

**Calculus of Variation**  
**Assignment 3**

**Q1** Which of the following is the form of extremals of the functional

$$F\{y(x), z(x)\} = \int_{x_0}^{x_1} (8yz - 5y^2 + y'^2 - 4z'^2) dx$$

- a)  $y(x) = 4A \cos 2x - 4B \sin 2x - C \cos x - D \sin x$
- b)  $y(x) = 4A \cos 2x + 4B \sin 2x + C \cos x + D \sin x$
- c)  $y(x) = 2A \cos 2x + 2B \sin 2x + 4C \cos x + D \sin x$
- d)  $y(x) = 4A \cos 2x + 2B \sin 2x + 2C \cos x + 2D \sin x$
- e)  $y(x) = 2A \cos 2x + 4B \sin 2x - C \cos x + D \sin x$
- f)  $y(x) = 2A \cos 2x - 4B \sin 2x - C \cos x - D \sin x$

where A, B, C and D are constants

**Q2** Find the extremal of the following functional

$$F\{y\} = \int_0^1 (y''^2 - 360x^2y) dx$$

subject to  $y(0) = 0$ ,  $y'(0) = 1$ ,  $y(1) = 1$  and  $y'(1) = 5/2$

- a)  $y(x) = \frac{1}{3}x^4 - \frac{1}{2}x^2 + x$
- b)  $y(x) = \frac{1}{2}x^4 - \frac{1}{3}x^2 + x$
- c)  $y(x) = \frac{1}{2}x^4 + \frac{1}{2}x^2 - x$
- d)  $y(x) = \frac{1}{3}x^6 - \frac{1}{2}x^3 + x$
- e)  $y(x) = \frac{1}{2}x^6 - \frac{1}{3}x^3 - x$
- f)  $y(x) = \frac{1}{2}x^6 - \frac{1}{2}x^3 + x$

**Q3** Determine the equation of the shortest arc in the first quadrant which passes through the points  $(0, 0)$  and  $(1, 0)$  and encloses a prescribed area A with the x-axis, where  $A \leq \frac{\pi}{8}$

- a)  $(x - k)^2 + (y + \frac{1}{2})^2 = k^2 + \frac{1}{4}$
- b)  $(x + k)^2 + (y - \frac{1}{2})^2 = k^2 + \frac{1}{4}$
- c)  $(x + k)^2 + (y + \frac{1}{2})^2 = k^2 - \frac{1}{4}$
- d)  $(x + \frac{1}{2})^2 + (y - k)^2 = k^2 + \frac{1}{4}$
- e)  $(x - \frac{1}{2})^2 + (y + k)^2 = k^2 + \frac{1}{4}$
- f)  $(x - \frac{1}{2})^2 + (y - k)^2 = k^2 - \frac{1}{4}$

where k is constant