1

$Z(P)$ means p belongs in $Z(-1, -2, \dots n)$

a] $\forall x \forall y \left(Z(x) \wedge Z(y \longrightarrow [x+y = y+x] \right)$

b] $\forall x \forall y \left(Z(x) \wedge Z(y) \longrightarrow Z(x+y) \wedge Z(x*y) \right)$

c] $\forall a \forall b \forall c \left(Z(a) \wedge Z(b) \wedge Z(c) \longrightarrow [(a+b)\times c = a\times c + b\times c] \right)$

d] $\forall a \forall b \forall c \left(Z(a) \wedge Z(b) \wedge Z(c) \longrightarrow [((a+b)+c = (a+c)+b) \wedge (a*(b*c) = (a*b)*c)] \right)$

~~e] $\forall x \; Z(x) \Rightarrow$~~

e] $\forall a \left(Z(a) \longrightarrow [(a+0=a) \wedge (a*1=a)] \right)$

Q-2] $\alpha = \forall x\,(P(x) \vee Q(x))$

$\beta = \forall x\, P(x) \vee \forall x\, Q(x)$

$\alpha \models \beta$??

Let

$x \in \mathbf{Z} - \{0\}$

and $P(x)$ — ~~check~~ if x is positive

$Q(x)$ — if x is odd

Now α in Z would be always true since numbers in domain x would be either positive or negative

Now for β $\forall x\, P(x)$ is not true for negative number

Therefore β is false.

Since α is true and β is false, therefore $\alpha \models \beta$ is false.

3

Predicates Used in this question:

1] $O(x)$ — x owns Millenium Falcon

2] $U(x)$ — x is unhappy

3] $Love(x,y)$ — x loves y

4] $V(x)$ — x visits Obi Wan Kenobi

5] $W(x)$ — x is wise

6) $T(x)$ — Obi Wan Kenobi teach x to use lightsaber

7] $R(x)$ — x joins rebel

8] $D(x,y)$ — x declares love for y

9] $F(x)$ $\longrightarrow$ x has chewbacca as friend.

