Calculus of Variation

Assignment 1

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Q1 Consider the functional

$$J(y) = \int_0^1 (1+x)(y^{'})^2 dx$$

where y is twice continuously differentiable and y(0) = 0 and y(1) = 1. Of all functions of the form

$$y(x) = x + c_1 x(1-x) + c_2 x^2 (1-x)$$

where c_1 and c_2 are constants, If we minimizes J, Then which of the following option is / are correct

a) $y = \frac{186}{131}x - \frac{77}{131}x^2 + \frac{20}{131}x^3$

b) $y = \frac{186}{131}x + \frac{77}{131}x^2 - \frac{20}{131}x^3$

c) $y = \frac{186}{131}x + \frac{77}{131}x^2 + \frac{20}{131}x^3$

d) $y = \frac{146}{151}x - \frac{97}{151}x^2 + \frac{20}{151}x^3$

e) $y = \frac{146}{151}x + \frac{97}{151}x^2 - \frac{20}{151}x^3$

f) $y = \frac{146}{151}x + \frac{97}{151}x^2 + \frac{20}{151}x^3$

Q2 Let $f = x^2 + y^2 + z^2$ subject to

$$\phi = xy + 1 - z = 0$$

Then Which of the following is / are correct options

a) Minimum of f is 1

b) Minimum of f is 2

c) Minimum of f is 1 at (1, 0, 0)

d) Minimum of f is 1 at (0, 0, 1)

e) Minimum of f is 2 at (0, 0, $\sqrt{2}$)

f) Minimum of f is 2 at (1, 0, 1)

Q3 Of all parabolas which pass through the points (0, 0) and (1, 1), determine that one which, when rotated about the x-axis, generates a solid of revolution with least possible volume between x = 0 and x = 1 [Notice that the equation may be taken in the form y = x + c x (1 - x), when c is to be determined]

a) minimum volume is $\frac{\pi}{2}$

b) minimum volume is $\frac{\pi}{4}$

c) minimum volume is $\frac{\pi}{8}$

d) minimum volume is $\frac{3\pi}{8}$

e) minimum volume is $\frac{5\pi}{16}$

f) minimum volume is $\frac{7\pi}{15}$