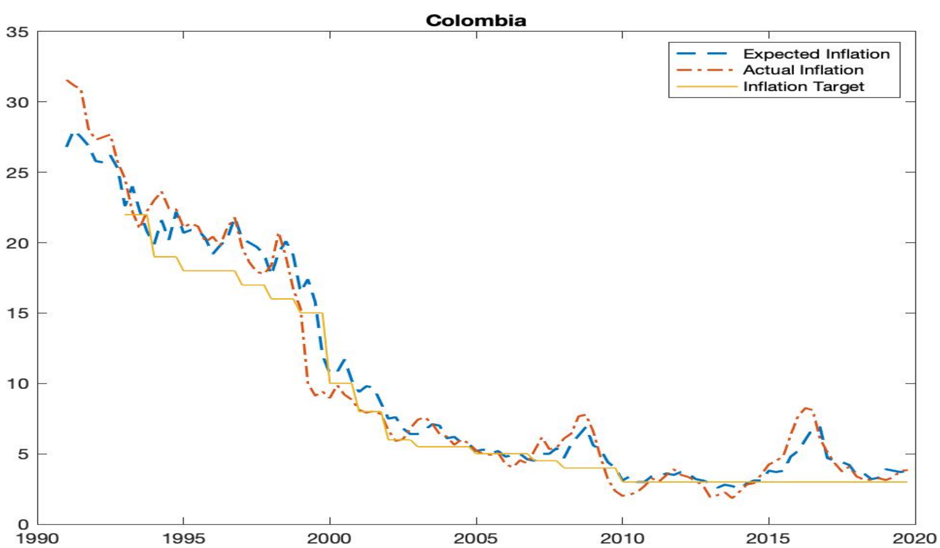


Figure 3: Inflation Target, Inflation and Inflation Expectations: Colombia



We hypothesise that a significant reduction in inflation came from using announced intermediate targets as a way to manage inflation expectations. Prior to the existence of an independent central bank and targets, the agents were familiar with what is commonly referred to as the Inflation Bias. However, the introduction of the new institution and policy objectives means the agents need to learn about a new policy environment. Moreover, if there is credibility in the institution with respect to the new policy, agents are consistently surprised and therefore, forced to adjust expectations. It is worth noting an important caveat here, we take for granted the relationship between inflation and output. That is, as demonstrated in the New Keynesian literature[9], with a higher price dispersion, there is a misallocation of resources which causes higher output loss.

Our model has three key predictions. First, high inflation volatility implies a higher disinflation cost and lower credibility of the central bank. This is driven by the difficulty for the agents to distinguish between introduction of the policy and objective of the central bank and exogenous variation of inflation. Thus, distorting the priors closer to the inflation bias levels. Second, higher the volatility of inflation, the more patient the government must be to introduce the reform. This is particularly true under the introduction of the independent central bank, alone. This is due to the central bank trying to reach optimal inflation (zero inflation) in every period from high levels of inflation.

Finally, when we introduce announced intermediate inflation targets, agents learn faster and the destabilisation to output is lower. The gains experienced from announcing the inflation targets come from a reduction in the inflation surprise that agents experience when forming expectations. That is, when agents observe actual inflation is close to announced inflation for a few periods, they adjust expectations in accordance to the announced policy. Thereby, increasing credibility for the central bank and reducing the cost of disinflation since deviation of inflation from expectations is small.

Therefore, by introducing announced intermediate targets the government is able to avoid a costly disinflation process or risk the economy falling into a recession. However, there is one caveat. On impact the welfare loss will be higher than that under the inflation bias case where there the government faces a trade-off between inflation and output. The gains from introducing an independent monetary authority and intermediate inflation targets will be long term gains.

Discussion of the Literature This paper speaks to four strands of literature. First, we build on the literature on optimal monetary policy rules and time inconsistency models by Kydland and Prescott (1977), Barro and Gordon (1983a), Barro and Gordon (1983b), and Barro (1986). As noted before, specifically building on Barro and Gordon (1983b). This paper also refers to the Inflation Bias which was first established in Kydland and Prescott (1977) (and later in Barro and Gordon (1983b)), which is the systematic difference between actual (realised) inflation and optimal inflation. We deviate from both papers by introducing an independent monetary authority which does not face a trade off between inflation and output. Therefore, agents must distinguish between the two institutions.

[9]See Clarida et al. (1999), Woodford and Walsh (2005), Galí (2015)

Second, this paper inserts self at the intersection of the the literature of disinflation and the literature on subjective expectations. Kostadinov and Roldán (2020) comes closest to the model we present in the subsequent sections however with some key deviations. The authors present a model where the government faces a trade-off between inflation and output but announces a sequence of inflation targets and the model is set up as a principal-agent model. In their paper, after the announcement of the targets, agents set expectations using Bayes's rule. Subsequently, the government then chooses inflation depending on the behavioural type it is. Therefore, agents must now distinguish whether the government is rational or of a behavioural type. On the other hand, the uncertainty in our paper is about the policy rather than the type of the agent. That is, from the perspective of the agents both the central bank and government are rational but they do not know the policy that is being followed by the new institution. Other papers which also build on type preferences of the government are Lu (2013) and Lu et al. (2016).

This paper also closely relates to Cukierman and Meltzer (1986) specifically the mechanism which prescribes that monetary surprises may lead to future higher inflation expectations. However, their paper assumes that agents are rational but have limited information about the monetary procedures. Moreover, they develop a model with discretionary policy. Our paper assumes that agents are Bayesian learners where they forecast the future taking into consideration all past information. Furthermore, the announcement of the future policy path acts as a commitment device which the central bank cannot renege on.

Ascari and Ropele (2012), Lamla and Vinogradov (2019) and Lamla et al. (2019) also ascertain the cost of disinflation and credibility. They do not do so from the context of the Latin American economies or the introduction of announcements of the policy. Using a model where agents lose trust in the announcements of the monetary authority, Lamla et al. (2019) show that it is possible to have an inflationary and deflationary bias. On the other hand, Lamla and Vinogradov (2019) looks at how central bank announcements effects consumers' beliefs using Micro data and 12 FOMC announcements. Ascari and Ropele (2012) employ money supply and interest based rules to test the different speeds at which disinflation can take place through a New Keynesian model.

Third, our paper ties into the literature on adaptive learning. Specifically, Marcet and Nicolini (2003) and Sargent et al. (2009). Both the above mentioned papers focus on the case of the South American context using bounded rationality. However, both papers study the hyperinflationary phases in these economies. Specifically, they explain how a combination of beliefs and debt dynamics were responsible for the hyperinflation experienced in these economies. That is, both papers are able to explain the behaviour of prices based on deviations from rationality. However, none of the papers focus on disinflation in the economies. Moreover, the period of analysis is a decade apart from our paper.

Finally, our paper adds to the discussion surrounding the Delphic and Odyssean view of forward guidance, see for instance Bassetto (2019). The Odyssean view refers to the announcement of a future course of action by the central bank. On the other hand, under the Delphic view, the central bank signals some private information about the state of the economy. Our set up, while closely related to the Odyssean view, adds an additional layer. The paper depicts that announced policy changes can help build credibility if the policy is delivered ex-post. This is true when ex-ante the participants in the economy do not believe the policy.

Road map The remainder of this paper is divided as follows. Section 2 presents four models which discuss the introduction of a central bank, followed by the introduction of intermediate targets which are announced or unannounced. Section 3 discusses the results of the model, detailing the welfare gains from the policy interventions and the role of inflation surprises. Finally, the paper concludes in section 4.

2 Model

This section details a model based on the early work of Barro and Gordon (1983b). We develop a model with a government which introduces an independent institution commonly known as the central bank. The central bank's sole responsibility is to minimise inflation. Subsequently, the central bank also introduces intermediate inflation targets which are announced. We aim to compute the welfare loss that occurs in both states of the world. In addition, understand when it is optimal to introduce an independent institution to help the government achieve its objective of lower welfare loss and output volatility.

2.1 Baseline Model

We begin by assuming an economy that only has a government that seeks to maximize social welfare. Which is approximated by a social welfare loss is given by,

$$L^G = \pi_t^2 - a\tilde{y} \tag{1}$$

Where, $\tilde{y}$[10] is the output gap and is weighted by a parameter a, and present inflation (π_t). The IS Curve is defined as the deviation of inflation from expectations weighted by a parameter c.

$$\tilde{y} = c(\pi_t - \pi_t^e) \tag{2}$$

The government can partly determine inflation. We assume the government has the capability to set the expected level of inflation $E_t(\pi_t)$ that we denote by $\bar{\pi}_t$. Realized

[10] Welfare function used in Woodford and Walsh (2005), Barro and Gordon (1983b), and that can be microfounded using a quadratic approximation to the utility function of the consumer's maximisation problem.

inflation is given by

$$\pi_t = \bar{\pi}_t + \epsilon_t \tag{3}$$

Where $\epsilon \sim iiN(0, \tilde{\sigma}_\epsilon^2)$ is a component of inflation the government cannot control and is independent of $\bar{\pi}_t$. The problem of the government is then to given by

$$\max_{\{\bar{\pi}_t\}_{t=1}^{\infty}} \bar{\pi}_t = \frac{ac}{2} \tag{4}$$

minimize the discounted Using the government's loss function we can compute optimal inflation which is,

$$\bar{\pi}_t = \frac{ac}{2} \tag{5}$$

Actual Inflation in this model is therefore,

$$\pi_t = \frac{ac}{2} + \epsilon_t \tag{6}$$

Solving for the welfare loss subject to a rational expectations equilibrium (REE). That is, agents already know a.

$$L^G = \frac{1}{1-\beta}\left[\frac{ac}{2} + \tilde{\sigma}_\epsilon^2\right] \tag{7}$$

Similar to Barro and Gordon (1983b), we assume that the government follows discretionary monetary policy and therefore faces an inflation bias. The inflation bias is the result of a gain in short-term welfare[11] by a reduction in output loss, allowing for inflation to rise.

To mitigate the welfare loss from excess inflation in the long run, as seen in practice, we now introduce an independent institution whose objective is to control inflation and prevent the deviation of inflation from the optimal level. That is, we now introduce an independent central bank. Thus, the loss function of the Central Bank is given by the following,

$$L^{CB} = \pi_t^2 \tag{8}$$

The rational expectation model implies that $\pi_t = \pi_t^e$. Therefore, the loss from the output gap is zero. Therefore, actual inflation is the only variable that impacts the welfare of the economy.