Now we use this to convert our sentences

1] $O(\text{Han Solo})$

2] $U(\text{Princess Leia})$

3] $Love(\text{Princess Leia}, \text{Han Solo})$

4] $\forall x\, (O(x) \vee U(x) \rightarrow V(x))$

5] $\forall x\, (V(x) \rightarrow W(x))$

6] $\forall x\, (O(x) \wedge V(x) \longrightarrow T(x)$

7) $\forall x\, (((U(x) \vee O(x)) \wedge T(x) \longrightarrow R(x))$

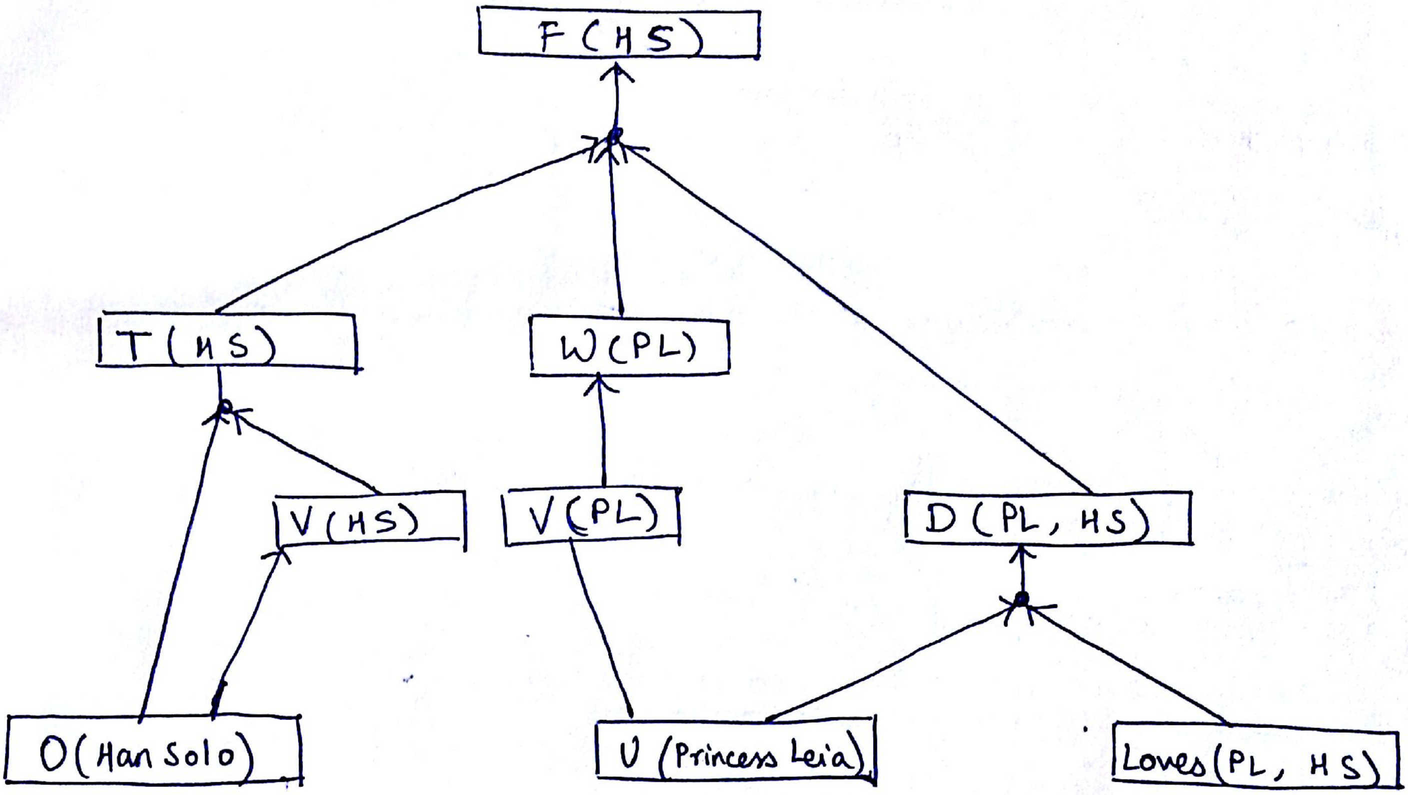
8) $\forall x, \forall y((U(x) \wedge Love(x, y)) \longrightarrow D(x, y))$

9] $\forall x \; \forall y \quad T(x) \wedge D(y, x) \wedge W(y) \longrightarrow F(x)$

we have to prove.

$F(\text{Han Solo})$.

