Calculus of Variation

Assignment 5

Q1 Consider the functional as the standard catenary with an isoperimetric constraint λ of the form,

$$F\{y\} = Ry_1 + \int_{x_0}^{x_1} (y - \lambda)\sqrt{1 + y'^2}dx.$$

Where, R is the resistance, the length is $L = \int_0^1 \sqrt{1 + y'^2}dx$ and the natural boundary conditions are $y(0) = y_0$ (known) and $y(1) = y_1$ (unknown). Then the value of y_1 is/are

a) $y_1 = c_1 \sinh(\frac{1-c_2}{c_1}) - \lambda$

b) $y_1 = c_2 \cosh(\frac{1-c_1}{c_2}) - \lambda$

c) $y_1 = c_1 \cosh(\frac{1-c_2}{c_1})$

d) $y_1 = c_1 \cosh(\frac{1-c_2}{c_1}) - \lambda$

e) $y_1 = c_1 \sinh(\frac{1}{c_1} - c_2) - \lambda$

f) $y = \frac{c_1}{c_2} \cosh(1 - c_2)$

Where, c_1 and c_2 are constants.

Q2 The curve joining the points $(0, 0)$ and $(1, 0)$ for which the integral is given by

$$\int_0^1 y''^2 dx$$

a) is a minimum if $y(0) = a$ and $y(1) = b$

b) is a maximum if $y'(0) = a$ and $y'(1) = b$

c) is a minimum if $y'(0) = a$ and $y'(1) = b$

d) is a minimum if no other conditions are prescribed

e) Both (c) and (d) are true

f) All of the above

Q3 Solve the Euler-Lagrange equation associated with

$$I = \int_a^b \left[y^2 - yy' + (y')^2\right] dx$$

a) arc cosh $\frac{x}{\sqrt{c_1}} + c_2 = \pm y$

b) arc cosh $\frac{y}{\sqrt{c_1}} + c_2 = \pm x$

c) arc sinh $\frac{x}{\sqrt{c_1}} + c_2 = \pm y$

d) arc sinh $\frac{y}{\sqrt{c_1}} + c_2 = \pm x$

e) arc tanh $\frac{x}{\sqrt{c_1}} + c_2 = \pm y$

f) arc tanh $\frac{y}{\sqrt{c_1}} + c_2 = \pm x$