Notice, the central bank is not constrained by output, unlike the government. Therefore, the incentive to deviate from a policy that requires commitment to the target inflation is zero. Moreover, optimal inflation in this environment is also zero.

However, this institutional environment is new for the agents, who are forming expectations about inflation. Therefore, they perceive that the central bank follows a

[11] Often for electoral gain

loss function similar to the one of the government. Thus, the agents' Perceived Loss Function is given by,

$$L^A = \pi_t^2 - \tilde{a}\tilde{y} \tag{9}$$

Agents therefore now have to learn about $\tilde{a}$ which is the weight that the central bank attaches to the output gap (zero, in this case). While knowing the government's loss function is an extreme assumption, we use the fact that the agents have been in the regime that produces an inflation bias for a long time such that they have learned the government's objective.

Let the prior be given by $a \sim \mathcal{N}(\tilde{a}_0, \tilde{\sigma}_0^2)$. We can now write the state space for the agents in the following way,

$$\pi_t = \frac{ac}{2} + \epsilon_t \tag{10}$$

$$a_t = a_{t-1} + \eta_t \tag{11}$$

Where $\epsilon \sim ii\mathcal{N}(0, \tilde{\sigma}_\epsilon^2)$ and $\eta \sim ii\mathcal{N}(0, \tilde{\sigma}_\eta^2)$.

With the updating equations given by,

$$\tilde{a}_t = \tilde{a}_{t-1} + K_t\Big(\pi_t - \frac{\tilde{a}_{t-1}c}{2}\Big) \tag{12}$$

$$\tilde{\sigma}_t^2 = \tilde{\sigma}_{t-1}^2 - K_t\Big(\frac{c}{2}\Big)\tilde{\sigma}_{t-1}^2 + \tilde{\sigma}_\eta^2 \tag{13}$$

The interpretation of the updating equations is the standard one where agents update $\tilde{a}$ based on the observed inflation adjusting with the optimal Kalman Gain, K_t, which is given by,

$$K_t = \Big(\frac{\tilde{\sigma}_{t-1}^2(\frac{c}{2})}{(\frac{c}{2})^2\tilde{\sigma}_{t-1}^2 + \tilde{\sigma}_\epsilon^2}\Big) \tag{14}$$

Based on the above information, we have that inflation expectations are given by,

$$\pi_{t+1|t}^e = \pi_{t|t-1}^e + \left(\frac{\tilde{\sigma}_{t-1}^2\,(c/2)^2}{\left((c/2)^2\,\tilde{\sigma}_{t-1}^2 + \tilde{\sigma}_\epsilon^2\right)}\right)\left(\bar{\pi}_t - \pi_{t|t-1}^e + \epsilon_t\right) \tag{15}$$

Therefore, the sequence of expected inflation is dependent on the sequence of $\bar{\pi}_t$ and the exogenous variation in inflation and the variance of the prior $\tilde{\sigma}_0^2$, which can be seen in equation 16.

As agents learn about a, there is a gradual decline in inflation expectations and overall inflation. Moreover, the variation in inflation comes from an exogenous shock, for example oil price shocks which were common in these economies for the period considered.

2.2 Welfare Loss

Since there is now an independent central bank with $a = 0$, and optimal inflation $\bar{\pi}_t = 0$, inflation is given by $\pi_t = \epsilon_t$ and therefore the sequence of inflation expectations are given by,

$$E_0 \left\{ \pi^e_{t|t-1} \right\} = \left(\frac{\tilde{\sigma}^2_\epsilon}{\left((c/2)^2 \, \tilde{\sigma}^2_{t-1} + \tilde{\sigma}^2_\epsilon \right)} \right) E_0 \left\{ \left(\pi^e_{t-1|t-2} \right) \right\} \tag{16}$$

To simplify the expectations, let us define the following,

$$\kappa_{t-1} = \frac{\tilde{\sigma}^2_\epsilon}{\left((c/2)^2 \, \tilde{\sigma}^2_{t-1} + \tilde{\sigma}^2_\epsilon \right)} \tag{17}$$

The above expression allows us to re-write expectations as,

$$E_0 \left\{ \pi^e_{t|t-1} \right\} = \kappa_{t-1} E_0 \left\{ \left(\pi^e_{t-1|t-2} \right) \right\} \tag{18}$$

Iterating backwards and writing expectations today as a function of the prior π^e_0 we have,

$$E_0 \left\{ \pi^e_{t|t-1} \right\} = \left(\prod_{j=1}^{t} \kappa_{t-j} \right) \left(\frac{\tilde{a}_0 c}{2} \right) \tag{19}$$

With $\pi^e_0 = \frac{ac}{2}$. We make the assumption that the prior of the agent here is the rational expectation equilibrium, which is the inflation bias when only the government exists in the economy. This prior is not unreasonable since agents have been in the regime with the inflation bias for a long time and therefore do not have reason to believe that the introduction of a new institution will change anything.

We interpret $\kappa_t \in (0, 1)$[12] as the persistence in inflation expectations.

Therefore, κ_t informs us about how the relationship between expectations in period t and $t + 1$ depend on the inflation expectations in the previous periods. As $t \to \infty$, inflation expectations gradually tend toward optimal inflation, which is zero in the case where we have independent central bank. This is because $\kappa \to 1$ and $\prod_{j=0}^{\infty} \kappa_t \to 0$.

Now that we have the building blocks of our model (equations 1 to 19), we can define the expected welfare loss in period $t = 0$.

[12] Notice, $\kappa_t = 1 - K_t$ (Kalman Gain)

$$E\{\mathcal{L}_0\} = E\left\{\sum_{t=0}^{\infty}\beta^t\left((\pi_t)^2 - \tilde{a}\tilde{y}_t\right)\right\}$$

$$E\{\mathcal{L}_0\} = E\left\{\sum_{t=0}^{\infty}\beta^t\left((\epsilon_t)^2 - \tilde{a}\tilde{y}_t\right)\right\}$$

$$= E\left\{\sum_{t=0}^{\infty}\beta^t\tilde{a}\tilde{y}_t\right\} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{20}$$

Substituting for $\tilde{y}_t = c(\epsilon_t - \pi^e_{t|t-1})$ and $E_0\left\{\pi^e_{t|t-1}\right\} = \left(\prod_{j=1}^{t}\kappa_{t-j}\right)\left(\frac{\tilde{a}_0 c}{2}\right)$ we have that,

$$E\{L_0^{G,CB}\} = \frac{ac^2}{2}\sum_{t=0}^{\infty}\beta^t\tilde{a}_0\left(\prod_{j=1}^{t}\kappa_{t-j}\right) + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{21}$$

The present value of the welfare loss given that agents learn $\tilde{a}$ over time, is given in equation 21.

Let us compute the present value of the welfare loss in the event there is no new institution and the government maintains an inflation bias. With no new institution (central bank) we will have that optimal inflation is given by, $\bar{\pi}_t = \frac{ac}{2}$ and agents don't learn a. Therefore,

$$E\left\{L_0^G\right\} = \left(\frac{ac}{2}\right)^2\sum_{t=0}^{\infty}\beta^t + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{22}$$

The difference then between equation 22 and 21 is the term in blue. Notice, the welfare loss when agents are learning a depends on the prior $\tilde{a}_0$ and κ_t. The term in blue suggests that the higher the prior the greater the loss of introducing a central bank. This is because, agents will take longer to learn about the new institution and its objective.

Moreover, an important distinction between the welfare loss with and without the central bank is optimal inflation. Prior to the introduction of a central bank, the government always has the incentive to deviate from any announced inflation for short term welfare gains thus resulting in the inflation bias. With the central bank now responsible for controlling inflation, this incentive to deviate for welfare gain not exist anymore. This leads to actual inflation now only being subject to exogenous variation.

To further understand whether the introduction of a central bank is beneficial for the economy we define a parameter ϑ_t. In order to do so, we re-write κ_t as,

$$\kappa_t = \frac{1}{\left((c/2)^2\frac{\tilde{\sigma}_{t-1}^2}{\tilde{\sigma}_\epsilon^2} + 1\right)} \tag{23}$$

Therefore, now we can define,

$$\frac{\tilde{\sigma}_t^2}{\tilde{\sigma}_\epsilon^2} = \left(\frac{1}{(c/2)^2 + \frac{\tilde{\sigma}_\epsilon^2}{\tilde{\sigma}_{t-1}^2}} \right) + \frac{\tilde{\sigma}_\eta^2}{\tilde{\sigma}_\epsilon^2} \tag{24}$$

We now define ϑ_t as the following,

$$\vartheta_t = \frac{\tilde{\sigma}_t^2}{\tilde{\sigma}_\epsilon^2} = \left(\frac{1}{(c/2)^2 + \frac{\tilde{\sigma}_\epsilon^2}{\tilde{\sigma}_{t-1}^2}} \right) + \frac{\tilde{\sigma}_\eta^2}{\tilde{\sigma}_\epsilon^2} \tag{25}$$

$$= \left(\frac{1}{(c/2)^2 + \frac{1}{\vartheta_{t-1}}} \right) + \frac{\tilde{\sigma}_\eta^2}{\tilde{\sigma}_\epsilon^2} \tag{26}$$

Now, the sequence of $\{\vartheta_t\}_{t=0}^{\infty}$ is determined by the initial value ϑ_0 which is given by $\vartheta_0 = \frac{\tilde{\sigma}_0^2}{\tilde{\sigma}_\epsilon^2}$.

$$\vartheta_t = \left(\frac{1}{(c/2)^2 \vartheta_{t-1} + 1} \right) \vartheta_{t-1} + \frac{\tilde{\sigma}_\eta^2}{\tilde{\sigma}_\epsilon^2} \tag{27}$$

ϑ_t can be interpreted as a persistence parameter which relies on the variation of the prior $\tilde{\sigma}_0^2$ (or how $\tilde{a}$ is centred around a) and the exogenous variation in inflation, $\tilde{\sigma}_\epsilon^2$. Therefore, the tighter the prior (small $\tilde{\sigma}_0^2$), the higher is the persistence in inflation expectations and the longer it will take the agents to learn about the objective of the central bank. Consequently, the welfare loss from having a central bank will be higher.

On the other hand, higher exogenous variation to inflation implies a lower ϑ_t and therefore higher persistence in inflation expectations. Which would also then lead to a higher welfare loss from the introduction of a central bank.