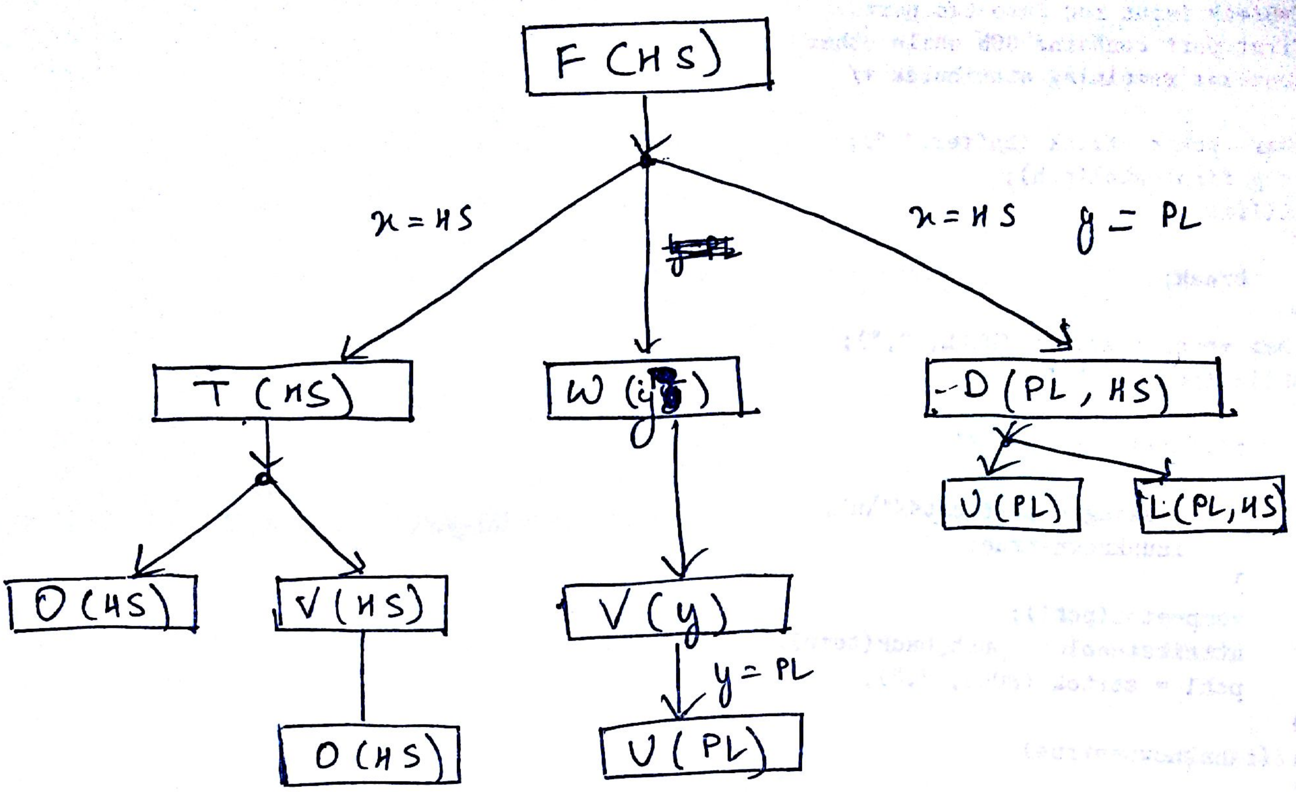
Forward Chaining



HS ≡ Han Solo

PL ≡ Princess Leia.

Back Tracking



Q-4

a) $A(Z) \longrightarrow$ Zeus were able.

$W(Z) \longrightarrow$ Willing to prevent evil.

$P(Z) \longrightarrow$ zeus prevents evil.

$I(Z) \longrightarrow$ zeus is impotent

$M(Z) \longrightarrow$ zeus is malo valent

$E(Z) \longrightarrow$ Zeus exist

So according to the above predicates

a) $A(Z) \land W(Z) \longrightarrow P(Z)$

Convert to CNF

Acc. to demorgans law

$$P \rightarrow Q \equiv \neg P \lor Q$$

so,

$$\neg(A(Z) \land W(Z)) \lor P(Z)$$

$$(\neg A(Z) \lor \neg W(Z)) \lor P(Z)$$

$$\boxed{\neg A(Z) \lor \neg W(Z) \lor P(Z)} \text{ — } (1)$$

b) $\neg A(z) \longrightarrow I(z)$

$\boxed{A(z) \vee I(z)}$ —— (2)

c) $\neg W(z) \longrightarrow M(z)$

$\boxed{W(z) \vee M(z)}$ —— (3)

d) $\boxed{\neg P(z)}$ —— (4)

