Calculus of Variation

Assignment 2

Q1 Assuming fixed end-points, Then which of the following is the extremals of the functional

$$F\{y\} = \int \sqrt{x^2+y^2}\sqrt{1+y'^2}\,dx,$$

a) $2xy\sin\alpha - (x^2-y^2)\cos\beta = \alpha$

b) $2xy\sin\alpha + (x^2-y^2)\cos\alpha = \beta$

c) $2xy\cos\alpha + (x^2-y^2)\sin\alpha = \beta$

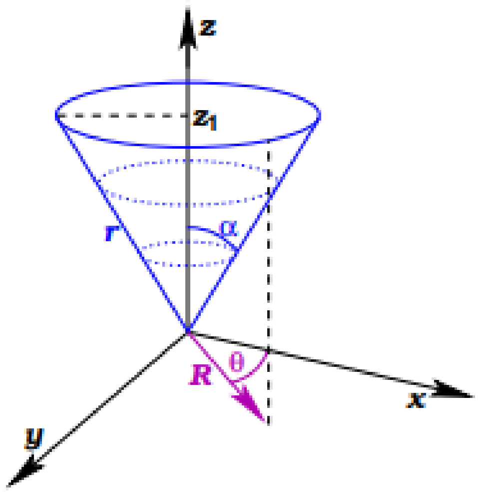
d) $y(x) = x\tan\alpha \pm \sqrt{x^2\sec^2\alpha + \beta\sec\alpha}$

e) $y(x) = x\tan\alpha \pm \sqrt{x^2\sec^2\alpha - \beta\sec\alpha}$

f) $y(x) = x\tan\alpha \pm \sqrt{x^2\sec^2\alpha - \beta\sec\beta}$

where α and β are arbitrary constants.

Q2 Which of the following is the geodesics on a right circular cone (as shown in the figure)



a) $r = \csc(\nu + \theta\sin\alpha)$

b) $r = \nu\csc(\nu + \theta\sin\alpha)$

c) $r = \mu\csc(\nu - \theta\sin\alpha)$

d) $r = \mu\csc(\nu + \theta\sin\alpha)$

e) $r = \nu\csc(\mu + \theta\sin\alpha)$

f) $r = \nu\csc(\mu - \theta\sin\alpha)$

Q3 If we minimize

$$I = \int_a^b \left[(y')^2 - y^2\right] dx$$

Then which of the following option is/are correct

a) $y = \pm\sqrt{-c}\sin x, c < 0$

b) $y = \pm\sqrt{c}\sin x, c > 0$

c) $y = \pm\sqrt{-c}\cos x, c < 0$

d) $y = \pm\sqrt{c}\cos x, c > 0$

e) $y = \pm\sqrt{-c}\tan x, c < 0$

f) $y = \pm\sqrt{c}\tan x, c > 0$