There are several things to note here. First, if two economies have the same ϑ_0, they will have the same welfare loss from changing the policy. Therefore, it is not only the variation of inflation that is important but also the prior that agents in the economy hold. Ideally, we would like that the agents have a lightly held prior ($\tilde{a}$ is far from a), that is they believe that the central bank is independent of the government and will commit and achieve the target that has been set.

Thus far, it seems that introducing a central bank if there is a tight prior implies a higher welfare cost for the economy. This leads to the natural question of whether there is a level of patience which would incline the government to introduce a central bank for each possible ϑ_0. That is, is it possible to find that a change in policy is optimal for a government irrespective of the persistence in expectations?

To answer this question we limit ourselves to the very simple case where $\vartheta_0 = 0$. This outcome is possible if $\tilde{a}_0 = a$ or if $\epsilon \to \infty$. That is, either the agents don't believe the new institution at all or external shocks to inflation are high. With $\vartheta_0 = 0$, the whole sequence $\{\vartheta_t\}_{t=0}^{\infty}$ will be zero (see equation 27). This implies $\kappa_t = 0$ (refer to equation 23). This implies that the welfare loss with the central bank is now,

$$E\left\{\mathcal{L}_0^{G,CB}\right\} = \frac{a\tilde{a}_0 c^2}{2}\sum_{t=0}^{\infty}\beta^t + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} = \frac{a\tilde{a}_0 c^2}{2}\frac{1}{1-\beta} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{28}$$

Now, the change in policy is optimal if the following is true,

$$\begin{aligned} \frac{a\tilde{a}_0 c^2}{2}\frac{1}{1-\beta} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} &\leq \left(\frac{ac}{2}\right)^2\frac{1}{1-\beta} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \\ \tilde{a}_0 &\leq \frac{a}{2} \end{aligned} \tag{29}$$

Therefore, even in the case where the agents do not learn about the introduction of a new institution and its objective, it is optimal to introduce a central bank as long as the prior after the change is low enough. Specifically, half the prior based on the government's objective.

Let us now assume that $\vartheta_0 = e$ where $e \to 0$. That is, agents hold a loose prior compared to the case where $\vartheta_0 = 0$. Let us now compute the welfare loss in this case. If this were the case, we know that the maximum value of $\prod_{j=1}^{t}\kappa_{t-j} \in [0,1]$. Therefore, the upper bound on the product is one. Therefore,

$$E\{\mathcal{L}_0^{G,CB}\} = \frac{a\tilde{a}_0 c^2}{2}\sum_{t=0}^{\infty}\beta^t\left(\prod_{j=1}^{t}\kappa_{t-j}\right) + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} < \frac{a\tilde{a}_0 c^2}{2}\left(1 + \sum_{t=1}^{\infty}\beta^t\kappa_0\right) + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{30}$$

$$\implies E\{\mathcal{L}_0^{C,GB}\} < \frac{a\tilde{a}_0 c^2}{2}\left(\frac{1}{1-\beta} - \beta\frac{1-\kappa_0}{1-\beta}\right) + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{31}$$

For the policy to be optimal, a sufficient condition would be, given $\tilde{a} = a$ we will have,

$$\frac{a\tilde{a}_0 c^2}{2}\left(\frac{1}{1-\beta} - \beta\frac{1-\kappa_0}{1-\beta}\right) + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} < \left(\frac{ac}{2}\right)^2\frac{1}{1-\beta} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta}\frac{a\tilde{a}_0 c^2}{2}\left(\frac{1}{1-\beta} - \beta\frac{1-\kappa_0}{1-\beta}\right) < \left(\frac{ac}{2}\right)^2\frac{1}{1-\beta} \tag{32}$$

This implies that,

$$\frac{1}{2}\frac{\left((c/2)^2\,\tilde{\sigma}_0^2 + \tilde{\sigma}_\epsilon^2\right)}{(c/2)^2\,\tilde{\sigma}_0^2} < \beta \tag{33}$$

This generalises the result that $\beta > \frac{1}{2}$ when $\tilde{\sigma}_\epsilon^2 = 0$. Therefore, when there is a tight prior $\tilde{a} \approx a$ and volatile inflation $\tilde{\sigma}_\epsilon^2$, the larger the patience of a planner to implement a new institution and a policy change. It is also important to note here, the larger the prior, the higher the welfare cost of introducing a central bank. Moreover, we do not have any cost of output volatility in this model. This is because a loose prior will imply larger volatility to output since expectations react more to new information. However, for the purposes of this paper, we shut down this channel.

The above discussion raises an important conclusion. Given the presence of a central bank, if agents notice high inflation in the previous period, they may be unable to discern if the higher inflation is a result of high exogenous variation to inflation or the central bank facing the same trade-off between inflation and output as the government. Thus, in order to aid the agents' expectation formation process, the central bank might want to use intermediate announcements as a way to gain credibility. We build on this idea in the following section where we introduce the idea of intermediate inflation targets which were used by the Latin American economies in order to bring inflation under control.

2.3 Disinflation with Intermediate Targets

Given the economic environment above, we now extend the model to now include intermediate inflation targets. This was one of the main features of the Latin American economies when they started the process of dis-inflating their economies. Introducing a simple parameter to the model discussed above we hope to rationalise the choice of introducing these targets as they lead to lower output costs and increased credibility.

We do this in two stages. First, we introduce targets. However, those targets are not announced to the public. Therefore, it as almost as a new policy does not exist for the agents. Notice, here we refer to the policy of the intermediate targets as the new policy. In the second stage, we introduce announced inflation targets and check how that effects inflation expectations and subsequently the welfare of the economy.

2.3.1 Without Announcements

We begin with intermediate targets which are not announced. As before, the independent central bank does not face a trade-off between inflation and output therefore $a = 0$ in the central bank loss function. Furthermore, it sets the sequence of inflation targets as $\bar{\pi}_t = 0$ for all t.

The loss function of the central bank is now given by,

$$\mathcal{L}^{CB} = (\pi_t - \pi_t^o)^2 \tag{34}$$

The perceived loss function by the agents is now given by,

$$\mathcal{L}^A = \gamma_t (\pi_t)^2 + (1 - \gamma_t) \left((\pi_t)^2 - a\tilde{y}_t\right) \tag{35}$$

If expectations are formed in accordance with equation 35, then optimal inflation by the central bank should be,

$$\bar{\pi}_t = \frac{ac}{2} - \gamma_t \frac{ac}{2} \tag{36}$$

Therefore, agents now must learn γ using the following state space,

$$\pi_t = \frac{ac}{2} + \gamma_t \frac{-ac}{2} + \epsilon_t \tag{37}$$

$$\gamma_t = \gamma_{t-1} + \eta_t \tag{38}$$

With the updating equations given by,

$$\tilde{\gamma}_t = \tilde{\gamma}_{t-1} + K_t \left(\pi_t - \frac{ac}{2} + \tilde{\gamma}_{t-1} \left(\frac{(ac)}{2} \right) \right) \tag{39}$$

$$K_t = \frac{\left(\tilde{\sigma}_{t-1|t-1}\right) \frac{(-ac)}{2}}{\left(\left(\frac{(-ac)}{2} \right)^2 \left(\tilde{\sigma}_{t-1|t-1}\right) + \tilde{\sigma}_\epsilon^2 \right)} \tag{40}$$

$$\tilde{\sigma}_{t|t} = \left(1 + \frac{1}{\left(1 + \frac{\tilde{\sigma}_\epsilon^2}{\left(\tilde{\sigma}_{t-1|t-1}\right)\left(\frac{(ac)}{2}\right)^2} \right)} \right) \tilde{\sigma}_{t-1|t-1} + \tilde{\sigma}_\eta^2 \tag{41}$$

Replace the fact that $\pi_t = \pi_t^o + \epsilon_t$ we get,

$$\tilde{\gamma}_t = \tilde{\gamma}_{t-1} + K_t \left(\pi_t^o - \frac{ac}{2} + \gamma_{t-1} \left(\frac{ac}{2} \right) + \epsilon_t \right) \tag{42}$$

Subsequently, inflation expectation are given by,

$$\pi_{t+1|t}^e = \pi_{t|t-1}^e - K_t \left(\frac{ac}{2} \right) \left(\pi_t^o + \epsilon_t - \pi_{t|t-1}^e \right) \tag{43}$$

Based on the equations above, we can now compute the welfare loss in an economy where there are inflation targets which are not announced,

$$\max_{\{\bar{\pi}_t\}_{t=0}^{\infty}} E\Big\{ \sum_{t=0}^{\infty} \beta^t \left(a\tilde{y}_t + (\pi_t)^2 \right) \Big\} \tag{44}$$

$$\text{s.t.} \qquad \pi_{t+1|t}^e = \pi_{t|t-1}^e - \left(\frac{\left(\tilde{\sigma}_{t-1}^2\right) (ac/2)^2}{\left((ac/2)^2 \, \tilde{\sigma}_{t-1}^2\right) + \tilde{\sigma}_\epsilon^2} \right) \left(\pi_t^o - \pi_{t|t-1}^e + \epsilon_t \right) \tag{45}$$

Therefore, having inflation targets but not announcements of those targets implies that inflation expectations will follow the same path as if there were no intermediate inflation targets. This implies that for the agents, their information set is no different between having an independent central bank and an independent central bank who has targets but are those which are not public information.