e) $E(z) \longrightarrow \neg I(z) \wedge \neg M(z)$

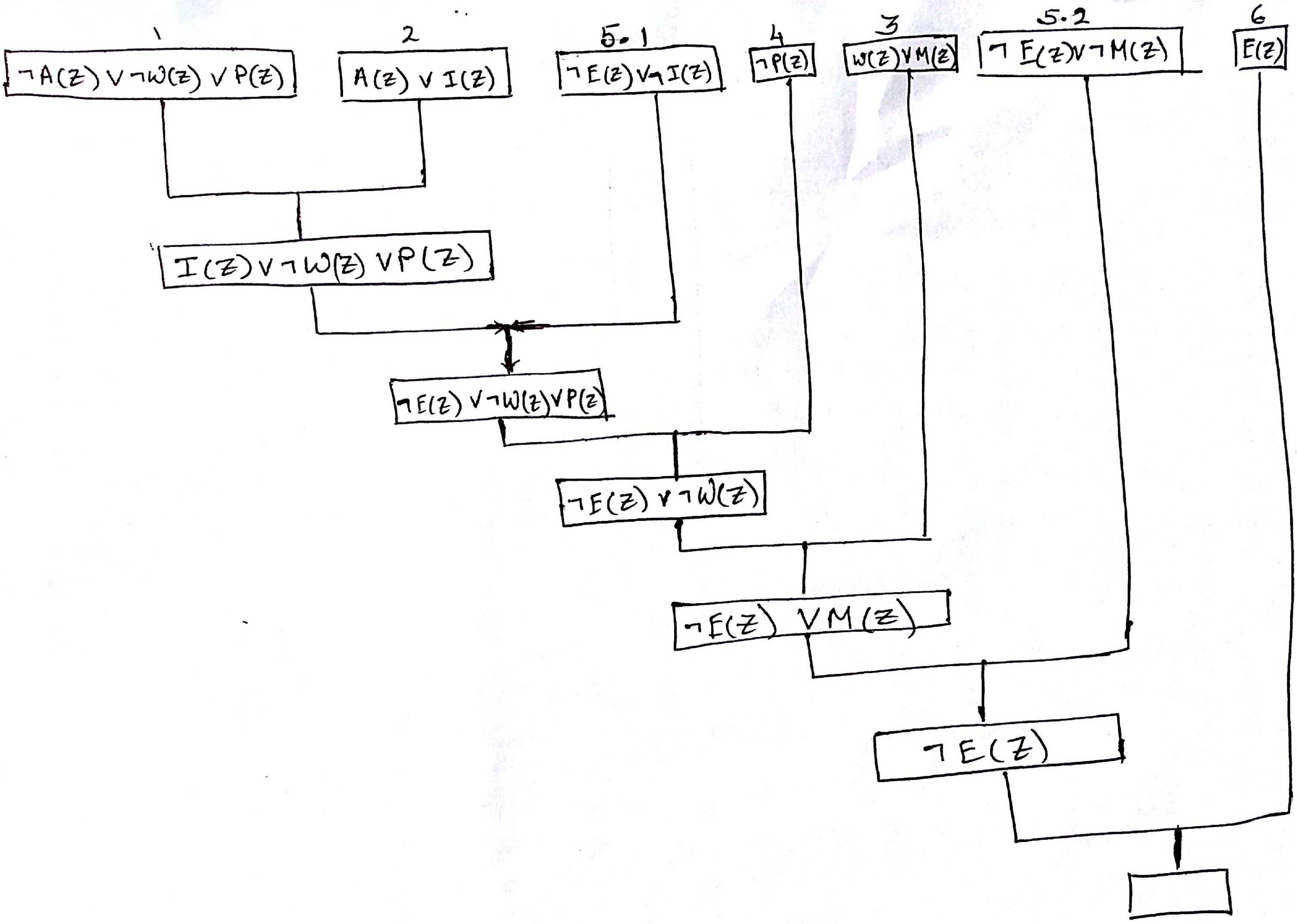
$\neg E(z) \vee (\neg I(z) \wedge \neg M(z))$

$\boxed{\underbrace{(\neg E(z) \vee \neg I(z))}_{5.1} \wedge \underbrace{(\neg E(z) \vee \neg M(z))}_{5.2}}$ —— (5)

To prove $\neg E(z)$

So we take contradiction of $\neg E(z)$ which gives

$E(z)$ —— (6)



Since we get empty set ∴ E(Z) is false i.e. Zeus does not exist

Q5

a] $\forall x \, (P(x) \rightarrow P(x))$

Applying de morgans.

$\forall x \, (\neg P(x) \vee P(x))$

Now

$A = \neg P(x) \vee P(x)$

Now to prove it by contradiction. we assume $\neg A$ is true.

So $\neg A = P(x) \wedge \neg P(x)$

which is alway false. Therefore $\forall x (P(x) \rightarrow P(x))$ is true.

b] $(\neg \exists x P(x)) \longrightarrow (\forall x \neg P(x)) = S$

Now

$\neg S = \neg (\exists x P(x) \vee \forall x \neg P(x))$

$= \forall x \neg P(x) \wedge \exists x P(x)$

Now we drop quantifiers.

$= \neg P(x) \wedge P(x)$

which is always false.

Therefore S is true.

c] $P = \left(\forall x \left(P(x) \vee Q(x)\right)\right) \rightarrow \left(\left(\forall x P(x)\right) \vee \left(\exists x Q(x)\right)\right)$

Applying demorgans law.

$\neg\left(\forall x \left(P(x) \vee Q(x)\right)\right) \vee \left(\left(\forall x P(x)\right) \vee \left(\exists x Q(x)\right)\right)$

$\left(\exists x \left(\neg P(x) \wedge \neg Q(x)\right)\right) \vee \left(\forall x P(x) \vee \exists x Q(x)\right)$

Removing Universal Quantifiers

$P = \left(\neg P(x) \wedge \neg Q(x)\right) \vee \left(P(x) \vee Q(x)\right)$

for contradiction $\neg P$ is true.

so

$\neg P = \left(P(x) \vee Q(x)\right) \wedge \left(\neg P(x) \wedge \neg Q(x)\right)$

$= \left(P(x) \vee Q(x)\right) \wedge \neg\left(P(x) \vee Q(x)\right)$

Now

$T(x) = P(x) \vee Q(x)$

so

$\neg P = T(x) \wedge \neg T(x)$

which is always false

So, P is always True.