Calculus of Variation

Assignment 4

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NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING

Q1 Solve the isoparametric problem

$$J(y) = \int_0^1 \left((y')^2 + x^2\right) dx, \quad y(0) = y(1) = 0$$

and

$$\int_0^1 y^2 dx = 2$$

a) $y = \pm 2\sin(n\pi x),\ n = 1, 3, 5, ...$

b) $y = \pm \sin(n\pi x),\ n = 1, 2, 3....$

c) $y = \pm 2\sin(n\pi x),\ n = 1, 2, 3....$

d) $y = \pm \cos(n\pi x),\ n = 1, 2, 3....$

e) $y = \pm 2\cos(n\pi x),\ n = 1, 3, 5....$

f) $y = \pm 2\cos(n\pi x),\ n = 1, 2, 3....$

Q2 Determine the radius and height of the right circular cylinder of maximum volume subject to the constraint of having a surface area of 1 m^2.

a) Radius $= \frac{1}{\sqrt{2\pi}}$, Height $= \frac{1}{\sqrt{3\pi}}$

b) Radius $= \frac{1}{\sqrt{2\pi}}$, Height $= \frac{2}{\sqrt{6\pi}}$

c) Radius $= \frac{1}{\sqrt{3\pi}}$, Height $= \frac{1}{\sqrt{2\pi}}$

d) Radius $= \frac{1}{\sqrt{6\pi}}$, Height $= \frac{3}{\sqrt{2\pi}}$

e) Radius $= \frac{1}{\sqrt{6\pi}}$, Height $= \frac{2}{\sqrt{6\pi}}$

f) Radius $= \frac{2}{\sqrt{6\pi}}$, Height $= \frac{1}{\sqrt{3\pi}}$

Q3 Among all the curves in C^2 joining a given point $(0, b)$ on the (positive) y-axis to a point on the (positive) x-axis, and enclosing a given area S together with the x- and y-axes, find the curve which generates the least area when rotated about the x-axis.

a) Straight line from (b, 0) to $(\frac{2S}{b}, 0)$

b) Straight line from (0, b) to $(\frac{2S}{b}, 0)$

c) Circle passing through (b, 0) to $(\frac{2S}{b}, 0)$

d) Circle passing through (0, b) to $(\frac{2S}{b}, 0)$

e) Any curve passing through (b, 0) to $(\frac{2S}{b}, 0)$

f) Any curve passing through (0, b) to $(\frac{2S}{b}, 0)$