We can now write the loss function in the following way,

$$E\{\mathcal{L}_0^G\} = E\left\{\sum_{t=0}^{\infty}\beta^t\left((\pi_t)^2 - \tilde{a}\tilde{y}_t\right)\right\} \tag{46}$$

Every period $\pi_t = \pi_t^o + \epsilon_t$ and $\pi_t^o = \rho^t\left(\frac{ac}{2}\right)$ consequently the expected loss we have is,

$$E\{\mathcal{L}_0\} = E\left\{\sum_{t=0}^{\infty}\beta^t\left(a\tilde{y}_t\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{47}$$

Where output is given by $\tilde{y}_t = c(\pi_t - \pi^e_{t|t-1}) = c\left(\rho^t\left(\frac{-ac}{2}\right) + \epsilon_t - \pi^e_{t|t-1}\right)$ and the expected value at $t=0$ is

$$E_0\left(\tilde{y}_t\right) = c\left(\rho^t\left(\frac{ac}{2}\right) - E_0\left\{\pi^e_{t|t-1}\right\}\right) \tag{48}$$

We know the following,

$$\pi^e_{t+1|t} = \gamma_t\frac{ac}{2} - \frac{ac}{2} \tag{49}$$

Plug in the value of γ_t from the updating equations,

$$\pi^e_{t+1|t} = \underbrace{\left(\gamma_{t-1} + \frac{\left(\tilde{\sigma}_{t-1|t-1}\right)\frac{(ac)}{2}}{\left(\left(\frac{(ac)}{2}\right)^2\left(\tilde{\sigma}_{t-1|t-1}\right) + \tilde{\sigma}_\epsilon^2\right)}\left(\pi_t + \frac{ac}{2} - \gamma_{t-1}\left(\frac{(ac)}{2}\right)\right)\right)}_{A}\frac{ac}{2} - \frac{ac}{2} \tag{50}$$

To compute the expected sequence of inflation expectations, we need to compute $E_0\gamma_{t-1}$ (given by A).

$$E_0\gamma_{t-1} = E_0\Bigg(\gamma_{t-2} + \frac{\left(\tilde{\sigma}_{t-2|t-2}\right)\frac{(ac)}{2}}{\left(\left(\frac{(ac)}{2}\right)^2\left(\tilde{\sigma}_{t-2|t-2}\right) + \tilde{\sigma}_\epsilon^2\right)}\left(\pi_{t-1} + \frac{ac}{2} - \gamma_{t-2}\left(\frac{(ac)}{2}\right)\right)\Bigg) \tag{51}$$

Which can be re-written the following way,

$$E_0\gamma_{t-1} = E_0\Big(\frac{\tilde{\sigma}_\epsilon^2}{\Big(\left(\frac{(ac)}{2}\right)^2\left(\tilde{\sigma}_{t-2|t-2}\right)+\tilde{\sigma}_\epsilon^2\Big)}\gamma_{t-2}+\frac{\left(\tilde{\sigma}_{t-2|t-2}\right)\frac{(ac)}{2}}{\Big(\left(\frac{(ac)}{2}\right)^2\left(\tilde{\sigma}_{t-2|t-2}\right)+\tilde{\sigma}_\epsilon^2\Big)}\Big(\pi_{t-1}+\frac{ac}{2}\Big)\Big) \tag{52}$$

Since, $\pi_{t-1} = \pi_{t-1}^o + \epsilon_{t-1}$, $\pi_{t-1}^o = \rho^{t-1}\left(\frac{ac}{2}\right)$ and $E_0\epsilon_{t-1} = 0$ we have the following,

$$E_0\gamma_{t-1} = E_0\Big(\frac{\tilde{\sigma}_\epsilon^2}{\Big(\left(\frac{(ac)}{2}\right)^2\left(\tilde{\sigma}_{t-2|t-2}\right)+\tilde{\sigma}_\epsilon^2\Big)}\gamma_{t-2}+\frac{\left(\tilde{\sigma}_{t-2|t-2}\right)\frac{(ac)}{2}}{\Big(\left(\frac{(ac)}{2}\right)^2\left(\tilde{\sigma}_{t-2|t-2}\right)+\tilde{\sigma}_\epsilon^2\Big)}\Big(\rho^{t-1}\frac{ac}{2}+\frac{ac}{2}\Big)\Big) \tag{53}$$

Let $\kappa_{t-1} = \frac{\tilde{\sigma}_\epsilon^2}{\left(\left(\frac{(ac)}{2}\right)^2\left(\tilde{\sigma}_{t-2|t-2}\right)+\tilde{\sigma}_\epsilon^2\right)}$. Therefore, re-writing the previous equation, we have the following,

$$E_0\gamma_{t-1} = E_0\Big(\kappa_{t-1}\gamma_{t-2} + (1-\kappa_{t-1})(1+\rho^{t-1})\Big) \tag{54}$$

Let's expand the previous equation in the following way,

$$\Rightarrow \kappa_{t-1}\kappa_{t-2}\kappa_{t-3}E_0\gamma_{t-4} + \kappa_{t-1}\kappa_{t-2}(1-\kappa_{t-3})(1+\rho^{t-3}) + \kappa_{t-1}(1-\kappa_{t-2})(1+\rho^{t-2}) + (1-\kappa_{t-1})(1+\rho^{t-1}) \tag{55}$$

$$\Rightarrow \kappa_{t-1}\kappa_{t-2}\kappa_{t-3}E_0\gamma_{t-4} + \kappa_{t-1}\kappa_{t-2}(1-\kappa_{t-3}) + \kappa_{t-1}(1-\kappa_{t-2}) + (1-\kappa_{t-1})$$
$$+ \kappa_{t-1}\kappa_{t-2}(1-\kappa_{t-3})\rho^{t-3} + \kappa_{t-1}(1-\kappa_{t-2})\rho^{t-2} + (1-\kappa_{t-1})\rho^{t-1} \tag{56}$$

$$\Rightarrow \kappa_{t-1}\kappa_{t-2}\kappa_{t-3}E_0\gamma_{t-4} + \kappa_{t-1}\kappa_{t-2}\kappa_{t-3} + \kappa_{t-1}\kappa_{t-2}(1-\kappa_{t-3})\rho^{t-3} + \kappa_{t-1}(1-\kappa_{t-2})\rho^{t-2} + (1-\kappa_{t-1})\rho^{t-1} \tag{57}$$

We now use this expression to build the welfare loss as in the previous cases. Details will follow in section 3.

2.3.2 With Announcements

We now introduce an alternative policy environment where there is an announcement of a sequence of inflation targets given by $\{\pi_t^o\}_{t=0}^{\infty}$. In order to make the model simple and comparable to the previous model we assume that the inflation targets are given by $\pi_t^o = \rho^t\left(\frac{ac}{2}\right)$ with $\rho \in (0,1)$. This is turn implies that, $\pi_t^o = \rho\pi_{t-1}^o$. That is, every subsequent period the target is a ratio ρ of the inflation target in the previous period. Under the presence of a central bank, the optimal inflation is still zero. However, the cost of zero inflation is very high for the government as it leads to a high loss in output.

Therefore, the government decides to introduce intermediate targets which allow for a gradual decline in inflation.

We make another assumption about who sets the inflation targets. In this model, the government does not delegate the decision of the targets to the central bank. The central bank is only responsible for the implementation of the inflation targets. This assumption is not unusual since most monetary policy committees have a few members who are from the government including the Finance Minister.

Given the addition of the new policy, the welfare loss function of the independent central bank is given by the deviation from these announced inflation targets,

$$\mathcal{L}^{CB} = (\pi_t - \pi_t^o)^2 \tag{58}$$

The perceived loss function by the agents is now given by,

$$\mathcal{L}^{A} = \gamma_t \left(\pi_t - \pi_t^o\right)^2 + (1-\gamma_t)\left((\pi_t)^2 - a\tilde{y}_t\right) \tag{59}$$

Thus, the agents think that the loss function is a weighted combination of the central bank loss function and the government's loss function. The loss function for the government and then process for actual inflation remain the same as in the previous model. Notice, agents now learn about γ instead of learning about a. Where γ measures the likelihood individuals think the central bank is only committed to the target.

This is so that we are able to compute optimal inflation when agents do not know if the central bank is following the implemented policy or face the same trade-off as the government.

The optimal inflation level for a given γ is given by

$$\implies \bar{\pi}_t = \gamma_t(\pi_t^o + \frac{ac}{2}) - \frac{ac}{2} \tag{60}$$

Learning about γ

As before, we have the agents learning about γ which is based on the observed level of inflation in the economy. As before, the agents are Bayesian and therefore the state space is given by,

$$\bar{\pi}_t = \gamma_t(\pi_t^o + \frac{ac}{2}) - \frac{ac}{2} + \epsilon_t \tag{61}$$

$$\gamma_t = \gamma_{t-1} + \eta_t \tag{62}$$

The agents observe the current level of inflation to make an inference about the policy being followed by the central bank. Therefore, the agents are learning about the fact that $\gamma = 0$ such that the only thing that matters for the central bank policy is for inflation to be at target.

The updating equations with respect to the Kalman Filter are given by,

$$\gamma_t = \gamma_{t-1} + K_t\left(\pi_t + \frac{ac}{2} - \gamma_{t-1}\left(\pi_t^o + \frac{(ac)}{2}\right)\right) \tag{63}$$

$$K_t = \frac{\left(\tilde{\sigma}_{t-1}^2\right)\left(\pi_t^o + \frac{(ac)}{2}\right)}{\left(\pi_t^o + \frac{(ac)}{2}\right)^2\left(\tilde{\sigma}_{t-1}^2\right) + \tilde{\sigma}_\epsilon^2} \tag{64}$$

$$\tilde{\sigma}_{t|t} = \left(1 - \frac{1}{\left(1 + \frac{\tilde{\sigma}_\epsilon^2}{\left(\tilde{\sigma}_{t-1|t-1}\right)\left(\pi_t^o + \frac{(ac)}{2}\right)^2}\right)}\right)\tilde{\sigma}_{t-1|t-1} + \tilde{\sigma}_\eta^2 \tag{65}$$

Using the updating equations, we can re-write the updating equations as below,

Replacing the equation for inflation ($\pi_t = \pi_t^o + \epsilon_t$) we have,

$$\tilde{\gamma}_t = \tilde{\gamma}_{t-1} + \frac{\left(\tilde{\sigma}_{t-1}^2\right)\left(\pi_t^o + \frac{(ac)}{2}\right)}{\left(\pi_t^o + \frac{(ac)}{2}\right)^2\left(\tilde{\sigma}_{t-1}^2\right) + \tilde{\sigma}_\epsilon^2}\left(\pi_t^o - \pi_{t|t-1}^e + \epsilon_t\right) \tag{66}$$

Replace the updating equation for γ_t in the previous equation to get the following,

$$\pi_{t+1|t}^e = \left(\tilde{\gamma}_{t-1} + \left(\frac{\left(\tilde{\sigma}_{t-1}^2\right)\left(\frac{(ac)}{2} + \pi_t^o\right)}{\left(\frac{(ac)}{2} + \pi_t^o\right)^2\left(\tilde{\sigma}_{t-1}^2\right) + \tilde{\sigma}_\epsilon^2}\right)\left(\pi_t + \frac{ac}{2} - \gamma_{t-1}\left(\pi_t^o + \frac{ac}{2}\right)\right)\right)\left(\frac{(ac)}{2} + \pi_{t+1}^o\right) - \frac{ac}{2}$$

$$\pi_{t+1|t}^e = \left(\frac{(ac)}{2} + \pi_{t+1}^o\right)\tilde{\gamma}_{t-1} + K_t\left(\pi_t + \frac{ac}{2} - \gamma_{t-1}\left(\pi_t^o + \frac{ac}{2}\right)\right)\left(\frac{(ac)}{2} + \pi_{t+1}^o\right) - \frac{ac}{2}$$

We can now add and subtract $\gamma_{t-1}\pi_t^o$ to write the following,

$$\pi_{t+1|t}^e = \pi_{t|t-1}^e + \gamma_{t-1}(\pi_{t+1}^o - \pi_t^o) + \left(\left(\frac{\left(\tilde{\sigma}_{t-1}^2\right)\left(\frac{(ac)}{2} + \pi_t^o\right)\left(\frac{(ac)}{2} + \pi_{t+1}^o\right)}{\left(\frac{(ac)}{2} + \pi_t^o\right)^2\left(\tilde{\sigma}_{t-1}^2\right) + \tilde{\sigma}_\epsilon^2}\right)\left(\pi_t - \pi_{t|t-1}^e\right)\right)$$

Unlike the previous model, where the only driving force of expectations was the prior of the agents and the exogenous volatility in inflation. Inflation expectations are now influenced by the change in the inflation target in the current and previous period, adjusted by the weight that agents attached to the loss function of the central bank in the previous period. The greater the value of γ, the tighter the prior or the lower the belief on the deviation of policy from the government's loss function.

