Assignment 1

Question 1 Report

1 System Model and Controller Design

The pendulum dynamics with external torque and friction:

$$\ddot{\theta} = \frac{-\frac{mg}{l}\sin\theta - k\dot{\theta} + \tau}{m}$$

where $m = 1$, $g = 10$, $l = 1$, $k = 1$ (friction coefficient).

1.1 Task 1: Stabilization at $\theta = \pi/1.5$

PID controller designed as:

$$\tau = -k_p(\theta - \theta_d) - k_d\dot{\theta} - k_i \int (\theta - \theta_d)dt$$

with gains: $k_p = 10$, $k_d = 5$, $k_i = 2$

Rationale: The proportional gain $k_p = 10$ is chosen to provide a sufficiently strong restoring torque to bring the pendulum towards the desired setpoint $\theta_d = \pi/1.5$ without causing excessive oscillations. The derivative gain $k_d = 5$ introduces damping, counteracting the natural tendency of the pendulum to overshoot due to inertia; the moderate value ensures critical-like damping, reducing oscillations while maintaining responsiveness. The integral gain $k_i = 2$ addresses steady-state errors that may arise from constant disturbances or modeling inaccuracies (e.g., friction), ensuring that the pendulum eventually settles exactly at the desired angle.

The relative magnitudes were selected based on a balance between fast convergence (higher k_p), minimal overshoot (adequate k_d), and elimination of residual offset (small k_i), which together ensure stable and smooth stabilization of the pendulum around θ_d.

1.2 Task 2: Frictionless Behavior

To achieve sustained oscillations, the controller cancels the natural friction:

$$\tau = -k_d\dot{\theta}, \quad k_d = -1$$

Substituting this into the dynamics:

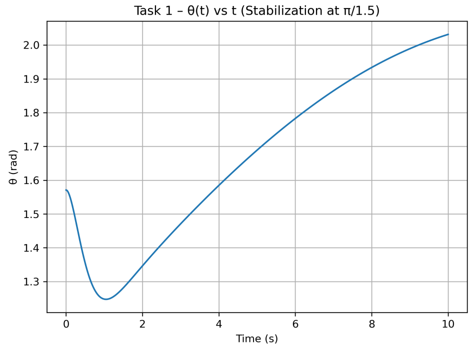
$$-\frac{mg}{l}\sin\theta - k\dot{\theta} + \tau = -\frac{mg}{l}\sin\theta - \dot{\theta} + \dot{\theta} = -\frac{mg}{l}\sin\theta$$

This results in a conservative pendulum with sustained oscillations.

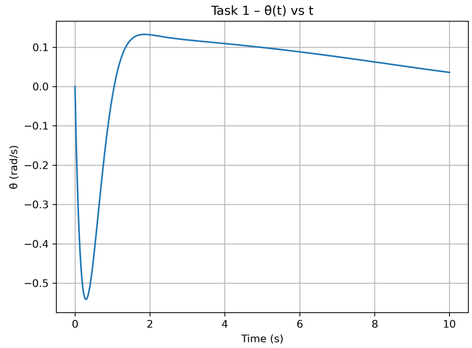
2 Results

2.1 Task 1: Stabilization

Initial conditions: $\theta(0) = \pi/2$, $\dot{\theta}(0) = 0$



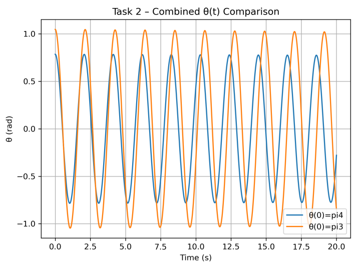
(a) θ vs t



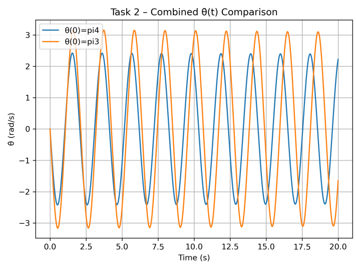
(b) $\dot{\theta}$ vs t

Figure 1: Pendulum stabilization at $\theta = \pi/1.5$ rad

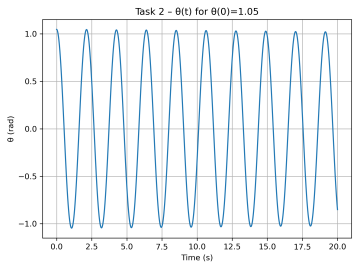
2.2 Task 2: Frictionless Behavior



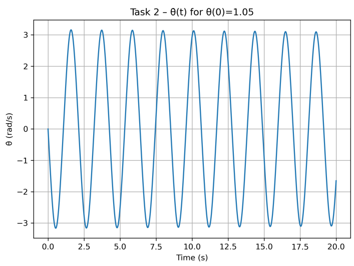
(a) Combined θ vs t



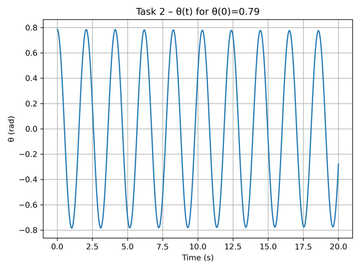
(b) Combined $\dot{\theta}$ vs t



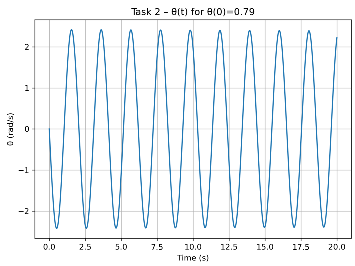
(c) θ vs t ($\theta_0 = \pi/3$)



(d) $\dot{\theta}$ vs t ($\theta_0 = \pi/3$)



(e) θ vs t ($\theta_0 = \pi/4$)



(f) $\dot{\theta}$ vs t ($\theta_0 = \pi/4$)

Figure 2: Frictionless pendulum oscillations for different initial conditions

Observation: The amplitude of oscillation is sensitive to the initial condition. For $\theta(0) = \pi/4$, the pendulum oscillates between approximately $-\pi/4$ and $+\pi/4$, while for $\theta(0) = \pi/3$, it oscillates between $-\pi/3$ and $+\pi/3$. The oscillations are sustained and symmetric, with no decay, as the system is frictionless.