Calculating bias

$$\text{Bias} = E_X[\text{Bias}(X)]$$
$$= E_X[(\bar{g}(X) - f(X))^2]$$
$$= E_X[(0 - x^2)^2] \quad \text{since } \bar{g}(X) = 0 \text{ from Part A}$$
$$= E_X[x^4]$$
$$= \frac{1}{2}\int_{-1}^{1} x^4 \, dx = \frac{1}{2}\left[\frac{x^5}{5}\right]_{-1}^{1} = \frac{1}{5}$$

$\therefore$ Bias $= \frac{1}{5}$.

$$\text{Variance} = E_X[\text{var}(X)]$$
$$= E_X[E_D[(g^{(D)}(X) - \bar{g}(X))^2]]$$

$g^{(D)}(X)$ has the form $ax+b$ and $\bar{g}(X) = 0$

$$\therefore \text{Variance} = E_X[E_D[(ax+b)^2]]$$
$$= E_X[E_D[a^2x^2 + b^2 + 2abx]]$$
$$= E_D\left[\frac{1}{2}\int_{x=-1}^{1} a^2x^2 + b^2 + 2abx \, dx\right]$$
$$= E_D\left[\frac{1}{2}\left[a^2 \times \frac{2}{3} + 2b^2\right]\right]$$
$$= E_D\left[\frac{a^2}{3} + b^2\right] = E_D\left[\frac{1}{3}(x_1^2 + x_2^2 + 2x_1x_2) + x_1^2x_2^2\right]$$
$$= \frac{1}{4}\iint \frac{1}{3}(x_1^2 + x_2^2 + 2x_1x_2) + x_1^2x_2^2 \, dx_1\, dx_2$$
$$= \frac{1}{4}\int_{-1}^{1} \frac{1}{3}\left[\frac{2}{3} + 2x_2^2\right] + \frac{2}{3}x_2^2 \, dx_2$$
$$= \frac{1}{4}\int_{-1}^{1} \frac{2}{9} + \frac{2}{3}x_2^2 + \frac{2}{3}x_2^2 \, dx_2$$

$$\text{variance} = \frac{1}{4}\int_{-1}^{1} \frac{2}{9} + \frac{4}{3} x_2^2 \quad dx_2$$

$$= \frac{1}{4}\left[\frac{4}{9} + \frac{4}{3} \times \frac{2}{3}\right]$$

$$= \frac{1}{9} + \frac{2}{9} = \frac{3}{9} = \frac{1}{3}$$

$$\therefore \text{Variance} = \frac{1}{3}.$$