The expected sequence of inflation expectations is given by

$$E_0\left\{\pi^e_{t+1|t}\right\} = \left(\frac{\tilde{\sigma}^2_\epsilon}{\left(\frac{(ac)}{2}+\pi^o_t\right)^2\tilde{\sigma}^2_{t-1}+\tilde{\sigma}^2_\epsilon}\right)E_0\left\{\pi^e_{t|t-1}\right\}$$
$$+\left(\frac{ac}{2}\rho^t\right)\left(\left(\frac{\tilde{\sigma}^2_{t-1}\left(\frac{(ac)}{2}+\pi^o_t\right)^2}{\left(\frac{(ac)}{2}+\pi^o_t\right)^2\tilde{\sigma}^2_{t-1}+\tilde{\sigma}^2_\epsilon}\right)-E_0\left\{\tilde{\gamma}_{t-1}\right\}(1-\rho)\right) \tag{67}$$

As before, inflation expectations depend on the variation of the prior and the variation in exogenous inflation. However, now the inflation targets set by the government is relevant for the expectations. In addition, ρ which is the speed at which the government resets the inflation target also defines expectations. γ which is the credibility parameter is also a determinant factor for inflation expectations.

As in section 2.1, let us define $\kappa_t = \frac{\tilde{\sigma}^2_\epsilon}{\left(\frac{(ac)}{2}+\pi^o_t\right)^2\tilde{\sigma}^2_{t-1}+\tilde{\sigma}^2_\epsilon}$ which allows us to re-formulate inflation expectations in the following way,

$$E_0\left\{\pi^e_{t+1|t}\right\} = \kappa_t E_0\left\{\pi^e_{t|t-1}\right\} + (1-\kappa_t)\left(\frac{ac}{2}\rho^t\right) - E_0\left\{\tilde{\gamma}_{t-1}\right\}\left(\frac{ac}{2}\rho^t\right)(1-\rho) \tag{68}$$

Thus, inflation expectations for tomorrow $(t+1)$ are expected to evolve as an average between expectations for t, the target at t with an adjustment on the change of the target $\left(\frac{ac}{2}\rho^t\right)(1-\rho)$ weighted by the expected credibility of the central bank $E_0\left\{\tilde{\gamma}_{t-1}\right\}$.

With a central bank that only cares about hitting the announced inflation target we have, $\gamma = 1 \rightarrow \bar{\pi}_t = \pi^o_t$. If $\bar{\pi}_t = \pi^o_t$ then there is exogenous variation in inflation still driving actual inflation. This would imply that the agent never believes the central bank? Unless, the agent is aware that the process of inflation is some mean inflation with shocks. Notice, agents are not learning about inflation they are learning about the objective of the central bank.

Loss from announcing targets

With this stochastic process for the dynamics of inflation, and inflation expectations we can now compute the present value of the policy change that is, the introduction of the intermediate central banks. The loss function is given by,

$$E\{\mathcal{L}^G_0\} = E\left\{\sum_{t=0}^{\infty}\beta^t\left(a\tilde{y}_t + (\pi_t)^2\right)\right\} \tag{69}$$

Every period $\pi_t = \pi_t^o + \epsilon_t$ and $\pi_t^o = \rho^t \left(\frac{ac}{2}\right)$ consequently the expected loss we have is,

$$E\{\mathcal{L}_0\} = E\left\{\sum_{t=0}^{\infty} \beta^t \left(a\tilde{y}_t\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{70}$$

Where output is given by $\tilde{y}_t = c(\pi_t - \pi^e_{t|t-1}) = c\left(\rho^t\left(\frac{ac}{2}\right) + \epsilon_t - \pi^e_{t|t-1}\right)$ and the expected value at $t = 0$ is

$$E_0\left(\tilde{y}_t\right) = c\left(\rho^t\left(\frac{ac}{2}\right) - E_0\left\{\pi^e_{t|t-1}\right\}\right) \, and \tag{71}$$

We can now use the updating equation for γ_t to compute the expected sequence of inflation expectations,

$$E_0\pi^e_{t+1|t} = E_0\Big(\tilde{\gamma}_t\left(\frac{(ac)}{2} + \pi^o_{t+1}\right) - \frac{ac}{2}\Big)$$

We can now compute the expected credibility of the central bank based on equation 66 and iterating backwards.

$$E_0\left\{\tilde{\gamma}_{t-1}\right\} = E_0\left\{\tilde{\gamma}_{t-2} + \left(\frac{(\tilde{\sigma}^2_{t-2})\left(\frac{(ac)}{2} + \pi^o_{t-1}\right)}{\left(\frac{(ac)}{2} + \pi^o_{t-1}\right)^2(\tilde{\sigma}^2_{t-2}) + \tilde{\sigma}^2_\epsilon}\right)\left(\pi^o_{t-1} + \frac{ac}{2} - \tilde{\gamma}_{t-2}\left(\frac{(ac)}{2} + \pi^o_{t-1}\right) + \epsilon_{t-1}\right)\right\} \tag{72}$$

$$= E_0\left\{\left(\frac{\tilde{\sigma}^2_\epsilon}{\left(\frac{(ac)}{2} + \pi^o_{t-1}\right)^2\tilde{\sigma}^2_{t-2} + \tilde{\sigma}^2_\epsilon}\right)\tilde{\gamma}_{t-2} + \left(\frac{\tilde{\sigma}^2_{t-2}\left(\frac{(ac)}{2} + \pi^o_{t-1}\right)^2}{\left(\frac{(ac)}{2} + \pi^o_{t-1}\right)^2\tilde{\sigma}^2_{t-2} + \tilde{\sigma}^2_\epsilon}\right)\right\} \tag{73}$$

$$\tag{74}$$

Iterating backwards we can write the following,

$$\begin{aligned}
&\kappa_{t-1}E_0\left\{\tilde{\gamma}_{t-2}\right\} + (1-\kappa_{t-1}) \\
&= \kappa_{t-1}\kappa_{t-2}\kappa_{t-3}E_0\left\{\tilde{\gamma}_{t-4}\right\} + \kappa_{t-1}\kappa_{t-2}(1-\kappa_{t-3}) + \kappa_{t-1}(1-\kappa_{t-2}) + (1-\kappa_{t-1}) \\
&= \left(\prod_{s=1}^{t-1}\kappa_{t-s}\right)(\tilde{\gamma}_0 - 1) + 1
\end{aligned} \tag{75}$$

We now define a parameter as $\alpha = 1 - \gamma$ to allow for comparability across the two models. Then, we can re-write equation 75 in the following way,

$$E_0\left\{\tilde{\alpha}_{t-1}\right\} = \left(\prod_{s=1}^{t-1}\kappa_{t-s}\right)\tilde{\alpha}_0 \tag{76}$$

Recall that we are able to compute the inflation expectations for the next period in the following way,

$$\begin{aligned}\pi^e_{t+1|t} &= \tilde{\gamma}_t\left(\frac{(ac)}{2} + \pi^o_{t+1}\right) - \frac{(ac)}{2} \\ &= \pi^o_{t+1} - \tilde{\alpha}_t\left(\frac{(ac)}{2} + \pi^o_{t+1}\right)\end{aligned} \tag{77}$$

Taking expectations and using 75 we get,

$$E_0\left\{\pi^e_{t+1|t}\right\} = \pi^o_{t+1} - \left(\prod_{s=1}^{t}\kappa_{t+1-s}\right)\tilde{\alpha}_0\left(\frac{(ac)}{2} + \pi^o_{t+1}\right) \qquad \text{for } t \geq 0 \tag{78}$$

With, $E_0\left\{\pi^e_{0|-1}\right\} = \pi^o_0 = \frac{ac}{2}$ as the initial expectations (Rational Expectations before the reform). Replacing this in the loss function we have,

$$E_0\{\mathcal{L}^{G,CB,O}_0\} = \frac{(ac)^2}{2}\tilde{\alpha}_0\sum_{t=1}^{\infty}\beta^t\left(\prod_{s=1}^{t-1}\kappa_{t-s}\right)(1-\rho^t) + \frac{\left(\frac{ac}{2}\right)^2}{1-\rho^2\beta} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{79}$$

We now have the welfare loss function that depends on the prior of the agents, exogenous variation in inflation, the inflation targets set by the government and the speed at which the targets are reset.

Recall the welfare loss from only introducing the central bank is given by,

$$E\{L^{G,CB}_0\} = \frac{ac^2}{2}\sum_{t=0}^{\infty}\beta^t\tilde{a}_0\left(\prod_{j=1}^{t}\kappa_{t-j}\right) + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta}$$

Thus, the terms marked in blue in equation 79 denote the differences in welfare loss when introducing a central bank and when introducing a central bank and intermediate targets. Therefore introducing inflation targets has two opposing forces on the welfare loss, a higher ρ implies a costly temporary inflation bias term $\frac{\left(\frac{ac}{2}\right)^2}{1-\rho^2\beta}$ but also a smaller output loss (term $(1-\rho^t)$) and faster learning (because of smaller κ_t values). We now turn to simulating the model which will allow us to see the effect of introducing the targets and compute the welfare loss.

