An Intuitive Breakdown of GARCH-Volatility Modeling

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I. INTRODUCTION

As of the first quarter of 2018, VIX has transformed from being low and stable to high and volatile, accompanying a transition from a bullish market to a bearish market.[1] With this transition, volatility forecasting is becoming more and more relevant. The algorithm forecasts volatility using a GARCH (1,1) process, and favors securities with low volatility forecasts. Securities from the S&P 500 are fed into a linear univariate GARCH model which is parametrized through a maximum likelihood estimation (MLE). The GARCH model predicts the volatility of each security. This, along with momentum, are run through a Monte Carlo simulation which in turn simulates potential price changes. Returns are then measured and ranked before being allocated into either a long or short portfolio.

II. GARCH FORECASTING

GARCH modeling is heavily used in time-series analyses for forecasting volatility. This model is used frequently because it takes into account multiple factors including financial data's heteroscedasticity, the principle that variances in error terms are not constant. The GARCH model used in this algorithm is a GARCH (1,1) model, signifying that there is only one autoregressive lag and one moving-average lag. Even though greater accuracy can be achieved with more terms, the risk of overfitting increases as well as more uncertainty in the MLE.

Typically, a GARCH (p,q) model takes the form:[2]

$$\sigma_t{}^2 = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2$$

A GARCH (1,1) model can be simplified to

$$\sigma_t{}^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$
$$\varepsilon_t = \sigma_t z_t$$
$$z_t = N(0,1)$$
$$\omega > 0, \alpha_i, \beta_j \geq 0$$

In the above simplification, ω and α represent the parameters for the ARCH component while β represents the parameter of the GARCH component. The ε_t component represents the standardized residuals. Stock returns can be measured with the forecasted variance using an AR-GARCH process like so:[3]

$$r_t = \mu + \sigma_t z_t$$

III. MLE FOR PARAMETRIZATION

The GARCH (1,1) model has three unknown parameters, $\omega, \alpha,$ and β. A Maximum Likelihood Estimation (MLE) determines how likely an outcome can be modeled as a function of multiple parameters, and maximizes this probability. In other words, the MLE determines which values for each parameter would most likely produce a distribution similar to that of the data given.

This process is similar to the Least Square Estimation (LSE) or Ordinary Least Squares (OLS) in that they both attempt to reduce error between a model and observed data. However, LSE focuses on reducing the sum of the squared residuals and is more useful for analyzing linear regression models. The MLE improves on LSE by asymptotically lowering the variance of parameter estimates while also maintaining invariance in the parametrization to ensure the MLE solution runs independent of the chosen parametrization while performing extremely well on large datasets. [4]

Given a data set of previous price points, $\{y_1, y_2 \ldots y_n\}$ for the given security, the MLE models the data *assuming* a parameter θ. This is denoted as follows: [5]

$$L(\theta) = f(y_1, y_2, y_3 \ldots y_n | \theta) = \prod_{i=1}^{n} f(y_i | \theta)$$

This denotes the probability of observing the given data with parameter θ. Obviously, our goal is to maximize this probability, we do so by maximizing the logarithm of the function as shown below:

$$l(\theta) = \sum_{i=1}^{n} log\,(f(y_i | \theta))$$

We can maximize this function by equating:

$$\frac{dl(\theta)}{d\theta} = 0$$

The solution to the above differential solves for the parameter θ. This is done for each unknown parameter $\omega, \alpha,$ and β.

IV. MONTE CARLO SIMULATION

When making a forecast for price points based on various uncertain variables, instead of taking the average of each variable and forecasting price, Monte Carlo simulations take into account the probability distributions of said variables into the price point forecast. The simulation chooses values for each factor based on its probability distribution.[6] Each time the values are chosen, the simulation runs in order to determine a price point change index.

This process is repeated 100 times for each security. Once the simulation is complete, a distribution for the index is determined and the mean is taken as the forecast.

V. ALLOCATION INTO LONG/SHORT

After the Monte Carlo simulation runs, a forecasted price point for each security is returned. This price point is divided by its current price point to give an estimate of potential returns. The algorithm then ranks the top n (subject to change) securities and allocates an order percentage based on the stock's returns compared to the total returns of the other stocks as shown below:

$$\zeta_i = \frac{r_i}{\sum_{i=1}^{n} r_n}$$

Where ζ_i represents the percentage of capital in that specific trade allocated to stock i given n stocks.

VI. PERFORMANCE ANALYSIS

The algorithm performed well against the market in a backtest from January 3, 2015 to November 11, 2016. The algorithm was able to outperform the market while maintaining a Beta value below 1. This shows that the algorithm was not as susceptible to sudden changes compared to the market. This proves helpful during market downturns with increased market volatility. The algorithm also produced favorable Sharpe and Sortino ratios - The algorithm was able to utilize volatility measures in order to optimize the risk-adjusted performance.

VII. REFERENCES

1 "VIX - CBOE Volatility Index." MarketWatch

2 Engle, Robert. 2001. "GARCH 101: The Use of ARCH/GARCH Models in Applied Econometrics." Journal of Economic Perspectives, 15 (4): 157-168.

3 Ferenstein, Elzbieta, and Miroslaw Gasowski. "Modelling stock returns with AR-GARCH processes." *SORT-Statistics and Operations Research Transactions* 28.1 (2004): 55-68.

4 Osbourne, M. R. Least Squares and Maximum Likelihood. Least Squares and Maximum Likelihood.

5 Geyer, Charles J.. (1991). Markov Chain Monte Carlo Maximum Likelihood. Interface Foundation of North America. Retrieved from the University of Minnesota Digital Conservancy.

6 "Monte Carlo Simulation of Stochastic Processes." Monte Carlo Simulation of Stochastic Processes, 10 Jan. 2004, marcoagd.usuarios.rdc.puc-rio.br/sim_stoc_proc.html.

7 Rutkowski, Marek. The Black-Scholes Model. pp. 27, The Black-Scholes Model.