Calculus of Variation

Assignment 5

Q1 Minimize

$$I = \int_0^{x_1} \left[y^2 - (y')^2 \right] dx$$

with left end point fixed and $y(x_1)$ is along the curve

$$x_1 = \frac{\pi}{4}$$

a) $y \equiv 0$

b) $y \equiv 1$

c) $y = x$

d) $y = x + 1$

e) $y = x^2 - 1$

f) $y = x^2 + 1$

Q2 Find the extremals for

$$I = \int_0^1 \left[\frac{1}{2}(y')^2 + yy' + y' + y \right] dx$$

where end values of y are free

a) $y = -\frac{1}{2}x + \frac{2}{3} + \frac{1}{3}x^2$

b) $y = -\frac{1}{3}x + \frac{1}{2} + \frac{1}{2}x^2$

c) $y = \frac{3}{2}x + \frac{2}{3} - \frac{1}{2}x^2$

d) $y = -\frac{3}{2}x + \frac{1}{2} + \frac{1}{2}x^2$

e) $y = -\frac{1}{2}x + \frac{1}{2} + \frac{3}{2}x^2$

f) $y = \frac{3}{2}x + \frac{1}{3} + \frac{1}{3}x^2$

Q3 Solve the Euler-Lagrange equation associated with

$$I = \int_a^b \left[y^2 - yy' + (y')^2 \right] dx$$

a) arc cosh $\frac{x}{\sqrt{c_1}} + c_2 = \pm y$

b) arc cosh $\frac{y}{\sqrt{c_1}} + c_2 = \pm x$

c) arc sinh $\frac{x}{\sqrt{c_1}} + c_2 = \pm y$

d) arc sinh $\frac{y}{\sqrt{c_1}} + c_2 = \pm x$

e) arc tanh $\frac{x}{\sqrt{c_1}} + c_2 = \pm y$

f) arc tanh $\frac{y}{\sqrt{c_1}} + c_2 = \pm x$