Characterising the sequence of κ_t

To be able to compare the welfare loss in the presence of inflation targets with the welfare loss under only a central bank. We now introduce the measure κ_t, which is the speed of learning. Recall it is given by $\kappa_t = \frac{1}{\left(\frac{(ac)}{2}+\pi^o_t\right)^2\frac{\tilde{\sigma}^2_{t-1}}{\tilde{\sigma}^2_\epsilon}+1}$

We also define $\vartheta_t = \frac{\tilde{\sigma}^2_{t-1}}{\tilde{\sigma}^2_{\epsilon}}$ Therefore, $\kappa_t = \frac{1}{\left(\frac{(ac)}{2}+\pi_t^o\right)^2 \vartheta_t + 1}$ and from the updating equation of the beliefs' variance we have find that,

$$\vartheta_{t+1} = \left(\frac{1}{1 + \vartheta_t \left(\frac{ac}{2}\right)^2 \left(1-\rho^t\right)^2} \right) \vartheta_t + \frac{\tilde{\sigma}^2_{\eta}}{\tilde{\sigma}^2_{\epsilon}} \tag{80}$$

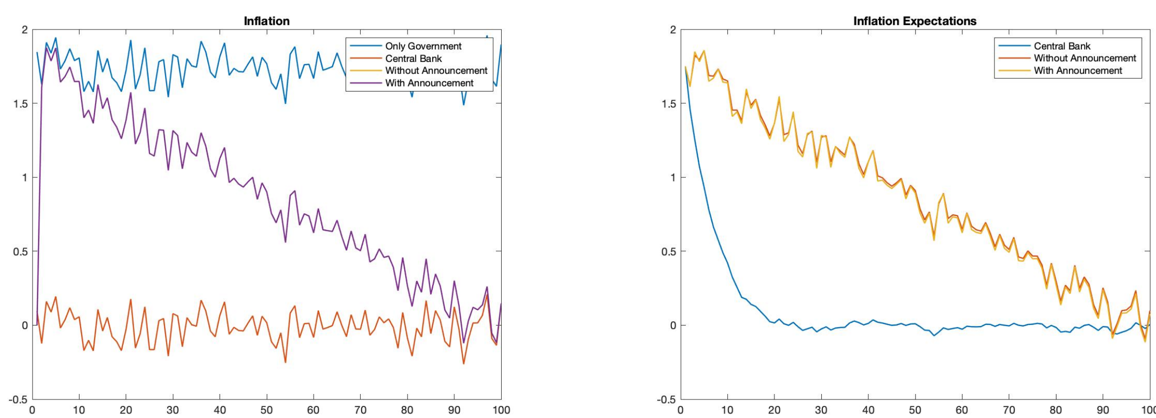
As before, ϑ_t is interpreted as the persistence parameter of beliefs. This persistence now depends on not only the prior and exogenous variation in inflation but also on the announced targets. As the central bank is able to commit to an inflation target credibly, the higher the belief the agents attach to the central bank in the perceived loss function. Thus, the speed of learning increases (declining κ_t).

3 Discussion

The previous section detailed four different models. First, the standard model of inflation bias[13], second a model which introduces an independent central bank with objectives which the agents of the economy must learn about, third a model where the government sets inflation targets but they are not announced, finally the model with announced inflation targets.

Figure 4a delineates key difference between the process for inflation between the 3 regimes (government, central bank and intermediate targets) is the reduction of inflation from the inflation bias level to the level the central bank targets using intermediate targets. The mechanism used to create the reduction in inflation is the announcement of the inflation targets causing a decline in inflation expectations (figure 4b).

Figure 4: Paths of Inflation and Inflation Expectations



(a) Inflation

(b) Inflation Expectations

[13]As seen in Barro and Gordon (1983b)

First when there is an independent central bank, inflation is given by the exogenous shocks to the economy, which the central bank cannot control. Second, in the presence of an independent central bank, expectations decline and adjust rapidly (blue line in figure 4b). In comparison, inflation expectations with and without announcements adjust at a slower pace.

While inflation expectations with announcements are close to those without announcements, their level is higher compared to when the intermediate inflation targets are announced. This is because the agents do not know the policy of the central bank. Therefore, they are uncertain about whether inflation is reducing to the policies or a favourable exogenous shock. This leads to higher inflation expectations.

At first glance, figure 4 suggests that if the government wants to control inflation, the ideal way to do it would be to introduce a central bank whose sole objective is to reduce inflation to zero. Thus, allowing for an immediate adjustment in inflation followed by an adjustment in inflation expectations in under one year with the sustained gain of low inflation from then on. However, the introduction of a central bank without any additional policies comes at an additional cost.

We now simulate the four different models to compare the welfare loss in all four regimes allowing for a rigorous analysis of the impacts of the policy.

3.1 Output and Welfare Loss

The previous discussion details the information of how inflation and inflation expectations evolve over time for each policy. However, while isolated, the efficacy of each policy is difficult to ascertain. To aid this discussion therefore, we plot the cumulative output loss (figure 5) and the cumulative welfare loss (figure 6) under the only government, independent central bank, no announcement and announcement of targets regimes, respectively.

We will split the discussion of this phenomenon in two parts. First, we compare the policies of the inflation bias, introduction of the independent central bank and the introduction of intermediate inflation targets (announced or unannounced). Second, we develop on the difference between the announced and unannounced inflation targets.

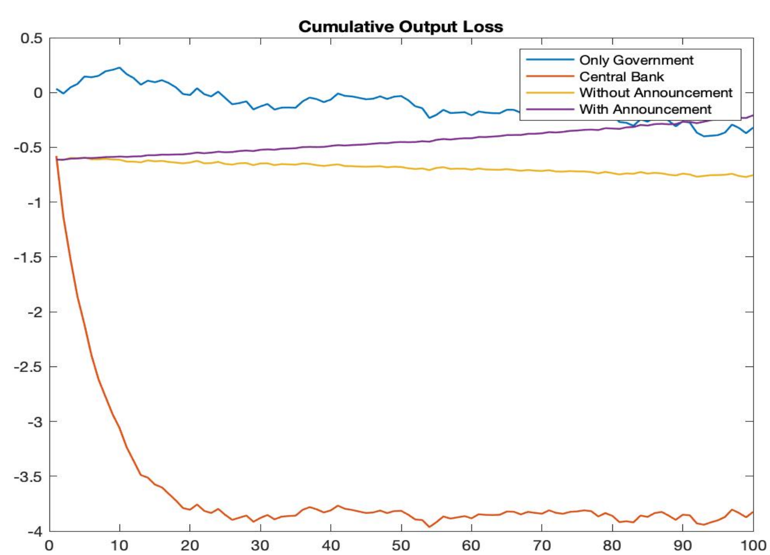
For a baseline calibration given in Table 2, in the appendix, figure 5 plots the cumulative output loss under the different regimes. The blue line presents the output loss under the inflation bias (only government) regime, the orange line presented the output loss under introducing a central bank, the yellow and purple lines represent the loss under the regime of intermediate inflation targets.

As measured by the output gap given by $\tilde{y} = c(\pi_t - \pi_t^e)$, the output loss from introducing the independent central bank is the highest followed by the loss from having intermediate targets. On the other hand, the loss from the government fluctuates around zero initially with a downward trend. Additionally, the volatility of the output gap is higher with the government. This suggests that in the regime where the government is responsible for monetary policy, they can manipulate prices such that they are able to satisfy their short term goals.

Let's turn our attention to the cumulative output loss under the independent central bank. We find that the output loss under this regime is the highest. This happens because the optimal inflation for the central bank is $\bar{\pi}_t = 0$. Moreover, the central bank does not attach any weight to the deviations from output in their loss function. Consequently, when a central bank is introduced, the inflation surprise experienced by the agents in the economy. This is the result of the agents not knowing what the objective of the central bank is and whether they will follow the same policy as the government. Therefore, the agents must learn about th objective of the central bank for at least one period before they adjust their inflation expectations, leading to a high output cost.

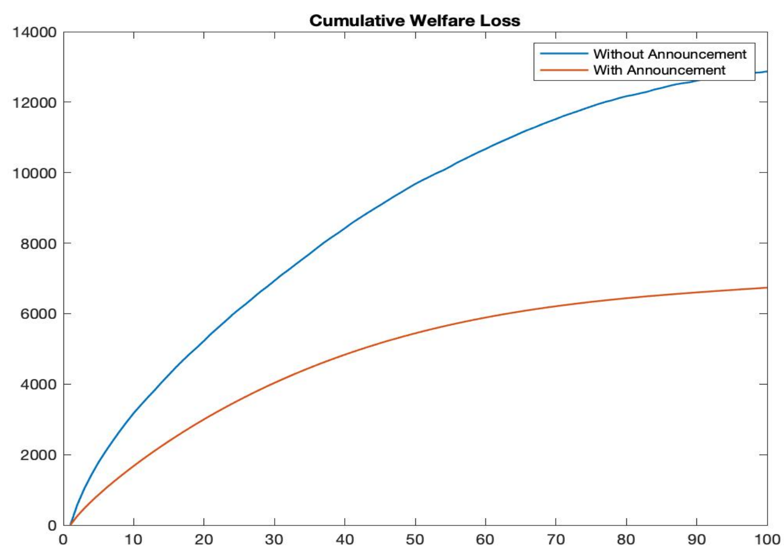
To avoid the cost of a recession in the economy and surprising the agents however, the central bank implements the intermediate inflation target. As seen in the case of the no announcement and announcement case both incur a lower cost of output compared to the introduction of the central bank. While the agents still need to learn about the objective of the central bank. The inflation surprise that is experienced by the agents in every period is smaller. Therefore, as the central bank starts to hit its inflation targets, the agents learn that there is a systematic change in inflation which leads to a decline in expectations and a smaller loss in output, over time.

Figure 5: Cumulative Output Loss



Given that the introduction of the inflation targets allows for more flexibility to the government to achieve lower inflation at a lower cost, announcing the inflation targets lead to a smaller output and welfare loss as portrayed in figure 6. As discussed in section 2, the difference between announcing the inflation targets and not announcing them stems from the introduction of the difference between the term given by $\gamma_{t-1}(\pi_t^o - \pi_{t-1}^o)$ in inflation expectations. That is, in addition to responding to observed inflation, agents are adjusting expectations according to the policy announced by the central bank. Therefore, when the central bank announces the sequence of inflation targets, the agents in the economy are able to adjust their forecasts taking into consideration the difference in announced inflation in the current and previous period on top of the forecast error from the last period. Thus, expanding the information set of the agents to generate forecasts.

Figure 6: Cumulative Welfare Loss

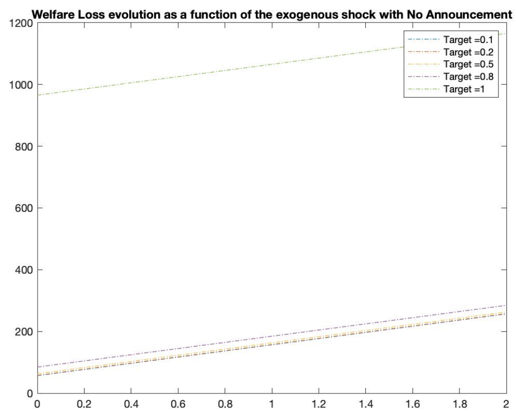


Given that the welfare loss is also largely dependent on the external shocks experienced by the economy, we now simulate the welfare loss for different exogenous shocks and different values of discounting the inflation bias (ρ) or in other words, the speed of reduction of the inflation target.

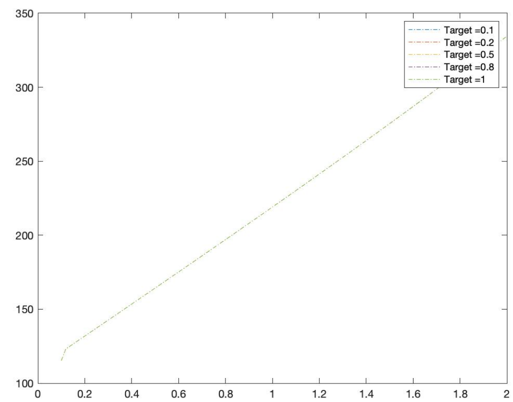
Figure 7 depicts the welfare loss under no announcements and announcements with varying exogenous shocks and inflation targets, respectively. We find that under both scenarios, the welfare loss increases as the external shocks increase, for the economy. Importantly however, the speed at which the economy disinflates matters for the welfare loss. As seen in figure 7a, a smaller rate of discounting allows for a lower welfare loss. That is the welfare loss when $\rho = 0.1$ is lower compared to when $\rho = 0.5$ when the intermediate inflation targets are not announced.

On the other hand, when the intermediate inflation targets are announced, the welfare cost attached to a different ρ is zero. That is, irrespective of how quickly the central bank disinflates does not interfere with the way agents learn and update their inflation expectations. This difference occurs because of two reasons, as discussed below.

Figure 7: Loss for different values of the exogenous shock and inflation target



(a) Without Announcement



(b) With Announcement

First, when the inflation targets are announced, the agents know the whole sequence of targets. Therefore, they are able to adjust their expectations according to the difference in inflation targets as given by $\gamma_{t-1}(\pi_t^o - \pi_{t-1}^o)$. The equation and graph suggests that the value of the actual targets is not relevant in the case of the targets. However, only the difference between the targets and difference of inflation from the target are relevant for the expectations and therefore, the welfare loss. It is therefore the change in target which becomes important for inflation expectations.

Second, the additional term in inflation expectations only contributes the adjustment for the target rather than a complete inference from observed inflation. Indeed, this is why in the first few periods the welfare loss for the announcement and no announcement case is similar. And as agents witness a systematic decline in inflation in accordance with the announced targets, they re-calibrate their belief γ on the policy that the central bank is following. Thus, also reducing their response to exogenous shocks.

Therefore, under the regime of no announcements, there is a higher degree of uncertainty about inflation in each period. The agents need to disentangle whether the decline in inflation is the result of a policy prescription or favourable exogenous shocks. Consequently, the closer the inflation targets to the inflation bias level of inflation even when not announced, the lower the inflation surprise, the agents face. Thus, lower the forecast error which allows the cost of disinflation to decline. The higher the surprise in inflation, the more the welfare cost and higher output loss.

4 Conclusion

This paper extends the model by Barro and Gordon (1983b) to include an independent central bank and intermediate inflation targets, taking into account that agents are Bayesian learners. We find that the introduction of an independent central bank can help circumvent the case of the inflation bias discussed in Barro and Gordon (1983b). However, this does come at the cost of a recession. In order to avoid the possibility of a high contraction in the economy, it is optimal for central banks to introduce inflation targets which are announced.

These targets serve to purposes. First, they allow agents to adjust expectations faster since there is less uncertainty with respect to the changes in inflation. Second, the central bank is able to build credibility over time which aids the disinflationary process by allowing persistence in expectations to increase and thus reducing the response of expectations to external shocks. Moreover, the inflation targets allow for the economy to face lower output costs compared to if the government was managing monetary policy or if there was only an independent central bank with an objective to adjust inflation to zero.

Given the tractability of the model, we are able to obtain analytical results about the effect of the intermediate inflation targets introduced by the central bank in a model economy. Nonetheless, there remains more to be done. Specifically, we would like to further this model and develop a New Keynesian model calibrated to Colombia, to allow for us to better capture the nuances of the Latin American economies and replicate their experience of the disinflation process. Moreover, we currently use a linear model for the inflation targets, while this makes it easy for us to derive analytical results, there is no economic rationale for us to limit the targets to a linear process. We leave this development for further research.

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Appendix

Table 1: Institutional Amendments

Country	Year of amendment
Brazil	1995
Chile	1989
Colombia	1991

Table 2: Parameter Values for both models

Parameter	Definition	Independent Central Bank	Intermediate Targets
β	Discount Factor	0.99	0.99
a	Weight on output gap	100	100
c	Weight on output and inflation	0.35	0.35

Mathematical Details

Welfare Loss without Announcements

$$E_0\gamma_t = 2 - \Big(\prod_{s=1}^{t-1}\kappa_{t-s}(1+\alpha_0)\Big) \tag{81}$$

Let's re-write the equation for inflation expectations,

$$E_0\pi^e_{t+1|t} = \Big(2 - \prod_{s=1}^{t-1}\kappa_{t-s}(1+\alpha_0)\Big)\frac{ac}{2} - \frac{ac}{2} \tag{82}$$

$$E_0\pi^e_{t+1|t} = \Big(ac - \frac{ac}{2}(1+\alpha_0)\prod_{s=1}^{t-1}\kappa_{t-s}\Big) - \frac{ac}{2} \tag{83}$$

$$E\{\mathcal{L}_0\} = E\left\{\sum_{t=0}^{\infty}\beta^t\,(a\tilde{y}_t)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{84}$$

$$E\{\mathcal{L}_0\} = E\left\{\sum_{t=0}^{\infty}\beta^t\left(ac\left(\rho^t\left(\frac{ac}{2}\right) - E_0\left\{\pi^e_{t|t-1}\right\}\right)\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{85}$$

$$E\{\mathcal{L}_0\} = E\left\{\sum_{t=0}^{\infty}\beta^t\left(ac\left(\rho^t\left(\frac{ac}{2}\right) - \left(\frac{2ac}{2} - \frac{ac}{2}(1+\alpha_0)\prod_{s=1}^{t-1}\kappa_{t-s} - \frac{ac}{2}\right)\right)\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{86}$$

$$E\{\mathcal{L}_0\} = E\left\{\sum_{t=0}^{\infty}\beta^t\left(ac\left(\rho^t\left(\frac{ac}{2}\right) - \frac{2ac}{2} + \frac{ac}{2}(1+\alpha_0)\prod_{s=1}^{t-1}\kappa_{t-s} + \frac{ac}{2}\right)\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{87}$$

$$E\{\mathcal{L}_0\} = E\left(\frac{ac^2}{2}\right)\left\{\sum_{t=0}^{\infty}\beta^t\left(\left(\rho^t - 2 + (1+\alpha_0)\prod_{s=1}^{t-1}\kappa_{t-s} + 1\right)\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{88}$$

$$E\{\mathcal{L}_0\} = E\left(\frac{ac^2}{2}\right)\left\{\sum_{t=0}^{\infty}\beta^t\left(\rho^t - 1 + (1+\alpha_0)\prod_{s=1}^{t-1}\kappa_{t-s}\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{89}$$

$$E\{\mathcal{L}_0\} = \left\{\sum_{t=0}^{\infty}\beta^t\left(\left(\frac{ac^2}{2}\right)\rho^t - \left(\frac{ac^2}{2}\right) + \left(\frac{ac^2}{2}\right)(1+\alpha_0)\prod_{s=1}^{t-1}\kappa_{t-s}\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{90}$$

$$E\{\mathcal{L}_0\} = \left\{\sum_{t=0}^{\infty}\beta^t\left(\frac{ac^2}{2}\right)\rho^t - \sum_{t=0}^{\infty}\beta^t\left(\frac{ac^2}{2}\right) + \sum_{t=0}^{\infty}\beta^t\left(\frac{ac^2}{2}\right)(1+\alpha_0)\prod_{s=1}^{t-1}\kappa_{t-s}\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{91}$$

Using equation 91 and 43 and plugging it into the the loss function we get the following,

$$E\{\mathcal{L}_0\} = \frac{(ac)^2}{2(1-\beta\rho)} + \frac{(ac)^2}{2(1-\beta)} + \frac{(ac)^2}{2}(1+\alpha_0)\sum_{t=0}^{\infty}\beta^t\prod_{s=1}^{t-1}\kappa_{t+s} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{92}$$

Welfare Loss with Announcements

We can now iterate on equation 93 to get the following,

$$E_0\pi^e_{t|t-1} = \prod_{s=1}^{t}\kappa_{t-s}\frac{-ac}{2} + (1-\kappa_{t-1})\rho^{t-1}\left(\frac{-ac}{2}\right) \tag{93}$$

Now the loss is given by,

$$E\{\mathcal{L}_0\} = E\left\{\sum_{t=0}^{\infty}\beta^t\left(a\tilde{y}_t\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}_\epsilon^2}{1-\beta} \tag{94}$$

$$E\{\mathcal{L}_0\} = \left\{\sum_{t=0}^{\infty} \beta^t \left(ac\left(\rho^t\left(\frac{ac}{2}\right) - E_0\left\{\pi^e_{t|t-1}\right\}\right)\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{95}$$

$$E\{\mathcal{L}_0\} = \left\{\sum_{t=0}^{\infty} \beta^t \left(ac\left(\rho^t\left(\frac{ac}{2}\right) - \prod_{s=1}^{t}\kappa_{t-s}\frac{-ac}{2} - (1-\kappa_{t-1})\rho^{t-1}\left(\frac{-ac}{2}\right)\right)\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{96}$$

$$E\{\mathcal{L}_0\} = \frac{(ac)^2}{2}\left\{\sum_{t=0}^{\infty} \beta^t \left(\left(\rho^t + \prod_{s=1}^{t}\kappa_{t-s} + (1-\kappa_{t-1})\rho^{t-1}\right)\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{97}$$

$$E\{\mathcal{L}_0\} = \frac{(ac)^2}{2}\left\{\sum_{t=0}^{\infty} \beta^t \left(\left(\rho^t + \rho^{t-1} + \prod_{s=1}^{t}\kappa_{t-s} - \kappa_{t-1}\rho^{t-1}\right)\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{98}$$

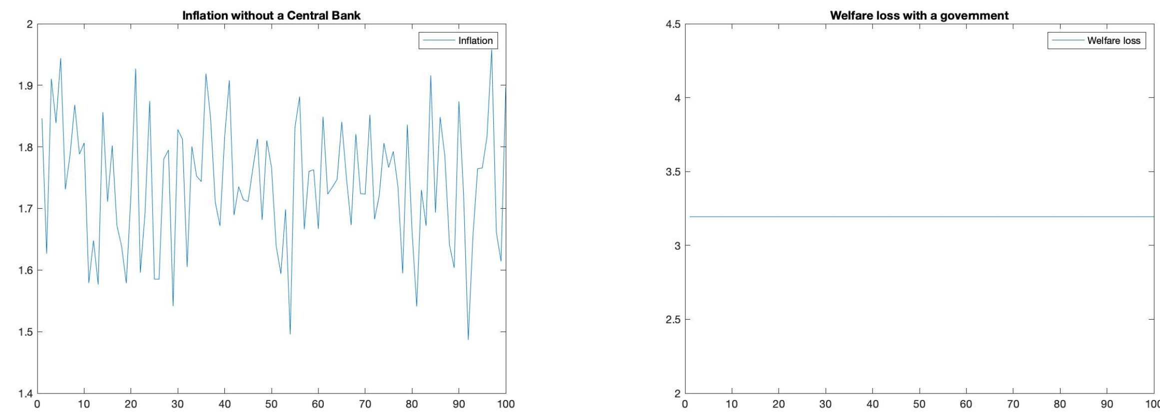
$$E\{\mathcal{L}_0\} = \frac{(ac)^2}{2}\left\{\sum_{t=0}^{\infty} \beta^t \left(\left(\rho^t + \rho^{t-1} + \prod_{s=1}^{t}\kappa_{t-s} - \prod_{s=1}^{t}\kappa_{t-s}\rho^0\right)\right)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{99}$$

$$E\{\mathcal{L}_0\} = \frac{(ac)^2}{2}\left\{\sum_{t=0}^{\infty} \beta^t (\rho^{t-1}(\rho+1)\right\} + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{100}$$

$$E\{\mathcal{L}_0\} = \frac{(ac)^2}{2}\left(\frac{1-\rho}{\rho(\beta\rho-1)}\right) + \frac{\left(\frac{ac}{2}\right)^2}{1-\beta\rho^2} + \frac{\tilde{\sigma}^2_\epsilon}{1-\beta} \tag{101}$$

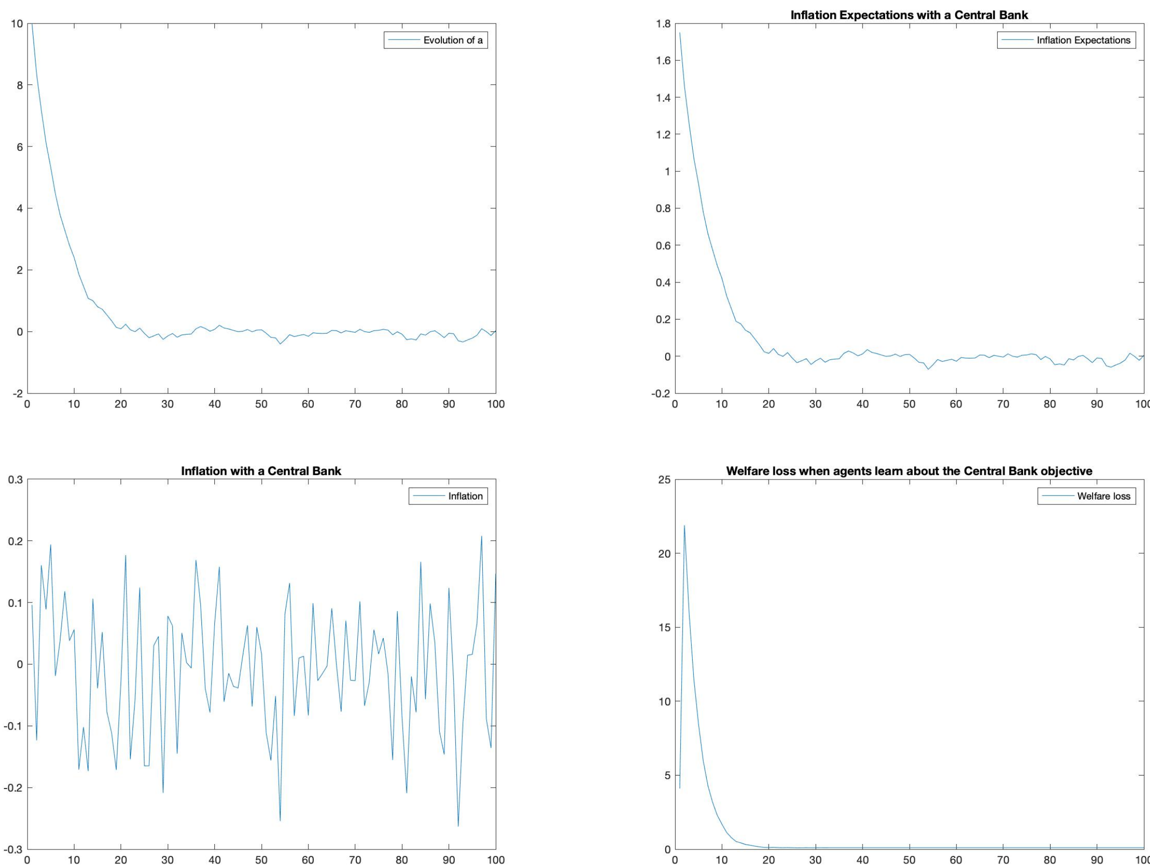
.1 Inflation Bias

Figure 8: Evolution of the variables over time - Only Government



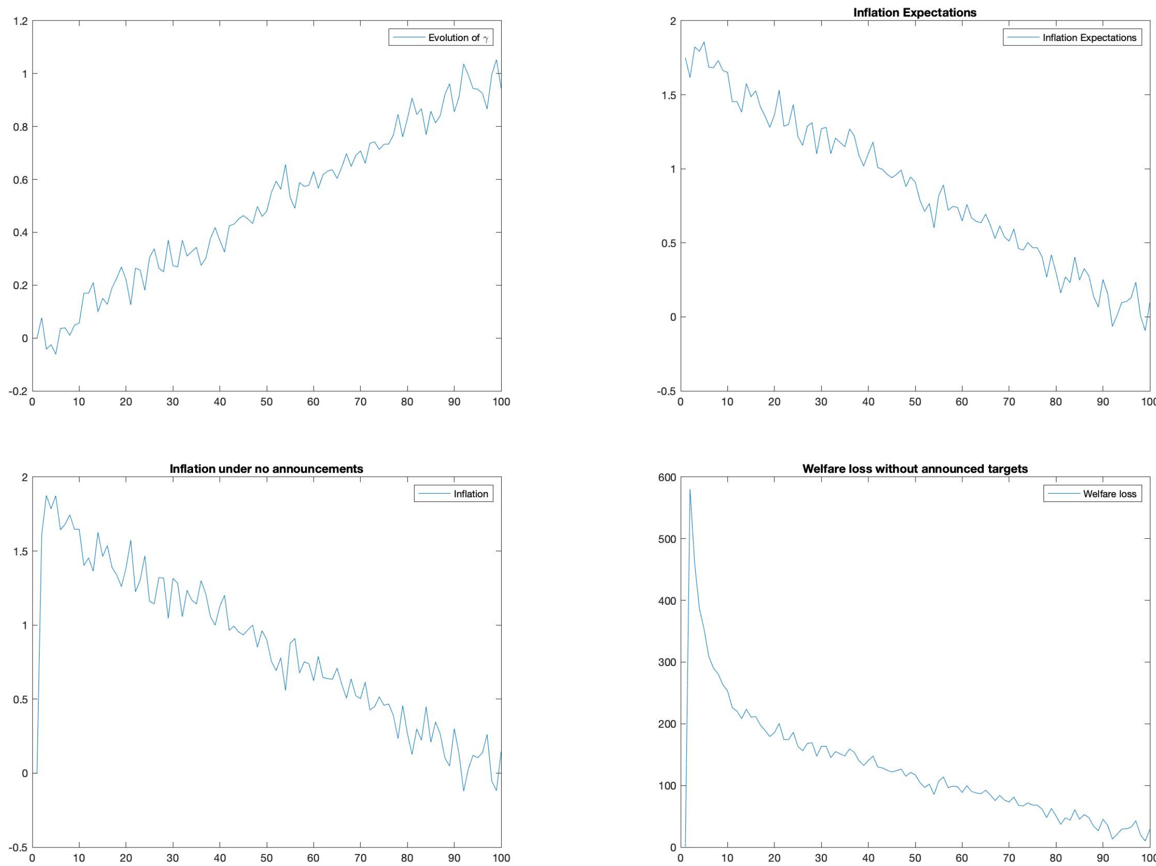
.2 Independent Central Bank

Figure 9: Evolution of the variables over time - Central Bank



.3 No Announcements

Figure 10: Evolution of the variables over time - No Announcements



.4 Announcements

Figure 11: Evolution of the variables over time - Announcements